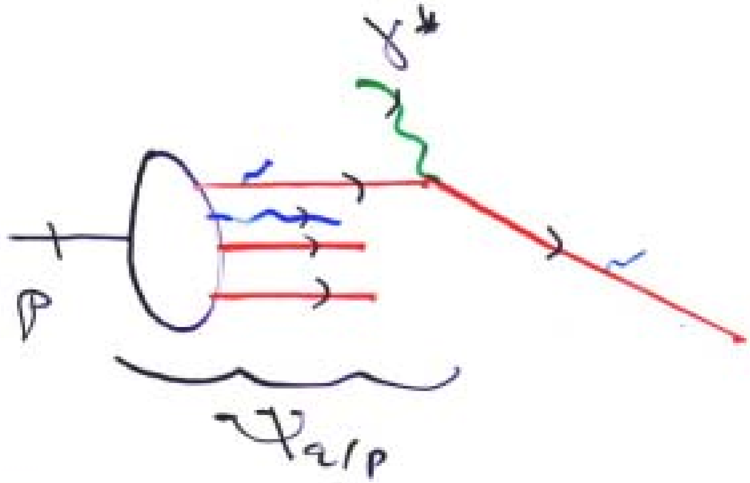
Feynman
Bjorken-Paschos

Foundation of Parton Model



proved in
Yukawa theory
Drell, Levy, Yan

DGLAP evol.

$$F_2(x,Q^2) = x \sum_{q\in p} e_q^2 \, q(x,Q^2)$$

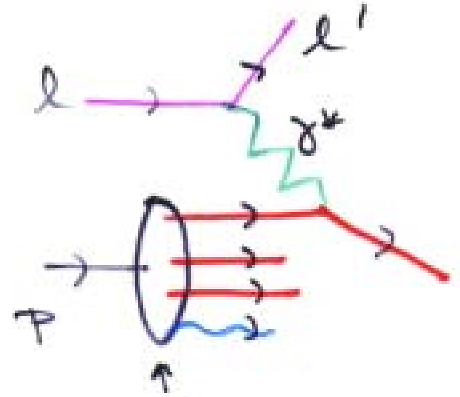
$q(x,Q^2)$ determined by target wavefunctions $\Psi_{q/p}$

All interactions after photon acts inconsequential !

→ phase or power-law suppressed !

BHMPS ⇒ QCD not parton model + DGLAP

Conventional Interpretation of DIS



$\Psi_{qqqg}(x_i, \vec{k}_{\perp i}, \lambda_i)$

$$F_2(x, Q^2) = \sum_{q \in p} e_q^2 \; x \; q(x, Q^2)$$

$$q(x, Q^2) = \sum_{n \geq 3} \int^{k_\perp^2 < Q^2} [d^2k_\perp][dx] \, |\Psi_n(x, k_\perp, \lambda)|^2 \; \delta(x_q - x)$$

$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$ eigenfunctions of H_{LF}^{QCD} (real)

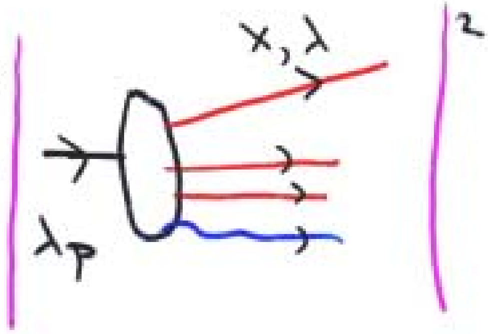
$q(x, Q^2)$ determined by target LFWFs + DGLAP

FSI : higher twist ; irrelevant phase

Light-Cone Wavefunctions

encode all helicity, transversity distributions

$q_{\lambda/\lambda_p} = \sum_f$



$q_{\lambda/\lambda_p}(x, \Lambda)$ $\left\{ \begin{array}{l} \text{transversity: density matrix} \\ \text{light-cone helicity} \end{array} \right.$

$$= \sum_{n,q} \int \left| \psi^{(\Lambda)}_{n,\lambda_p}(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2 \prod_{j=1} dx_j \prod_{j=1} d^2 k_{\perp j}$$

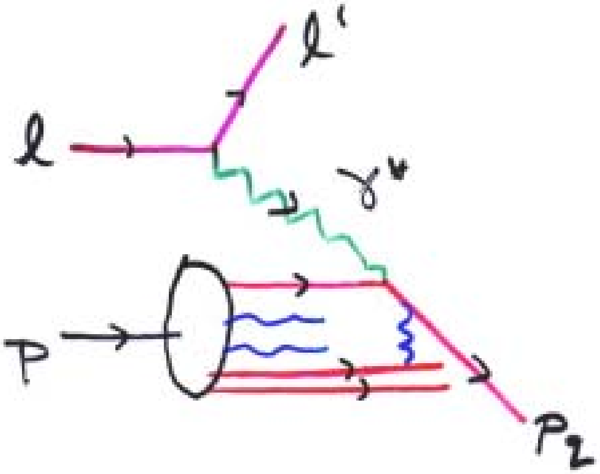
$$\delta\left(\sum_i^n x_i - 1\right) \delta\left(\sum_i^n \vec{k}_{\perp i}\right)$$

$$\delta(x - x_q)\, \delta_{\lambda, \lambda_q}$$

Factorization
Light-cone Scheme

$$\Theta(\Lambda^2 - \mathcal{M}_n^2)$$

Unexpected Role of Final State Interactions in Deep Inelastic Scattering



gluon exchange after photon acts

not in LF WF

* Single-spin asymmetry $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Bjorken-scaling

* Diffraction at Leading Twist

* Nuclear Shadowing (interference of diff channels)

* Energy Loss, p_T Broadening

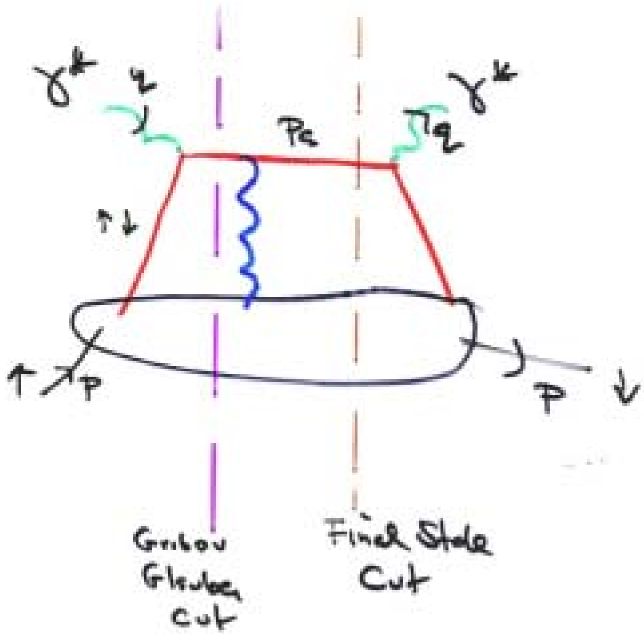
Diffraction, Nuclear Shadowing, Pomeron not in nuclear wavefunction!

* $$\Delta L_{Ioffe} = \frac{2\nu}{Q^2} = \frac{1}{M x_{Bj}}$$

Explicit calculation of FSI SSA

D.S. Hwang
I. A. Schmidt
SJB
hep/ph/0201296

Collins, X. Ji



Overlap of wavefunctions with $\Delta L_z = 1$

$$\left[e^{i(\chi_1 - \chi_2)}\right]$$

$\chi_1 - \chi_2$: IR Finite

$$i\, \vec{S}_p \cdot \vec{q} \times \vec{p}_q = i\, \vec{S}_p \cdot \vec{q} \times \vec{r}$$

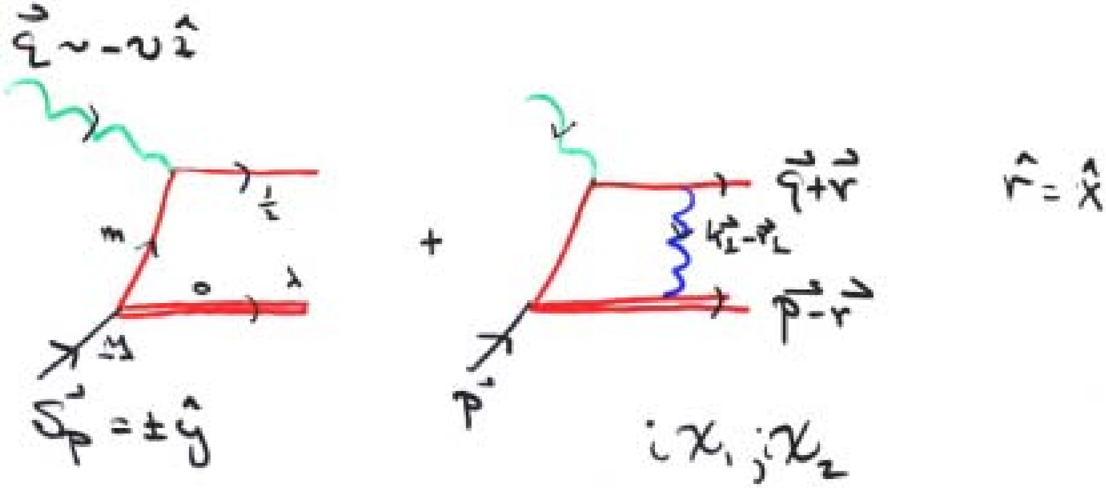
$$\vec{p}_q = \vec{q} + \vec{r}$$

Approximate at large $\vec{r}_\perp$:

$$P_y = A_n \simeq \frac{\alpha_s(r_\perp^2)\; x_{Bj}\; M\, |\vec{r}_\perp|\, \ln r_\perp^2}{r_\perp^2 + M^2}$$

Bjorken scaling for finite $r_\perp$

Same matrix elements as $a_p = F_2(0)$



$$\sigma \propto \epsilon^{\mu abc} P_\mu S_a q_b r_c = M \vec{S} \cdot \vec{q} \times \vec{r}$$

$$A_n = P_y = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

$$= \frac{C_F \alpha_s(\mu^2)(\Delta M + m) r_x}{(\Delta M + m)^2 + r_\perp^2}$$

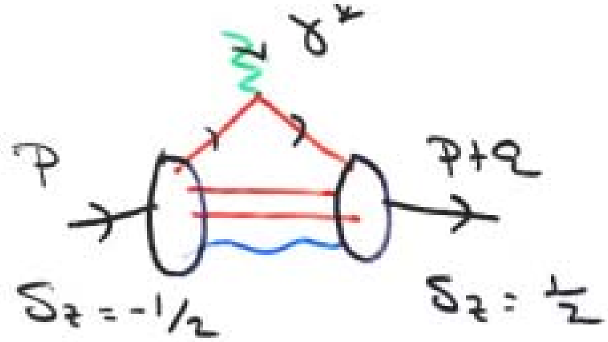
$$\otimes \left[r_\perp^2 + \Delta(1-\Delta)\left(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta}\right) \right]$$

$$\otimes \left[\frac{1}{r_\perp^2} \ln \frac{r_\perp^2 + \Delta(1-\Delta)\left(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta}\right)}{\Delta(1-\Delta)\left(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta}\right)} \right]$$

$$\Delta = x_B, \qquad \mu^2_{MS} = \langle e^{-5/3} (\vec{k}_\perp - \vec{r}_\perp)^2 \rangle$$

BHS

$\boxed{\text{Pauli Form Factor } F_2(q^2)}$ $\quad \kappa = F_2(0)$



Required overlap of LCWFs with $\underline{\Delta L_z = 1}$

Drell + SJB

e.g. (3q)

$$\psi^{1/2}_{-1/2\,-1/2\,-1/2} \otimes \psi^{-1/2}_{-1/2\,-1/2\,-1/2}$$

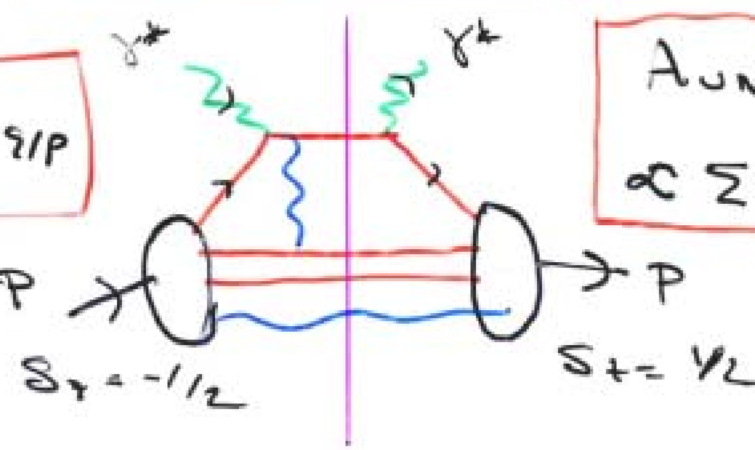
$L_z = +2 \qquad L_z = +1$

$$\psi^{1/2}_{1/2\,1/2\,-1/2} \otimes \psi^{-1/2}_{\frac{1}{2}\,1/2\,-1/2}$$

$L_z = 0 \qquad L_z = -1$

<u>Same matrix elements appear in SSA</u>

$$\boxed{\kappa_p = \sum_{q \in p} e_q \kappa_{q/p}}$$

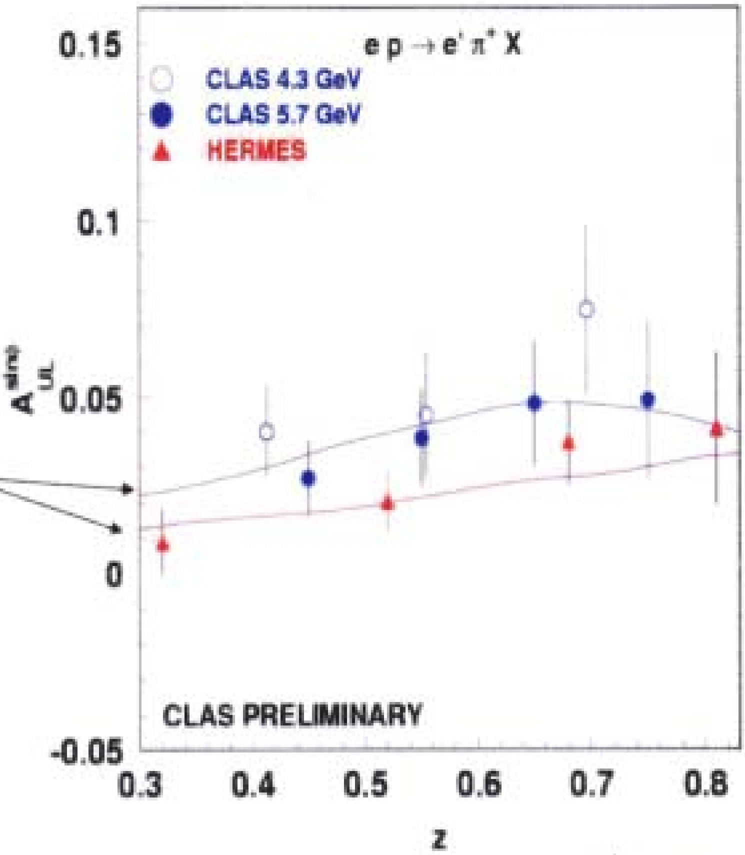


$$\boxed{\begin{aligned} A_{UN} &= SSA \\ &\propto \sum e_q^2 \kappa_{q/p}\, \alpha_s \end{aligned}}$$

Longitudinally Pol. Target: SSA for π^+

Target SSA: CLAS (4.3 GeV, and 5.7 GeV) consistent with HERMES (27.5 GeV)

predictions for **Sivers effect** from BHS Phys.Lett.B53099,2002
Sivers only interpretation require large A_{UT}



A_{UL} **z**-dependence also consistent both in magnitude and sign with predictions based on **Collins mechanism**

$$A_{UL} \propto \sin\theta_\gamma \times A_{UT} \propto \sin\theta_\gamma \frac{f_{1T}^{\perp u}(x)}{u(x)}$$

Single-Spin Asymmetry in Semi-Inclusive DIS

Distinguish Sivers vs Collins Effects

- Sivers: $\vec{S}_{\vec{p}} \cdot \vec{q} \times \vec{P}_{jet}$

  * no hadronization necessary
  * observe quark direction $\vec{P}_q = \vec{P}_{jet}$

- Collins: T-odd fragmentation necessary

  $H_1^\perp$: $\vec{S}_q \cdot \vec{P}_H \times \vec{P}_q$

- Sivers: $\sin(\phi_H^l - \phi_{S_p}^l)$ indep. of $\phi_{l'}$

- Collins: $\sin(\phi_H^l + \phi_{S_p}^l)$

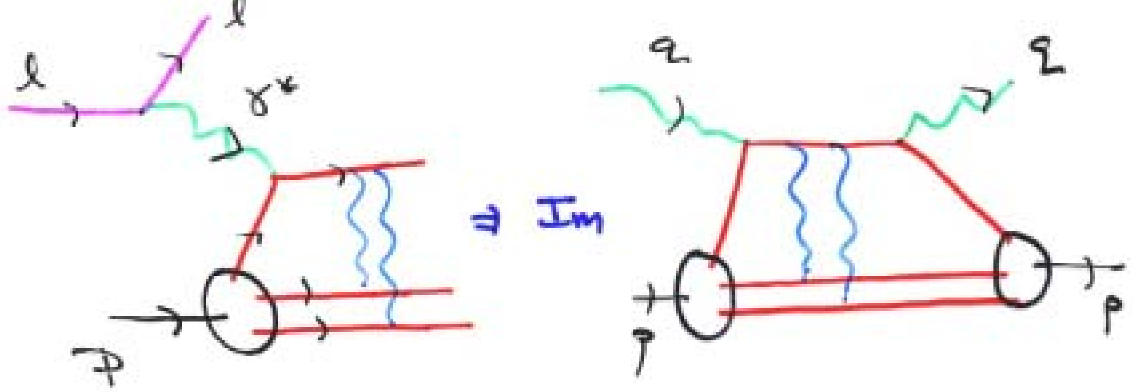
- Sivers: A_{UL}, A_{UT} same in

  $\nu p_\uparrow \to H X$ charged current *

  Collins: $A_{UL}, A_{UT} = 0$ *

  $\propto \dfrac{2 c_L c_R}{c_L + c_R}$ Z^0 neutral current

New perspectives on Final-State-Interactions



$$f_{q/N}(x_{Bj}, Q^2) = \frac{1}{8\pi}\int dy^- \, e^{-i x_B p^+ y^-}$$

$$\langle N(p)|\bar{q}(y^-)\gamma^+ P\, e^{ig\int_0^{y^-} dw^- A^+(w^-)} q(0)|N(p)\rangle$$

Usual argument $A^+ = 0$ gauge

no effect from FSI!

phase irrelevant

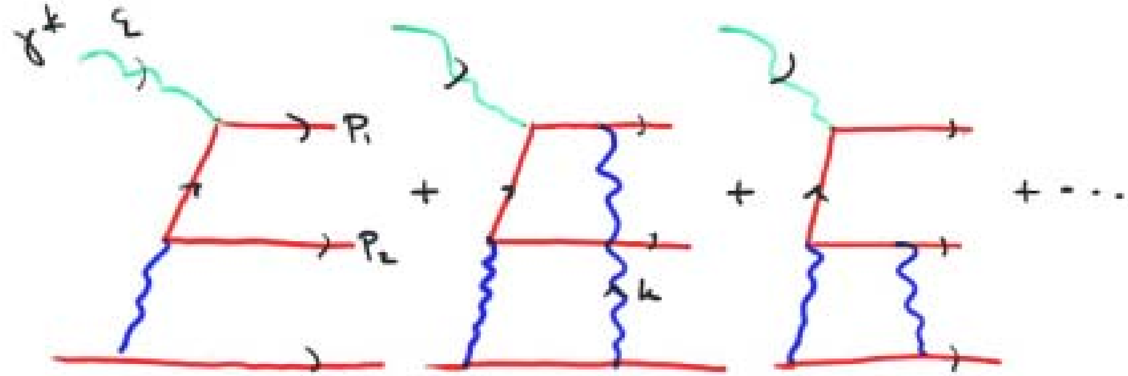
∴ Identify $f_{q/N}$ with LC Prob. Dist

$$P_{q/N}(x_B, Q^2) = \sum_n \int^{k_{\perp i}^2 < Q^2} \left[\prod dx\, d^2k_\perp\right] |\psi_n(x, k_{\perp i})|^2 \sum_j \delta(x_B - x_j)$$

Explicit calculation of FSI

BHMPS

$q^+ \leq 0$ frame



LFTOPT : gluon exchanged after photon acts

Find non-zero leading-twist FSI effect

Leading twist diffraction , Eikonal form

Shadowing correction , not Coulomb phase

Gauge - independent :

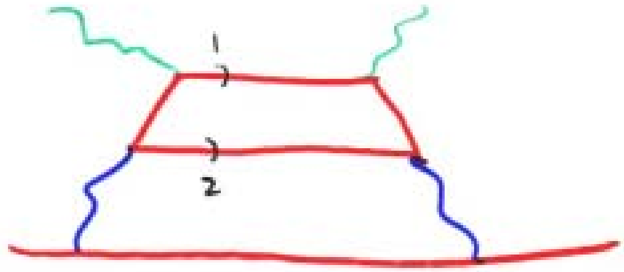
Checked: Feynman , LCG $[A^+=0]$ $\left\{ \begin{array}{l} ML \\ PV \\ k \end{array} \right.$ prescriptions

Lesson for LCG :

$$g^{\mu\nu} - \frac{n^\mu k^\nu + k^\nu n^\mu}{n \cdot k}$$

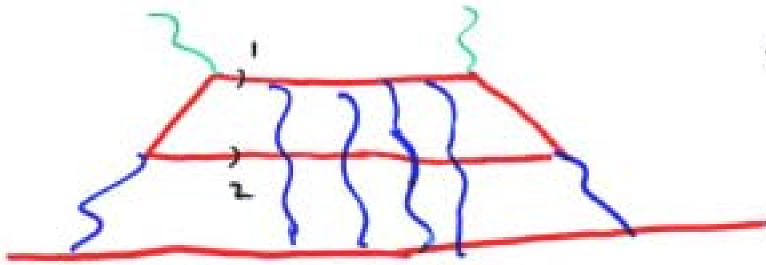
At pole: $n \cdot k = k^+ = O(\frac{1}{\nu})$.

Consider



$\xi^+ \simeq 0$

In Feynman gauge, must keep



Shadowing effect from on-shell states

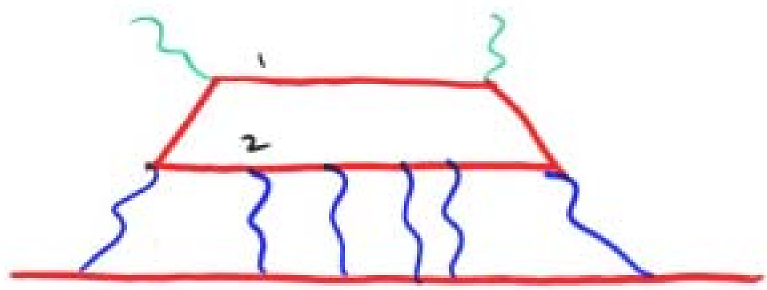
$$|M|^2 \Rightarrow \left| \frac{\sin[g^2 W(r_\perp, R_\perp)/2]}{g^2 W(r_\perp, R_\perp)/2} \; M_0 \right|^2$$

$$W(r_\perp, R_\perp) = \int d^2k_\perp \, \frac{1 - e^{+i r_\perp \cdot \vec{k}_\perp}}{k_\perp^2} \, e^{i\vec{R}_\perp \cdot \vec{k}_\perp}$$

$$= \frac{1}{2\pi} \log\left(\frac{|\vec{R}_\perp + \vec{r}_\perp|}{|R_\perp|}\right)$$

In Light-Cone gauge ("Kovchegov" prescription)

must keep



Final state rescattering of p_2 line

These graphs are suppressed in Feynman gauge

but in l.c.g

$$d_{LC}^{\mu\nu} = \frac{i}{k^2+i\epsilon}\left[-g^{\mu\nu} + \frac{n^\mu k^\nu + n^\nu k^\mu}{n\cdot k}\right]$$

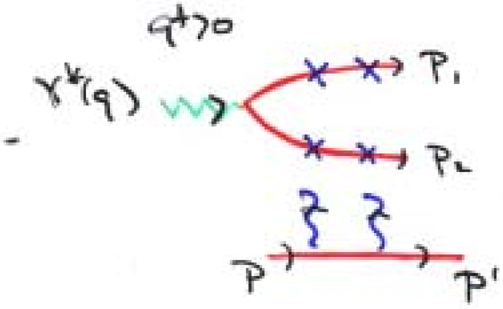
and $n\cdot k = k^+ = O\left(\frac{1}{\nu}\right)$ for on-shell states !

* Result: identical answer as Feynman gauge

* Not included in l.c.g wavefunctions !

M-L, PV prescriptions differ by resid. g. trans

Effect of Rescatterings on the DIS Cross Section



$+ \cdots$

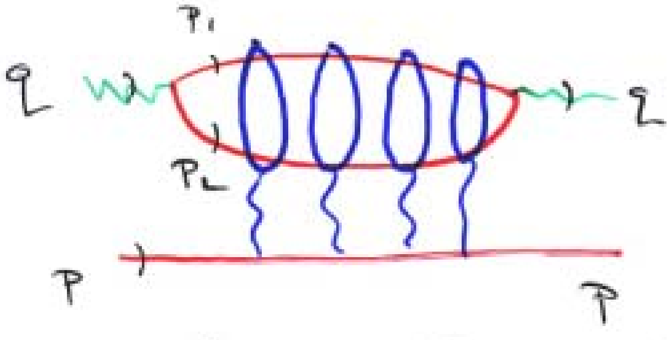
P_1: aligned jet
P_2: spectator jet

$$Q^4 \frac{d\sigma}{dQ^2\, dx_B} = \frac{\alpha}{16\pi^2} \frac{1-y}{y^2} \frac{1}{2M\nu} \int \frac{dp_2^-}{p_2^-} d^2r_\perp\, d^2R_\perp |M|^2$$

$$|M| = \left| \frac{\sin[g^2 W(\vec{r}_\perp, \vec{R}_\perp)/2]}{g^2 W(\vec{r}_\perp, \vec{R}_\perp)/2} M_{Born}(p_2^-, \vec{r}_\perp, \vec{R}_\perp) \right|$$

< 1 for all $\vec{r}_\perp, \vec{R}_\perp$

Equiv. to sum of cuts of forward virt. Compt. ampl.



Find shadowing only arises from diagrams involving attachments to p_1 in F.G.!

cuts give Glauber-Gribov shadowing

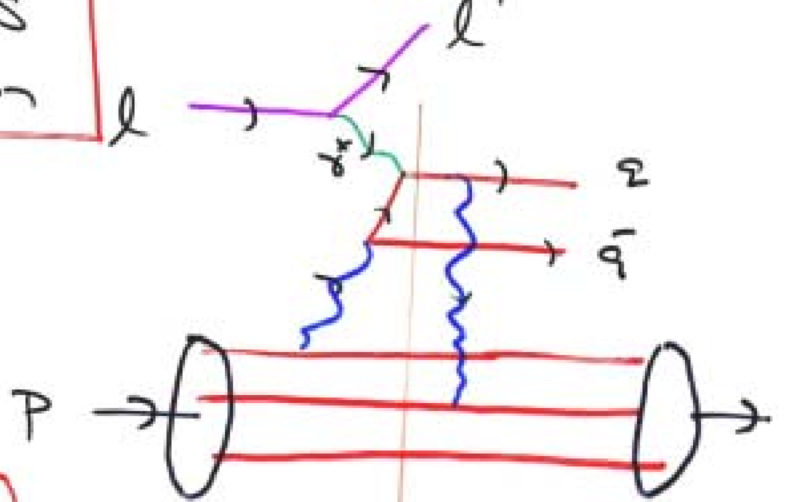
Same result in Feynman, l.c.g. (ML)

↑ from $\frac{2\vec{p}_{1\perp} - \vec{k}_\perp}{k^+}$ term

Non-universal Pomeron Coupling

Hoyer, Peigne, Mueller, Sannino, SJB

DIS Diffraction



$\ell p \to \ell' p' X$

(rap gap)

β-dep like $g \to q\bar{q}$

- 2nd gluon occurs after γ^* interaction exchange

- Cut gives imaginary phase !

- Coupling depends on color dipole moment of $q\bar{q}$

Hebecker, Quack, SJB

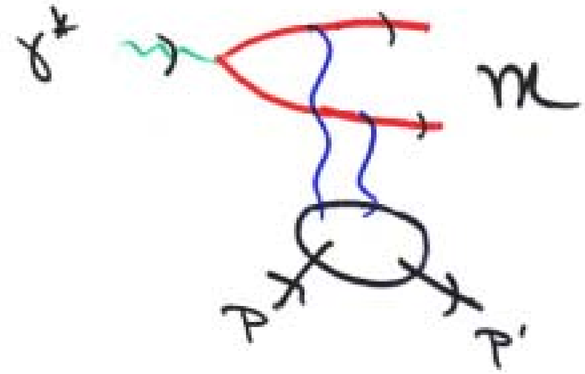
$\therefore$ Pomeron not part of proton w.f.

Not universal !

$P_{\gamma/p}(x)$, $P_{q/p}(x)$, $P_{g/p}(x)$, $\cancel{P_{pom/p}(x)}$ No!

Diffractive Dissociation (large rapidity gaps)

is leading twist in QCD



$$\frac{d\sigma}{dM^2}(\gamma^* p \to M p')$$

Review by Hebecker

Kopeliovich, Nicolaev

$$\frac{d\sigma_T}{dM^2} \sim \left(\frac{1}{M^2+Q^2}\right)^2, \quad \sigma_T \sim \frac{1}{Q^2}$$

aligned jet regime Bj + Kogut

large color dipole Hoyer, Magnea, SJB

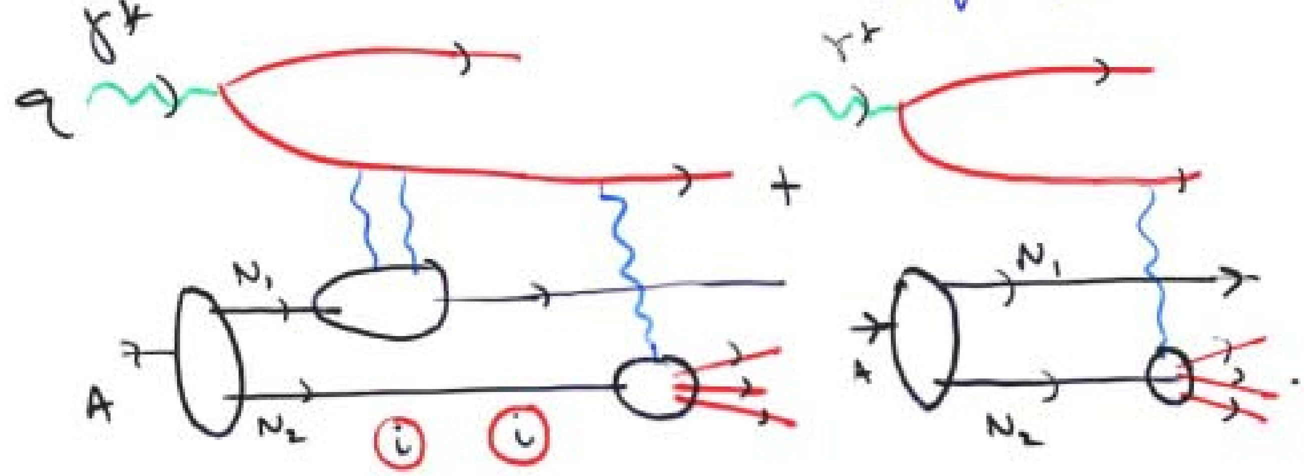
— Coherence for $L_{Ioffe} = \frac{2\nu}{Q^2} > R_N$

— Shadowing in nuclei for

$$L_{Ioffe} = \frac{2\nu}{Q^2} > R_A$$

Diffraction leads to nuclear shadowing:

Gribov, Glauber
Pumplin SJB
Gottfried, Yennie γA



- Destructive interference leads to shadowing at low x.

  Frankfurt, Strikman

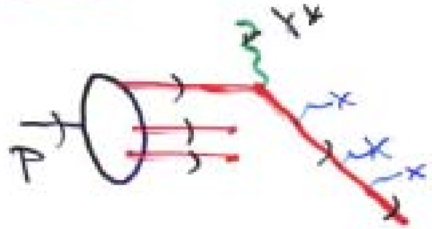
- Shadowing of quark, gluon distributions

- Reggeon exchange leads to antishadowing
  + Odderon

  Lu, SJB

  Schmidt, Yang, SJB

Proposal by Ji and Yuan (also Collins)



$$\Psi \to \Psi \, L$$

complex phase

Augment LFWF (lcg) with phase

$$L = \mathcal{P} \exp\left[i g \int_0^\infty d\vec{z}_\perp \cdot \vec{A}_\perp (z^- = \infty, \vec{z}_\perp) \right]$$

where $\vec{A}_\perp = -\frac{g}{2\pi} \theta(z^-) \, \vec{\nabla}_\perp \ln \mu \vec{r}_\perp$ $\left\{ \begin{array}{l} \text{finite} \\ \text{at} \\ z^- = 0 \end{array} \right.$

$A^+ = A^- = 0$

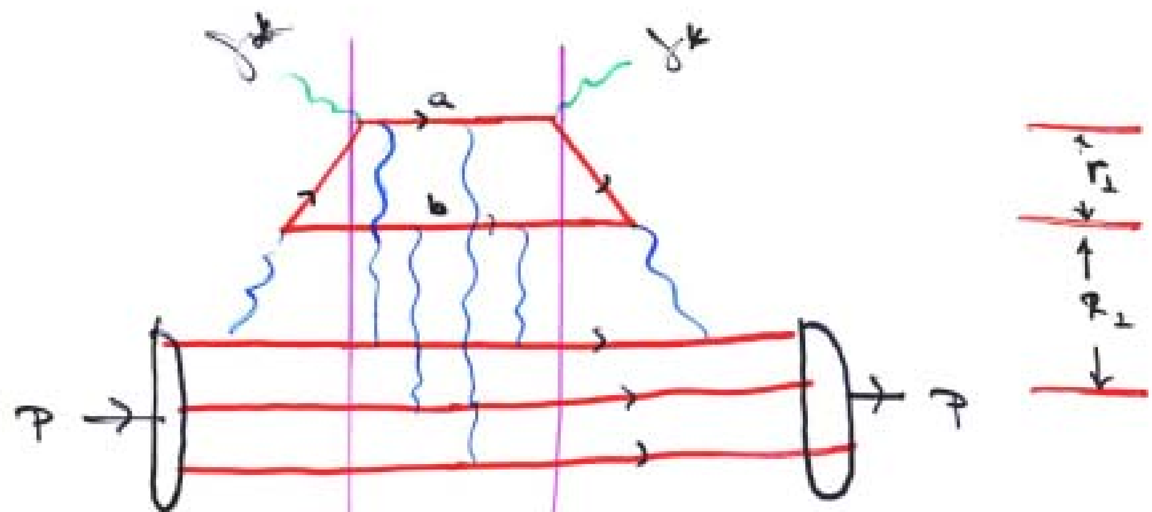
- corresponds to coulomb field of charged particle moving at $v = c$

Equivalently

$$D^{\mu\nu}(q) = -\frac{i}{q^2} \left(g^{\mu\nu} - \frac{q^\mu n^\nu + q^\nu n^\mu}{q \cdot n + i\epsilon} \right)$$

✗ Non-causal b.c. (BHMPS)

✗ Process specific - not universal



Ψ_{LF} $\Leftarrow$ FSIs $\Rightarrow$ $\Psi_{LF}(x_i, \vec{k}_{\perp i}, \lambda_i)$

$$W = \frac{g^2}{2\pi} \log \frac{|\vec{R}_\perp + \vec{r}_\perp|}{|\vec{R}_\perp|}$$

phase $e^{iW_a}\, e^{iW_b} = e^{iW}$

subtract tree gluon

Augment LFWF

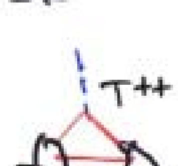
$$\Psi_{LF} \Rightarrow \Psi_{LF}\, \frac{e^{iW} - 1}{iW}$$

$$F_2 \Rightarrow F_2 \left| \frac{\sin W/2}{W} \right|^2$$

Two Wilson lines!: like external field. (BJy)

* Shadowing not contained in Ψ_{LFWF}

- not in LGTh or Ψ_{BS}
- not in local operators



Conclusions

* T-odd Physics
  ⇒ new insight into QCD
  role of ISI, FSI

* Sivers + Collins effects both present in QCD
  BHS, Collins

* Mechanisms understood at perturbative level

* $\alpha_s(\mu^2)$?

* SSA related to Diffractive DIS, Shadowing

* $k_\perp$ distributions, energy loss
  from ISI, FSI

  Bodwin, Lepage, SJB ; Boer, Hwang, SJB
  Metz, Polyakov et al ; Boer, Mulders

* DIS structure functions measure augmented LFWFs
  BHMPS
  Ji, Yang
  ⇒ Solve QCD in external field

* Many questions of principle
  Sum rules; factorization (process indep?); DHG in nuclei

$\boxed{\text{When is } \psi_{LC} \text{ maximal?}}$

$$\psi_n = \frac{\Gamma_n}{M^2 - \mathcal{M}_n^2} \quad , \quad \mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{m_{\perp i}^2}{x_i}$$

Denominator minimum when

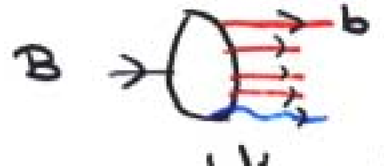
$$\boxed{x_i = \hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}}$$

and $\quad \mathcal{M}_n \to \hat{\mathcal{M}}_n = \sum_j^n m_{\perp j}$

In rest frame: $\quad x_i = \frac{k_i^0 + k_i^z}{P^+} \Rightarrow \hat{x}_i$

when $\underline{k_i^z = 0}$!

* Thus maximal w.f. is equal y_i
* Heavy partons have most of x_i
* Isgur - Wise symmetry

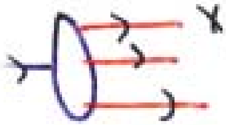


$$x_b \cong \frac{m_{\perp b}}{\mathcal{M}_n}$$

$\psi_{n/B}(x_i, \vec{k}_{\perp i}, \lambda_i)$

Intuition on LC Wavefunctions

$0 < x_i < 1$

Ψ  $x_1, \vec{k}_{\perp 1}, \lambda_1$

$\sum_i x_i = 1 \, , \, \sum_i k_{\perp i} = 0$

$$\Psi = \frac{\Gamma(x_i, k_{\perp i})}{M^2 - \sum_i \frac{k_\perp^2 + m^2}{x_i} + i\epsilon}$$

$\leftarrow \mathfrak{m}_i^2 = \left(\sum_i k_i^\mu\right)^2$

Ψ peaks at $x_i = \frac{m_{\perp i}}{\sum m_{\perp i}}$ $\quad (m_{\perp i}^2 = k_{\perp i}^2 + m_i^2)$

"equal velocity" $\Leftrightarrow$ minimum inv. mass

$$x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

$$\Delta y_i = \ln x_i = y_P - y_i$$

Non-relativistic:

$$x_i = \frac{k^0 + k^z}{M} \simeq \frac{m + k^z}{M}$$ peaks at $\frac{m}{M}$

$x \Rightarrow 0$ is $k_z \Rightarrow -\infty$ for $m_\perp \neq 0$.

Other Applications of LF Quantization

Light-Front Thermodynamics

Set boundary conditions at fixed $\tau = t + z/c$
not t

$$Z_{LF} = \sum_n \exp - \frac{m_n^2}{T_{LF}}$$

$m_n^2 = P_n^- P^+ + P_\perp^2$

SSB
Das et al

Light-Front Lippmann - Schwinger

$$T = H_I + H_I \frac{1}{M^2 - \sum \frac{m_\perp^2}{x} + i\epsilon} T$$

Variational Solutions to Bound-State Probs:

minimize $\langle \psi_{trial} | H_{LF} | \psi_{trial} \rangle$

Construct $\langle \psi_{trial} | n \rangle = \psi_n^{trial}(x_i, k_{\perp i}, \lambda_i)$

using numerator from L_z
ladder relations

(discussions with Capstick)

Variational Method:

Minimize $\langle p | H_{LF}^{QCD} | p \rangle$

by varying trial LFWFS !

Construct model

$$\psi_{3q}(\mathcal{M}^2) = \frac{f_B}{(\mathcal{M}^2 + \zeta)^2} P(x_i)$$

$$\mathcal{M}^2 = \sum_{i=1}^{3} \left(\frac{k_\perp^2 + m^2}{x} \right)_i$$

Construct higher Fock states, orbital excitations via $\frac{1}{D} H_{\pm}^{LC} |\psi\rangle$.

Calculate observables, $F_i(t)$, $q(x_i)$, etc.

$$H_{LC} |\Psi\rangle = \mathcal{M}^2 |\Psi\rangle$$

$$\langle m | H_{LC} | n \rangle \langle n | \Psi \rangle = \mathcal{M}^2 \langle m | \Psi \rangle$$

Discretized Light-Cone Quantization

general solutions obtained: mass spectrum, wavefunctions

* QCD(1+1), QED(1+1) — Eller, Hornbostel, Pauli, SJB
* QCD(1+1) adjoint matter — Klebanov, Dalley, Antonuccio …
* QED(3+1) — Kaluza, Pauli; Krautgärtner, Wölz; van de Sande; Martinovic

Given $\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$, compute

* Form Factors
* Structure Functions, helicity structure
* Decay constants
* Exclusive Amplitudes
* High $\vec{k}_\perp$, $x \to 1$ from PQCD

Summary

Light-Front Quantization and Gauge Theory

* "Rigorous" repn of quantum field theory

  $H_{LF}|\Psi\rangle = M^2|\Psi\rangle \Rightarrow$

  $\langle n|\Psi\rangle = \Psi_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)$ essential for QCD phenom

* Boost-invariant, ghost-free repn of hadrons in terms of q, g d.o.f.

* LFQ of Standard Model $\Rightarrow$ zero mode of ϕ_H

  $\Rightarrow$ new formulation of SSB, Higgs, N.G.

* χSB of QCD $\rightarrow$ Effective Theory N.G.

  $\rightarrow$ needs fundamental QCD basis

* New solns of $H_{LF}|\Psi\rangle = M^2|\Psi\rangle$

  renormalization, models, DLCQ, T.L.

* FSI in DIS $\Rightarrow$ SSA, Diff., Shadowing

  not in ψ_{LF}

Some advantages of Light-Front-Quantized QCD

* No Fermion doubling
* Multiple fermion flavors
* Minkowski space
* Gauge-Fixed: Physical Degrees of Freedom
* Manifest Frame-Independence: $P^+, \vec{P}_\perp$ arbitrary
  J_z kinematical
* LF. Vacuum Trivial + Zero modes
* Vanishing anomalous gravitomagnetic moment

  $$B(0) \equiv 0 \; ; \; \lim_{HR \to 0} M_A = 0$$

* DLCQ discretization retains symmetries
  continuum limit: $K \to \infty$
* LF Wavefunctions, continuum solns., amplitudes, phases as well as spectra
* Solns for QCD(1+1), SUSY(1+1)
  QCD(3+1): Challenging!