The AdS/CFT Correspondence and Light-Front QCD

Stan Brodsky, SLAC
Columbia University

February 18, 2008
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances.

- Analogous to the Schrödinger Theory for Atomic Physics.

- AdS/QCD Holographic Model.

- Light-Front Hadron Wavefunctions.
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

Fixed \( \tau = t + z/c \)

\[ \sum_{i}^{n} x_i = 1 \]

\[ \sum_{i}^{n} \vec{k}_{\perp i} = \vec{0}_{\perp} \]

Invarient under boosts! Independent of \( P^\mu \)
Dirac’s Amazing Idea: The Front Form

Evolve in ordinary time

\[
\sigma = ct - z
\]

Evolve in light-front time!

\[
\tau = t + \frac{z}{c}
\]

Instant Form

Front Form

P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)
‘Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
‘tis an immortal thing

...A moment standing still for ever.

James Montgomery 1833
The poetical works of James Montgomery

TIME: A RHAPPSODY.

Sod fugit, interea fugit irreparabile tempus.
Vivo. Georg. iii. 384.

'Tis a mistake: time flies not,
He only hovers on the wing;
Once born, the moment dies not,
'Tis an immortal thing;
While all is change beneath the sky,
Fix'd like the sun as learned ages prove,
Though from our moving world he seems to move,
'Tis time stands still, and we that fly.

There is no past; from nature's birth,
Days, months, years, ages, till the end
Of these revolving heavens and earth,
All to one centre tend;
And, having reach'd it late or soon,
Converge, — as in a lens, the rays,
Caught from the fountain-light of noon,
Blend in a point that blinds the gaze:
— What has been is, what is shall last;
The present is the focus of the past;
The future, perishing as it arrives,
Becomes the present, and itself survives.

TIME. 138

MISCELLANIES.

Time is not progress, but amount;
One vast accumulating store,
Laid up, not lost; — we do not count
Years gone but added to the score
Of wealth untold, to clime nor class confined,
Riches to generations lent,
For ever spending, never spent,
The' augst inheritance of all mankind.
Of this, from Adam to his latest heir,
All in due turn their portion share,
Which, as they husband or abuse,
Their souls they win or lose.

Though history, on her faded scrolls,
Fragments of facts, and wrecks of names enrols,
Time's indefatigable fingers write
Men's meanest actions on their souls,
In lines which not himself can blot:
These the last day shall bring to light,
Though through long centuries forgot,
When hearts and sepulchres are bared to sight.

Then, having fill'd his measure up,
Amidst his own assembled progeny,
(All that have been, that are, or yet may be,) Before the great white throne,
To Him who sits thereon,
Time shall present—'the amalgamating cup,
In which, as in a crucible,
He hid the moments as they fell,

More precious than Golconda's gems, Or stars in angels' diadems,
Though to our eyes they seem'd to pass Like sands through his symbolic glass: But now, the process done,
Of millions multiplied by millions, none Shall there be wanting,—while by change Ineffable and strange, All shall appear at once, all shall appear as one.

Ah! then shall each of Adam's race,
In that center'd instant, trace,
Upon the tablet of his mind,
His whole existence in a thought combined,
Thenceforth to part no more, but be Impicted on his memory;
— As in the image-chamber of the eye, Seen at a glance, in clear perspective, lie Myriads of forms of ocean, earth, and sky.

Then shall be shown, that but in name Time and eternity were both the same; A point which life nor death could sever, A moment standing still for ever.

1833.

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_{\bot})$$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD}|\psi\rangle \geq M^2|\psi\rangle$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k^2_\perp) \]

"Soft Wall" model

de Teramond, sjb

\[
\frac{\notdef .g0001\text{Soft Wall}.notdef.g0002}{\notdef .g0006\text{GeV}.notdef.g0007}
\]

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
A Unified Description of Hadron Structure

GPDs

Parton momentum distributions

Elastic form factors

Real Compton scattering at high

Deeply Virtual Compton Scattering

Deeply Virtual Meson production

Light Front Wavefunctions

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
Some Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm

- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules

- Exclusive weak decay amplitudes

- Single spin asymmetries: Role if ISI and FSI

- Factorization theorems, DGLAP, BFKL, ERBL Evolution

- Quark interchange amplitude

- Relation of spin, momentum, and other distributions to physics of the hadron itself.
\[ |p, S_z > = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i > \]

\textit{sum over states with } n = 3, 4, \ldots \text{ constituents}

The Light Front Fock State Wavefunctions

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction

\[ x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z} \]

are boost invariant.

\[ \sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}. \]

\textbf{Intrinsic heavy quarks}  \( \bar{u}(x) \neq \bar{d}(x) \)

\textbf{Mueller: BFKL DYNAMICS}  \( \bar{s}(x) \neq s(x) \)

\textbf{Fixed LF time}

\textbf{Columbia Seminar}
February 18, 2008

\textbf{QCD on the LF}

\textbf{Stan Brodsky, SLAC}
\[ L^{QCD} \rightarrow H_{LF}^{QCD} \]

\[ H_{LF}^{QCD} = \sum_i \left[ \frac{m^2 + k^2_{\perp}}{x} \right]_i + H_{LF}^{int} \]

\( H_{LF}^{int} \): Matrix in Fock Space

\[ H_{LF}^{QCD} |\Psi_h > = \mathcal{M}_h^2 |\Psi_h > \]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation
Light-Front QCD

Heisenberg Matrix Formulation

\[ H_{QCD}^{LF} |\Psi_h \rangle = M_h^2 |\Psi_h \rangle \]

DLCQ

Discretized Light-Cone Quantization

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

H.C. Pauli & sjb

**DLCQ:** Frame-independent, No fermion doubling; Minkowski Space
Light-Front Wave Functions in QCD

- Hadronic bound state expanded in n-particle Fock eigenstates $|\psi_h\rangle = \sum_n \psi_{n/h} |\psi_h\rangle$ of the LF Hamiltonian $H_{LF} = P^2 = P^+P^- - P^2_\perp$, $H_{LF} |P\rangle = \mathcal{M}^2 |P\rangle$, at fixed LF time $\tau = t + z/c$ (Dirac ’49; Pauli and Pinsky, sjb Phys. Rept. 1988).

- Fock components
  
  $$\psi_{n/h}(x_i, k_{\perp i}) = \langle n; x_i, k_{\perp i}, | \psi_h(P^+, P_\perp) \rangle,$$

  frame independent and encode hadron properties in high momentum-transfer collisions.

- Momentum fraction $x_i = k_i^+/P^+$ and $k_{\perp i}$ are the relative coordinates of parton $i$ in Fock-state $n$

  $$\sum_{i=1}^n x_i = 1 \quad \sum_{i=1}^n k_{\perp i} = 0.$$

- Define transverse position coordinates $x_i r_{\perp i} = x_i R_{\perp} + b_{\perp i}$

  $$\sum_{i=1}^n b_{\perp i} = 0, \quad \sum_{i=1}^n x_i r_{\perp i} = R_{\perp}.$$
Discretized Light-Cone Quantization

- Periodic boundary condition in x-
- discrete positive plus momenta
- frame-independent formulation
- no fermion doubling
- covariant limit on Fock space

\[
\begin{align*}
P^+ &= \frac{2\pi}{L} K \\
\sum_{i} \frac{2\pi}{L} n_i &= P^+ \\
\sum_{i} n_i &= K
\end{align*}
\]
QCD\((1+1)\) Interaction vertices.

\[
\frac{L}{2\pi} \frac{g^2}{\pi} \frac{1}{2} \left[ \frac{1}{N} \delta_{c_4} \delta_{c_1} - \frac{1}{\delta_{c_4} \delta_{c_1}} \right]
\]

\[
\times \sum_{n_i = 1/2, 3/2, \ldots} \frac{\delta_{n_1 + n_2, n_3 + n_4}}{(n_1 + n_3)^2} b_{n_4}^{\dagger} b_{n_3, c_3} d_{n_2, c_2}^{\dagger} d_{n_1}^{c_1}.
\]
Light-cone-quantized QCD in 1 + 1 dimensions

Kent Hornbostel
Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Stanley J. Brodsky
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

Hans-Christian Pauli
Max Plank Institute for Nuclear Physics, D-6900 Heidelberg 1, Germany
(Received 5 February 1990)

The QCD light-cone Hamiltonian in one space and one time dimension is diagonalized in a discrete momentum-space basis. The hadronic spectrum and wave functions for various coupling constants, numbers of color, and baryon number are computed.

FIG. 2. Spectra for $N = 3$, baryon number $B = 0, 1, $ and $2$ as a function of $g/m$; $K$ fixed.
FIG. 4. Higher-Fock contributions to $N = 3$ structure functions. (a) Lightest meson. (b) Lightest baryon, including antiquarks. (c) Baryon: contribution from two extra quark pairs. The curves are intended to guide the eye.
FIG. 5. (a)–(c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, $2K = 24$. (d) Eleventh state.
\[
\left( M_\pi^2 - \sum_i \frac{k_i^2}{x_i} \right) \begin{bmatrix}
\psi_{q\bar{q}/\pi} \\
\psi_{q\bar{q}g/\pi}
\end{bmatrix} = \begin{bmatrix}
\langle q\bar{q} | V | q\bar{q} \rangle \\
\langle q\bar{q}g | V | q\bar{q} \rangle \\
\langle q\bar{q}g | V | q\bar{q}g \rangle \\
\vdots
\end{bmatrix} \begin{bmatrix}
\psi_{q\bar{q}/\pi} \\
\psi_{q\bar{q}g/\pi}
\end{bmatrix}
\]

\[A^+ = 0\]

G.P. Lepage, sjb

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
Angular Momentum on the Light-Front

\[ J^z = \sum_{i=1}^{n} s^z_i + \sum_{j=1}^{n-1} l^z_j. \]

Conserved LF Fock state by Fock State

\[ l^z_j = -i(k^1_j \frac{\partial}{\partial k^2_j} - k^2_j \frac{\partial}{\partial k^1_j}) \]

n-1 orbital angular momenta

Nonzero Anomalous Moment --> Nonzero orbital angular momentum
Hadronization at the Amplitude Level

\( \tau = x^+ \)

Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
Light-Front Wavefunctions

\[ \sum_{i} x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp \]

\[ \sum_{i} x_i = 1 \]

\[ \sum_{i} \vec{k}_\perp = \vec{0}_\perp \]

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

Fixed \( \tau = t + z/c \)

\( p^+ = p^0 + p^z \)

Invariant under boosts! Independent of \( p^\mu \)
Calculation of Form Factors in Equal-Time Theory

Calculation of Form Factors in Light-Front Theory

Need vacuum fluctuations

zero for $q^+ = 0$
Problems with the Instant Form

- Need $n$ simultaneous measurements to specify initial condition for system of $n$ partons

- Boosts are dynamical and complex; not product of Wigner boosts! McGee, Primack, Osborne, sjb

- Wavefunction insufficient to determine current matrix elements -- need to couple to vacuum

- $N!$ time-ordered frame-dependent diagrams to calculate covariant Feynman amplitude of order $g^N$

$$D = E_{\text{init}} - \sum_{n} \sqrt{\vec{k}^2 + m^2 + i\epsilon}$$
Lorentz boost of composite systems

\[ S_a(\Lambda) = \exp \left( \frac{i}{\sqrt{2}} \vec{a}_a \cdot \vec{V} \tanh^{-1} \frac{V}{E+m} \right) \]

\[ = \sqrt{\frac{E+m}{2m}} \left( 1 + \frac{\vec{a}_a \cdot \vec{P}}{E+m} \right). \]

\[
\begin{pmatrix}
1 + \frac{\vec{a}_a \cdot \vec{P}}{m+E} & \frac{\vec{a}_a \cdot \vec{p}}{2m_a + k_a} \\
\frac{\vec{a}_a \cdot (\vec{P} + \vec{p})}{m+E} & \frac{\vec{a}_a \cdot \vec{p}}{2m_a + k_a}
\end{pmatrix} \otimes
\begin{pmatrix}
1 - \frac{\vec{a}_b \cdot \vec{P}}{m+E} & \frac{\vec{a}_b \cdot \vec{p}}{2m_b + k_b} \\
\frac{\vec{a}_b \cdot (\vec{P} - \vec{p})}{m+E} & \frac{\vec{a}_b \cdot \vec{p}}{2m_b + k_b}
\end{pmatrix}.
\]


\[ \frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times \]
\[ \left[ -\frac{1}{q^L} \psi^\dagger_\alpha(x_i, k'_{\perp i}, \lambda_i) \psi^\dagger_\alpha(x_i, k_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi^\dagger_\alpha(x_i, k'_{\perp i}, \lambda_i) \psi^\dagger_\alpha(x_i, k_{\perp i}, \lambda_i) \right] \]
\[ k'_{\perp i} = k_{\perp i} - x_i q_\perp \]
\[ k'_{\perp j} = k_{\perp j} + (1 - x_j) q_\perp \]

Must have \( \Delta \ell_z = \pm 1 \) to have nonzero \( F_2(q^2) \)
We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$
\langle P', S'_z | J^\mu (0) | P, S_z \rangle = \\
\bar{U}(P', \lambda') \left[ F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i}{2M} \sigma^{\mu\alpha} q_\alpha + F_3(q^2) \frac{-1}{2M} \sigma^{\mu\alpha} \gamma_5 q_\alpha \right] U(P, \lambda)
$$

$$
\kappa = \frac{e}{2M} [F_2(0)], \quad d = \frac{e}{M} [F_3(0)]
$$

Find: $F_3(q^2) = F_2(q^2) \times \tan \phi$
Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$, $q^+ = 0$ frame, imply ($q^R/L \equiv q^I \pm i q^2$):

$$
\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times

\left[ - \frac{1}{q^L} \psi_a^\dag(x_i, k'_\perp i, \lambda_i) \psi_a^\dag(x_i, k_\perp i, \lambda_i) + \frac{1}{q^R} \psi_a^\dag(x_i, k'_\perp i, \lambda_i) \psi_a^\dag(x_i, k_\perp i, \lambda_i) \right],
$$

$$
\frac{F_3(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{i}{2} \times

\left[ - \frac{1}{q^L} \psi_a^\dag(x_i, k'_\perp i, \lambda_i) \psi_a^\dag(x_i, k_\perp i, \lambda_i) - \frac{1}{q^R} \psi_a^\dag(x_i, k'_\perp i, \lambda_i) \psi_a^\dag(x_i, k_\perp i, \lambda_i) \right],
$$

$k'_\perp j = k_\perp j + (1 - x_j)q_\perp$ for the struck constituent $j$ and $k'_\perp i = k_\perp i - x_i q_\perp$ for each spectator ($i \neq j$). $q^+ = 0 \implies$ only $n' = n$. Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

$$F_3(q^2) = F_2(q^2) \times \tan \phi$$
Anomalous gravitomagnetic moment $B(0)$

Okun, Kobzarev, Teryaev: $B(0)$ Must vanish because of Equivalence Theorem

Hwang, Schmidt, sjb; Holstein et al

$B(0) = 0$

Each Fock State

QCD on the LF

Stan Brodsky, SLAC
Advantages of LF QCD

- Lorentz frame invariant; no boosts
- No fermion doubling
- Minkowski space
- Complete set of eigensolutions, bound state and continuum spectroscopy
- LFWFs, observables, simple spin properties
- Physical gauge: ghost free
- Zero modes instead of VEVs
- QED(3+1), QCD(1+1) ......
- Perturbation theory tractable, renormalization (alternate denominators)
- Relativistic statistical physics

QCD at the Amplitude Level

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
LFD in Exclusive Processes

\[ \sum |q^2| >> \Lambda_{\text{QCD}}^2 \]

\[ T_H = \frac{\alpha_s^2}{Q^4} f(x_i, y_i) \]

\[ q^+ \text{ Absent in } q^+ = 0 \]

\[ \text{Absent} \]
Hadron Distribution Amplitudes

\[ \phi_H(x_i, Q) \]

\[ \sum_i x_i = 1 \]

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

\[ \phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp) \]

Lepage, sjb
Frishman, Lepage, Sachrajda, sjb
Peskin, Braun
Efremov, Radyushkin, Chernyak etal
GPDs & Deeply Virtual Exclusive Processes
- New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)

\[ H(x,\xi,t), E(x,\xi,t), \ldots \]

“Generalized Parton Distributions”

Quark angular momentum (Ji sum rule)

\[ J^q = \frac{1}{2} - J^G = \frac{1}{2} \int dx \left[ H^q(x,\xi,0) + E^q(x,\xi,0) \right] \]

Light-cone wavefunction representation of deeply virtual Compton scattering

Stanley J. Brodsky\textsuperscript{a}, Markus Diehl\textsuperscript{a,1}, Dae Sung Hwang\textsuperscript{b}
\[
\frac{1}{\sqrt{1 - \xi}} \frac{\Delta_1 - i \Delta_2}{2M} E_{(n \to n)}(x, \xi, t)
\]
\[
= (\sqrt{1 - \xi})^{2-n} \sum_{n, \lambda_i} \int \prod_{i=1}^{n} \frac{dx_i \, d^2 \vec{k}_{\perp i}}{16\pi^3} \, 16\pi^3 \delta \left(1 - \sum_{j=1}^{n} x_j\right) \delta^{(2)} \left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right) \times \delta(x - x_1) \psi_{(n)}^{\uparrow \ast}(x_1', \vec{k}_{\perp 1}, \lambda_1, \lambda_i) \psi_{(n)}^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i),
\]

where the arguments of the final-state wavefunction are given by

\[
x_1' = \frac{x_1 - \xi}{1 - \xi}, \quad \vec{k}_{\perp 1}' = \vec{k}_{\perp 1} - \frac{1 - x_1}{1 - \xi} \Delta_{\perp}
\]
for the struck quark,

\[
x_i' = \frac{x_i}{1 - \xi}, \quad \vec{k}_{\perp i}' = \vec{k}_{\perp i} + \frac{x_i}{1 - \xi} \Delta_{\perp}
\]
for the spectators \(i = 2, \ldots, n\).
Light-Front Wave Function Overlap Representation

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll

\[ k = \bar{k} - \frac{\Delta}{2} \]
\[ k = \bar{k} + \frac{\Delta}{2} \]
\[ p = \bar{p} + \frac{\Delta}{2} \]
\[ p = \bar{p} - \frac{\Delta}{2} \]

\[ \sum_{N} \]
\[ 1 + \xi \]
\[ 1 - \xi \]

\[ \sum_{N} \]
\[ 1 + \xi \]
\[ -1 - \xi \]

\[ -\xi < \bar{x} < \xi \]

N=3 VALENCE QUARK \Rightarrow Light-cone Constituent quark model

N=5 VALENCE QUARK + QUARK SEA \Rightarrow Meson-Cloud model

DVCS/GPD

QCD on the LF

Stan Brodsky, SLAC
Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

\[ H^q(x,0,0) = q(x), \quad \bar{q}(-x) \]

\[ \tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta \bar{q}(-x) \]

Form factors (sum rules)

\[ \int dx \sum_q \left[ H^q(x,\xi,t) \right] = F_1(t) \] Dirac f.f.

\[ \int dx \sum_q \left[ E^q(x,\xi,t) \right] = F_2(t) \] Pauli f.f.

\[ \int dx \tilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int dx \tilde{E}^q(x,\xi,t) = G_{P,q}(t) \]

Quark angular momentum (Ji’s sum rule)

\[ J^q = \frac{1}{2} - J^G = \frac{1}{2} \int x dx \left[ H^q(x,\xi,0) + E^q(x,\xi,0) \right] \]


Verified using LFWFs
Diehl,Hwang, sbj

QCD on the LF

Columbia Seminar
February 18, 2008

Stan Brodsky, SLAC
Diffractive Dissociation of Pion into Quark Jets

Mueller, sjb
Frankfurt Miller Strikman

E791 Ashery et al.

Diffractive dissociation of pion into quark jets

\[ M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp) \]

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus
Nucleus left Intact!
Single-spin asymmetries

\[ i \vec{S}_p \cdot \vec{q} \times \vec{p}_q \]

\textbf{Pseudo-} T-\textbf{Odd}

Leading-Twist Sivers Effect

\[ \vec{q}^* \]

\[ e^– \]

\[ e^– \]

current quark jet

QCD S- and P- Coulomb Phases

D. S. Hwang, I. A. Schmidt, sjb

J. Collins

Light-Front Wavefunction

S and P- Waves

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
\[ \phi(x) = \frac{1}{\sqrt{2}} v + \varphi = \frac{1}{\sqrt{2}} \left( [v + h(x)] + i\eta(x) \right) \]

No Higgs VEV!

Goldstone field

\[ k^+ = 0 \] zero mode

A Unitary and renormalizable theory of the standard model in ghost free light cone gauge.

*P. Srivastava and sjb*  
hep-ph/0202141

**Decoupling of gravity to the Higgs zero mode**

Columbia Seminar  
February 18, 2008  
QCD on the LF  
43  
Stan Brodsky, SLAC
\[ |p, S_z \rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_\perp, \lambda_i) |n; \vec{k}_\perp, \lambda_i \rangle \]

sum over states with \( n = 3, 4, \ldots \) constituents

The Light Front Fock State Wavefunctions
\[ \Psi_n(x_i, \vec{k}_\perp, \lambda_i) \]
are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction
\[ x_i = \frac{k^+_i}{p^+} = \frac{k^0_i + k^z_i}{P^0 + P^z} \]
are boost invariant.

\[ \sum^n_i k^+_i = P^+, \sum^n_i x_i = 1, \sum^n_i \vec{k}_\perp^i = \vec{0}_\perp. \]

Intrinsic heavy quarks
\[ \bar{u}(x) \neq \bar{d}(x) \]
\[ \bar{s}(x) \neq s(x) \]

Mueller: BFKL DYNAMICS

Fixed LF time

Columbia Seminar
February 18, 2008

QCD on the LF

Stan Brodsky, SLAC
Light Antiquark Flavor Asymmetry

- Naïve Assumption from gluon splitting:

\[ \bar{d}(x) = \bar{u}(x) \]

- E866/NuSea (Drell-Yan)
\[|uudc\bar{c}\rangle \text{ Fluctuation in Proton} \]

\[QCD: \text{ Probability } \frac{\Lambda_{QCD}^2}{M_Q^2} \]

\[|e^+e^-\ell^+\ell^- \rangle \text{ Fluctuation in Positronium} \]

\[QED: \text{ Probability } \frac{(m_\ell\alpha)^4}{M_\ell^4} \]

\[\langle p | \frac{G_{\mu\nu}^3}{m_Q^2} | p \rangle \text{ vs. } \langle p | \frac{F_{\mu\nu}^4}{m_\ell^4} | p \rangle \]

\[c\bar{c} \text{ in Color Octet} \]

Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

\[\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}} \]

\textbf{High x charm!}

Hoyer, Peterson, Sakai, sjb
Measure $c(x)$ in Deep Inelastic Lepton-Proton Scattering