Light-Front Holography:
A New Approximation to QCD

High Energy Physics in the LHC Era
Third International Workshop

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Universidad Tecnica Federico Santa Maria
Valparaiso, Chile, January 4-8, 2010
Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining
\[ \mathbf{H}_{QED} \]

\[(H_0 + H_{int}) |\Psi\rangle \geq E |\Psi\rangle \]

\[ \left[ -\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r}) \]

\[ \left[ -\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{l(l+1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r) \]

\[ V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r} \]

QED atoms: positronium and muonium

Coupled Fock states

Effective two-particle equation

Includes Lamb Shift, quantum corrections

Spherical Basis \( r, \theta, \phi \)

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED
Goal: Use AdS/QCD duality to construct a first approximation to QCD

**Hadron Spectrum**
**Light-Front Wavefunctions, Form Factors, DVCS, etc**

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

in collaboration with Guy de Teramond and Alexandre Deur

**Central problem for strongly-coupled gauge theories**
\[ H_{QCD}^{LF} \]

\[ (H_{LF}^0 + H_{LF}^I)|\Psi \rangle \geq M^2 |\Psi \rangle \]

\[ \left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp) \]

\[ \left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta) \]

\[ U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2) \]

**QCD Meson Spectrum**

**Coupled Fock states**

**Effective two-particle equation**

**Azimuthal Basis** \( \zeta, \phi \)

**Confining AdS/QCD potential**

**Semiclassical first approximation to QCD**
Dirac’s Amazing Idea: The Front Form

Evolve in ordinary time

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)
Each element of flash photograph illuminated at same Light Front time

\[ \tau = t + \frac{z}{c} \]

Evolve in LF time

\[ P^- = i \frac{d}{d\tau} \]

Causal

DIS, Form Factors, DVCS, etc. measure proton WF at fixed
• QCD Lagrangian

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr} (G^{\mu\nu} G_{\mu\nu}) + i \bar{\psi} D_\mu \gamma^\mu \psi + m \bar{\psi} \psi \]

• LF Momentum Generators \( P = (P^+, P^-, \mathbf{P}_\perp) \) in terms of dynamical fields \( \psi, A_\perp \)

\[
\begin{align*}
P^- &= \frac{1}{2} \int dx^- d^2x_\perp \bar{\psi} \gamma^+ \frac{(i \nabla_\perp)^2 + m^2}{i \partial^+} \psi + \text{interactions} \\
P^+ &= \int dx^- d^2x_\perp \bar{\psi} \gamma^+ i \partial^+ \psi \\
\mathbf{P}_\perp &= \frac{1}{2} \int dx^- d^2x_\perp \bar{\psi} \gamma^+ i \nabla_\perp \psi
\end{align*}
\]

• LF Hamiltonian \( P^- \) generates LF time translations

\[
[\psi(x), P^-] = i \frac{\partial}{\partial x^+} \psi(x)
\]

and the generators \( P^+ \) and \( \mathbf{P}_\perp \) are kinematical
Light-Front Bound State Hamiltonian Equation

\[ \tau = t + \frac{z}{c} \]

- Construct light-front invariant Hamiltonian for the composite system: \( H_{LF} = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 \)

\[ H_{LF} \mid \psi_H \rangle = \mathcal{M}_H^2 \mid \psi_H \rangle \]

- State \( \mid \psi_H(P^+, \mathbf{P}_\perp, J_z) \rangle \) is expanded in multi-particle Fock states \( \mid n \rangle \) of the free LF Hamiltonian:

\[ \mid \psi_H \rangle = \sum_n \psi_{n/H} \mid n \rangle, \quad \mid n \rangle = \begin{cases} \mid uud \rangle \\ \mid uudg \rangle \\ \mid uud\bar{q}q \rangle \end{cases} \]

where \( k_i^2 = m_i^2, \quad k_i = (k_i^+, k_i^-, \mathbf{k}_\perp) \), for each component \( i \)

- Fock components \( \psi_{n/H}(x_i, \mathbf{k}_\perp, \lambda^Z_i) \) are independent of \( P^+ \) and \( \mathbf{P}_\perp \) and depend only on relative partonic coordinates: momentum fraction \( x_i = k_i^+ / P^+ \), transverse momentum \( \mathbf{k}_\perp \) and spin \( \lambda^Z_i \)

\[ \sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_\perp = 0. \]

DIS, Form Factors, DVCS, etc. measure proton LFWF
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

Fixed \( \tau = t + z/c \)

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

Process Independent
Direct Link to QCD Lagrangian!

\[ \sum_i^n x_i = 1 \]
\[ \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp} \]

Invariant under boosts! Independent of \( P \)
Angular Momentum on the Light-Front

\[ J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z. \]

Conserved LF Fock state by Fock State!

LF Spin Sum Rule

\[ l_j^z = -i(k_1^j \frac{\partial}{\partial k_2^j} - k_2^j \frac{\partial}{\partial k_1^j}) \]

n-1 orbital angular momenta

Nonzero Anomalous Moment \(\rightarrow\) Nonzero orbital angular momentum!
\[ |p, S_z \rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_\perp, \lambda_i) |n; \vec{k}_\perp, \lambda_i \rangle \]

*sum over states with \( n = 3, 4, \ldots \) constituents*

The Light Front Fock State Wavefunctions

\[ \Psi_n(x_i, \vec{k}_\perp, \lambda_i) \]

are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction

\[ x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z} \]

are boost invariant.

\[ \sum_{i}^{n} k_i^+ = P^+, \sum_{i}^{n} x_i = 1, \sum_{i}^{n} \vec{k}_\perp = \vec{0}_\perp. \]

**Intrinsic heavy quarks**

\[ c(x), b(x) \text{ at high } x \]

\[ \bar{s}(x) \neq s(x) \]

\[ \bar{u}(x) \neq \bar{d}(x) \]

*Fixed LF time*
QCD and the LF Hadron Wavefunctions

AdS/QCD
Light-Front Holography
LF Schrödinger Eqn

Heavy Quark Fock States
Intrinsic Charm

Coordinate space
representation

Quark & Flavor Structure

J=0 Fixed Pole
DVCS, GPDs, TMDs
LF Overlap, incl ERBL

Initial and Final State
Rescattering
DDIS, DDIS, T-Odd

Non-Universal
Antishadowing

Baryon Excitations

Gluonic properties
DGLAP

Orbital Angular Momentum
Spin, Chiral Properties
Crewther Relation

Hard Exclusive Amplitudes
Form Factors
Counting Rules

Distribution amplitude
ERBL Evolution

\[ \phi_p(x_1, x_2, Q^2) \]

Nuclear Modifications
Baryon Anomaly
Color Transparency

Hadronization at
Amplitude Level

Baryon Decay
\[
\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times
\]

\[
\left[ -\frac{1}{q_L} \psi^\dagger_a(x_i, k'_\perp i, \lambda_i) \psi^\dagger_a(x_i, k_\perp i, \lambda_i) + \frac{1}{q_R} \psi^\dagger_a(x_i, k'_\perp i, \lambda_i) \psi^\dagger_a(x_i, k_\perp i, \lambda_i) \right]
\]

\[
k'_\perp i = k_\perp i - x_i q_\perp
\]

\[
k'_\perp j = k_\perp j + (1 - x_j)q_\perp
\]

\[
q_{R,L} = q^x \pm iq^y
\]

 Must have \(\Delta \ell_z = \pm 1\) to have nonzero \(F_2(q^2)\)

**Same matrix elements appear in Sivers effect**

--- connection to quark anomalous moments
Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem

Graviton

Hwang, Schmidt, sjb; Holstein et al

$B(0) = 0$

Each Fock State
Calculation of Form Factors in Equal-Time Theory

**Instant Form**

\[ \sum \] Absent for \( q^+ = 0 \)

Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

**Front Form**

\[ \sum \] Complete Answer

Absent for \( q^+ = 0 \)
Light-Front QCD

\[ L^{QCD} \rightarrow H^{QCD}_{LF} \]

\[ H^{QCD}_{LF} = \sum_i \left[ \frac{m^2 + k_\perp^2}{x} \right]_i + H^{int}_{LF} \]

\[ H^{int}_{LF}: \text{Matrix in Fock Space} \]

\[ H^{QCD}_{LF} |\Psi_h >= M^2_h |\Psi_h > \]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions
Light-Front QCD Features and Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme
Light-Front QCD

Heisenberg Matrix Formulation

\[ H_{LF}^{QCD} |\Psi_h\rangle = M_h^2 |\Psi_h\rangle \]

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation
Example of LFWF representation of GPDs \((n \rightarrow n)\)

\[
\frac{1}{\sqrt{1-\zeta}} \frac{\Delta_1 - i \Delta_2}{2M} E_{(n \rightarrow n)}(x, \zeta, t)
\]

\[
= (\sqrt{1-\zeta})^{2-n} \sum_{n, \lambda_i} \int \prod_{i=1}^{n} \frac{dx_i}{16\pi^3} \frac{d^2k_{\perp i}}{16\pi^3} 16\pi^3 \delta \left(1 - \sum_{j=1}^{n} x_j\right) \delta^{(2)} \left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)
\]

\[
\times \delta(x - x_1) \psi_{(n)}^{\uparrow*}(x_1', \vec{k}_{\perp 1}', \lambda_1) \psi_{(n)}^{\downarrow}(x_1, \vec{k}_{\perp 1}, \lambda_1),
\]

where the arguments of the final-state wavefunction are given by

\[
x_1' = \frac{x_1 - \zeta}{1 - \zeta}, \quad \vec{k}_{\perp 1}' = \vec{k}_{\perp 1} - \frac{1 - x_1}{1 - \zeta} \vec{\Delta}_{\perp}
\]

for the struck quark,

\[
x_i' = \frac{x_i}{1 - \zeta}, \quad \vec{k}_{\perp i}' = \vec{k}_{\perp i} + \frac{x_i}{1 - \zeta} \vec{\Delta}_{\perp}
\]

for the spectators \(i = 2, \ldots, n\).
DIS at $\xi = t = 0$

$H^q(x, 0, 0) = q(x), \quad \bar{q}(-x)$

$\tilde{H}^q(x, 0, 0) = \Delta q(x), \quad \Delta \bar{q}(-x)$

**Form factors (sum rules)**

\[
\int dx \sum_q [H^q(x, \xi, t)] = F_1(t) \quad \text{Dirac f.f.}
\]

\[
\int dx \sum_q [E^q(x, \xi, t)] = F_2(t) \quad \text{Pauli f.f.}
\]

\[
\int dx \tilde{H}^q(x, \xi, t) = G_{A,q}(t), \quad \int dx \tilde{E}^q(x, \xi, t) = G_{V,q}(t)
\]

**Quark angular momentum (Ji’s sum rule)**

\[
J^q = \frac{1}{2} - J^G = \frac{1}{2} \int x dx [H^q(x, \xi, 0) + E^q(x, \xi, 0)]
\]


**Verified using LFWFs**

Diehl, Hwang, sjb

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**Chile LHC**

**January 8, 2010**

**LF Holography and QCD**

**Stan Brodsky**

**SLAC**
**J=0 Fixed pole in real and virtual Compton scattering**

- Effective two-photon contact term
- Seagull for scalar quarks
- Real phase
  \[ M = s^0 \sum e_q^2 F_q(t) \]
- Independent of Q^2 at fixed t
- \( <1/x> \) Moment: Related to Feynman-Hellman Theorem
- Fundamental test of local gauge theory

**Q^2-independent contribution to Real DVCS amplitude**

\[ s^2 \frac{d\sigma}{dt} (\gamma^* p \rightarrow \gamma p) = F^2(t) \]
<table>
<thead>
<tr>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Square of Target LFWFs</td>
<td>Modified by Rescattering: ISI &amp; FSI</td>
</tr>
<tr>
<td>• No Wilson Line</td>
<td>Contains Wilson Line, Phases</td>
</tr>
<tr>
<td>• Probability Distributions</td>
<td>No Probabilistic Interpretation</td>
</tr>
<tr>
<td>• Process-Independent</td>
<td>Process-Dependent - From Collision</td>
</tr>
<tr>
<td>• T-even Observables</td>
<td>T-Odd (Sivers, Boer-Mulders, etc.)</td>
</tr>
<tr>
<td>• No Shadowing, Anti-Shadowing</td>
<td>Shadowing, Anti-Shadowing, Saturation</td>
</tr>
<tr>
<td>• Sum Rules: Momentum and $J^z$</td>
<td>Sum Rules Not Proven</td>
</tr>
<tr>
<td>• DGLAP Evolution; mod. at large $x$</td>
<td>DGLAP Evolution</td>
</tr>
<tr>
<td>• No Diffractive DIS</td>
<td>Hard Pomeron and Odderon Diffractive DIS</td>
</tr>
</tbody>
</table>

\[
\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \quad \left( 2 \right)
\]
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond

Chile LHC
January 8, 2010

LF Holography and QCD

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SLAC
Application of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

String Theory

Bottom-Up

Top-Down
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu\nu}, P^\mu, D, K^\mu, \]

the generators of \( \text{SO}(4,2) \)

\( \text{SO}(4,2) \) has a mathematical representation on AdS\(_5\)
\textbf{AdS/CFT:} Anti-de Sitter Space / Conformal Field Theory

Maldacena:

\textit{Map }AdS_5 \times S_5\textit{ to conformal }N=4\textit{ SUSY}

- **QCD is not conformal:** however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** \( \alpha_s(Q^2) \approx \text{const \ at \ small \ } Q^2 \)

- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**
• Does $\alpha_s$ develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur …

• Recent lattice simulations: evidence that $\alpha_s$ becomes constant and is not small in the infrared Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE).
Deur, Korsch, et al.

\[ \frac{\alpha_{s,g}}{\pi} \]

- Fit
- pQCD evol. eq.

\[ \Gamma_{JLab}, \] \[ \text{GDH limit} \]

- Burkert-Ioffe
- Cornwall

- Godfrey-Isgur
- Bloch et al.

\[ \alpha_{s,g}/\pi \]

- Bhagwat et al.
- Maris-Tandy

\[ Q (\text{GeV}) \]

- DSE gluon couplings
- Lattice QCD

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LF Holography and QCD
IR Conformal Window for QCD

- **Dyson-Schwinger Analysis:** QCD gluon coupling has IR Fixed Point

- Evidence from Lattice Gauge Theory

- **Stability of** $\Upsilon \rightarrow ggg$  
  Shrock, sjb

- Define coupling from observable: **indications of IR fixed point for QCD effective charges**  
  Deur, Chen, Burkert, Korsch,

- Confined gluons and quarks have maximum wavelength:  
  **Decoupling of QCD vacuum polarization at small** $Q^2$  
  Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 << 4m^2$$

- **Justifies application of** AdS/CFT in strong-coupling conformal window
Maximal Wavelength of Confined Fields

- Colored fields confined to finite domain \((x - y)^2 < \Lambda_{QCD}^{-2}\)
- All perturbative calculations regulated in IR
- High momentum calculations unaffected
- Bound-state Dyson-Schwinger Equation
- Analogous to Bethe’s Lamb Shift Calculation

Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

Shrock, sjb

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).
Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
LF Holography and QCD
- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).
\( LF(3+1) \quad AdS_5 \)

\[
\psi(x, \vec{b}_\perp) \quad \leftrightarrow \quad \phi(z)
\]

\[
\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}
\]

\[
\psi(x, \zeta) = \sqrt{x(1-x)\zeta^{-1/2}} \phi(\zeta)
\]

**Holography:** Unique mapping derived from equality of LF and AdS formula for current matrix elements
Relativistic LF radial equation

\[
\left[- \frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[\zeta^2 = x(1 - x)b^2_\perp.\]

\[U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)\]
Light-Front Quantization of QCD and AdS/CFT

• Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure gives an unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...

• Frame-independent LF Hamiltonian equation: similar structure as AdS EOM

\[ P^\mu P_\mu |P> \geq (P^- P^+ - \vec{P}_\perp^2)|P> \geq M^2 |P> \]

• First semiclassical approximation to the bound-state LF Hamiltonian equation in QCD is equivalent to equations of motion in AdS and can be systematically improved

GdT and Sjb PRL 102, 081601 (2009)
AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5

- Scale Transformations represented by wavefunction in 5th dimension
  \[ x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z \]

- Match solutions at small \( z \) to conformal twist dimension of hadron wavefunction at short distances
  \[ \psi(z) \sim z^\Delta \text{ at } z \rightarrow 0 \]

- Hard wall model: Confinement at large distances and conformal symmetry in interior

- Truncated space simulates “bag” boundary conditions
  \[ 0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}} \]
Bosonic Solutions: Hard Wall Model

- Conformal metric: \( ds^2 = g_{\ell m} dx^\ell dx^m \). \( x^\ell = (x^\mu, z) \), \( g_{\ell m} \to \left( \frac{R^2}{z^2} \right) \eta_{\ell m} \).

- Action for massive scalar modes on AdS\(_{d+1}\):

\[
S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \frac{1}{2} \left[ g_{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to \left( \frac{R}{z} \right)^{d+1}.
\]

- Equation of motion

\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g_{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.
\]

- Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z) \), \( P_\mu P^\mu = M^2 \):

\[
\left[ z^2 \partial_z^2 - (d-1)z \partial_z + z^2 M^2 - (\mu R)^2 \right] \Phi(z) = 0.
\]

- Solution: \( \Phi(z) \to z^\Delta \) as \( z \to 0 \),

\[
\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z M) \quad \Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4
\]
Let \( \Phi(z) = z^{3/2} \phi(z) \)

**AdS Schrodinger Equation for bound state of two scalar constituents:**

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = \mathcal{M}^2 \phi(z)
\]

\( L \): light-front orbital angular momentum

**Derived from variation of Action in AdS\(_5\)**

**Hard wall model: truncated space**

\( \phi(z = z_0 = \frac{1}{\Lambda_c}) = 0. \)
Match fall-off at small \( z \) to conformal twist-dimension
at short distances

- Pseudoscalar mesons: \( \mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\ell_1 \ldots D_{\ell_m}} \psi \) (\( \Phi_\mu = 0 \) gauge). \( \Delta = 2 + L \)

- 4-\( d \) mass spectrum from boundary conditions on the normalizable string modes at \( z = z_0 \), \( \Phi(x, z_0) = 0 \), given by the zeros of Bessel functions \( \beta_{\alpha,k} \): \( M_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD} \)

- Normalizable AdS modes \( \Phi(z) \)

\[ S' = 0 \quad \text{Meson orbital and radial AdS modes for } \Lambda_{QCD} = 0.32 \text{ GeV.} \]
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
**Soft-Wall Model**

\[ S = \int d^4x \, dz \sqrt{g} \, e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2 \]

**Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field**

Karch, Katz, Son and Stephanov (2006)]

- Equation of motion for scalar field \( \mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2) \)

\[
\left[ z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) \right. z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \left. \right] \Phi(z) = 0
\]

with \((\mu R)^2 \geq -4\).

- LH holography requires ‘plus dilaton’ \( \varphi = +\kappa^2 z^2 \). Lowest possible state \((\mu R)^2 = -4\)

\[
\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}
\]

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion
AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

\[
\left[ - \frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = M^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Derived from variation of Action

Dilaton-Modified AdS$_5$

\[
e^{\Phi(z)} = e^{+\kappa^2 z^2}
\]

Positive-sign dilaton
\[ ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2) \]

\[ V(z) = mc^2 \sqrt{g_{00}} \]

Agrees with Klebanov and Maldacena for positive-sign exponent of dilaton
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$Pion\ has\ zero\ mass!$

$m_q = 0$
Higher-Spin Hadrons

- Obtain spin-\(J\) mode \(\Phi_{\mu_1 \cdots \mu_J}\) with all indices along 3+1 coordinates from \(\Phi\) by shifting dimensions

\[
\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)
\]

- Substituting in the AdS scalar wave equation for \(\Phi\)

\[
\left[ z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0
\]

- Upon substitution \(z \rightarrow \zeta\)

\[
\phi_J(\zeta) \sim \zeta^{-3/2 + J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)
\]

we find the LF wave equation

\[
\left( - \frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}
\]

with \((\mu R)^2 = -(2 - J)^2 + L^2\)
Higher Spin Bosonic Modes SW

• Effective LF Schrödinger wave equation

\[
\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1)\right] \phi_S(z) = \mathcal{M}^2 \phi_S(z)
\]

with eigenvalues \( \mathcal{M}^2 = 2\kappa^2(2n + 2L + S) \).

• Compare with Nambu string result (rotating flux tube):

\[
M_n^2(L) = 2\pi\sigma (n + L + 1/2).
\]

Same slope in \( n \) and \( L \)

Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \) GeV.

• Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).
Quark separation increases with $L$

(a) $\Phi(z)$ vs $\tilde{z}$

(b) $\Phi(z)$ vs $\tilde{z}$

(c) $M^2$ (GeV$^2$) vs $L$

(d) $S = 1$

$\rho_3 (1690)$

$a_2 (1320)$

$\omega_3 (1670)$

$f_2 (1270)$

$\omega (782)$

$\rho (770)$

$\rho (1700)$

$\rho (1450)$

$\rho (770)$
\[ M^2 = 2\kappa^2(2n + 2L + S). \]

\[ S = 1 \]
Parent and daughter Regge trajectories for the $I = 1$ \( \rho \)-meson family (red) and the $I = 0$ \( \omega \)-meson family (black) for $\kappa = 0.54$ GeV.
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = zQ K_1(zQ) \]

\[ F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

High \( Q^2 \) from small \( z \sim 1/Q \)

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi \) scales as \( \Phi^{(n)} \sim z^{\Delta n} \). Thus:

\[ F(Q^2) \to \left[ \frac{1}{Q^2} \right]^{\tau - 1}, \]

where \( \tau = \Delta n - \sigma_n \), \( \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).  

Polchinski, Strassler, de Teramond, sjb

Dimensional Quark Counting Rules:
General result from AdS/CFT and Conformal Invariance
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

\( q^2(GeV^2) \)

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

Data Compilation
Baldini, Kloe and Volmer

de Teramond, sjb
See also: Radyushkin

Chile LHC
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Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

\[ [z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2] J_\kappa(Q, z) = 0. \]

- Solution bulk-to-boundary propagator

\[ J_\kappa(Q, z) = \Gamma \left(1 + \frac{Q^2}{4\kappa^2}\right) U \left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right), \]

where \( U(a, b, c) \) is the confluent hypergeometric function

\[ \Gamma(a) U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt. \]

- Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)

\[ F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z). \]

- For large \( Q^2 \gg 4\kappa^2 \)

\[ J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z), \]

the external current decouples from the dilaton field.
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

\[ q^2(GeV^2) \]

Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement
Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

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de Teramond, sjb
See also: Radyushkin
Stan Brodsky
• Analytical continuation to time-like region $q^2 \rightarrow -q^2$. $M_{\rho} = 2\kappa = 750$ MeV

• Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

• Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).
Dressed soft-wall current bring in higher Fock states and more vector meson poles