AdS/QCD, Light-Front Holography, and the Chiral Condensate

\[ \Psi_n(x_i, \bar{k}_i, \lambda_i) \]

Origin Of Mass
May 3-7, 2010

Stan Brodsky  SLAC  & CP³-Origins

CP³ - Origins  Odense, Denmark
Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates
- Systematically improvable
**Light-Front Holography and Non-Perturbative QCD**

**Goal:**
*Use AdS/QCD duality to construct a first approximation to QCD*

**Hadron Spectrum**
**Light-Front Wavefunctions, Form Factors, DVCS, etc**

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

*in collaboration with Guy de Teramond and Alexandre Deur*

**Central problem for strongly-coupled gauge theories**
Dirac’s Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!

P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Instant Form

Front Form

CP3
May 7, 2009

AdS/QCD, LF Holography, & Chiral Condensate

Stan Brodsky
SLAC-CP3
Each element of flash photograph illuminated at same Light Front time

\[ \tau = t + \frac{z}{c} \]

Evolve in LF time

\[ P^- = i \frac{d}{d\tau} \]

Causal, Trivial Vacuum

DIS, Form Factors, DVCS, etc. measure proton WF at fixed
QCD Lagrangian

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr} \left( G^{\mu\nu} G_{\mu\nu} \right) + i\bar{\psi} D_\mu \gamma^\mu \psi + m\bar{\psi}\psi \]

- LF Momentum Generators \( P = (P^+, P^-, \mathbf{P}_\perp) \) in terms of dynamical fields \( \psi, A_\perp \)

\[
P^- = \frac{1}{2} \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+} \psi + \text{interactions}
\]

\[
P^+ = \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ i\partial^+ \psi
\]

\[
\mathbf{P}_\perp = \frac{1}{2} \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ i\nabla_\perp \psi
\]

- LF Hamiltonian \( P^- \) generates LF time translations

\[
[\psi(x), P^-] = i\frac{\partial}{\partial x^+} \psi(x)
\]

and the generators \( P^+ \) and \( \mathbf{P}_\perp \) are kinematical
Light-Front Bound State Hamiltonian Equation

- Construct light-front invariant Hamiltonian for the composite system: $H_{LF} = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$

\[ H_{LF} |\psi_H\rangle = \mathcal{M}_H^2 |\psi_H\rangle \]

- State $|\psi_H(P^+, \mathbf{P}_\perp, J_z)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian:

\[ |\psi_H\rangle = \sum_n \psi_{n/H} |n\rangle, \quad |n\rangle = \begin{cases} 
|uud\rangle \\
|uudg\rangle \\
|uud\bar{q}q\rangle 
\end{cases} \]

where $k_i^2 = m_i^2, \quad k_i = (k_i^+, k_i^-, \mathbf{k}_\perp i), \text{ for each component } i$

- Fock components $\psi_{n/H}(x_i, \mathbf{k}_\perp i, \lambda_i^z)$ are independent of $P^+$ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+ / P^+$, transverse momentum $\mathbf{k}_\perp i$ and spin $\lambda_i^z$

\[ \sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_\perp i = 0. \]

\textit{DIS, Form Factors, DVCS, etc. measure proton LFWF}
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

Fixed \( \tau = t + z/c \)

\[ \sum_{i=1}^{n} x_i P^+ + x_i \vec{P}_\perp + \vec{k}_\perp i \]

Process Independent Direct Link to QCD Lagrangian!

\[ \psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

\[ \sum_{i=1}^{n} x_i = 1 \]

\[ \sum_{i=1}^{n} \vec{k}_\perp i = \vec{\sigma}_\perp \]

Invariant under boosts! Independent of \( P \)

---

CP3
May 7, 2009

AdS/QCD, LF Holography, & Chiral Condensate

Stan Brodsky
SLAC-CP3
\[ |p, S_z> = \sum_{n=3}^{\infty} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i > \]

**sum over states with n=3, 4, ... constituents**

The Light Front Fock State Wavefunctions

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction

\[ x_i = \frac{k_i^+}{p^+} = \frac{k_0^i + k_z^i}{P^0 + P^z} \]

are boost invariant.

\[ \sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_{\perp i} = 0_{\perp}. \]

**Intrinsic heavy quarks**

\( c(x), b(x) \) at high \( x \)

\[ \bar{s}(x) \neq s(x) \]

\[ \bar{u}(x) \neq \bar{d}(x) \]

**Fixed LF time**

Stan Brodsky

SLAC-CP3
QCD and the LF Hadron Wavefunctions

- AdS/QCD
  - Light-Front Holography
  - LF Shr"odinger Eqn

- Heavy Quark Fock States
  - Intrinsic Charm

- Coordinate space representation

- Quark & Flavor Structure

- J=0 Fixed Pole
  - DVCS, GPDs, TMDs
  - LF Overlap, incl ERBL

- Initial and Final State Rescattering
  - DDIS, DDIS, T-Odd

- Non-Universal Antishadowing

- Baryon Excitations

- Gluonic properties
  - DGLAP

- Orbital Angular Momentum
  - Spin, Chiral Properties
  - Crewther Relation

- Hard Exclusive Amplitudes
  - Form Factors
  - Counting Rules

- Nuclear Modifications
  - Baryon Anomaly
  - Color Transparency

- Distribution amplitude
  - ERBL Evolution
  \( \phi_p(x_1, x_2, Q^2) \)

- Hadronization at Amplitude Level

- Baryon Decay
Angular Momentum on the Light-Front

\[ J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z. \]

Conserved at every vertex and LF Fock state by Fock State!

LF Spin Sum Rule

\[ l_j^z = -i(k^1_j \frac{\partial}{\partial k^2_j} - k^2_j \frac{\partial}{\partial k^1_j}) \]

n-1 orbital angular momenta

Nonzero Anomalous Moment  -->  Nonzero quark orbital angular momentum!
Light-Front QCD

Heisenberg Matrix Formulation

\[ L^{QCD} \rightarrow H^{QCD}_{LF} \]

\[ H^{QCD}_{LF} = \sum_{i} \left[ \frac{m^2 + k^2}{x} \right]_{i} + H^{int}_{LF} \]

\( H^{int}_{LF} \): Matrix in Fock Space

\[ H^{QCD}_{LF} |\Psi_{h}> = \mathcal{M}^2_{h} |\Psi_{h}> \]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions
Light-Front QCD

Heisenberg Matrix Formulation

\[ H_{LF}^{QCD} |\Psi_h \rangle = M^2_h |\Psi_h \rangle \]

Discretized Light-Cone Quantization

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation
Light-Front QCD Features and Phenomenology

- Trivial Vacuum
- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme
Calculation of Form Factors in Equal-Time Theory

**Instant Form**

Calculation of Form Factors in Light-Front Theory

**Front Form**

Need vacuum-induced currents

Complete Answer

Absent for $q^+ = 0$

zero!!

Stan Brodsky
SLAC-CP3
\[ F_2(q^2) = \sum_a \int [dx][d^2k_\perp] \sum e_j \frac{1}{2} \times \]

\[ \left[ - \frac{1}{q_L} \psi_a^\dagger(x_i, k'_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) + \frac{1}{q_R} \psi_a^\dagger(x_i, k'_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) \right] \]

\[ k'_{\perp i} = k_{\perp i} - x_i q_\perp \]

\[ k'_{\perp j} = k_{\perp j} + (1 - x_j) q_\perp \]

Must have \( \Delta \ell_z = \pm 1 \) to have nonzero \( F_2(q^2) \)

**Same matrix elements appear in Sivers effect**

**connection to quark anomalous moments**
Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem

Hwang, Schmidt, sjb; Holstein et al: $B(0) = 0$  Each Fock State

$B(0) = 0$  Each Fock State
Light-cone wavefunction representation of deeply virtual Compton scattering

Stanley J. Brodsky\textsuperscript{a}, Markus Diehl\textsuperscript{a,1}, Dae Sung Hwang\textsuperscript{b}
Example of LFWF representation of GPDs \((n \Rightarrow n)\)

\[
\frac{1}{\sqrt{1 - \zeta}} \frac{\Delta^1 - i \Delta^2}{2M} E_{(n \Rightarrow n)}(x, \zeta, t)
\]

\[
= (\sqrt{1 - \zeta})^{2-n} \sum_{n, \lambda_i} \int \prod_{i=1}^{n} \frac{dx_i \, d^2 \vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta \left( 1 - \sum_{j=1}^{n} x_j \right) \delta^{(2)} \left( \sum_{j=1}^{n} \vec{k}_{\perp j} \right) \delta(x - x_1) \psi_{(n)}^{\uparrow*}(x_i', \vec{k}_{\perp i}^{'}, \lambda_i) \psi_{(n)}^\downarrow(x_i, \vec{k}_{\perp i}, \lambda_i),
\]

where the arguments of the final-state wavefunction are given by

\[
x_1' = \frac{x_1 - \zeta}{1 - \zeta}, \quad \vec{k}_{\perp 1}^{' \perp} = \vec{k}_{\perp 1} - \frac{1 - x_1}{1 - \zeta} \vec{\Delta}_{\perp} \quad \text{for the struck quark},
\]

\[
x_i' = \frac{x_i}{1 - \zeta}, \quad \vec{k}_{\perp i}^{' \perp} = \vec{k}_{\perp i} + \frac{x_i}{1 - \zeta} \vec{\Delta}_{\perp} \quad \text{for the spectators } i = 2, \ldots, n.
\]
Link to DIS and Elastic Form Factors

Form factors (sum rules)

\[ \int dx \sum_q [H^q(x, \xi, t)] = F_1(t) \] Dirac f.f.
\[ \int dx \sum_q [E^q(x, \xi, t)] = F_2(t) \] Pauli f.f.
\[ \int dx \bar{\tilde{H}}^q(x, \xi, t) = G_{A,q}(t), \quad \int dx \bar{\tilde{E}}^q(x, \xi, t) = G_{P,q}(t) \]

Verified using LFWFs
Diehl, Hwang, sjb

Quark angular momentum (Ji’s sum rule)

\[ J^q = \frac{1}{2} - J^G = \frac{1}{2} \int_{-1}^{1} dx \left[ H^q(x, \xi, 0) + E^q(x, \xi, 0) \right] \]

$J=0$ Fixed pole in real and virtual Compton scattering

Effective two-photon contact term

Seagull for scalar quarks

Real phase

$$M = s^0 \sum e_q^2 F_q(t)$$

Independent of $Q^2$ at fixed $t$

$<1/x>$ Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory

$Q^2$-independent contribution to Real DVCS amplitude

$$s^2 \frac{d\sigma}{dt} (\gamma^* p \rightarrow \gamma p) = F^2(t)$$
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

AdS/QCD, LF Holography, & Chiral Condensate

Stan Brodsky
SLAC-CP3

CP3
May 7, 2009
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances

- Analogous to the Schrödinger Theory for Atomic Physics

- AdS/QCD Light-Front Holography

- Hadronic Spectra and Light-Front Wavefunctions
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu \nu}, P^\mu, D, K^\mu, \]

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5
**AdS/CFT:** Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S_5$ to conformal $N=4$ SUSY

- **QCD is not conformal;** however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- Use mathematical mapping of the conformal group $SO(4,2)$ to $AdS_5$ space
Deur, Korsch, et al.

\[ \frac{\alpha_s g_1}{\pi} JLab \quad \text{GDH limit} \quad \text{Burkert-Ioffe} \]

Fit \quad pQCD evol. eq.

\[ \alpha_s, g_1 \text{/ pQCD} \]

Cornwall

Godfrey-Isgur

Bloch et al.

Bhagwat et al.

Maris-Tandy

Fischer et al.

DSE gluon couplings

\( Q (\text{GeV}) \)

\( \alpha_s g_1 / \pi JLab \)

Lattice QCD

CP3
May 7, 2009

AdS/QCD, LF Holography, & Chiral Condensate

Stan Brodsky
SLAC-CP3
Conformal Behavior of QCD in Infrared

- Does $\alpha_s$ develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur . . .

- Recent lattice simulations: evidence that $\alpha_s$ becomes constant and is not small in the infrared
  Furui and Nakajima, hep-lat/0612009  (Green dashed curve: DSE).
Nearly conformal QCD?

Define $s$ from Björkén sum,

$$\Gamma_{1}^{p-n} \equiv \int_{0}^{1} dx \left( g_{1}^{p}(x, Q^{2}) - g_{1}^{n}(x, Q^{2}) \right) = \frac{1}{6} g_{A} \left( 1 - \frac{\alpha_{s} g_{1}}{\pi} \right)$$

$g_{1}$ = spin dependent structure function (from inelastic ep scattering)

Data from EG1 exp., at JLab CLAS (2008)

$s$ runs only modestly at small $Q^{2}$

Fig. from 0803.4119, Duer et al.
Lesson from QED and Lamb Shift:

maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\Lambda_{QCD}} \]
\[ \lambda < \Lambda_{QCD} \]

B-Meson

gluon and quark propagators cutoff in IR
because of color confinement

R. Shrock, sjb
Maximal Wavelength of Confined Fields

- Colored fields confined to finite domain \((x - y)^2 < \Lambda_{QCD}^{-2}\)
- All perturbative calculations regulated in IR
- High momentum calculations unaffected
- Bound-state Dyson-Schwinger Equation
- Analogous to Bethe’s Lamb Shift Calculation

Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

Shrock, sjb

J. D. Bjorken, SLAC-PUB 1053 Cargese Lectures 1989

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).
Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$ 

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
AdS/QCD, LF Holography, & Chiral Condensate

May 7, 2009

Stan Brodsky
SLAC-CP3
• Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

• Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).
AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5

- Scale Transformations represented by wavefunction in 5th dimension
  \[ x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z \]

- Match solutions at small \( z \) to conformal twist dimension of hadron wavefunction at short distances
  \[ \psi(z) \sim z^\Delta \quad \text{at} \quad z \rightarrow 0 \]

- Hard wall model: Confinement at large distances and conformal symmetry in interior

- Truncated space simulates “bag” boundary conditions
  \[ 0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}} \]
Bosonic Solutions: Hard Wall Model

• Conformal metric: \( ds^2 = g_{\ell m} dx^\ell dx^m \). \( x^\ell = (x^\mu, z) \), \( g_{\ell m} \to (R^2/z^2) \eta_{\ell m} \).

• Action for massive scalar modes on \( \text{AdS}_{d+1} \):

\[
S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g_{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.
\]

• Equation of motion

\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.
\]

• Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z) \), \( P_\mu P^\mu = \mathcal{M}^2 \):

\[
\left[ z^2 \partial_z^2 - (d - 1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.
\]

• Solution: \( \Phi(z) \to z^\Delta \) as \( z \to 0 \),

\[
\Phi(z) = Cz^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).
\]

\[
\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4
\]
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrodinger Equation for bound state of two scalar constituents:**

$$\left[ - \frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = M^2 \phi(z)$$

$L$: light-front orbital angular momentum

**Derived from variation of Action in AdS$_5$**

**Hard wall model: truncated space**

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$
Match fall-off at small $z$ to conformal twist-dimension at short distances

- Pseudoscalar mesons: $O_{2+L} = \bar{\psi} \gamma_5 D_{\ell_1} \cdots D_{\ell_m} \psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$

- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $M_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

\[ \phi(z) = \begin{cases} \Delta & z \approx 3 \\ 0 & z_0 = \frac{1}{\Lambda_{QCD}} \end{cases} \]

$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
**Soft-Wall Model**

\[ S = \int d^4 x \, d z \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2 \]

*Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field*

Karch, Katz, Son and Stephanov (2006)]

- Equation of motion for scalar field
  \[ \mathcal{L} = \frac{1}{2} \left( g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right) \]
  \[ \left[ z^2 \partial^2_z - \left( 3 \mp 2 \kappa^2 z^2 \right) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0 \]
  with \((\mu R)^2 \geq -4\).

- LH holography requires ‘plus dilaton’ \( \varphi = + \kappa^2 z^2 \). Lowest possible state \((\mu R)^2 = -4\)
  
  \[ \mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2} \]

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion
$ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} \left( dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2 \right)$

$ds^2 = e^{A(y)} \left( -dx_0^2 + dx_1^2 + dx_3^2 + dx_3^2 \right) + dy^2$
\[ ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2) \]

\[ V(z) = m c^2 \sqrt{g_{00}} \]

\[ V(z) \sim \frac{R}{z} e^{+\kappa^2 z^2/2} \]

\[ V(z) \sim \frac{R}{z} e^{-\kappa^2 z^2/2} \]

Agrees with Klebanov and Maldacena for positive-sign exponent of dilaton
AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Derived from variation of Action

Dilaton-Modified AdS\(_5\)

Positive-sign dilaton

\[
e^{\Phi(z)} = e^{+\kappa^2 z^2}
\]
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$m_q = 0$

Pion has zero mass!

Quark separation increases with $L$

Pion mass automatically zero!
Higher-Spin Hadrons

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from $\Phi$ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for $\Phi$

$$\left[z^2 \partial^2_z - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2 + J} e^{\kappa^2 \zeta^2/2} \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(- \frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$
Quark separation increases with $L$

(a) $\Phi(\tilde{z})$

(b) $\Phi(\tilde{z})$

$S = 1$

$\rho_3 (1690)$

$f_4 (2050)$

$a_4 (2040)$

$\omega (782)$

$\omega_3 (1670)$

$\rho (770)$

$f_2 (1270)$

$\rho (1700)$

$\rho (1450)$

$\rho (770)$

$M^2 (\text{GeV}^2)$

$L$

$n$

May 7, 2009

Stan Brodsky

SLAC-CP3
\[ M^2 = 2\kappa^2 (2n + 2L + S). \]

\[ S = 1 \]
Parent and daughter Regge trajectories for the $I = 1$ $\rho$-meson family (red) and the $I = 0$ $\omega$-meson family (black) for $\kappa = 0.54$ GeV.
**Hadron Form Factors from AdS/CFT**

Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = z Q K_1(zQ) \]

\[ F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi \) scales as \( \Phi^{(n)} \sim z^{\Delta_n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1} \]

where \( \tau = \Delta_n - \sigma_n \), \( \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).
**Spacelike pion form factor from AdS/CFT**

\[ F_\pi(q^2) \]

- **Soft Wall: Harmonic Oscillator Confinement**
- **Hard Wall: Truncated Space Confinement**

*One parameter - set by pion decay constant.*

Data Compilation
Baldini, Kloe and Volmer

See also: Radyushkin

**AdS/QCD, LF Holography, & Chiral Condensate**
Stan Brodsky
SLAC-CP3

**CP3**
May 7, 2009
Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor
  \[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \psi_P^*(x, k_\perp - xq_\perp) \psi_P(x, k_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)
  \[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find \( b = |\vec{b}_\perp| \):
  \[ F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \]
  \[ = 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bq_x) \left| \tilde{\psi}(x, b) \right|^2, \]
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[ F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{1-x} \right) \tilde{\rho}(x, \zeta), \]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

- Transversality variable

\[ \zeta = \sqrt{x(1-x) \bar{b}_\perp^2} \]

- Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[ \int_0^1 dx J_0 \left( \zeta Q \sqrt{1-x} \right) = \zeta Q K_1(\zeta Q), \]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \).
• Electromagnetic form-factor in AdS space:

\[ F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2, \]

where \( J(Q^2, z) = zQ K_1(zQ). \)

• Use integral representation for \( J(Q^2, z) \)

\[ J(Q^2, z) = \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) \]

• Write the AdS electromagnetic form-factor as

\[ F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_{\pi^+}(z)|^2 \]

• Compare with electromagnetic form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = R^3 \frac{x(1-x) |\Phi_{\pi}(\zeta)|^2}{2\pi \zeta^4} \]

with \( \zeta = z, \quad 0 \leq \zeta \leq \Lambda_{\text{QCD}} \)
LF(3+1) \hspace{5cm} AdS_5

\[ \psi(x, \vec{b}_\perp) \leftrightarrow \phi(z) \]

\[ \zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \leftrightarrow z \]

\[ \psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2} \phi(\zeta) \]

**Holography**: Unique mapping derived from equality of LF and AdS formula for current matrix elements