Hadronic Light-Front Wavefunctions from AdS/QCD

Stan Brodsky, SLAC National Accelerator Laboratory

PANICo8 Eilat, Israel

9-14 November, 2008
• Quarks and Gluons: Fundamental constituents of hadrons and nuclei

• *Quantum Chromodynamics (QCD)*

• New Insights from higher space-time dimensions: *AdS/QCD*

• *Light-Front Holography: First Approximation to QCD*

• *Hadronization at the Amplitude Level*

• *Light Front Wavefunctions:* Analogous to Schrödinger wavefunctions of atomic physics

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[
x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}
\]

\[\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\]

Fixed \(\tau = t + z/c\)

\[\sum_i^n x_i = 1\]

\[\sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}\]

Process Independent

Direct Link to QCD Lagrangian!

Invariant under boosts! Independent of \(p^\mu\)
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

$x_i = \frac{k_i^+}{P^+}$

Invariant under boosts. Independent of $P^\mu$

\[ H_{LF}^{QCD}|\psi >= M^2|\psi > \]

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Prediction from AdS/QCD: Meson LFWF

\psi_M(x, k^2_{\perp})

“Soft Wall” model

de Teramond, sjb

Light-Front Wavefunctions from AdS/QCD

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SLAC
A Unified Description of Hadron Structure

\[ \Psi_n(x_i, \vec{k}_\perp, \lambda_i) \]

- Elastic form factors
- B-Decays
- Real Compton scattering at high
- GPDs
- TMDs
- Parton momentum distributions
- Hadronization at the amplitude level
- Deeply Virtual Compton Scattering
- Deeply Virtual Meson production
- Distribution Amplitudes

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\[ \text{November 10, 2008} \]

Light-Front Wavefunctions from AdS/QCD

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Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Event amplitude generator

Off-shell T-matrix
Each element of flash photograph illuminated at same LF time

\[ \tau = t + \frac{z}{c} \]

Evolve in LF time

\[ P^- = i \frac{d}{d\tau} \]

Eigenstate -- independent of \( \tau \)
Deep Inelastic Electron-Proton Scattering

Nonperturbative wavefunction
color confinement
spin, momenta, orbital angular
momentum ....

Gluonic Bremmstrahlung
DGLAP Evolution

Light-Front Quantization:
Rigorous realization of IMF

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Deep Inelastic Electron-Proton Scattering
Dynamic Structure Functions

Nonperturbative wavefunction
color confinement
spin, momenta, orbital angular momentum ....

Final-State Rescattering
Leading-Twist
Sivers Effect, DDIS, Shadowing, Antishadowing
<table>
<thead>
<tr>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Square of Target LFWFs</td>
<td>Modified by Rescattering: ISI &amp; FSI</td>
</tr>
<tr>
<td>• No Wilson Line</td>
<td>Contains Wilson Line, Phases</td>
</tr>
<tr>
<td>• Probability Distributions</td>
<td>No Probabilistic Interpretation</td>
</tr>
<tr>
<td>• Process-Independent (in isolation)</td>
<td>Process-Dependent - From Collision</td>
</tr>
<tr>
<td>• T-even Observables</td>
<td>T-Odd (Sivers, Boer-Mulders, etc.)</td>
</tr>
<tr>
<td>• No Shadowing, Anti-Shadowing</td>
<td>Shadowing, Anti-Shadowing, Saturation</td>
</tr>
<tr>
<td>• Sum Rules: Momentum and $J^z$</td>
<td>Sum Rules Not Proven</td>
</tr>
<tr>
<td>• DGLAP Evolution; mod. at large $x$</td>
<td>DGLAP Evolution</td>
</tr>
<tr>
<td>• No Diffractive DIS</td>
<td>Hard Pomeron and Odderon Diffractive DIS</td>
</tr>
</tbody>
</table>

\[ \psi_n(x_i, \vec{k}_i, \lambda_i) \]
Angular Momentum on the Light-Front

\[ J^z = \sum_{i=1}^{n} s^z_i + \sum_{j=1}^{n-1} l^z_j. \]

\[ l^z_j = -i \left( k^1_j \frac{\partial}{\partial k^2_j} - k^2_j \frac{\partial}{\partial k^1_j} \right) \]

\[ A^+ = 0 \text{ gauge:} \]

No unphysical degrees of freedom

Conserved LF Fock state by Fock State

Nonzero Anomalous Moment requires Nonzero orbital angular momenta

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Light-Front Wavefunctions from AdS/QCD

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Calculation of Form Factors in Equal-Time Theory

**Instant Form**

Calculation of Form Factors in Light-Front Theory

**Front Form**

Need vacuum-induced currents

Absent for $q^+ = 0$
\[ \frac{F_2(q^2)}{2M} = \sum_a \int [dx] [d^2k_\perp] \sum_j e_j \frac{1}{2} \times \]

\[ \left[ -\frac{1}{q_L} \psi_a^\dagger(x_i, k'_\perp, \lambda_i) \psi_a(x_i, k_\perp, \lambda_i) + \frac{1}{q_R} \psi_a^\dagger(x_i, k'_\perp, \lambda_i) \psi_a(x_i, k_\perp, \lambda_i) \right] \]

\[ k'_\perp = k_\perp - x_i q_\perp \]

\[ k'_\perp = k_\perp + (1 - x_j) q_\perp \]

\[ q^2 = -q_\perp^2 \]

\[ q^+ = 0 \]

\[ q_{R,L} = q^x \pm iq^y \]

Must have \( \Delta \ell_z = \pm 1 \) to have nonzero \( F_2(q^2) \)

**Checked to \( O(\alpha^3) \) in QED**

Roskies, Suaya, sjb
**Light-Front Holography**

\[ \phi(z) \]

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

**Light Front Wavefunctions:**

Schrödinger Wavefunctions of Hadron Physics
Applications of AdS/CFT to QCD

in collaboration with Guy de Teramond

Changes in physical length scale mapped to evolution in the 5th dimension z
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

de Teramond, sjb

String Theory

Bottom-Up

Top-Down
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances.

- Analogous to the Schrödinger Theory for Atomic Physics.

- AdS/QCD Light-Front Holography.

- Hadronic Spectra and Light-Front Wavefunctions.
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu\nu}, P^\mu, D, K^\mu, \]

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5
Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (dx^\mu dx_\mu - dz^2)$$

$x^\mu \to \lambda x^\mu$, $z \to \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z$$

$x^2 = x^\mu dx_\mu$ : invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.
We will consider both holographic models

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map \( \text{AdS}_5 \times S_5 \) to conformal \( N=4 \) SUSY

- **QCD is not conformal;** however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** \( \alpha_s(Q^2) \approx \text{const} \) at small \( Q^2 \)

- **Use mathematical mapping of the conformal group** \( \text{SO}(4,2) \) to \( \text{AdS}_5 \) space
Deur, Korsch, et al.

\[ \alpha_{s,gl}/\pi \text{ JLab} \quad \ldots \quad \text{GDH limit} \]

- **Fit**
- **pQCD evol. eq.**

- **Cornwall**
- **Godfrey-Isgur**
- **Bloch et al.**
- **Bhagwat et al.**
- **Maris-Tandy**

- **DSE gluon couplings**
- **Lattice QCD**

\( Q \) (GeV)
Lesson from QED and Lamb Shift:

maximum wavelength of bound quarks and gluons

$$k > \frac{1}{\Lambda_{QCD}}$$

$$\lambda < \Lambda_{QCD}$$

B-Meson

Shrock, sjb

gluon and quark propagators cutoff in IR
because of color confinement
Consequences of Maximum Quark and Gluon Wavelength

- Infrared integrations regulated by confinement
- Infrared fixed point of QCD coupling
  \[ \alpha_s(Q^2) \text{ finite, } \beta \to 0 \text{ at small } Q^2 \]
- Bound state quark and gluon Dyson-Schwinger Equation
- Quark and Gluon Condensates exist within hadrons

Shrock, sbj
AdS/CFT

• Use mapping of conformal group SO(4,2) to AdS5

• Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension
  
  $x^2_{\mu} \rightarrow \lambda^2 x^2_{\mu} \quad z \rightarrow \lambda z$

• Match solutions at small $z$ to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$

• Hard wall model: Confinement at large distances and conformal symmetry in interior

• Truncated space simulates “bag” boundary conditions

  $0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$
Conformal metric: \( ds^2 = g_{\ell m} dx^{\ell} dx^{m}. \ x^{\ell} = (x^{\mu}, z), \ g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}. \)

Action for massive scalar modes on AdS_{d+1}:

\[
S[\Phi] = \frac{1}{2} \int d^{d+1} x \sqrt{g} \frac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.
\]

Equation of motion

\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^{\ell}} \left( \sqrt{g} g^{\ell m} \frac{\partial}{\partial x^{m}} \Phi \right) + \mu^2 \Phi = 0.
\]

Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z), \ P_\mu P^\mu = M^2 \):

\[
\left[ z^2 \partial_z^2 - (d-1)z \partial_z + z^2 M^2 - (\mu R)^2 \right] \Phi(z) = 0. \quad d = 4
\]

Solution: \( \Phi(z) \rightarrow z^\Delta \) as \( z \rightarrow 0 \),

\[
\Phi(z) = Cz^{d/2} J_{\Delta - d/2}(z M), \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right) \quad \text{(Orbital Angular Momentum)}.
\]

\[
(\mu R)^2 = L^2 - 4
\]
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrödinger Equation for bound state of two scalar constituents:**

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = M^2 \phi(z)$$

$L$: orbital angular momentum

**Derived from variation of Action in AdS$_5$**

**Hard wall model: truncated space**

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$
**Match fall-off at small $z$ to conformal twist-dimension at short distances**}

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 \{D_{\ell_1} \ldots D_{\ell_m}\} \psi \quad (\Phi_{\mu} = 0 \text{ gauge}). \quad \Delta = 2 + L$

- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $M_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

\[ \Delta = 2 + L \]

\[ S = 0 \quad \text{Meson orbital and radial AdS modes for} \ \Lambda_{QCD} = 0.32 \text{ GeV.} \]
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
AdS Schrodinger Equation for bound state of two scalar constituents:

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = M^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Derived from variation of Action Dilaton-Modified AdS$_5$
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Soft Wall Model

Pion mass automatically zero!

$m_q = 0$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \right] \phi_S(z) = M^2 \phi_S(z) \]

with eigenvalues \( M^2 = 2\kappa^2 (2n + 2L + S) \).

- Compare with Nambu string result (rotating flux tube):

\[
M_n^2(L) = 2\pi\sigma (n + L + 1/2) .
\]

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).

Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \) GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).
$\alpha(t) \approx \frac{1}{2} + 0.9t$

**AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories**
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = zQ K_1(zQ) \]

\[ F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

High \( Q^2 \)
from
small \( z \sim 1/Q \)

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi^{(n)} \) scales as \( z^{\Delta_n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1} \]

where \( \tau = \Delta_n - \sigma_n, \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).

Polchinski, Strassler
de Teramond, sjb

Dimensional Quark Counting Rule
General result from AdS/CFT
• Phenomenological success of dimensional scaling laws for exclusive processes

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^{n-2}}, \quad n = n_A + n_B + n_C + n_D, \]

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev et al. (1973).

• Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space
