Light-Front Quantization Approach to the Gauge/Gravity Correspondence and Applications to the Light Hadron Spectrum

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Niccolò Cabeo 2012

Hadronic Spectroscopy

Ferrara, May 21 - 26, 2011

The objective of these lectures is to describe recent analytical insights into the nonperturbative nature of the strong coupled dynamics of light-hadron bound states. The holographic approach described in the lectures is at the confluence of phenomenology, light-front physics and the gauge/gravity correspondence. Important dynamical properties of hadrons such as their light-front wavefunctions, form factors and the systematics of their excitation spectrum are well described in this framework.
1 General Introduction

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   Quantum Chromodynamics
   Lattice QCD
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   Pion Transition Form Factor
   Higher Fock Components in LF Holographic QCD

Conclusions
1 Introduction

Internal Structure of the Proton

- High Energy (20 GeV) scattering at SLAC (1969) revealed the internal structure of the proton

- Deep inelastic scattering experiments (1967-1973): Bjorken and Feynman partons identified with Gell-Mann and Zweig quarks

- Quarks were not just hypothetical mathematical entities but the true building blocks of hadrons
Quantum Chromodynamics (QCD)

- Quarks should have an additional quantum number “color”
  \[ \psi(x)_i, \quad i = R, G, B \]

- QCD Lagrangian follows from the gauge invariance of the theory
  \[ \psi(x) \rightarrow e^{i\alpha^a(x)T^a} \psi(x), \quad \left[T^a, T^b\right] = if_{abc}T^c \]
  \[ (T^a)_{ij}, \quad i, j = 1, 2, 3, \quad a, b = 1, 2, \ldots 8 \]

- Find QCD Lagrangian
  \[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr} (G^{\mu\nu} G_{\mu\nu}) + i\overline{\psi} D_\mu \gamma^\mu \psi + m\overline{\psi} \psi \]
  \[ \text{where } D_\mu = \partial_\mu - igT^a A^a_\mu, \quad G^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f_{abc} A^b_\mu A^c_\nu \]

- Quarks and gluons interactions from color charge, but ... gluons also interact with each other: strongly coupled non-abelian gauge theory → confinement!

- But how could the actual existence of gluons be demonstrated experimentally?
• In the mid 1970s QCD was referred as the “candidate” theory of the strong interactions

• Asymptotically free QFT which can describe scaling

• First hint of gluons from deep-inelastic scattering experiments:
  only half of a proton’s momentum is carried by the quarks

• PETRA storage ring at DESY (1979):
  first direct experimental proof of the existence of the gluon

• QCD becomes fundamental theory of quarks and gluons:
  interactions of quarks and gluons at high energies is well described by QCD

• Most challenging problem of strong interaction dynamics: determine
  the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom

• “Enormous complexity out of a very simple Lagrangian” (H. Fritsch)
Lattice QCD

- Lattice numerical simulations at the petaflop/sec scale (resolution $\sim L/a$)
- Sums over quark paths with billions of dimensions
- Dynamical properties in Minkowski space-time not amenable to Euclidean lattice computations
- Computational complexity of hadronic excitation spectrum beyond ground state configuration
**General Theory of Relativity to the Rescue?**

- Space curvature determined by the mass-energy present following Einstein’s equations

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}
\]

- Matter curves space and space determines how matter moves
- Perhaps, but in a holographic sense: duality between theories in different number of space-time dimensions!
Gauge/Gravity Correspondence and QCD

- Recent developments inspired by the AdS/CFT correspondence [Maldacena (1998)] between gravity in AdS space and conformal field theories in physical space-time provide physical insights into the non-perturbative dynamics of QCD

- Description of strongly coupled gauge theory using a dual gravity description in a higher dimensional space (holographic)

- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

  Isometry group: most general group of transformations which leave invariant the distance between two points $E_j$: $S^N \sim O(N+1)$

  Dimension of isometry group of AdS$_{d+1}$ is $\frac{(d+1)(d+2)}{2}$

- Mapping of AdS gravity to QCD quantized at fixed light-front time gives a precise relation between wave functions in AdS space and the LF wavefunctions describing the internal structure of hadrons
• AdS$_5$ metric:

\[
\frac{ds^2}{L_{\text{AdS}}} = \frac{R^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right)_{L_{\text{Minkowski}}}
\]

• A distance $L_{\text{AdS}}$ shrinks by a warp factor $z/R$ as observed in Minkowski space ($dz = 0$):

\[
L_{\text{Minkowski}} \sim \frac{z}{R} L_{\text{AdS}}
\]

• Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, the variable $z$ acts like a scaling variable in Minkowski space

• Short distances $x_\mu x^\mu \rightarrow 0$ maps to UV conformal AdS$_5$ boundary $z \rightarrow 0$

• Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda_{\text{QCD}}^2$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\text{QCD}}$, thus there is a maximum separation of quarks and a maximum value of $z$

• Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS
2 Light Front Dynamics

• Different possibilities to parametrize space-time [Dirac (1949)]

• Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results

• Forms of Relativistic Dynamics: dynamical vs. kinematical generators [Dirac (1949)]

• *Instant form*: hypersurface defined by $t = 0$, the familiar one

\[ H, K \text{ dynamical, } L, P \text{ kinematical} \]

• *Point form*: hypersurface is an hyperboloid

\[ P^\mu \text{ dynamical, } M^{\mu\nu} \text{ kinematical} \]

• *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

\[ P^-, L^x, L^y \text{ dynamical, } P^+, P_\perp, L^z, K \text{ kinematical} \]

\[ P^\pm = P^0 \pm P^3 \]
• LF coordinates

\[ x^+ = x^0 + x^3 \]  
light-front time

\[ x^- = x^0 - x^3 \]  
longitudinal space variable

\[ \mathbf{x}_\perp = (x^1, x^2) \]  
transverse space variable

\[ P^+ = P^0 + P^3 \]  
longitudinal momentum  
\( (P^+ > 0) \)

\[ P^- = P^0 - P^3 \]  
light-front energy

\[ \mathbf{P}_\perp = (P^1, P^2) \]  
transverse momentum

• Compute \( P \cdot x \) to identify Hamiltonian as conjugate to LF time \( x^+ \)

\[ P_\mu x^\mu = \frac{1}{2} \left( P^+ x^- + P^- x^+ \right) - \mathbf{P}_\perp \cdot \mathbf{x}_\perp \]

• On shell relation \( P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 = M^2 \) leads to dispersion relation for LF Hamiltonian \( P^- \)

\[ P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+} \]
• LF quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of partonic content of a hadron → LFWFs

• Calculation of matrix elements $\langle P + q | J | P \rangle$ requires boosting the relativistic hadronic bound state from $|P\rangle$ to $|P + q\rangle$: extremely complicated in the instant form but boosts are trivial in the LF

• No coupling to vacuum-induced currents or off-diagonal contributions in the LF

• Form factors in LF expressed as convolution of frame-independent LFWFs (Drell-Yan-West formula)

• Mapping to AdS transition amplitudes possible (Polchinski-Strassler formula: overlap of AdS WFs)

• Hamiltonian equation for bound states similar structure of AdS equations: direct connection of QCD and AdS/CFT possible

Image credit: M. Vanderhaeghen
Light-Front Quantization of QCD

• Express the hadron four-momentum generator $P = (P^+, P^-, P_\perp)$ in terms of dynamical fields

\[
P^- = \frac{1}{2} \int dx^- d^2x_\perp \bar{\psi}_+ \gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+} \psi + \text{(interactions)},
\]

\[
P^+ = \int dx^- d^2x_\perp \bar{\psi}_+ \gamma^+ i\partial^+ \psi_+,
\]

\[
P_\perp = \frac{1}{2} \int dx^- d^2x_\perp \bar{\psi}_+ \gamma^+ i\nabla_\perp \psi_+,
\]

where the integrals are over the null plane $\tau = x^+ = x^0 + x^3$

• LF Hamiltonian $P^-$ generates LF time translations and the generators $P^+$ and $P_\perp$ are kinematical

• Hamiltonian equation for the relativistic bound state

\[
P^- |\psi(P)\rangle = \frac{\mathcal{M}^2 + P_\perp^2}{P^+} |\psi(P)\rangle
\]

• Construct LF Lorentz invariant Hamiltonian $H_{LF} \equiv P^2 = P^- P^+ - P_\perp^2$

\[
H_{LF} |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle
\]
Light-Front Fock Representation

• Dirac field $\psi$, expanded in terms of ladder operators on the initial surface

$$P^- = \sum_\lambda \int \frac{dq^+d^2q_\perp}{(2\pi)^3} \left( \frac{q_\perp^2 + m^2}{q^+} \right) b_\lambda^\dagger(q)b_\lambda(q) + \text{interactions}$$

• LF Lorentz invariant Hamiltonian equation for the relativistic bound state

$$P_\mu P^\mu |\psi(P)\rangle = M^2 |\psi(P)\rangle$$

• State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \quad |n\rangle = \{ |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \cdots \}$$

with $k_i^2 = m_i^2$, $k_i = (k_i^+, k_i^-, k_{\perp i})$, for each constituent $i$ in state $n$

• Fock components $\psi_n(x_i, k_{\perp i}, \lambda_i^z)$ independent of $P^+$ and $P_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+ / P^+$, transverse momentum $k_{\perp i}$ and spin $\lambda_i^z$

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n k_{\perp i} = 0.$$
Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

• Compute $\mathcal{M}^2$ from hadronic matrix element
  \[
  \langle \psi(P') | P_\mu P^\mu | \psi(P) \rangle = \mathcal{M}^2 \langle \psi(P') | \psi(P) \rangle
  \]

• Find
  \[
  \mathcal{M}^2 = \sum_n \int [dx_i] [d^2k_{\perp i}] \sum_q \left( \frac{k^2_{\perp q} + m_q^2}{x_q} \right) |\psi_n(x_i, k_{\perp i})|^2 + \text{interactions}
  \]

• LFWF $\psi_n$ represents a bound state which is off the LF energy shell $\mathcal{M}^2 - \mathcal{M}_n^2$

  \[
  \mathcal{M}^2_n = \left( \sum_{i=1}^{n} k_i^\mu \right)^2 = \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i}
  \]

  with $k_a^2 = m_a^2$ for each constituent

• Invariant mass $\mathcal{M}_n^2$ key variable which controls the bound state

• Semiclassical approximation to QCD:

  \[
  \psi_n(k_1, k_2, \ldots, k_n) \rightarrow \phi_n \left( \frac{(k_1 + k_2 + \cdots + k_n)^2}{\mathcal{M}_n^2} \right), \quad m_q \rightarrow 0
  \]
• In terms of $n - 1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n - 1$,

$$
\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2 \mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_q \left( \frac{-\nabla^2 b_{\perp q} + m_q^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}
$$

• Relevant variable conjugate to invariant mass $\mathcal{M}_n^2$ (Cluster decomposition)

$$
\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|
$$

the $x$-weighted transverse impact coordinate of the spectator system ($x$ active quark)

• For a two-parton system $\zeta^2 = x(1-x) \mathbf{b}_{\perp}^2$

• To first approximation LF dynamics depend only on the invariant variable $\zeta$, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$
\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}
$$

factoring angular $\varphi$, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$ ($P^+, P_{\perp}$ and $J_z$ commute with $P^-$)
• Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple ($L = L^z$)

\[ \mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \]

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

• LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for $\phi$

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} \right) + \frac{U(\zeta)}{\zeta^2} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

• Effective relativistic and frame-independent LF Schrödinger equation: $U$ is instantaneous in LF time

• Eigenmodes $\phi(\zeta)$ represent the probability amplitude to find $n$-massless partons at transverse impact separation $\zeta$ within the hadron at equal LF time

• The $SO(2)$ Casimir $L^2$ corresponds to group of rotations in transverse LF plane
  Casimir operator for $SO(N)$ is $L(L + N - 2)$

• Semiclassical approximation to LF QCD does not account for particle creation and absorption
Meson Spectrum in Hard Wall Model

[LF Hard wall model: GdT and S. J. Brodsky, PRL 94, 201601 (2005)]

- Conformal model up to the confinement scale $1/\Lambda_{QCD}$ [Polchinski and Strassler (2002)]

\[
U(\zeta) = \begin{cases} 
0 & \text{if } \zeta \leq \frac{1}{\Lambda_{QCD}} \\
\infty & \text{if } \zeta > \frac{1}{\Lambda_{QCD}}
\end{cases}
\]

- Confinement scale $\frac{1}{\Lambda_{QCD}} \sim 1$ Fm, $\Lambda_{QCD} \sim 200$ MeV

- Covariant version of MIT bag model: quarks permanently confined inside a finite region of space

- Normalized eigenfunctions

\[
\langle \phi | \phi \rangle = \int_{0}^{\Lambda_{QCD}^{-1}} d\zeta \phi^2(z) = 1
\]

\[
\phi_{L,k}(\zeta) = \sqrt{2\Lambda_{QCD}} \frac{J_{1+L}(\beta_{L,k})}{J_{1+L}(\beta_{L,k})} \sqrt{\zeta} J_L(\zeta \beta_{L,k} \Lambda_{QCD})
\]

- Eigenvalues

\[
\mathcal{M}_{L,k} = \beta_{L,k} \Lambda_{QCD}
\]
Table 1: $I = 1$ mesons. For a $q\bar{q}$ state $P = (-1)^{L+1}, C = (-1)^{L+S}$

<table>
<thead>
<tr>
<th>$L$</th>
<th>$S$</th>
<th>$n$</th>
<th>$J^{PC}$</th>
<th>$I = 1$ Meson</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$0^{-+}$</td>
<td>$\pi(140)$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$0^{-+}$</td>
<td>$\pi(1300)$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>$0^{-+}$</td>
<td>$\pi(1800)$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$1^{--}$</td>
<td>$\rho(770)$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$1^{--}$</td>
<td>$\rho(1450)$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>$1^{--}$</td>
<td>$\rho(1700)$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$1^{+-}$</td>
<td>$b_1(1235)$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$0^{++}$</td>
<td>$a_0(980)$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$0^{++}$</td>
<td>$a_0(1450)$</td>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$1^{++}$</td>
<td>$a_1(1260)$</td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>$2^{++}$</td>
<td>$a_2(1320)$</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>$2^{-+}$</td>
<td>$\pi_2(1670)$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>$2^{-+}$</td>
<td>$\pi_2(1880)$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>$3^{--}$</td>
<td>$\rho_3(1690)$</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>$4^{++}$</td>
<td>$a_4(2040)$</td>
</tr>
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</table>
Orbital and radial excitations for the $\pi$ and the $\rho$ $I=1$ meson families ($\Lambda_{QCD} = 0.32$ GeV)

- Pion is not chiral
- $\mathcal{M} \sim 2n + L$ in contrast to usual Regge dependence $\mathcal{M}^2 \sim n + L$
- Important $J - L$ splitting (different $J$ for same $L$) in mesons not described by hard-wall model
- Radial modes not well described in hard-wall model
3 Light-Front Holographic Mapping

Higher Spin Wave Equations in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)
- Spin-$J$ in AdS represented by totally symmetric rank $J$ tensor field $\Phi_{M_1...M_J}$
- Action for spin-$J$ field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ \( x^M = (x^\mu, z) \)

\[
S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \left( g^{MN} g^{M_1M_1'} \cdots g^{M_JM_J'} D_M \Phi_{M_1...M_J} D_N \Phi_{M_1'...M_J'} - \mu^2 g^{M_1M_1'} \cdots g^{M_JM_J'} \Phi_{M_1...M_J} \Phi_{M_1'...M_J'} + \cdots \right)
\]

where $D_M$ is the covariant derivative which includes parallel transport (affine connection)

\[
D_M \Phi_{M_1...M_J} = \partial_M \Phi_{M_1...M_J} - \Gamma^K_{MM_1} \Phi_{K...M_J} - \cdots - \Gamma^K_{MM_J} \Phi_{M_1...K}
\]

- Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates

\[
\Phi_P(x, z)_{\mu_1...\mu_J} = e^{-i P \cdot x} \Phi(z)_{\mu_1...\mu_J}, \quad \Phi_{z\mu_2...\mu_J} = \cdots = \Phi_{\mu_1\mu_2...z} = 0
\]

with four-momentum $P_\mu$ and invariant hadronic mass $P_\mu P^\mu = M^2$
• Construct effective action in terms of spin-\(J\) modes \(\Phi_J\) with only physical degrees of freedom

[H. G. Dosch, S. J. Brodsky and GdT]

• Introduce fields with Lorentz (tangent) indices

\[
g_{MN}(x) = \eta_{AB} e^A_M(x) e^B_N(x)
\]

\[
\hat{\Phi}_{A_1 A_2 \cdots A_J} = e^{M_1}_{A_1} e^{M_2}_{A_2} \cdots e^{M_J}_{A_J} \Phi_{M_1 M_2 \cdots M_J} = \left(\frac{z}{R}\right)^J \Phi_{A_1 A_2 \cdots A_J}
\]

where \(M, N = 1, \cdots, d + 1\) curved space indices, \(A, B = 1, \cdots, d + 1\) tangent indices

• Find effective action for the Lorentz spin-\(J\) mode \(\hat{\Phi}_J = \hat{\Phi}_{\mu_1 \cdots \mu_J}\)

\[
S = \frac{1}{2} \int d^d x \, d z \sqrt{g} e^\varphi(z) \left( g^{NN'} \partial_N \hat{\Phi}_{J} \partial_{N'} \hat{\Phi}_{J} - \mu^2 \hat{\Phi}_{J} \hat{\Phi}_{J}\right)
\]

upon \(\mu\)-rescaling

• Variation of the action gives AdS wave equation for spin-\(J\) mode \(\Phi_J = \Phi_{\mu_1 \cdots \mu_J}\)

\[
\left[-\frac{z^{d-1-2J}}{e^\varphi(z)} \partial_z \left( \frac{e^\varphi(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)
\]

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 85, 076003 (2012)]
Dual QCD Light-Front Wave Equation

[GrT and S. J. Brodsky, PRL 102, 081601 (2009)]

- Upon substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2} + J e^{\varphi(z)/2} \Phi_J(\zeta)$ in AdS WE

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \frac{\partial}{\partial z} \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = M^2 \Phi_J(z)$$

find LFWE ($d = 4$)

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \varphi''(z) + \frac{1}{4} \varphi'(z)^2 + \frac{2J - 3}{2z} \varphi'(z)$$

and $(\mu R)^2 = -(2 - J)^2 + L^2$

- LF Schrödinger equation from AdS$_5$ mapping to physical 3+1 Minkowski space at fixed LF time $x^+$
- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension $\tau$ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition
Meson Spectrum in Soft Wall Model

- Linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]
- Dilaton profile $\varphi(z) = +\kappa^2 z^2$
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)$
- LF WE

\[
\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)\right) \phi_J(\zeta) = M_J^2 \phi_J(\zeta)
\]

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z) = 1$

\[
\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^{L}(\kappa^2 \zeta^2)
\]

- Eigenvalues

\[
\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J + L}{2}\right)
\]
LFWFs $\phi_{n,L}(\zeta)$ in physical space-time: (L) orbital modes and (R) radial modes
• $J = L + S$, $I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Orbital and radial excitations for the $\pi$ ($\kappa = 0.59$ GeV) and the $\rho$ $l=1$ meson families ($\kappa = 0.54$ GeV)

• Triplet splitting for the $L = 1$, $J = 0, 1, 2$, $I = 1$ vector meson $a$-states

$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$

• $J - L$ splitting in mesons and radial excitations are well described in soft-wall model

Fermionic Modes in AdS Space and Baryon Spectrum

• Lattice calculations of the ground state hadron masses agree very well with experimental values.

• However, excitation spectrum of nucleon represents an important challenge to LQCD due to enormous computational complexity beyond ground state configuration and multi-hadron thresholds.

• Large basis of interpolating operators required in LQCD since excited nucleon states are classified according to irreducible representations of the lattice, not the angular momentum.

• The gauge/gravity duality can give important insights into the strongly coupled dynamics of nucleons using simple analytical methods.

• Analytical exploration of systematics of light-baryon resonances and nucleon form factors.

• Extension of holographic ideas to spin-$\frac{1}{2}$ (and higher half-integral $J$) hadrons by considering propagation of RS spinor field $\Psi_{\alpha M_1 \ldots M_{J-1}/2}$ in AdS space.
Higher Spin Wave Equations in AdS Space

- For fermion fields in AdS one cannot break conformality with introduction of dilaton background since it can be scaled away leaving the action conformally invariant [I. Kirsch (2006)]

- Introduce an effective confining potential \( V(z) \) in the action for a Dirac field in AdS \( d+1 \)

\[
S_F = \int d^d x \, dz \sqrt{g} g^{M_1M'_1} \cdots g^{M_TM'_T} (\overline{\Psi}_{M_1\ldots M_T} (ie^M_A \Gamma^A D_M - \mu - V(z)) \Psi_{M'_1\ldots M'_T} + \cdots)
\]

where \( D_M \) is the covariant derivative of the spinor field \( \Psi_{\alpha M_1\ldots M_T} \), \( T = J - \frac{1}{2} \)

\[
D_M \Psi_{M_1\ldots M_T} = \partial_M \Psi_{M_1\ldots M_T} - \frac{i}{2} \omega_M^{AB} \Sigma_{AB} \Psi_{M_1\ldots M_T} - \Gamma^K_{MM_1} \Psi_{K\ldots M_T} - \cdots - \Gamma^K_{MM_M} \Psi_{M_1\ldots K}
\]

- \( M, N = 1, \cdots, d+1 \) curved space indices, \( A, B = 1, \cdots, d+1 \) tangent indices

- \( e^M_A \) is the vielbein, \( w_M^{AB} \) spin connection, \( \Sigma_{AB} \) generators of the Lorentz group, \( \Sigma_{AB} = \frac{i}{4} [\Gamma_A, \Gamma_B] \)

- \( \Gamma^A \) tangent space Dirac matrices \( \{ \Gamma^A, \Gamma^B \} = \eta^{AB} \)

- For \( d \) even we choose \( \Gamma_A = (\Gamma_\mu, \Gamma_z) \) with \( \Gamma_z = -\Gamma^z = \Gamma_0 \Gamma_1 \cdots \Gamma_{d-1} \)

- For \( d = 4 \): \( \Gamma_A = (\gamma_\mu, -i\gamma_5) \)
• Physical hadron has plane-wave, spinors, and polarization along $3+1$ physical coordinates

$$\Psi_P(x, z)_{\mu_1\cdots\mu_T} = e^{-iP \cdot x} \Psi(z)_{\mu_1\cdots\mu_T}, \quad \Psi_{z\mu_2\cdots\mu_T} = \cdots = \Psi_{\mu_1\mu_2\cdots z} = 0$$

with four-momentum $P_\mu$ and invariant hadronic mass $P_\mu P^\mu = M^2$

• Construct effective action in terms of spin-$J$ modes $\hat{\Psi}_J$ with only physical degrees of freedom

[H. G. Dosch, S. J. Brodsky and GdT]

• Introduce fields with Lorentz indices \((T = J - \frac{1}{2})\)

$$\hat{\Psi} A_1 A_2 \cdots A_T = e_{A_1}^M e_{A_2}^M \cdots e_{A_T}^M \Psi_{M_1 M_2 \cdots M_T} = \left( \frac{z}{R} \right)^T \Psi_{A_1 A_2 \cdots A_T}$$

• Find effective action for the Lorentz spin-$J$ mode $\hat{\Psi}_J = \hat{\Phi}_{\mu_1\cdots\mu_{J-1/2}}$

$$S_F = \int d^d x \, d z \sqrt{g} \left( \bar{\hat{\Psi}}_J \left( i z \eta^{MN} \Gamma_M \partial_N + \frac{i}{2} \Gamma_z - \mu R - RV(z) \right) \hat{\Psi}_J \right)$$

• Variation of the action gives AdS wave equation for spin-$J$ mode $\Phi_J = \Phi_{\mu_1\cdots\mu_{J-1/2}}$

$$\left[ i \left( z \eta^{MN} \Gamma_M \partial_N + \frac{d}{2} \Gamma_z \right) - \mu R - RV(z) \right] \Psi_J = 0$$

upon $\mu$-rescaling

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 85, 076003 (2012)]
Baryon Spectrum in Soft-Wall Model

- Upon substitution $z \rightarrow \zeta$ and
  \[
  \Psi_J(x, z) = e^{-iP \cdot x} z^2 \psi^J(z) u(P),
  \]
  find LFWE for $d = 4$
  \[
  \frac{d}{d\zeta} \psi^J_+ + \frac{\nu + \frac{1}{2}}{\zeta} \psi^J_+ + U(\zeta) \psi^J_+ = \mathcal{M} \psi^J_+,
  \]
  \[
  -\frac{d}{d\zeta} \psi^J_- + \frac{\nu + \frac{1}{2}}{\zeta} \psi^J_- + U(\zeta) \psi^J_- = \mathcal{M} \psi^J_-,
  \]
  where $U(\zeta) = \frac{R}{\zeta} V(\zeta)$

- Choose linear potential $U = \kappa^2 \zeta$

- Eigenfunctions
  \[
  \psi^J_+(\zeta) \sim \zeta^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2/2} L^n(\kappa^2 \zeta^2), \quad \psi^J_-(\zeta) \sim \zeta^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2/2} L^{n+1}(\kappa^2 \zeta^2)
  \]

- Eigenvalues
  \[
  \mathcal{M}^2 = 4\kappa^2(n + \nu + 1), \quad \nu = L + 1 \quad (\tau = 3)
  \]

- Full $J - L$ degeneracy (different $J$ for same $L$) for baryons along given trajectory!
<table>
<thead>
<tr>
<th>$L$</th>
<th>$S$</th>
<th>$n$</th>
<th>Baryon State</th>
</tr>
</thead>
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<tr>
<td>0</td>
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</tr>
<tr>
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<td>$\frac{1}{2}$</td>
<td>1</td>
<td>$N_{\frac{1}{2}}^1$ (1440)</td>
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<tr>
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<td>2</td>
<td>$N_{\frac{1}{2}}^1$ (1710)</td>
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<tr>
<td>0</td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$\Delta_{\frac{3}{2}}^3$ (1232)</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{3}{2}$</td>
<td>1</td>
<td>$\Delta_{\frac{3}{2}}^3$ (1600)</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$N_{\frac{1}{2}}^{-1}$ (1535) $N_{\frac{3}{2}}^{-1}$ (1520)</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$N_{\frac{1}{2}}^{-1}$ (1650) $N_{\frac{3}{2}}^{-1}$ (1700) $N_{\frac{5}{2}}^{-1}$ (1675)</td>
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<tr>
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<td>0</td>
<td>$\Delta_{\frac{1}{2}}^3$ (1620) $\Delta_{\frac{3}{2}}^3$ (1700)</td>
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<tr>
<td>2</td>
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<td>$N_{\frac{5}{2}}^5$ (1900)</td>
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<tr>
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<td>$\Delta_{\frac{1}{2}}^3$ (1910) $\Delta_{\frac{3}{2}}^3$ (1920) $\Delta_{\frac{5}{2}}^3$ (1905) $\Delta_{\frac{7}{2}}^3$ (1950)</td>
</tr>
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<tr>
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</tr>
<tr>
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<td>$N_{\frac{9}{2}}^9$ $N_{\frac{11}{2}}^9$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$N_{\frac{7}{2}}^9$ $N_{\frac{9}{2}}^9$ $N_{\frac{11}{2}}^9$ (2600) $N_{\frac{13}{2}}^9$</td>
</tr>
</tbody>
</table>
• Gap scale $4\kappa^2$ determines trajectory slope and spectrum gap between plus-parity spin-$\frac{1}{2}$ and minus-parity spin-$\frac{3}{2}$ nucleon families!

• No $J - L$ splitting!
• Fix the energy scale to the proton mass for the lowest state $n = 0, L = 0$

• Subtraction to mass scale may be understood as displacement required to describe nucleons with $N_C = 3$ as composite system with twist $3 + L$ instead of a quark-squark bound state with twist $2 + L$

• Phenomenological rules for increase in mass $\mathcal{M}^2$ to construct full baryon spectrum from proton state

\[
\begin{align*}
4\kappa^2 & \quad \text{for } \Delta n = 1 \\
4\kappa^2 & \quad \text{for } \Delta L = 1 \\
2\kappa^2 & \quad \text{for } \Delta S = 1 \\
2\kappa^2 & \quad \text{for } \Delta P = \pm 
\end{align*}
\]

• Eigenvalues

\[
\begin{align*}
\mathcal{M}^{2(+)}_{n,L,S} &= 4\kappa^2 \left( n + L + S/2 + 3/4 \right) \\
\mathcal{M}^{2(-)}_{n,L,S} &= 4\kappa^2 \left( n + L + S/2 + 5/4 \right)
\end{align*}
\]
Orbital and radial excitations for positive parity $N$ and $\Delta$ baryon families ($\kappa = 0.49 - 0.51$ GeV)

[See also: H. Forkel, M. Beyer and T. Frederico, JHEP 0707, 077 (2007)]
Baryon orbital trajectories for $n = 0$ and $\kappa = 0.49 - 0.51$ GeV

- $\Delta(1930)$ quantum number assignment (E. Klempt and J. M. Richard (2010): $S = 3/2$, $L = 1$, $n = 1$

- Find $M_{\Delta(1930)} = 4\kappa \simeq 2$ GeV compared with experimental value 1.96 GeV
4 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL 96, 201601 (2006)] Mapping of EM currents
[S. J. Brodsky and GdT, PRD 78, 025032 (2008)] Mapping of energy-momentum tensor

• EM transition matrix element in QCD: local coupling to pointlike constituents

\[ \langle P' | J^\mu | P \rangle = (P + P')^\mu F(Q^2) \]

where \( Q = P' - P \) and \( J^\mu = e_q q q_\gamma^\mu q \)

• EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode \( \Phi(x, z) \)

\[ \int d^4x \, dz \, \sqrt{g} \, A^M(x, z) \Phi^*_P(x, z) \partial_\mu \Phi_P(x, z) \]

\[ \sim (2\pi)^4 \delta^4(P' - P) \epsilon_\mu (P + P')^\mu F(Q^2) \]

• How to recover hard pointlike scattering at large \( Q \) out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

• Mapping of \( J^+ \) elements at fixed light-front time: \( \Phi_P(z) \leftrightarrow |\psi(P)\rangle \)
• Compare with electromagnetic FF in LF QCD for arbitrary $Q$. Expressions can be matched only if LFWF is factorized

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

• Find

$$X(x) = \sqrt{x(1-x)}, \quad \phi(\zeta) = \left(\frac{\zeta}{R}\right)^{-3/2} \Phi(\zeta), \quad z \rightarrow \zeta$$

• Form factor in soft-wall model expressed as $\tau - 1$ product of poles along vector radial trajectory (twist $\tau = N + L$) [Brodsky and GdT, Phys.Rev. D77 (2008) 056007]

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M^2_\rho}\right) \left(1 + \frac{Q^2}{M^2_{\rho'}}\right) \cdots \left(1 + \frac{Q^2}{M^2_{\rho^{\tau-2}}}\right)}$$

• Analytical form $F(Q^2)$ incorporates correct scaling from constituents and mass gap from confinement

• $M_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$ since VM is twist-2 $q\bar{q}$ and not twist 3 squark-squark with $L = 1$

- **Nucleon EM form factor**
  
  \[
  \langle P' | J^\mu(0) | P \rangle = u(P') \left[ \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q^\nu}{2M} F_2(q^2) \right] u(P)
  \]

- **EM hadronic matrix element in AdS space from non-local coupling of external EM field in AdS with fermionic mode** \(\Psi_P(x, z)\)

  \[
  \int d^4x \, dz \, \sqrt{g} \, \overline{\Psi}_{P'}(x, z) \, e_M^A A^M_A(x, z) \Psi_P(x, z)
  \sim (2\pi)^4 \delta^4 (P' - P - q) \epsilon_\mu u(P') \gamma^\mu F_1(q^2) u(P)
  \]

- **Effective AdS/QCD model: additional term in the 5-dim action**
  

  \[
  \int d^4x \, dz \, \sqrt{g} \, \overline{\Psi} \, e_M^A e_N^B [\Gamma_A, \Gamma_B] \, F^{MN} \Psi
  \sim (2\pi)^4 \delta^4 (P' - P - q) \epsilon_\mu u(P') \frac{i\sigma^{\mu\nu} q^\nu}{2M} F_2(q^2) u(P)
  \]

- **Generalized Parton Distributions in AdS/QCD**
  
  [Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]
Using $SU(6)$ flavor symmetry and normalization to static quantities
Nucleon Transition Form Factors

\[ F_{1 \, N \rightarrow N^*}(Q^2) = \frac{\sqrt{2}}{3} \frac{Q^2}{M^2_{\rho}} \left( 1 + \frac{Q^2}{M^2_{\rho}} \right) \left( 1 + \frac{Q^2}{M^2_{\rho'}} \right) \left( 1 + \frac{Q^2}{M^2_{\rho''}} \right). \]

Proton transition form factor to the first radial excited state. Data from JLab
Flavor Decomposition of Elastic Nucleon Form Factors


- Proton SU(6) WF: \( F_{u,1}^p = \frac{5}{3} G_+ + \frac{1}{3} G_- \), \( F_{d,1}^p = \frac{1}{3} G_+ + \frac{2}{3} G_- \)
- Neutron SU(6) WF: \( F_{u,1}^n = \frac{1}{3} G_+ + \frac{2}{3} G_- \), \( F_{d,1}^n = \frac{5}{3} G_+ + \frac{1}{3} G_- \)

\[
G_+(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{\rho}^2}\right)\left(1 + \frac{Q^2}{M_{\rho'}^2}\right)}
\]

and

\[
G_-(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{\rho}^2}\right)\left(1 + \frac{Q^2}{M_{\rho'}^2}\right)\left(1 + \frac{Q^2}{M_{\rho''}^2}\right)}
\]
Pion Transition Form-Factor


- Definition of $\pi - \gamma$ TFF from $\gamma^* \pi^0 \rightarrow \gamma$ vertex in the amplitude $e\pi \rightarrow e\gamma$
  \[ \Gamma^\mu = -ie^2 F_{\pi\gamma}(q^2)\epsilon_{\mu\nu\rho\sigma}(p_\pi)\nu\epsilon_\rho(k)q_\sigma, \quad k^2 = 0 \]

- Asymptotic value of pion TFF is determined by first principles in QCD:
  \[ Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2f_\pi \quad \text{[Lepage and Brodsky (1980)]} \]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]
  \[ \int d^4x \int dz \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q \]
  \[ \sim (2\pi)^4 \delta^{(4)}(p_\pi + q - k) F_{\pi\gamma}(q^2)\epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q)(p_\pi)\nu\epsilon_\rho(k)q_\sigma \]

- Find for $A_z \propto \Phi_\pi(z)/z$
  \[ F_{\pi\gamma}(Q^2) = \frac{1}{2\pi} \int_0^\infty \frac{dz}{z} \Phi_\pi(z)V(Q^2, z) \]
  with normalization fixed by asymptotic QCD prediction

- $V(Q^2, z)$ bulk-to-boundary propagator of $\gamma^*$
Higher Fock Components in LF Holographic QCD

- Effective interaction leads to \( qq \rightarrow qq, q\bar{q} \rightarrow q\bar{q} \) but also to \( q \rightarrow qq\bar{q} \) and \( \bar{q} \rightarrow \bar{q}qq \).

- Higher Fock states can have any number of extra \( q\bar{q} \) pairs, but surprisingly no dynamical gluons.

- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale:

\[
|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{qqq\bar{q}}|qqq\bar{q}\rangle_{\tau=4} + \cdots
\]

- Modify form factor formula introducing finite width: \( q^2 \rightarrow q^2 + \sqrt{2iM\Gamma} \) \( (P_{q\bar{q}qq} = 13\%) \)
5 Conclusions

- The gauge/gravity duality leads to a simple analytical frame-independent nonperturbative semiclassical approximation to the light-front Hamiltonian problem for QCD: “Light-Front Holography”

- Unlike usual instant-time quantization the Hamiltonian equation in the light-front is frame independent and has a structure similar to eigenmode equations in AdS

- AdS transition matrix elements (overlap of AdS wave functions) map to current matrix elements in LF QCD (convolution of frame-independent light-front wave functions)

- Mapping of AdS gravity to boundary QFT quantized at fixed light-front time gives a precise relation between holographic wave functions in AdS and LFWFs describing the internal structure of hadrons

- No constituent gluons

- Improve the semiclassical approximation: introduce nonzero quark masses and short-range Coulomb-like gluonic corrections (heavy and heavy-light quark systems)

- Apply Lippmann-Schwinger methods to systematically improve the light-front Hamiltonian of the semiclassical holographic approximation
Despite some limitations of AdS/QCD, the light-front holographic approach to the gauge/gravity duality has provided so far significant physical insight into the strongly-coupled nature and internal structure of hadrons. The resulting model provides a simple framework for describing nonperturbative hadron dynamics: the systematics of the excitation spectrum of hadrons: the mass spectrum, observed multiplicities and degeneracies. It also provides powerful new analytical tools for computing hadronic transition amplitudes, incorporating the scaling behavior and the transition from the hard-scattering perturbative domain, where quark and gluons are the relevant degrees of freedom, to the long range confining hadronic region. The holographic mapping provides the basis for a profound connection between physical QCD quantized in the light-front and the physics of hadronic modes in a higher dimensional AdS space.