E. Klempt et al.: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

\[
F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,
\]
\[
F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,
\]

where the effective charges \(g_+\) and \(g_-\) are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have \(S^z = +1/2\). The two AdS solutions \(\psi_+(\zeta)\) and \(\psi_-(\zeta)\) correspond to nucleons with \(J^z = +1/2\) and \(-1/2\).

- For SU(6) spin-flavor symmetry

\[
F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,
\]
\[
F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\]

where \(F_1^p(0) = 1, F_1^n(0) = 0\).
• Scaling behavior for large $Q^2$: $Q^4 F_1^p(Q^2) \to \text{constant}$

Proton $\tau = 3$

- Scaling behavior for large $Q^2$: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$

Neutron $\tau = 3$

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

Harmonic Oscillator
Confinement
Normalized to anomalous moment

$F_2^p(Q^2)\n\kappa = 0.49 \text{ GeV}$

G. de Teramond, sjb

Ruhr-University Bochum
June 22, 2010

AdS/QCD and Light-Front Holography

Stan Brodsky
SLAC-CP3

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**String Theory**

**AdS/CFT**
- Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
- Conformal behavior at short distances + Confinement at large distance

**AdS/QCD**
- QCD at the Amplitude Level
- Counting rules for Hard Exclusive Scattering
- Regge Trajectories

**Semi-Classical QCD / Wave Equations**

**Boost Invariant 3+1 Light-Front Wave Equations**
- $J = 0, 1, 1/2, 3/2$ plus $L$
- Integrable!

**Hadron Spectra, Wavefunctions, Dynamics**

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**Ruhr-University Bochum**

**AdS/QCD and Light-Front Holography**

**Stan Brodsky**

SLAC-CP3

June 22, 2010
Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \text{ or } g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

Coupling measured at momentum scale $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta \, d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$
\[ \frac{\alpha_s(Q)}{\pi} = e^{-Q^2/4\kappa^2} \]

\(\kappa = 0.54 \text{ GeV} \)

Deur, de Teramond, sjb
\[ \beta_{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4 \kappa^2} e^{-Q^2/4\kappa^2} \]
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$. Spectrum is independent of $S$
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

Hamiltonian light-front field theory within an AdS/QCD basis.

J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,
G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb

Pauli, Hornbostel, Hiller, McCartor, sjb
**Light-Front QCD**

**Heisenberg Equation**

\[ H_{QCD}^{LC} |\Psi_h\rangle = M_h^2 |\Psi_h\rangle \]

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**Use AdS/QCD basis functions**

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**AdS/QCD and Light-Front Holography**

**Stan Brodsky**

**SLAC-CP3**
Formation of Relativistic Anti-Hydrogen

**Measured at CERN-LEAR and FermiLab**

\[ \bar{p} \rightarrow \gamma^* \rightarrow e^+ + e^- \]

Munger, Schmidt, sjb

\[ \bar{H}(\bar{p}e^+) \]

\[ b_\perp \leq \frac{1}{m_{red\alpha}} \]

\[ y_{\bar{p}} \simeq y_{e^+} \]

Coalescence of off-shell co-moving positron and antiproton.

Wavefunction maximal at small impact separation and equal rapidity

“Hadronization” at the Amplitude Level
Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Event amplitude generator
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

\[ e^+ \rightarrow \gamma^* \rightarrow g u d \psi(x, \vec{k}_\perp, \lambda_i) \]

Baryon Production

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs
Hadronization at the Amplitude Level

Higher Fock State Coalescence \(|uuds\bar{s}\rangle\)

Asymmetric Hadronization! \(D_{s\to p}(z) \neq D_{s\to \bar{p}(z)}\)

B-Q Ma, sjb

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AdS/QCD and Light-Front Holography
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Features of LF T-Matrix Formalism

“Event Amplitude Generator”

- Coalesce color-singlet cluster to hadronic state if
  \[ M_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2 \]

- The coalescence probability amplitude is the LF wavefunction
  \[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

- No IR divergences: Maximal gluon and quark wavelength from confinement

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AdS/QCD and Hadronic Physics

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Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

Collision can produce 3 collinear quarks

\( uu \rightarrow p\bar{d} \)

\( \phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2 \)

\( n_{active} = 6 \)

\( n_{eff} = 2n_{active} - 4 \)

\( n_{eff} = 8 \)

\( qq \rightarrow B\bar{q} \)

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Small color-singlet
Color Transparent
Minimal same-side energy

Sickles; sjb
$\pi N \rightarrow \mu^+ \mu^- X$ at high $x_F$

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \rightarrow \infty$

Light-Front Wavefunctions from AdS/CFT

Entire pion wf contributes to hard process

Virtual photon is longitudinally polarized

"Direct" Subprocess

Berger, sjb
Khoze, Brandenburg, Muller, sjb
Hoyer Vanttinen

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AdS/QCD and Light-Front Holography

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\[ \pi^- N \rightarrow \mu^+ \mu^- X \text{ at } 80 \text{ GeV/c} \]

\[ \frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \rho \sin 2\theta \cos \phi + \omega \sin^2 \theta \cos 2\phi. \]

\[ \frac{d^2\sigma}{dx_\pi d\cos \theta} \propto x_\pi \left( (1-x_\pi)^2 (1+\cos^2 \theta) + \frac{4}{9} \frac{\langle k_f^2 \rangle}{M^2} \sin^2 \theta \right) \]

\[ \langle k_f^2 \rangle = 0.62 \pm 0.16 \text{ GeV}^2/c^2 \]

\[ Q^2 = M^2 \]

Dramatic change in angular distribution at large \( x_F \)

Example of a higher-twist direct subprocess

Direct Subprocess Prediction.

Chicago-Princeton Collaboration

Crucial Test of Leading-Twist QCD: Scaling at fixed $x_T$

$$x_T = \frac{2p_T}{\sqrt{s}}$$

$$E \frac{d\sigma}{d^3p}(pN \rightarrow \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

**Parton model:** $n_{eff} = 4$

As fundamental as Bjorken scaling in DIS

Conformal scaling: $n_{eff} = 2n_{active} - 4$
\[
pp \rightarrow \gamma X
\]

\[
E \frac{d\sigma}{d^3p} (pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}
\]

\[
gu \rightarrow \gamma u
\]

\[
n_{active} = 4
\]

\[
n_{eff} = 2n_{active} - 4
\]

\[
n_{eff} = 4
\]
\( \sqrt{s}^n \frac{d\sigma}{d^3p} (pp \rightarrow \gamma X) \) at fixed \( x_T \)

**xt-scaling of direct photon production is consistent with PQCD**
Leading-Twist Contribution to Hadron Production

\[ \frac{d\sigma}{d^3p/E} = \alpha_s^2 \frac{F(x_{\perp},y)}{p_{\perp}^4} \]

Parton model and Conformal Scaling:

\[ G_{q/p}(x_1, p_{\perp}^2) \]

\[ G_{q/p}(x_2, p_{\perp}^2) \]

\[ D_{\pi/q}(z, p_{\perp}^2) \]
**QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling**

\[
\frac{d\sigma}{d^3p/E} = \frac{F(x_\perp,y)}{p^n(x_\perp)}
\]

**Key test of PQCD:** power-law fall-off at fixed $x_T$

- **Arleo, Aurenche, Hwang, Sickles, sjb**
  - $5 < p_\perp < 20 \, \text{GeV}$
  - $70 \, \text{GeV} < \sqrt{s} < 4 \, \text{TeV}$

**Inclnlo**

- Photon: $pp \rightarrow \pi X$
- Pion: $pp \rightarrow \gamma X$

**CTEQ6.6 PDF**
- DSS/BFG FF
- scales=$p_\perp$
- y=0

**DSS (De Florian-Sassot-Stratmann)**
\[ [\sqrt{s}]^n \frac{d\sigma}{d^3p/E} (pp \rightarrow \pi^0 X) \text{ at fixed } x_T = \frac{2p_T}{\sqrt{s}} \]

\[
\pi^0 \text{ from } p+p
\]

\( n = 6.38 \)

M. J. Tannenbaum

PHENIX 62.4 and 200 GeV data
\[ E \frac{d\sigma}{d^3p} (pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{eff}}} \]

- $\sqrt{s}=38.8/31.6$ GeV E706
- $\sqrt{s}=62.4/22.4$ GeV PHENIX/FNAL
- $\sqrt{s}=62.8/52.7$ GeV R806
- $\sqrt{s}=52.7/30.6$ GeV R806
- $\sqrt{s}=200/62.4$ GeV PHENIX
- $\sqrt{s}=500/200$ GeV UA1
- $\sqrt{s}=900/200$ GeV UA1
- $\sqrt{s}=1800/630$ GeV CDF

- $n_{eff}$

- $E d\sigma d^3p (pp \to HX)$

- $F(x_T, \theta_{CM} = \pi/2)$

- $p_T^{n_{eff}}$

- $x_T = 2p_T/\sqrt{s}$

\[ E \frac{d\sigma}{d^3p} (pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{eff}}} \]
Significant increase of the hadron $n_{\text{exp}}$ with $x_\perp$
- $n_{\text{exp}} \approx 8$ at large $x_\perp$

Huge contrast with photons and jets!
- $n_{\text{exp}}$ constant and slight above 4 at all $x_\perp$
Direct Contribution to Hadron Production

\[ \frac{d\sigma}{d^3 p/E} = \alpha_s^3 f_\pi^2 F(x_\perp, y) \frac{p_\perp^6}{p_\perp^6} \]

No Fragmentation Function
Direct Proton Production

Explains “Baryon anomaly” at RHIC

$E \frac{d\sigma}{d^3p}(p \ p \rightarrow p \ X) \sim \frac{F(x_{\perp}, y_{\text{cm}})}{p_{\perp}^8}$

$\text{n}_{\text{active}} = 6$
Baryon can be made directly within hard subprocess

Collision can produce 3 collinear quarks

Coalescence within hard subprocess

$uu \rightarrow p \bar{d}$

$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$

$\text{Baryon anomaly}$

$qq \rightarrow B \bar{q}$

$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$


Sickles; sjb

Small color-singlet
Color Transparent
Minimal same-side energy

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AdS/QCD and Light-Front Holography

Stan Brodsky
SLAC-CP3
Chiral Symmetry Breaking in AdS/QCD

- Chiral symmetry breaking effect in AdS/QCD depends on weighted $z^2$ distribution, not constant condensate

$$\delta M^2 = -2m_q \langle \bar{\psi}\psi \rangle \times \int dz \, \phi^2(z) z^2$$

- $z^2$ weighting consistent with higher Fock states at periphery of hadron wavefunction

- AdS/QCD: confined condensate

- Suggests “In-Hadron” Condensates

Erlich et al.

de Teramond, Shrock, sjb
In presence of quark masses the Holographic LF wave equation is $(\zeta = z)$

\[
\left[-\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2}\right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta),
\]  

(1)

and thus

\[
\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.
\]  

(2)

The parameter $a$ is determined by the Weisberg term

\[
a = \frac{2}{\sqrt{x}}.
\]

Thus

\[
X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x} \langle \bar{\psi} \psi \rangle z^3,
\]  

(3)

and

\[
\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2 \sum_i m_i \langle \bar{\psi} \psi \rangle \langle z^2 \rangle + \langle \bar{\psi} \psi \rangle^2 \langle z^4 \rangle,
\]  

(4)

where we have used the sum over fractional longitudinal momentum $\sum_i x_i = 1$. 

\textbf{Mass shift from dynamics inside hadronic boundary}
Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind

Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon. Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame. A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.
Bethe-Salpeter Analysis

\[ f_H P^\mu = Z_2 \int_\Lambda^{d^4q} \frac{d^4q}{(2\pi)^4} \frac{1}{2}[T_H \gamma_5 \gamma^\mu S(\frac{1}{2}P + q))\Gamma_H(q; P)S(\frac{1}{2}P - q))] \]

\( f_H \) Meson Decay Constant
\( T_H \) flavor projection operator,
\( Z_2(\Lambda), Z_4(\Lambda) \) renormalization constants
\( S(p) \) dressed quark propagator
\( \Gamma_H(q; P) = F.T. \langle H|\psi(x_a)\bar{\psi}(x_b)|0\rangle \)
Bethe-Salpeter bound-state vertex amplitude.

\[ i\rho^H_\zeta \equiv -\frac{\langle q\bar{q} \rangle^H_\zeta}{f_H} = Z_4 \int_\Lambda^{d^4q} \frac{d^4q}{(2\pi)^4} \frac{1}{2}[T_H \gamma_5 S(\frac{1}{2}P + q))\Gamma_H(q; P)S(\frac{1}{2}P - q))] \]

In-Hadron Condensate!

\[ f_H m^2_H = -\rho^H_\zeta \mathcal{M}_H \quad \mathcal{M}_H = \sum_{q \in H} m_q \]

\[ m_\pi^2 \propto (m_q + m_{\bar{q}})/f_\pi \quad \text{GMOR} \]
Simple physical argument for “in-hadron” condensate

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

\[ < B | \bar{q}q | B > \text{ not } < 0 | \bar{q}q | 0 > \]
Higher Light-Front Fock State of Pion Simulates DCSB

\[
f_{\pi} P^+ = \langle 0 | \bar{q} \gamma^5 \gamma^+ q | \pi \rangle
\]

Instantaneous quark propagator contribution to \( \bar{q} \pi \) derived from higher Fock state

\[
i \rho_{\pi} = \langle 0 | \bar{q} \gamma^5 q | \pi \rangle
\]

Higher Fock state acts like mass insertion

Roberts, Tandy, Shrock, sjb
Essence of the vacuum quark condensate

Stanley J. Brodsky,¹,² Craig D. Roberts,³,⁴ Robert Shrock,⁵ and Peter C. Tandy⁶

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⁶Center for Nuclear Research, Department of Physics, Kent State University, Kent OH 44242, USA

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

PACS numbers: 11.30.Rd; 14.40.Be; 24.85.+p; 11.15.Tk

ArXiv: 10005.4610
Quark and Gluon condensates reside within hadrons, not vacuum

- Bound-State Dyson Schwinger Equations
- AdS/QCD
- Analogous to finite size superconductor
- Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict

Casher and Susskind, Maris, Roberts, Tandy, Shrock and sjb

R. Shrock, sjb
PNAS
ArXiv:0905.1151
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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\[
(\Omega_\Lambda)_{QCD} \sim 10^{45} \quad \Omega_\Lambda = 0.76 (expt)
\]

\[
(\Omega_\Lambda)_{EW} \sim 10^{56}
\]

QCD Problem Solved if Quark and Gluon condensates reside
within hadrons, not vacuum!

R. Shrock, sjb

“Condensates in Quantum Chromodynamics and the Cosmological Constant.”
Quark and Gluon condensates reside within hadrons, not LF vacuum

- Bound-State Dyson-Schwinger Equations
- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs
- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!
- Implications for cosmological constant

"Confined QCD Condensates"
Determinations of the vacuum Gluon Condensate

\[< 0 | \frac{\alpha_s}{\pi} G^2 | 0 > [\text{GeV}^4]\]

\[-0.005 \pm 0.003 \text{ from } \tau \text{ decay.}\]
\[+0.006 \pm 0.012 \text{ from } \tau \text{ decay.}\]
\[+0.009 \pm 0.007 \text{ from charmonium sum rules}\]

Consistent with zero vacuum condensate

Davier et al.
Geshkenbein, Ioffe, Zyablyuk
Ioffe, Zyablyuk
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite $N_c = 3$: Baryons built on 3 quarks -- Large $N_c$ limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General ‘classical’ potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
$H_{QCD}^{LF}$

$\langle H^0_{LF} + H^I_{LF} \rangle |\Psi\rangle \geq M^2 |\Psi\rangle$

$\left[ \vec{k}_\perp^2 + m^2 \over x(1-x) + V_{eff}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$

$\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$

$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$

\textbf{QCD Meson Spectrum}

\textbf{Coupled Fock states}

\textbf{Effective two-particle equation}

$\zeta^2 = x(1-x)b^2_{\perp}$

\textbf{Azimuthal Basis $\zeta, \phi$}

\textbf{Confining AdS/QCD potential}

\textbf{Semiclassical first approximation to QCD}
An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- As Simple as Schrödinger Theory in Atomic Physics
- LF radial variable $\zeta$ conjugate to invariant mass squared
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales: Essential for Gauge Link phenomena
- Hadron Spectroscopy and Dynamics from one parameter $\kappa$
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates: Zero cosmological constant!
- Systematically improvable with DLCQ Methods
GPDs & Deeply Virtual Exclusive Processes  
- New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)

GPDs  
$H, \tilde{H}, E, \tilde{E}$

$H(x, \xi, t), E(x, \xi, t), \ldots$  “Generalized Parton Distributions”

$\gamma^* \rightarrow N \leftarrow N$  
$\gamma \rightarrow x + \xi \rightarrow x - \xi$

$x$ - quark momentum fraction

$\xi$ - longitudinal momentum transfer

$\sqrt{t}$ - Fourier conjugate to transverse impact parameter
Light-Front Wave Function Overlap Representation

Diehl, Hwang, sjb, NPB596, 2001
See also: Diehl, Feldmann, Jakob, Kroll

Diehl, Hwang, sjb, NPB596, 2001
See also: Diehl, Feldmann, Jakob, Kroll

Light-Front Wave Function Overlap Representation

DVCS/GPD

DGLAP region

ERBL region

DGLAP region

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AdS/QCD and Light-Front Holography

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SLAC-CP3

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Example of LFWF representation of GPDs \((n+1 \Rightarrow n-1)\)

\[
\frac{1}{\sqrt{1 - \zeta}} \frac{\Delta^1 - i \Delta^2}{2M} E_{(n+1 \rightarrow n-1)}(x, \zeta, t)
\]

\[
= \left(\sqrt{1 - \zeta}\right)^{3-n} \sum_{n, \lambda_i} \int \prod_{i=1}^{n+1} \frac{dx_i \, d^2 \vec{k}_\perp i}{16\pi^3} \left(1 - \sum_{j=1}^{n+1} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n+1} \vec{k}_\perp j\right)
\]

\[
\times 16\pi^3 \delta(x_{n+1} + x_1 - \zeta) \delta^{(2)}\left(\vec{k}_\perp n+1 + \vec{k}_\perp 1 - \vec{\Delta}_\perp\right)
\]

\[
\times \delta(x - x_1) \psi_{(n-1)}^{\uparrow*}(x'_i, \vec{k}'_\perp i, \lambda_i) \psi_{(n+1)}^{\downarrow}(x_i, \vec{k}_\perp i, \lambda_i) \delta_{\lambda_1 - \lambda_{n+1}},
\]

where \(i = 2, \ldots, n\) label the \(n - 1\) spectator partons which appear in the final-state hadron wavefunction with

\[
x'_i = \frac{x_i}{1 - \zeta}, \quad \vec{k}'_\perp i = \vec{k}_\perp i + \frac{x_i}{1 - \zeta} \vec{\Delta}_\perp.
\]
\textbf{J=0 Fixed Pole Contribution to DVCS}

- J=0 fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator

Real amplitude, independent of $Q^2$ at fixed $t$

\textit{Szczepaniak, Llanes-Estrada, sjb}
\textit{Close, Gunion, sjb}
Deeply Virtual Compton Scattering

\[ \gamma^* p \rightarrow \gamma p \]

Reflects elementary coupling of two photons to quarks

\[ T(\gamma^*(q)p \rightarrow \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s_R^\alpha(t) \beta_R(t) \]

\[ \alpha_R(t) \rightarrow 0 \]

\[ \beta_R(t) \sim \frac{1}{t^2} \quad \frac{d\sigma}{dt} \sim \frac{1}{s^2} \frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]
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**Exclusive Electroproduction**

\[ ep \to e' \pi^+ n \]

**Hard Reggeon Domain**

\[ s \gg -t, Q^2 \gg \Lambda_{QCD}^2 \]

\[ T(\gamma^* p \to \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^\alpha(t) \beta_R(t) \]

\[ \alpha_R(t) \to -1 \]

\[ \beta_R(t) \sim \frac{1}{t^2} \]

**Reflects elementary exchange of quarks in t-channel**

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]
Regge domain

\[ T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_{R} s^\alpha_R(t) \beta_R(t) \quad s \gg -t, Q^2 \]

\[ \alpha_R(t) \rightarrow -1 \text{ at } t \rightarrow -\infty \]

**Reflects elementary exchange of quarks in t-channel**

\[ \frac{d\sigma}{dt} (\gamma^* p \rightarrow \pi^+ n) \rightarrow \frac{1}{s^3 \beta^2_R(t)} \]

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^3} \frac{1}{t^4} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]

Fundamental test of QCD
**J=0** Fixed pole in real and virtual Compton scattering

Effective two-photon contact term

Seagull for scalar quarks

Real phase

\[ M = s^0 \sum e_q^2 F_q(t) \]

Independent of \( Q^2 \) at fixed \( t \)

\( \langle 1/x \rangle \) Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory

No ambiguity in D-term

\( Q^2 \)-independent contribution to Real DVCS amplitude

\[ s^2 \frac{d\sigma}{dt}(\gamma^* p \rightarrow \gamma p) = F^2(t) \]
QCD and the LF Hadron Wavefunctions

AdS/QCD
Light-Front Holography
LF Schrödinger Eqn

Heavy Quark Fock States
Intrinsic Charm

Coordinate space
representation

Quark & Flavor Structure

J=0 Fixed Pole
DVCS, GPDs, TMDs
LF Overlap, incl ERBL

Nuclear Modifications
Baryon Anomaly
Color Transparency

Initial and Final State
Rescattering
DDIS, DDIS, T-Odd

Non-Universal
Antishadowing

Baryon Excitations

Gluonic properties
DGLAP

Orbital Angular Momentum
Spin, Chiral Properties
Crewther Relation

Hard Exclusive Amplitudes
Form Factors
Counting Rules

Distribution amplitude
ERBL Evolution

Hadronization at
Amplitude Level

Baryon Decay
Single-spin asymmetries

Pseudo-\( T \)-Odd

\[ i \vec{S}_p \cdot \vec{q} \times \vec{p}_q \]

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

QCD S- and P-Coulomb Phases

--Wilson Line

Leading-Twist Rescattering Violates pQCD Factorization!
Final-State Interactions Produce Pseudo $T$-Odd (Sivers Effect)

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;
- Wilson line effect -- gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!
- Alternate: Retarded and Advanced Gauge: Augmented LFWFs

$\mathbf{i} \mathbf{S} \cdot \mathbf{p}_{jet} \times \mathbf{q}$

Pasquini, Xiao, Yuan, sjb
Mulders, Boer
Qiu, Sterman

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June 22, 2010

AdS/QCD and Light-Front Holography

Stan Brodsky
SLAC-CP3
Final State Interactions Produce T-Odd (Sivers Effect)

- Bjorken Scaling!
- Arises from Interference of Final-State Coulomb Phases in S and P waves
- Relate to the quark contribution to the target proton anomalous magnetic moment
- Sum of Sivers Functions for all quarks and gluons vanishes. (Zero anomalous gavitomagnetic moment)

$$\vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

Hwang, Schmidt.
sjb; Burkardt
Measure single-spin asymmetry $A_N$ in Drell-Yan reactions.

Leading-twist Bjorken-scaling $A_N$ from $S, P$-wave initial-state gluonic interactions.

Predict: $A_N(DY) = -A_N(DIS)$
Opposite in sign!

$pp \uparrow \rightarrow \ell^+ \ell^- X$

$\vec{S} \cdot \vec{q} \times \vec{p}$ correlation

Collins; Hwang, Schmidt, sjb
Drell-Yan angular distribution

Unpolarized DY

- Experimentally, a violation of the Lam-Tung sum rule is observed by sizeable \( \cos 2\Phi \) moments
- Several model explanations
  - higher twist
  - spin correlation due to non-trivial QCD vacuum
  - Non-zero Boer Mulders function

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)
\]

Experiment: \( \nu \approx 0.6 \)

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B. Seitz
Measurement of Angular Distributions of Drell-Yan Dimuons in $p + d$ Interaction at 800 GeV/c
(FNAL E866/NuSea Collaboration)

![Graph showing parameter $\nu$ vs. $p_T$ in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4$ GeV/c$^2$ are also shown.]

Parameter $\nu$ vs. $p_T$ in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4$ GeV/c$^2$ are also shown.
DY $\cos 2\phi$ correlation at leading twist from double ISI

Product of Boer-Mulders Functions

$$h_1^{\perp}(x_1, p_\perp^2) \times \bar{h}_1^{\perp}(x_2, k_\perp^2)$$
Unpolarized Distribution
Bj Sum Rule
Transversity
Sivers Function
Boer-Mulders Function

$T$-Odd: Require ISI or FSI
Double Initial-State Interactions generate anomalous \( \cos 2 \phi \): Boer, Hwang, sjb

Drell-Yan planar correlations

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)
\]

PQCD Factorization (Lam Tung): \( 1 - \lambda - 2\nu = 0 \)

\[
\frac{\nu}{2} \propto h_1^\pi(\pi) h_1^N(N).
\]

\[\pi N \rightarrow \mu^+ \mu^- X \]

NA10

Violates Lam-Tung relation!

Double ISI

Hard gluon radiation

Model: Boer, Stan Brodsky

SLAC-CP3

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AdS/QCD and Light-Front Holography
DY \cos 2\phi \text{ correlation at leading twist from double ISI}

\[ h_1^+(x_1, p_\perp^2) \times \bar{h}_1^+(x_2, k_\perp^2) \]

Product of Boer-Mulders Functions

\[ F = \mathcal{F}[ (2 \hat{h} \cdot p_\perp \hat{h} \cdot k_\perp - p_\perp \cdot k_\perp) h_1^+ \bar{h}_1^+] \]

\[ = \int d^2p_\perp d^2k_\perp \delta^2(p_\perp + k_\perp - q_\perp)(2 \hat{h} \cdot p_\perp \hat{h} \cdot k_\perp - p_\perp \cdot k_\perp) \]

\[ \times h_1^+(\Delta, p_\perp^2) \bar{h}_1^+(\bar{\Delta}, k_\perp^2), \]

\[ G = \mathcal{F}[f_1 \bar{f}_1] \]

\[ = \int d^2p_\perp d^2k_\perp \delta^2(p_\perp + k_\perp - q_\perp) f_1(\Delta, p_\perp^2) \bar{f}_1(\bar{\Delta}, k_\perp^2), \]

Boer, Hwang, sjb

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AdS/QCD and Light-Front Holography

Stan Brodsky SLAC-CP3
### Static
- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and $J^z$
- DGLAP Evolution; mod. at large $x$
- No Diffractive DIS

\[
\Psi_n(x_i, \vec{k}_\perp i, \lambda_i)^2
\]

### Dynamic
- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Sum Rules Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon Diffractive DIS

**Diagram:**
- Electron beam
- Proton
- Current quark jet
- Final state interaction
- Spectator system
Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer
Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions


The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.
Problem for factorization when both ISI and FSI occur
$\cos 2\phi$ correlation for quarkonium production at leading twist from double ISI
Enhanced by gluon color charge
M. Hirai, S. Kumano and T. H. Nagai,  
“Nuclear parton distribution functions and their uncertainties,”  
Nuclear Shadowing in QCD

Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF!

Dynamical effect due to virtual photon interacting in nucleus
The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken $x_B$:

$$\frac{1}{M x_B} = 2 \nu / Q^2 \geq L_A.$$

If the scattering on nucleon $N_1$ is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\bar{q}$ flux reaching $N_2$.

→ Shadowing of the DIS nuclear structure functions.

**Observed HERA DDIS produces nuclear shadowing**
Integration over on-shell domain produces phase $i$

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate $T$-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

Antishadowing (Reggeon exchange) is not universal!

Schmidt, Yang, sjb
$Q^2 = 5 \text{ GeV}^2$

SLAC/NMC data

Extrapolations from NuTeV

Scheinbein, Yu, Keppel, Morfin, Olness, Owens
Antiquark interacts with target nucleus at energy $\hat{s} \propto \frac{1}{x_{bj}}$

Regge contribution: $\sigma_{\bar{q}N} \sim \hat{s}^{\alpha R - 1}$

Nonsinglet Kuti-Weisskoff $F_{2p} - F_{2n} \propto \sqrt{x_{bj}}$ at small $x_{bj}$.

Shadowing of $\sigma_{\bar{q}M}$ produces shadowing of nuclear structure function.

Landshoff, Polkinghorne, Short
Close, Gunion, sjb
Schmidt, Yang, Lu, sjb
Reggeon Exchange

Phase of two-step amplitude relative to one step:

\[ \frac{1}{\sqrt{2}} (1 - i) \times i = \frac{1}{\sqrt{2}} (i + 1) \]

Constructive Interference

Depends on quark flavor!

Thus antishadowing is not universal

Different for couplings of \( \gamma^*, Z^0, W^\pm \)

**Critical test: Tagged Drell-Yan**
Shadowing and Antishadowing in Lepton-Nucleus Scattering

- Shadowing: Destructive Interference of Two-Step and One-Step Processes
  
  *Pomeron Exchange*

- Antishadowing: Constructive Interference of Two-Step and One-Step Processes!
  
  *Reggeon and Odderon Exchange*

- Antishadowing is Not Universal!
  
  Electromagnetic and weak currents: different nuclear effects!
  
  Potentially significant for NuTeV Anomaly

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Schmidt, Yang; sjb

Modifies NuTeV extraction of

\[ \sin^2 \theta_W \]

Nuclear Antishadowing not universal!
- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Conformal Template (BLM, CSR, BFKL scale)
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict with Standard Model

Roberts, Shrock, Tandy, and sjb