Light-Front Holography and Non-Perturbative QCD

Berlin -- Brandenburgische Akademie der Wissenschaften

QCD: The Modern View of the Strong Interactions

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

Stan Brodsky, SLAC National Accelerator Laboratory

October 8, 2009
Goal: Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions, Form Factors, DVCS, etc

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

in collaboration with Guy de Teramond
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

\[ P^+, \vec{P}_\perp \]

Fixed \( \tau = t + z/c \)

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp i \]

Process Independent
Direct Link to QCD Lagrangian!

\[ \Psi_n (x_i, \vec{k}_\perp i, \lambda_i) \]

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n \vec{k}_\perp i = \vec{0}_\perp \]

Invariant under boosts! Independent of \( P^\mu \)
Calculation of Form Factors in Equal-Time Theory

**Instant Form**

\[ \sum = \text{Need vacuum-induced currents} \]

Calculation of Form Factors in Light-Front Theory

**Front Form**

\[ \sum = \text{Complete Answer} \]

Absent for \( q^+ = 0 \)

\( \text{zero} !! \)
\[
\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_{e_j} \frac{1}{2} \times \\
\left[ - \frac{1}{q_L} \psi_a^\dagger(x_i, k'_\perp, \lambda_i) \psi_a(x_i, k_\perp, \lambda_i) + \frac{1}{q_R} \psi_a^\dagger(x_i, k'_\perp, \lambda_i) \psi_a(x_i, k_\perp, \lambda_i) \right] \\
\]

\[k'_\perp = k_\perp - x_i q_\perp\]
\[k'_j = k_\perp + (1 - x_j) q_\perp\]

Must have \(\Delta \ell_z = \pm 1\) to have nonzero \(F_2(q^2)\)

Same matrix elements appear in Sivers effect
-- connection to quark anomalous moments

Drell, sjb

\[q_{R,L} = q^x \pm iq^y\]
Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem

graviton

sum over constituents

Hwang, Schmidt, sjb; Holstein et al: $B(0) = 0$

$B(0) = 0$

Each Fock State

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LF Holography/QCD

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The Light Front Fock State Wavefunctions

\[ |p, S_z \rangle = \sum_{n=3}^{n} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i \rangle \]

are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction

\[ x_i = \frac{k_0^i + k_z^i}{P^0 + P^z} \]

are boost invariant.

Intrinsic heavy quarks

\[ c(x), b(x) \text{ at high } x \quad \bar{s}(x) \neq s(x) \quad \bar{u}(x) \neq \bar{d}(x) \]
Light-Front QCD Features and Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme
QCD and the LF Hadron Wavefunctions

- AdS/QCD
  - Light-Front Holography
  - LF Schrödinger Eqn
- Initial and Final State Rescattering
  - DDIS, DDIS, T-Odd
  - Non-Universal Antishadowing
- Baryon Excitations
- Gluonic properties
  - DGLAP
- Heavy Quark Fock States
  - Intrinsic Charm
- Coordinate space representation
- Quark & Flavor Structure
- $J=0$ Fixed Pole
  - DVCS, GPDs, TMDs
  - LF Overlap, incl ERBL
- Hadronization at Amplitude Level
- Nuclear Modifications
  - Baryon Anomaly
  - Color Transparency
- Orbital Angular Momentum
  - Spin, Chiral Properties
  - Crewther Relation
- Hard Exclusive Amplitudes
  - Form Factors
  - Counting Rules
- Distribution amplitude
  - ERBL Evolution
  - $\phi_p(x_1, x_2, Q^2)$
- Baryon Decay

$\Psi_n(x_i, k_{i\perp}, \lambda_i)$
<table>
<thead>
<tr>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Square of Target LFWFs</td>
<td>Modified by Rescattering: ISI &amp; FSI</td>
</tr>
<tr>
<td>- No Wilson Line</td>
<td>Contains Wilson Line, Phases</td>
</tr>
<tr>
<td>- Probability Distributions</td>
<td>No Probabilistic Interpretation</td>
</tr>
<tr>
<td>- Process-Independent</td>
<td>Process-Dependent - From Collision</td>
</tr>
<tr>
<td>- T-even Observables</td>
<td>T-Odd (Sivers, Boer-Mulders, etc.)</td>
</tr>
<tr>
<td>- No Shadowing, Anti-Shadowing</td>
<td>Shadowing, Anti-Shadowing, Saturation</td>
</tr>
<tr>
<td>- Sum Rules: Momentum and ( J^z )</td>
<td>Sum Rules Not Proven</td>
</tr>
<tr>
<td>- DGLAP Evolution; mod. at large ( x )</td>
<td>DGLAP Evolution</td>
</tr>
<tr>
<td>- No Diffractive DIS</td>
<td>Hard Pomeron and Odderon Diffractive DIS</td>
</tr>
</tbody>
</table>

\[
\Psi_n(x_i, k_{\perp i}, \lambda_i) = 2
\]
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

String Theory

Bottom-Up

Top-Down
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances

- Analogous to the Schrodinger Theory for Atomic Physics

- AdS/QCD Light-Front Holography

- Hadronic Spectra and Light-Front Wavefunctions
Conformal Theories are invariant under the Poincare and conformal transformations with 

\[ M^{\mu\nu}, P^\mu, D, K^\mu, \]

the generators of \( \text{SO}(4,2) \)

\( \text{SO}(4,2) \) has a mathematical representation on AdS5
Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).
\( \psi(x, \vec{b}_\perp) \quad \leftrightarrow \quad \phi(z) \)

\[ \zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \leftrightarrow \quad z \]

\[ \psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta) \]

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map AdS\(_5\) \times S\(_5\) to conformal N=4 SUSY

- **QCD is not conformal**: however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window**: \(\alpha_s(Q^2) \simeq \text{const} \) at small \(Q^2\)

- Use mathematical mapping of the conformal group SO(4,2) to AdS\(_5\) space
Conformal QCD Window in Exclusive Processes

• Does $\alpha_s$ develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur . . .

• Recent lattice simulations: evidence that $\alpha_s$ becomes constant and is not small in the infrared
  Furui and Nakajima, hep-lat/0612009  (Green dashed curve: DSE).
Deur, Korsch, et al.

\[ \frac{\alpha_s g_1}{\pi} \]

- JLab
- GDH limit
- Burkert-Ioffe

\[ \text{Fit} \]

\[ pQCD \text{ evol. eq.} \]

\[ \text{Corwall} \]

\[ \text{Godfrey-Isgur} \]

\[ \text{Bloch et al.} \]

\[ \text{Bhagwat et al.} \]

\[ \text{Maris-Tandy} \]

\[ \text{Fischer et al.} \]

\[ \text{DSE gluon couplings} \]

\[ Q \,(\text{GeV}) \]

\[ 10^{-1} \quad 1 \quad 10^{-1} \quad 1 \]
Maximal Wavelength of Confined Fields

- Colored fields confined to finite domain \((x - y)^2 < \Lambda_{QCD}^{-2}\)
- All perturbative calculations regulated in IR
- High momentum calculations unaffected
- Bound-state Dyson-Schwinger Equation
- Analogous to Bethe’s Lamb Shift Calculation

Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

Shrock, sjb

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).
IR Conformal Window for QCD

• **Dyson-Schwinger Analysis:** QCD gluon coupling has IR Fixed Point

• **Evidence from Lattice Gauge Theory**

• **Stability of** \( \Upsilon \to ggg \)

• Define coupling from observable: **indications of IR fixed point for QCD effective charges**

• Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small** \( Q^2 \)

\[
\Pi(Q^2) \to \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 << 4m^2
\]

• Justifies application of AdS/CFT in strong-coupling conformal window

Shrock, sjb

Deur, Chen, Burkert, Korsch,

\[\ell^+ \quad \ell^-\]
AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5

- Scale Transformations represented by wavefunction in 5th dimension
  \[ x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z \]

- Match solutions at small z to conformal twist dimension of hadron wavefunction at short distances
  \[ \psi(z) \sim z^\Delta \text{ at } z \rightarrow 0 \]

- Hard wall model: Confinement at large distances and conformal symmetry in interior

- Truncated space simulates “bag” boundary conditions
  \[ 0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}} \]
Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to (R^2/z^2) \eta_{\ell m}$.

- Action for massive scalar modes on $\text{AdS}_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1} x \sqrt{g} \frac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right] , \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along $x^\mu$-coordinates, $\Phi_P(x,z) = e^{-iP\cdot x} \Phi(z)$, $P_\mu P^\mu = M^2$:

$$[z^2 \partial_z^2 - (d - 1)z \partial_z + z^2 M^2 - (\mu R)^2] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$, $\Delta = 2 + L$

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z M) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4 \mu^2 R^2} \right).$$

$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrödinger Equation for bound state of two scalar constituents:**

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = M^2 \phi(z)$$

$L$: light-front orbital angular momentum

**Derived from variation of Action in AdS$_5$**

**Hard wall model: truncated space**

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$
Match fall-off at small $z$ to conformal twist-dimension $\Delta$ at short distances

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D \{ \ell_1 \ldots D \ell_m \} \psi$ \hspace{1em} (\Phi_\mu = 0 \text{ gauge}) \hspace{1em} $\Delta = 2 + L$

- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $M_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

\[ S = 0 \hspace{1em} \text{Meson orbital and radial AdS modes for } \Lambda_{QCD} = 0.32 \text{ GeV.} \]
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
Higher Spin Bosonic Modes HW

- Each hadronic state of integer spin $S \leq 2$ is dual to a normalizable string mode
  \[
  \Phi(x, z)_{\mu_1 \mu_2 \cdots \mu_S} = \epsilon_{\mu_1 \mu_2 \cdots \mu_S} e^{-iP \cdot x} \Phi_S(z).
  \]
  with four-momentum $P_\mu$ and spin polarization indices along the 3+1 physical coordinates.

- Wave equation for spin $S$-mode  
  \[
  \left[ z^2 \partial_z^2 - (d+1-2S)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_S(z) = 0,
  \]

- Solution
  \[\tilde{\Phi}(z)_S = \left( \frac{z}{R} \right)^S \Phi(z)_S = C e^{-iP \cdot x} z^{\frac{d}{2}} J_{\Delta - \frac{d}{2}}(z \mathcal{M}) \epsilon(P)_{\mu_1 \mu_2 \cdots \mu_S},\]

- We can identify the conformal dimension:
  \[
  \Delta = \frac{1}{2} \left( d + \sqrt{(d - 2S)^2 + 4\mu^2 R^2} \right).
  \]

- Normalization:
  \[
  R^{d-2S-1} \int_0^{\Lambda_{\text{QCD}}^{-1}} dz \frac{dz}{z^{d-2S-1}} \Phi^2_S(z) = 1.
  \]

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October 8, 2009

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Soft-Wall Model

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce a smooth cutoff which depends on the profile of a dilaton background field \( \varphi(z) = \pm \kappa^2 z^2 \)

\[
S = \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L},
\]

- Equation of motion for scalar field

\[
\mathcal{L} = \frac{1}{2} \left( g^{\ell m} \partial_{\ell} \Phi \partial_{m} \Phi - \mu^2 \Phi^2 \right)
\]

\[
\left[ z^2 \partial_z^2 - (d - 1 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0
\]

with \((\mu R)^2 \geq -4\). See also [Metsaev (2002), Andreev (2006)]

- LH holography requires ‘plus dilaton’ \( \varphi = +\kappa^2 z^2 \). Lowest possible state \((\mu R)^2 = -4\)

\[
\mathcal{M}^2 = 4\kappa^2 n, \quad \Phi_n(z) \sim z^2 e^{-\kappa^2 z^2} L_n(\kappa^2 z^2)
\]

\( \Phi_0(z) \) a chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

\textbf{Massless pion}
AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = M^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Derived from variation of Action

Dilaton-Modified AdS$_5$

\[
e^{\Phi(z)} = e^{+\kappa^2 z^2}
\]
\[ ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 -dz^2) \]

\[ V(z) = mc^2 \sqrt{g_{00}} \]

\[ V(z) \sim \frac{R}{z} e^{\kappa^2 z^2 / 2} \]

\[ V(z) \sim \frac{R}{z} e^{-\kappa^2 z^2 / 2} \]

Agrees with Klebanov and Maldacena for positive-sign exponent of dilaton
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion mass automatically zero!

$m_q = 0$
Quark separation increases with $L$

(a) $\Phi(z)$

(b) $\Phi(z)$

Quark separation increases with $L$

(a) $S = 1$

(b) $S = 1$

$M^2$ (GeV$^2$) vs. $L$

$M^2$ (GeV$^2$) vs. $n$

Quark separation increases with $L$
Parent and daughter Regge trajectories for the $I = 1 \rho$-meson family (red) and the $I = 0 \omega$-meson family (black) for $\kappa = 0.54$ GeV.
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[
\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1)\right] \phi_S(z) = \mathcal{M}^2 \phi_S(z)
\]

with eigenvalues \( \mathcal{M}^2 = 2\kappa^2 (2n + 2L + S) \).

- Compare with Nambu string result (rotating flux tube): \( M_n^2(L) = 2\pi\sigma (n + L + 1/2) \).

Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \text{ GeV} \).

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).

To do: compare with Ebert, Faustov, & Galkin, Plessas, et al.
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = zQK_1(zQ) \]

\[ F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi \) scales as \( \Phi^{(n)} \sim z^{\Delta_n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1} , \]

where \( \tau = \Delta_n - \sigma_n, \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

- **Soft Wall**: Harmonic Oscillator Confinement
- **Hard Wall**: Truncated Space Confinement

*One parameter - set by pion decay constant.*

Data Compilation
Baldini, Kloe and Volmer

de Teramond, sjb
See also: Radyushkin

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Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation
  \[
  \left[ z^2 \partial_z^2 - z \left( 1 + 2\kappa^2 z^2 \right) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.
  \]

- Solution bulk-to-boundary propagator
  \[
  J_\kappa(Q, z) = \Gamma \left( 1 + \frac{Q^2}{4\kappa^2} \right) U \left( \frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2 \right),
  \]
  where \( U(a, b, c) \) is the confluent hypergeometric function
  \[
  \Gamma(a) U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt.
  \]

- Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)
  \[
  F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).
  \]

- For large \( Q^2 \gg 4\kappa^2 \)
  \[
  J_\kappa(Q, z) \to zQ K_1(zQ) = J(Q, z),
  \]
  the external current decouples from the dilaton field.

**Soft Wall Model**
Spacelike pion form factor from AdS/CFT

\[ F_{\pi}(q^2) \]

- Soft Wall: Harmonic Oscillator Confinement
- Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

Data Compilation
Baldini, Kloe and Volmer

See also: Radyushkin

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LF Holography/QCD
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• Analytical continuation to time-like region $q^2 \rightarrow -q^2$ 
  \[ M_\rho = 2\kappa = 750 \text{ MeV} \]

• Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

![Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.](image)

• Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).
Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension $\tau$, $\Phi_\tau$ in the SW model
  \[ F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}. \]

- For $\tau = N$, $\Gamma(N + z) = (N - 1 + z)(N - 2 + z) \ldots (1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles
  \[ F(Q^2) = \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2, \]
  \[ F(Q^2) = \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3, \]
  \[ \ldots \]
  \[ F(Q^2) = \frac{(N - 1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right) \ldots \left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N. \]

- For large $Q^2$:
  \[ F(Q^2) \to (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}. \]
Spacelike and timelike pion form factor

$\kappa = 0.534 \text{ GeV}$

$|\pi > = \psi_{qq}|q\bar{q}> + \psi_{qqqq}|q\bar{q}q\bar{q}>$

$Q^2 (\text{GeV}^2)$

$\Gamma_\rho = 120 \text{ MeV}, \Gamma'_\rho = 300 \text{ MeV}$

$P_{q\bar{q}q\bar{q}} = 15\%$
Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_P^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find \( b = |\vec{b}_\perp| \):

\[
\begin{align*}
F(q^2) &= \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\
&= 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bqx) \, |\tilde{\psi}(x, b)|^2.
\end{align*}
\]

\[ \vec{q}_\perp^2 = Q^2 = -q^2 \]
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[
F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta \, d\zeta \, J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),
\]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

- Transversality variable

\[
\zeta = \sqrt{x(1-x)b_{\perp}^2}
\]

- Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[
\int_0^1 dx \, J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),
\]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \)!
• Electromagnetic form-factor in AdS space:

\[ F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2, \]

where \( J(Q^2, z) = zQ K_1(zQ) \).

• Use integral representation for \( J(Q^2, z) \)

\[ J(Q^2, z) = \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) \]

• Write the AdS electromagnetic form-factor as

\[ F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_{\pi^+}(z)|^2 \]

• Compare with electromagnetic form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \hat{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_{\pi}(\zeta)|^2}{\zeta^4} \]

with \( \zeta = z, \ 0 \leq \zeta \leq \Lambda_{\text{QCD}} \)
Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

**LF(3+1) \hspace{2cm} AdS_5**

\[ \psi(x, \vec{b}_\perp) \leftrightarrow \phi(z) \]

\[ \zeta = \sqrt{x(1-x)b_\perp^2} \]

\[ \psi(x, \vec{b}_\perp) = \sqrt{x(1-x)} \phi(\zeta) \]

**Light-Front Holography/QCD**
Gravitational Form Factor in AdS space

• Hadronic gravitational form-factor in AdS space

\[ A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2, \]

where \( H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ) \)

• Use integral representation for \( H(Q^2, z) \)

\[ H(Q^2, z) = 2 \int_0^1 x dx J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) \]

• Write the AdS gravitational form-factor as

\[ A_\pi(Q^2) = 2R^3 \int_0^1 x dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_\pi(z)|^2 \]

• Compare with gravitational form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{qq/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}, \]

Identical to LF Holography obtained from electromagnetic current
Light-Front Holography:  
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

\[ \left[ - \frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = M^2 \phi(\zeta) \]

\[ \zeta^2 = x(1 - x)b_\perp^2. \]

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \]

G. de Teramond, sjb

soft wall confining potential:
Derivation of the Light-Front Radial Schrödinger Equation directly from LF QCD

\[ \mathcal{M}^2 = \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \frac{k_\perp^2}{x(1-x)} |\psi(x, k_\perp)|^2 + \text{interactions} \]

\[ = \int_0^1 \frac{dx}{x(1-x)} \int d^2 b_\perp \psi^*(x, b_\perp) \left( -\nabla_{b_\perp}^2 \right) \psi(x, b_\perp) + \text{interactions} \]

Change variables \((\tilde{\zeta}, \varphi), \tilde{\zeta} = \sqrt{x(1-x)} b_\perp: \nabla^2 = \frac{1}{\tilde{\zeta} d\tilde{\zeta}} \left( \frac{d}{d\tilde{\zeta}} \left( \frac{d}{d\tilde{\zeta}} \right) + \frac{1}{\tilde{\zeta}^2} \frac{\partial^2}{\partial \varphi^2} \right) \]

\[ \mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \]

\[ + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \]

\[ = \int d\zeta \phi^*(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \]
\[ H_{\text{QED}} \]

\[
(H_0 + H_{\text{int}}) |\Psi\rangle = E |\Psi\rangle
\]

\[
\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r})\right] \psi(\vec{r}) = E \psi(\vec{r})
\]

\[
\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell + 1)}{r^2} + V_{\text{eff}}(r, S, \ell)\right] \psi(r) = E \psi(r)
\]

\[ V_{\text{eff}} \rightarrow V_{\text{C}}(r) = -\frac{\alpha}{r} \]

QED atoms: positronium and muonium

Coupled Fock states

Effective two-particle equation

Includes Lamb Shift, quantum corrections

Spherical Basis \( r, \theta, \phi \)

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED
Coupled Fock states

\[ (H_{LF}^0 + H_{LF}^I)\Psi \geq M^2\Psi \]

Effective two-particle equation

\[ \left( \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right) \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp) \]

Azimuthal Basis \( \zeta, \phi \)

Semiclassical first approximation to QCD

\[ U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2) \]

Confining AdS/QCD potential

QCD Meson Spectrum
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k^2_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k^2_\perp}{2\kappa^2 x(1-x)}} \]

Note coupling \( k^2_\perp, x \)

\[ \phi_M(x, Q_0) \propto \sqrt{x(1-x)} \]

“Soft Wall” model

\( \kappa = 0.375 \text{ GeV} \)

massless quarks

Connection of Confinement to TMDs

Berlin
October 8, 2009

LF Holography/QCD
56

Stan Brodsky
SLAC
Hadron Distribution Amplitudes

\[ \phi_H(x_i, Q) \]

\[ \sum_i x_i = 1 \]

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

\[ \phi_M(x, Q) = \int Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp) \]
Second Moment of Pion Distribution Amplitude

\[
< \xi^2 > = \int_{-1}^{1} d\xi \ \xi^2 \phi(\xi)
\]

\[
\xi = 1 - 2x
\]

\[
< \xi^2 >_\pi = \frac{1}{5} = 0.20
\]

\[
\phi_{asymp} \propto x(1 - x)
\]

\[
< \xi^2 >_\pi = \frac{1}{4} = 0.25
\]

\[
\phi_{AdS/QCD} \propto \sqrt{x(1 - x)}
\]

Lattice (I) \[
< \xi^2 >_\pi = 0.28 \pm 0.03
\]

Lattice (II) \[
< \xi^2 >_\pi = 0.269 \pm 0.039
\]

Donnellan et al.  
Braun et al.

Stan Brodsky  
SLAC
Photon-to-pion transition form factor

$Q^2 F_{\gamma \rightarrow \pi^0}(Q^2)$

F. Cao, GdT, sjb (preliminary)
ERBL Evolution of Pion Distribution Amplitude

\[ \phi(x, Q^2)/f_\pi \]

\[ x(1 - x) \]

\[ Q^2 = 100 \text{ GeV}^2 \]

\[ Q^2 = 2 \text{ GeV}^2 \]

\[ \sqrt{x(1 - x)} \]

F. Cao, GdT, sjb (preliminary)
Photon-to-pion transition form factor with ERBL evolution

\[ Q^2 F_{\gamma \rightarrow \pi^0}(Q^2) \]

\begin{center}
\begin{tabular}{c}
\text{BaBar} \\
\text{CLEO} \\
\text{CELLO} \\
\text{asymptotic} \\
\text{AdS, evolved} \\
\text{CZ, evolved} \\
\text{Flat, evolved}
\end{tabular}
\end{center}

F. Cao, GdT, sbj (preliminary)
Baryons Spectrum in "bottom-up" holographic QCD


Baryons in Ads/CFT

- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\bar{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g} \bar{\Psi}(x, z) \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z).$$

- Equation of motion:

$$\left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z) = 0$$

$$\left[ i \left( z\eta^\ell m \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0.$$
Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha \Pi(\zeta) \psi(\zeta) = \mathcal{M} \psi(\zeta),$$

where $H_{LF} = \alpha \Pi$ and the operator

$$\Pi_L(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{L}{\zeta} + \frac{1}{2} \zeta \gamma_5 \right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[ \Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$
• Note: in the Weyl representation \((i\alpha = \gamma_5\beta)\)

\[
i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.
\]

• Baryon: twist-dimension \(3 + L \ (\nu = L + 1)\)

\[
\mathcal{O}_{3+L} = \psi D_{\ell_1} \cdots D_{\ell_q} \psi D_{\ell_{q+1}} \cdots D_{\ell_m} \psi, \quad L = \sum_{i=1}^{m} \ell_i.
\]

• Solution to Dirac eigenvalue equation with UV matching boundary conditions

\[
\psi(\zeta) = C \sqrt{\zeta} [J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-].
\]

Baryonic modes propagating in AdS space have two components: orbital \(L\) and \(L + 1\).

• Hadronic mass spectrum determined from IR boundary conditions

\[
\psi_{\pm}(\zeta = 1/\Lambda_{QCD}) = 0,
\]
given by

\[
\mathcal{M}^{\pm}_{\nu,k} = \beta_{\nu,k} \Lambda_{QCD}, \quad \mathcal{M}^{-}_{\nu,k} = \beta_{\nu+1,k} \Lambda_{QCD},
\]
with a scale independent mass ratio.
Fig: Light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV in the HW model. The 56 trajectory corresponds to $L$ even $P = +$ states, and the 70 to $L$ odd $P = -$ states.
Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

• We write the Dirac equation

\[(\alpha \Pi(\zeta) - \mathcal{M}) \psi(\zeta) = 0,\]

in terms of the matrix-valued operator \(\Pi\)

\[\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),\]

and its adjoint \(\Pi_\nu^\dagger\), with commutation relations

\[\left[ \Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.\]

• Solutions to the Dirac equation

\[\psi_+(\zeta) \sim z^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2),\]

\[\psi_-(\zeta) \sim z^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2).\]

• Eigenvalues

\[\mathcal{M}^2 = 4\kappa^2 (n + \nu + 1).\]
\begin{align*}
\text{Parent and daughter } & 56 \text{ Regge trajectories for the } N \text{ and } \Delta \text{ baryon families for } \kappa = 0.5 \text{ GeV}
\end{align*}
E. Klempt et al.: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

<table>
<thead>
<tr>
<th>$SU(6)$</th>
<th>$S$</th>
<th>$L$</th>
<th>Baryon State</th>
</tr>
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<tbody>
<tr>
<td>56</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$N_{\frac{1}{2}}^{1^+}(939)$</td>
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<tr>
<td></td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$\Delta_{\frac{3}{2}}^{3^+}(1232)$</td>
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<td>1</td>
<td>$N_{\frac{1}{2}}^{1^-}(1535)$ $N_{\frac{3}{2}}^{3^-}(1520)$</td>
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<tr>
<td></td>
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<td>1</td>
<td>$N_{\frac{1}{2}}^{1^-}(1650)$ $N_{\frac{3}{2}}^{3^-}(1700)$ $N_{\frac{5}{2}}^{5^-}(1675)$</td>
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<tr>
<td></td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>$\Delta_{\frac{1}{2}}^{1^-}(1620)$ $\Delta_{\frac{3}{2}}^{3^-}(1700)$</td>
</tr>
<tr>
<td>56</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>$N_{\frac{3}{2}}^{3^+}(1720)$ $N_{\frac{5}{2}}^{5^+}(1680)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}$</td>
<td>2</td>
<td>$\Delta_{\frac{1}{2}}^{1^+}(1910)$ $\Delta_{\frac{3}{2}}^{3^+}(1920)$ $\Delta_{\frac{5}{2}}^{5^+}(1905)$ $\Delta_{\frac{7}{2}}^{7^+}(1950)$</td>
</tr>
<tr>
<td>70</td>
<td>$\frac{1}{2}$</td>
<td>3</td>
<td>$N_{\frac{5}{2}}^{5^-}$ $N_{\frac{7}{2}}^{7^-}$</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
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<td>$N_{\frac{7}{2}}^{7^+}$ $N_{\frac{9}{2}}^{9^+}(2220)$</td>
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<tr>
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<tr>
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<td>$N_{\frac{9}{2}}^{9^-}$ $N_{\frac{11}{2}}^{11^-}(2600)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}$</td>
<td>5</td>
<td>$N_{\frac{7}{2}}^{7^-}$ $N_{\frac{9}{2}}^{9^-}$ $N_{\frac{11}{2}}^{11^-}$ $N_{\frac{13}{2}}^{13^-}$</td>
</tr>
</tbody>
</table>
Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

\[ F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2, \]
\[ F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2, \]

where the effective charges \( g_+ \) and \( g_- \) are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have \( S^z = +1/2 \). The two AdS solutions \( \psi_+(\zeta) \) and \( \psi_-(\zeta) \) correspond to nucleons with \( J^z = +1/2 \) and \(-1/2\).

- For \( SU(6) \) spin-flavor symmetry

\[ F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2, \]
\[ F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right], \]

where \( F_1^p(0) = 1, \ F_1^n(0) = 0. \)
• Scaling behavior for large $Q^2$: $Q^4 F^p_1(Q^2) \rightarrow \text{constant}$

Proton $\tau = 3$

• Scaling behavior for large $Q^2$: $Q^4F_1^n(Q^2) \rightarrow \text{constant}$  

[Graph showing the relationship between $Q^4F_1^n(Q^2)$ and $Q^2$ (GeV$^2$)].

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

$$F_2^p(Q^2)$$

Harmonic Oscillator
Confinement
Normalized to anomalous moment

$\kappa = 0.49$ GeV

G. de Teramond, sjb

$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m\pi m_p}$
in chiral perturbation theory

AdS/QCD No chiral divergence!
Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

Five dimensional action in presence of dilaton background

\[ S = -\frac{1}{4} \int d^4xdz \sqrt{g} \ e^{\phi(z)} \frac{1}{g_5^2} G^2 \]

where \( \sqrt{g} = \left( \frac{R}{z} \right)^5 \) and \( \phi(z) = +\kappa^2 z^2 \)

Define an effective coupling \( g_5(z) \)

\[ S = -\frac{1}{4} \int d^4xdz \sqrt{g} \frac{1}{g_5^2 (z)} G^2 \]

Thus \( \frac{1}{g_5^2 (z)} = e^{\phi(z)} \frac{1}{g_5^2 (0)} \) or \( g_5^2 (z) = e^{-\kappa^2 z^2} g_5^2 (0) \)

Light-Front Holography:
\[ z \rightarrow \zeta = b_\perp \sqrt{x(1-x)} \]

\[ \alpha_s(q^2) \propto \int_0^\infty d\zeta J_0(\zeta Q) \alpha_s(\zeta) \] where \( \alpha_s(z) = e^{-\kappa^2 z^2} \alpha_s(0) \)
Running Coupling from Modified AdS/QCD

\[
\frac{\alpha_s^{\text{eff}}(Q^2)}{\pi} = e^{-Q^2/8\kappa^2} I_0\left(\frac{Q^2}{8\kappa^2}\right)
\]

Deur, de Teramond, sbj, (preliminary)}
Running Coupling from Modified AdS/QCD

\[ \beta_{\text{eff}} = \frac{d}{d \log Q^2} \alpha_{\text{eff}} = \frac{\pi Q^2}{8 \kappa^2} e^{-Q^2/8\kappa^2} \left[ -\frac{I_0(Q^2)}{8\kappa^2} + \frac{I_1(Q^2)}{8\kappa^2} \right] \]

Deur, de Teramond, sjb, (preliminary)
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite \( N_c = 3 \): Baryons built on 3 quarks -- Large \( N_c \) limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General ‘classical’ potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$. Spectrum is independent of $S$
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
Light-Front QCD

\[ L^{QCD} \rightarrow H^{QCD}_{LF} \]

\[ H^{QCD}_{LF} = \sum_i \left[ \frac{m^2 + k^2}{x} \right]_i + H^{int}_{LF} \]

\[ H^{int}_{LF} \]: Matrix in Fock Space

Physical gauge: \( A^+ = 0 \)

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions
Light-Front QCD

Heisenberg Equation

\[ H_{LC}^{QCD} |\psi_h\rangle = M_h^2 |\psi_h\rangle \]

Use AdS/QCD basis functions

Berlin
October 8, 2009
Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs
Features of LF T-Matrix Formalism

- Only positive + momenta; no backward time-ordered diagrams
- Frame-independent! Independent of $P^+$ and $P_z$
- LC gauge: No ghosts; physical helicity
- $J^z = L^z + S^z$ conservation at every vertex
- Sum all amplitudes with same initial-and final-state helicity, then square to get rate
- Renormalize each UV-divergent amplitude using “alternating denominator” method
- Multiple renormalization scales (BLM)
**Goal: First Approximant to QCD**

- Counting rules for Hard Exclusive Scattering
- Regge Trajectories

**AdS/QCD**

- QCD at the Amplitude Level
- Mapping of Poincaré' and Conformal $SO(4,2)$ symmetries of $3+1$ space to AdS5 space
- Conformal behavior at short distances + Confinement at large distance

**Semi-Classical QCD / Wave Equations**

**Boost Invariant 3+1 Light-Front Wave Equations**

$J = 0, 1, 1/2, 3/2$ plus $L$

**Integrable!**

**Hadron Spectra, Wavefunctions, Dynamics**

---

Berlin
October 8, 2009

LF Holography/QCD

Stan Brodsky
SLAC
Chiral Symmetry Breaking in AdS/QCD

- Chiral symmetry breaking effect in AdS/QCD depends on weighted $z^2$ distribution, not constant condensate

\[ \delta M^2 = -2m_q \langle \bar{\psi}\psi \rangle \times \int dz \, \phi^2(z) z^2 \]

- $z^2$ weighting consistent with higher Fock states at periphery of hadron wavefunction

- AdS/QCD: confined condensate

- Suggests “In-Hadron” Condensates

Erlich et al.

de Teramond, Shrock, sjb
Use Dyson-Schwinger Equation for bound-state quark propagator:

\[ < \bar{b} | \bar{q}q | \bar{b} > \text{ not } < 0 | \bar{q}q | 0 > \]
Pion mass and decay constant.
e-Print: nucl-th/9707003

Pi- and K meson Bethe-Salpeter amplitudes.
e-Print: nucl-th/9708029

Concerning the quark condensate.
e-Print: nucl-th/0301024

"In-Meson Condensate"

\[ - \langle \bar{q}q \rangle_\zeta^\pi = f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle. \]

Valid even for \[ m_q \to 0 \]
\[ f_\pi \text{ nonzero} \]
Chiral magnetism (or magnetohadronicrons)

Aharon Casher and Leonard Susskind

Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.\(^1\) Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.\(^2\) A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron’s wave function and not to the vacuum.\(^3\)
Quark and Gluon condensates reside within hadrons, not LF vacuum

- Bound-State Dyson-Schwinger Equations
- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs
- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!
- Implications for cosmological constant -- reduction by 45 orders of magnitude!

“Confined QCD Condensates”
Determinations of the vacuum Gluon Condensate

\[ \langle 0 \mid \frac{\alpha_s}{\pi} G^2 \mid 0 \rangle [\text{GeV}^4] \]

\(-0.005 \pm 0.003\) from \(\tau\) decay.  \(\text{Davier et al.}\)

\(+0.006 \pm 0.012\) from \(\tau\) decay.  \(\text{Geshkenbein, Ioffe, Zyablyuk}\)

\(+0.009 \pm 0.007\) from charmonium sum rules  \(\text{Ioffe, Zyablyuk}\)

Consistent with zero vacuum condensate
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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\[(\Omega_\Lambda)_{QCD} \sim 10^{45}\]
\[(\Omega_\Lambda)_{EW} \sim 10^{56}\]

\[\Omega_\Lambda = 0.76(\text{expt})\]

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not LF vacuum

Shrock, sjb
• Color Confinement: Maximum Wavelength of Quark and Gluons

• Conformal symmetry of QCD coupling in IR

• Conformal Template (BLM, CSR, ...)

• Motivation for AdS/QCD

• QCD Condensates inside of hadronic LFWFs

• Technicolor: confined condensates inside of technihadrons -- alternative to Higgs

• Simple physical solution to cosmological constant conflict with Standard Model

Shrock and sjb