Maximum Wavelength of Confined Quarks and Gluons and Properties of QCD

Stan Brodsky, SLAC

JTI Workshop on Dynamics of Symmetry Breaking

Argonne National Laboratory, IL
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In Collaboration with Robert Shrock and Guy de Teramond


QCD is not conformal; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** \( \alpha_s(Q^2) \sim \text{const at small } Q^2 \)

- Use mathematical mapping of the conformal group \( \text{SO}(4,2) \) to \( \text{AdS}_5 \) space

**AdS/CFT:** Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map \( \text{AdS}_5 \times S_5 \) to conformal \( N=4 \) SUSY
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu\nu}, P^\mu, D, K^\mu, \]

the generators of \( \text{SO}(4,2) \)

\( \text{SO}(4,2) \) has a mathematical representation on AdS5
Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

Invariant measure

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$  

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

ψ (x, z) ~ z^Δ

ψ (x, z) ~ z^Δ

5-Dimensional Anti-de Sitter Spacetime

Confinement Radius

AdS Boundary (z=0)

4-Dimensional Flat Spacetime (hologram)

de Teramond, sjb

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Maximal Wavelength and QCD Properties

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Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances.

- Analogous to the Schrodinger Theory for Atomic Physics.

- AdS/QCD Light-Front Holography.

- Hadronic Spectra and Light-Front Wavefunctions.
• *Light-Front Holography*

\[ \phi(z) \]

\[ \psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

• *Light Front Wavefunctions:*

Schrödinger Wavefunctions of Hadron Physics
Prediction from AdS/QCD: Meson LFWF

\[ \psi_M(x, k^2_\perp) \]

"Soft Wall" model

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Goal: First Approximant to QCD
Counting rules for Hard Exclusive Scattering Regge Trajectories QCD at the Amplitude Level

AdS/CFT
Mapping of Poincare’ and Conformal SO(4,2) symmetries of 3 +1 space to AdS5 space

AdS/QCD
Conformal Invariance + Confinement at large distances

Semi-Classical QCD / Wave Equations
Light Front Holography

Boost Invariant 3+1 Light-Front Wave Equations
Integrable!

Hadron Spectra, Wavefunctions, Dynamics

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Verifying Asymptotic Freedom

\[ \frac{\sigma(e^+e^-\rightarrow \text{three jets})}{\sigma(e^+e^-\rightarrow \text{two jets})} \]

Ratio of rate for \( e^+e^- \rightarrow q\bar{q}g \) to \( e^+e^- \rightarrow q\bar{q} \)

at \( Q = E_{CM} = E_{e^-} + E_{e^+} \)

proportional to \( \alpha_s(Q) \)

\[ \alpha(Q^2) \simeq \frac{4\pi}{\beta_0} \frac{1}{\log Q^2/\Lambda_{QCD}^2} \]
\[ \Gamma_{bj}^{p-n} (Q^2) \equiv \frac{g_A}{6} \left[ 1 - \frac{\alpha_s g_1 (Q^2)}{\pi} \right] \]

**Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule**

**IR conformal window**

\[ \alpha_s^{g_1} (Q^2) \]

**GDH constraint**

- JLab CLAS
- JLab PLB 650 4 244
- $\alpha_{s,g_1}/\pi$ world data
- $\alpha_{s,F_3}/\pi$
- GDH limit
- pQCD evol. eq.
- $\alpha_{s,\tau}/\pi$ OPAL

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**Maximal Wavelength and QCD Properties**

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Deur, Korsch, et al.

\[ \frac{\alpha_s}{\pi} J_{\text{Lab}} \quad \text{GDH limit} \quad \text{Fit} \quad pQCD \text{ evol. eq.} \]

Cornwall

\[ \text{Bhagwat et al.} \quad \text{Maris-Tandy} \quad \text{DSE gluon couplings} \]

\[ \text{Bloch et al.} \quad \text{Godfrey-Isgur} \quad \text{Lattice QCD} \]

\( Q \text{ (GeV)} \)

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IR Conformal Window for QCD

• Dyson-Schwinger Analysis: QCD gluon coupling has IR Fixed Point

• Evidence from Lattice Gauge Theory

• Define coupling from observable: indications of IR fixed point for QCD effective charges

• Confined gluons and quarks have maximum wavelength: Decoupling of QCD vacuum polarization at small $Q^2$

\[ \Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2 \]

• Justifies application of AdS/CFT in strong-coupling conformal window

Serber-Uehling

Shrock, de Teramond, sjb

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\[ M_{ee \rightarrow ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u) \]

\[ \alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)} \]

Gell Mann-Low Effective Charge for QED
QED One-Loop Vacuum Polarization

\[ \Pi(Q^2) = \frac{\alpha(0)}{3\pi} \left[ \frac{5}{3} - \frac{4m^2}{Q^2} - (1 - \frac{2m^2}{Q^2}) \sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{1 - \sqrt{1 + \frac{4m^2}{Q^2}}} \right] \]

\[ \Pi(Q^2) \sim \frac{\alpha(0)}{3\pi} \log \frac{Q^2}{m^2} \quad Q^2 >> 4m^2 \]

\[ \beta = \frac{d(\frac{\alpha}{4\pi})}{d\log Q^2} = \frac{4}{3}(\frac{\alpha}{4\pi})^2 n_\ell > 0 \]

\[ \Pi(Q^2) = \frac{\alpha(0) Q^2}{15\pi m^2} \quad Q^2 << 4m^2 \quad \text{Serber-Uehling} \]

\[ \beta \propto \frac{Q^2}{m^2} \quad \text{vanishes at small momentum transfer} \]
Lesson from QED:

**Lamb Shift in Hydrogen**

\[ \Delta E \sim \alpha (Z \alpha)^4 \ln (Z \alpha)^2 m_e \]

\[ \lambda < \frac{1}{Z \alpha m_e} \]

\[ k > Z \alpha m_e \]

Maximum wavelength of bound electron

Infrared divergence of free electron propagator removed because of atomic binding
Lesson from QED and Lamb Shift:
maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\Lambda_{QCD}} \]

\[ \lambda < \Lambda_{QCD} \]

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gluon and quark propagators cutoff in IR because of color confinement
Maximal Wavelength of Confined Fields

- Colored fields confined to finite domain \((x - y)^2 < \Lambda_{QCD}^{-2}\)
- All perturbative calculations regulated in IR
- High momentum calculations unaffected
- Bound-state Dyson-Schwinger Equation
- Analogous to Bethe’s Lamb Shift Calculation

Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale \(\Lambda_{QCD}^{-1}\) (whether or not the segment is spacelike or timelike).
Single-spin asymmetries

Pseudo-\(T\)-Odd

\[ i \vec{S}_p \cdot \vec{q} \times \vec{p}_q \]

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

QCD S- and P- Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves

Maximal Wavelength and QCD Properties

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Final-State Interactions Produce Pseudo $T$-Odd (Sivers Effect)

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in $S$- and $P$-waves; Wilson line effect; gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale: IR Fixed Point!
- New window to QCD coupling and running gluon mass in the IR
- QED $S$ and $P$ Coulomb phases infinite -- difference of phases finite

$i \, \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$

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Maximal Wavelength and QCD Properties

\[ h^{\perp}_1(x_1, p^2_\perp) \times \bar{h}^{\perp}_1(x_2, k^2_\perp) \]

**DY cos 2\phi correlation at leading twist from double ISI**

**Product of Boer-Mulders Functions**

---

**Kopeliovich**

**Boer, Hwang, sjb**

---

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**Maximal Wavelength and QCD Properties**

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Double Initial-State Interactions generate anomalous $\cos 2\phi$:

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)
\]

Drell-Yan planar correlations

\[
\frac{\nu}{2} \propto h_1^+(\pi) h_1^+(N).
\]

PQCD Factorization (Lam Tung):

\[
1 - \lambda - 2\nu = 0
\]

Double Initial-State Interactions generate anomalous $\cos 2\phi$:

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)
\]

PQCD Factorization (Lam Tung):

\[
1 - \lambda - 2\nu = 0
\]

\[
\pi N \rightarrow \mu^+ \mu^- X \quad \text{NA10}
\]

\[
\nu(Q_T)
\]

Hard gluon radiation.

Double ISI.

Volates Lam-Tung relation!

Model: Boer, Hwang, sjb

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Remarkable observation at HERA

10% to 15% of DIS events are diffractive!

Fraction $r$ of events with a large rapidity gap, $\eta_{\text{max}} < 1.5$, as a function of $Q_{\text{DA}}^2$ for two ranges of $x_{\text{DA}}$. No acceptance corrections have been applied.

Deep Inelastic Electron-Proton Scattering

Conventional wisdom wrong:
Final-state interactions of struck quark cannot be neglected

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Final-State Interaction Produces Diffractive DIS

Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHM Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron
QCD Mechanism for Rapidity Gaps

Wilson Line: \( \bar{\psi}(y) \int_0^y dx \, e^{iA(x)\cdot dx} \psi(0) \)

 Origin of Diffractive DIS
Reproduces lab-frame color dipole approach

Hoyer, Marchal, Peigne, Sannino, sjb

QCD Mechanism for Rapidity Gaps

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Final State Interactions in QCD

\[ \begin{array}{c}
\gamma^* \\
q \\
k_1 \quad k_2
\end{array} \quad \begin{array}{c}
\gamma^* \\
q \\
k_1 \quad k_2
\end{array} \]

Feynman Gauge  Light-Cone Gauge

Result is Gauge Independent
Integration over on-shell domain produces phase $i$

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target
Conformal symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses

- Eigensolutions of ERBL evolution equation for distribution amplitudes

- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

- Fix Renormalization Scale (BLM, Effective Charges)

- Use AdS/CFT

V. Braun et al; Frishman, Lepage, Sachrajda, sjb

H. J. Lu, sjb

Kataev, Gabadadze, Rathsman, Lu, sjb

Grunberg

Lepage, Mackenzie, Binger, sjb

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Maximal Wavelength and QCD Properties

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Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

\[
R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right].
\]

\[
\int_0^1 dx \left[ g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha g_1(Q)}{\pi} \right].
\]
Generalized Crewther Relation

\[
[1 + \frac{\alpha_R(s^*)}{\pi}] \left[1 - \frac{\alpha g_1(q^2)}{\pi}\right] = 1
\]

\[\sqrt{s^*} \approx 0.52Q\]

Conformal relation true to all orders in perturbation theory

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

Analytic matching at quark thresholds

No renormalization scale ambiguity!
\[
\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{MS}(Q)}{\pi} + \left( \frac{\alpha_{MS}(Q)}{\pi} \right)^2 \left[ \left( \frac{41}{8} - \frac{11}{3} \zeta_3 \right) C_A - \frac{1}{8} C_F + \left( -\frac{11}{12} + \frac{2}{3} \zeta_3 \right) f \right] \\
+ \left( \frac{\alpha_{MS}(Q)}{\pi} \right)^3 \left\{ \left( \frac{90445}{2592} - \frac{2737}{108} \zeta_3 - \frac{55}{18} \zeta_5 - \frac{121}{432} \pi^2 \right) C_A^2 + \left( -\frac{127}{48} - \frac{143}{12} \zeta_3 + \frac{55}{3} \zeta_5 \right) C_A C_F - \frac{23}{32} C_F^2 \\
+ \left[ \left( \frac{970}{81} + \frac{224}{27} \zeta_3 + \frac{5}{9} \zeta_5 + \frac{11}{108} \pi^2 \right) C_A + \left( -\frac{29}{96} + \frac{19}{6} \zeta_3 - \frac{10}{3} \zeta_5 \right) C_F \right] f \\
+ \left( \frac{151}{162} - \frac{19}{27} \zeta_3 - \frac{1}{108} \pi^2 \right) f^2 + \left( \frac{11}{144} - \frac{1}{6} \zeta_3 \right) \frac{d^{abc} d^{abc}}{C_F d(R)} \frac{\left( \sum_f Q_f \right)^2}{\sum_f Q_f^2} \right\}. 
\]

\[
\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{MS}(Q)}{\pi} + \left( \frac{\alpha_{MS}(Q)}{\pi} \right)^2 \left[ \frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f \right] \\
+ \left( \frac{\alpha_{MS}(Q)}{\pi} \right)^3 \left\{ \left( \frac{5437}{648} - \frac{55}{18} \zeta_5 \right) C_A^2 + \left( -\frac{1241}{432} + \frac{11}{9} \zeta_3 \right) C_A C_F + \frac{1}{32} C_F^2 \\
+ \left[ \left( \frac{3535}{1296} - \frac{1}{2} \zeta_3 + \frac{5}{9} \zeta_5 \right) C_A + \left( \frac{133}{864} + \frac{5}{18} \zeta_3 \right) C_F \right] f + \frac{115}{648} f^2 \right\}. 
\]

Eliminate MSbar, Find Amazing Simplification

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\[ R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right]. \]

\[ \int_0^1 dx \left[ g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right] \]

\[ \frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left( \frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left( \frac{\alpha_R(Q^{***})}{\pi} \right)^3 \]

**Geometric Series in Conformal QCD**

**Generalized Crewther Relation**

Lu, Kataev, Gabadadze, Sjb
Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation
Transitivity Property of Renormalization Group

Relation of observables independent of intermediate scheme $C$

$A \rightarrow C \quad C \rightarrow B \quad \text{identical to} \quad A \rightarrow B$
Leading Order Commensurate Scales

\[ \alpha_\tau (1.36Q) \]
\[ \alpha_\eta b (1.67Q) \]
\[ \alpha_{GLS} (1.18Q) \]
\[ \alpha_M (0.904Q) \]
\[ \alpha_p (Q) \]
\[ \alpha_{MS} (0.435Q) \]
\[ \alpha_T (2.77Q) \]
\[ \alpha_R (0.614Q) \]
\[ \alpha_g (1.18Q) \]

Translation between schemes at LO

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Leading-Twist PQCD Factorization for form factors, exclusive amplitudes

\[ M = \int \prod dx_i dy_i \phi_F(x_i, \bar{Q}) \times T_H(x_i, y_i, \bar{Q}) \times \phi_I(y_i, \bar{Q}) \]

If \( \alpha_s(\bar{Q}^2) \approx \text{constant} \)

\[ Q^4 F_1(Q^2) \approx \text{constant} \]
• Scaling behavior for large $Q^2$: $Q^4F^p_1(Q^2) \to \text{constant}$

\[ 
\begin{array}{c}
\text{Proton } \tau = 3
\end{array}
\]

Conformal behavior: $Q^2 F_\pi (Q^2) \rightarrow \text{const}$

\[ \begin{align*}
&\text{Determination of the Charged Pion Form Factor at} \\
&\text{Q2=1.60 and 2.45 (GeV/c)^2.} \\
&\text{By Fpi2 Collaboration (T. Horn et al.). Jul 2006. 4pp.} \\
&\text{e-Print Archive: nucl-ex/0607005}
\end{align*} \]
Consequences of Maximum Quark and Gluon Wavelength

- Infrared integrations regulated by confinement
- Infrared fixed point of QCD coupling
  \[ \alpha_s(Q^2) \text{ finite, } \beta \to 0 \text{ at small } Q^2 \]
- Bound state quark and gluon Dyson-Schwinger Equation
- Quark and Gluon Condensates exist within hadrons

Casher, Susskind
Shrock, sjb
Maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\Lambda_{\text{QCD}}} \]

\[ \lambda < \Lambda_{\text{QCD}} \]

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

\[ \langle \bar{b} | \bar{q}q | \bar{b} \rangle \text{ not } \langle 0 | \bar{q}q | 0 \rangle \]

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