Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Event amplitude generator

\( \psi(x, k_\perp, \lambda_i) \)
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

Event amplitude generator

Capture if \( \zeta^2 = x(1 - x)b^2 \perp > \frac{1}{\Lambda_{QCD}^2} \)

i.e.,

\[ M^2 = \frac{k^2}{x(1-x)} < \Lambda_{QCD}^2 \]

AdS/QCD

Hard Wall Confinement:
Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb

\[ \bar{H}(\bar{p}e^+) \]

\[ b_\perp \leq \frac{1}{m_{\text{red}}\alpha} \]

\[ y_{\bar{p}} \simeq y_{e^+} \]

Coalescence of off-shell co-moving positron and antiproton.

Wavefunction maximal at small impact separation and equal rapidity

“Hadronization” at the Amplitude Level
**Features of LF T-Matrix Formalism**

**“Event Amplitude Generator”**

- Coalesce color-singlet cluster to hadronic state if
  \[
  \mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2
  \]
  \[
  \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)
  \]

- The coalescence probability amplitude is the LF wavefunction

- No IR divergences: Maximal gluon and quark wavelength from confinement

\[
x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}
\]

\[
P^+ = p^0 + p^z
\]
Features of LF T-Matrix Formalism

“Event Amplitude Generator”

If $M_n^2 \geq \Lambda_{QCD}^2$ use PQCD hard gluon exchange

- DGLAP and ERBL Evolution from gluon emission and exchange
- Factorization Scale for structure functions and fragmentation functions set: $\mu_{fact} = \Lambda_{QCD}$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp \ni$

$P^+ = P^0 + P^z$
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

Event amplitude generator for DIS

Wilson Line Effect: Rescattering: FSI

\[ \psi_{n/H}^{LF}(x_i, \vec{k}_{\perp i}, \lambda_i) \]

\[ p \]

\[ \gamma^* \]
Features of \textit{LF T-Matrix Formalism}

- Only positive \( P^+ \) momenta; no backward time-ordered diagrams
- Frame-independent! Independent of \( P^+ \) and \( P^z \)
- LC gauge: No ghosts; physical helicity
- \( J^z = L^z + S^z \) conservation at every vertex
- Sum all amplitudes with same initial-and final-state helicity, then square to get rate
- Renormalize each UV-divergent amplitude using “alternating denominator” method
- Multiple renormalization scales (BLM)
Features of LF T-Matrix Formalism

“Event Amplitude Generator”

• Same principle as antihydrogen production: off-shell coalescence

• coalescence to hadron favored at equal rapidity, small transverse momenta

• leading heavy hadron production: D and B mesons produced at large z

• hadron helicity conservation if hadron LFWF has $L^z = 0$

• Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp i$
\[ \pi N \rightarrow \mu^+ \mu^- X \text{ at high } x_F \]

In the limit where \((1-x_F)Q^2\) is fixed as \(Q^2 \rightarrow \infty\)

Entire pion wave function contributes to hard process

Virtual photon is longitudinally polarized

"Direct" Subprocess

Berger, sjb
Khoze, Brandenburg, Muller, sjb
Hoyer Vanttinen

Stan Brodsky
SLAC
\[ \pi^{-} N \rightarrow \mu^{+} \mu^{-} X \text{ at } 80 \text{ GeV/c} \]

\[ \frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \rho \sin 2\theta \cos \phi + \omega \sin^2 \theta \cos 2\phi. \]

\[ \frac{d^2\sigma}{dx_\pi d\cos \theta} \propto x_\pi \left( (1 - x_\pi)^2 (1 + \cos^2 \theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2 \theta \right) \]

\[ \langle k_T^2 \rangle = 0.62 \pm 0.16 \text{ GeV}^2/c^2 \]

Dramatic change in angular distribution at large \( x_T \)

Example of a higher-twist direct subprocess

Chicago-Princeton Collaboration


JTI Workshop ANL
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AdS/QCD and LF Holography

Stan Brodsky
SLAC

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Crucial Test of Leading-Twist QCD: Scaling at fixed $x_T$

$$x_T = \frac{2p_T}{\sqrt{s}}$$

$$E \frac{d\sigma}{d^3p} (pN \rightarrow \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

**Parton model:** $n_{eff} = 4$

**As fundamental as Bjorken scaling in DIS**

**Conformal scaling:** $n_{eff} = 2n_{active} - 4$
Scaling of direct photon production consistent with PQCD
\[ pp \rightarrow \gamma X \]

\[ E \frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4} \]

\[ gu \rightarrow \gamma u \]

\[ n_{active} = 4 \]

\[ n_{eff} = 2n_{active} - 4 \]

\[ n_{eff} = 4 \]
Higher-Twist Contribution to Hadron Production

\[ gq \rightarrow \pi q \]

\[ \phi_{\pi}(x, p_{\perp}^2) \propto f_{\pi} \]

\[ \frac{d\sigma}{d^3 p/E} = \alpha_s^3 f_{\pi}^2 \frac{F(x_{\perp}, y)}{p_{\perp}^6} \]

No Fragmentation Function
Clear evidence for higher-twist contributions

Fermilab, ISR data

Continuous Rise of $n_{\text{eff}}$

$n_{\text{eff}}$

$n_{\text{eff}}$

$E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^{n_{\text{eff}}}}$

$x_T = 2p_T/\sqrt{s}$

$E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^{n_{\text{eff}}}}$
QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

\[
\frac{d\sigma}{d^3 p/E} = \frac{F(x_\perp, y)}{p_{\perp}^n(x_\perp)}
\]

DSS
(De Florian-Sassot-Stratmann)

\[
pp \to \pi X
\]

\[
pp \to \gamma X
\]

5 < p_{\perp} < 20 \text{ GeV}

70 \text{ GeV} < \sqrt{s} < 4 \text{ TeV}
$5 < p_\perp < 20 \text{ GeV} \quad 70 \text{ GeV} < \sqrt{s} < 4 \text{ TeV}$

\[ \frac{d\sigma}{d^3p/E} = \frac{F(x_\perp, y)}{p_\perp} \]

$p p \to \pi X$

$p p \to p X$

$p p \to \bar{p} X$

\[ x_\perp = \frac{2p_\perp}{\sqrt{s}} \]

Arleo, Aurenche

De Florian-Sassot-Stratmann
\[ \frac{E}{d^3 p} (pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{\text{eff}}} \]
Baryon Anomaly: Particle ratio changes with centrality!

Protons less absorbed in nuclear collisions than pions


Central

Peripheral

Sickles, sjb
\[ \sqrt{s_{NN}} = 130 \text{ and } 200 \text{ GeV} \]

Proton power changes with centrality!
$pp \rightarrow HX$ at high $p_T$

Proton created from jet fragmentation

Color Opaque

$n_{\text{active}} = 4$

$n = 2n_{\text{active}} - 4$

$n = 4$

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AdS/QCD and LF Holography

Stan Brodsky
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Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

\[ b_\perp \simeq 1/p_T \]

Collision can produce 3 collinear quarks

\[ uu \rightarrow p\bar{d} \]

\[ \phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2 \]

Small color-singlet

Color Transparent

Minimal same-side energy

\[ n_{active} = 6 \]

\[ n_{eff} = 2n_{active} - 4 \]

\[ n_{eff} = 8 \]

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles, sjb

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AdS/QCD and LF Holography

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Baryon made directly within hard subprocess

\[ b_\perp \simeq 1 \text{ fm} \]

\[ b_\perp \simeq 1/p_T \]

\[ uu \rightarrow p\bar{d} \]

\[ n_{\text{active}} = 6 \]

\[ n_{\text{eff}} = 2n_{\text{active}} - 4 \]

\[ n_{\text{eff}} = 8 \]
Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process.
Proton production more dominated by color-transparent direct high-$n_{\text{eff}}$ subprocesses

Proton trigger: $2.5 < p_T < 4.0$ GeV/c
Associated: $1.8 < p_T < 2.5$ GeV/c

Anne Sickles

Stan Brodsky
SLAC
Central Au+Au: PHENIX
Central Au+Au: STAR
p+p NSD: STAR
e^+e^- → ggg: ARGUS
e^+e^- → q\bar{q}: ARGUS

Central Au+Au: STAR
40%-60% central: STAR
200 GeV p+p: STAR
630 GeV p+p: UA1

Baryon to Meson Ratios

Transverse Momentum $p_T$ (GeV/c)
Power-law exponent $n(x_T)$ for $\pi^0$ and $h$ spectra in central and peripheral Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV


**Proton production dominated by color-transparent direct high $n_{\text{eff}}$ subprocesses**
Lambda can be made directly within hard subprocess

Coalescence within hard subprocess

$ud \rightarrow \Lambda \bar{s}$

Small color-singlet
Color Transparent
Minimal same-side energy

$\bar{s}$ produced on away side

$n_{\text{active}} = 6$

$n_{\text{eff}} = 2n_{\text{active}} - 4$

$n_{\text{eff}} = 8$

Sickles, sjb

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AdS/QCD and LF Holography

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Baryon Anomaly: Evidence for Direct, Higher-Twist Subprocesses

- Explains anomalous power behavior at fixed $x_T$
- Protons more likely to come from direct higher-twist subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of color transparency
- Predicts increasing proton to pion ratio in central collisions
- Proton power $n_{\text{eff}}$ increases with centrality since leading twist contribution absorbed
- Fewer same-side hadrons for proton trigger at high centrality
- Exclusive-inclusive connection at $x_T = 1$
**Chiral Symmetry Breaking in AdS/QCD**

We consider the action of the $X$ field which encodes the effects of CSB in AdS/QCD:

$$S_X = \int d^4x dz \sqrt{g} \left( g^{\ell m} \partial_\ell X \partial_m X - \mu_X^2 X^2 \right),$$

with equations of motion

$$z^3 \partial_z \left( \frac{1}{z^3} \partial_z X \right) - \partial_\rho \partial^\rho X - \left( \frac{\mu_X R}{z} \right)^2 X = 0. \quad (2)$$

The zero mode has no variation along Minkowski coordinates

$$\partial_\mu X(x, z) = 0,$$

thus the equation of motion reduces to

$$[z^2 \partial_z^2 - 3z \partial_z + 3] X(z) = 0. \quad (3)$$

for $(\mu_X R)^2 = -3$, which corresponds to scaling dimension $\Delta_X = 3$. The solution is

$$X(z) = \langle X \rangle = Az + Bz^3, \quad (4)$$

where $A$ and $B$ are determined by the boundary conditions.

$$A \propto m_q \quad B \propto < \bar{\psi} \psi >$$

*Expectation value taken inside hadron*
In presence of quark masses the Holographic LF wave equation is \((\zeta = z)\)

\[
\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2} \right] \phi(\zeta) = M^2 \phi(\zeta),
\]

and thus

\[
\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.
\]

The parameter \(a\) is determined by the Weisberger term

\[
a = \frac{2}{\sqrt{x}}.
\]

Thus

\[
X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x} \langle \bar{\psi} \psi \rangle z^3,
\]

and

\[
\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2 \sum_i m_i \langle \bar{\psi} \psi \rangle \langle z^2 \rangle + \langle \bar{\psi} \psi \rangle^2 \langle z^4 \rangle,
\]

where we have used the sum over fractional longitudinal momentum \(\sum_i x_i = 1\).
Chiral Symmetry Breaking in AdS/QCD

- Chiral symmetry breaking effect in AdS/QCD depends on weighted $z^2$ distribution, not constant condensate

$$\delta M^2 = -2m_q \langle \bar{\psi}\psi \rangle \times \int dz \, \phi^2(z) z^2$$

- $z^2$ weighting consistent with higher Fock states at periphery of hadron wavefunction

- AdS/QCD supports confined condensate picture

de Teramond, Shrock, sjb
Quark and Gluon condensates reside within hadrons, not vacuum

- **Bound-State Dyson-Schwinger Equations**
- LF vacuum trivial up to $k^+ = 0$ zero modes
- Analogous to finite size superconductor
- Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict

Casher and Susskind  Roberts et al.  Shrock and sjb

Roberts et al.

Shrock and sjb
Pion mass and decay constant.

Pi- and K meson Bethe-Salpeter amplitudes.

Concerning the quark condensate.

"In-Meson Condensate"

\[ - \langle \bar{q}q \rangle_{\pi} \zeta = f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle. \]

Valid even for \( m_q \to 0 \)

\( f_\pi \) nonzero
QCD Symmetries

• Color Confinement: Maximum Wavelength of Quark and Gluons

• Conformal symmetry of QCD coupling in IR

• Provides Conformal Template

• Motivation for AdS/QCD

• QCD Condensates inside of hadronic LFWFs

• Technicolor: confined condensates inside of technihadrons -- alternative to Higgs

• Simple physical solution to cosmological constant conflict
LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.

- Relation of spin, momentum, and other distributions to physics of the hadron itself.

- Connections between observables, orbital angular momentum

- Role of FSI and ISIs--Sivers effect

- Higher Fock States give GMOR Relations, Chiral Symmetry Breaking
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level

- Holographic Model from AdS/CFT: Confinement at large distances and conformal behavior at short distances

- Model for LFWFs, meson and baryon spectra: many applications!

- New basis for diagonalizing Light-Front Hamiltonian

- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.

- Quark Interchange dominant force at short distances
Quark Interchange  
(Spin exchange in atom-atom scattering)

\[
\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}
\]

\[M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}\]

Gluon Exchange  
(Van der Waal -- Landshoff)

\[M(s, t)_{\text{gluon exchange}} \propto sF(t)\]

MIT Bag Model (de Tar), large \( N_c \), ('t Hooft), AdS/CFT  
all predict dominance of quark interchange:

CIM: Blankenbecler, Gunion, sjb

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AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions.

\[ M(t, u)_{\text{interchange}} \propto \frac{1}{u t^2} \]

Non-linear Regge behavior:

\[ \alpha_R(t) \to -1 \]
Why is quark-interchange dominant over gluon exchange?

Example: \( M(K^+p \rightarrow K^+p) \propto \frac{1}{ut^2} \)

Exchange of common \( u \) quark

\[
M_{QIM} = \int d^2k_\perp dx \, \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B
\]

Holographic model (Classical level):

Hadrons enter 5th dimension of \( AdS_5 \)

Quarks travel freely within cavity as long as separation \( z < z_0 = \frac{1}{\Lambda_{QCD}} \)

LFWFs obey conformal symmetry producing quark counting rules.
Comparison of Exclusive Reactions at Large $t$

B. R. Baller, (a) G. C. Blazey, (b) H. Courant, K. J. Heller, S. Heppelmann, (c) M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl (d)

University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue (e) and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747

(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^\pm p \rightarrow p\pi^\pm, p\rho^\pm, \pi^+\Delta^\pm, K^+\Sigma^\pm, (\Lambda^0/\Sigma^0)K^0; K^\pm p \rightarrow pK^\pm; p^\pm p \rightarrow pp^\pm$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^\pm p \rightarrow p\pi^\pm,$
$K^\pm p \rightarrow pK^\pm,$
$\pi^\pm p \rightarrow p\rho^\pm,$
$\pi^\pm p \rightarrow \pi^+\Delta^\pm,$
$\pi^\pm p \rightarrow K^+\Sigma^\pm,$
$\pi^- p \rightarrow \Lambda^0K^0, \Sigma^0K^0,$
$p^\pm p \rightarrow pp^\pm.$
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$. Spectrum is independent of $S$
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
String Theory

AdS/CFT

AdS/QCD

Semi-Classical QCD / Wave Equations

Boost Invariant 3+1 Light-Front Wave Equations

Hadron Spectra, Wavefunctions, Dynamics

Goal: First Approximant to QCD
Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

Mapping of Poincare’ and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
Conformal behavior at short distances + Confinement at large distances
Holography
Integrable!