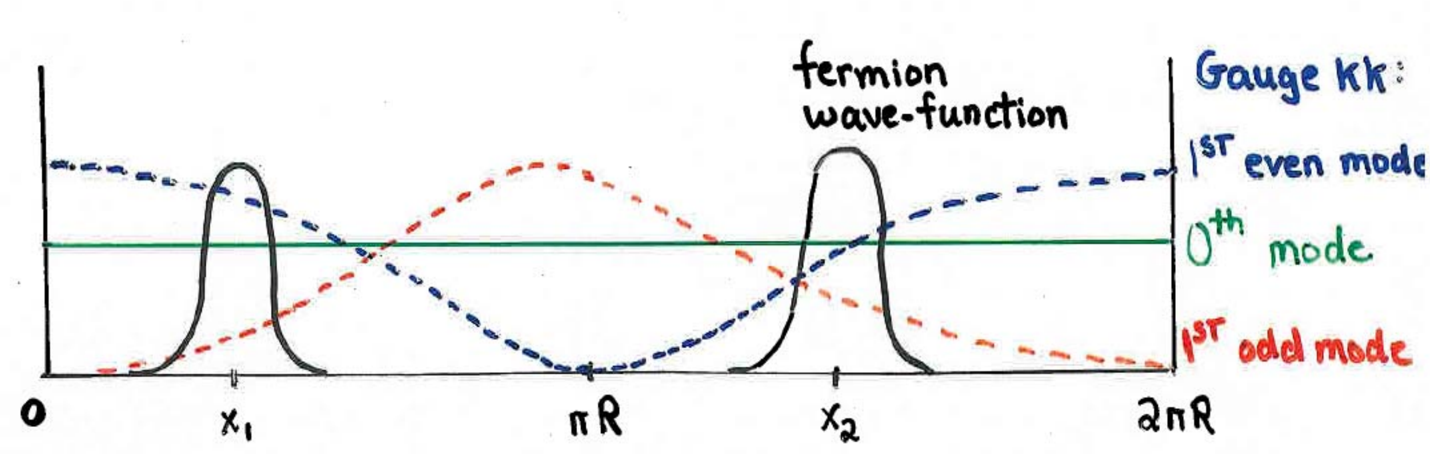
# A Taste of Flavor From Extra Dimensions





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10/29/07

# Extra Dimensions at B-Factories

Given the low center of mass energies of B-factories the most likely signatures of extra dimensions there will be in the flavor sector…this can be easily seen by a few scaling arguments…

Like all arguments there are always caveats…

# Scaling laws for Higher Dimensional Operators

Below threshold, the effects of new physics can be described by a set of higher dimensional operators that have a fixed mass dimension d:

$$\sim O/M^{(d-4)}$$

These will contribute in a *relatively* suppressed manner to low energy observables, e.g., cross sections and asymmetries as

$$\sim [s/M^2]^p \qquad \text{where} \quad p=d/2-2$$

Since cross sections scale as 1/s, we can write a scaling law for the discovery `reach' for the mass scale M as a function of s, d and the integrated luminosity, L:

# Reach ~ $(s^{d-5} L)^{1/(2d-8)}$

For example: for d=6 ~ $(sL)^{1/4}$ while for d=8 ~ $(s^3L)^{1/8}$

Compare LEPII at $E_{cm}$ ~200 GeV and L ~1 $fb^{-1}$ with a B-Factory for equal mass reach…what is the required luminosity ???

For d=6 we need ~$(20)^2$=400x more lumi OK !

For d=8 we need ~$(20)^6$=6.4 x $10^7$x more ! No way…

→ thus, e.g., ADD graviton exchange is not observable at 10 and graviton emission is not observable by similar arguments

Note that in *some* cases we may get a bit of a reprieve by comparing SM loop effects with tree-level higher dim operators due to $g^2/16\pi^2$ as well as possible mixing angle factors…

There are many extra-dimensional models which can lead to `significant' signals in the Flavor sector…here we will concentrate on just two:

- Universal Extra Dimensions (UED)

- Warped Extra Dimensions/Randall-Sundrum models with the Standard Model fields in the Bulk (RS)

These models were chosen because they are both quite popular, have totally different signatures at the LHC and their impact on Flavor physics is quite different--though they have some common features.

The most important aspect of these models is the existence of Kaluza-Klein excitations of SM fields

## Reminder:

We will not be looking for or expecting huge new flavor effects as the B-factories have already told us that the SM `works' quite well for most processes where the calculations can be made reliably….

In *most* cases this means searching for O(10%) modifications to SM predictions → high precision measurements will likely be needed to see anything new.

→ A motivation Super-B factories

# UNIVERSAL EXTRA DIMENSIONS - basics

- One extra dimension of radius R with $-\pi R \leq y \leq \pi R$ and a parity $y \rightarrow -y$ symmetry, i.e., *even* or *odd* states

- All SM fields are `in the bulk', i.e., will have KK excitations

- $M_n^2 = m_0^2 + (n/R)^2$ , n=0,1,2,… where $m_0$ is the SM particle mass, are the excitation/tower masses. Even and odd parity states are degenerate.

  → The usual SM particles are the zero modes of the KK tower

- SM Gauge and Higgs bosons are parity even, i.e., have zero modes (obviously)

EVEN ODD

## KK Tower Structure

etc…

| | EVEN | ODD |
|---|---|---|
| n = 2 | --------- | --------- |
| n = 1 | --------- | --------- |
| n = 0 | --------- | X |

… only the even tower has a zero mode

$\psi_n$ ~ cos ny/R ~ sin ny/R ← KK wavefunctions, periodic BCs

- For fermions, even and odd towers BOTH exist and have opposite helicity. E.g., for SM doublets (singlets), even tower fields are LH (RH)

→ **KK fermion excitations are similar to `vector-like' fermions**

# Model Parameters : only two → very predictive!

•1/R, the KK mass scale How large is it ?

• Λ – the cutoff scale → UED is an effective theory and needs a cutoff. The practical application is that the tree level spectrum is highly degenerate so loop corrections to masses are important. They behave as a sum of terms that go like

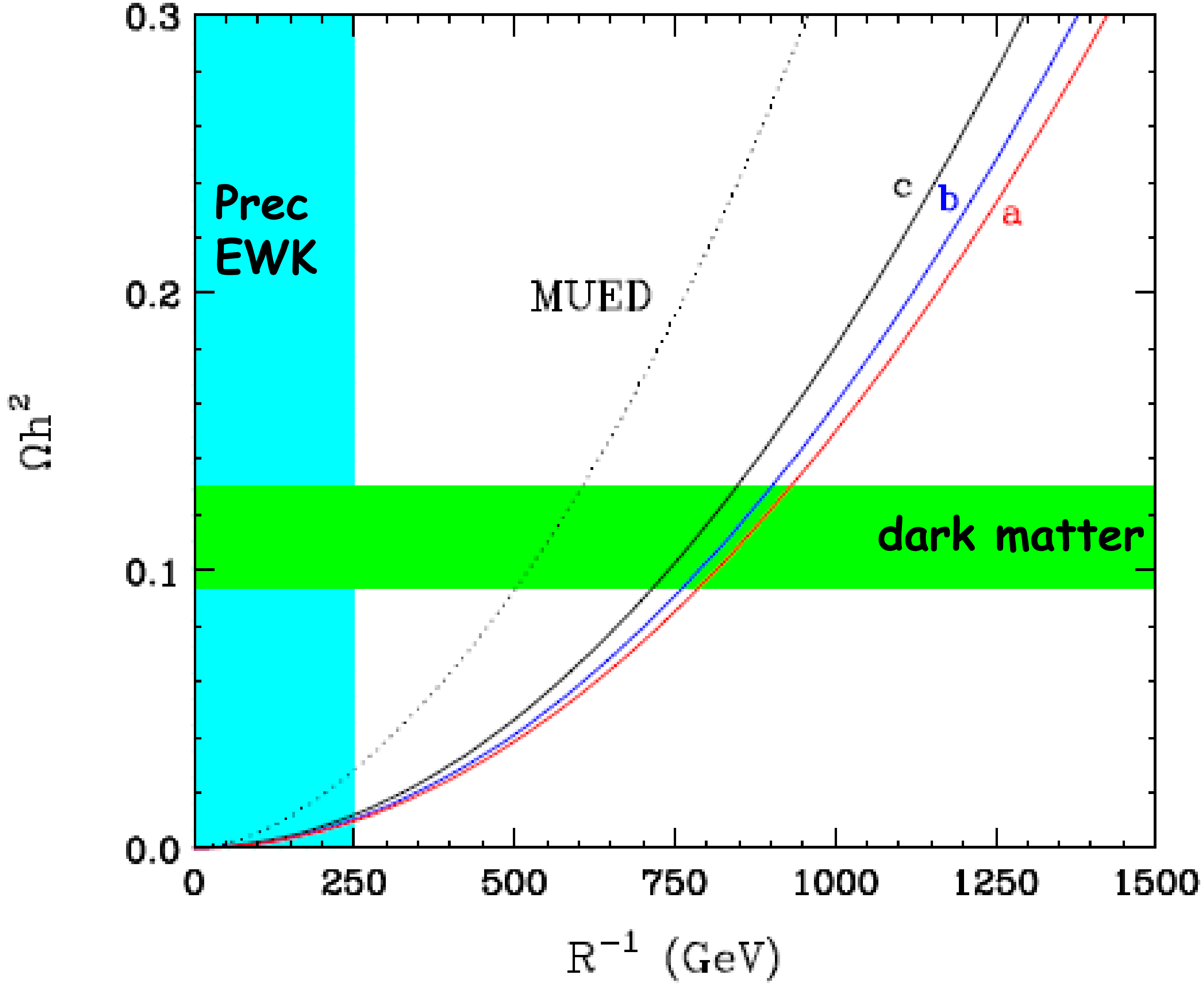
$$\Delta m^2 \sim N_i/R^2 \; (\alpha_i/4\pi) \log(\Lambda R) \qquad (\text{ with } \Lambda R \sim 20)$$

Note that there is only log sensitivity to Λ !

→ How large is 1/R and what does a realistic spectrum look like???

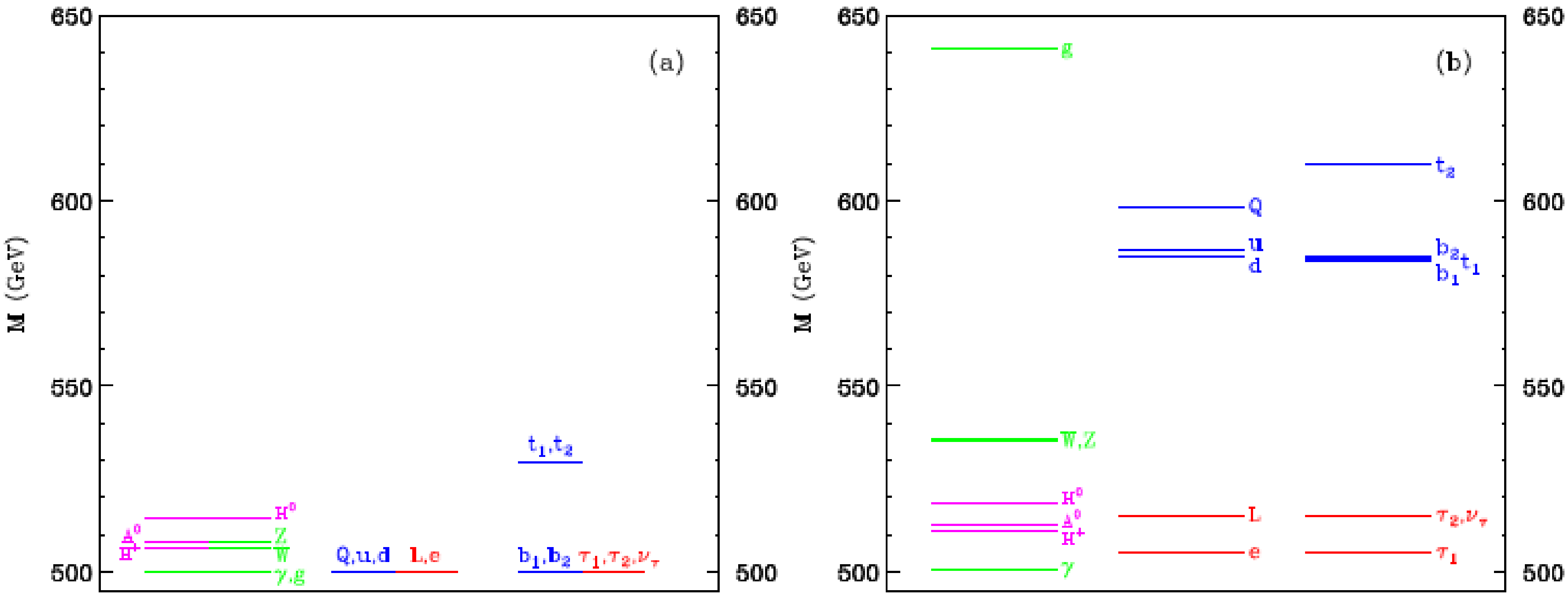
Precision EWK data → 1/R > 250 - 300 GeV

Dark Matter density → 450 < 1/R < 600 GeV (preferred)



Here is the effect of radiative corrections on the various particle masses for 1/R=500 GeV and ΛR=20

As can be seen these are substantial and important to the phenomenology




before

after

Since all fermions have the same wavefunctions etc. and differ only in their zero mode masses there is an active GIM mechanism and all flavor interactions are controlled *only* by the CKM matrix…

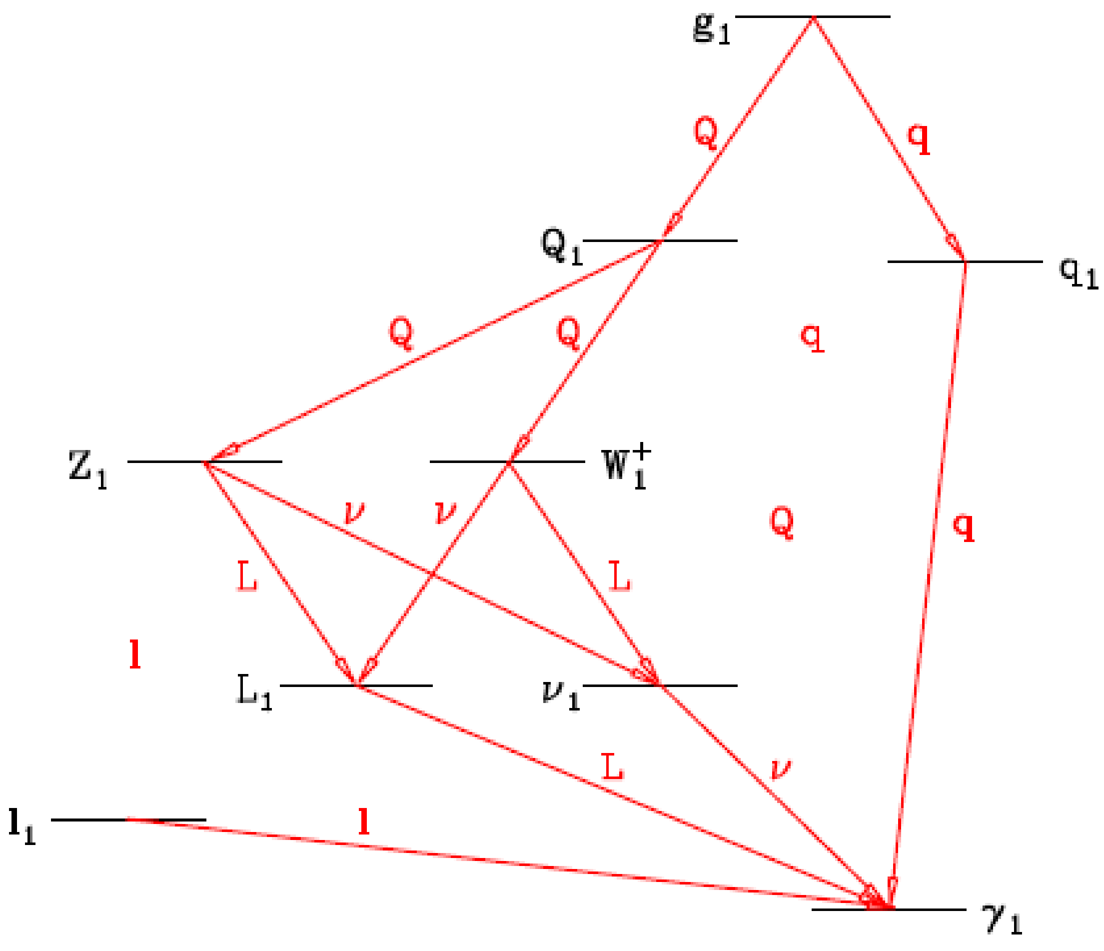
→ All the new flavor physics comes in loops with KK tower fields in them

Comments:

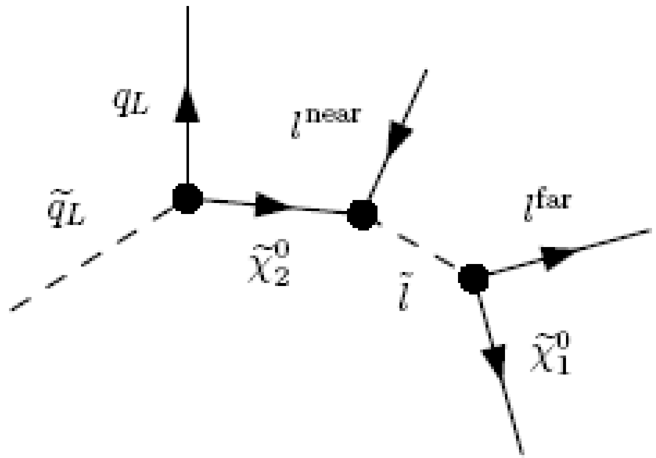
- KK towers can talk to each other via Yukawa couplings and the usual Higgs field that generate SM fermion masses

- The KK towers of the Higgs doublet whose zero modes are eaten by the SM W/Z remain in the spectrum

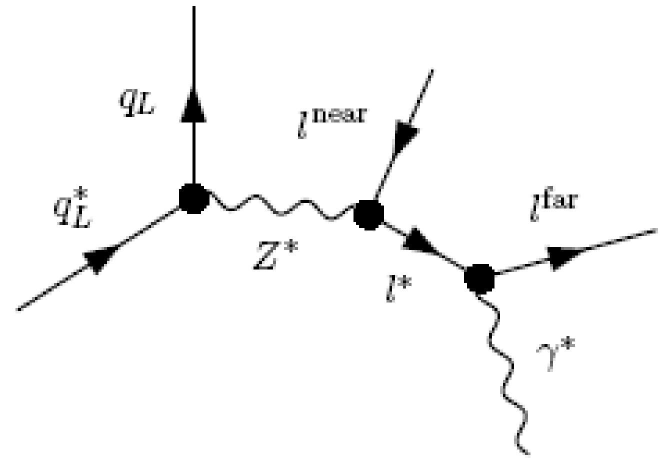
The production and long decay chains of UED KK excitations at the LHC with masses in the above range looks *a lot* like SUSY...



## SUSY




## UED




These decay patterns look very similar except for the particle spin…note that because of the parity symmetry the lightest UED KK state is stable (LKP) like the LSP in SUSY...that's why it can be the dark matter.

These models are generally indistinguishable at LHC & might require ILC to do the job…info from the flavor sector may also be of some help here.

# Applications * Example: B-mixing

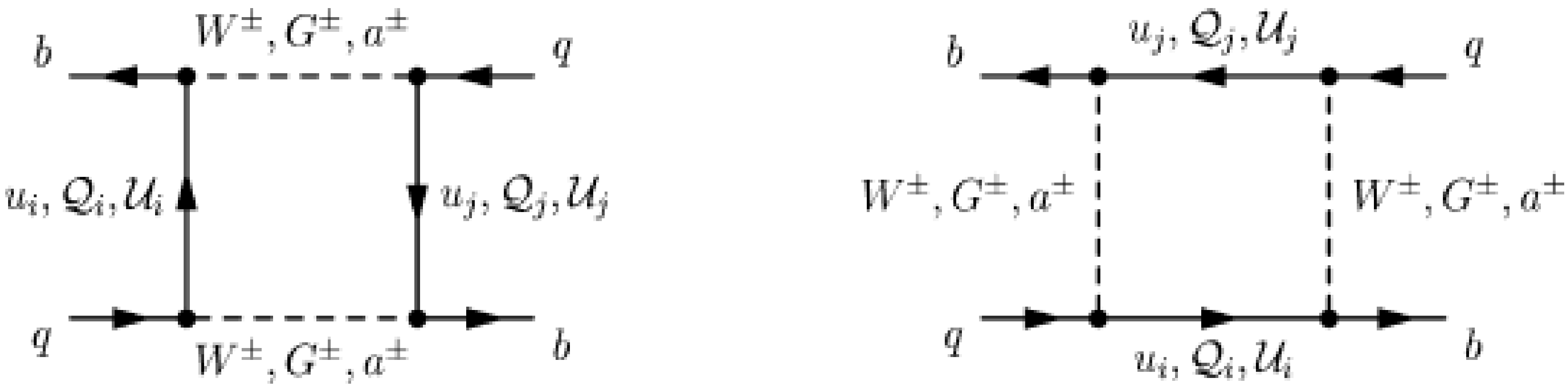



Fig. 1. Box diagrams contributing to $S_n(x_t, x_n)$. We suppress the KK mode number.

These look like our usual, familiar boxes but now there are KK excitations of Ws and quarks running through them as well as the charged Higgs-like, *physical* Goldstone boson KK excitations.

$$\Delta M_d = 0.50/\mathrm{ps}\left[\frac{\sqrt{\hat{B}_{B_d}}\,F_{B_d}}{230\ \mathrm{MeV}}\right]^2\left[\frac{|V_{td}|}{7.8\times 10^{-3}}\right]^2\left[\frac{\eta_B}{0.55}\right]\left[\frac{S(x_t, 1/R)}{2.34}\right]$$

* for details see Buras et.al. hep-ph/0212143,0306158

The `loop function' S is larger, by ~10% for 1/R=300 GeV, than in the SM due to the additional KKs..note decoupling occurs rapidly..

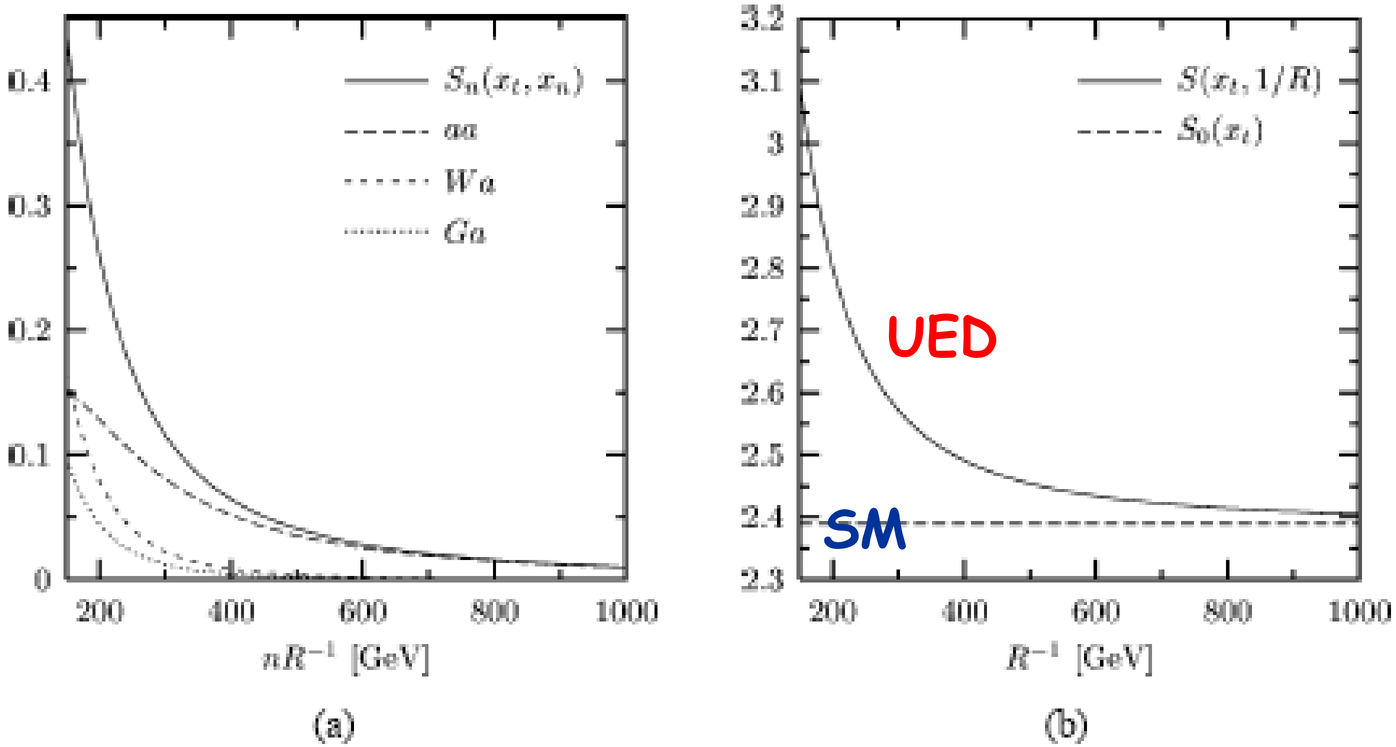
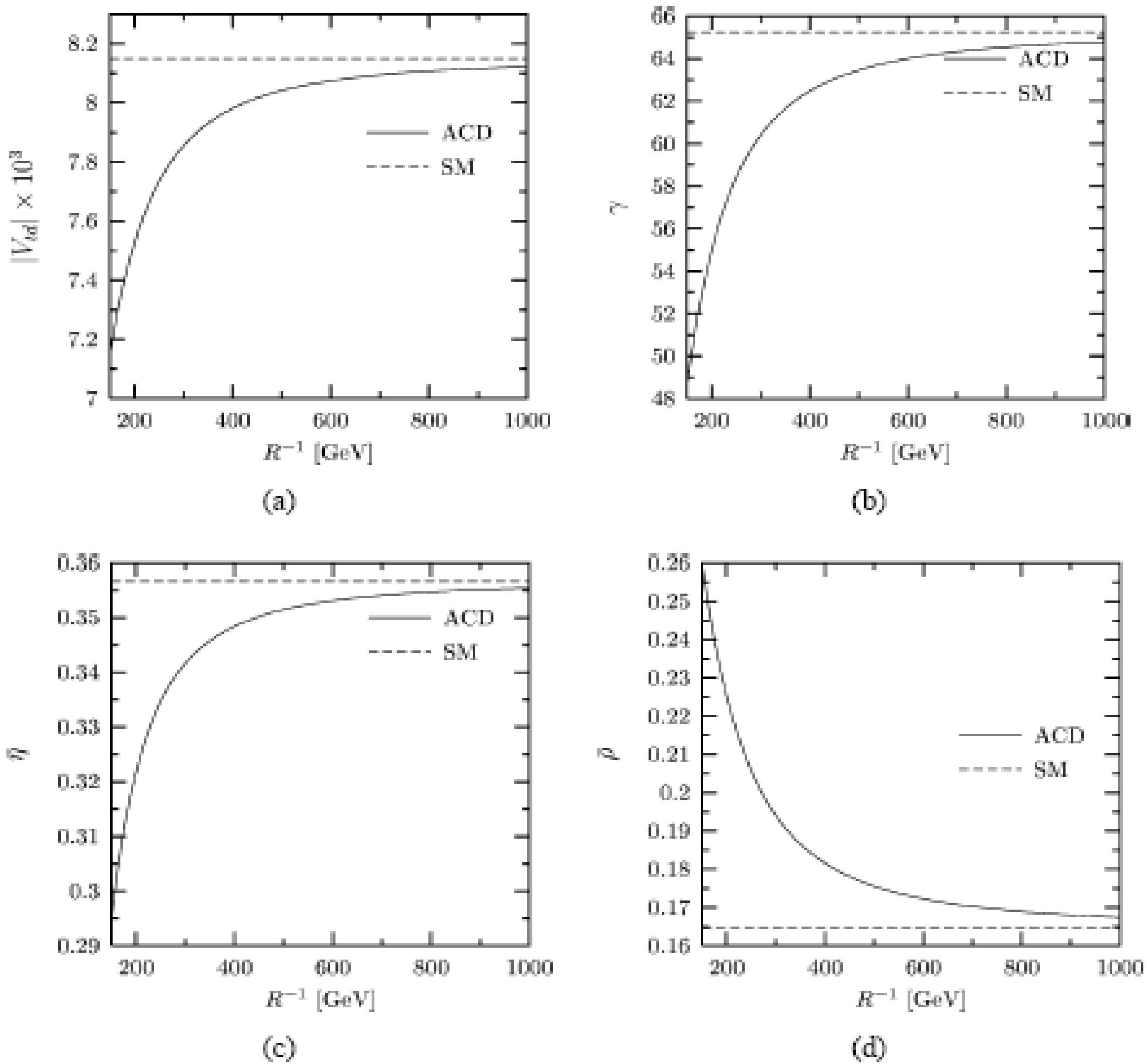



Fig. 2. (a) Contribution $S_n$ of the $n$th KK mode to $S(x_t, 1/R)$. The contributions with $a^{\pm}$ dominate, those with only $G^{\pm}$ and $W^{\pm}$ are negligible and not shown. (b) The functions $S(x_t, 1/R)$ and $S_0(x_t)$.

# How are the extracted values of parameters modified ?



## This results in a small shift in the apex of the Unitarity Triangle..

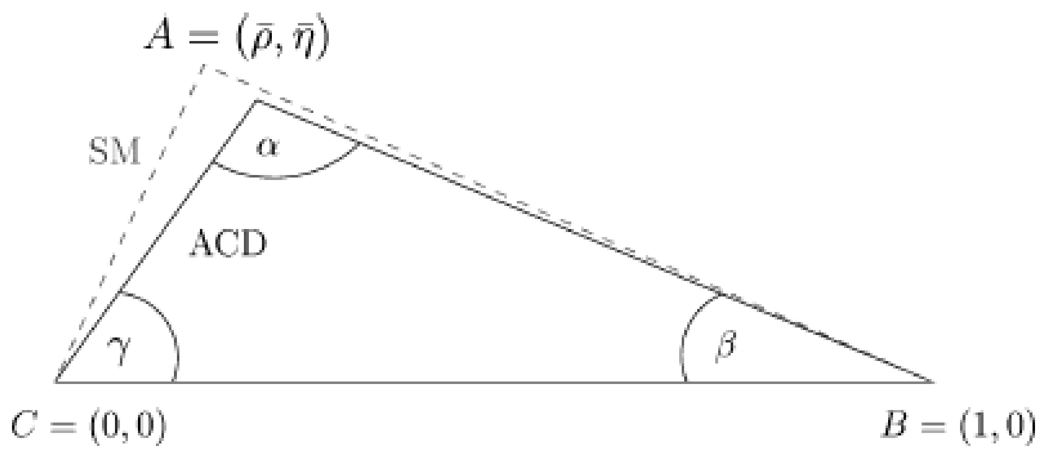
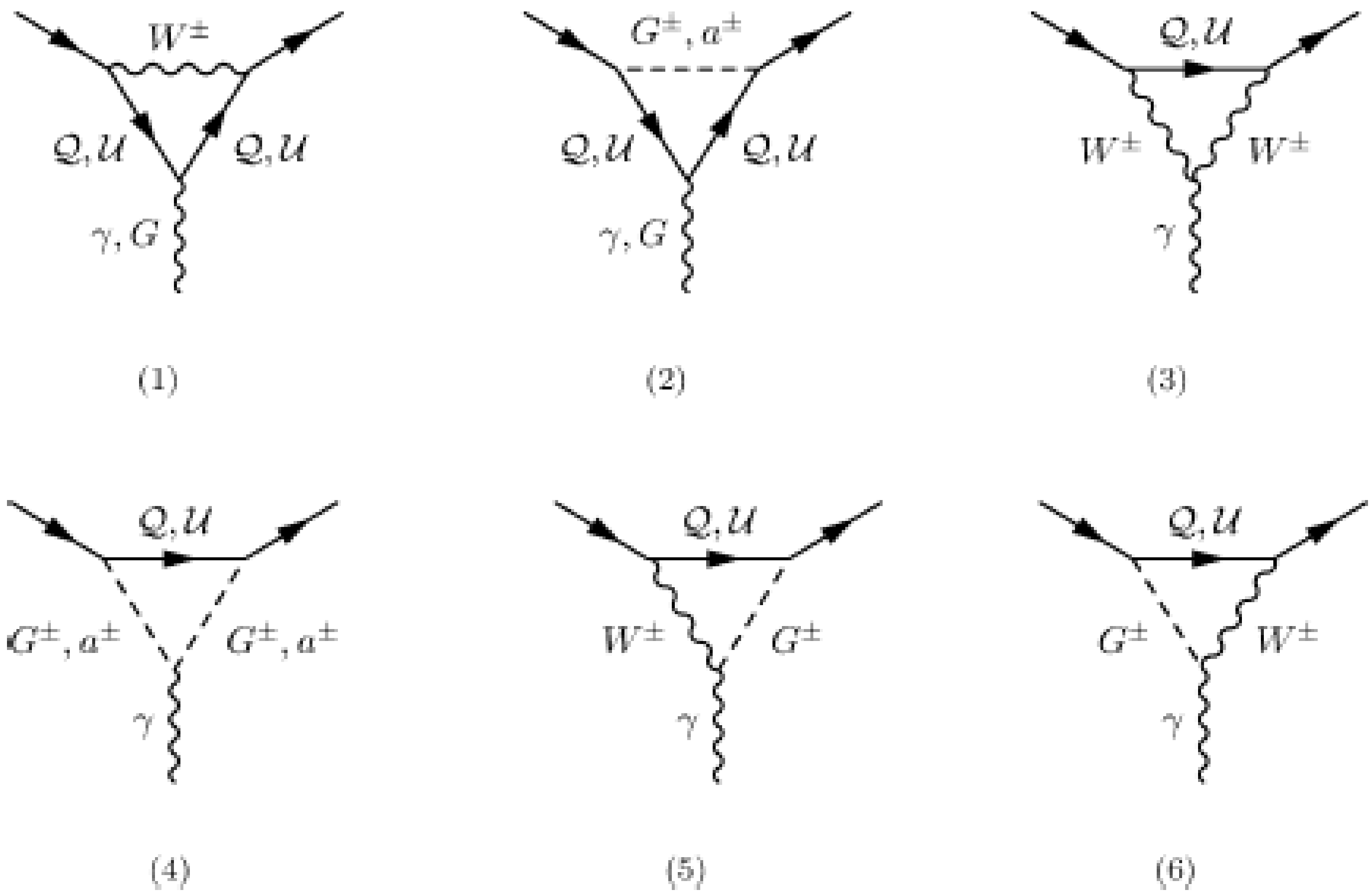


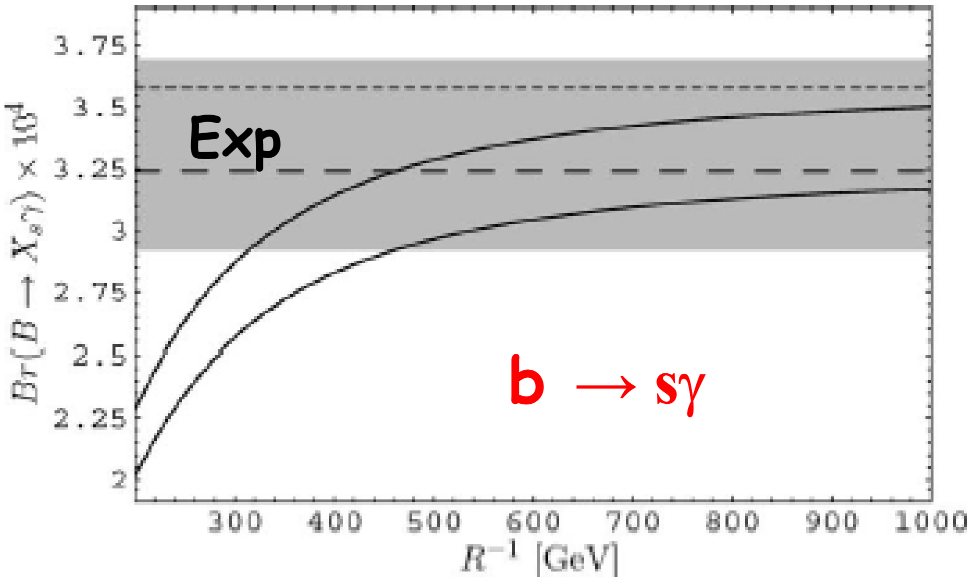

Fig. 5. Unitarity triangle in the ACD model for $1/R = 200$ GeV and in the SM.

**How about rare B decays ? Again the amplitudes get new loop-order contributions from KK tower states…**

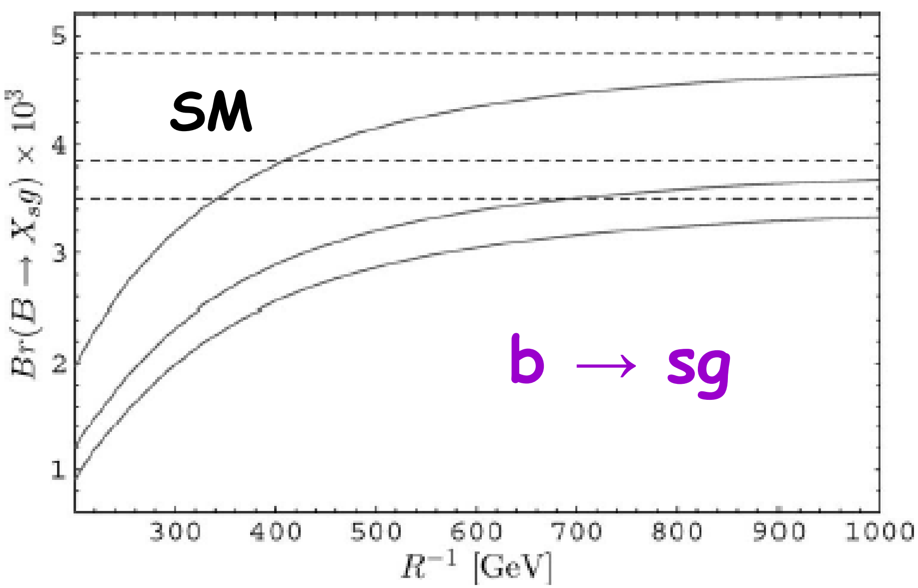
## $b \to s\gamma$, $sg$ :

**Again the loop graphs look very familiar but now there are KK states everywhere inside…**






UED predicts values for both these rates which are below those of the SM…

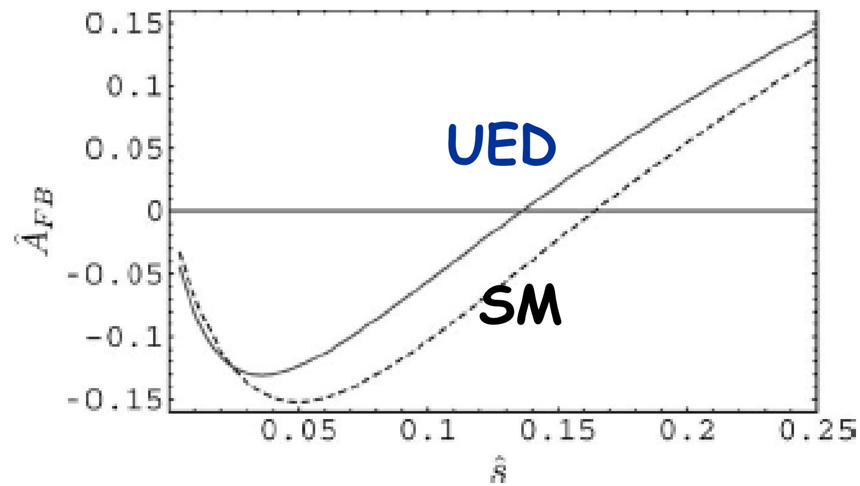



But once 1/R exceeds 400 GeV or so the difference is quite small

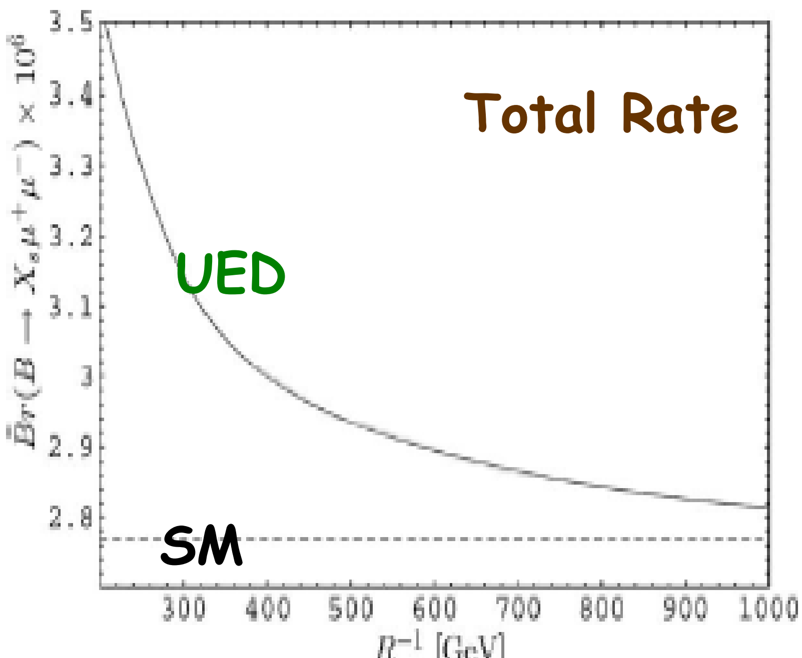
# b → s $l^+ l^-$ :

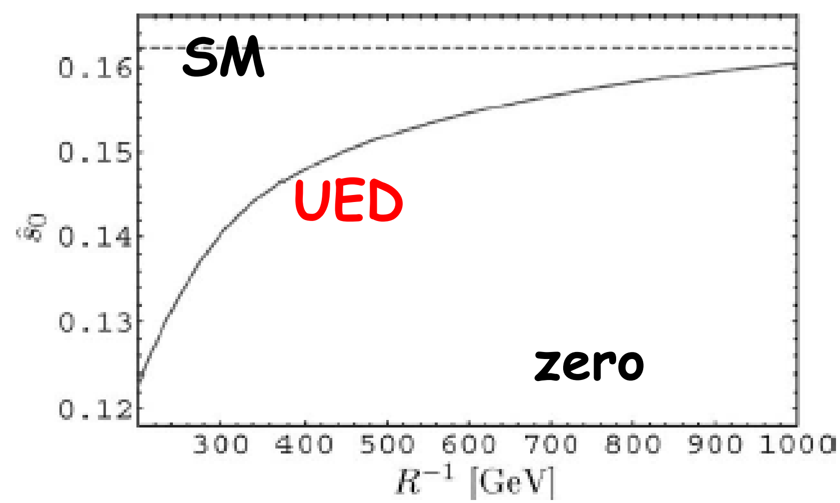
## F-B Asymmetry

Again, UED predicts O(10%) shifts in both the rate and the Forward-Backward Asymmetry




(a)



# Warped Extra Dimensions

The space has a constant negative curvature *unlike* in UED which is flat

$$ds^2 = e^{-2k|y|}\,\eta_{\mu\nu}dx^\mu\,dx^\nu\ -dy^2$$

$$-\pi R \leq y \leq \pi R\ ,\quad kR \sim 11$$

Solves Hierarchy Problem

$M_{5D} \sim k \sim M_{Pl}$

UV brane at y=0

IR brane at y=πR

Unlike in UED where wavefunctions are sines and cosines things are more complicated here…

- Zero mode gauge fields are essentially y-independent, i.e., flat like UED but
- Gauge KK excitations are Bessel functions peaked at the IR brane. Gauge KK masses are fixed by a series of Bessel roots apart from an overall scale factor

Zero mode gravitons are peaked at the UV brane → very weak

## How does 'warping' work ?

- imagine the Higgs field on the TeV brane....

$$S = \int d^4x \, dy \, \sqrt{-g} \, \{ g^{\mu\nu} \partial_\mu \hat{H}^\dagger \partial_\nu \hat{H} - \lambda (\hat{H}^2 - v_0^2)^2 \} \, \delta(y - \pi r_c)$$

$\sqrt{-g} = \{ [e^{-2ky}]^4 \}^{1/2}$  $g^{\mu\nu} \to e^{2ky} g^{\mu\nu}$  $v_0$: Higgs vev $\sim M_{pl}$

$0 \le y \le \pi r_c$

$$S = \int d^4x \{ e^{-2kr_c\pi} \partial_\mu \hat{H}^\dagger \partial^\mu \hat{H} - e^{-4kr_c\pi} \lambda (\hat{H}^2 - v_0^2)^2 \}$$

now rescale $\hat{H} \to e^{kr_c\pi} H$

$$S = \int d^4x \{ \partial_\mu H^\dagger \partial^\mu H - \lambda ( H^2 - \underbrace{v_0^2 e^{-2kr_c\pi}}_{v} )^2 \}$$

"Canonically" normalized! (kinetic term)  v is TeV scale now

The Higgs on the TeV brane gets a TeV scale vev ... even though we started at $\sim M_{pl}$!

- Warping modifies all energy scales.

**The original Lagrangian only has Planck scale parameters. But if the Higgs field is located near the IR brane its Lagrangian parameters are warped down to the electroweak scale which produces a TeV scale Higgs vev.**

Unlike UED, zero mode fermions do *not* have flat wave functions but ones which are exponentially peaked in a way which depends on their 5D mass:

$m_{fL,fR} = k\, c_{fL,fR}$ with c's being O(1) of either sign

$\psi(c,y) \sim N(c)\, e^{(2-c)k|y|}$ This lets us generate the fermion mass hierarchy w/ no small parameters using exponentials !

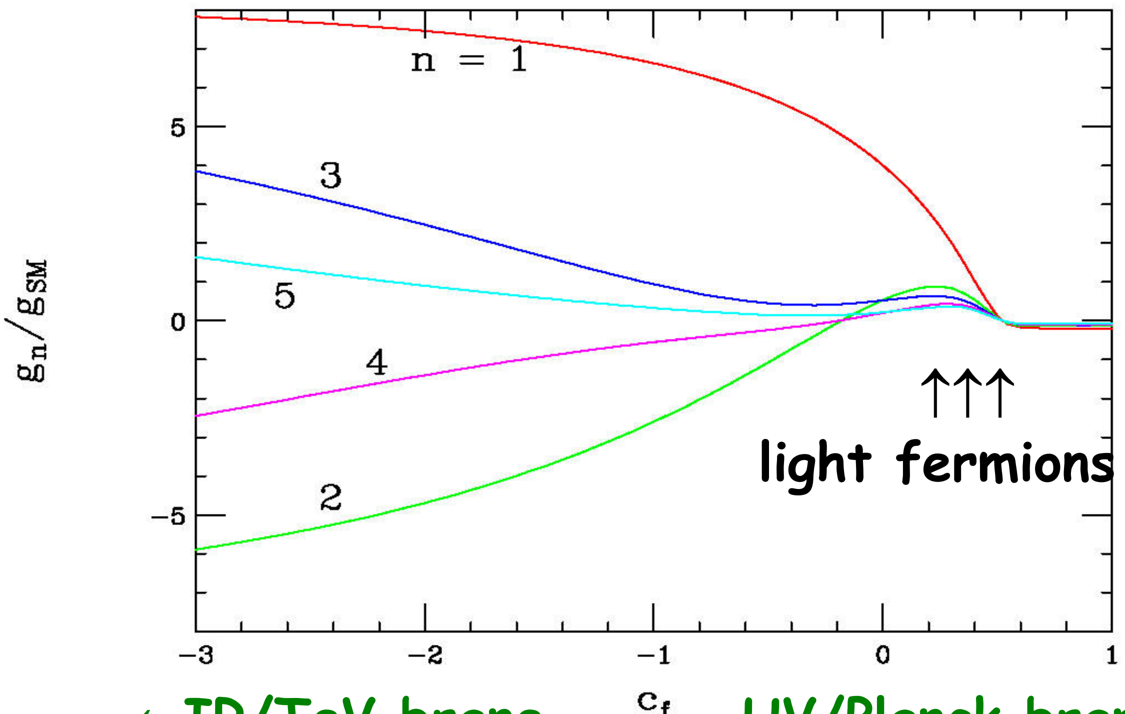
$m \sim \int dy\, g^{1/2}\, \psi(c_L, y)\, \psi(c_R, y)\, H(y) \sim N(c_L)\, N(c_R) e^{(1-CL-CR)\pi kR}$

where $N(c) \sim e^{(1-2c)\pi kR}$ ...correlated c's give the right mass, light (heavy) fermions must be localized near the UV(IR)

→ But we can't choose the c's *any* way we want...

For some values of c, the fermion will be strongly coupled to the gauge KK excitations which have TeV scale masses. This will disturb precision EWK data…

1st & 2nd generation fermions must be `localized' closer to the UV brane since EWK constrains them the most




←IR/TeV brane UV/Planck brane→

The 3rd generation can be localized closer to the IR brane

This can be done and still generate the mass hierarchy for fermions.. there is some tension mostly alleviated by an additional custodial $SU(2)_R$ symmetry.

The couplings of fermions (f) to gauge fields (G) is given by an integral

$$g_{fG}/g_{SM} = \int dy\, w(y)\, \psi(c,y)\, \psi(c,y)\, G(y)$$

If G is a constant this is just 1 by orthonormality. OK! But gauge KKs are not flat so these ratios depend on c as was seen in the figure…

→→ ***GIM violation !!***

Since different fermions have different c's the field rotations to the mass basis will induce FCNC's mediated by gauge bosons (and KK gravitons) with masses in the few TeV range

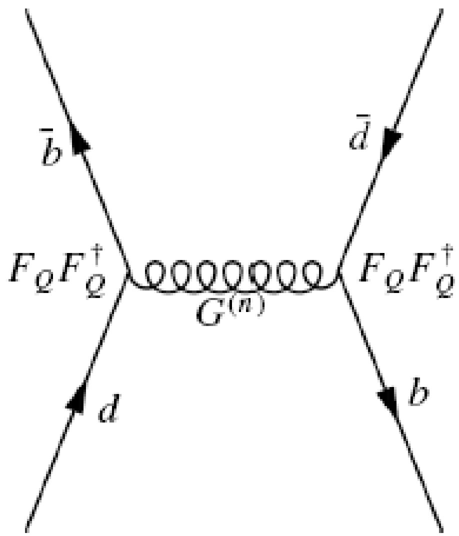
→ c's of light fermions can't be too different

Of course one *also* has to suppress higher dimensional operators generated in the 5D theory by terms like

$$\int dy\, g^{1/2}\, \psi_1(c1,y)\psi_2(c2,y)\psi_3(c3,y)\psi_4(c4,y)/M_5^3 \quad \to$$

$$\sim K\, \psi_1\psi_2\psi_3\psi_4/(1\ \mathrm{TeV})^2$$

which are still allowed by the SM gauge symmetries. K *can* be made exponentially small provided the values of the c's are wisely chosen.




An example of these gauge FCNCs is the KK gluon contribution to B-Bbar mixing…

Agashe, Perez & Soni
hep-ph/0408134

A rough estimate for this matrix element is given by the ratio

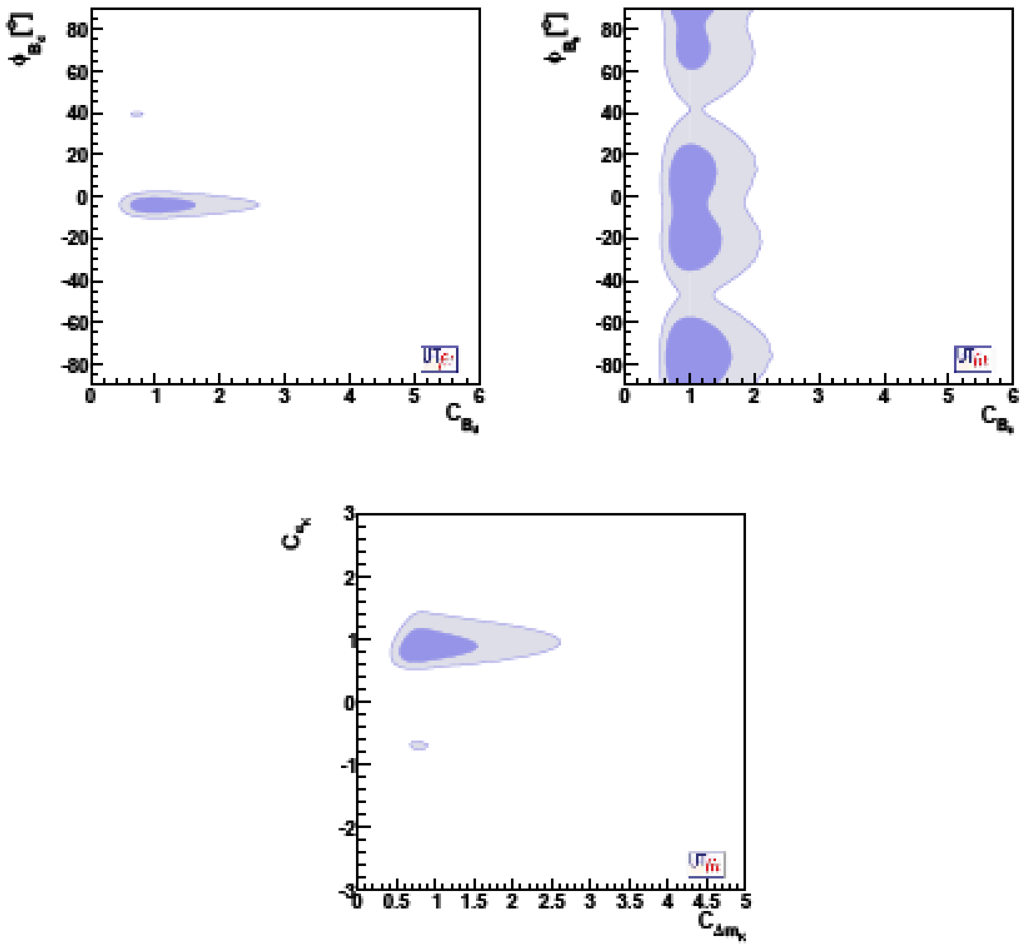
$$\frac{M_{12}^{\rm RS}}{M_{12}^{\rm SM}} \sim \frac{16\pi^2}{N_c}\,\frac{8g_s^2}{g_2^4 S_0(m_t)}\,\frac{M_W^2}{m_{\rm KK}^2}\,\frac{k\pi r_c}{f_{Q^3}^4}$$

$$\sim 0.5 \times \left(\frac{3\ {\rm TeV}}{m_{\rm KK}}\right)^2 \left(\frac{3}{f_{Q^3}}\right)^4,$$

Which implies a contribution, with a different phase of O(0.1-1) !

Of course a similar calculation can be done for other systems with mixing and strong constraints exist…

$$C_{B_q}\, e^{2i\phi_{B_q}} = \frac{\langle B_q | H_{\rm eff}^{\rm full} | \bar{B}_q \rangle}{\langle B_q | H_{\rm eff}^{\rm SM} | \bar{B}_q \rangle}$$

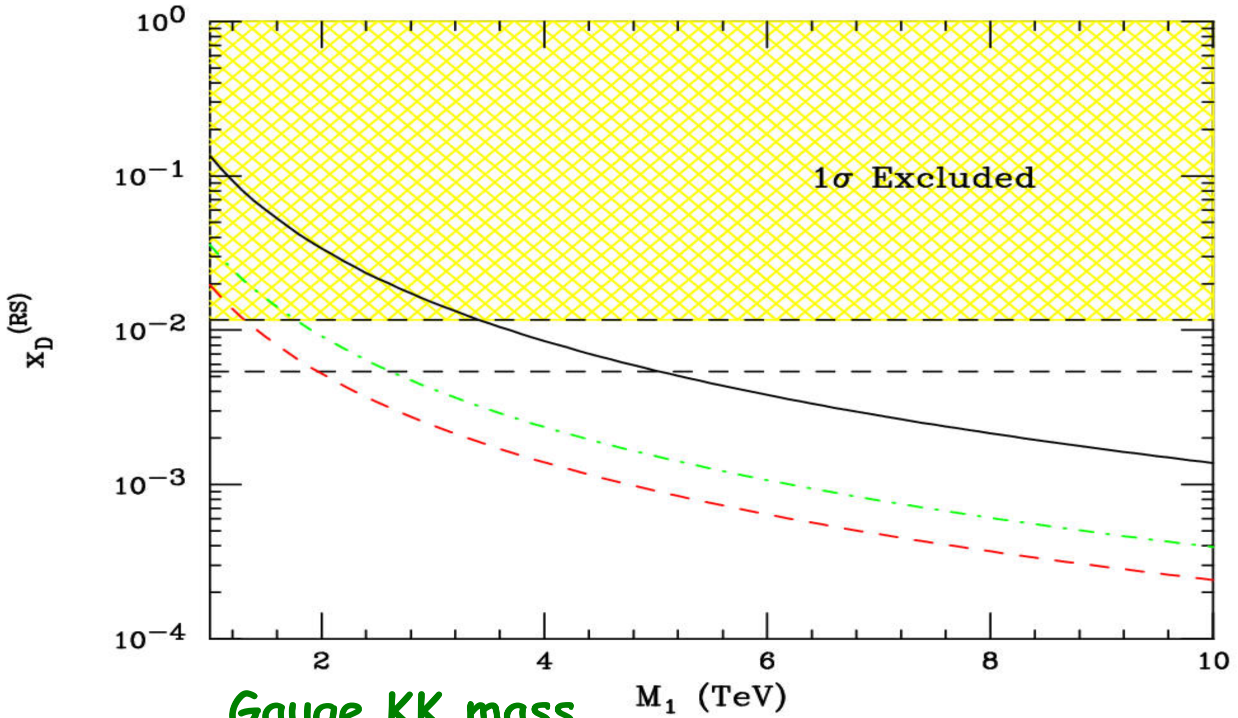



**UT*fit***
**0707.0636**

FIG. 2: Constraints on $\phi_{B_d}$ vs. $C_{B_d}$, $\phi_{B_s}$ vs. $C_{B_s}$ and $C_{\epsilon_K}$ vs $C_{\Delta m_K}$ from the NP generalized analysis.

The third generation couplings are the largest and least constrained by EWK data…but other generations can show effects as well.

→ RS predicts possibly `large' FCNC signals involving *fundamental* dimension-6 operators as well as those generated by KK gauge (as well as graviton KK) exchange




Gauge KK mass

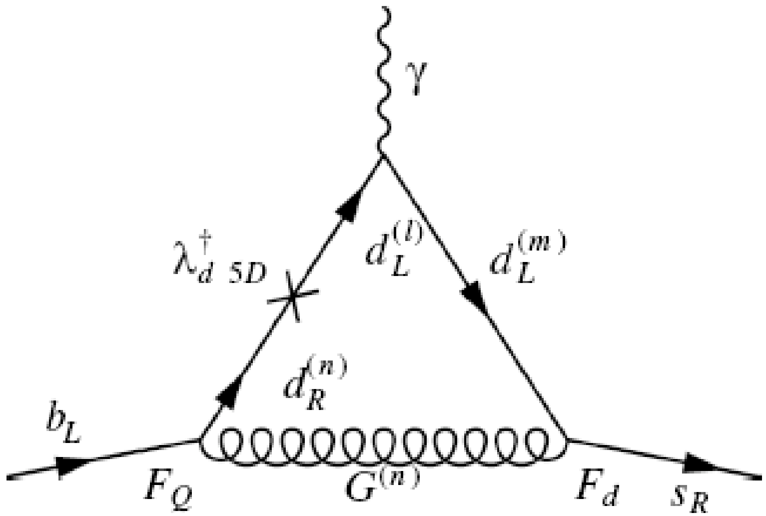
e.g. D-Dbar mixing induced by KK exchange

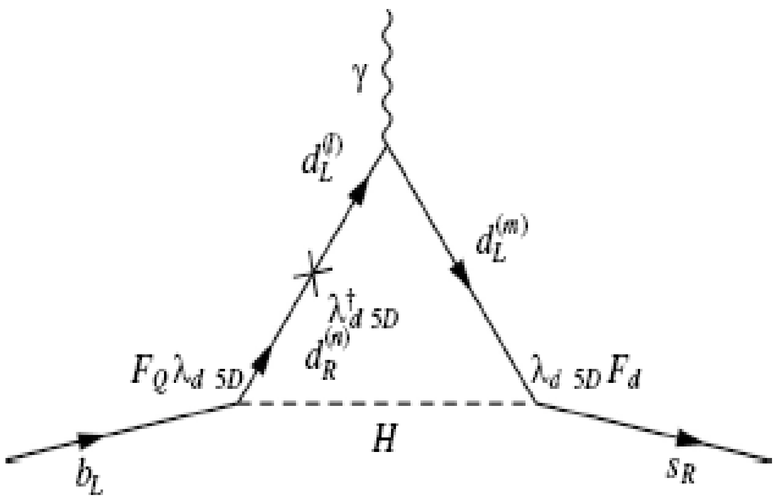
← Curves for different sets of c values

Golowich etal

Given the RS framework it seems more likely that we will see new physics effects in b ->$s\mu\mu$ or b->$s\tau\tau$ than in b->$s\gamma$ or b->sg but the loop contributions to these processes can also be enhanced…

Here are just two of the many new diagrams that now contribute to, e.g., b->$s\gamma$…




Agashe et al

Again one finds that these contributions can be O(0.1) and have an arbitrary phase…

$$\frac{C'_{7\gamma}}{C^{\rm SM}_{7\gamma}} \sim \frac{M_W^2}{m_{\rm KK}^2} \left| \frac{(D_R)_{23}}{V_{tb}V^*_{ts}} \right| \frac{(\lambda_{5\rm D}k)^2}{D'_0(m_t)g_2^2}$$

$$\sim 1 \times \left(\frac{3\ \rm TeV}{m_{\rm KK}}\right)^2 \frac{(D_R)_{23}}{2}, \qquad (44)$$

where $D'_0(m_t) \sim 0.4$ [26] and in the above estimate we used $(D_R)_{23} \sim f_{d^3}/f_{d^2} \sim 0.5$ [see (12)] and $\lambda_{5\rm D}k \sim 4$,

→ Possibly large contributions to CP asymmetries in radiative decays

# SUMMARY

There are many interesting models from extra dims that lead to new physics in the flavor sector but the signatures are not always unique…

UED predicts all new effects arise from loops

RS predicts tree-level FCNCs… but also new loop contributions that can be significant

These NP effects should/might be observable at the next level of precision, i.e., a Super-B factory, and may be helpful with the interpretation of LHC results