

Hadron Spectroscopy and Form Factors in AdS/QCD for Experimentalists

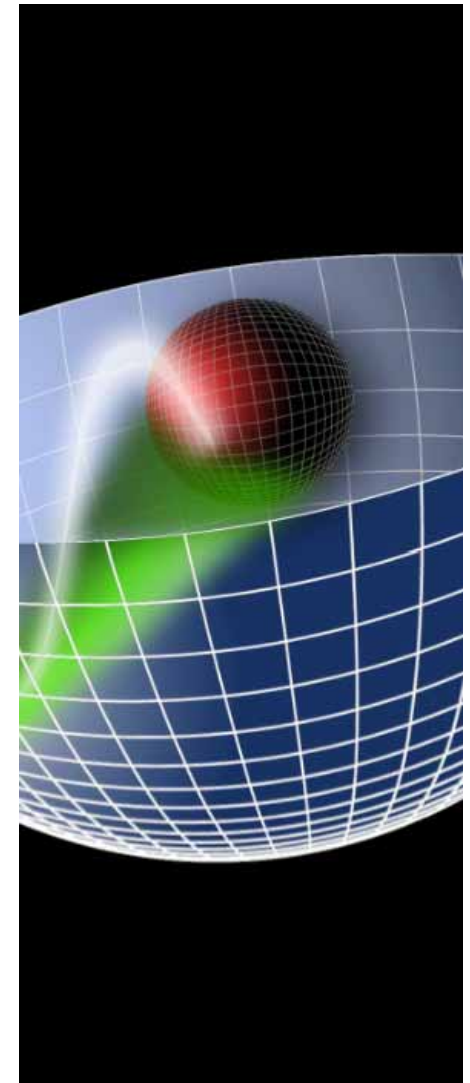
Guy F. de Téramond

University of Costa Rica and SLAC

High Energy Physics Group

Imperial College London

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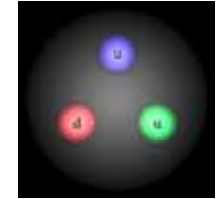
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I. Introduction



- QCD fundamental theory of quarks and gluons
- QCD Lagrangian follows from the gauge invariance of the theory

$$\psi(x) \rightarrow e^{i\alpha^a(x)T^a} \psi(x), \quad [T^a, T^b] = if_{abc}T^c$$

- Find QCD Lagrangian

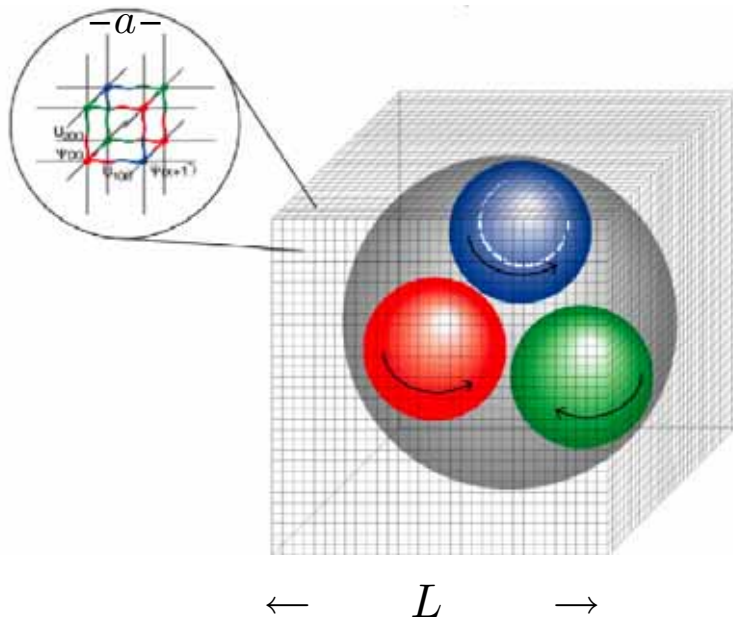
$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr} (G^{\mu\nu} G_{\mu\nu}) + i\bar{\psi} D_\mu \gamma^\mu \psi + m\bar{\psi}\psi$$

where $D_\mu = \partial_\mu - igT^a A_\mu^a$, $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{abc}A_\mu^b A_\nu^c$

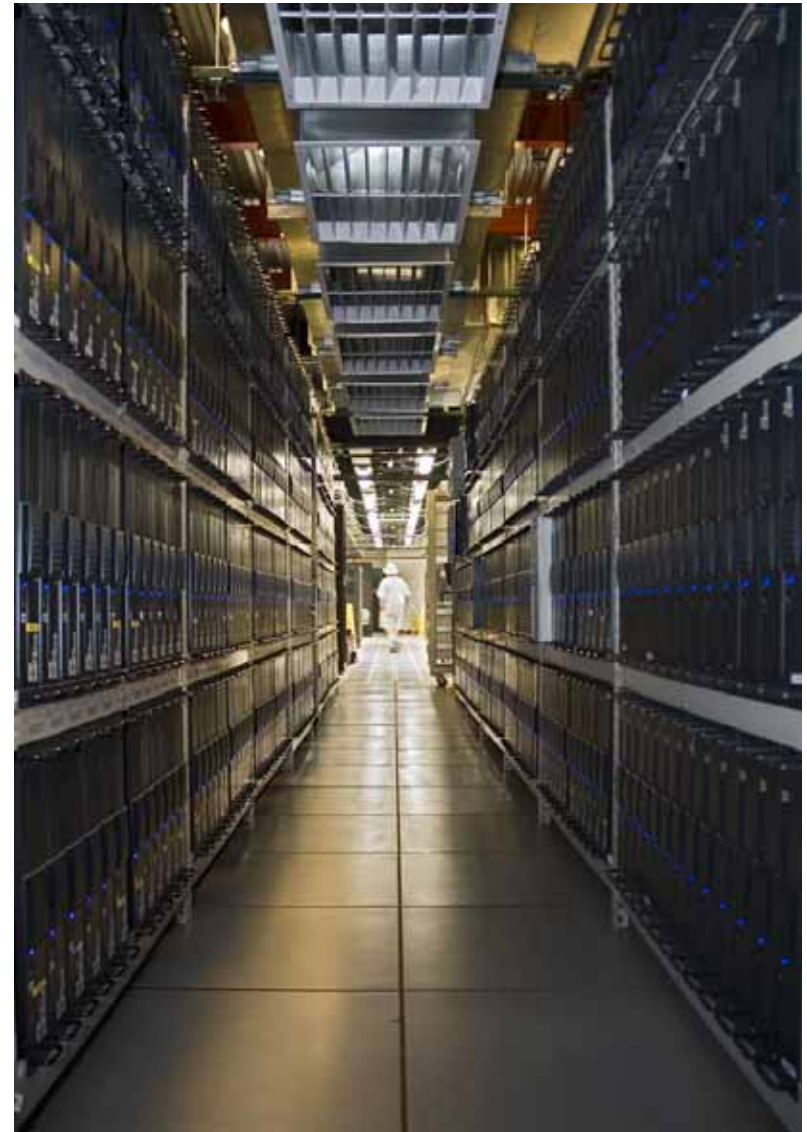
- Quarks and gluons interactions from color charge, but ... gluons also interact with each other:
strongly coupled non-abelian gauge theory \rightarrow color confinement
- Most challenging problem of strong interaction dynamics: determine the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom

Lattice QCD

- Lattice numerical simulations at the teraflop/sec scale (resolution $\sim L/a$)
- Sums over quark paths with billions of dimensions
- LQCD (2009) > 1 petaflop/sec



- Dynamical properties in Minkowski space-time not amenable to Euclidean lattice computations



Gravity

- Space curvature determined by the mass-energy present following Einstein's equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu}}_{\text{geometry}} = \kappa \underbrace{T_{\mu\nu}}_{\text{mater}}$$

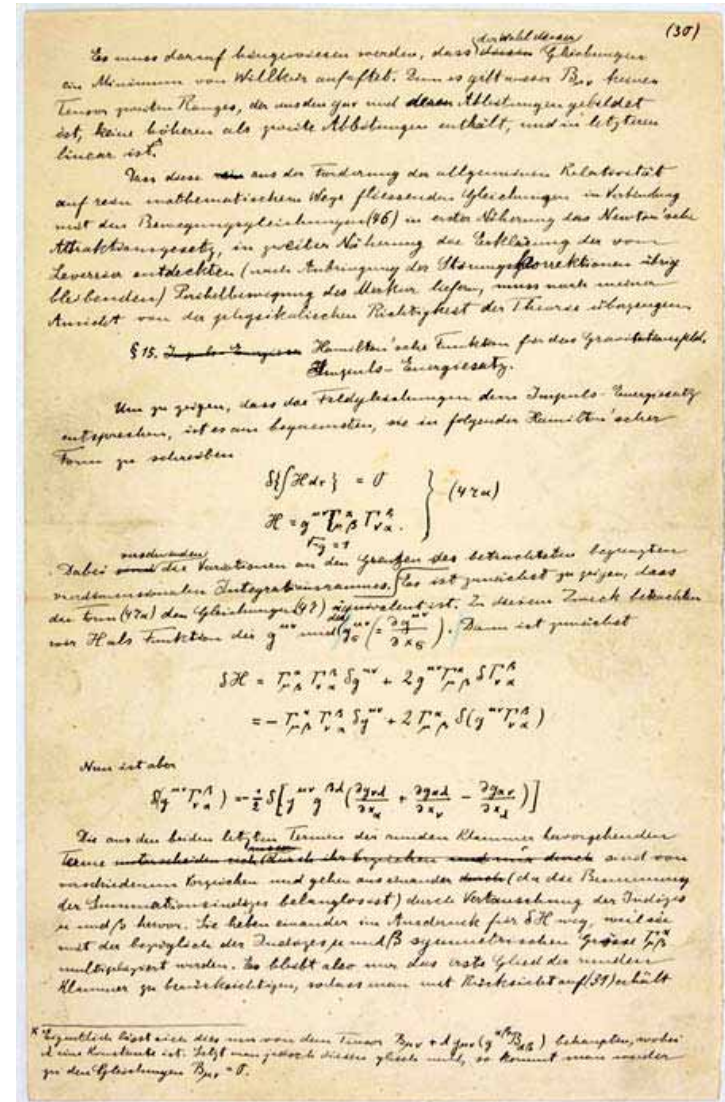
$R_{\mu\nu}$ Ricci tensor , R space curvature

$g_{\mu\nu}$ metric tensor ($ds^2 = g_{\mu\nu}dx^\mu dx^\nu$)

$T_{\mu\nu}$ energy-momentum tensor

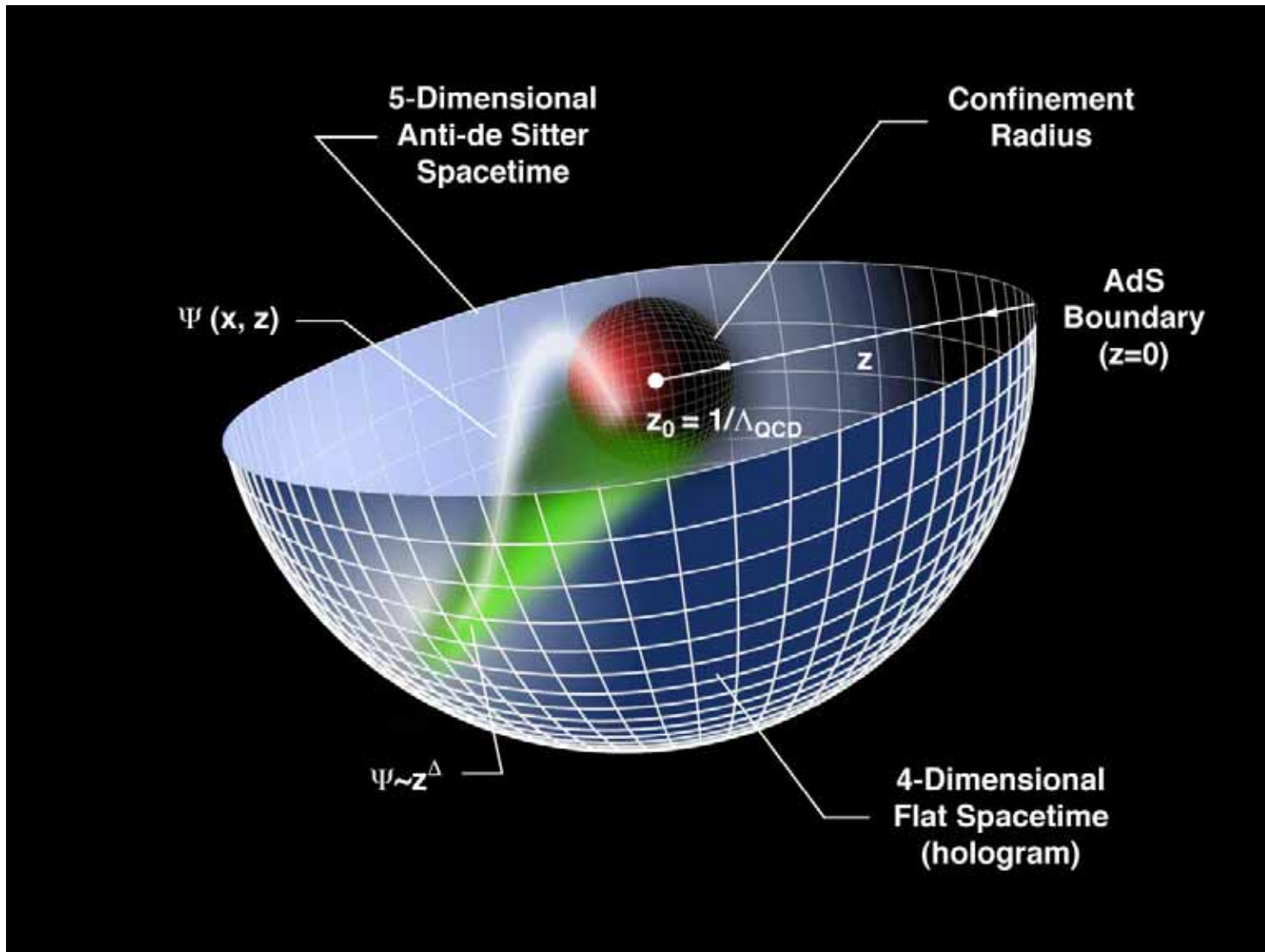
$$\kappa = 8\pi G/c^4,$$

- Matter curves space and space determines how matter moves !



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Holographic Correspondence



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II. Gauge Gravity Correspondence and Light-Front QCD

- The AdS/CFT correspondence [Maldacena (1998)] between gravity on AdS space and conformal field theories in physical spacetime has led to a semiclassical approximation for strongly-coupled QCD, which provides analytical insights into the confining dynamics of QCD
- Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...
- Light-front holography provides a remarkable connection between the equations of motion in AdS and the bound-state LF Hamiltonian equation in QCD [GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]
- Isomorphism of $SO(4, 2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS_5 , a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

Isometry group: most general group of transformations which leave invariant the distance between two points

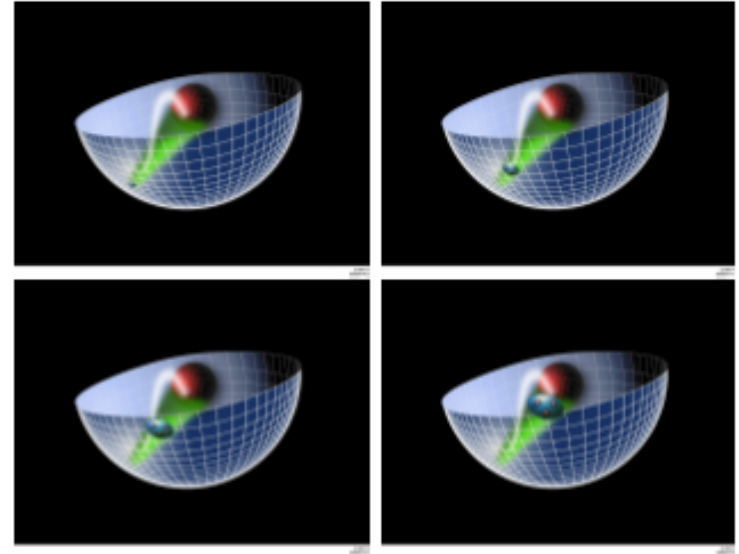
Dim isometry group of AdS_{d+1} is $\frac{(d+1)(d+2)}{2}$

- AdS₅ metric:

$$\underbrace{ds^2}_{L_{\text{AdS}}} = \frac{R^2}{z^2} \left(\underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{L_{\text{Minkowski}}} - dz^2 \right)$$

- A distance L_{AdS} shrinks by a warp factor z/R as observed in Minkowski space ($dz = 0$):

$$L_{\text{Minkowski}} \sim \frac{z}{R} L_{\text{AdS}}$$



- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, the variable z acts like a scaling variable in Minkowski space
- Short distances $x_\mu x^\mu \rightarrow 0$ maps to UV conformal AdS₅ boundary $z \rightarrow 0$
- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda_{\text{QCD}}^2$ maps to large IR region of AdS₅, $z \sim 1/\Lambda_{\text{QCD}}$, thus there is a maximum separation of quarks and a maximum value of z
- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

- Nonconformal metric dual to a confining gauge theory

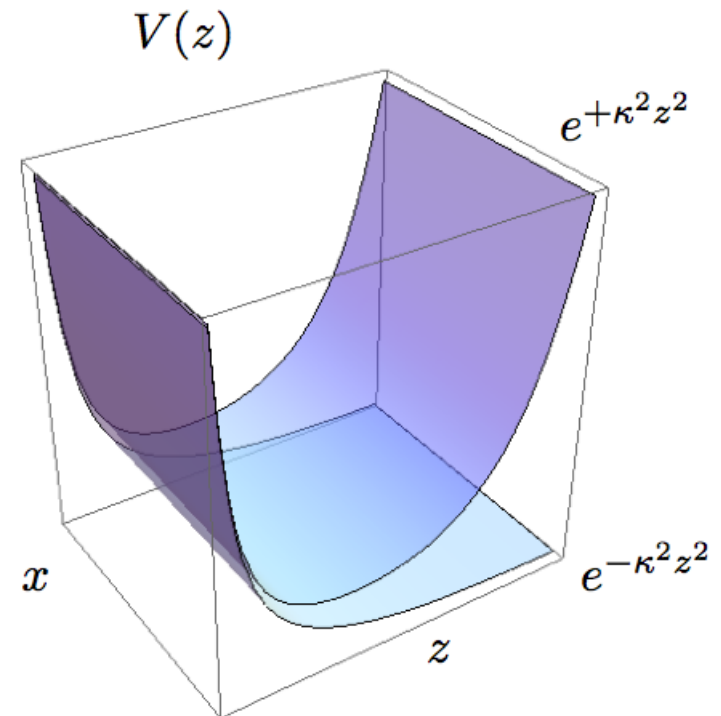
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

where $\varphi(z) \rightarrow 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Higher Spin Modes in AdS Space

(Fronsdal, Fradkin and Vasiliev)

- Lagrangian for scalar field in AdS_{d+1} in presence of dilaton background $\varphi(z)$ ($x^M = (x^\mu, z)$)

$$S = \int d^d x dz \sqrt{g} e^{\varphi(z)} (g^{MN} \partial_M \Phi^* \partial_N \Phi - \mu^2 \Phi^* \Phi)$$

- Factor out plane waves along 3+1: $\Phi_P(x^\mu, z) = e^{-iP \cdot x} \Phi(z)$

$$\left[-\frac{z^{d-1}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

where $P_\mu P^\mu = \mathcal{M}^2$ invariant mass of physical hadron with four-momentum P_μ

- Define spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 and shifted dimensions $\Phi_J(z) \sim z^{-J} \Phi(z)$
- Find AdS wave equation

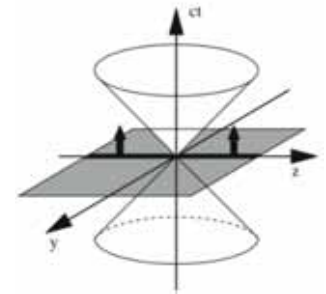
$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$



III. Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one

- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$



$$x^+ = x^0 + x^3 \quad \text{light-front time}$$

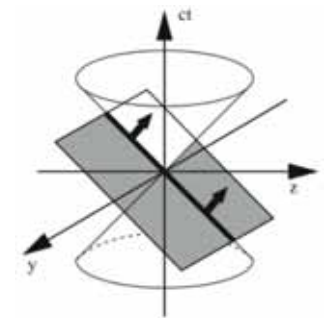
$$x^- = x^0 - x^3 \quad \text{longitudinal space variable}$$

$$k^+ = k^0 + k^3 \quad \text{longitudinal momentum} \quad (k^+ > 0)$$

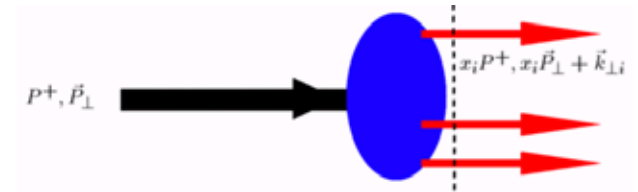
$$k^- = k^0 - k^3 \quad \text{light-front energy}$$

$$k \cdot x = \frac{1}{2} (k^+ x^- + k^- x^+) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$



Light-Front Fock Representation



- LF Lorentz invariant Hamiltonian equation for the relativistic bound state system

$$P_\mu P^\mu |\psi(P)\rangle = (P^- P^+ - \mathbf{P}_\perp^2) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \quad |n\rangle = \{ |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \dots \}$$

with $k_i^2 = m_i^2$, $k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each constituent i in state n

- Fock components $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ independent of P^+ and \mathbf{P}_\perp and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+ / P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute \mathcal{M}^2 from hadronic matrix element

$$\langle \psi(P') | P_\mu P^\mu | \psi(P) \rangle = \mathcal{M}^2 \langle \psi(P') | \psi(P) \rangle$$

- Find

$$\mathcal{M}^2 = \sum_n \int [dx_i] [d^2\mathbf{k}_{\perp i}] \sum_\ell \left(\frac{\mathbf{k}_{\perp \ell}^2 + m_\ell^2}{x_\ell} \right) |\psi_n(x_i, \mathbf{k}_{\perp i})|^2 + \text{interactions}$$

- Semiclassical approximation to QCD:

$$\psi_n(k_1, k_2, \dots, k_n) \rightarrow \phi_n \left(\underbrace{(k_1 + k_2 + \dots + k_n)^2}_{\mathcal{M}_n^2} \right)$$

with $k_i^2 = m_i^2$ for each constituent

- Functional dependence of Fock state $|n\rangle$ given by invariant mass

$$\mathcal{M}_n^2 = \left(\sum_{a=1}^n k_a^\mu \right)^2 = \sum_a \frac{\mathbf{k}_{\perp a}^2 + m_a^2}{x_a}$$

Key variable controlling bound state: off-energy shell $\mathcal{E} = \mathcal{M}^2 - \mathcal{M}_n^2$

- In terms of $n - 1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}, j = 1, 2, \dots, n - 1,$

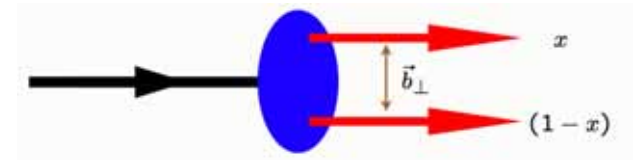
$$\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2 \mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_{\ell} \left(\frac{-\nabla_{\mathbf{b}_{\perp \ell}}^2 + m_{\ell}^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Relevant variable conjugate to invariant mass

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

the x -weighted transverse impact coordinate of the spectator system (x active quark)

- For a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_{\perp}^2$



- To first approximation LF dynamics depend only on the invariant variable ζ , and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular φ , longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

- Ultra relativistic limit $m_q \rightarrow 0$ longitudinal modes $X(x)$ decouple ($L = L^z$)

$$\mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

where the confining forces from the interaction terms is summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\left(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{\text{kinetic energy of partons}} + \underbrace{U(\zeta)}_{\text{confinement}} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$



- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable
- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n -massless partons at transverse impact separation ζ within the hadron at equal light-front time
- Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in the LF Hamiltonian EOM or by applying the L-S formalism

Light-Front Holographic Mapping

$$\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

- LF Holographic mapping found originally matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD [Brodsky and GdT (2006, 2008)]
- Upon substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2} \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE ($d = 4$)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \varphi''(z) + \frac{1}{4} \varphi'(z)^2 + \frac{2J-3}{2z} \varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

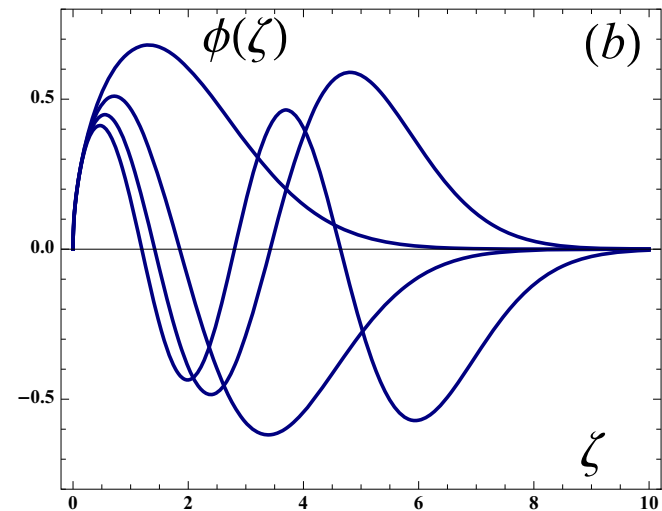
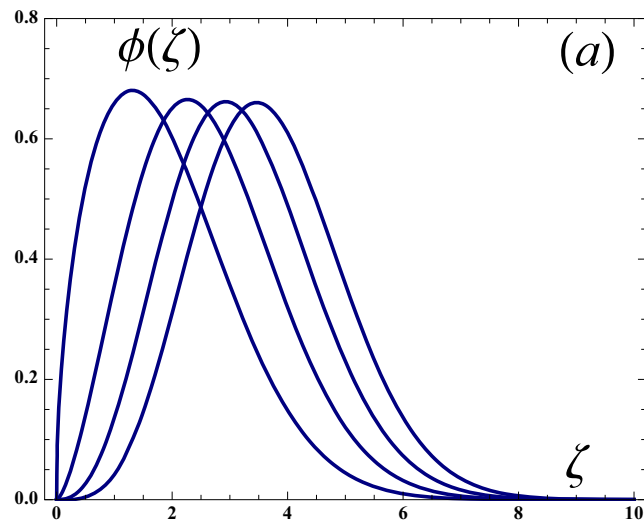
- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode Φ_J is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD

- Positive dilaton background $\varphi = \kappa^2 z^2$: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$
- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta |\phi(z)|^2 = 1$

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

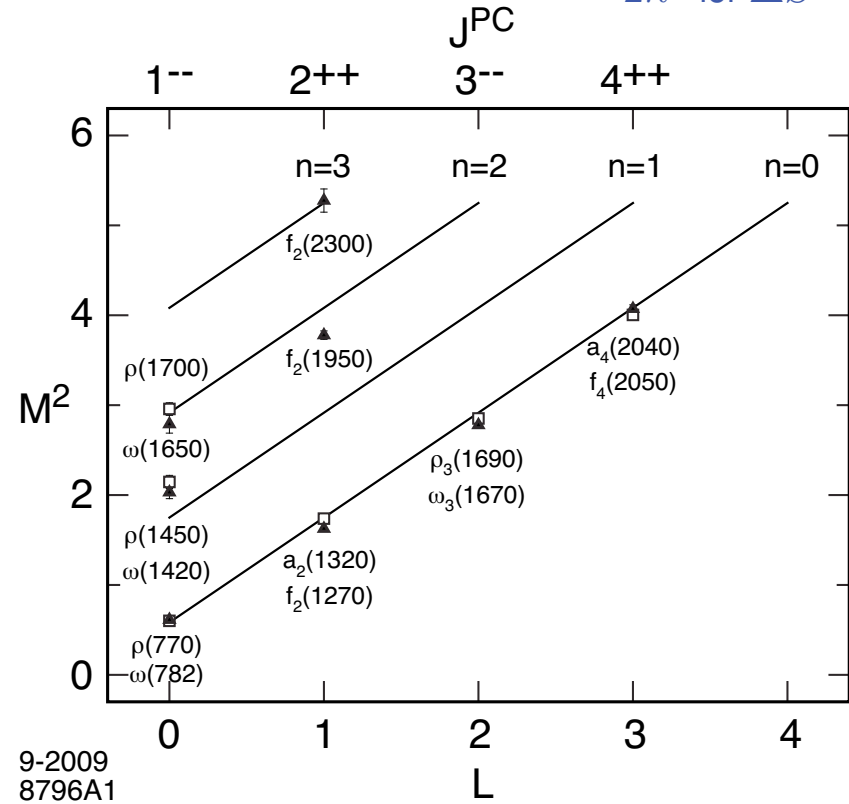
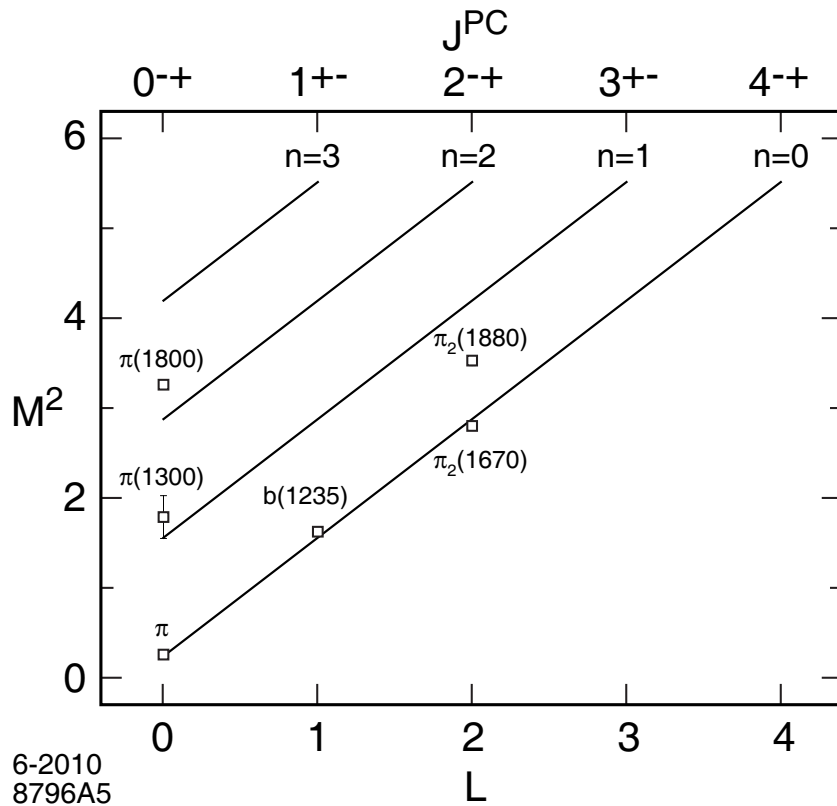
$$\mathcal{M}_{n,L,S}^2 = 4\kappa^2 (n + L + S/2)$$



LFWFs $\phi_{n,L}(\zeta)$ in physical spacetime for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes

Light Meson and Baryon Spectrum

$4\kappa^2$ for $\Delta n = 1$
 $4\kappa^2$ for $\Delta L = 1$
 $2\kappa^2$ for $\Delta S = 1$



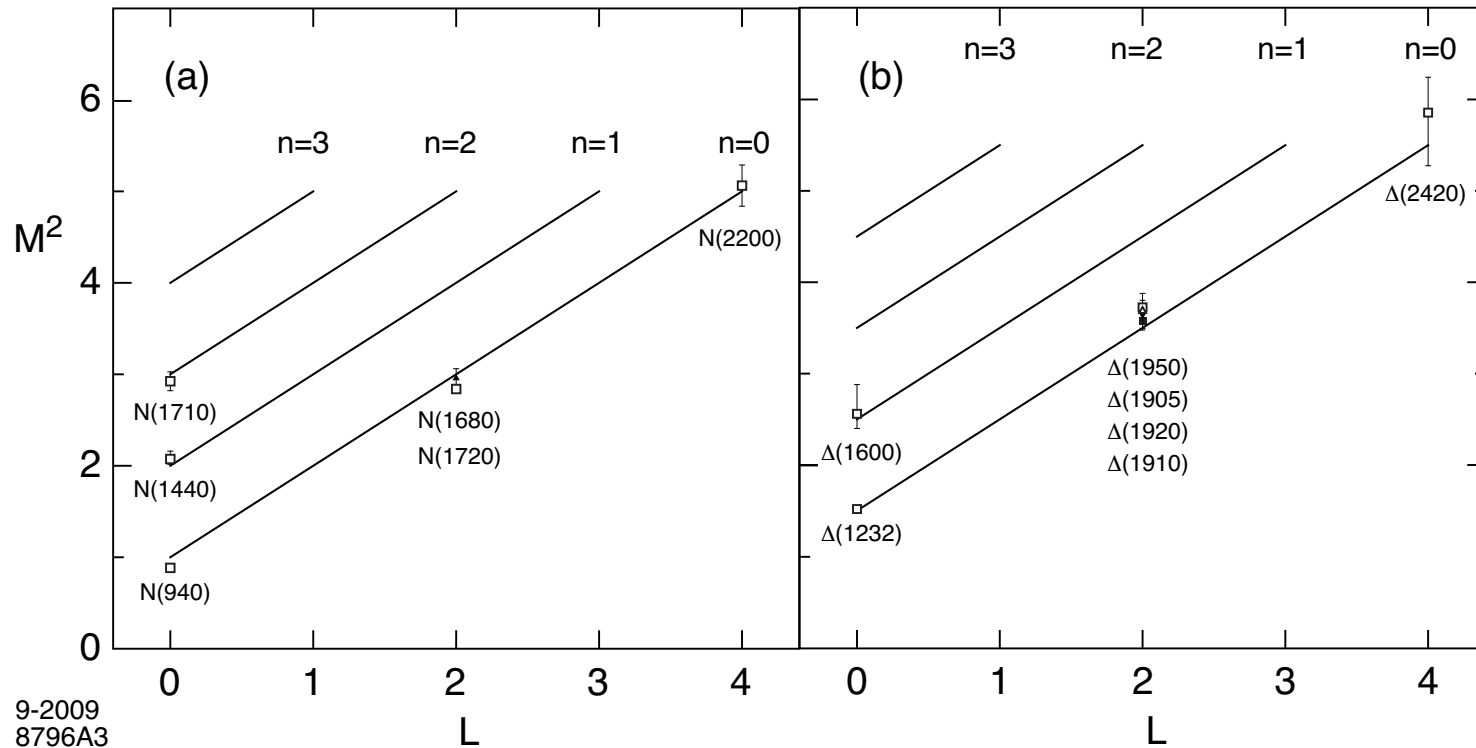
Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ -meson and $I = 0$ ω -meson families ($\kappa = 0.54$ GeV)

Same multiplicity of states for mesons and baryons!

$$4\kappa^2 \text{ for } \Delta n = 1$$

$$4\kappa^2 \text{ for } \Delta L = 1$$

$$2\kappa^2 \text{ for } \Delta S = 1$$



Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

- Δ spectrum identical to Forkel, Beyer and Frederico and Forkel and Klempt

IV. Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P') F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \bar{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from non-local coupling of external EM field propagating in AdS with extended mode $\Phi(x, z)$

$$\int d^4x dz \sqrt{g} A^\ell(x, z) \Phi_{P'}^*(x, z) \overleftrightarrow{\partial}_\ell \Phi_P(x, z)$$

- Are the transition amplitudes related ?
- How to recover hard pointlike scattering at large Q out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

- Mapping at fixed light-front time: $\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$

- Electromagnetic probe polarized along Minkowski coordinates, ($Q^2 = -q^2 > 0$)

$$A(x, z)_\mu = \epsilon_\mu e^{-iQ \cdot x} V(Q, z), \quad A_z = 0$$

- Propagation of external current inside AdS space described by the AdS wave equation

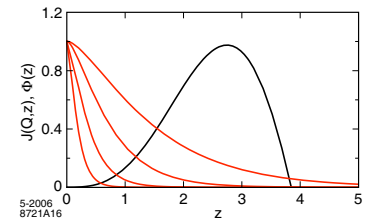
$$[z^2 \partial_z^2 - z \partial_z - z^2 Q^2] V(Q, z) = 0$$

- Solution $V(Q, z) = zQ K_1(zQ)$
- Substitute hadronic modes $\Phi(x, z)$ in the AdS EM matrix element

$$\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z), \quad \Phi(z) \rightarrow z^\tau, \quad z \rightarrow 0$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with the non-normalizable mode $V(Q, z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q, z) \Phi_J^2(z) \rightarrow \left(\frac{1}{Q^2} \right)^{\tau-1}$$



At large Q important contribution to the integral from $z \sim 1/Q$ where $\Phi \sim z^\tau$ and power-law point-like scaling is recovered [Polchinski and Susskind (2001)]

Electromagnetic Form-Factor

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006); PRD **77**, 056007 (2008)]

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\left(i\mathbf{q}_{\perp} \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

- Consider a two-quark π^+ Fock state $|u\bar{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_{\bar{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_{\perp} e^{i\mathbf{q}_{\perp} \cdot \mathbf{b}_{\perp}(1-x)} \left| \psi_{u\bar{d}/\pi}(x, \mathbf{b}_{\perp}) \right|^2$$

with normalization $F_{\pi^+}(q=0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left| \psi_{u\bar{d}/\pi}(x, \zeta) \right|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_{\perp}^2$

- Compare with electromagnetic FF in AdS space

$$F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q, z) \Phi_{\pi^+}^2(z)$$

where $V(Q, z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q, z) = \int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right)$$

- Find

$$F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right) \Phi_{\pi^+}^2(z)$$

- Compare with electromagnetic FF in LF QCD for arbitrary Q . Expressions can be matched only if LFWF is factorized

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

$$X(x) = \sqrt{x(1-x)}, \quad \phi(\zeta) = \left(\frac{\zeta}{R} \right)^{-3/2} \Phi(\zeta), \quad \zeta \rightarrow z$$

- Same results from mapping of gravitational form factor [S. J. Brodsky and GdT, PRD **78**, 025032 (2008)]

- Expand $X(x)$ in Gegenbauer polynomials (DA evolution equation [Lepage and Brodsky (1980)])

$$X(x) = \sqrt{x(1-x)} = x(1-x) \sum_{n=0}^{\infty} a_n C_n^{3/2}(2x-1)$$

- Normalization

$$\int_0^1 \frac{dx}{x(1-x)} X^2(x) = \sum_n P_n = 1$$

$$\begin{aligned} \langle C_n^\lambda | C_m^\lambda \rangle &= \int_0^1 dx x^{\lambda-1/2} (1-x)^{\lambda-1/2} C_n^\lambda(2x-1) C_m^\lambda(2x-1) \\ &= \frac{2^{1-4\lambda} \pi \Gamma(n+2\lambda)}{n!(n+\lambda)\Gamma^2(\lambda)} \end{aligned}$$

- Compute asymptotic probability $P_{n=0}$ ($Q \rightarrow \infty$)

$$P_{n=0} = \frac{\pi^2}{32} \simeq 0.3 \quad \left(a_0 = \frac{3\pi}{4} \right)$$

V. Higher Fock Components

(S. Brodsky and GdT)

- LF Lorentz invariant Hamiltonian equation for the relativistic bound state system

$$P_\mu P^\mu |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle,$$

where $P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 \equiv H_{LF}$

- H_{LF} sum of kinetic energy of partons H_{LF}^0 plus an interaction H_I , $H_{LF} = H_{LF}^0 + H_I$
- Expand in Fock eigenstates of H_{LF}^0 : $|\psi\rangle = \sum_n \psi_n |n\rangle$,

$$\left(\mathcal{M}^2 - \sum_{i=1}^n \frac{\mathbf{k}_{\perp i}^2 + m^2}{x_i} \right) \psi_n = \sum_m \langle n|V|m\rangle \psi_m$$

an infinite number of coupled integral equations

- Only interaction in AdS/QCD is the confinement potential
- In QFT the resulting LF interaction is the 4-point effective interaction $H_I = \bar{\psi}\psi V(\zeta^2)\bar{\psi}\psi$ which leads to $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$, $q \rightarrow qq\bar{q}$ and $\bar{q} \rightarrow \bar{q}q\bar{q}$
- Create Fock states with extra quark-antiquark pairs. No mixing with $q\bar{q}g$ Fock states ($g_s \bar{\psi}\gamma \cdot A\psi$)
- Explain the dominance of quark interchange in large angle elastic scattering

Detailed Structure of Space-and Time Like Pion Form Factor

- Holographic variable for n-parton hadronic bound state

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

the x -weighted transverse impact coordinate of the spectator system (x active quark)

- Form factor in soft-wall model expressed as $N - 1$ product of poles along vector radial trajectory [Brodsky and GdT (2008)] ($\mathcal{M}_\rho^2 \rightarrow 4\kappa^2(n + 1/2)$)

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}$$

- Higher Fock components in pion LFWF

$$|\pi\rangle = \psi_{q\bar{q}/\pi} |q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}/\pi} |q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

- Expansion of LFWF up to twist 4 (monopole + tripole)

$$\kappa = 0.54 \text{ GeV}, \Gamma_\rho = 130, \Gamma_{\rho'} = 400, \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\bar{q}q\bar{q}} = 13\%$$

