

# Setting the Renormalization Scale in QCD: The Principle of Maximum Conformality PMC

SLAC Exp Seminar, 21th July 2011

Based on the work done in collaboration with:

Stanley J. Brodsky

SLAC-PUB-14425, Apr 2011;

Stanley J. Brodsky, L.D.G.: [arXiv:1107.0338 \[hep-ph\]](https://arxiv.org/abs/1107.0338);

# Outlook

- The Renormalization Scale Setting
- The QED scenario
- The BLM approach
- The PMC scale setting procedure in details :  
Analytic&Num
- Examples: 3-jet, Exclusive decays,...
- Properties of the PMC (CSR, Conformal  
Template,...)

# The QCD Scale Setting: a Key Issue

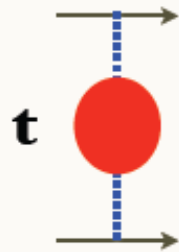
- Test QCD to maximum precision
- High precision determination of  $\alpha_s(Q^2)$
- Relate observable to observable --no scheme or scale ambiguity
- Eliminate renormalization scale ambiguity in a scheme-independent manner
- Maximize sensitivity to new physics at the LHC

# No scale ambiguity in QED

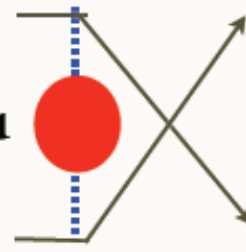
- The Gell Mann-Low scheme

$$\mathcal{M}_{ee \rightarrow ee}(++; ++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$

e-e scattering



**t**



**u**

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

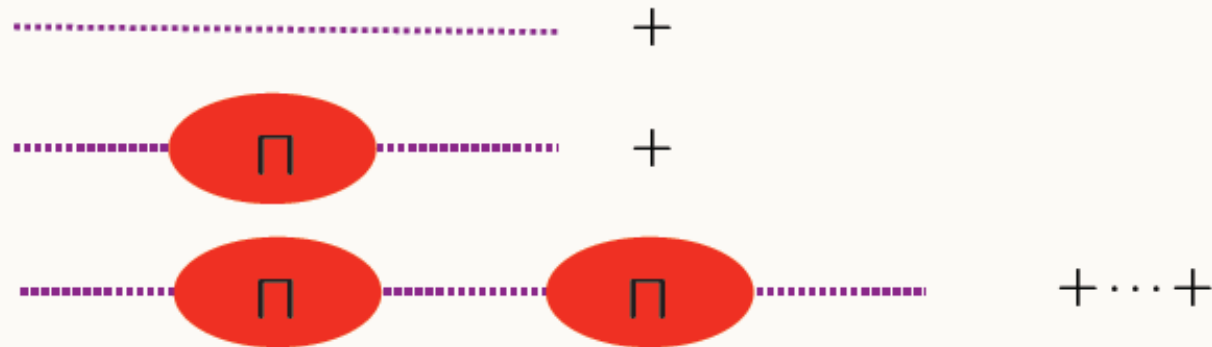
The Vacuum Polarization

Gell-Mann-Low Effective Charge

# The Renormalization scale in QED is not arbitrary

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

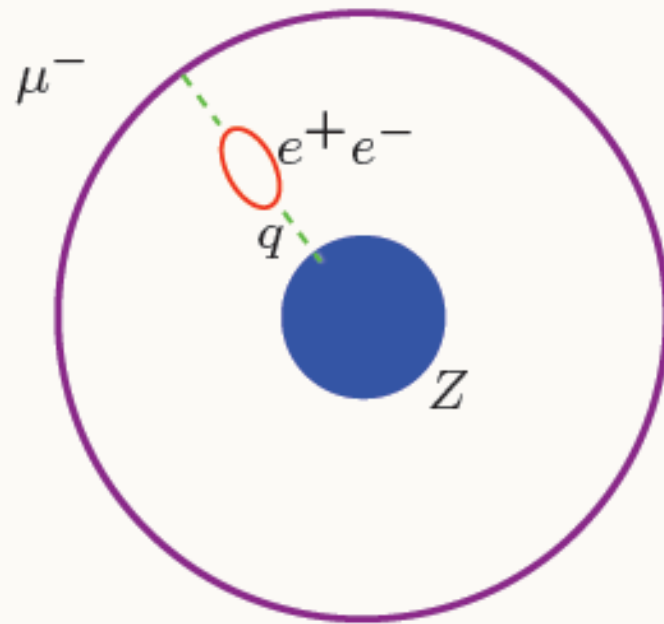
*All-orders lepton loop corrections to dressed photon propagator*



$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \quad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

**Initial scale  $t_0$  is arbitrary -- Variation gives RGE Equations**  
**Physical renormalization scale  $t$  not arbitrary!**

# QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

**Scale is unique: Tested to ppm**

Gyulassy: Higher Order VP verified to 0.1% precision in  $\mu$  Pb

## Relation between scales of the Gell-Mann-Low and MS schemes

$$\log \frac{\mu_0^2}{m_\ell^2} = 6 \int_0^1 x(1-x) \log \frac{m_\ell^2 + Q_0^2 x(1-x)}{m_\ell^2}$$

$$\log \frac{\mu_0^2}{m_\ell^2} = \log \frac{Q_0^2}{m_\ell^2} - 5/3$$

$$\mu_0^2 = Q_0^2 e^{-5/3}$$

$$\text{when } Q_0^2 \gg m_\ell^2$$

Space-Like  $q^2$   
Dimensional Reg.

D.S. Hwang, S.J. Brodsky  
M. Binger

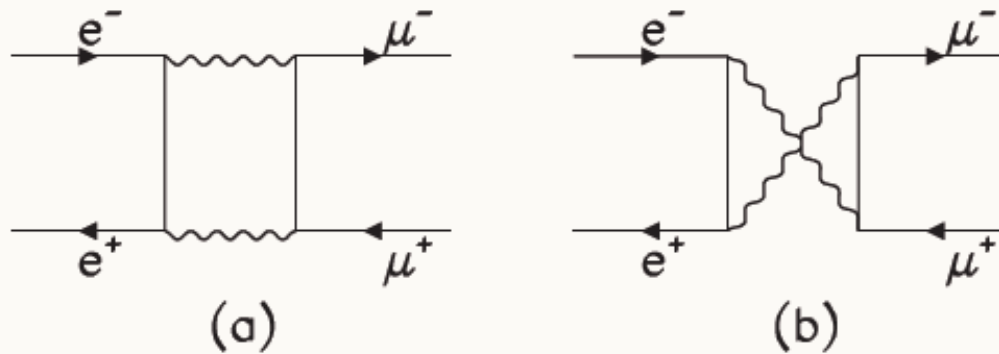
*Can use MS scheme in QED; answers are scheme independent  
Analytic extension: coupling is complex for timelike argument*

**Target normal spin asymmetry and charge asymmetry for  $e\mu$  elastic scattering and the crossed processes**

E. A. Kuraev, V. V. Bytev, and Yu. M. Bystritskiy  
 JINR-BLTP, 141980 Dubna, Moscow region, Russian Federation

E. Tomasi-Gustafsson  
 DAPNIA/SPHn, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France

*Physics of conformal series; not associated with renormalization*

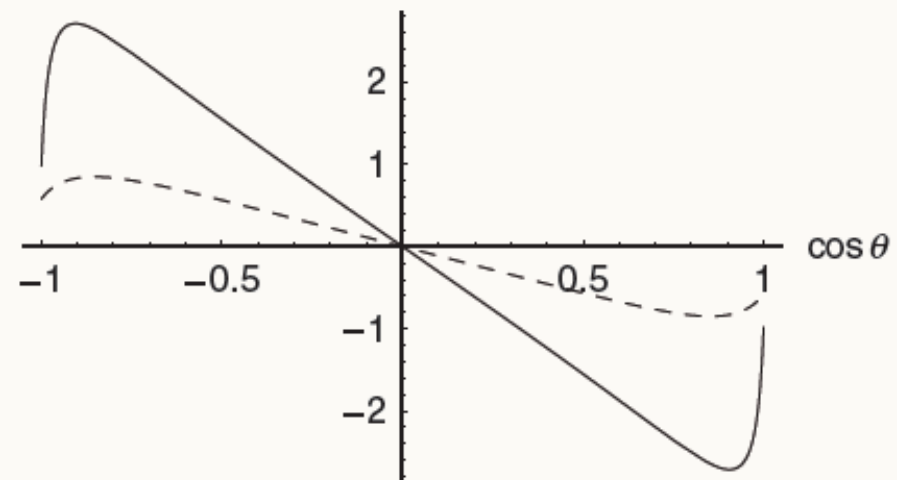


$$A(\theta, \Delta E) = \frac{d\sigma(\theta) - d\sigma(\pi - \theta)}{d\sigma_B(\theta)} = \frac{4\alpha}{\pi} \Upsilon \Phi(s, \cos \theta)$$

$$\frac{d\sigma_{\text{ann}}}{d\Omega} = \frac{\alpha^3 \beta}{2\pi s} (2 - \beta^2 + \beta^2 c^2) \Upsilon,$$

$$\Upsilon = 2 \ln \frac{1 + \beta c}{1 - \beta c} \ln \left( \frac{2\Delta E}{m} \right) + \Phi(s, \cos \theta)$$

$$\Phi(s, \cos \theta) = \mathcal{D}_S^{\text{ann}} - \frac{\mathcal{D}_V^{\text{ann}}}{2 - \beta^2 + \beta^2 c^2}.$$



# The Renormalization Scale Problem in QED

- No renormalization scale ambiguity in QED
- Gell Mann-Low QED Coupling defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds – number of active leptons set
- Examples: muonic atoms,  $g^{-2}$ , Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion
- Results are scheme independent!

# QCD: The Abelian Limit Huet, S.J.Brodsky

$$\lim N_C \rightarrow 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

QCD  $\rightarrow$  Abelian Gauge Theory

*Analytic Feature of SU(Nc) Gauge Theory*

*Scale-Setting procedure for QCD  
must be applicable to QED*

# Legends on the scale setting in QCD

- Renormalization scale “unphysical”: No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess  $\mu_R = Q$  with an arbitrary range  $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale  $\mu_F = \mu_R$

**These assumptions are untrue in QED  
and thus they cannot be true for QCD**

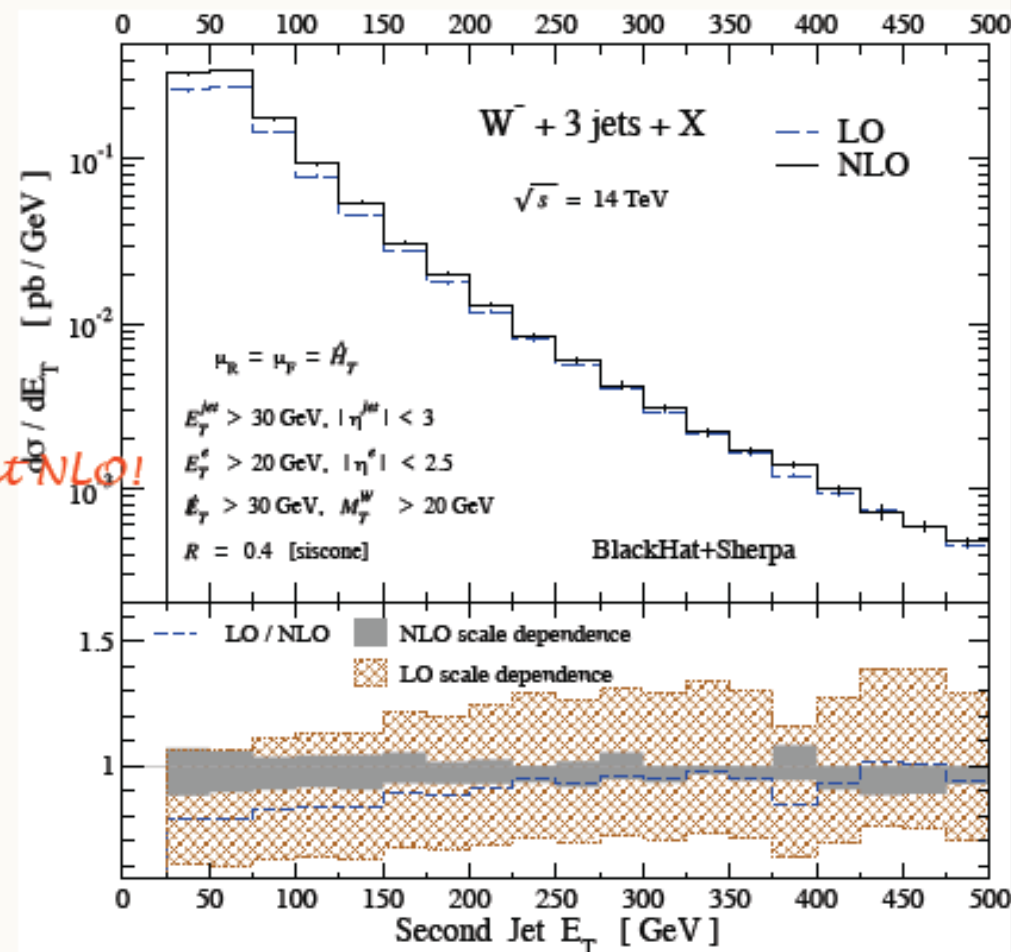
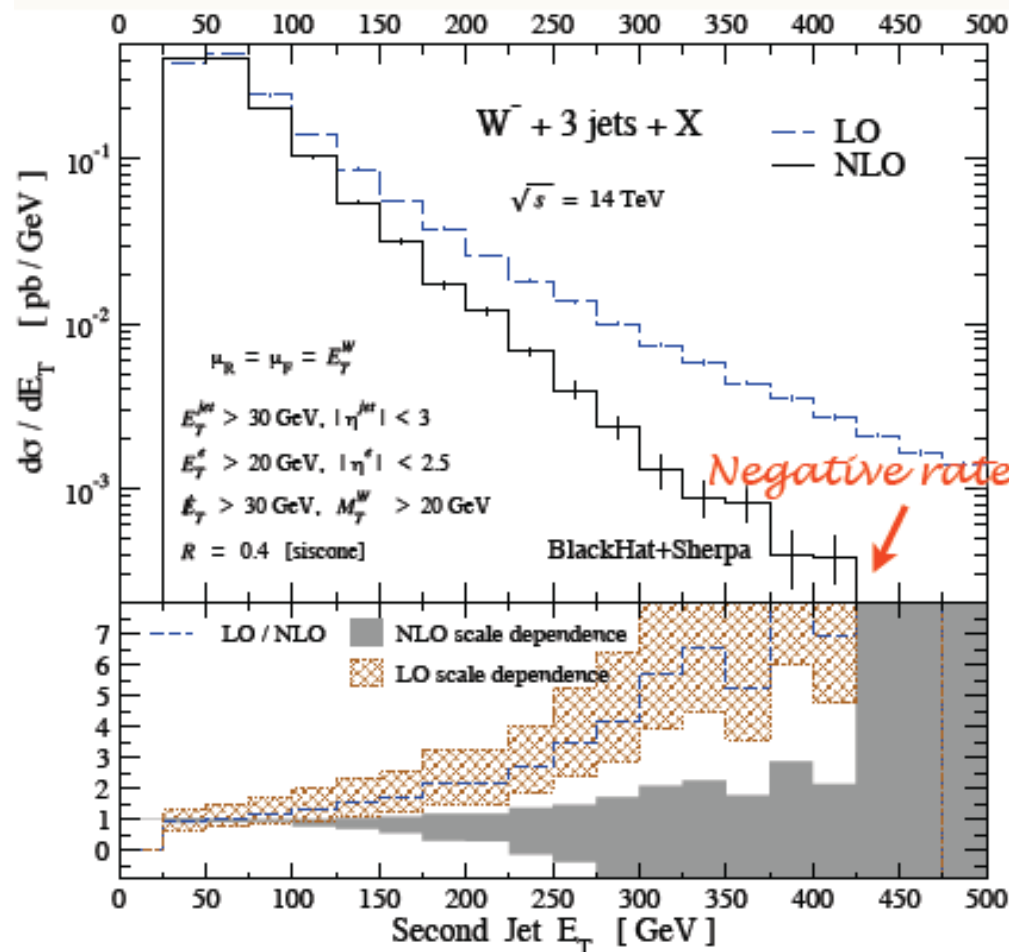
**Wrong in QED, Scheme dependent!**

# NLO QCD predictions for $W \rightarrow 3\text{Jet}$ distributions at LHC

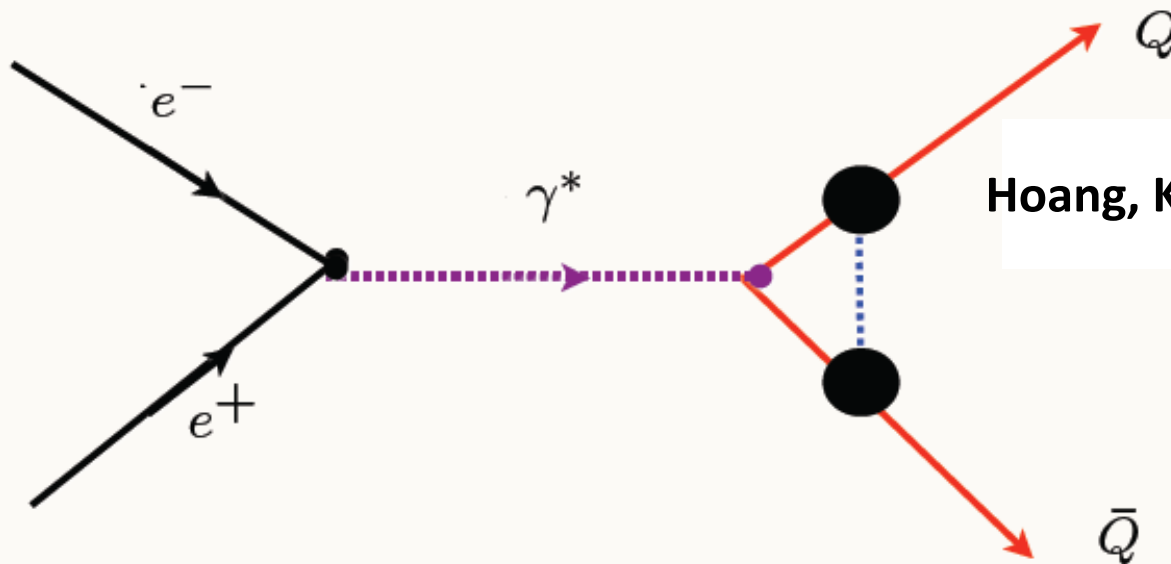
*Black Hat*

$$\mu_R = \mu_F = E_T^W$$

$$\mu_R = \mu_F = \hat{H}_T$$



F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower, and D. Maitre



Hoang, Kuhn, Teubner, S.J. Brodsky

$$F_1 + F_2 = \left[ 1 - 2 \frac{\alpha_s (s e^{3/4} / 4)}{\pi} \right] \times \left[ 1 + \frac{\pi \alpha_s (s v^2)}{4v} \right]$$

Angular distributions of massive quarks close to threshold.

*Example of Multiple BLM Scales*

**Need QCD coupling at small scales at low relative velocity  $v$**

# Features of the BLM/PMC scale

- All terms associated with the beta-function are included into the running coupling;
- BLM/PMC scale sets the number of active flavors;
- Only the nf-terms are required to fix the BLM/PMC scale at NLO;
- Results are scheme independent +Transitivity Property is valid
- Correct Abelian limit;
- No renormalon growth  $n!$  in PQCD associated with the beta function;
- In general the BLM/PMC scale depends on all the invariants;
- Resulting series is identical to conformal series! (CSR - Crewther Relation ;)

# The BLM scale fixing

$$\rho = C_0 \alpha_{\overline{\text{MS}}}(Q) \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \left( -\frac{3}{2} \beta_0 A_{\text{VP}} + \frac{33}{2} A_{\text{VP}} + B \right) + \dots \right]$$

*$n_f$  dependent coefficient identifies quark loop VP contribution*

by

$$\rho = C_0 \alpha_{\overline{\text{MS}}}(Q^*) \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q^*)}{\pi} C_1^* + \dots \right],$$

where

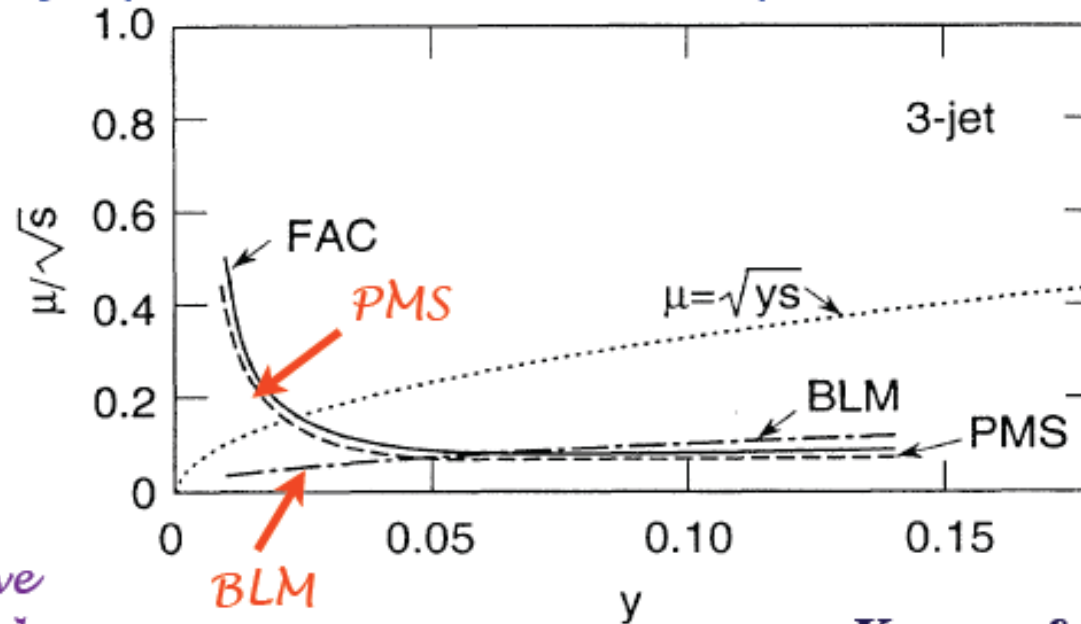
Conformal coefficient - independent of  $\beta$

$$Q^* = Q \exp(3A_{\text{VP}}),$$

$$C_1^* = \frac{33}{2} A_{\text{VP}} + B.$$

The term  $33A_{\text{VP}}/2$  in  $C_1^*$  serves to remove that part of the constant  $B$  which renormalizes the leading-order coupling.

## Three-jet production in electron-positron annihilation



Jet Invariant mass squared:

$$\mathcal{M}^2 = ys$$

PMS & FAC have wrong physical dependence!

Kramer & Lampe

The scale  $\mu/\sqrt{s}$  according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and  $\sqrt{y}$  (dotted) procedures for the three-jet rate in  $e^+e^-$  annihilation, as computed by Kramer and Lampe. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low  $y$ . In particular, the latter two methods predict increasing values of  $\mu$  as the jet invariant mass  $\mathcal{M} < \sqrt{(ys)}$  decreases.

Other Jet Observables using BLM: Rathsmann

# The Principle of Maximum Conformality

Stanley J. Brodsky, L.D.G.:  
[arXiv:1107.0338 \[hep-ph\]](https://arxiv.org/abs/1107.0338);

- BLM: Set scale in each skeleton graph to absorb all nonzero beta terms.
- In practice easier to set a single global scale
- Consider general hard subprocess:  $a + b \rightarrow c + d + e + \dots$

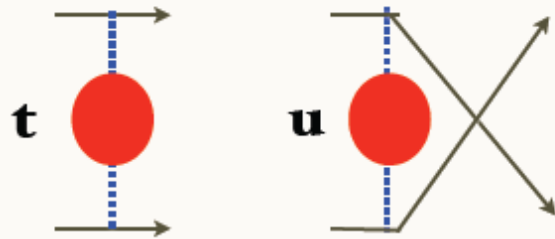
$$p_{ij}^2 = p_i \cdot p_j \quad \text{e.g., } pp \rightarrow W + 3 \text{ jets} + X$$

$$\hat{\mu}^2 = C \times \prod_{ij} [p_{ij}^2]^{w_{ij}} \quad \log \hat{\mu}^2 = \sum_{i \neq j} w_{ij} \log p_{ij}^2 + \log C$$

$$w_{ij} = \frac{f_{ij}}{\sum_{i \neq j} f_{ij}}. \quad C \text{ is the scheme displacement}$$

$$C = e^{-5/3} \text{ for } \overline{MS}$$

# Example: spinless electron-electron scattering



$$M = \frac{s-t}{t} \alpha(t) + \frac{s-u}{u} \alpha(u)$$

*Scales sum VP to all orders*

$$M \simeq \left[ \frac{s-t}{t} + \frac{s-u}{u} \right] \alpha(\mu_0^2) + \left[ \frac{s-t}{t} \right] \frac{\alpha^2(\mu_0^2)}{3\pi} n_\ell \log\left(\frac{t}{\mu_0^2}\right) + \left[ \frac{s-u}{u} \right] \frac{\alpha^2(\mu_0^2)}{3\pi} n_\ell \log\left(\frac{u}{\mu_0^2}\right)$$

$$M = \left[ \frac{s-t}{t} + \frac{s-u}{u} \right] \alpha(\hat{\mu}^2)$$

$$\hat{\mu}^2 = t^{w_t} \times u^{w_u}$$

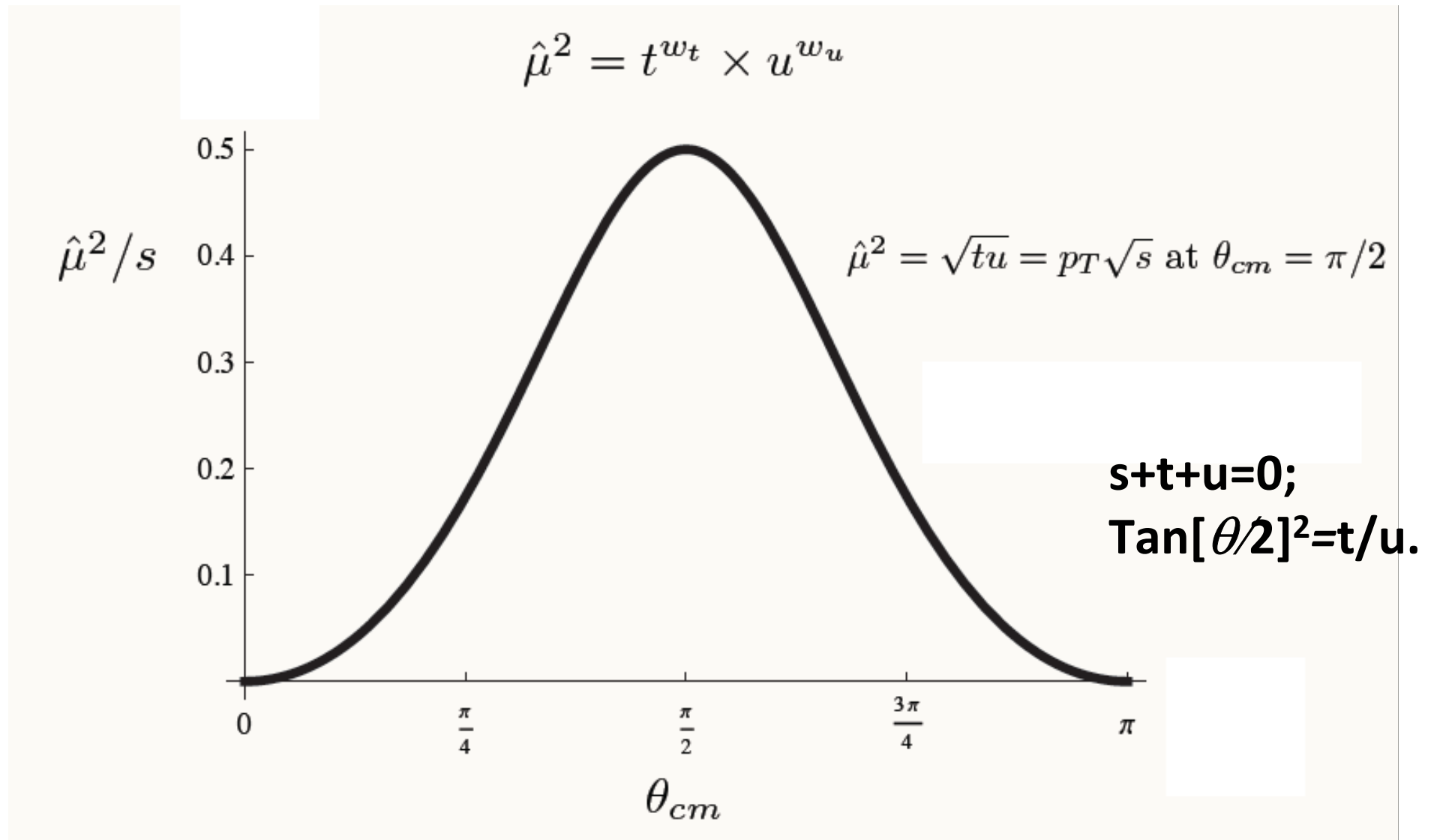
$$w_t = \frac{\frac{s-t}{t}}{\frac{s-t}{t} + \frac{s-u}{u}}$$

$$w_u = \frac{\frac{s-u}{u}}{\frac{s-t}{t} + \frac{s-u}{u}}$$

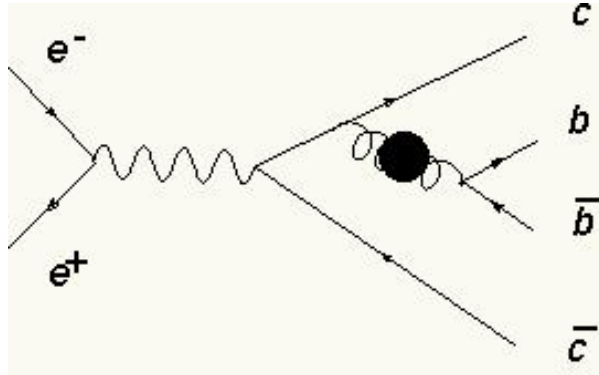
Identify  $w_t$  from  $\frac{dM}{d \log t}$

Remaining  $\mathcal{O}(\alpha^2)$  correction is conformal

# Spinless electron-electron scattering



# PMC – Exclusive Case



All flavors and momenta in the final state are identified

- The nf terms at NLO come from the quark loop in the gluon propagator.
- The PMC scale for the differential cross section in the  $\overline{\text{MS}}$  scheme is given by the scheme displacement of the gluon virtuality:

$$\mu_{PMC}^2 = e^{-5/3} (p_b + p_{\bar{b}})^2$$

# 3-Jet case: Integrated results

\* R. K. Ellis *et Al*, Nucl. Phys. B178, 421-456 (1981)

$$\frac{1}{\sigma_0} \frac{d\sigma^{(s)} + d\sigma^3}{dy} = \int_y^{1-2y} dz \int_y^{1-y-z} dx T[1-x-z, x, z] \alpha_s(Q^2) (1 - \beta_0 \alpha_s(Q^2) (\log[x] + \log[z] - \frac{5}{3} \dots))$$

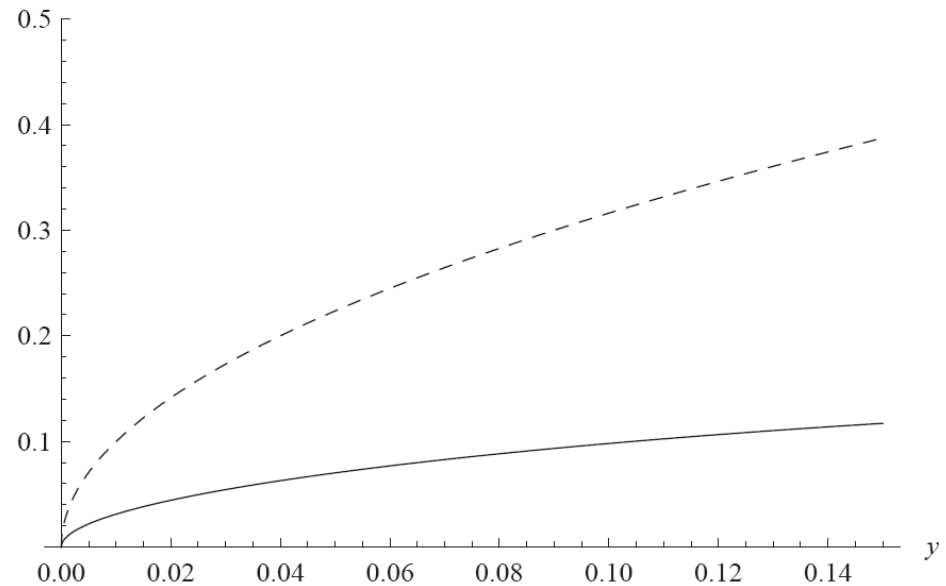
**y**: the maximum  
virtuality of the jet

$$= \alpha_s(Q^2) (T(y) - \beta_0 \alpha_s(Q^2) (C(y) + \dots))$$

$$\equiv T(y) \alpha_s(Q^2) (1 - \beta_0 \alpha_s(Q^2) 2 \log[\frac{\mu_{BLM}}{\sqrt{s}}]) = T(y) \alpha_s(\mu_{BLM}^2);$$

$$\mu_{PMC/BLM}^2 = s \times e^{-\frac{5}{3}} + \frac{C(y)}{T(y)}$$

For comparison the dashed line is  $\sqrt{y}$



**We recover the  
Kramer and Lampe results**

# 3-jet case: differential X-section

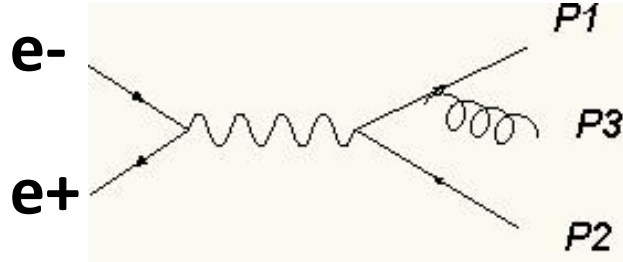
$$\frac{1}{\alpha_s(Q^2)} \frac{d}{d\beta_0} \left. \frac{d\sigma^3(Q^2)}{dz dy dx} \right|_{Born}^{-1} \left( \frac{d\sigma^{(s)} + d\sigma^3}{dz dy dx} \right) = \left[ \log[xy] - \frac{5}{3} \right] + O(\alpha_s).$$

$$\omega_i = \frac{d}{d \log(x_i)} \frac{1}{\alpha_s(Q^2)} \frac{d}{d\beta_0} \left. \frac{d\sigma^3(Q^2)}{dz dy dx} \right|_{Born}^{-1} \left( \frac{d\sigma^{(s)} + d\sigma^3}{dz dy dx} \right)$$

The **Analytic Form** of the PMC scale can be determined by isolating the nf term and then varying the subprocess amplitude with respect to each invariant and thus determining the **weights  $\omega_i$**  for each invariant.

$$\mu_{PMC}^2 \simeq Q^2 \times C \times \prod_i x_i^{\omega_i} = Q^2 x y e^{-\frac{5}{3}}.$$

# 3-Jet case: Numerical PMC scale fixing



\* K. Fabricius *et Al*, Z. Phys. C 11, 315 (1981)

$$\frac{d^2 \sigma^{(3)}(\epsilon, \delta)^*}{dx_1 dx_2} = \sigma_0 \frac{\alpha_s(q^2)}{2\pi} C_F \times$$

$$\left\{ B^V(x_1, x_2) \left[ 1 - \alpha_s(q^2) \beta_0 \left( \log\left(\frac{1 - \cos \delta}{2}\right) + \log \hat{x}_3^2 - \frac{13}{3} \right) \right] - B^S(x_1, x_2) \alpha_s(q^2) \frac{\beta_0}{2} \right\} + \mathcal{O}(\delta^2) + \dots$$

$$y_{ij} = s_{ij}/q^2 = (p_i + p_j)^2/q^2 \quad x_1 = 1 - y_{23} \quad x_2 = 1 - y_{13},$$

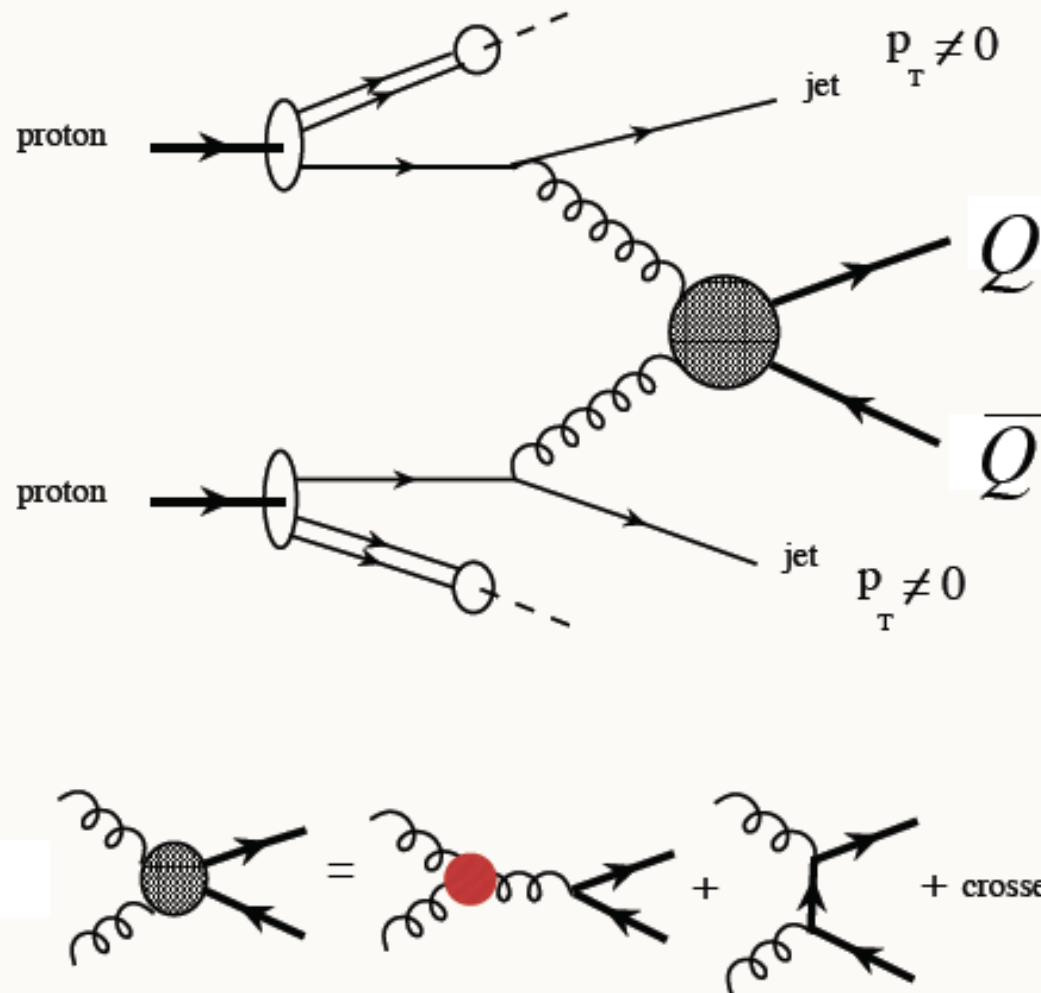
$$x_1 + x_2 + x_3 = 2.$$

$$\left[ \frac{d\sigma^{(3)}(\mu^2)}{dx_1 dx_2} \Big|_{\text{Born}}^{-1} \cdot \left( \frac{d}{dn_f} \frac{d^2 \sigma^{(3)}(\epsilon, \delta; \mu^2)}{dx_1 dx_2} \right) \Big|_{n_f=0} \right] \Big|_{\mu^2 = \mu_{PMC}^2} = 0$$

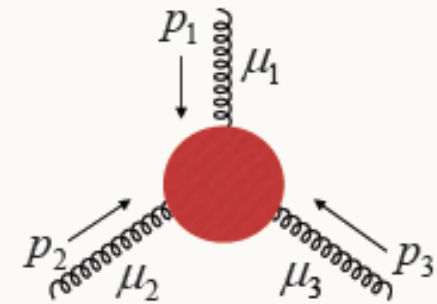
$$\frac{df}{d\beta_0} = \frac{df}{dn_f} \times \frac{d\beta_0}{dn_f}^{-1} \quad \mu_{PMC}^2 \simeq q^2 (2 - x_1 - x_2)^2 \frac{\delta^2}{4} e^{-\frac{13}{3} + \frac{B_S(x_1, x_2)}{2 B_V(x_1, x_2)}}$$

## 3-jet PMC scale : Analytic form

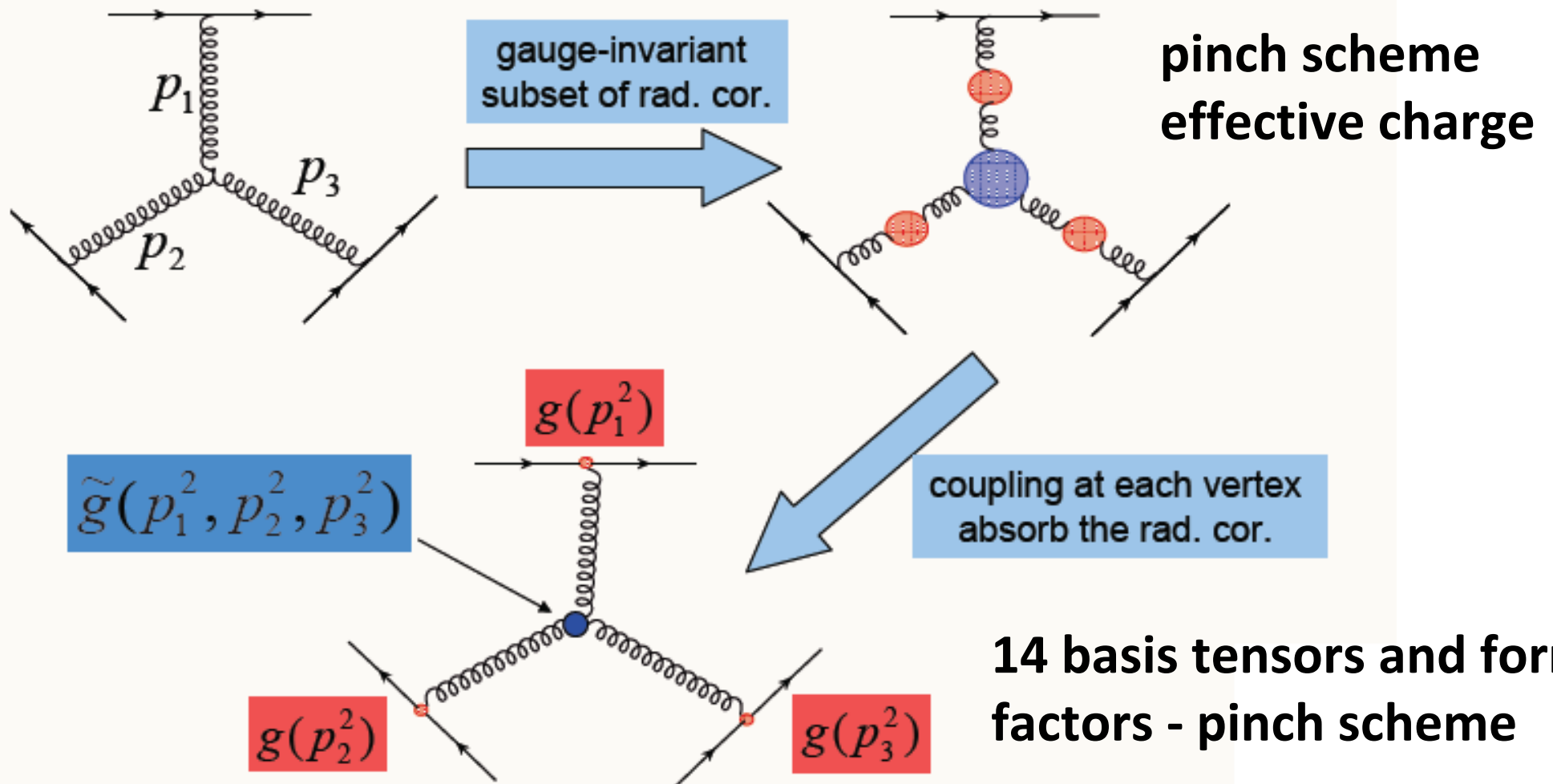
# Heavy Quark Hadroproduction



**3-gluon  
coupling  
depends on 3  
physical scales**



# Multi-scale renormalization



M.Binger , S.J. Brodsky  
 Phys.Rev D 74, 054016 (2006)

# 3 Scale Effective Charge

$$\tilde{\alpha}(a,b,c) \equiv \frac{\tilde{g}^2(a,b,c)}{4\pi}$$

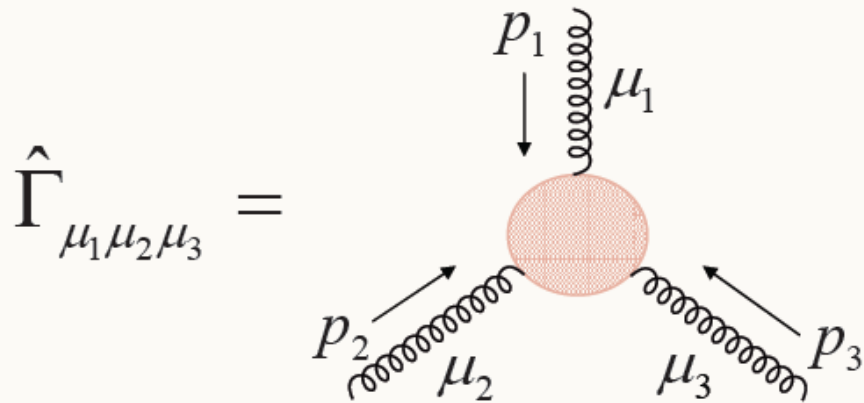
$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi} \beta_0 \left( L(a,b,c) - \frac{1}{\varepsilon} + \dots \right)$$

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi} \beta_0 [L(a,b,c) - L(a_0,b_0,c_0)]$$

$L(a,b,c)$  = 3-scale “log-like” function

$L(a,a,a) = \log(a)$

# Effective Scale properties



$$Q_{\text{eff}}^2(a, b, c) = Q_{\text{eff}}^2(-a, -b, -c)$$

$$Q_{\text{eff}}^2(\lambda a, \lambda b, \lambda c) = |\lambda| Q_{\text{eff}}^2(a, b, c)$$

$$Q_{\text{eff}}^2(a, a, a) = |a|$$

$$Q_{\text{eff}}^2(a, -a, -a) \approx 5.54 |a|$$

$$Q_{\text{eff}}^2(a, a, c) \approx 3.08 |c| \quad \text{for } |a| \gg |c|$$

$$Q_{\text{eff}}^2(a, -a, c) \approx 22.8 |c| \quad \text{for } |a| \gg |c|$$

$$Q_{\text{eff}}^2(a, b, c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for } |a| \gg |b|, |c|$$

$$\mu_R^2 \simeq \frac{p_{\min}^2 p_{\text{med}}^2}{p_{\max}^2}$$

H. J. Lu

*Scale determines effective number of flavors*

# QCD observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

**Scale-Free  
Conformal Series**

**Running Coupling  
Effects**

**Higher Twist from  
Hadron Dynamics**

**Intrinsic Heavy  
Quarks**

**Light by Light  
Loops**

***BLM: Absorb  $\beta$  terms  
into running coupling***

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

**BLM/PMC: Scheme independent**

# Define QCD coupling from observables

**Grunberg**

**Effective Charges: analytic at quark mass thresholds, finite at small momenta**

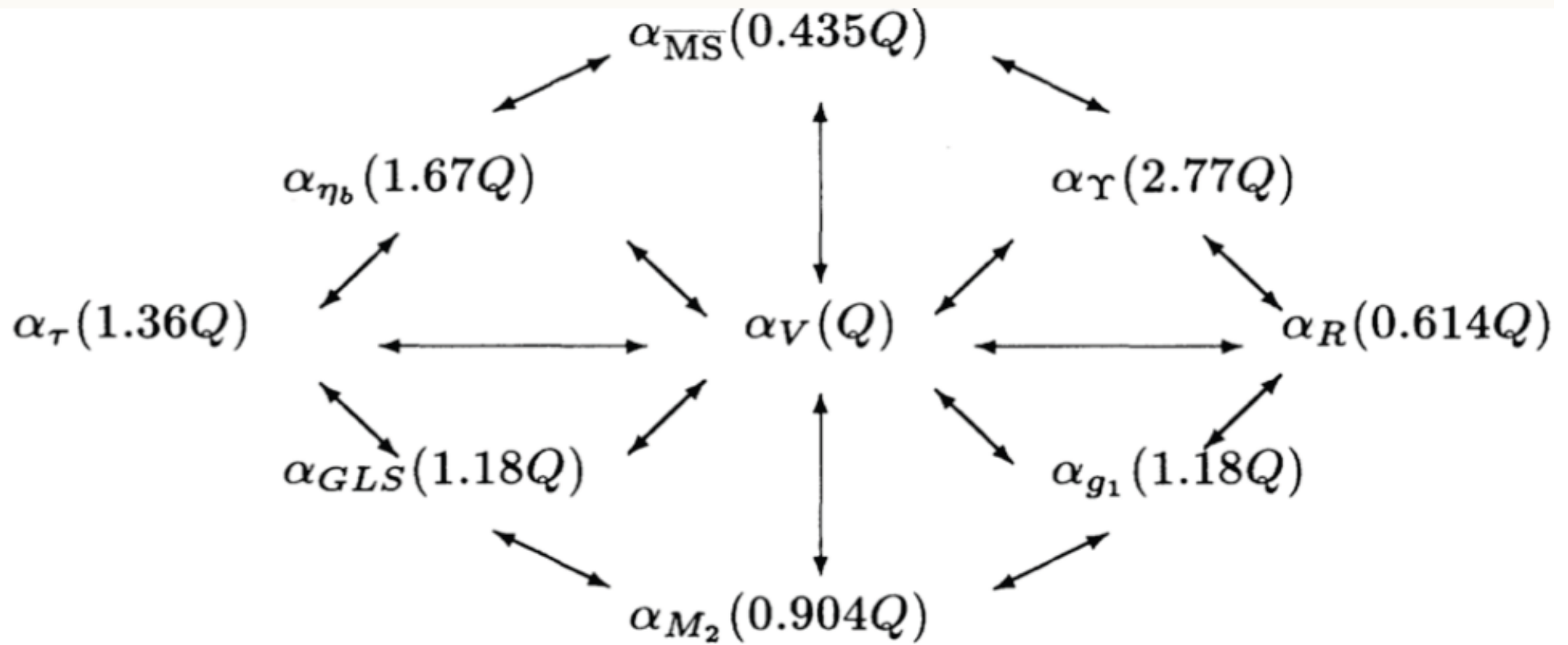
$$R_{e^+e^- \rightarrow X}(s) \equiv 3 \sum_q e_q^2 \left[ 1 + \frac{\alpha_R(s)}{\pi} \right]$$

$$\Gamma(\tau \rightarrow X e \nu)(m_\tau^2) \equiv \Gamma_0(\tau \rightarrow u \bar{d} e \nu) \times \left[ 1 + \frac{\alpha_\tau(m_\tau^2)}{\pi} \right]$$

**Commensurate scale relations:**

**Relate observable to observable at commensurate scales**

**H. Lu, Rathsmann, S.J. Brodsky**

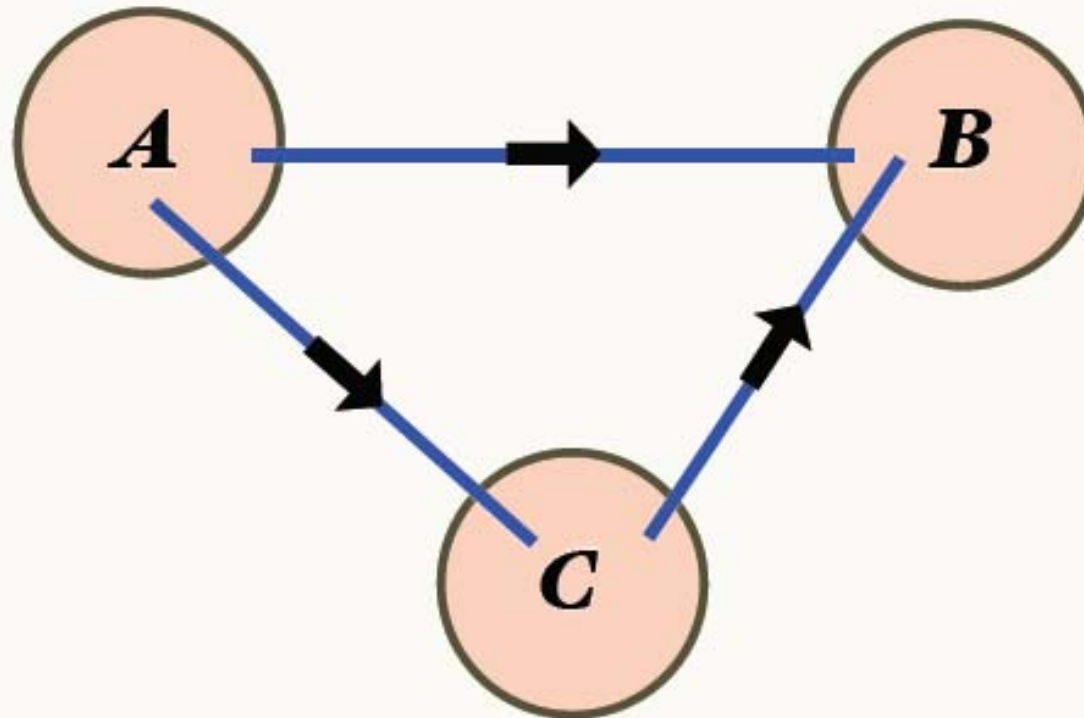


$$\frac{\alpha_T(M_T)}{\pi} = \frac{\alpha_R(Q^*)}{\pi},$$

$$Q^* = M_T \exp \left[ -\frac{19}{24} - \frac{169}{128} \frac{\alpha_R(M_T)}{\pi} \right]$$

# *Transitivity Property of Renormalization Group*

*Relation of observables must be independent of intermediate scheme*



**$A \rightarrow C$**      **$C \rightarrow B$**     *identical to*     **$A \rightarrow B$**

*Violated by PMS!*

# BLM/PMC: Relate Observables to each other

- Eliminate Intermediate schemes
- No Scale Ambiguity
- Transitivity
- Commensurate Scale Relations
- Conformal Template
- Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\begin{aligned}
\frac{\alpha_R(Q)}{\pi} = & \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi}\right)^2 \left[ \left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8}C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f \right] \\
& + \left(\frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi}\right)^3 \left\{ \left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32}C_F^2 \right. \\
& + \left[ \left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F \right] f \\
& \left. + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2} \right\}.
\end{aligned}$$

$$\begin{aligned}
\frac{\alpha_{g_1}(Q)}{\pi} = & \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi}\right)^2 \left[ \frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f \right] \\
& + \left(\frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi}\right)^3 \left\{ \left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32}C_F^2 \right. \\
& \left. + \left[ \left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F \right] f + \frac{115}{648}f^2 \right\}.
\end{aligned}$$

**Eliminate MS**  
**Find Amazing Simplification**

$$R_{e+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left( \frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left( \frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

*Geometric Series in Conformal QCD*

*Generalized Crewther Relation*

**Lu, Kataev, Gabadadze, S.J. Brodsky**

## PMC: Generalized Crewther Relation

$$\left[1 + \frac{\alpha_R(s^*)}{\pi}\right] \left[1 - \frac{\alpha_{g_1}(q^2)}{\pi}\right] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

*Conformal relation true to all orders in  
perturbation theory*

*No radiative corrections to axial anomaly*

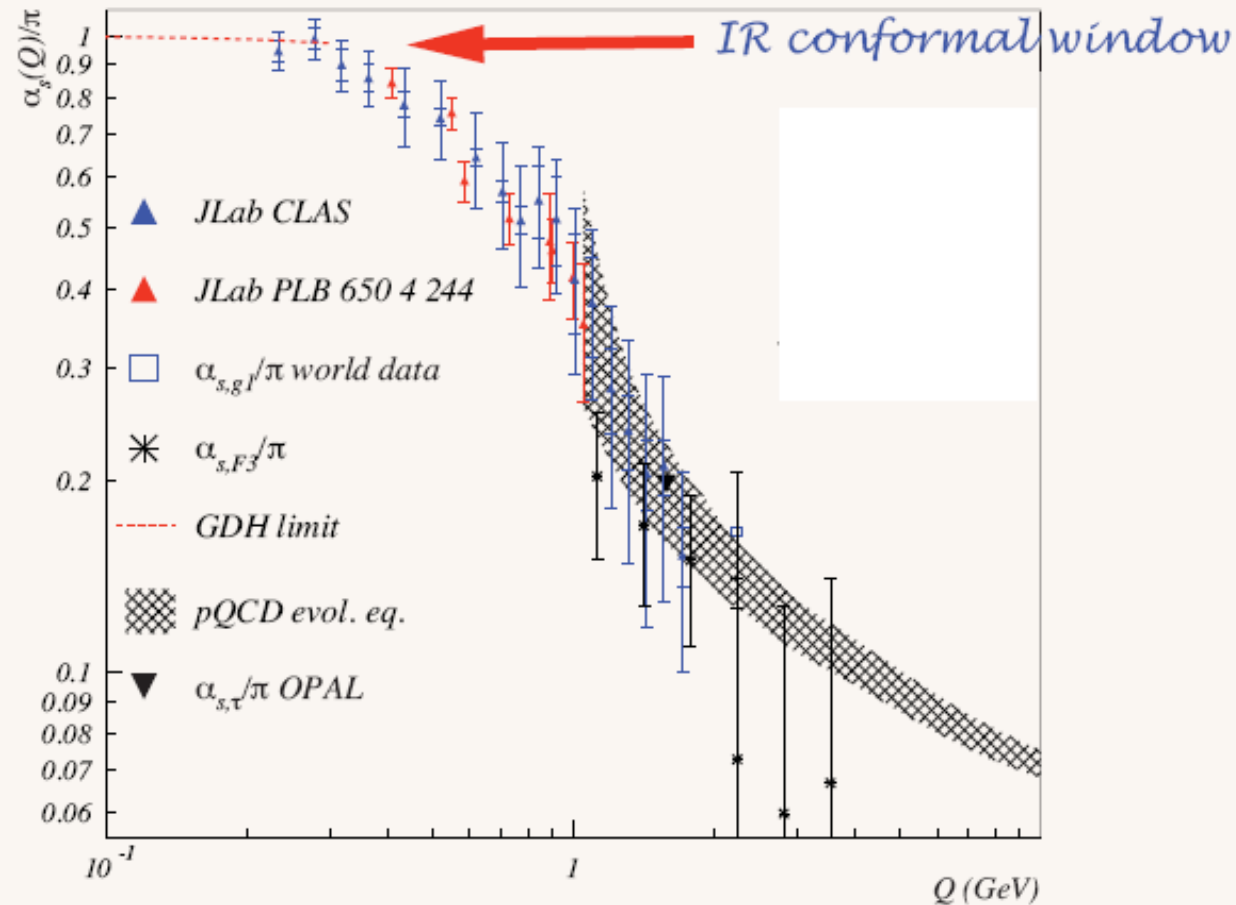
*Nonconformal terms set relative scales (BLM)*

*No renormalization scale ambiguity!*

**Both observables go through new quark thresholds  
at commensurate scales!**

# Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6} \left[ 1 - \frac{\alpha_s^{g1}(Q^2)}{\pi} \right]$$



**Conformal window:**  $\alpha_s(Q^2) \simeq \text{const}$  at small  $Q^2$

Deur, Korsch, et al

# Hints for IR Fixed Point for QCD

- Effective gluon mass – maximal wavelength
- Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point
- Lattice Gauge Theory
- Define coupling from observable, indications of IR fixed point for QCD effective charges
- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small  $Q^2$
- Justifies application of AdS/CFT in strong-coupling conformal window

**QCD is not conformal; however, it has manifestations of a scale invariant theory**

# Conformal Symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for nonzero beta function and quark masses
- Eigensolutions of ERBL evolution equation for distribution amplitudes
- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation
- Fix Renormalization Scale PMC/BLM
- IR Fixed Point -- A Conformal Domain
- Use AdS/CFT

# Conclusions: PMC points

- PMS/FAC - incorrectly sums conformal terms – even minimizes physical asymmetries!
- PMC/BLM: exposes conformal series - no renormalons
- Conformal series has new physics – not associated with renormalization
- PMC: No need to analyze diagrams or codes – simply identify non-conformal logarithms – then shift scale
- PMC: Applies to subprocesses with multiple final particles- recursive procedure
- PMC/BLM: Agrees with QED in Abelian limit
- PMC/BLM: Result is independent of scheme and initial scale choice