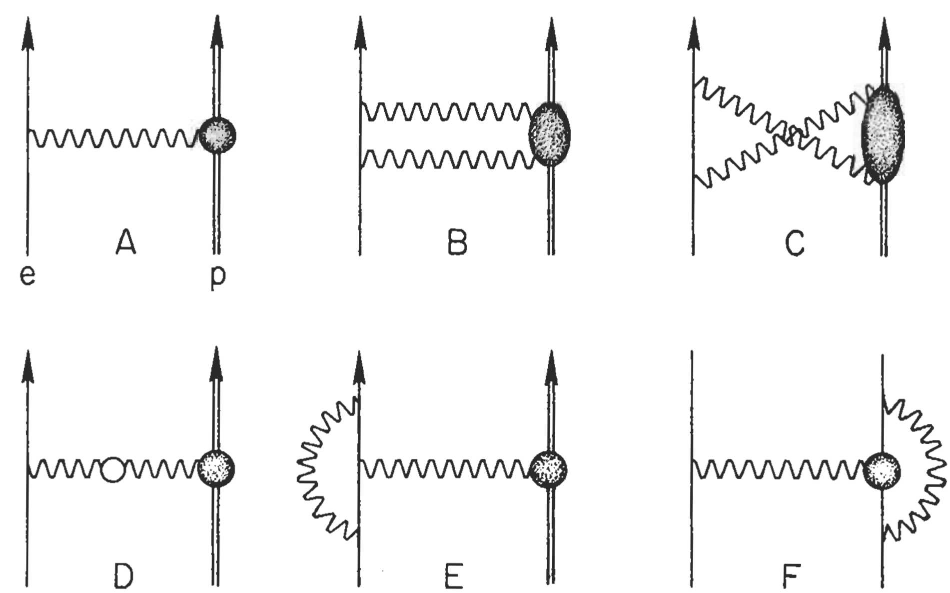
## III. COMPARISON OF POSITRON-PROTON AND ELECTRON-PROTON ELASTIC SCATTERING

### A. Introduction

In Part I of this report we have proposed to separate the electric and magnetic form factors of the proton by analyzing the data assuming the validity of the first order Born approximation (one-photon exchange). Two-photon exchange processes can be directly investigated in electron-proton elastic scattering by measuring the ratio of positron-proton to electron-proton scattering. Such results will not only be interesting in themselves but will provide useful information to assist the analysis of the data from the experiment described in Part I.

The Feynman diagrams shown below are involved in elastic electron scattering to order $\alpha^3$.




The amplitude A is used to calculate the Rosenbluth formula. If the corrections to this formula are small, they should come mainly from the interference between A and the other five diagrams. Diagrams D, E, and F can be calculated with straightforward electrodynamics and are unimportant. However, B and C involve the structure of the proton, virtual Compton scattering, etc., and their possible importance has been the subject of considerable speculation.[1, 2, 3] In view of the central role played by the Rosenbluth formula in our interpretation of electron scattering data, it is important to verify the assumption that the two-photon exchanges (B and C) can be ignored, or to discover if they are important.

A comparison of positron-proton and electron-proton elastic scattering is very directly sensitive to the interference between the diagram A and diagrams B and C, since the interference terms change sign relative to $|A|^2$ when positrons are substituted for electrons. In such an experiment we would measure the ratio r, defined by

$$r = \frac{\sigma_+}{\sigma_-} ,$$

where $\sigma_+$ and $\sigma_-$ are the differential scattering cross sections for positrons and electrons at identical angles and energies. However, we prefer to consider the data as a ratio R, defined by

$$R = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{r - 1}{r + 1} .$$

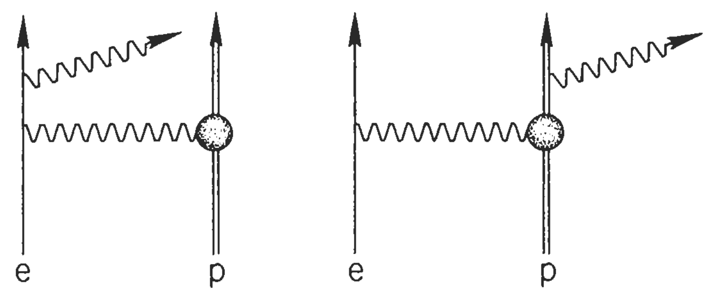
If the amplitudes D, E and F are much less than A, then

$$R = \frac{2 \,\mathrm{Re}\, |B + C|}{A} .$$

Since it is experimentally practical to compare the cross sections to a few percent or better, this technique is sensitive to very small values of the real parts of B or C. A number of measurements of R have been made,[2, 4] but they were limited by the available positron beam intensities to relatively small momentum transfers and to a few angles. Browman, et al.[2] report the value $R = (4.0 \pm 1.5)\%$, after taking account of radiative corrections. The experiment was done at 0.85 BeV and 90° $\left[q^2 = 0.76\,(\mathrm{BeV/c})^2\right]$. Apart from this result, there is no evidence for two-photon effects in the experiments performed thus far. It seems worthwhile to make additional checks at the high energies and momentum transfers available at SLAC.

Other two-photon investigations have been made in two ways. First, the angular dependence of the scattering at fixed $q^2$ can be studied and compared with that predicted by the Rosenbluth formula. In principle, this may not show up a two-photon exchange effect since the angular distribution need not necessarily be different. A second technique is to measure the polarization of the recoil proton in elastic electron scattering.[(5)] This polarization is directly sensitive to the imaginary part of diagrams B and C. However, such an experiment is technically quite difficult.

The Feynman diagrams below (and similar ones) give rise to an interference in the radiative corrections which changes sign when positrons are substituted for electrons.




According to the calculations of Tsai[(6)] and of Meister and Yennie[(7)], such terms give rise to experimental differences of several percent in the positron and electron cross sections at SLAC energies. In order to analyze the data, these corrections will be studied and verified. The calculations are expected to be quite reliable.

Finally, the importance of two-photon effects may be significant outside the context of the Rosenbluth formula. An example is the recent work of Drell and Sullivan[(8)] in which existing positron-proton scattering data appear to be inconsistent with a proposed mechanism for explaining a discrepancy between theory and experiment for the ground state hyperfine splitting in hydrogen by a hypothetical axial vector meson[(9)].

B. The Experimental Method

In Fig. 1 we show the predicted variation of the electron-proton elastic scattering cross section with angle for fixed values of $q^2$. These curves have been calculated assuming the validity of the Rosenbluth formula and using the CEA expressions[10] for the proton form factors. The experimental design is similar to that for the experiment in Part I except that the high momentum resolution requirement can be relaxed in favor of an increase in the intensity of the positron beam.

The primary beam, which will be electrons or positrons, will strike a 20 cm (0.024 radiation lengths) liquid hydrogen target and the momentum spectrum of the elastically scattered events will be recorded using the 8-BeV/c spectrometer over a range of angles and incident energies. Some of the program could be pursued fruitfully with the 20-BeV/c spectrometer but the small solid angle, and its limited angular travel make it much less suitable. In Fig. 1 the broken lines show the limits defined by a pair of recoil momenta p', and a pair of incident energies $E_o$. The counting rates shown have been calculated assuming 1 $\mu$A average beam current ($\sim 6 \times 10^{12}$ positrons or electrons/sec) and using the 8-BeV/c spectrometer. The only previous positron scattering data which can be shown is the experiment of Browman, et al.[2], which lies just at the top of the figure.

A preliminary design study of the positron beam has been reported.[11] The basic ideas have stayed the same, although the details have changed. Positrons are made in a radiator struck by 5-BeV electrons located one-third of the way along the SLAC machine. The radiator is about 4 radiation lengths thick, and low energy positrons are reaccelerated in the second two-thirds of the machine and focused during the early stages of their acceleration by a special magnet system consisting of a tapered solenoid, a uniform solenoid, and 13 special quadrupole triplets.

We estimate that the positron beam with momentum spread of 1% will have an intensity of about 5% of the electron beam incident on the radiator up to a maximum of about one microampere average current ($\sim 6 \times 10^{12}$ positrons/sec). The radiator heating establishes this limit. Phase slip between radiator and accelerator produces an inherently broad energy spectrum of the positrons. The intensity will probably depend linearly on $\Delta p/p$ for $\Delta p/p \leq 1\%$. Because the

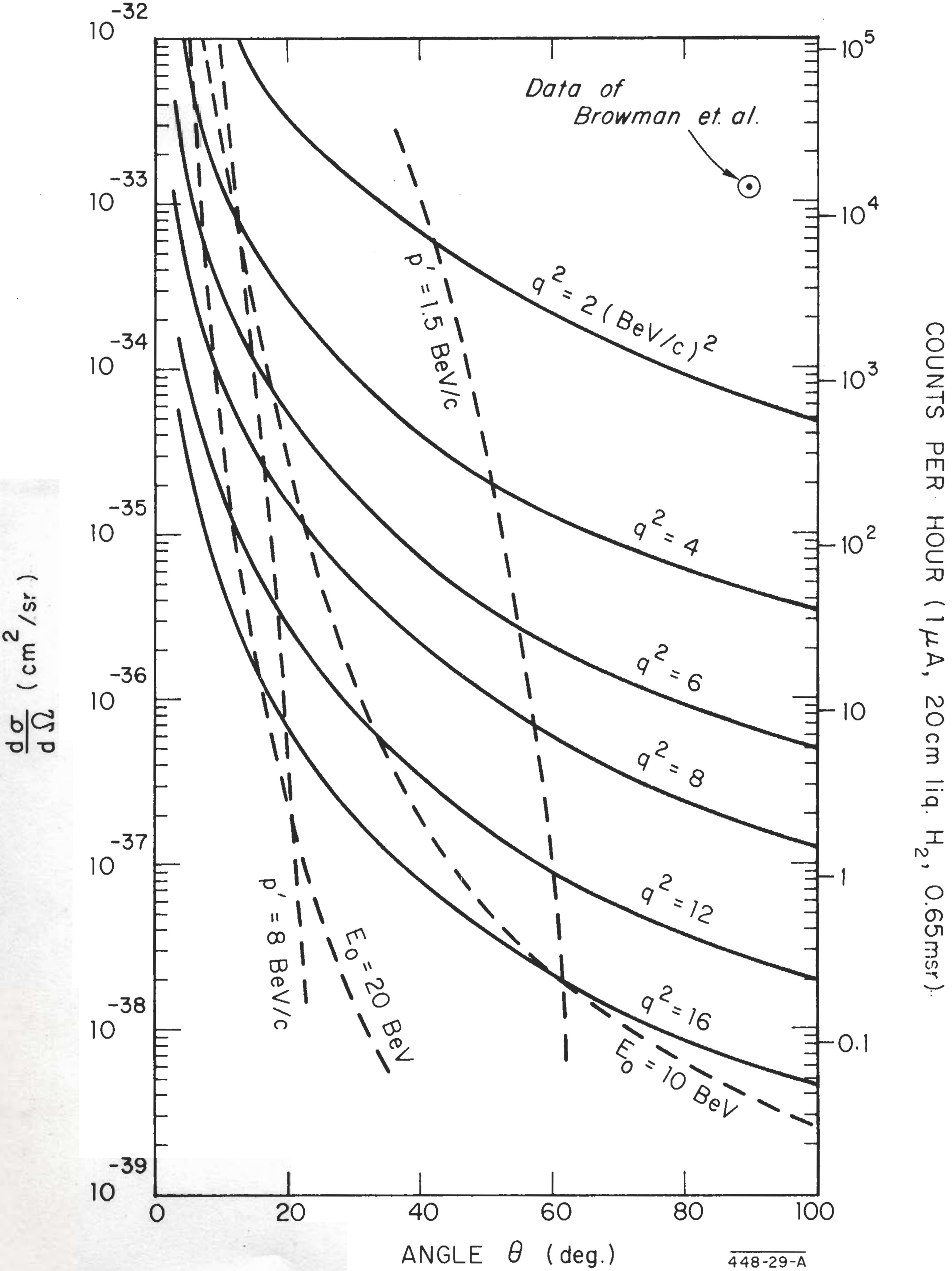



**Fig. 1 – PREDICTED e – p CROSS SECTIONS USING THE CEA FORM FACTORS**

beam intensity is limited to currents which are like the electron currents at Stanford Mark III and elsewhere, the monitoring and targeting problems which arise from beam heating are not great. Currently conventional systems will be satisfactory.

The nature of the backgrounds, discussed in Section C below, makes high-resolution spectroscopy very useful. For these measurements we can afford to relax our requirements on the primary beam momentum resolution and thus increase the useful intensity of the positron beam. We will thus require a momentum resolution of about ± 0.5% from the Beam Switchyard.

The data will be recorded using the 8-BeV/c spectrometer hodoscope being built at MIT (see Appendix III) in conjunction with the SDS 9300 computer system (Appendix IV). The $\pi$-e discrimination system could be the one built by the MIT group. Most likely, a rejection of 100 to 1 will be quite adequate (see Section C) so that a rather simple system will suffice, perhaps a part of the MIT system.

C. Backgrounds

Strictly speaking, the ideal elastic electron scattering experiment faces no problem from confusion with other processes. Figure 2 summarizes qualitatively the main features of the momentum spectrum of scattered particles at a given angle. Curves 1 and 2 are electrons from elastic and inelastic scattering. Curve 3 is for photoproduction from annihilation gamma rays, and 4 is for pions from bremsstrahlung photoproduction and electroproduction. The figure has been sketched assuming an incident energy $E_o$ of about 10 BeV, and a beam momentum spread of 1% full width at half maximum. The relative magnitudes of the four processes are estimated below.

Electroproduction

There are experimental data from CEA[12] at 31° and $q^2$ values of 1.17, 1.75 and 3.89 $(\mathrm{BeV/c})^2$. We will use the semi-empirical formula due to Hand and Wilson[13] to estimate the relative size of the inelastic peak (curve 2 in Fig. 2) over the kinematic region of interest. Considering only the (33) resonance of 1.238 BeV, the cross section is approximately given by the following formula,

$$\frac{d^2\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{MOTT}} ST \left| G_{MV}(q^2) \right|^2 (9 \times 10^{26})\ \mathrm{cm^2/sr\text{-}BeV}$$

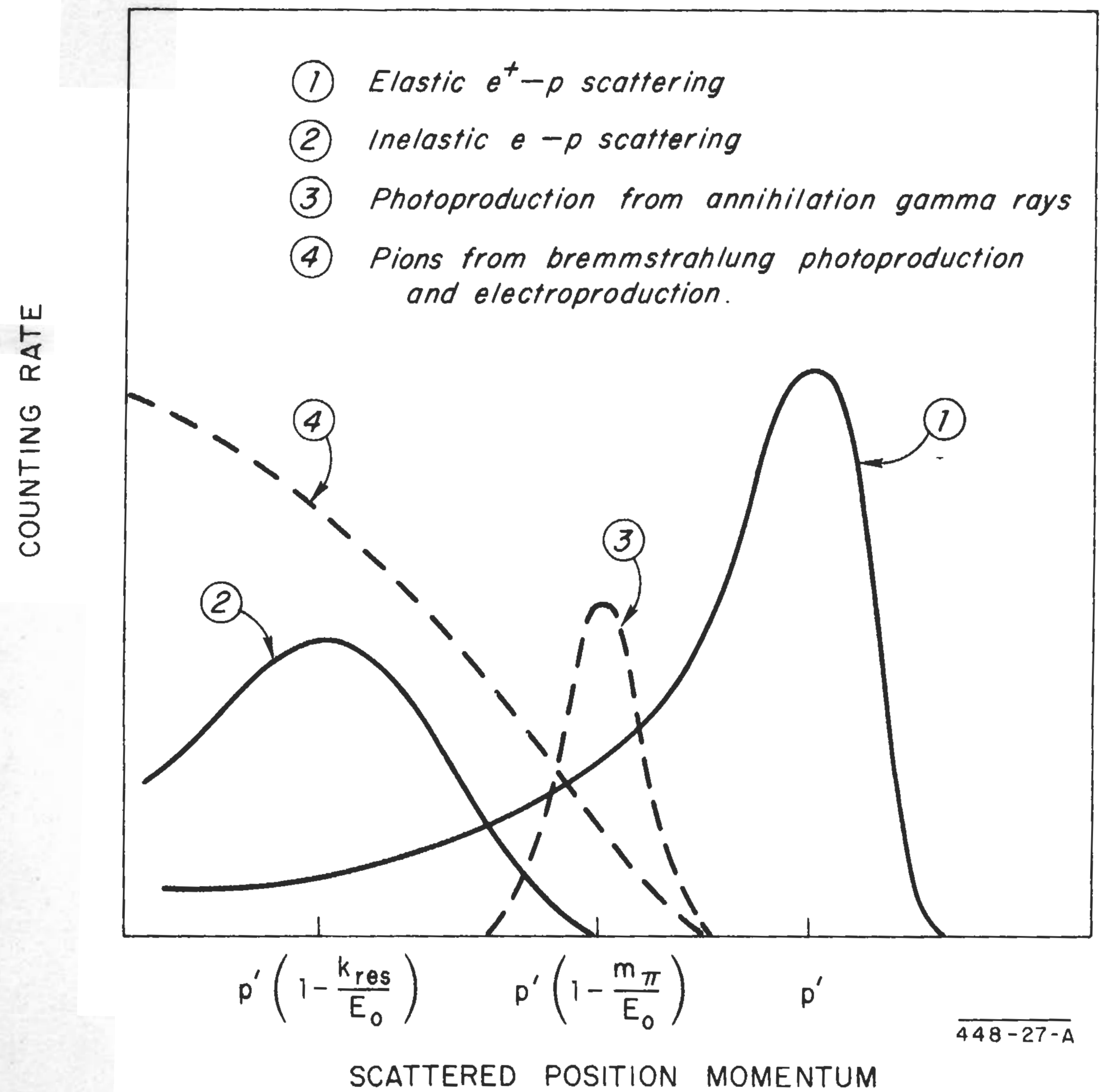



Fig. 2 - SCHEMATIC MOMENTUM SPECTRUM FROM $e^+$-p SCATTERING AT FIXED ANGLE

where

$$S = k_{res} E_o , \quad k_{res} = 0.34 \text{ BeV.}$$

$$T = \frac{1}{1 + \tau (1 + \frac{k_{res}}{2M\tau})}$$

$$\tau = \frac{q^2}{4M^2} \quad .$$

For our purpose, we make unimportant errors by taking $T \cong \frac{1}{1+\tau}$. Then the ratio of the inelastic to the elastic cross sections, $f_{in}$, has the form

$$f_{in} \equiv \frac{\sigma_{in}(1.238)}{\sigma_{el}}$$

$$= C \left(\frac{E_o}{\tau}\right) \frac{\left(\frac{\tau}{1+\tau} + 2\tau \tan^2 \frac{\theta}{2}\right)|G_{MV}|^2}{\left[\left(\frac{\tau}{1+\tau} + 2\tau \tan^2 \frac{\theta}{2}\right)|G_{Mp}|^2 + \frac{1}{1+\tau}|G_{Ep}|^2\right]}$$

where $C$ is a constant. If the elastic scattering from $G_{Ep}$ is neglected, we can get an upper limit on $f_{in}$:

$$f_{in} \lesssim C \left(\frac{E_o}{\tau}\right) \frac{|G_{MV}|^2}{|G_{Mp}|^2} \approx C \frac{E_o}{\tau} .$$

If we take the relation $f_{in} = \frac{1}{3} \frac{(E_o/M)}{\tau}$, we get a reasonable fit to the CEA data. A "worst case" for the proposed experiment might involve $f_{in} \sim 3$ at $E_o = 10$ BeV, $\tau = 1$. Cases like this, where $f_{in}$ is largest, have such large elastic cross sections that the beam momentum spread can be much reduced and the experimental resolution thereby improved somewhat. There appears to be no serious problem of interference between the inelastic and elastic peaks. When the inelastic scattering is comparable to the elastic, determination of R for this process may be a by-product of the experiment.

Photoproduction

The magnitudes of curves 3 and 4 set requirements on our ability to distinguish between pions and electrons. In contrast to the inelastic electrons, these pions are expected to be very asymmetric for positrons and electrons. For example, curve 3 exists only for positron scattering.

Table I summarizes our estimates for these processes assuming only single pion photoproduction, and that the pion lies in a momentum band within $\pm$ 1% of the electron elastic peak.

TABLE I

| $q^2$ $(\mathrm{BeV/c})^2$ | $\theta$ | $E_o$ (BeV) | $\frac{\sigma_\pi}{\sigma_{ep}}$ | $\left(\frac{I_\gamma}{I_e}\right)_{\mathrm{Brem}}$ | $\left(\frac{I_\gamma}{I_e}\right)_{\mathrm{Ann}}$ | $\frac{\pi_{\mathrm{Brem}}}{e_{\mathrm{elastic}}}$ | $\frac{\pi_{\mathrm{Ann}}}{e_{\mathrm{elastic}}}$ |
|---|---|---|---|---|---|---|---|
| 4 | $35^o$ | 4.6 | 75 | $3 \times 10^{-4}$ | $10^{-4}$ | 0.01 | 0.004 |
| 4 | $90^o$ | 2.8 | $10^3$ | $3 \times 10^{-4}$ | $1.5 \times 10^{-4}$ | 0.15 | 0.08 |
| 16 | $20^o$ | 16.5 | $\lesssim 10^4$ | $3 \times 10^{-4}$ | $3.5 \times 10^{-5}$ | $\lesssim 1.5$ | $\lesssim 0.18$ |
| 16 | $55^o$ | 10.3 | $\lesssim 10^4$ | $3 \times 10^{-4}$ | $5 \times 10^{-5}$ | $\lesssim 1.5$ | $\lesssim 0.25$ |

Table I. Comparison of high energy pions from photoproduction with electron elastic scattering. (The notation used here is defined in the text.)

The photoproduction cross sections are rather wild guesses from CEA data.[14] The ratio $(\sigma_\pi/\sigma_{ep})$ is an estimate of the single pion photoproduction to e-p elastic cross sections. The intensity of produced bremsstahlung gammas, $(I_\gamma)_{\mathrm{Brem}}$, and annihilation gammas, $(I_\gamma)_{\mathrm{Ann}}$, are expressed in terms of the number of incident electrons, $I_e$. The term $(I_\gamma)_{\mathrm{Brem}}$ includes a rough estimate of electroproduction by assuming the latter process is equivalent to an effective radiator of thickness 0.02 radiation lengths. The last two columns compare the pion photoproduction with electron-proton elastic scattering in the 20 cm liquid hydrogen target, where, for example

$$\frac{\pi_{\mathrm{Brem}}}{e_{\mathrm{elastic}}} = \frac{1}{2}\left(\frac{\sigma_\pi}{\sigma_{ep}}\right)\left(\frac{I_\gamma}{I_e}\right)_{\mathrm{Brem}} .$$

The factor $\frac{1}{2}$ is because the backgrounds are calculated for gamma ray production in 10 cm of liquid hydrogen and photoproduction in a like amount.

The four kinematical points shown in Table I seemed representative to us. From the ratios in the last two columns it appears that this background will be quite manageable for the elastic scattering, since it is easy to obtain about 100 to 1 pion rejection with shower counters at these energies. The instrument being built by Friedman and Kendall for use with the spectrometer may give $10^4:1$. This is relevant if one wants to measure the inelastic scattering, since curve 4 rises to very high values when double pion production comes in.

D. Objectives of the Experiment

There are a variety of reasons why this experiment is particularly suitable for early running at SLAC. Some are:

1) The low beam currents exert little pressure on the heat-dissipating and radiation-shielding capabilities of both Beam Switchyard and End Station A.
2) Accurate absolute calibrations are not required, by the nature of the ratio measurement. This applies, for example, to beam energy, spectrometer momentum and angle, solid angle, target length, etc.
3) The interesting region of data includes cross sections so large that the experiment can proceed at only a small fraction of the design positron beam.
4) The spectrometer optics need only approach design performance to about a factor 3 to 5. The spectrometer shielding need not be at its best. The $\pi$-e identification system can be simple.

From the predicted counting rates shown in Fig. 1 it seems practical in the initial stages to make about ten measurements of r $(= \sigma_+/\sigma_-)$ to an accuracy of $\pm$ 4% or better (i.e., a sensitivity to 2% two-photon effect) for a range of angles at $q^2 \lesssim 4\,(\mathrm{BeV/c})^2$. These runs will involve cross sections of $\gtrsim 10^{-35}\ \mathrm{cm^2/sr}$ (or ~ 4 counts/min, assuming a 1$\mu$A beam). This program can be performed quite easily even if the positron beam is running at ~ 1/10 of design intensity.

With the experience obtained in the early stages of running the positron beam we would then propose to extend the measurements to higher values of $q^2$, up to $q^2 \sim 12\,(\mathrm{BeV/c})^2$, but with decreasing accuracy. These runs will involve cross sections of $\gtrsim 10^{-37}\ \mathrm{cm^2/sr}$ ( or $\gtrsim$ 10 counts/hour, assuming a 1 $\mu$A beam).

It is difficult to estimate the required running time due to the uncertainty in the counting rates at high $q^2$ values. We request 200 hours to pursue this program, plus time using the electron beam at reduced repetition rates for testing (~50 hours). If operation permits, this last item could be parasitic time.

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