APPENDIX II

MOMENTUM RESOLUTION REQUIREMENTS

The energy resolution required for the 8- and 20-BeV/c spectrometers depends on the physics programs to be carried out with the use of these instruments. Though it is impossible to envisage all programs that will ultimately be pursued, the experiments that are now being contemplated provide important guidelines for determining what parameters will be useful. We will use electron scattering to illustrate the criteria for the energy and angular resolution of the spectrometer.

If G is the energy gap in the spectrum of scattered electrons between the elastic scattering energy and the threshold energy for producing a pion then

$$\frac{G}{p} = \frac{m_\pi + \frac{m_\pi^2}{2M}}{p_o}$$

p_o is the momentum of the incoming electrons and p is the momentum of the elastically scattered electrons; M is the target mass. For $p_o = 20$ BeV/c we thus find $\frac{G}{p} = 0.7\%$.

A. Source of Momentum Spread

In general, there exist the following sources which cause momentum spread:

1) The beam height causes a percentage spread in detected momentum $\frac{\delta p}{p}$ denoted as δ_r.

2) There will also be a finite spread in θ corresponding to a given point in the focal plane, denoted θ_r. From the kinematic relation $p = p(\theta)$ we get

$$\left(\frac{dp}{p}\right)_\theta = \frac{d}{d\theta}\left[\ell n\ p(\theta)\right]\theta_r$$

3) The angular spread of the primary beam from the switchyard also contributes a $\left(\frac{dp}{p}\right)_{\theta_s}$ where

$$\left(\frac{dp}{p}\right)_{\theta_s} = \frac{d}{d\theta}\left[\ell n\ p(\theta)\right]\theta_s$$

and θ_s represents the angular spread of the primary beam.

4) The momentum spread of the beam from the switchyard contributes a $\left(\frac{dp}{p}\right)_s$.

5) The detector widths in the horizontal direction cause a spread in θ which thus contributes a $\left(\frac{dp}{p}\right)_x$.

6) The detector widths in the vertical direction cause a spread in p and thus contribute a $\left(\frac{dp}{p}\right)_y$.

7) The multiple scattering of the electrons in the target contributes an additional angular spread. This angular spread gives rise to a $\left(\frac{dp}{p}\right)_{MS}$.

8) Radiative processes cause a non-symmetric broadening of a delta function in momentum.

9) Landau straggling also causes a non-symmetric broadening of momentum. This is a negligible factor in the resolution.

B. Total Resolution

Effects (1) through (7) are symmetric and can be assigned widths. Effects (8) and (9) are skew and have long tails.

The distributions for (2), (3) and (4) are assumed to be Gaussians; (7) is Gaussian to a good approximation; triangular for (1); and square for (5) and (6). The fold of each of these effects, considered together, will approximately give rise to a Gaussian distribution in p whose variance is δ_T^2 . It follows that

$$\delta_T^2 = a_1 \delta_r^2 + a_2 \left[\left(\frac{dp}{p}\right)_x^2 + \left(\frac{dp}{p}\right)_y^2 \right] + \left[\left(\frac{dp}{p}\right)_\theta^2 + \left(\frac{dp}{p}\right)_{\theta_s}^2 + \left(\frac{dp}{p}\right)_s^2 + \left(\frac{dp}{p}\right)_{MS}^2 \right] .$$

To be conservative, a_1 and a_2 will be taken as unity although formally a_1 is less than 1/3 and a_2 is exactly equal to 1/3. There is a question both of the exact shapes of the distribution functions and the rapidity with which the multiple fold converges to a Gaussian distribution.

C. Values of Half Widths

The values of the half widths used in the calculation of the total variance, δ_T, will be specifically evaluated for the e^--p elastic scattering process as follows:

1) δ_r: Both the 20- and the 8-BeV/c spectrometers have been designed to have an intrinsic resolution of $\sim \pm 0.05\%$ which is adequate for the contemplated physics program.

2) $\left(\frac{dp}{p}\right)_\theta$: The resolution in θ is set by matching $\left(\frac{dp}{p}\right)_\theta$ to δ_r for E_o = 20-BeV. Using the e-p elastic scattering kinematics, we have

$$\left(\frac{dp}{p}\right)_\theta = -\frac{p}{M} \sin\theta \ \theta_r .$$

From Fig. 1, we see that the minimum angular resolution required is $\theta_r = \pm 0.15$ mr for $\left(\frac{dp}{p}\right)_{\theta\,max} = \pm 0.05\%$. The single pion photo-production kinematics give the same result to a very good approximation.

3) $\left(\frac{dp}{p}\right)_{\theta s}$: If the four quadrupoles, Q-13, 14, 20 and 21, in the A beam are used to focus the electron beam onto the spectrometer target, a ribbon beam with $x = \pm 0.5$ cm and $y = \pm 0.15$ cm can be obtained. The respective angles θ and ϕ can be minimized consistent with the output phase space of the accelerator. Second order calculations of the Beam Switchyard optics show that the angular spread of the primary beam, θ_s, should be less than 8×10^{-5} rad for a momentum slit width of the Beam Switchyard of $\pm 1.0\%$ or less.

4) $\left(\frac{dp}{p}\right)_s$: The momentum resolution of the Beam Switchyard is related to the momentum resolution of the elastically scattered electrons by the following formula:

$$\left(\frac{dp}{p}\right)_s = -\frac{1}{\left(1 + \frac{2p_o}{M} \sin^2 \frac{\theta}{2}\right)} \left(\frac{dp_o}{p_o}\right) .$$

Figure 2 shows the momentum resolution of the primary electrons required to maintain $\left(\frac{dp}{p}\right)_s = \pm 0.05\%$. It shows clearly that the momentum slits in the Beam Switchyard can be widened when the spectrometer is operating at large angles.

5) $\left(\frac{dp}{p}\right)_x$: The finite width of the plastic scintillators in the hodoscope will introduce some spreads in the resolutions, even if the intrinsic resolutions of the spectrometer are infinitesimally small. The x-displacement per milliradian at the focal plane has been designed to be 1.534 cm/mr for the 20-BeV/c spectrometer, and 4.24 cm/mr for the 8-BeV/c spectrometer. Accordingly, the detector channel widths will correspond to $\left(\frac{dp}{p}\right)_x = \pm 0.05\%$.

6) $\left(\frac{dp}{p}\right)_y$: The y-displacement per percent momentum variation at the focal plane has been set as 2.826 cm/% for the 20-BeV/c spectrometer, and 2.955 cm/% for the 8-BeV/c spectrometer. Accordingly, the detector channel width will correspond to $\left(\frac{dp}{p}\right)_y = \pm 0.05\%$.

7) $\left(\frac{dp}{p}\right)_{MS}$: The rms multiple scattering angle due to the target should be less than ±0.15 mr in order to match the other angular spreads. This allows a reasonable target length. The angular spread due to multiple scattering of the primary beam in the target is given by

$$\Delta\theta = \frac{0.015}{E_o \text{ (in BeV)}} \sqrt{\ell}$$

where ℓ is the target length in radiation lengths. The corresponding momentum spread is given by

$$\left(\frac{\Delta p}{p}\right)_{MS}^{\text{Primary}} = \frac{0.015\, p}{Mp_o} \sin\theta \sqrt{\ell}$$

where p_o is the primary electron momentum, p is the scattered electron momentum, and θ is the scattering angle. The elastically scattered electrons, because of their lower energy, will undergo larger multiple scattering than the primary electrons. The momentum spread due to the multiple scattering of the scattered electrons is given by

$$\left(\frac{\Delta p}{p}\right)_{MS}^{\text{Scatt}} = \frac{0.015}{M} \sqrt{R\,|\sin\theta|}$$

where R is the radius of a cylindrical target in units of radiation length and the target axis is assumed to be along the beam line. The result for a target of radius $\frac{1}{200}$ radiation length is shown in Fig. 3.

D. Total Resolution

Thus each of the symmetric spreads has been nominally set to result individually in a percentage spread of momentum of ± .05% except for the effects of multiple scattering which are somewhat larger for $\theta \gtrsim 20^{\circ}$.

Thus for $\left(\frac{\delta p}{p}\right)_T$ we have approximately

$$\delta_T = \left(\frac{\delta p}{p}\right)_T \simeq \pm\sqrt{7}\ (.05)\,\% = \pm\ 0.13\%\ .$$

Note that if any one of the distributions is reduced in width by a factor of two $\left(\frac{\delta p}{p}\right)_T = \pm\ 0.125\%$. Hence the total resolution is highly insensitive to a great reduction in the widths of any one of these factors. Thus a reduction of δ_r by a factor of two (if the physics required it) would only be justified if it is feasible to reduce all other widths. On the other hand, if any width is increased by a factor of two $\left(\frac{\delta p}{p}\right)_T = .16\%$, reflecting the fact that the total resolution is more sensitive to a change in this direction.

E. Effect of Radiation Processes on Total Resolution

Bremsstrahlung before and after scattering and radiation during the scattering process produce a non-symmetric broadening of a detected peak in addition to a long tail. An important question is how the radiation processes affect the total resolution of the system. This has been estimated by folding the expected resolution curve with the function which describes the radiation straggling. Assumed were an elastic e-p scattering with a momentum transfer of $q^2 = 19.4\ (\text{BeV/c})^2$ and a target .03 radiation lengths thick. It should be pointed out that for this condition, only about 35% of the straggling results from bremsstrahlung before and after scattering. The target thus can be as thick as .05, or 40 cm of liquid H_2, of a radiation length without being the dominant source of straggling. In Figs. 4 and 5 are shown the resolution curves as they would appear in detectors of full width 0.1%, with and without radiation folding. The straggling function used was an approximate form of that derived by Yennie and Meister (Phys. Rev. 130, 1210 (1963)). Both curves have been arbitrarily normalized to have the same peak value for purposes of comparison. Because of the dissimilar shapes it probably is most useful to compare their half-widths on the lower momentum side. The radiation straggling adds about 0.10% in momentum to the unfolded half-width. In the separation of peaks this broadening (unlike resolution broadening) only appears once. The curves indicate that the radiation straggling decreases but does not greatly deteriorate the total resolution for a $(\delta p/p)_T = \pm .13\%$.

F. Conclusions

In Figs. 4 and 5 for purposes of comparison, the resolution curves are separated from another peak of equal height by a momentum difference of 0.7% (which corresponds to the threshold for pion production). The curves indicate that there would be reasonable separation even if the inelastic continuum were to rise rapidly. We could conclude that the resolution is adequate.

A factor of two improvement in resolution would involve a considerable increase in cost and effort since, effectively all contributions of momentum spread would have to be improved by about this factor.

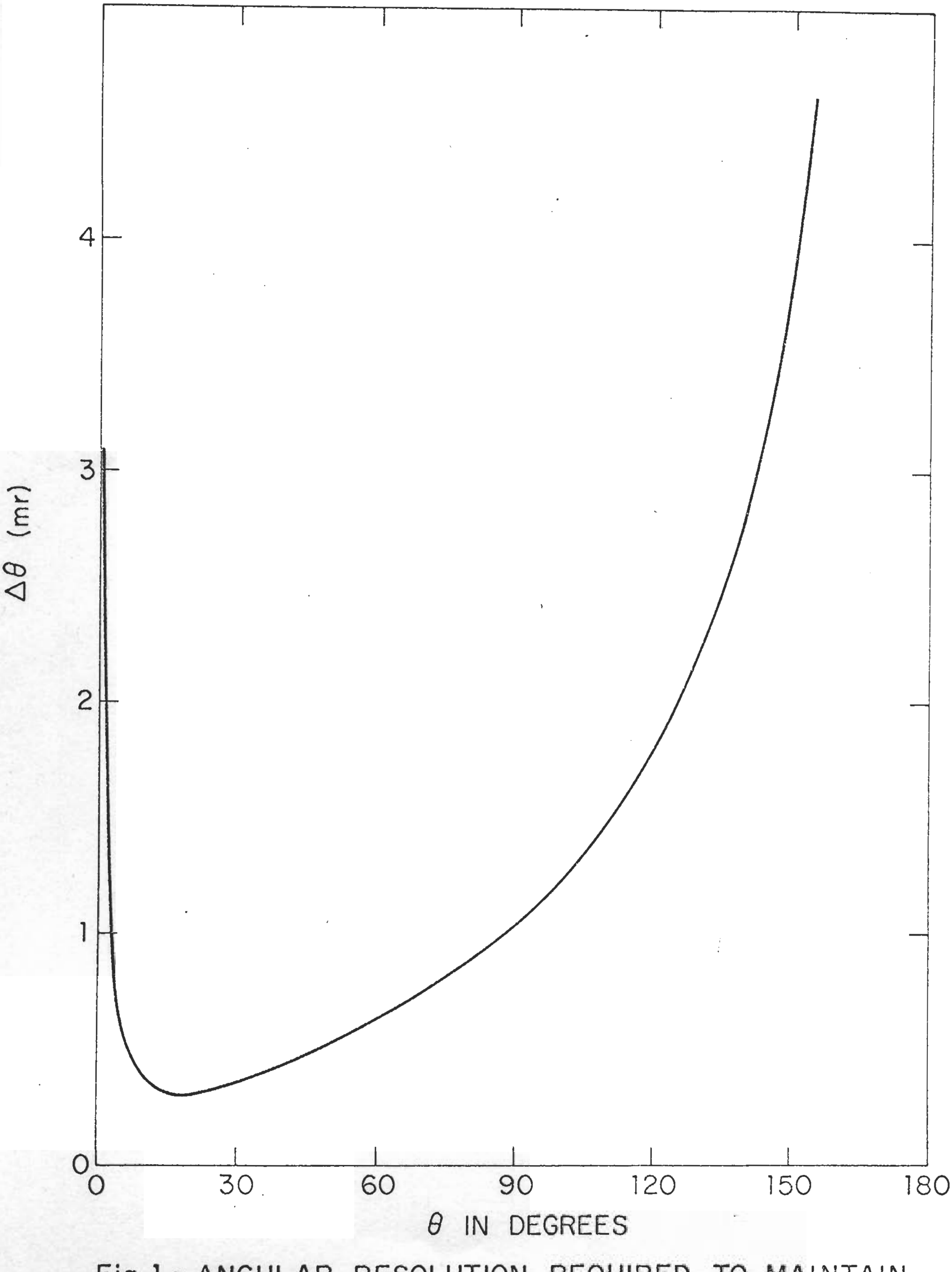

Fig. 1 - ANGULAR RESOLUTION REQUIRED TO MAINTAIN $\frac{\Delta p}{p} = 0.1\%$ FOR PRIMARY ELECTRON ENERGY OF 20-BeV

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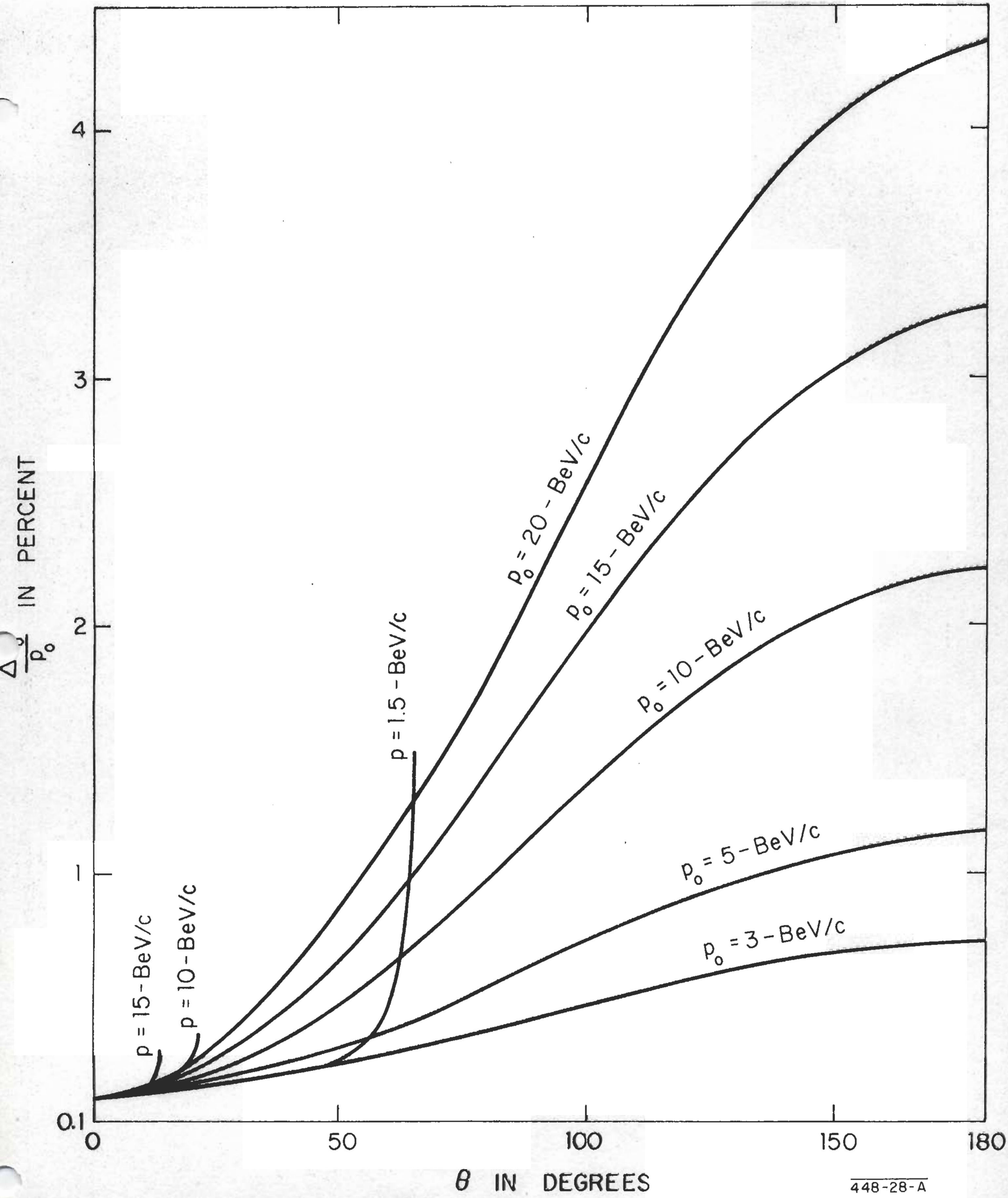

Fig. 2 — MOMENTUM RESOLUTION OF THE PRIMARY ELECTRONS REQUIRED TO MAINTAIN $\frac{\Delta p}{p} = 0.1\%$ FOR THE SCATTERED ELECTRONS

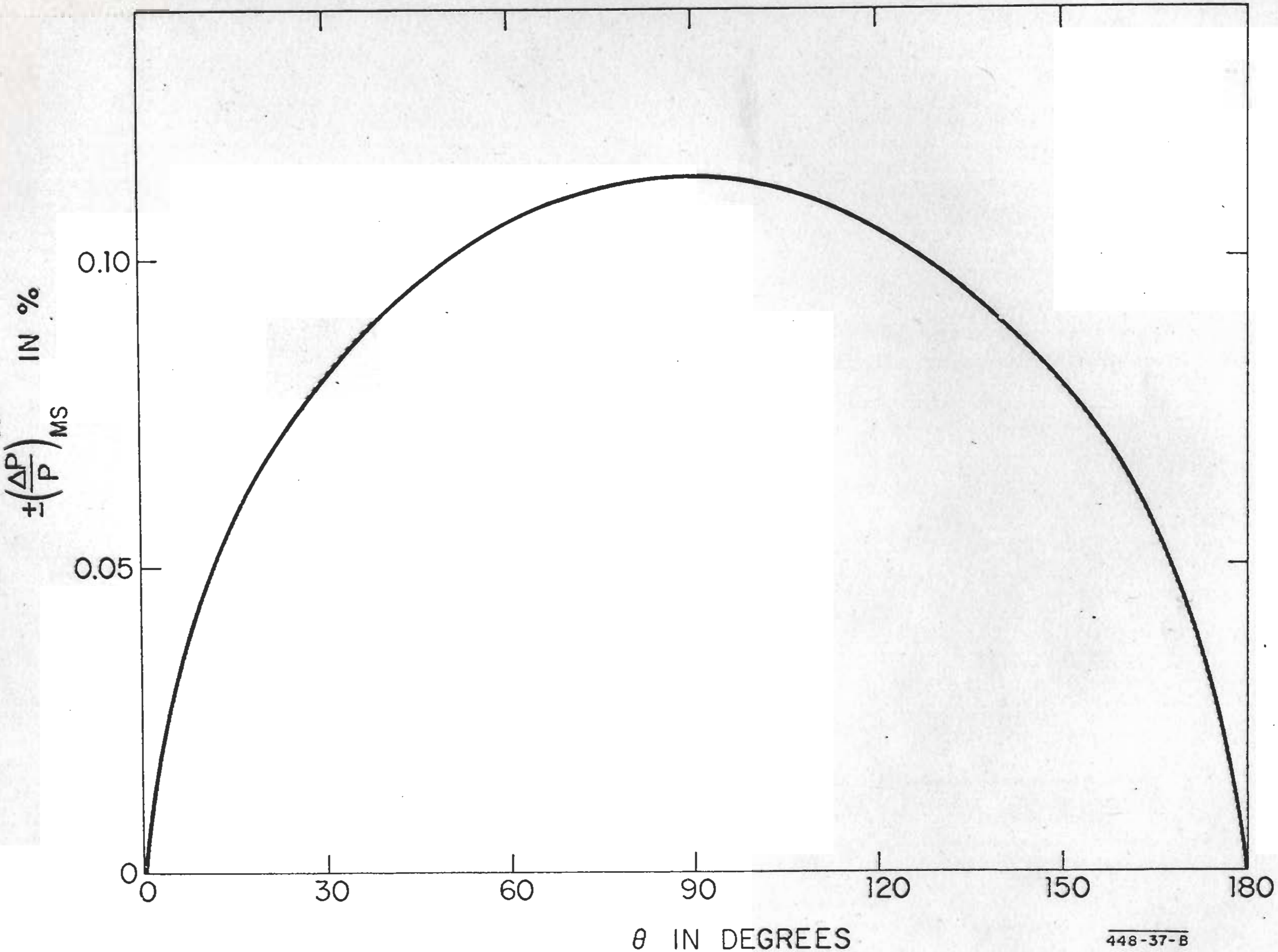

Fig. 3 - DEGRADATION IN MOMENTUM RESOLUTION DUE TO THE MULTIPLE SCATTERING OF THE ELASTICALLY SCATTERED - ELECTRONS IN A CYLINDRICAL LIQUID HYDROGEN TARGET OF RADIUS $\frac{1}{200}$ r.l.

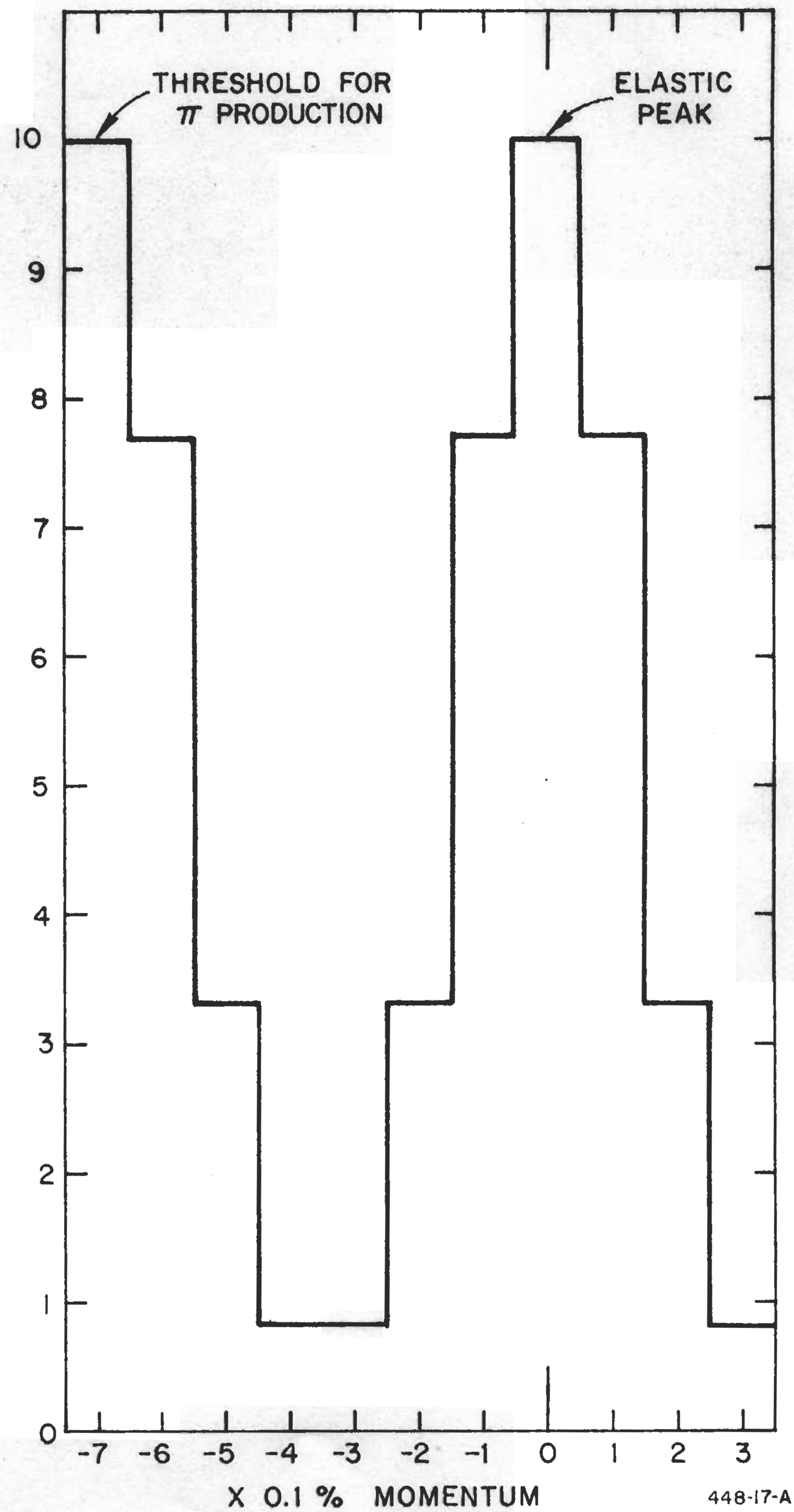

FIG. 4 -- RESOLUTION CURVE FOR $\sigma_T = \pm 0.13\%$ AND WITHOUT RADIATION STRAGGLING ($E_0 = 20$ BeV, $q^2 = 20$ (BeV/c)^2, 0.03 r.l. TARGET).

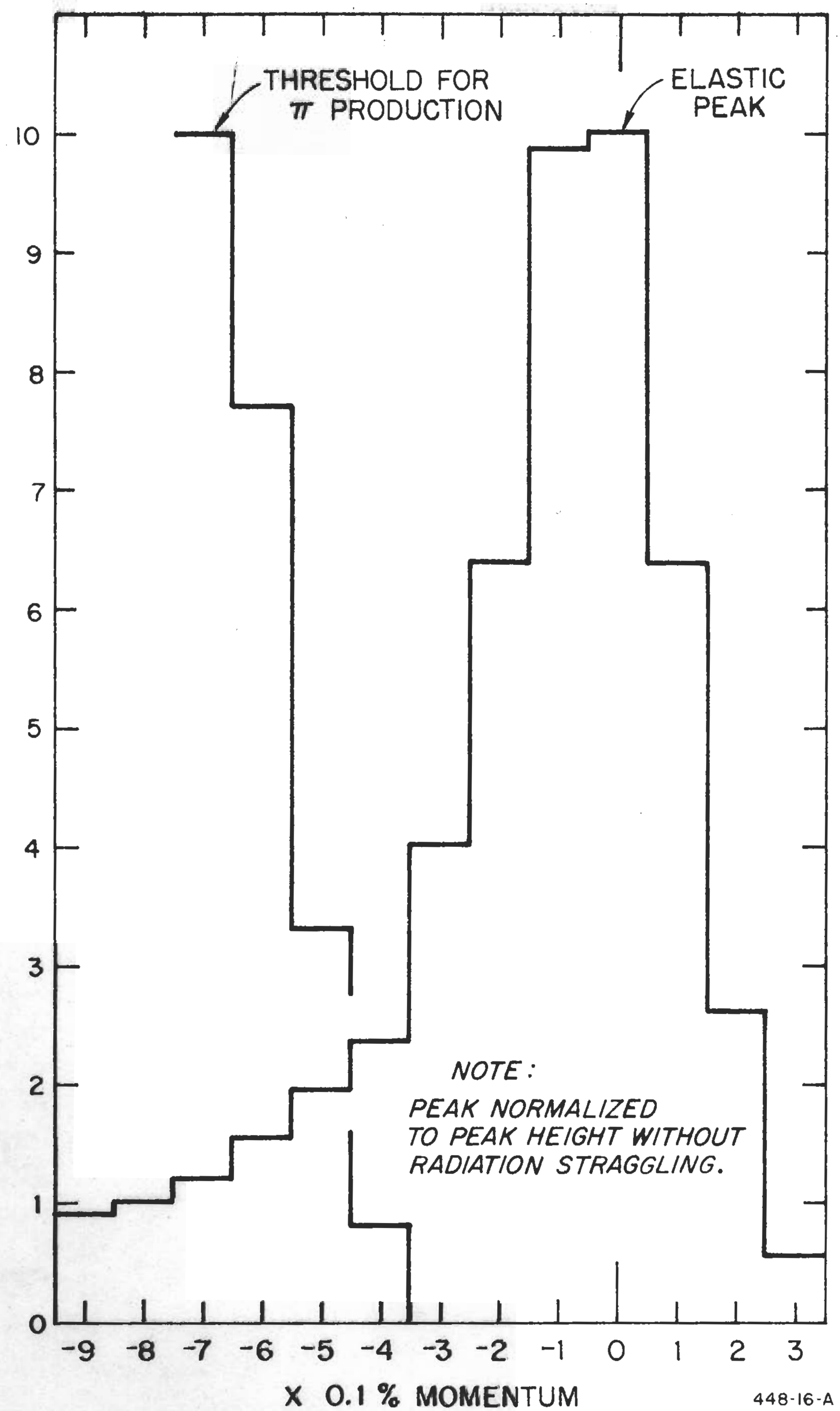

FIG. 5-- RESOLUTION CURVE FOR $\sigma_T = \pm 0.13$ % WITH RADIATION STRAGGLING ($E_0 = 20$ BeV, $q^2 = 20$ (BeV/c)^2; 0.03 r.l. TARGET).