SLAC Proposal 123-A
12/75

A PROPOSAL TO STUDY MUON PAIR PRODUCTION IN $\pi^{\pm}$ INTERACTIONS

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and

SLAC Proposal 123-B
12/75

A PROPOSAL TO SEARCH FOR NARROW MASS STATES ASSOCIATED WITH SINGLE MUON PRODUCTION IN $\pi^{\pm}$ INTERACTIONS

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Summary

We propose to use the SLAC two-meter streamer chamber to make a detailed study of direct muon pair production in π - p interactions. Data taking runs are proposed with both positive and negative pions incident on a liquid hydrogen target at momenta of 17 and 12 GeV/c.

A photon calorimeter combined with measurement of the charged tracks will allow a measurement of missing energy of greater than 1 GeV making possible a recognition of the presence of energetic neutrinos.

With a 1,000 hour run at 180 pps the experiment will be sensitive at the level of 2.5×10^3 events per microbarn assuming a trigger efficiency of 10%.

To understand the background of this experiment it will be necessary to obtain a data sample of events without trigger selection and also of events with a single muon trigger.

These events will be combined with an additional 100,000 events taken with a single muon trigger designed to enhance the $p_\perp$ region greater than 400 MeV/c to use in a search for narrow mass states, possibly charmed mesons and baryons, associated with single muon production.

Introduction:

Direct lepton production in hadron-hadron interactions was originally studied to search for new particles. It was expected that the signals for these particles would be observed above a background continuum of Bethe-Heitler and quark-antiquark annihilation mechanisms. Several first generation experiments observed copious production of single leptons and lepton pairs.[1,2,3,4,5] Conflicting results were reported at low energies.[5,6,7,8] Some interesting patterns arise which we will list as follows: (See also Table I.)

1. Transverse momentum dependence: (Fig. 1) Originally experiments were designed to study the large transverse momentum region; however, data seem to indicate that the signal of prompt leptons could be a factor of 2-3 higher at $p_\perp = 0.2$ than at a larger $p_\perp$ region, $p_\perp > 1$ GeV/c. This may indicate that the low $p_\perp$ signal may come from a low mass object.[9]

2. Longitudinal momentum dependence: (Fig. 2) Again, a factor of 5 is observed when $x_\mu = p_\mu / p_{msx}$ varies from 0.5 to 0.1. This may indicate that these signals come from non-diffractive sources.[10]

3. Energy dependence: (Fig. 3) There are conflicting reports on this subject. The Penn.-Stony Brook preliminary results[8] show no threshold effect for proton beam momenta of 10, 15, and 24 GeV/c. However, background problems in this experiment are severe and there is conflicting evidence from Winter et al.[7] and a Serpukov experiment.[5,6]

Some of the unpublished results of the BNL-Yale experiment[11] reported at the BNL Charm-Direct Lepton Symposium (BNL Nov. 18-19, 1975) indicate that the observed single muons are a result of pair production.

$$\int_{x=0.125}^{1} [f(x)dx]_{\text{dimuon}} \Big/ \int_{x=0.125}^{1} [f(x)dx]_{\text{single}} = 1.10 \pm 0.15$$

From their Monte-Carlo calculations it appears that these pairs populate a low mass region probably below the ρ mass. Their experiment has poor mass resolution and cannot give a definitive answer concerning this.

Another report by G. Sanders[12] of an experiment at 150 GeV/c also seemed

TABLE I

Group	$e^{\pm}/\pi^{0}$ x10^{4}	$u^{\pm}\pi^{0}$x10^{4}	$p_{\perp}$	$\sqrt{s}$	θ^*	Comment
Serpukhov		0.25	~1	12	90°	$\langle + \rangle / \langle - \rangle \sim 1.75$
Columbia-Fermilab	0.8+0.2	1.0± 0.2	1.6-3.5	23 26	75,90 100°	$\langle + \rangle = \langle - \rangle$
Chicago-Princeton		0.8±0.2	1.5-5.5	23	90°	
CCRS	0.8-1.3		0.6-3.0	23-62	90°	$\langle - \rangle > \langle + \rangle$ in $P_{\perp} < 1.6$
Chicago-Harvard Penn-Wisc.		~ 0.7	1-2.2	10-20	Var.	
Yale-BNL		1.0 @ X=0.15	0	26	0	Large $X_{\parallel}$ Depen.
Penn-Stony Brook	1.0±0.15		0.6-1.3	4.5-7	90°	e^{+} only
Winter (PS)	~ 0		0.5	5	37°	Old
Yale-BNL		~1.0 @ X~0.4	0	7	0	Large $X_{\parallel}$ Depen. old
MIT-BNL	~1			7	90°	Prelim.
CHGRM	1.5-5.0		0.2-1.0	53	30°	Prelim.
Serpukhov	≤ 0.35	0.35±0.1	1.8-2.8	8-12	90°	

Fig. 1

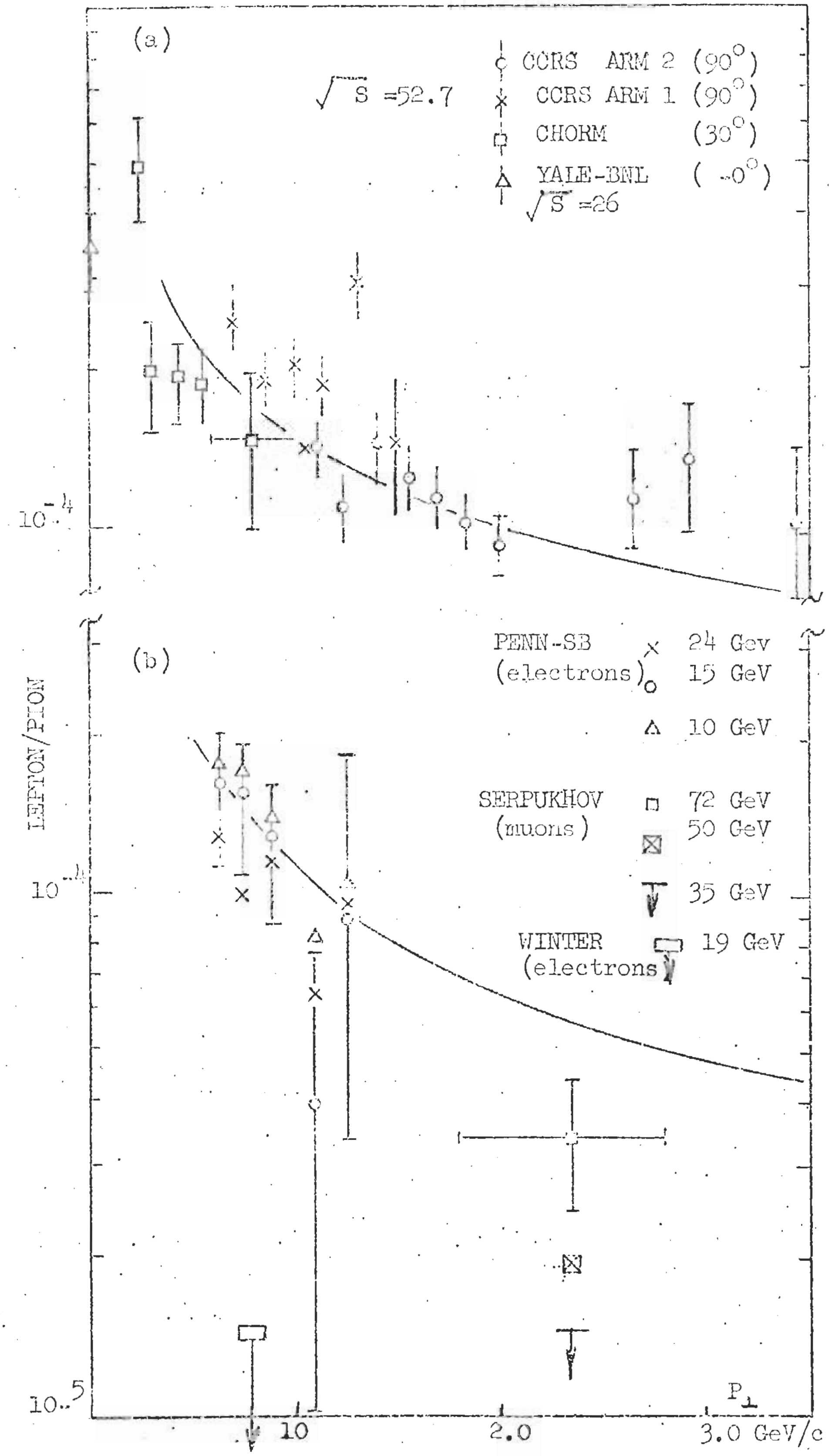


Fig. 1 Upper graph: Low $p_\perp$ data at ISR with Yale-BNL point Lower graph: Penn-SB, Serpukhov data and PS data.

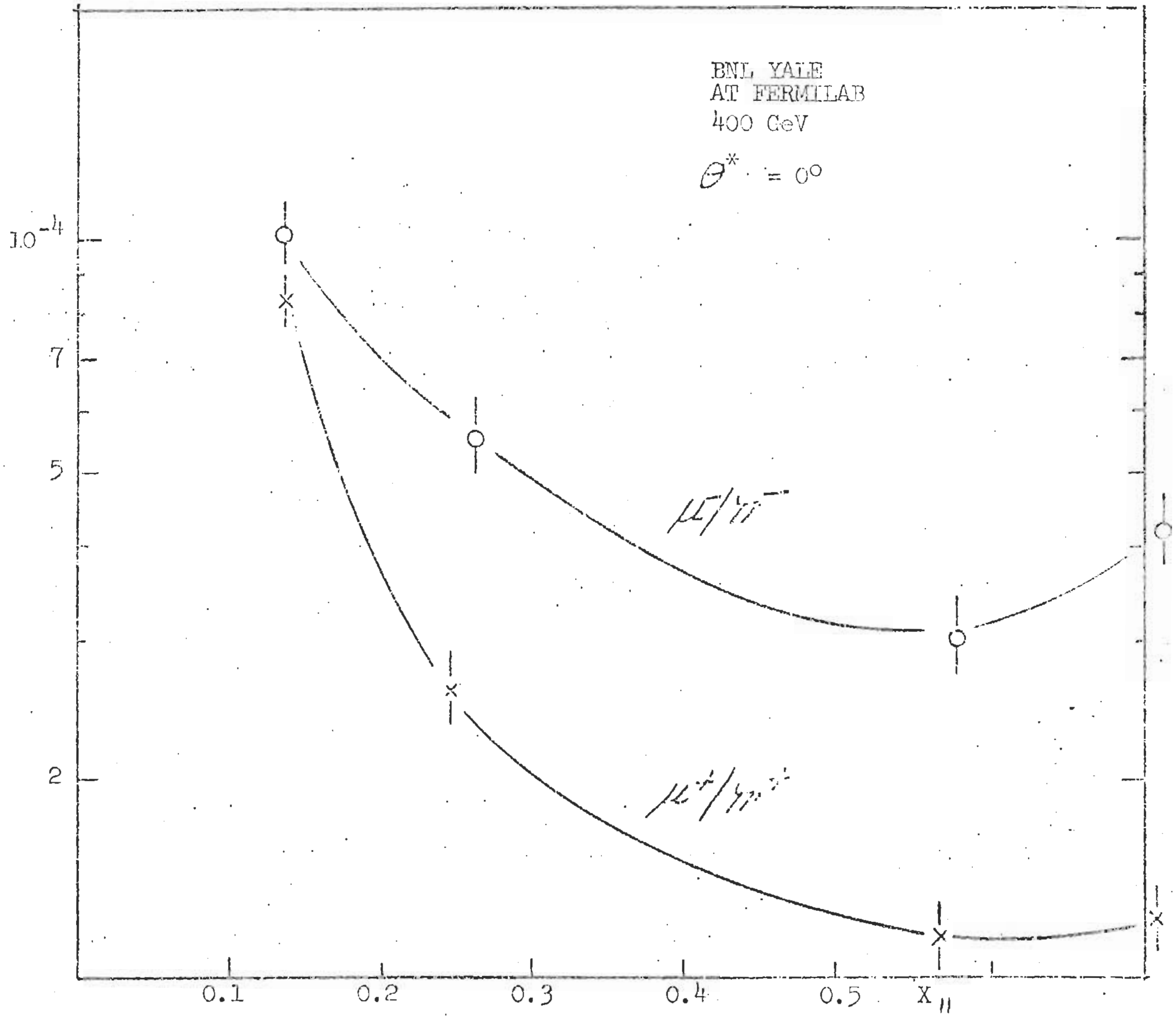
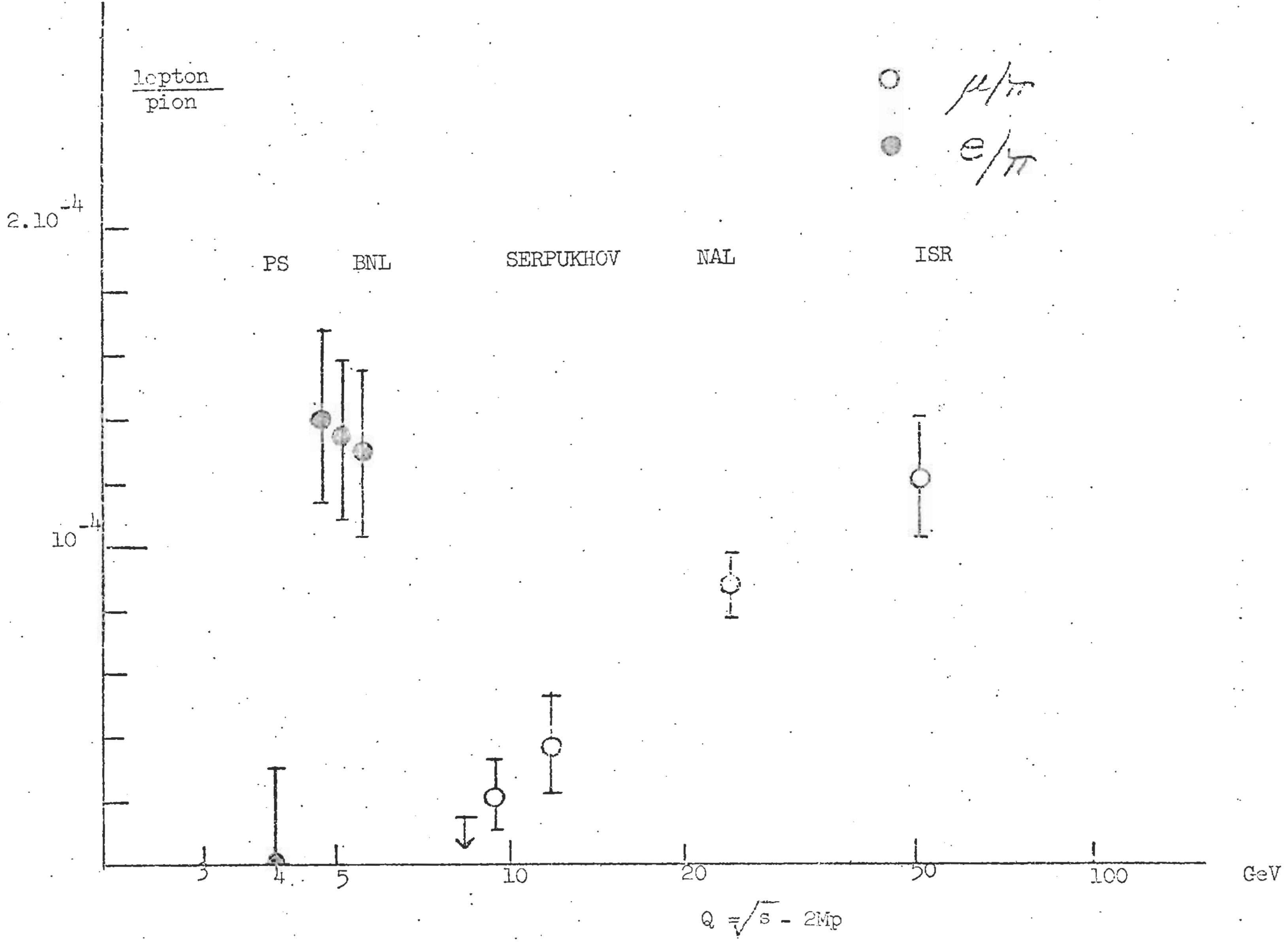


Fig. 2 Direct muons in Yale-BNL experiment

Fig. 3 ENERGY DEPENDENCE



to indicate an enhancement below the ρ mass, and a poor resolution study by R. Weinstein et al.[13] indicates a signal in the ρ region appreciably higher than expected.

It is clear that an experiment is required with excellent resolution of the dimuon mass and production and decay angular distributions. The cross section for dimuons is ample and the yield at low $p_\perp$ good. In spite of conflicting estimates of the energy dependence the apparent low mass of the dimuon would make the relatively low energy of the SLAC accelerator adequate and a study of the threshold behavior appropriate.

We propose to use the SLAC two-meter streamer chamber facility to study muon pair production. We would utilize both π^+ and π^- beams at 17 GeV/c and at 12 GeV/c. In a run of 1,000 hours (750 effective hours) at 180 pps we would obtain $2.5 \times 10^4 \times \epsilon$ events per microbarn (where ϵ is the trigger efficiency estimated at between 10% and 20% for various processes). Accompanying any anomalous muon pairs would be a ρ signal of about 1,900 events and an ω signal of about 100.

The streamer chamber will provide a mass resolution considerably superior to previous experiments. In the ρ region the σ will be of the order of 6 MeV/c^2. Its angular resolution will allow an accurate study of the muon pair production and decay angular distributions. If the production is due to π^+, π^- annihilation as suggested by Blankenbecler[14] and Chu and Kuplik[15] the observed lepton pairs should have a decay distribution $1 - \cos^2\Theta$, while if the pair is produced by quark-antiquark annihilation it should have a decay distribution $1 + \cos^2\Theta$. The proposed experiment with its large trigger acceptance can clearly distinguish between these distributions.

If it is a $1 + \cos^2\Theta$ distribution indicating $q\bar{q}$ annihilation, the forward directed hadrons detected in the virtually 4π acceptance should be from a single quark fragmentation (Fig. 4).

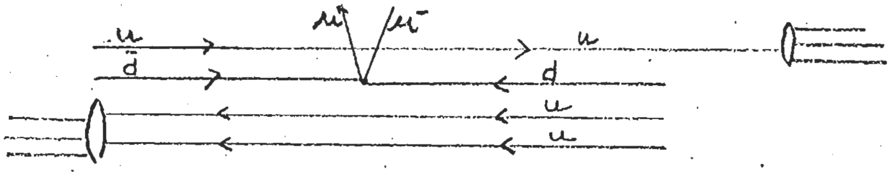


Fig. 4

Even if one assumed a mixture of processes the $q\bar{q}$ annihilation process should dominate in those events for which the decay is at small angles. Detailed studies of these fragmentation functions can be compared with colliding beam data where jets are observed. See Fig. 5.

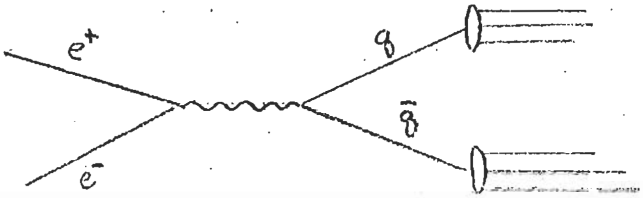


Fig. 5

Our μp data also shows that it is possible to compare the e^+e^- and μp interactions by means of such diagrams.[16] See Fig. 6.

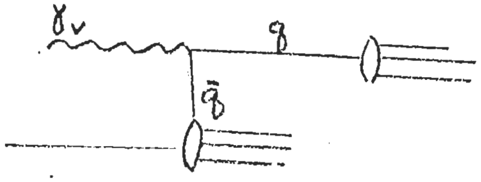


Fig. 6

For a π^+ beam one should view u quark fragmentation in the forward direction yielding charge asymmetries similar to those seen in μp scattering while in the backward direction the uu valence quarks would dominate the final state. The π^- data can be studied in the same way. If the interaction involves $q\bar{q}$ annihilation into photons and thence into muon pairs, one would expect a negative pion beam to be possibly 3 - 5 times as effective as a positive beam. The interaction could be considered as involving leading quark annihilation only in the case of the negative pion beam and hence the characteristics of the entire interaction may change greatly with pion beams of differing signs.

It may of course be true that the dimuon signal comes from the decay of two particles which have a large branching ratio into muons. Our measurement at 12 GeV would establish a possible threshold if this were a pair of 2 GeV/c^2 particles.

We plan a calorimeter to detect 90% of the average energy carried by π^0's. This means that on the average only 3% of the total energy is left unaccounted for if we add the calorimeter reading to the visible energy measured. (Comparisons must be restricted to those events with a proton identified by ionization.) If we assume that the prompt muons come from decays of D's (Charmed mesons), calculations show that the energy of the neutrinos may peak at 1/3 of the D mass (Goldman-Duong-van)[17] of possibly 1/2 of the D mass (Lederman).[18] In the laboratory this energy will be well over one GeV. One may determine with this calorimeter whether there is an excessive missing energy when we trigger with prompt lepton pairs.

In addition to these very topical studies, the signal from ρ decay should be sufficient to allow a study of inclusive ρ production in the forward direction, a study which is normally made difficult by combinatorial background. With data from four constraint fitted final states from our background data a comparison with muon pair data will allow a study of the ρ to 2μ branching ratio possibly superior to other comparisons in this field. No good study of the ω to 2μ branching ratio has been made and here it may be possible even with the few events available, although necessary assumptions as to the effect of the $\rho - \omega$ interference may cause appreciable error.

It is clear from the great activity in this field that it is impossible to predict now what mix of beam particle and energy would be optimum even a few months from now. At present we propose that the run be predominately with π^- mesons and at the highest energy possible, 17 GeV. For a low mass dimuon the yield should be relatively energy-independent. For a quark-antiquark annihilation process one would expect π^- mesons to be most effective. Our suggested run would be as follows:

π^-	17 GeV/c	600 hours
π^+	17 GeV/c	200 hours
π^-	12 GeV/c	200 hours

In addition we request a month of 10 pps checkout running.

Experimental Technique:

A beam of 17 GeV pions will be incident on a 1.5 meter liquid hydrogen target located in the SLAC two-meter streamer chamber (see Fig. 7). An intensity of 8 pions per pulse will produce an average of one interaction per pulse or 30 events per microbarn per hour at 180 pps.

The downstream end of the streamer chamber will be shortened to about 50 cm outside of the visible region to reduce the region in which pion decay can take place. Immediately outside the electric field region of the chamber a 20 cm thick photon calorimeter will be placed. This will consist of a 10 section hodoscope of alternating 1 cm scintillators 1.5 cm lead layers. This will cover an area about 2 meters x 1 meter and according to Monte Carlo calculations will intercept about 90% of the π^0 energy. In such a sandwich an energy measure-

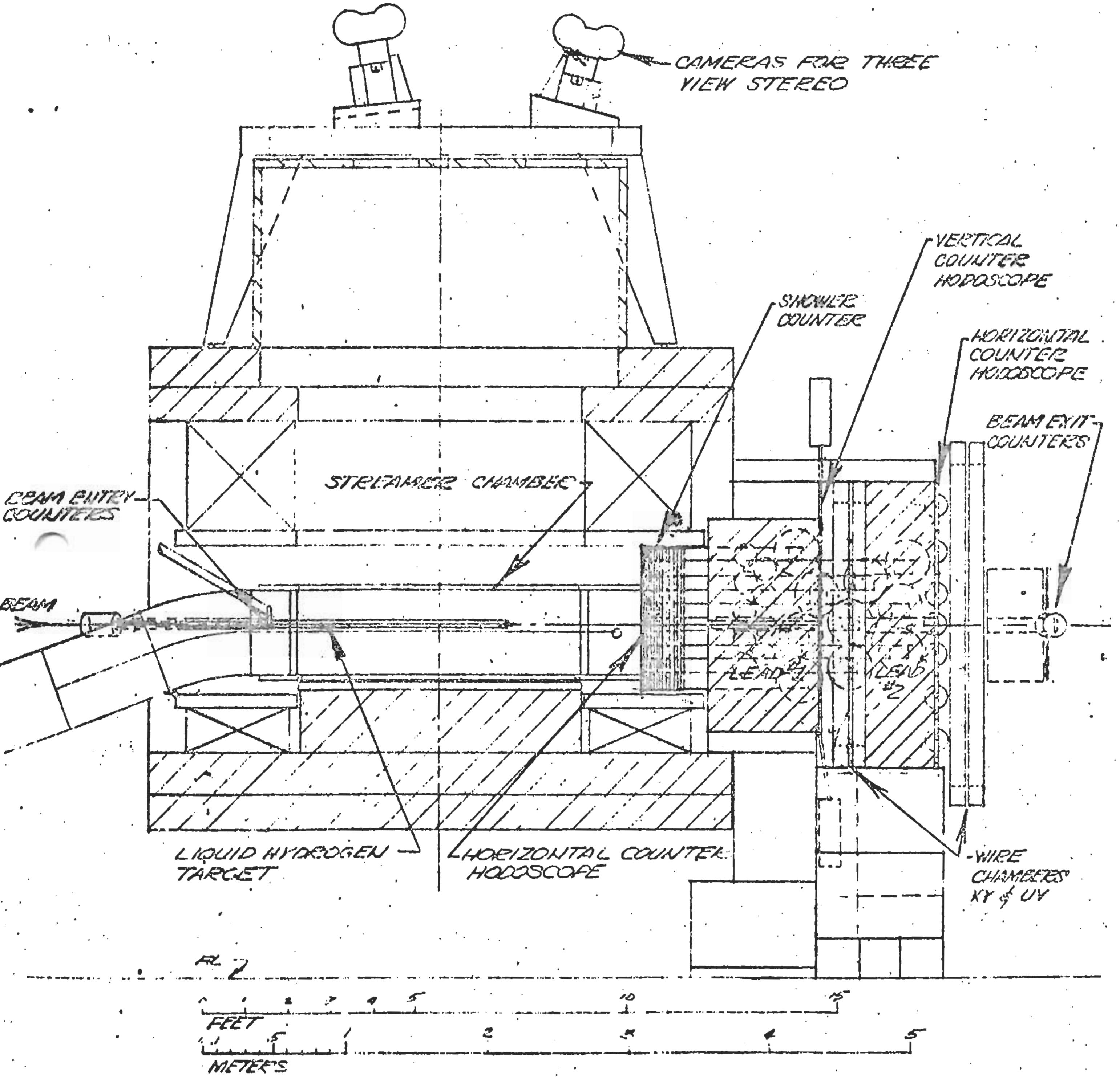

CAMERAS FOR THREE VIEW STEREO
VERTICAL COUNTER HODOSCOPE
SHOWER COUNTER
HORIZONTAL COUNTER HODOSCOPE
BEAM EXIT COUNTERS
BEAM ENTRY COUNTERS
STREAMER CHAMBER
BEAM
LEAD #1
LEAD #2
LIQUID HYDROGEN TARGET
HORIZONTAL COUNTER HODOSCOPE
WIRE CHAMBERS XY & UV
FL
0 1 2 3 4 5 10 15
FEET
0 .5 1 2 3 4 5
METERS

FIG. 7A

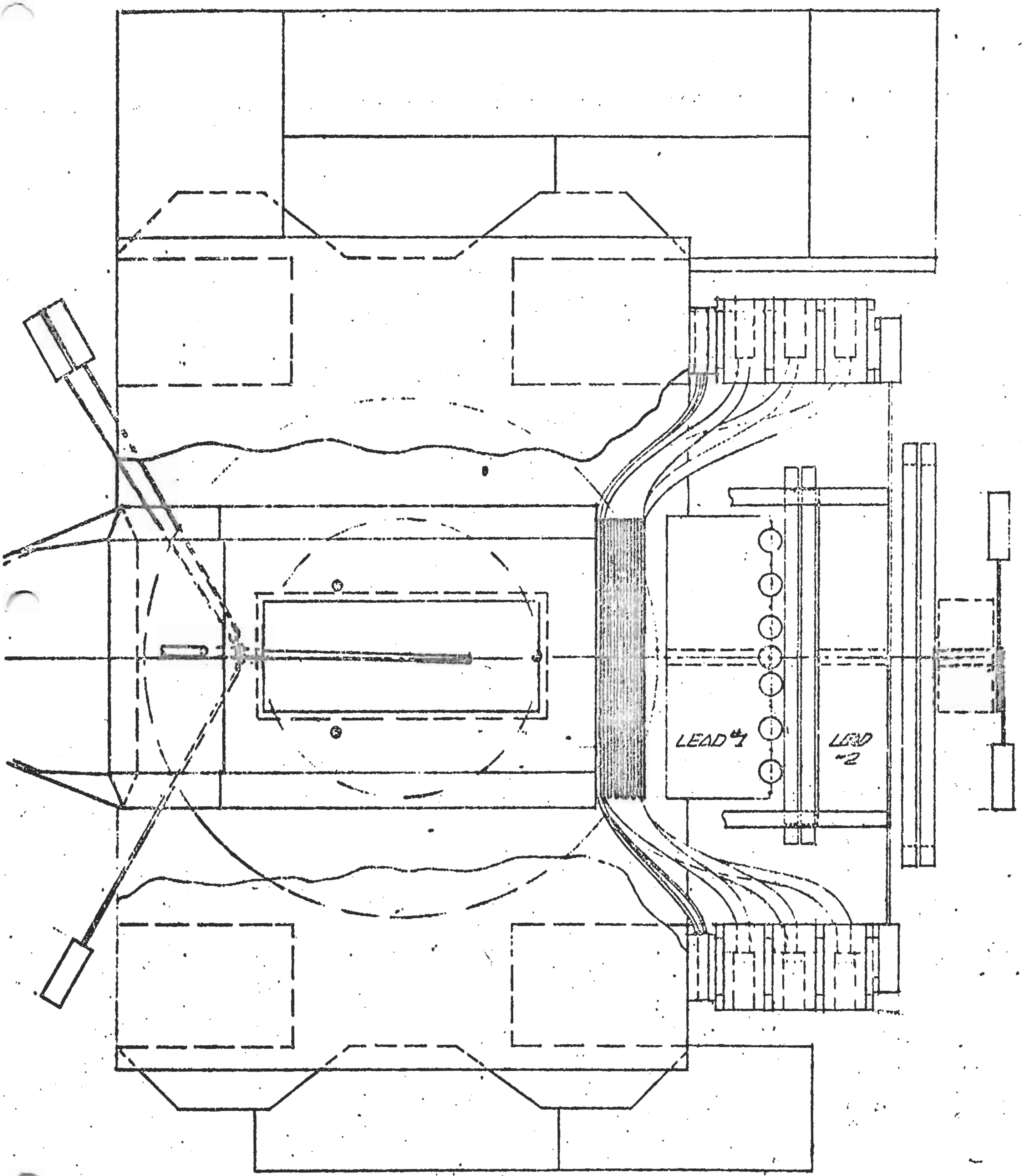


FIG. 7B

ment accuracy of about $\frac{25\%}{\sqrt{E}}$ has been obtained.

Directly behind the calorimeter will be a lead wall of sufficient thickness to absorb muons of up to 2 GeV in energy. This wall will extend 2.2 meters horizontally and 1.5 meters vertically and be as close to the streamer chamber as possible to reduce the path for pion decay. Scintillation counters will be located in front of the wall, at 1.5 GeV depth, and behind the wall at a 2 GeV depth. In the area directly downstream the wall can extend to 3 GeV in equivalent thickness with a corresponding layer of scintillation counters. The counters will be in a crude hodoscope arrangement and at least two hits per particle will be required. Depending on background rates triggers could either require that two muons each over 2 GeV or 2 muons with a summed energy of over 4 GeV be present.

In a low energy experiment of this nature pions can punch through the lead wall at a level implying an attenuation length at least double that given by inelastic scattering data. These cannot easily be removed from the trigger but the use of wire chambers located behind the lead wall will allow subsequent removal from the data. (As indicated in a study by A. Grant.[19]) The punch-through pions and kaons characteristically have interacted sufficiently to be well outside the spatial spread caused by multiple scattering of muons.

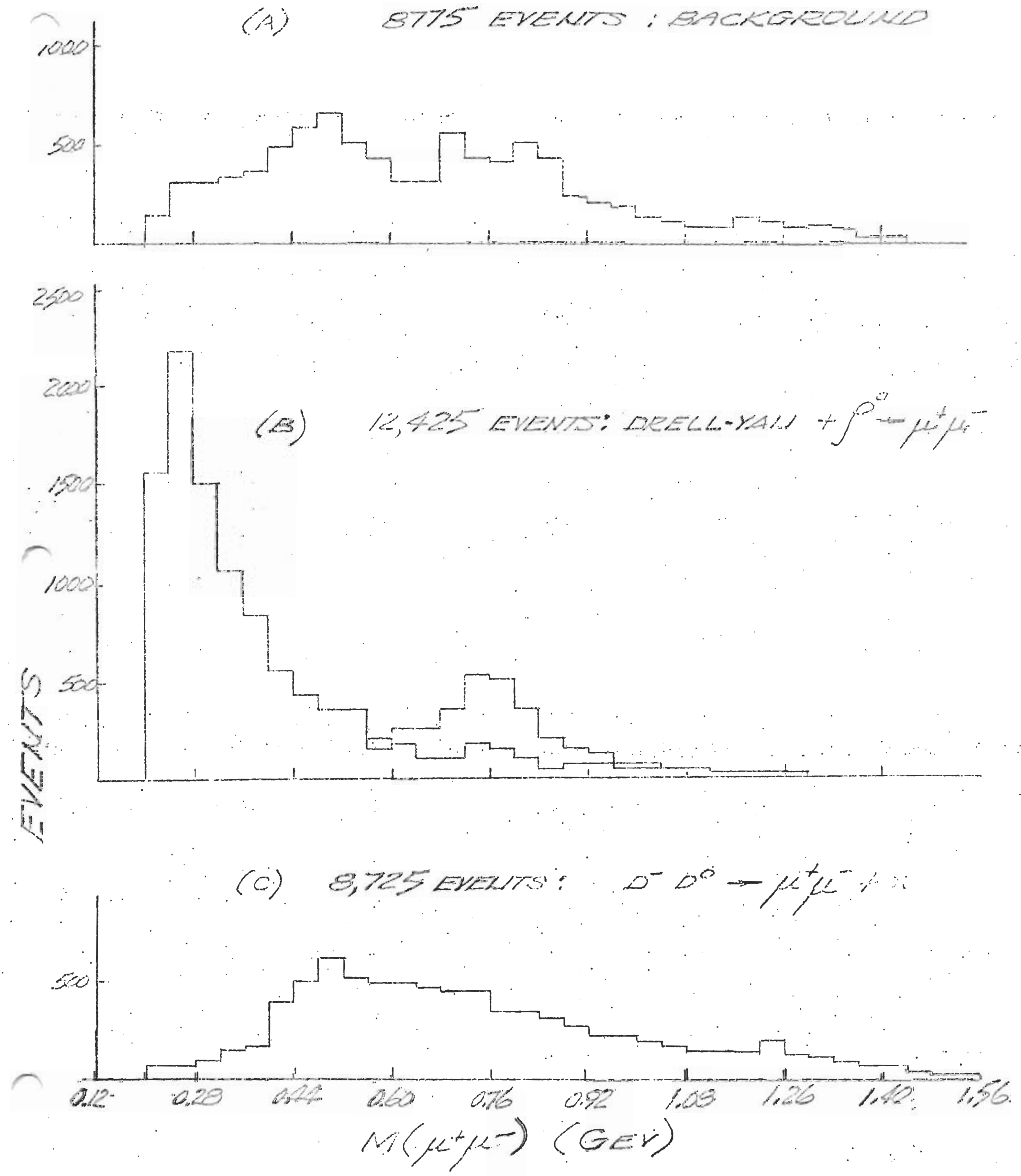
Pion and kaon decays and punch-through will largely determine the trigger rate. The average pion or kaon will have a 125 cm path in the visible region of the chamber, a 50 cm path through the termination region of the chamber and then be incident on the π^0 calorimeter. The whole path length will allow decays which can cause triggers. However we estimate that over half of those decaying in the visible region can be identified. In the kaons the decay vertex will be obvious in scanning while for the pions the procedure is to establish the event vertex which in our μp scattering experiment has been done to 0.1 x 1.0 x 1.2 mms. The candidate tracks must then undergo intensive measurement. Track sections of about 20 cm in length must then be extrapolated back to the vertex. A pion decay should cause a false extrapolation. Preliminary measurements would be done on events in the downstream end of the target while the kink removal process would be performed at a later period in the measurement and only for muon candidate pairs in an interesting mass region. As a result of this the total path length for decay in the visible region can be reduced to about 50 cm. Calculations for pion background assume this 50 cm plus 50 cm of invisible region, 20 cm of Pb and scintillator and 10 additional cm of Pb.

The size of the pion decay contamination can be accurately estimated after the experiment by use of data taken on runs with no selective trigger and with a single muon trigger. With the single muon trigger one can assume that all but ~ 1% of the triggers are from pion and kaon decays. Combining this with the untriggered data should allow a background estimate of better than 10% accuracy and should allow the determination of the presence of a dimuon signal at a level of 10^{-5} of the pion flux averaged over the entire mass region and appreciably better for low mass dimuons from a Drell-Yan process.

Table II shows our estimated background based on our own pion measurements at 15 GeV. Numbers are given as the number of events assuming 2.5×10^4 events per microbarn. Three categories are tabulated: (1) triggers (2) dimuon candidates after wire chamber analysis of punch-through and (3) after detailed measurements of $\mu^+\mu^-$ candidates. The numbers shown for dimuons are averaged over all masses while for a dimuon model for example the signal to noise in the low mass region is about 4 to 1. Fig. 8(a) shows the mass dependence of this background. This assumes a 50 cm visible decay region and 80 cm invisible region with the rest of the visible region removed by track fitting.

Although it is possible to estimate with fair accuracy the expected background the present situation regarding the expected signal is so confusing that one can only show the results of different models. If a Drell-Yan type of process is involved one obtains the results shown in Fig. 8(b). Here the total cross section is assumed to be 4 μb and the angular and energy distribution of decay come from the Drell-Yan process. Ordinary calculations of Drell-Yan production ignoring color give only a 1.0 μb cross section.

Another model is to assume that two charmed mesons of 2.2 GeV mass are made with a 4 μb cross section yielding 2.0 muons (4 body decay). One obtains the results shown in Fig. 8(c). The small yield shown, ~ 9%, is due to the wide angular distribution of the muons.


(A) 8775 EVENTS : BACKGROUND

(B) 12,425 EVENTS: DRELL-YAN + $\rho^0 \to \mu^+\mu^-$

(C) 8,725 EVENTS: $D^- D^0 \to \mu^+\mu^- + X$

FIG. 8

TABLE II: Event Yield in a 1,000 Hour Run at 180 pps with 75% Efficiency

Process	Triggers	Dimuon Candidates*	After Detailed Measurement
1. Background			
Same charge	32,000	16,000	-
Opposite charge	40,000	20,000	8,800
TOTAL	72,000	36,000	8,800
2. Drell-Yan+	10,500	-	10,500
3. CHARM+	8,700	-	8,700
4. ρ^o	1,900	-	1,900
Total Triggers	84,000		

*After wire chamber analysis

+Assume a total cross section of 10^{-4} of $\frac{\pi^+ + \pi^-}{2}$

The addition of the π^o detectors shown in Fig. 7 increases the versatility of the experiment immensely. Calculations indicate that over 90% of the total π^o energy will be deposited in these counters, assuming that the number of π^o's is the same as that of $\frac{\pi^+ + \pi^-}{2}$. Since about 30% of the total energy is carried by π^o's and since only 10% of this is missing, the lost energy is about 3%. If other tracks are measured and if protons are identified the total energy of the interaction should be reconstructable to about 1 GeV. Comparison of dimuon events and ordinary events must be made only in those cases where protons are identified which should encompass about half of events the remainder having neutron final states or unidentified protons.

An additional loss will result from pions which cause showers and hence large pulses in the shower counters. If a shower counter with a charged particle incident on it shows an energy deposit of greater than about 300 MeV the event must be eliminated from consideration. This will occur in about 30% of the remaining events. The shower counter of course has utility for studies of π^o production allowing model dependent estimates of the numbers and direction.

Search for Narrow Mass States:

In June 1974 we completed data taking in an experiment searching for charmed meson production in π^+ nucleon interactions. This experiment utilized the equipment from our μ-p and μ-d scattering experiments, E-72 and E-104. A 15 GeV/c pion beam was incident on a series of 1/2 cm polyethelene targets located in the streamer chamber. Triggers came from any particle passing through 1.5 meters of lead downstream. Our original conception of the experiment had implied an examination only of those events showing a visible strange particle decay in the chamber; the logic being that a charmed meson or baryon would decay preferentially into strange particles, accompanied by a nearly instantaneous lepton in many cases.

On analysis it very soon became apparent that our muon trigger did not appreciably enhance strange particle production. In addition, the difference in event yield observed between targets close to the lead absorber and at a greater distance was that expected from the trigger acceptance and meson decay into muons. In this part of the investigation it was clear that the distance of the lead absorber (chosen for the μp scattering experiments) considerably decreased the selectivity for seeing prompt muon production. We obtained an enhancement of high multiplicity events which had an appreciable chance for decay of at least one pion.

The chamber's large acceptance allowed us to determine high multiplicity effective mass distribution with a mass resolution ~ 1%. Thus we began a search for narrow mass states, hoping that the "muon" signal gave us some selection on one member of a charm pair, while the other member decayed into all charged particles. Given sufficient statistics, this experiment has the unique advantage of being able to look for high multiplicity decay modes of narrow states.

We have found more narrow mass states over four standard deviations above a smooth background than one would expect given the total number of time we have looked at it. The total number of events we now have in any of these peaks does not allow us to divide the events up in a search for systematics. The combinatorial backgrounds in the current data are also very high because there is no particle identification.

This proposed experiment would allow us to measure at least ten times as many events with $p_\perp \geq 400$ MeV/c to confirm or kill already observed narrow mass enhancements. This would allow us to set σ x Br. limit at the 1 μb level for a charm system decaying into any number of charged particles.

The proposed experiment would improve on our previous one in many ways. The lead wall would be much broader and closer. This would reduce the path for pion decay by a factor of three and increase the solid angle for a muon trigger by the same factor for muons of one sign and by a factor of six for muons of the opposite sign. In addition we plan a greater interaction rate.

Although in earlier experiments the streamer chamber had proven effective in mass identification, being able to distinguish between pions and protons of up to 800 MeV/c, new film used to reduce the size of halation produced flares also reduced the track size increase caused by ionization. This made scanning table recognition of dense ionization impossible. New equipment for densitometer measurement is planned for these experiments and this will allow us to restore and possibly even improve on the earlier ionization measurements. This would reduce the combinatorial background for narrow mass searches.

It is proposed that this experiment will run simultaneously with the two muon experiment and take data during the same beam time. We propose to take 25,000 pictures with no $p_\perp$ selection; 55,000 pictures with $p_\perp$ (of the muon) $\geq$ 400 MeV/c; and 20,000 pictures with a higher $p_\perp$ cutoff. The data with non-selective triggers from the dimuon experiment will also be utilized for background studies in this experiment.

Scanning and Measuring:

We propose to take a total of approximately 300,000 triads for both experiments in categories as shown in Table III.

TABLE III

	Triads	After spark chamber analysis
Dimuon Experiment		
Dimuon candidates	85,000	50,000
Interaction triggers	25,000	25,000
Single muon triggers no $p_\perp$ selection	25,000	15,000
Narrow mass search		
Single muon triggers no $p_\perp$ selection	25,000	15,000
Single muon with $p_\perp$ selection	75,000	45,000
Junk	65,000	0
TOTAL	300,000	150,000

Spark chamber information used after data taking should allow removal of the general background triggers and 40% of the dimuon and single muon candidates, leaving about half of the triads for further analysis.

Among the dimuon candidates about a third will be from $\pi - \mu$ decays leading to like signed muons. These can be removed by a very crude measurement. If there is a single event on a frame a crude vertex location should allow mass and charge determination using data from external spark chambers. If there is more than one event per frame, it will be necessary to measure the vertex and the end of each track. With this information combined with time information from scintillation counters it will be possible to select the correct event and determine mass and the charges.

All 50,000 must undergo this crude measurement which will allow the determination of the interesting 32,000 events. 20,000 of these will be background under this assumption of signal. To reduce the background an additional measurement of the identified muon tracks sampling each at least fifty times must be done. This could be done at the second measurement stage or on selected events at a third stage. It will result in reducing the background by a factor of 2.3 yielding a final sample of 21,000 events including about 8,800 background. A preliminary estimate would indicate the need for measuring about 10,000 of the interaction trigger and single muon trigger events for background calculations. The remainder would be a reserve in case of the need of greater accuracy in part of the background subtraction. These events could be crudely measured with possibly only three points per track. These can then be incorporated into a Monte Carlo calculation of the size of the background. To estimate the effectiveness of the "kink" removal procedure a sample of the "single muon" tracks must be measured. In 99% of the events these will be single pions decaying into muons.

For the Charm experiment we presently estimate the need of measuring an additional 20,000 with $p_\perp$ selection leaving a reserve of 40,000 unmeasured. Thus the total detailed measurement required is 50,000 triads for the dimuon experiment and 20,000 for the narrow mass search.

Approximately 300 hours of computing time will be required.

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