Crab crossing and crab waist at super KEKB

K. Ohmi (KEK)

Super B workshop at SLAC

15-17, June 2006

Thanks, M. Biagini, Y. Funakoshi, Y. Ohnishi, K.Oide,
E. Perevedentsev, P. Raimondi, M Zobov

Super bunch approach

K. Takayama et al, PRL, (LHC)
F. Ruggiero, F. Zimmermann (LHC)
P.Raimondi et al, (super B)

Short bunch

$$\frac{\sqrt{\varepsilon_x \beta_x}}{\theta \sigma_z} > 1$$

$$L \sim \frac{N^2}{\sqrt{\varepsilon_x \beta_x \varepsilon_y \beta_y}}$$

$$\xi_x \sim \frac{N}{\varepsilon_x}$$

$$\xi_y \sim N \sqrt{\frac{\beta_y}{\varepsilon_x \beta_x \varepsilon_y}}$$

$$\beta_y > \sigma_z$$

$$\times \frac{\sqrt{\varepsilon_x \beta_x}}{\theta \sigma_z} \quad (<1)$$

Overlap factor

Long bunch

$$L \sim \frac{N^2}{\theta \sigma_z \sqrt{\varepsilon_y \beta_y}}$$

$$\xi_x \sim \frac{N}{\theta \sigma_z} \sqrt{\frac{\beta_x}{\varepsilon_x}}$$

$$\xi_y \sim \frac{N}{\theta \sigma_z} \sqrt{\frac{\beta_y}{\varepsilon_y}}$$

$$\beta_y > \frac{\sqrt{\varepsilon_x \beta_x}}{\theta}$$

ξ_x is smaller due to cancellation of tune shift along bunch length

Essentials of super bunch scheme

$$L \sim \frac{N^2}{\theta\sigma_z\sqrt{\varepsilon_y\beta_y}}$$

$$\xi_x \sim \frac{N}{\theta\sigma_z}\sqrt{\frac{\beta_x}{\varepsilon_x}}$$

$$\xi_y \sim \frac{N}{\theta\sigma_z}\sqrt{\frac{\beta_y}{\varepsilon_y}}$$

$$\beta_y > \frac{\sqrt{\varepsilon_x\beta_x}}{\theta}$$

Keep $\sqrt{\dfrac{\beta_y}{\varepsilon_y}}$, $\sqrt{\dfrac{\beta_x}{\varepsilon_x}}$ and $\dfrac{\sqrt{\varepsilon_x\beta_x}}{\beta_y}$.

$$\varepsilon_y\beta_y \to 0$$

$$L \to \infty$$

- Bunch length is free.
- Small beta and small emittance are required.

Short bunch scheme

$$L \sim \frac{N^2}{\sqrt{\varepsilon_x \beta_x \varepsilon_y \beta_y}}$$

$$\xi_x \sim \frac{N}{\varepsilon_x}$$

$$\xi_y \sim N \sqrt{\frac{\beta_y}{\varepsilon_x \beta_x \varepsilon_y}}$$

$$\beta_y > \sigma_z$$

Keep ε_x, β_x and $\sqrt{\frac{\beta_y}{\varepsilon_y}}$.

$$\varepsilon_y \beta_y \to 0$$

$$L \to \infty$$

- Small coupling
- Short bunch

- Another approach: Operating point closed to half integer

$$\nu_x \to +0.5 \qquad \xi_y \to \infty \qquad L \to \infty$$

We need $L=10^{36}\ cm^{-2}s^{-1}$

- Not infinity.
- Which approach is better?

- Application of lattice nonlinear force
- Traveling waist, crab waist

Nonlinear map at collision point

$$H = a_i x_i + b_{ij} x_i x_j + c_{ijk} x_i x_j x_k$$

$$\mathbf{x} = x_i = (x, p_x, y, p_y, z, \delta (= \Delta E / E))$$

$$M = \exp(- : H :) \mathbf{x}^* = \mathbf{x} - [H, \mathbf{x}] + \frac{1}{2} [H, [H, \mathbf{x}]] + ...$$

$$= a_i + b_{ij} x_j + c'_{ijk} x_j x_k + ...$$

- 1st orbit
- 2nd tune, beta, crossing angle
- 3rd chromaticity, transverse nonlinearity. z-dependent chromaticity is now focused.

Waist control-I traveling focus

$$\mathbf{M} = e^{-:H_I:}\mathbf{M}_0 e^{:H_I:}$$

$$H_I = ap_y^2 z$$

$$\bar{y} = y + \frac{\partial H_I}{\partial P_y} = y + azP_y \qquad \bar{\delta} = \delta - \frac{\partial H}{\partial z} = \delta - ap_y^2$$

- Linear part for y. z is constant during collision.

$$\begin{pmatrix} \bar{\beta} & -\bar{\alpha} \\ -\bar{\alpha} & \bar{\gamma} \end{pmatrix} = T \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} T^t = \begin{pmatrix} \beta + \frac{a^2 z^2}{\beta} & \frac{az}{\beta} \\ \frac{az}{\beta} & \frac{1}{\beta} \end{pmatrix}$$

α=0

$$T = \begin{pmatrix} 1 & az \\ 0 & 1 \end{pmatrix}$$

Waist position for given z

- Variation for s

$$M(s)\begin{pmatrix} \beta+\dfrac{a^2z^2}{\beta} & \dfrac{az}{\beta} \\ \dfrac{az}{\beta} & \dfrac{1}{\beta} \end{pmatrix} M^t(s) = \begin{pmatrix} \beta+\dfrac{\left(s+az\right)^2}{\beta} & \dfrac{s+az}{\beta} \\ \dfrac{s+az}{\beta} & \dfrac{1}{\beta} \end{pmatrix}$$

- Minimum β is shifted s=-az

Realistic example- I

- Collision point of a part of bunch with z, <s>=z/2.
- To minimize β at s=z/2, a=-1/2
- Required H

$$H_I = -\frac{1}{2} p_y^2 z$$

- RFQ TM210
-

$$V = \frac{1}{2\beta^* \beta} \frac{c^2}{\omega^2} \frac{pc}{e}$$

V~10 MV or more

Waist control-II crab waist

(P. Raimondi et al.)

$$\mathbf{M} = e^{-:H_I:}\mathbf{M}_0 e^{:H_I:}$$

$$H_I = axp_y^2$$

$$\overline{y} = y + \frac{\partial H_I}{\partial P_y} = y + axP_y \qquad \overline{p_x} = p_x - \frac{\partial H}{\partial x} = p_x - ap_y^2$$

- Take linear part for y, since x is constant during collision.

$$\begin{pmatrix} \overline{\beta} & -\overline{\alpha} \\ -\overline{\alpha} & \overline{\gamma} \end{pmatrix} = T \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} T^t = \begin{pmatrix} \beta + \frac{a^2x^2}{\beta} & \frac{ax}{\beta} \\ \frac{ax}{\beta} & \frac{1}{\beta} \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & ax \\ 0 & 1 \end{pmatrix}$$

Waist position for given x

- Variation for s

$$M(s)\begin{pmatrix} \beta+\dfrac{a^2x^2}{\beta} & \dfrac{ax}{\beta} \\ \dfrac{ax}{\beta} & \dfrac{1}{\beta} \end{pmatrix} M^t(s) = \begin{pmatrix} \beta+\dfrac{(s+ax)^2}{\beta} & \dfrac{s+ax}{\beta} \\ \dfrac{s+ax}{\beta} & \dfrac{1}{\beta} \end{pmatrix}$$

- Minimum β is shifted to s=-ax

Realistic example- II

- To complete the crab waist, a=1/θ, where θ is full crossing angle.
- Required H

$$H_I = \frac{1}{\theta} x p_y^2$$

- Sextupole strength

$$K_2 = \frac{1}{2} \frac{B''L}{p/e} \approx \frac{1}{\theta} \frac{1}{\beta_y^* \beta_y} \sqrt{\frac{\beta_x^*}{\beta_x}}$$

K_2~30-50

Not very strong

Crabbing beam in sextupole

- Crabbing beam in sextupole can give the nonlinear component at IP
- Traveling waist is realized at IP.

$$H_I = a p_y^2 z$$

$$z^* = \sqrt{\frac{\beta(s)}{\beta^*}}\zeta(s)x(s)$$

$$K_2 = \frac{1}{2}\frac{B''L}{p/e} \approx \frac{1}{\theta}\frac{1}{\beta_y^*\beta_y}\sqrt{\frac{\beta_x^*}{\beta_x}}$$

K_2~30-50

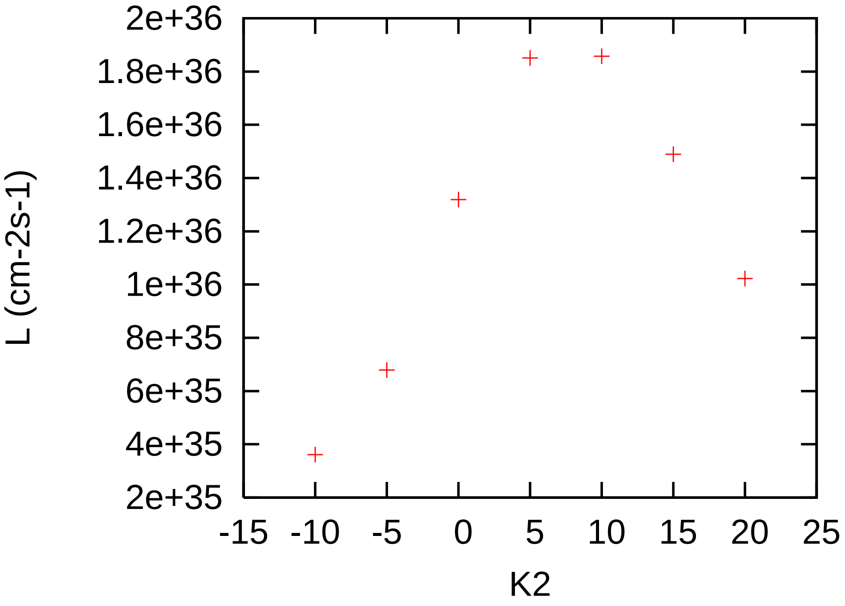
Super B (LNF-SLAC)

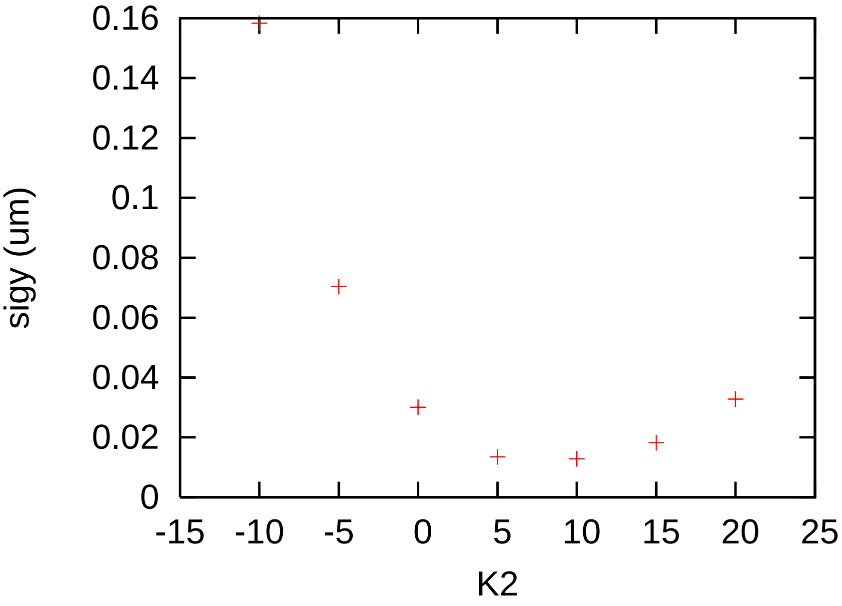
	Base	PEP-III
C	3016	2200
εx	4.00E-10	2.00E-08
εy	2.00E-12	2.00E-10
βx (mm)	17.8	10
βy (mm)	0.08	0.8
σz (mm)	4	10
ne	2.00E+10	3.00E+10
np	4.40E+10	9.00E+10
ϕ/2 (mrad)	25	14
ξx	0.0025	
ξy	0.1	

$$\xi_y = \int_{-\infty}^{\infty} \left. \frac{\partial F_y}{\partial y} \right|_{x=\phi z} \rho(z)\delta(s+z/2)\,dz\,ds$$

Luminosity for the super B

- Luminosity and vertical beam size as functions of K2
- L>1e36 is achieved in this weak-strong simulation.





DAFNE upgrade

	DAFNE
C	97.7
εx	3.00E-07
εy	1.50E-09
βx (mm)	133
βy (mm)	6.5
σz (mm)	15
ne	1.00E+11
np	1.00E+11
$\phi/2$ (mrad)	25
ξx	0.033
ξy	0.2479

Luminosity for new DAFNE

- L (x10^{33}) given by the weak-strong simulation
- Small ν_s was essential for high luminosity

ne	tune	νs	L(K2=10)	15	20
6.00E+10	0.53, 0.58	0.012		4.27	3.79
6.00E+10	0.53, 0.58	-0.01		4.35	3.93
1.00E+11	0.53, 0.58	-0.01	4.07	5.66	5.53
6.00E+10	0.53, 0.58	0.012	σz=10mm	4.32	2.47
6.00E+10	0.53, 0.58	-0.01	σz=10mm	4.65	2.7
6.00E+10	0.057, 0.097	-0.01		5.19(3.3)	
1.00E+11	0.057, 0.097	-0.01		13.21(4.8)	

() strong-strong , horizontal size blow-up

Super KEKB

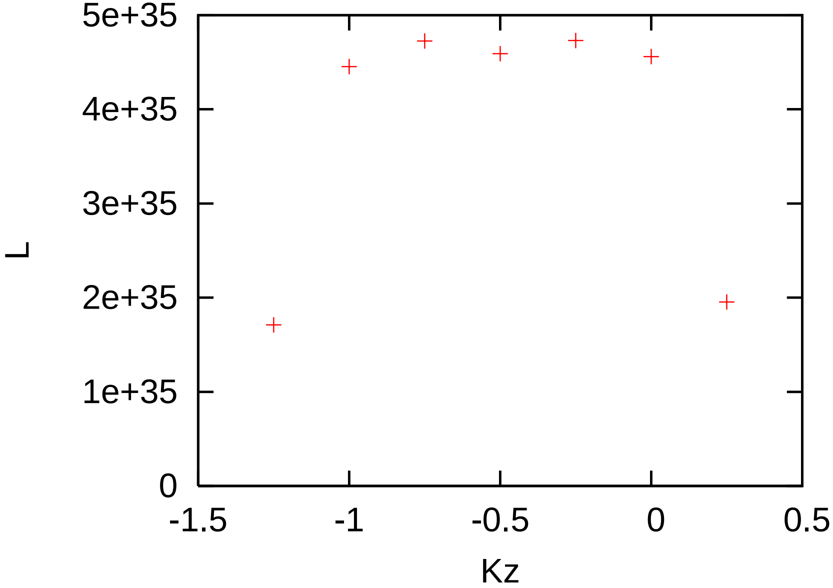
	SuperKEKB	Crab waist			
εx	9.00E-09	6.00E-09	6.00E-09	6.00E-09	6.00E-09
εy	4.50E-11	6.00E-11	6.00E-11	6.00E-11	6.00E-11
βx (mm)	200	100	50	100	50
βy (mm)	3	1	0.5	1	0.5
σz (mm)	3	6	6	4	4
νs	0.025	0.01	0.01	0.01	0.01
ne	5.50E+10	5.50E+10	5.50E+10	3.50E+10	3.50E+10
np	1.26E+11	1.27E+11	1.27E+11	8.00E+10	8.00E+10
ϕ/2 (mrad)	0	15	15	15	15
ξx	0.397	0.0418	0.022	0.0547	0.0298
ξy	0.794->0.24	0.1985	0.179	0.178	0.154
Lum (W.S.)	8E+35	6.70E+35	1.00E+36	3.95E+35	4.80E+35
Lum (S.S.)	8.25E35	4.77E35	9E35	3.94E35	4.27E35

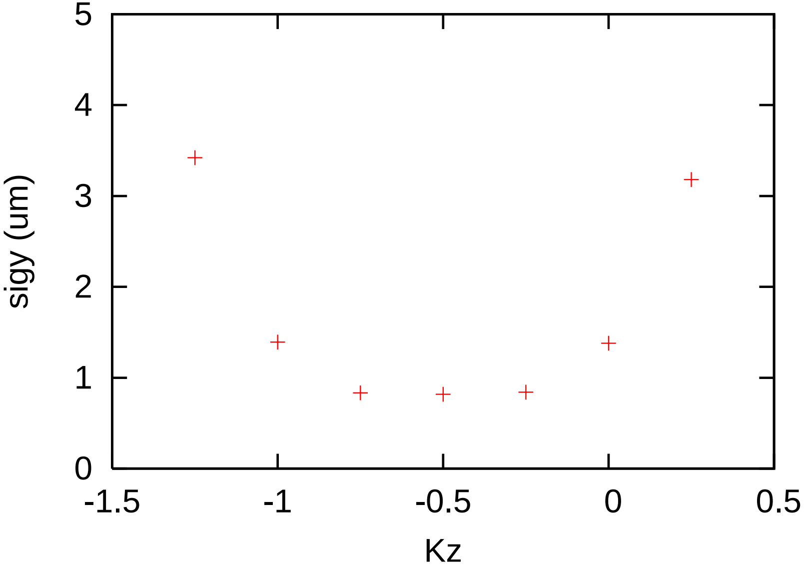
Horizontal blow-up is recovered by choice of tune. (M. Tawada)

Traveling waist

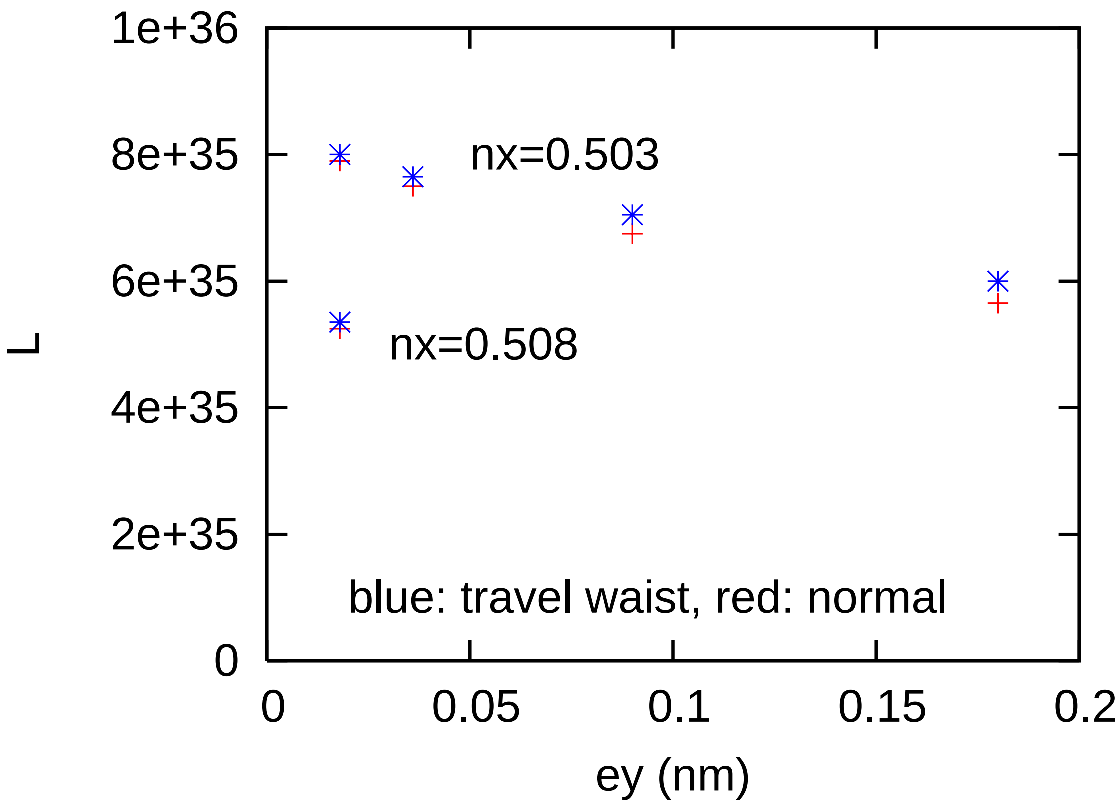
- Particles with z collide with central part of another beam. Hour glass effect still exists for each particles with z.
- No big gain in Lum.!
- Life time is improved.

ε_x=24 nm
ε_y=0.18nm βx=0.2m
βy=1mm σz=3mm

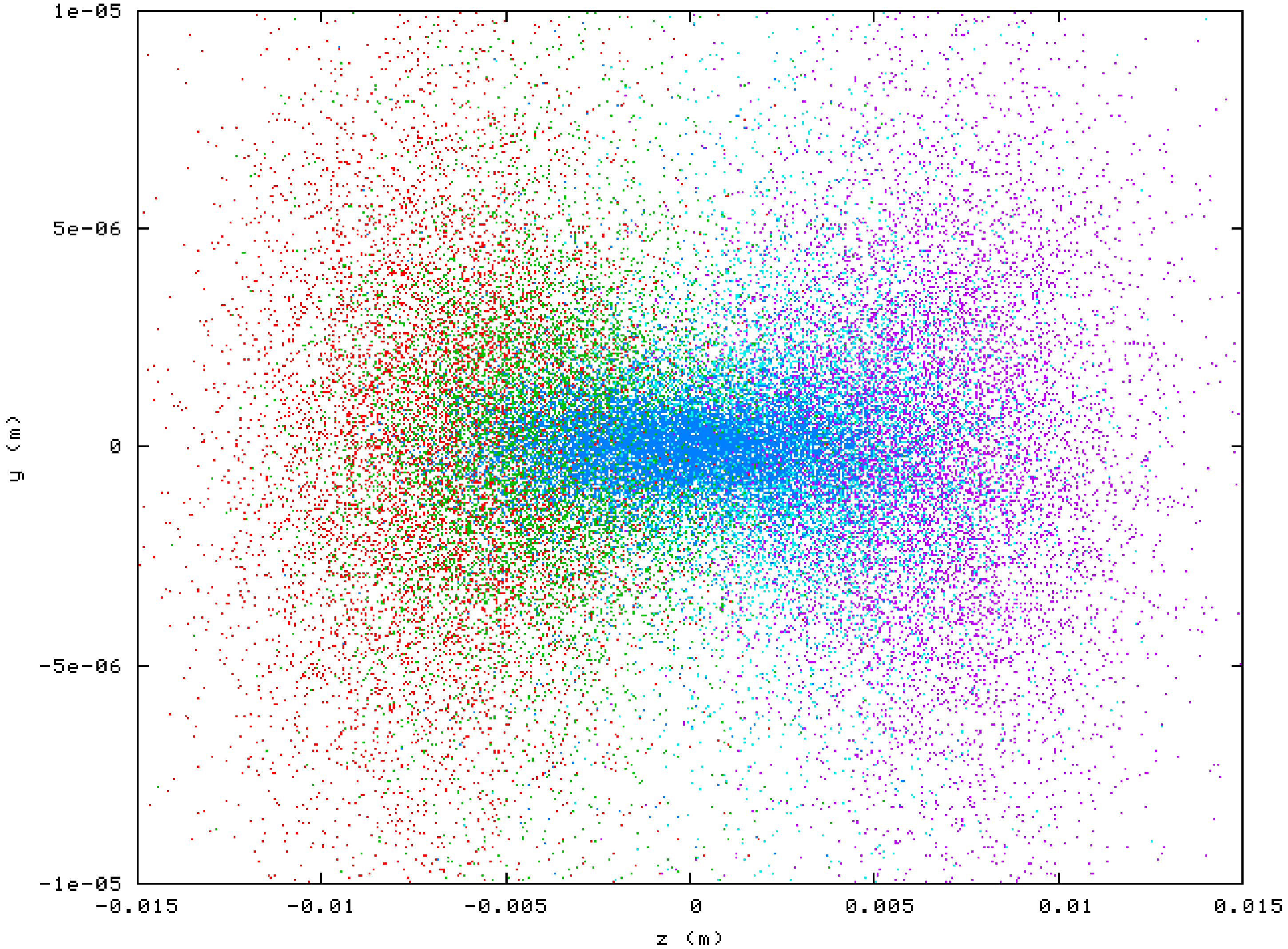




Small coupling



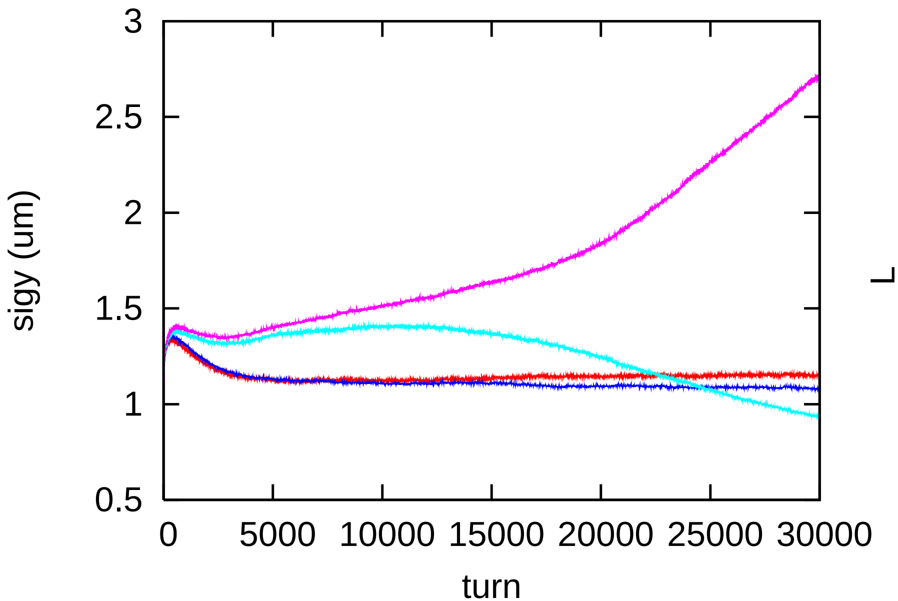
Traveling of positron beam

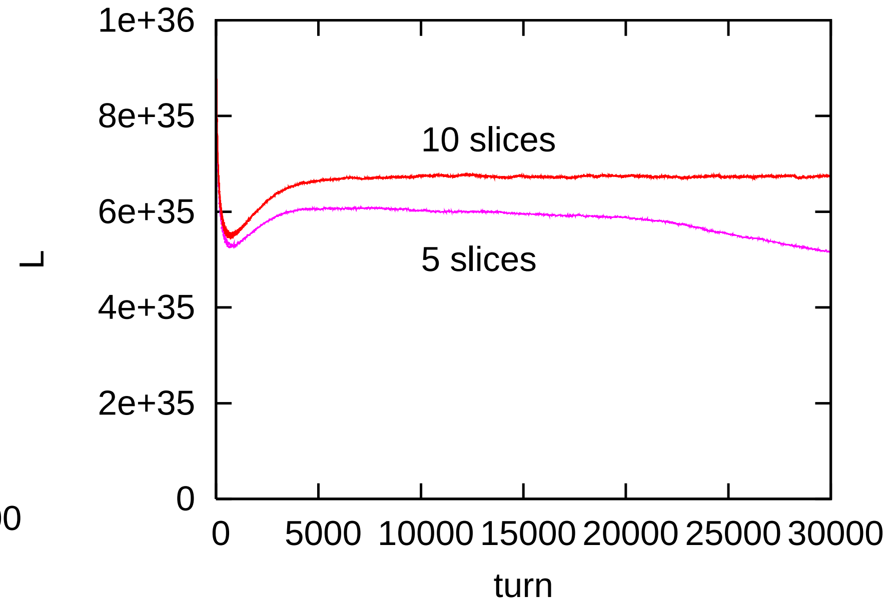


Increase longitudinal slice

ε_x=18nm, ε_y=0.09nm, β_x=0.2m β_y=3mm σ_z=3mm

Lower coupling becomes to give higher luminosity.





Why the crab crossing and crab waist improve luminosity?

- Beam-beam limit is caused by an emittance growth due to nonlinear beam-beam interaction.
- Why emittance grows?
- Studies for crab waist is just started.

Weak-strong model

- 3 degree of freedom
- Periodic system
- Time (s) dependent

$$H(x, p_x, y, p_y, z, p_z; s) = H'(J_1, \varphi_1, J_2, \varphi_2, J_3, \varphi_3; s)$$

$$\varphi(s+L) = \varphi(s) + 2\pi\nu$$

Solvable system

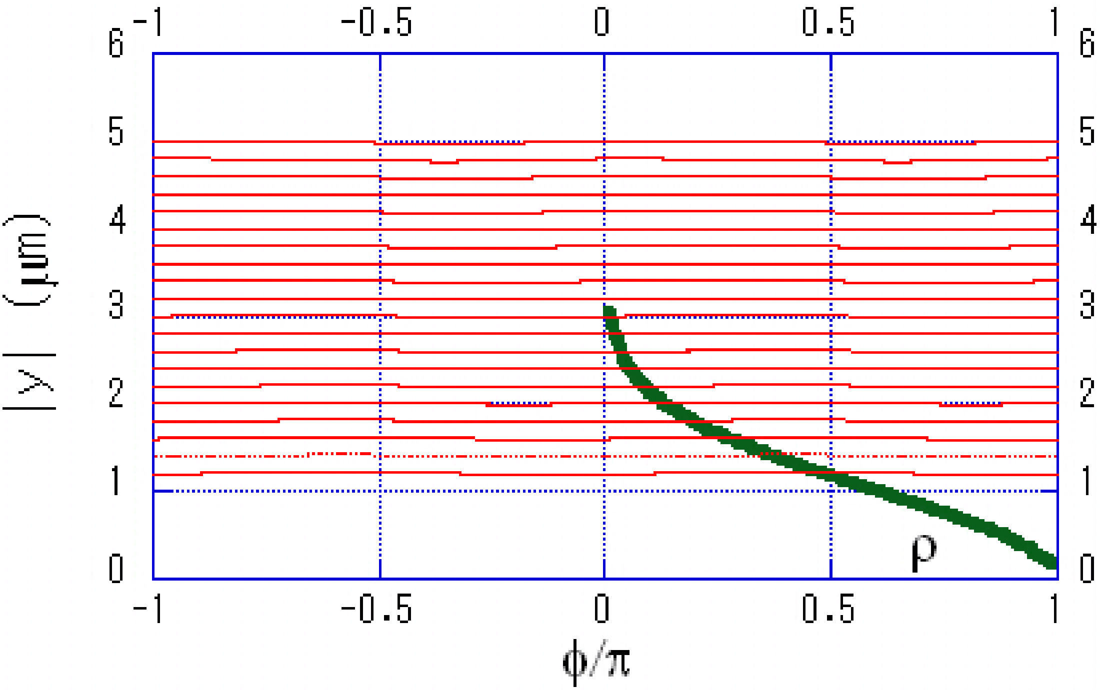
- Exist three J's, where H is only a function of J's, not of φ's.
- For example, linear system.
- Particles travel along J. J is kept, therefore no emittance growth, except mismatching.

$$\frac{dJ}{ds} = -\frac{\partial H}{\partial \varphi} = 0 \qquad J = \mathrm{const}$$

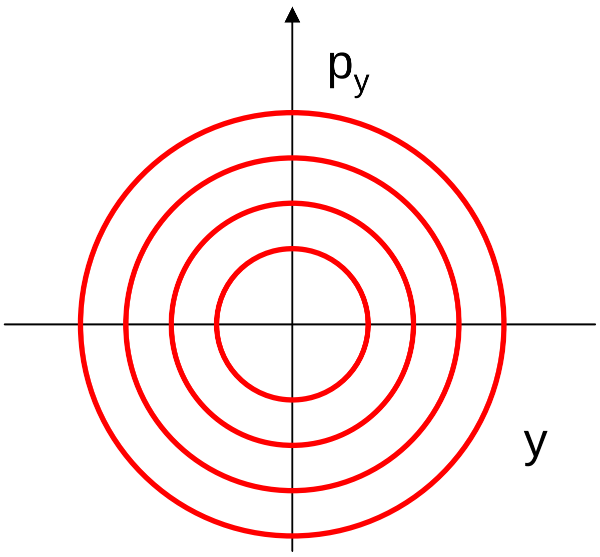
$$\frac{d\varphi}{ds} = \frac{\partial H(J)}{\partial J} \qquad \oint d\varphi = 2\pi\nu(J)$$

- Equilibrium distribution

$$\psi(J) \approx \exp\left(-\frac{J_1}{\varepsilon_1} - \frac{J_2}{\varepsilon_2} - \frac{J_3}{\varepsilon_3}\right) \qquad \varepsilon\text{: emittance}$$



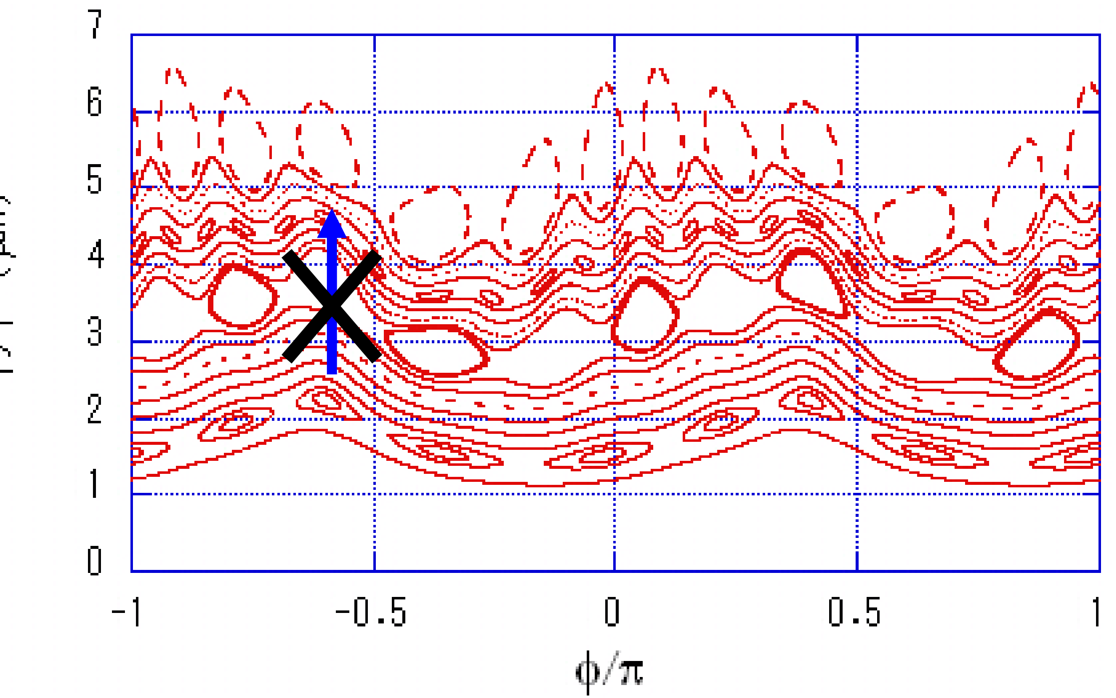
-1
-0.5
0
0.5
1
6
5
4
3
2
1
0
|y| (μm)
ρ
φ/π

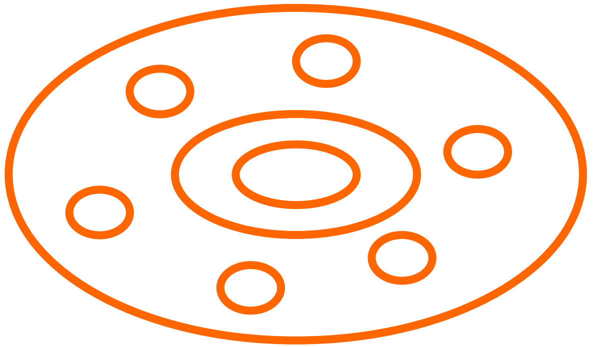


p_y
y

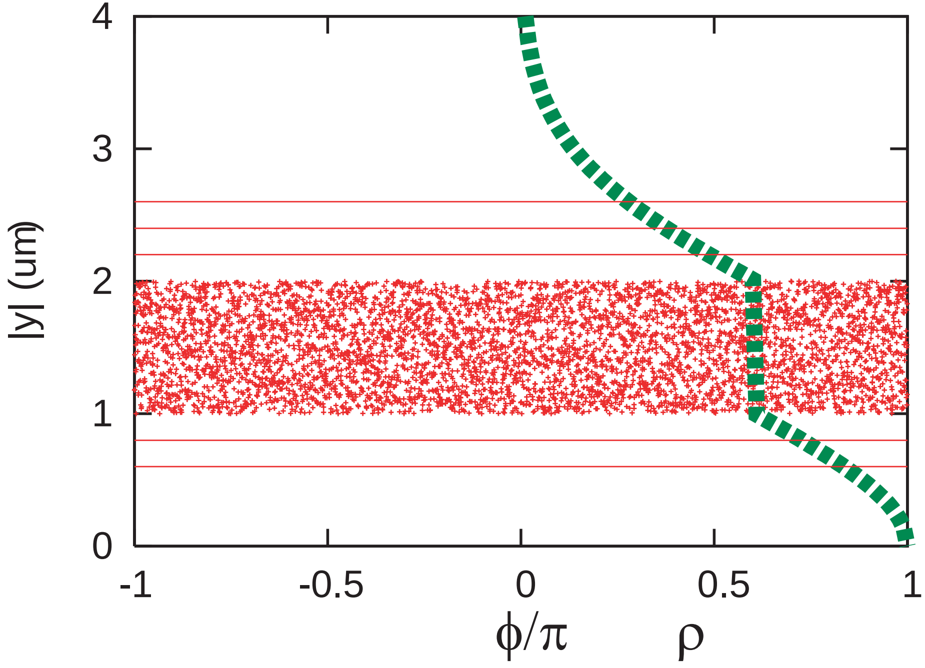
One degree of freedom

- Existence of KAM curve
- Particles can not across the KAM curve.
- Emittance growth is limited. It is not essential for the beam-beam limit.





- Schematic view of equilibrium distribution
- Limited emittance growth

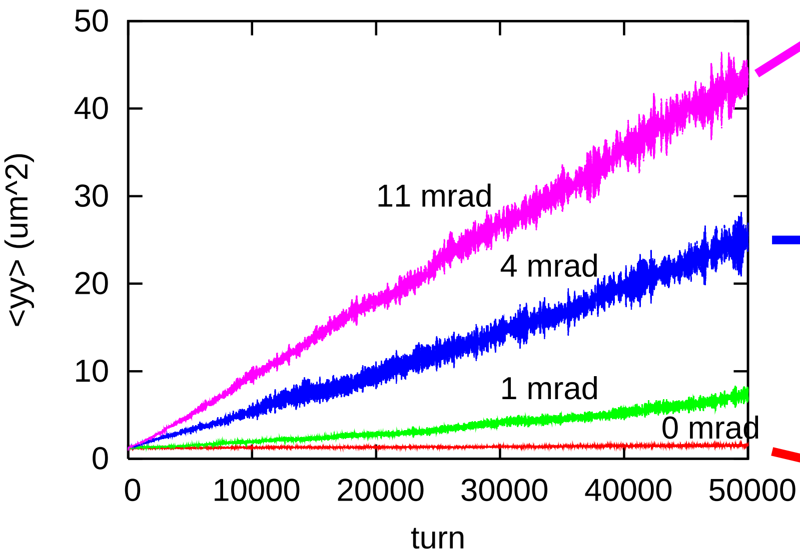


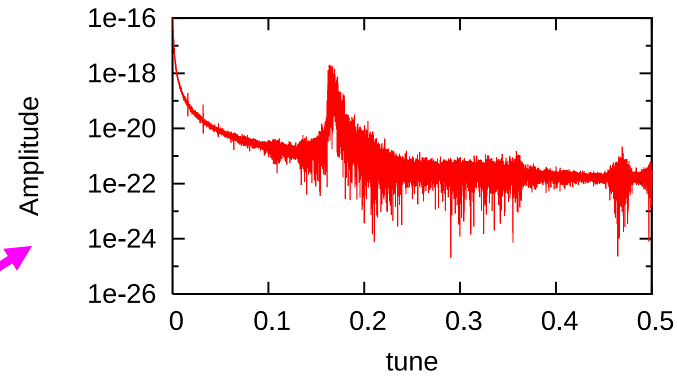
More degree of freedom

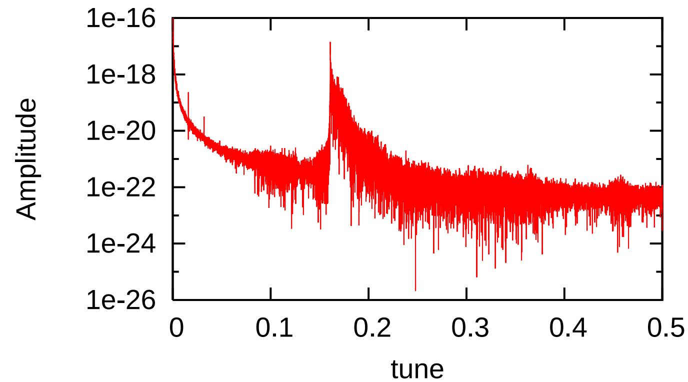
Gaussian weak-strong beam-beam model

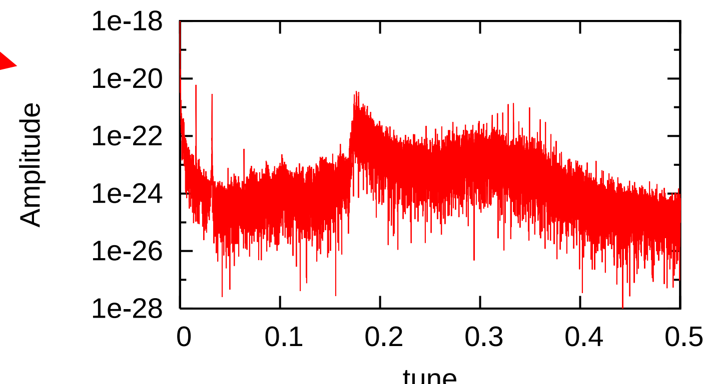
- Diffusion is seen even in sympletic system.

Diffusion due to crossing angle and frequency spectra of $<y^2>$







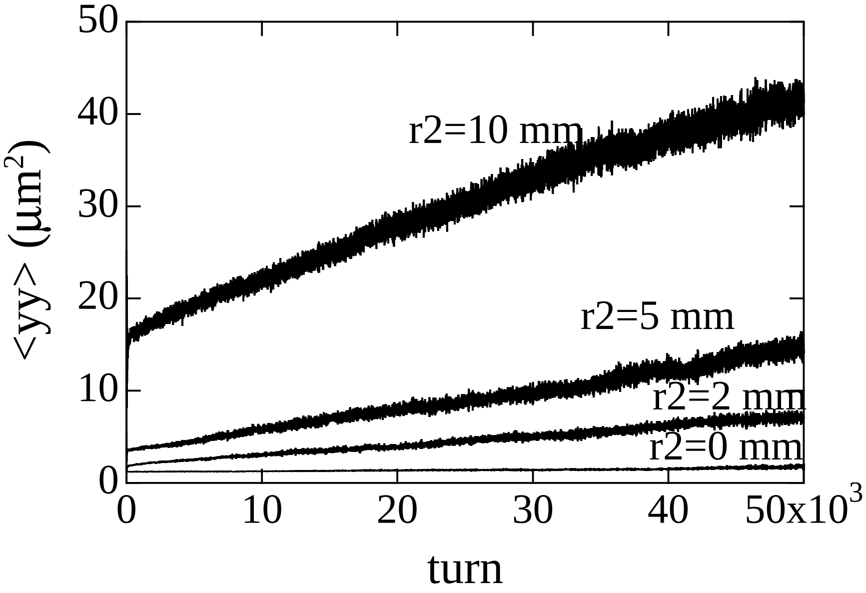


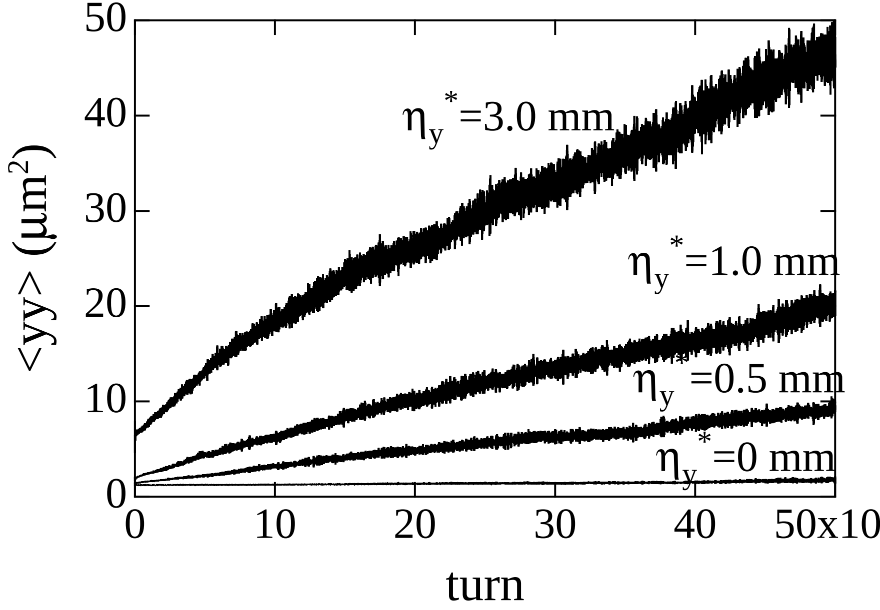
Only $2\nu_y$ signal was observed.

Linear coupling for KEKB

- Linear coupling (r's), dispersions, (η, ζ=crossing angle for beam-beam) worsen the diffusion rate.

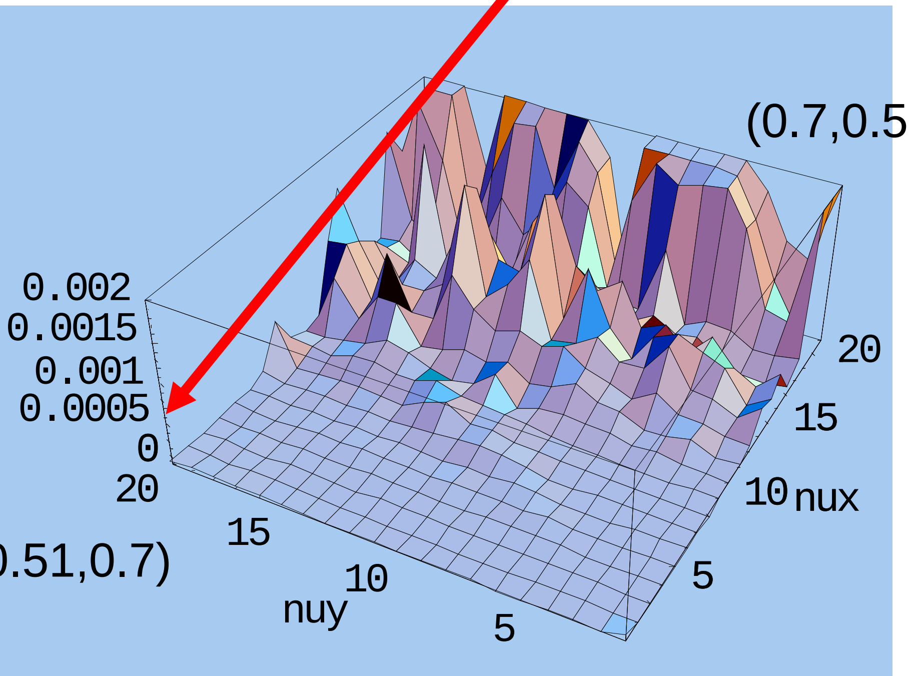
M. Tawada et al, EPAC04



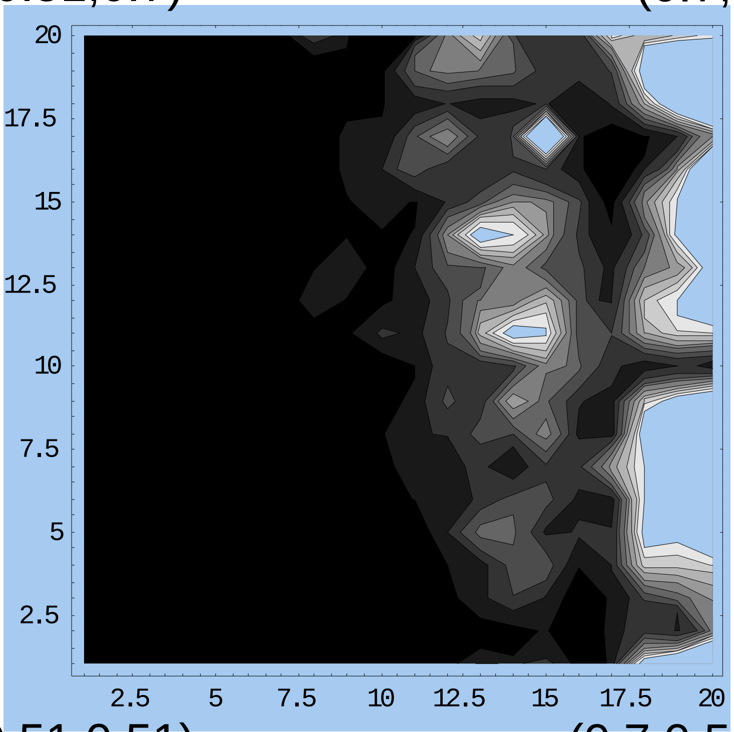


Two dimensional model

- Vertical diffusion for ξ=0.136
- D_C<<D_γ for wide region (painted by black).
- No emittance growth, if no interference. Actually, simulation including radiation shows no luminosity degradation nor emittance growth in the region.
- Note KEKB $D_\gamma=5\times10^{-4}$ /turn DAFNE $D_\gamma=1.8\times10^{-5}$ /turn



KEKB D_γ=0.0005

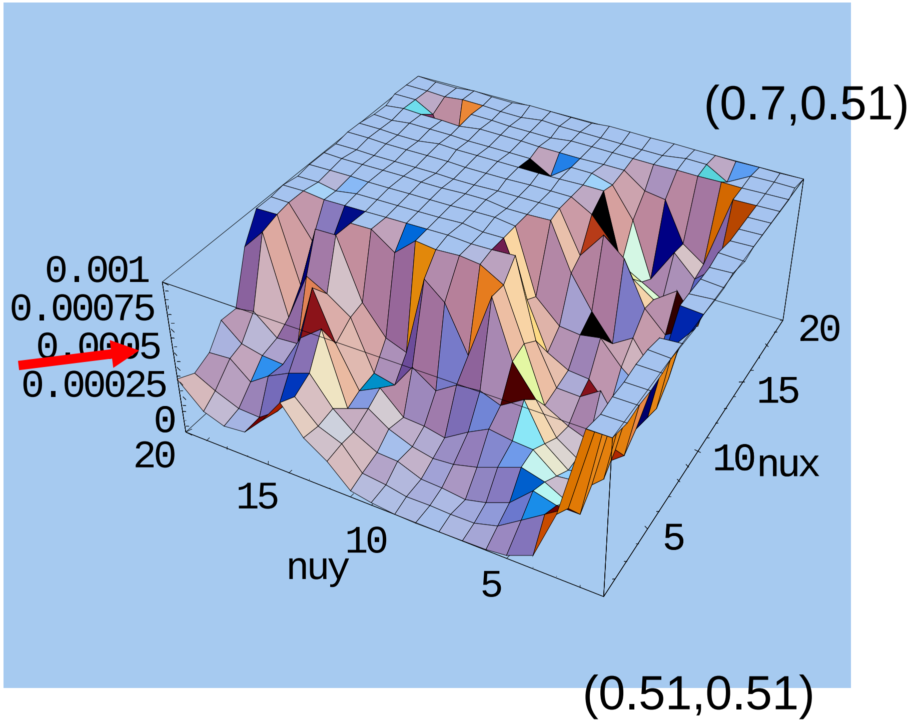


3-D simulation including bunch length ($\sigma_z \sim \beta_y$)

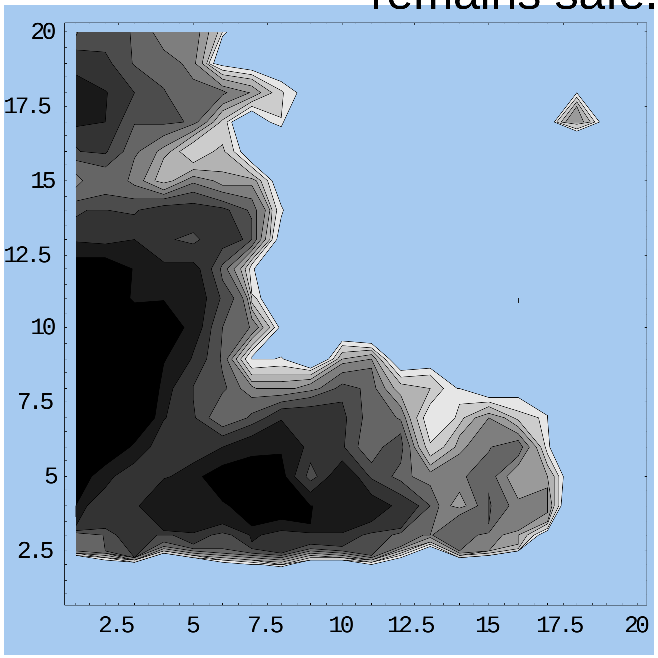
Head-on collision

Global structure of the diffusion rate.

Fine structure near ν_x=0.5



Contour plot

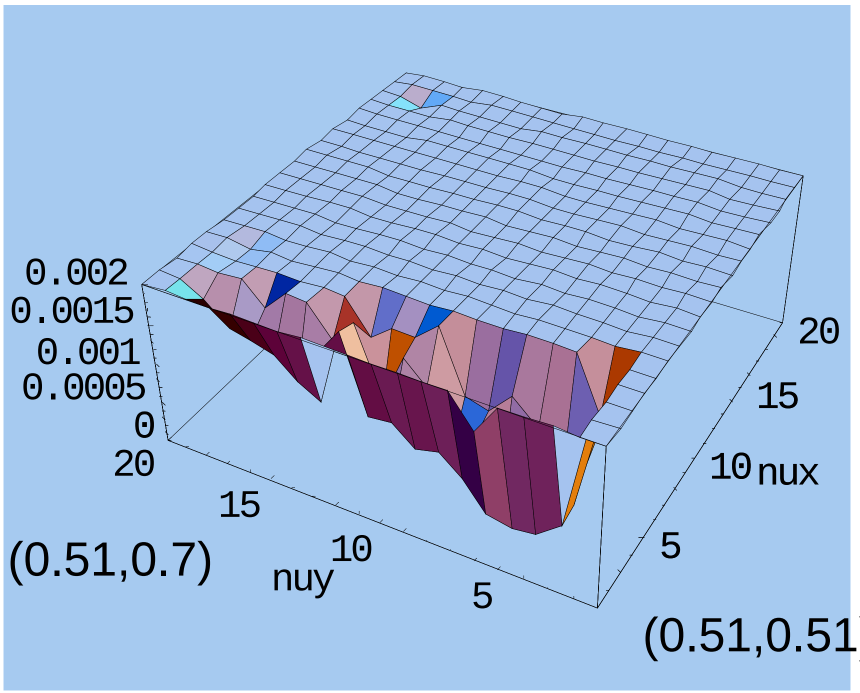


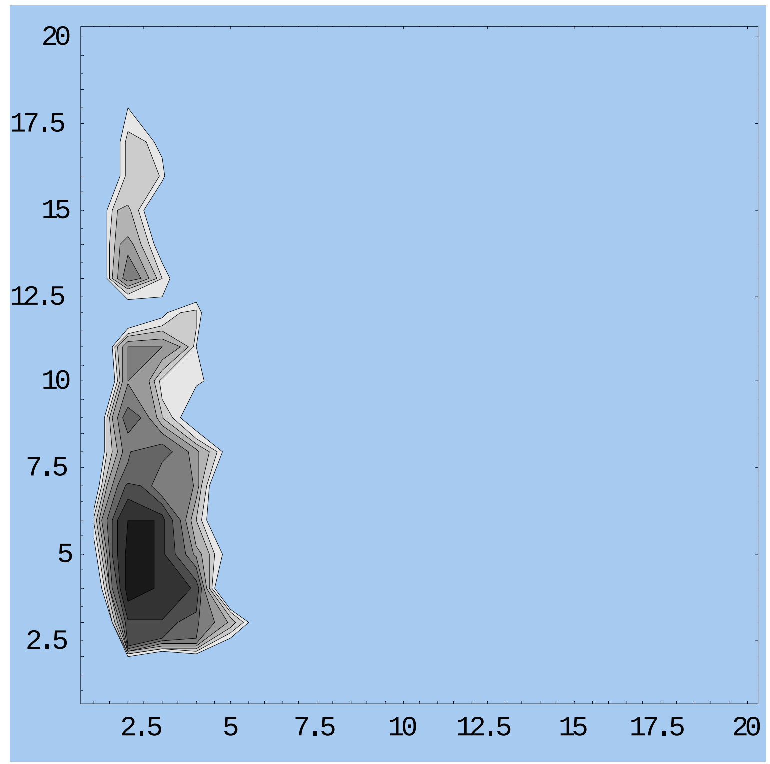
- Good region shrunk drastically.
- Synchrobeta effect near νy~0.5.
- νx~0.5 region remains safe.

Crossing angle ($\phi\sigma_z/\sigma_x$~1, σ_z~β_y)

- Good region is only (ν_x, ν_y)~(0.51,0.55).

(0.7,0.51)





Reduction of the degree of freedom.

- For νx~0.5, x-motion is integrable.

(work with E. Perevedentsev)

$$\lim_{\nu_x \to 0.5+} D_{C,y} = 0$$

if zero-crossing angle and no error.

$$L \propto \frac{1}{\Delta \nu_x + \text{(crossing angle)+(coupling)+(fast noise)}}$$

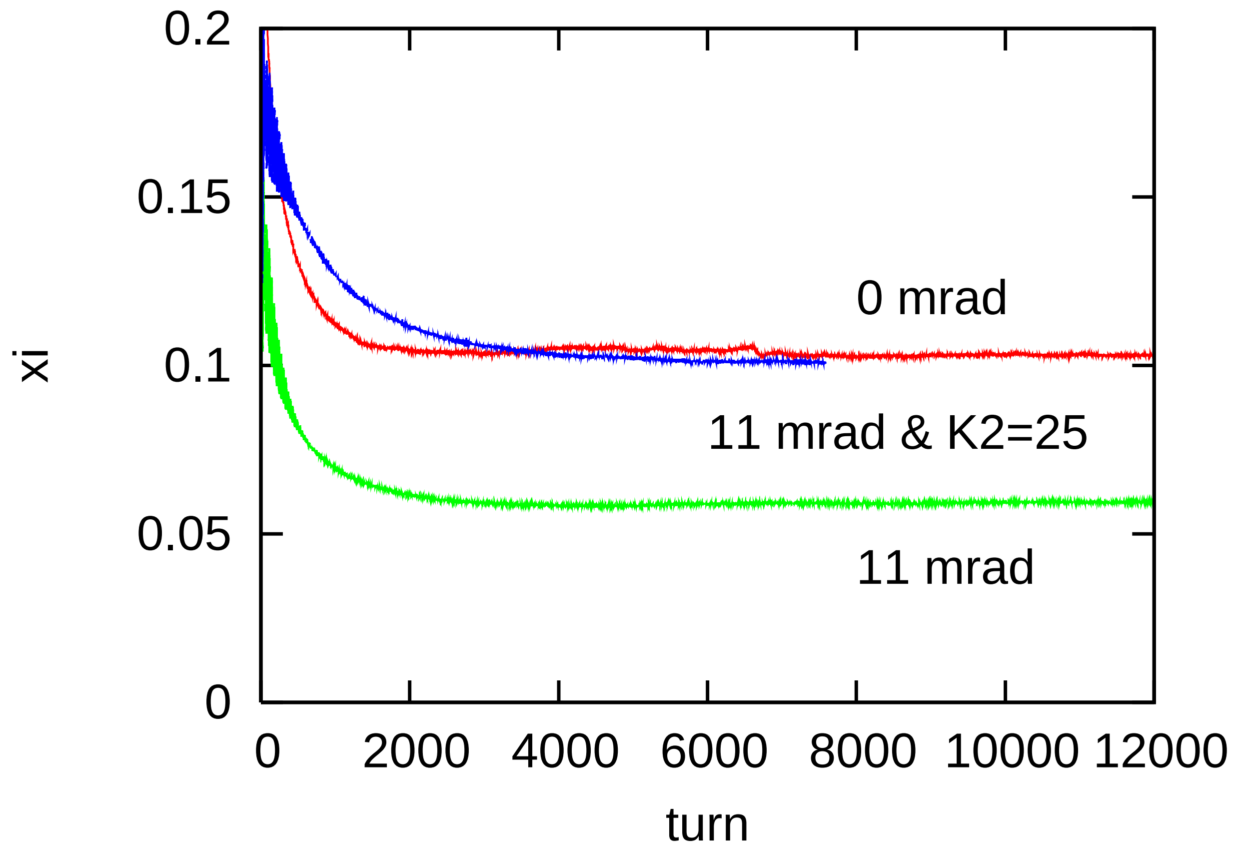
- Dynamic beta, and emittance

$$\lim_{\nu_x \to 0.5+} \left\langle x^2 \right\rangle < \sigma_{x,0}^2 \qquad \lim_{\nu_x \to 0.5+} \left\langle p_x^2 \right\rangle = \infty$$

- Choice of optimum ν_x

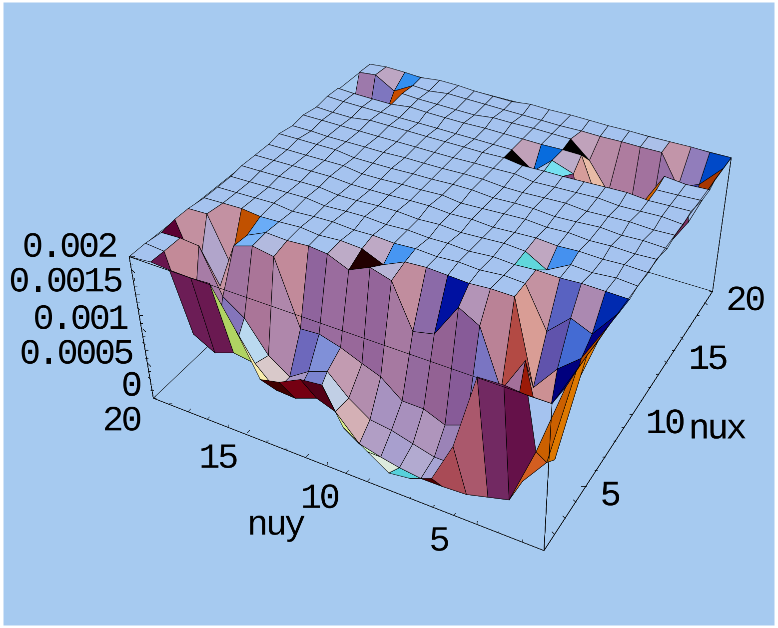
Crab waist for KEKB

- H=25 x p_y^2.
- Crab waist works even for short bunch.



Sextupole strength and diffusion rate

K2=20



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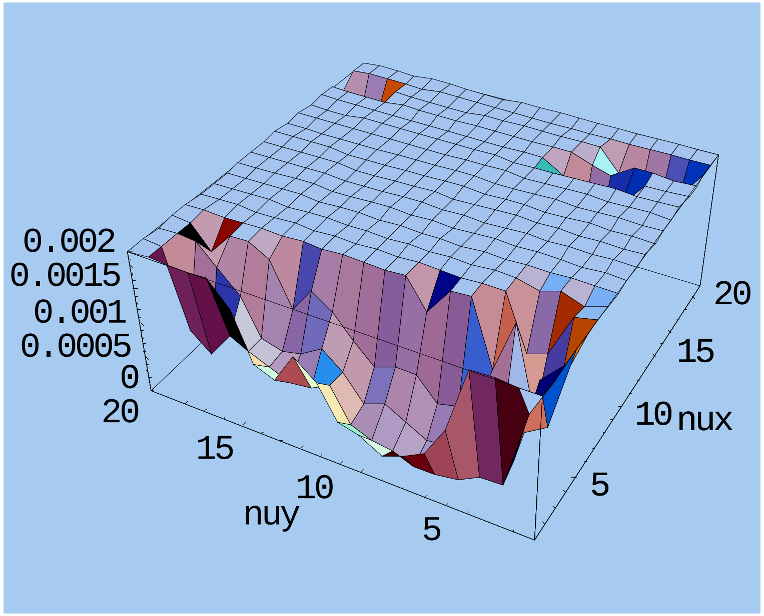

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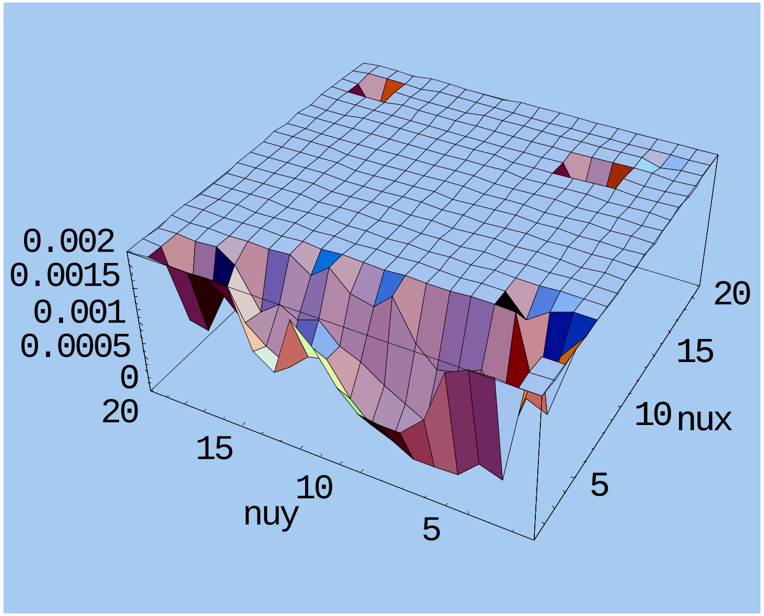

Why the sextupole works?

- Nonlinear term induced by the crossing angle may be cancelled by the sextupole.

- Crab cavity

$$\exp(-:F:) = \exp(-:\theta p_x z:)\exp(:\theta p_x z:) = 1$$

- Crab waist

$$\exp(-:F:) = \exp(-:\theta p_x z:)\exp(-K_2 :xp_y^2:) = \ldots$$

Need study

$$\exp(:F:)M_{BB}\exp(-:F:)$$

Conclusion

- Crab-headon

  Lpeak=8×10^{35}. Bunch length 2.5mm is required for L=10^{36}.

- Small beam size (superbunch) without crab-waist.

  Hard parameters are required ε_x=0.4nm β_x=1cm β_y=0.1mm.

- Small beam size (superbunch) with crab-waist.

  If a possible sextupole configuration can be found, L=10^{36} may be possible.
- Crab waist scheme is efficient even for shot bunch scheme.