Precision measurements of R and their uses

J. Swain and L. Taylor
Northeastern University, Boston, Mass.

March 1999, SLAC
Workshop on the τ-charm Factory

Introduction

A reminder that there are good reasons to measure

$$R = \frac{\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{hadrons})}{\sigma(\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-)}$$

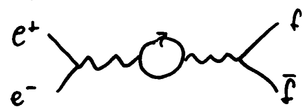
as a function of centre-of-mass energy.

- **Applications to radiative corrections and physics at different energies :**
  - $\alpha(M_Z^2)$
  - **Muon g-2**
- **Tests of CVC and QCD**
- **Running of α in the τ-charm region**

Radiative Corrections

Typically one has to calculate diagrams like:

To go into calculations like:



But since we don't know how to do strong-interaction physics, use the optical theorem to connect (the imaginary part of) this amplitude to measurable processes:

$$\mathrm{Im}\Pi(s) = -\frac{\alpha}{3}R(s)$$

Analyticity gives us the full amplitude.

Radiative Corrections

$$\Delta\alpha_{\rm had}(M_Z^2) = -\frac{\alpha(0)M_Z^2}{3\pi}\,\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds\,\frac{R(s)}{s(s-M_Z^2)-i\epsilon}$$

$$a_\mu^{\rm had} = \frac{\alpha^2(0)}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\,\frac{K(s)}{s}R(s)$$

where

$$K(s) = x^2\left(1-\frac{x^2}{2}\right) + (1+x)^2\left(1+\frac{1}{x^2}\right) \times\left(\ln(1+x)-x+\frac{x^2}{2}\right) + \frac{(1+x)}{(1-x)}x^2\ln x\ ,$$

Current Uncertainties in $\sin^2\theta_W$

☛ **Recall that $\sin^2\theta_W$ as measured from precision electroweak data, plus M_Z and M_W is overconstrained – a test of consistency ! (but note the Higgs)**

From Davier, LAL 98-97 and TAU98:

Table 1: Uncertainties of the electroweak input expressed in terms of $\Delta\sin^2\theta_W$ ($\times 10^{-4}$). Downward arrows indicate future experimental improvement.

Input	$\Delta\sin^2\theta_W$	Uncertainty/Source
Exp.	18	(LEP+SLD)[1] ↓
$\alpha(M_Z)$	23	$\Delta\alpha^{-1}(M_Z^2) = 0.09$ [2]
m_t	15	$\Delta m_t = 5.0$ GeV (CDF+D0)[3] ↓
Theory	5-10	2-loop EW prediction[4]
M_H	150	65 – 1000 GeV

1) D. Karlen, talk at ICHEP, Vancouver, 1998.

2) Eidelman and Jegerlehner, Z. Phys. C67 (1995) 585.

3) R. Partridge, talk at ICHEP, Vancouver, 1998.

4) G. Degrassi, P. Gambino, M. Passera, A. Sirlin, Phys. Lett. B418 (1998) 209; G. Degrassi, talk at Zeuthen Workshop on Elementary Particle Theory (1998), hep-ph/9807293.

Error Sources Today

From Davier, LAL 98-97 and TAU98:

Table 2: Contributions to $\Delta\alpha_{\mathrm{had}}(M_Z^2)$, a_μ^{had} and to a_e^{had} from the different energy regions. The subscripts in the first column give the quark flavours involved in the calculation.

Energy (GeV)	$\Delta\alpha_{\mathrm{had}}(M_Z^2) \times 10^4$	$a_\mu^{\mathrm{had}} \times 10^{10}$	$a_e^{\mathrm{had}} \times 10^{14}$
$(2m_\pi - 1.8)_{uds}$	$56.36 \pm 0.70_{\mathrm{exp}} \pm 0.18_{\mathrm{th}}$	$634.3 \pm 5.6_{\mathrm{exp}} \pm 2.1_{\mathrm{th}}$	$173.67 \pm 1.7_{\mathrm{exp}} \pm 0.6_{\mathrm{th}}$
$(1.8 - 3.700)_{uds}$	$24.53 \pm 0\ 28_{\mathrm{th}}$	$33.87 \pm 0.46_{\mathrm{th}}$	$8.13 \pm 0.11_{\mathrm{th}}$
$\psi(1S, 2S, 3770)_c + (3.7 - 5)_{udsc}$	$24.75 \pm 0.84_{\mathrm{exp}} \pm 0.50_{\mathrm{th}}$	$14.31 \pm 0.50_{\mathrm{exp}} \pm 0.21_{\mathrm{th}}$	$3.41 \pm 0.12_{\mathrm{exp}} \pm 0.05_{\mathrm{th}}$
$(5 - 9.3)_{udsc}$	$34.95 \pm 0.29_{\mathrm{th}}$	$6.87 \pm 0.11_{\mathrm{th}}$	$1.62 \pm 0.03_{\mathrm{th}}$
$(9.3 - 12)_{udscb}$	$15.70 \pm 0.28_{\mathrm{th}}$	$1.21 \pm 0.05_{\mathrm{th}}$	$0.28 \pm 0.02_{\mathrm{th}}$
$(12 - \infty)_{udscb}$	$120.68 \pm 0.25_{\mathrm{th}}$	$1.80 \pm 0.01_{\mathrm{th}}$	$0.42 \pm 0.01_{\mathrm{th}}$
$(2m_t - \infty)_t$	$-0.69 \pm 0.06_{\mathrm{th}}$	≈ 0	≈ 0
$(2m_\pi - \infty)_{udscbt}$	$276.3 \pm 1.1_{\mathrm{exp}} \pm 1.1_{\mathrm{th}}$	$692.4 \pm 5.6_{\mathrm{exp}} \pm 2.6_{\mathrm{th}}$	$187.5 \pm 1.7_{\mathrm{exp}} \pm 0.7_{\mathrm{th}}$

Examples of R near the ρ

From Davier, LAL 98-97 and TAU98:

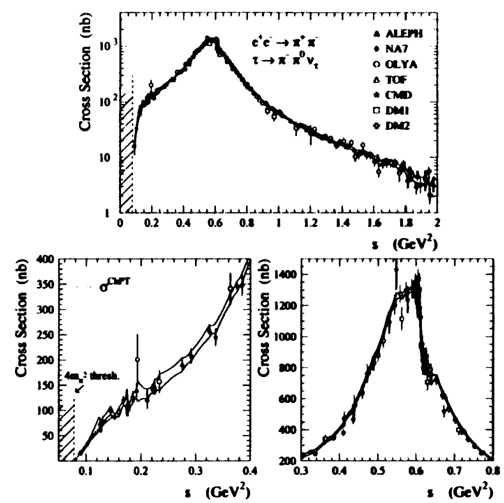


Figure 1: Two-pion cross section as a function of the c.m. energy-squared. The band represents the result of the averaging procedure described in the text within the diagonal errors. The lower left hand plot shows the chiral expansion of the two-pion cross section used (see Alemany, Davier, and Höcker, Europ. J. Phys. **C2** (1998) 123.)

R as we know it today

From Davier, LAL 98-97 and TAU98:

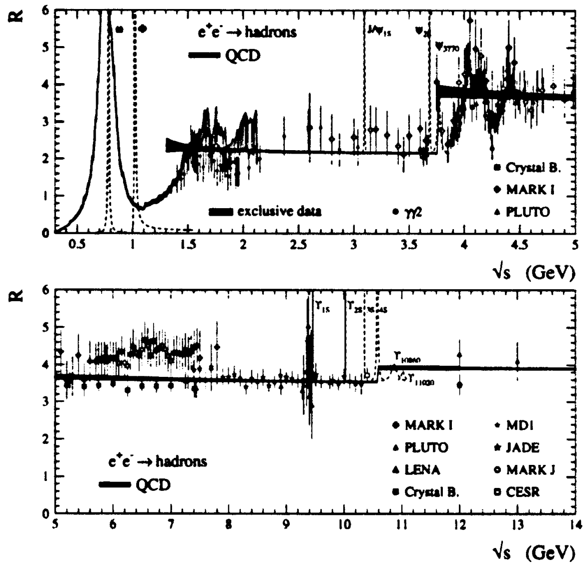
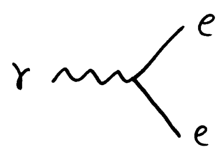


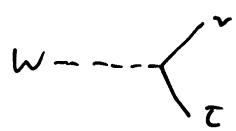
Figure 2: Inclusive hadronic cross section ratio in e^+e^- annihilation versus the c.m. energy $\sqrt{s}$. Additionally shown is the QCD prediction of the continuum contribution as explained in the text. The shaded areas indicate regions were experimental data are used for the evaluation of $\Delta\alpha_{\rm had}(M_Z^2)$ and $a_\mu^{\rm had}$ in addition to the measured narrow resonance parameters. The exclusive e^+e^- cross section measurements at low c.m. energies are taken from DM1,DM2,M2N,M3N,OLYA,CMD,ND and τ data from ALEPH (see Alemany *et al. (op. cit.)* for detailed information).

CVC and QCD Tests

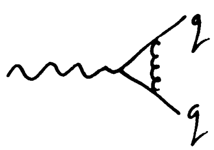
Two assumptions used to date and tested to the $\sim$ 1% level

☛ Relate the hadronic vector current to EM current so we can use τ decays to simulate low energy e^+e^- annihilation





☛ QCD calculations used now in tau-charm region – compare direct measurements to use of theory.



An Idea

It would be nice to be able to see α run. So far we have

- $\alpha \sim 1/128$ at the Z
- CKM unitarity at low energies (Sirling, Phys. Rev. Lett. **72** (1994) 1786.)
- running of α_s (including the $\alpha_s(m_\tau)$)

Can we see clear evidence of running in one place?

☛ **Note that most of the "running" happens at steps when new channels open up – look at $\sigma(e^+e^- \to \mu^+\mu^-)$**

☛ **Back of the envelope:**

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{86.8\mathrm{nb}}{\mathrm{E_{cm}(GeV)^2}}$$

For $10\mathrm{fb}^{-1}$ we get $\sim 5 \times 10^6\ \mu^+\mu^-$ per year at 4GeV

☛ **Implies per mil precision on σ, and per mil precision on α.**

$$\frac{\delta\sigma}{\sigma} \sim \frac{\delta(\alpha^2)}{\alpha^2} \sim 2\frac{\delta\alpha}{\alpha}$$

More studies needed, but looks like a decent chance to see α run in one place ("improved Born approximation") !

... and there's more ... !

- $e^+e^- \to \overset{(-)}{p} X_s$ asymmetries

  Brodsky & Ma 1996

- θ dist'ns of $\tau^+\tau^-$ & "$q\bar{q}$" near threshold : α_s & $\frac{(g-2)}{2}$ for τ

} not really R, but in here anyway.

Brodsky, Hoang, Kühn, Teubner 1995

- QCD tests from R
  "Hypothetical τ's"
  Brodsky, Peláez, Toumbas 1998

- QCD tests from R (more!)
  (DIS & R)
  Brodsky, Gabadadze, Kataev, Lu 1995

Conclusions

- A tau-charm factory can dramatically improve our knowledge of R in the tau-charm region.
- Knowing R better is useful to lots of people – a τcF isn't a stand-alone experiment, but rather part of the integrated whole of particle physics
- Possibility to observe running α, α_s...
- And lots more!