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An Overview of $D^0\bar{D}^0$ Mixing Search Techniques: Current Status and Future Prospects *

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LBL

Abstract

The search for $D^0\bar{D}^0$ mixing may carry a large discovery potential for new physics since the $D^0\bar{D}^0$ mixing rate is expected to be small in the Standard Model. The past decade has seen significant experimental progress in sensitivity. This paper discusses the techniques, current experimental status, and future prospects for the mixing search. Some new ideas, applicable to future mixing searches, are introduced. In this paper, the importance of separately measuring the decay rate difference and the mass difference of the two CP eigenstates (in order to observe New Physics) has been emphasized, since the theoretical calculations for long distance effects are still plagued by large uncertainties.

1 Introduction

Particle-antiparticle mixing has always been of fundamental importance in testing the Standard Model and constraining new physics. This is because mixing is responsible for the small mass differences between the mass eigenstates of neutral mesons. Being a flavor changing neutral current (FCNC) process, it often involves heavy quarks in loops. Such higher order processes are of great interest since the amplitudes are sensitive to any weakly-coupling quark flavor running around the loop. Historically, $K^0\bar{K}^0$ mixing is the rare (FCNC) process that has been experimentally examined in the greatest detail. It has been amply demonstrated that in spite of many inherent uncertainties of strong interaction physics, the Standard Model predicts the correct phenomenology of the $K^0\bar{K}^0$ mixing. In fact, based on the calculation of the K_L - K_S mass difference, Gaillard and Lee [1] were able to estimate the value of the charm quark mass before the discovery of charm. Moreover, $B^0\bar{B}^0$ mixing gave the first indication of a large top quark mass.

Although $D^0\bar{D}^0$ mixing is very similar to $K^0\bar{K}^0$ and $B^0\bar{B}^0$ mixing, as all are FCNC processes, there are significant differences which make $D^0\bar{D}^0$ mixing a possible unique place

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$D^0\bar{D}^0$ mixing

- CP eigenstates: D_1 and D_2

		mass	decay rate
even	$\|D_1>=\frac{1}{\sqrt{2}}\,(\,\|D^0> \;+\|\bar{D}^0>\,)$	m_1	γ_1
odd	$\|D_2>=\frac{1}{\sqrt{2}}\,(\,\|D^0> \;-\|\bar{D}^0>\,)$	m_2	γ_2

- Define $\Delta m \equiv m_2 - m_1$, and $\Delta\gamma \equiv \gamma_2 - \gamma_1$, one can characterize $D^0\bar{D}^0$ mixing in terms of the two dimensionless variables ($\gamma \equiv (\gamma_2+\gamma_1)/2$):

$$x \equiv \frac{\Delta m}{\gamma}, \quad y \equiv \frac{\Delta\gamma}{2\gamma}$$

- Assuming $x, y \ll 1$ and CP invariance, we have

$$\mathrm{R_{mixing}} \equiv \frac{BR(D^0 \to \bar{D}^0 \to \bar{f})}{BR(D^0 \to f)} = \frac{1}{2}(x^2+y^2)$$

- In general (particle-antiparticle mixing):

$$\mathrm{R_{mixing}} \sim \frac{x^2+y^2}{2+x^2-y^2}$$

$$\left[\begin{array}{ll} K^0\bar{K}^0: & x \approx 0.5 \\ & y \simeq 1 \\ R_{mixing} & \approx 1 \end{array}\right.$$

$$\left[\begin{array}{l} B_d\bar{B}_d: \quad x \simeq 0.7 \gg y \\ R_{mixing} = \frac{x^2}{2+x^2} \end{array}\right.$$

$$\left[\begin{array}{l} B_s\bar{B}_s: \quad x \gg 1, \; y \sim 0.1 \\ R_{mixing} \simeq 1 \end{array}\right.$$

- Mixing caused by decay rate difference:

$$|D^0\rangle \xrightarrow{t} \frac{1}{\sqrt{2}}\left(|D_1\rangle + |D_2\rangle\right)$$

$t=0$

$D^0\bar{D}^0$ mixing search techniques

~23 years

Right after the discovery of the D^0 meson at SPEAR (1976), experimentalists began to search for $D^0\bar{D}^0$ mixing

- The **Semileptonic** method: $D^0 \to \bar{D}^0 \to X\ell\bar{\nu}$
  1. Theoretically unambiguous (no DCSD)
  2. Usually suffers from large backgrounds (missing neutrino)
  3. One exception (at Tau Charm Factory): $e^+e^- \to D^-D^{*+} \to (K^+\pi^-\pi^-)(K^+l^-\nu)\pi_s^+$
- The **hadronic** method: $D^0 \to \bar{D}^0 \to K^+\pi^-(X)$
  1. Significantly less background
  2. Need to distinguish between $D^0\bar{D}^0$ mixing and DCSD

$$\begin{tikzcd} & \bar{D}^0 \arrow[dr, "\text{CFD}"] & \\ D^0 \arrow[ur, "\text{mixing}"] \arrow[rr, bend right, "\text{DCSD}"'] & \text{interfere} & K^+\pi^- \end{tikzcd}$$

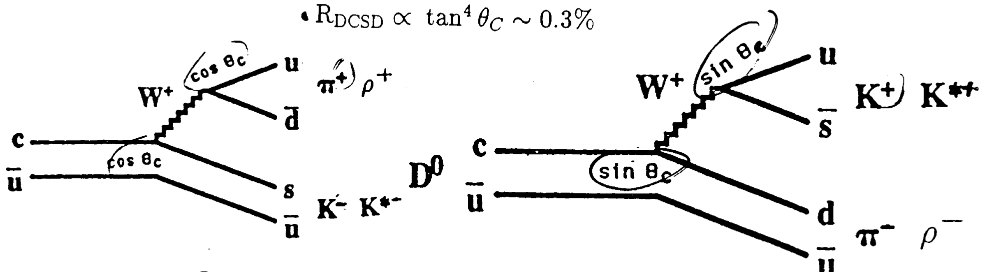
3. • One exception (at Tau – Charm Factory)

Doubly Cabibbo Suppressed Decays (DCSD): $D^0 \overset{DCSD}{\rightarrow} K^+\pi^-$, $K^+\pi^-\pi^0$ ...

mixing $\searrow \bar{D}^0 \nearrow$ CFD

- Cabibbo Favored Decay (CFD) vs. DCSD

  - $R_{DCSD} \propto \tan^4\theta_C \sim 0.3\%$



- CFD

DCSD

- $R_{DCSD} = |\eta|^2 \sim (2-3)tan^4\theta_C$ where

$$\eta = \frac{Amp(D^0 \to K^+\pi^-)}{Amp(\bar{D}^0 \to K^+\pi^-)}$$

- DCSD are interesting on their own: can be used to extract CKM angle γ in $B^- \to D^0(\bar{D}^0)K^-$ modes (Atwood, Dunietz, Soni, PRL 78, 1997)

Hadronic Method: $D^0 \to K^+\pi^-$

wrong sign $\ulcorner\!\rightarrow K^+\pi^-$ $\qquad\qquad$ $\ulcorner\!\rightarrow K^+\pi^+$

- Use $D^{*+} \to D^0\pi_s^+$ and $D^{*-} \to \bar{D}^0\pi_s^-$

right sign $\llcorner\!\rightarrow K^-\pi^+$ $\qquad\qquad$ $\llcorner\!\rightarrow K^+\pi^-$

- The charge of the π_s can be used to tag whether a D^0 or $\bar{D}^0$ was produced
- Small Q value of the $D^{*+} \to D^0\pi_s^+$ decay $\Longrightarrow$ good mass resolution (low background)
  $\Delta M = M_{K\pi\pi_s} - M_{K\pi} - M_{\pi_s} \sim 6\ \mathrm{MeV}$
- The right sign signal can be used to provide a model-independent normalization
- Mixing has a unique attribute, namely the decay time-dependence, which can be used to distinguish mixing form DCSD
- Potentially small mixing signature could be enhanced by DCSD through interference

Mixing Search: Hadronic Methods

- Decay time-dependence

$$D^0 \to K^+\pi^-$$

- Resonance substructure

$$\begin{tikzcd} & K^+\pi^-\pi^0 \\ D^0 \arrow[ur] \arrow[dr] & \\ & K^+\pi^-\pi^+\pi^- \end{tikzcd}$$

- Quantum statistics

$$\begin{tikzcd} & & & K^+\pi \\ e^+e^- \arrow[r] & 1r \arrow[r] & D\bar{D}^0 \arrow[ur] \arrow[dr] & \\ & & & K^+\pi \end{tikzcd}$$

- Use CP eigenstates to measure $y \equiv \frac{\Delta\gamma}{2\gamma}$

$$D^0 \to K^+K^-, \pi^+\pi^- \ldots$$

- Phenomenology
- status
- prospects

- advantage

&

- limitations

Time dependence of $D^0 \to K^+\pi^-$

- A state that is purely $|D^0>$ at $t = 0$ will evolve to $|D^0_{\text{phys}}(t)>$:

$$I(\,|D^0_{\text{phys}}(t)> \to f\,) = |\bar{a}(f)|^2 \, \underline{\underline{|\, \frac{1}{2}(i\delta m + \frac{1}{2}\delta\gamma)\, t + \eta \,|^2}} \, e^{-\gamma t}$$

- Look at the equation in a different way $(\underline{\gamma t \to t})$:

• $x = \frac{\delta m}{\gamma}$

• $y = \frac{\delta\gamma}{2\gamma}$

$$I(|D^0_{\text{phys}}(t)> \to f) = |\bar{a}(f)|^2 \,|\, \frac{ix+y}{2}\, t + \eta \,|^2 \, e^{-t}$$

$$|\, \sqrt{\mathrm{R_{mixing}}/2}\ t + \sqrt{\mathrm{R_{DCSD}}}\ e^{i\phi} \,|^2$$

• $R_{DCSD} = |\eta|^2$

• $R_{mixing} = \frac{x^2+y^2}{2}$

$$\phi = Arg(ix+y) - \underline{Arg(\eta)}$$

✓ $$\left[\mathrm{R_{DCSD}} + \underline{\underline{\sqrt{2\mathrm{R_{mixing}}\mathrm{R_{DCSD}}}\ t\ \cos\phi}} + \frac{1}{2}\mathrm{R_{mixing}}\ t^2\right] e^{-t}$$

DCSD　　interference term　　mixing

$$\eta = \frac{Amp(D^0 \to K^+\pi^-)}{Amp(\bar{D}^0 \to K^+\pi^-)}(\underline{\frac{p}{q}})$$

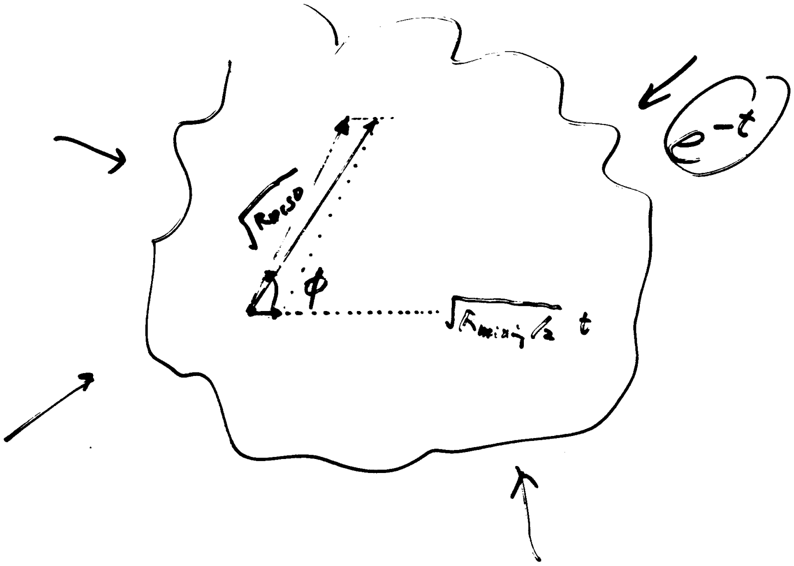
$$\underline{Arg(\eta)} = \delta\ (FSI) + \phi_m$$

δ (FSI) ↓ unknown

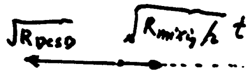
ϕ_m ↓ $\not{CP}$ phase ≈ 0 in D system (SM)

$$I(|D^0_{phys}(t)\rangle \to f) \propto \left|\sqrt{R_{mixing}/2}\; t + \sqrt{R_{DCSD}}\, e^{i\phi}\right|^2 e^{-t}$$

$$\to \left[R_{DCSD} + \sqrt{2R_{mixing}R_{DCSD}}\; t\cos\phi + \tfrac{1}{2}R_{mixing}t^2\right]e^{-t}$$

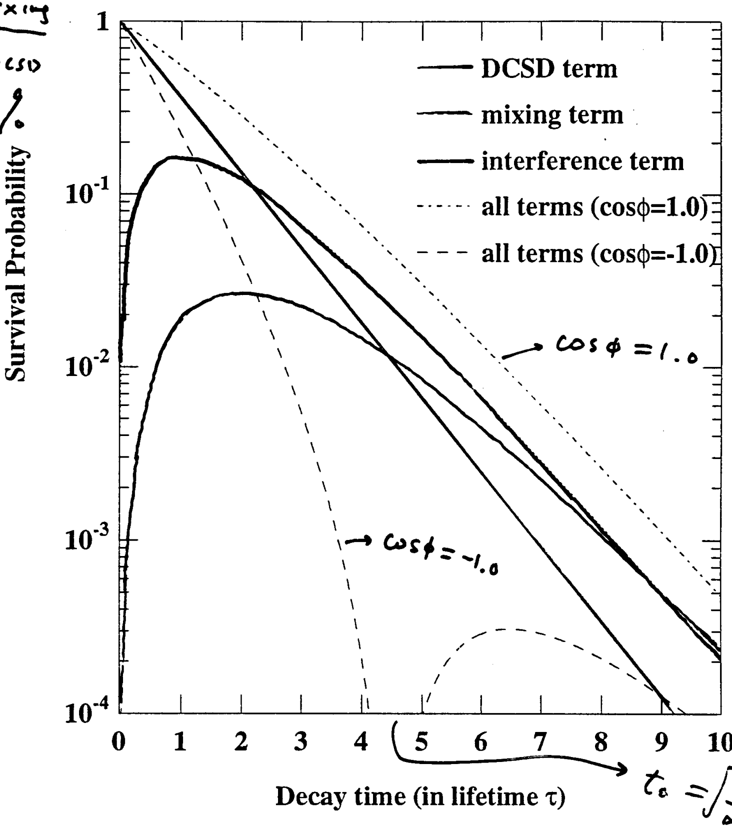


$\cos\phi = -1$ case:



$$I(t) \propto (\underline{R_{DCSD}} + \underline{\sqrt{2R_{DCSD}R_{mixing}}}\, t\cos\phi + \underline{\frac{1}{2}\, t^2 R_{mixing}})\, e^{-t}$$

$$\alpha = \frac{R_{mixing}}{R_{DCSD}} = 10\%$$



$$t_0 = \sqrt{\frac{2}{\alpha}} = \sqrt{\frac{2R_{DCSD}}{R_{mixing}}}$$

- our ability to observe the signature of a potentially small mixing signal depends on the number of $D^0 \to K^+\pi^-$ events we will have

Observing $D^0 \to K^+\pi^-$ would be an important step

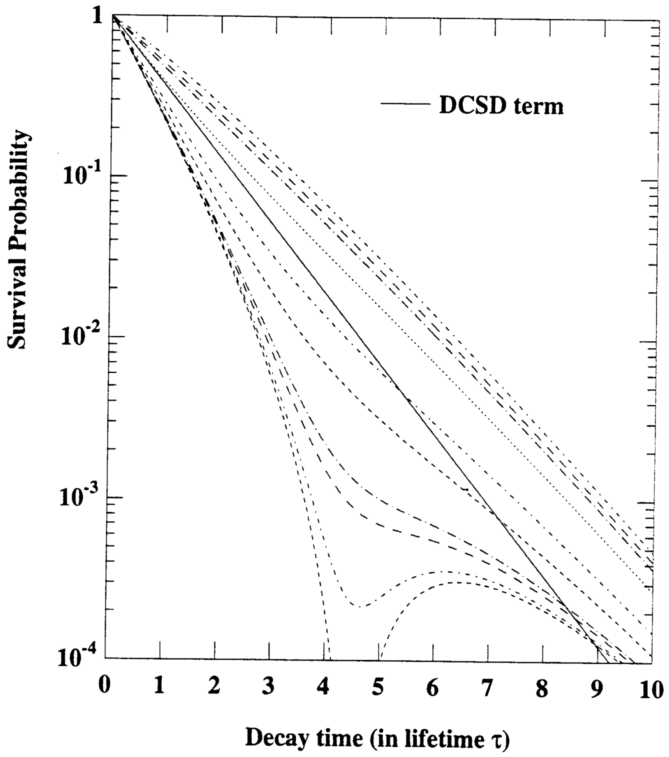


Figure 6: The decay time dependence of DCSD and mixing with $\alpha = R_{\mathrm{mixing}}/R_{\mathrm{DCSD}} = 10\%$. For different $cos\phi$ values: from bottom to top, $cos\phi = -1.0, -0.99, -0.96, -0.94, -0.80, -0.6, 0.0, 0.5, 0.7, 1.0$. The solid line is the DCSD term, as a reference line.

Implications for the design of detectors

$D^{*+} \longrightarrow D^0 \pi_s^{\oplus}$, $D^0 \hookrightarrow K^+\pi^-$

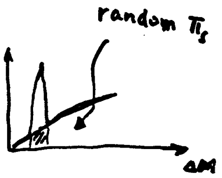
- Excellent vertexing capabilities

  Fixed Target: $\frac{L}{\sigma} \sim 8\text{-}10$. BF: $\frac{L}{\sigma} \sim 3$

- Good acceptance at short decay times

  interference term peaks at $t=1$

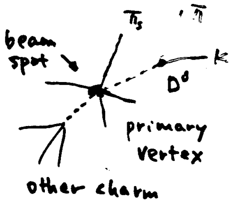
- Low background around the primary vertex $\Longrightarrow$ low nasty background from random slow π_s^+ combined with real D^0s $\Longrightarrow$ the figure of merit for the background is:

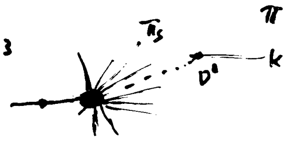


$$\frac{B}{S} \simeq \underbrace{\left[\frac{1}{S}\frac{dB}{dQ}\right]}_{\text{bkg density } \rho} (2\sigma_Q)$$

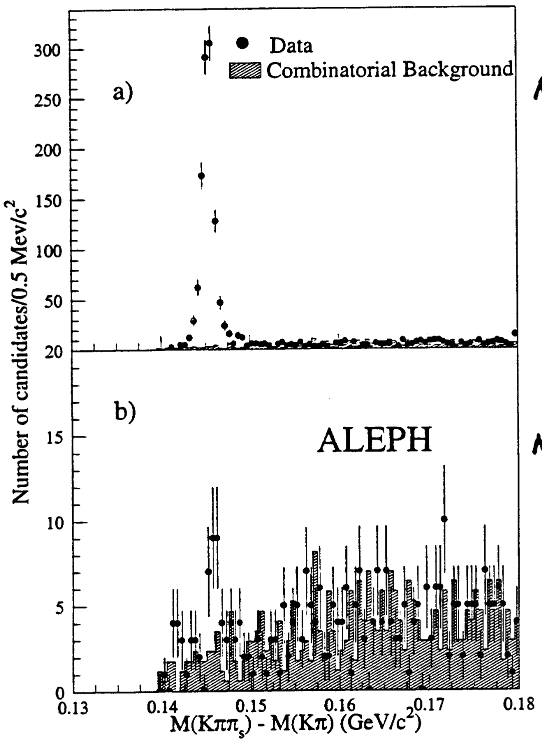
S: right sign signal
B: wrong sign bkg
Q: D^*-D mass diff.
σ_Q: mass diff. resolution

	Examples.	ρ	σ_Q (MeV)	$\frac{B}{S}$
e^-	ARGUS :	0.002/MeV	0.61	$\sim 2.5\times10^{-3}$
	CLEO1.5-2:	0.002/MeV	0.7	$\sim 3\times10^{-3}$
	CLEO2.5-3: (BF.)	"	$\lesssim 0.3$	$\lesssim 1\times10^{-3}$
	ALEPH:	0.004/MeV	0.65	$\sim 5\times10^{-3}$
T:	(E691, E831 photoproduction)	~ 0.004/MeV	~ 1	$\sim 8\times10^{-3}$
	(E791 hedroproduction)	Worse		Worse





HC LHC TEVATRON ? ideas for improvement ...



$N = 1038.8 \pm 32.5$
± 4.4

$N = 19.1 \pm 6.1 \pm 3.5$

Figure 1: Mass-difference distribution (a) for candidates of the decay channel $D^{*+} \rightarrow D^0\pi_s^+$, $D^0 \rightarrow K^-\pi^+$ and (b) candidates of the decay channel $D^{*+} \rightarrow D^0\pi_s^+$, $D^0 \rightarrow K^+\pi^-$. The dots with error bars are data while the hatched histogram represents the distribution of the combinatorial background.

R_{mixing} :

$< 0.92\%$ $\cos\phi = 0$

$< 0.96\%$ $\cos\phi = 1$

$< 3.6\%$ $\cos\phi = -1$

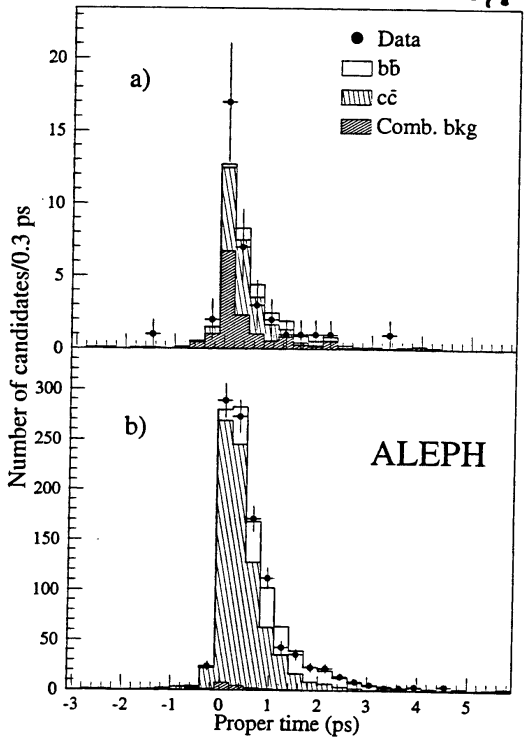


Figure 3: Proper time distribution for (a) the $D^0 \to K^+\pi^-$ candidates, and (b) for the $D^0 \to K^-\pi^+$ candidates. The dots with error bars are the data. The histograms are the contributions of $c\bar{c}$, $b\bar{b}$ and combinatorial background events resulting from the unconstrained fit when no interference is assumed.

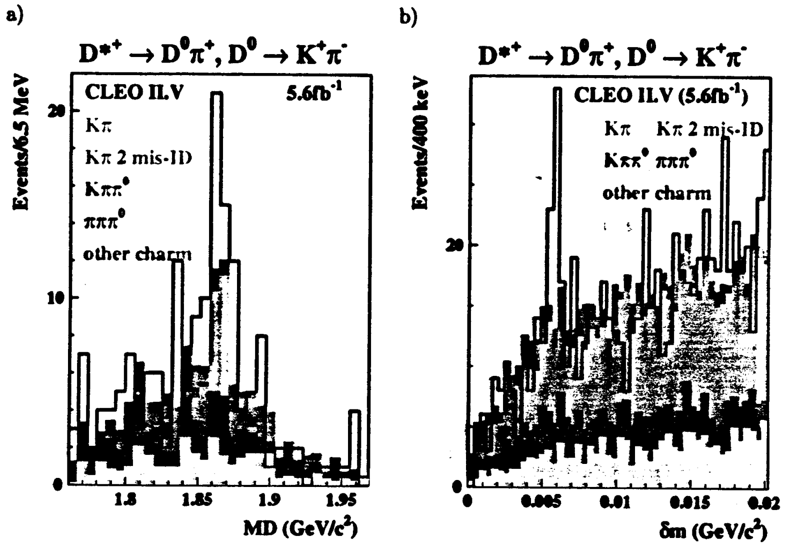
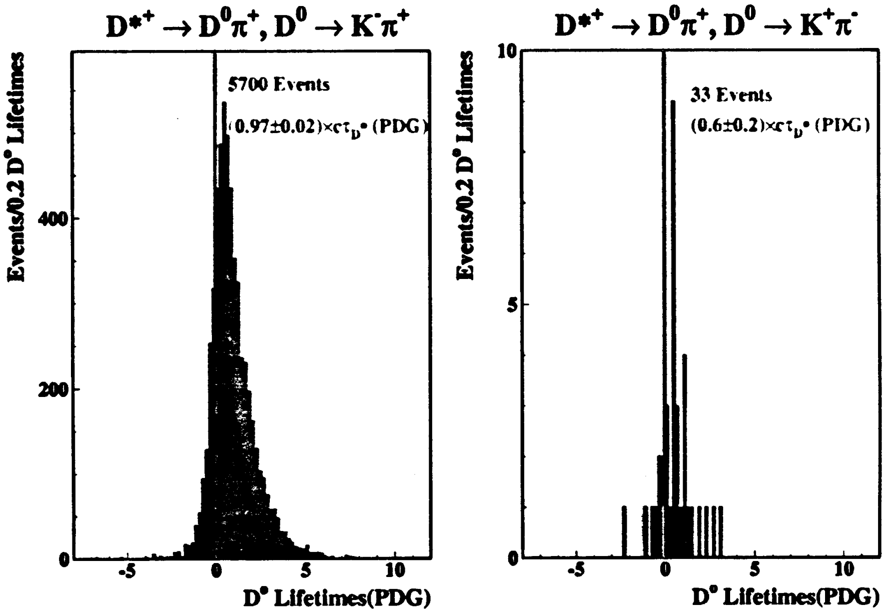


FIG. 20. a) Background composition determined from the generator information for a)projection on to $M_{K\pi}$ axis b) projection on to δm axis mass for $D^{*+} \to D^0\pi_s^+, D^0 \to K^+\pi^-$.

"Back of the envelope" Lifetimes



Mode	N_{fit}	τ (τ_{D^o})
Signal	17.3 ± 5.4	?
Real D^o	8.9 ± 0.8	1.0
Other	5.8 ± 0.0	0.0

Mode	τ (τ_{D^o})
DCSD	1.0
Mixing	3.0

$$\tau_{\text{wrong sign}} = (0.65 \pm 0.33(\text{stat}) \pm 0.5(\text{syst})) \times \tau_{D^o}$$

$R_{mixing} < 0.85\%$ 1996

Fermilab E791 hadronproduction

A search for $D^0 - \overline{D}^0$ mixing and doubly-Cabibbo-suppressed decays of the D^0 in hadronic final states

$D^{*+} \to D^0 \pi_s^+$, $\hookrightarrow K^-\pi^+$ right sign

$D^{*+} \to D^0 \pi_s^+$, $\hookrightarrow K^-\pi^+\pi^-\pi^+$

$D^{*+} \to D^0 \pi_s^+$, $\hookrightarrow K^+\pi^-$ wrong sign

$D^{*+} \to D^0 \pi_s^+$, $\hookrightarrow K^+\pi^-\pi^+\pi^-$

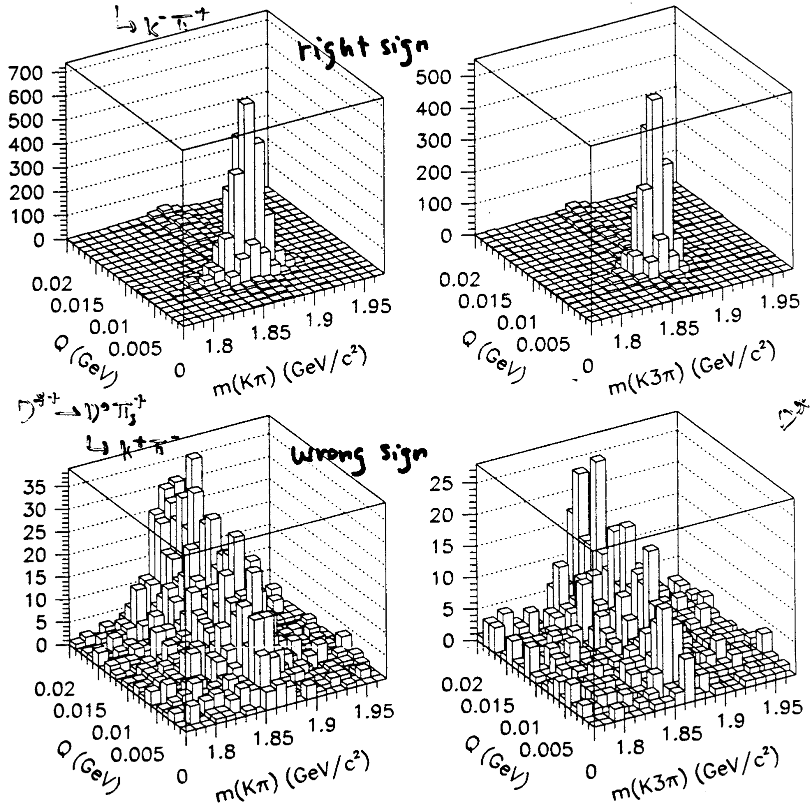


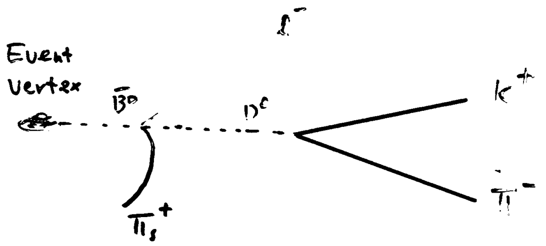
FIG. 1. Plots of Q (defined in the text) versus the candidate D mass for right-sign $D \to K\pi$ (top-left), right-sign $D \to K\pi\pi\pi$ (top-right), wrong-sign $D \to K\pi$ (bottom-left), and wrong-sign $D \to K\pi\pi\pi$ (bottom-right). Clean signals are apparent in both right-sign plots. In all four plots, the bands of events at $m(K\pi)$, $m(K\pi\pi\pi) \approx 1.87$ GeV/c^2 are due to real D decays combining with random pions to give false D^* candidates.

One possible way to improve $\frac{B}{S}$: at Fixed Target Exps. or Hadron Machine...

$B_r \sim 4.4\%$

- Use $\bar{B}^0 \to D^{*+} l^- \nu$ followed by $D^{*+} \to D^0 \pi_s^+$

$\hookrightarrow K^+\pi^-$



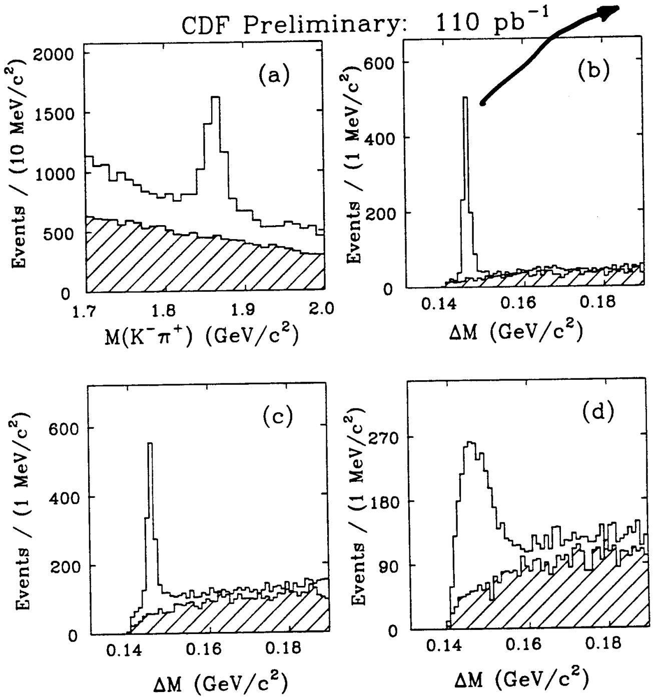
- The primary D^{*+} decay vertex can be determined by the l^- and π_s^+

- The background level around the primary vertex is intrinsically low $\Longrightarrow$ low $\frac{B}{S}$

THE CDF II DETECTOR

TECHNICAL DESIGN REPORT

FERMILAB-Pub-96/390-E



$\bar{B} \to D^{*+} \ell^- \nu X$
↳ $D^0 \pi^+$
↳ $K^- \pi^+$

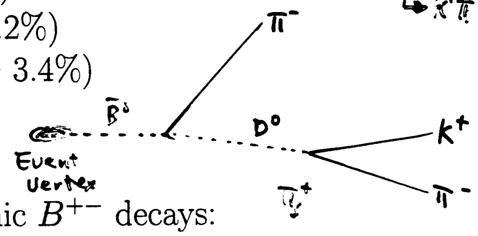
Figure 2.59: Charm signals reconstructed in association with a high-p_T lepton.
(a): Signal for $\bar{B} \to D^0 \ell^- \bar{\nu} X; D^0 \to K^- \pi^+$ (+ c.c.)
(b): Signal for $\bar{B} \to D^{*+} \ell^- \bar{\nu} X; D^{*+} \to D^0 \pi^+; D^0 \to K^- \pi^+$ (+ c.c.)
(c): Signal for $\bar{B} \to D^{*+} \ell^- \bar{\nu} X; D^{*+} \to D^0 \pi^+; D^0 \to K^- \pi^+ \pi^- \pi^+$ (+ c.c.)
(d): Signal for $\bar{B} \to D^{*+} \ell^- \bar{\nu} X; D^{*+} \to D^0 \pi^+; D^0 \to K^- \pi^+ X$ (+ c.c.)
Mode (a) is dominated by B^- decays and modes (b) – (d) by $\bar{B}^0$ decays (ΔM is the mass difference between the $D^0 \pi^+$ and the D^0). Shaded histograms show wrong-charge combinations (*e.g.*, $\ell^- K^+$); in (a) these are scaled by 0.5 for display purposes.

Other possible ways to improve $\frac{B}{S}$: at Fixed Target Exps. or Hadron Machine...

just a thought!

- Maybe possible to use hadronic B^0 or $\bar{B}^0$ decays:
  $\bar{B}^0 \to D^{*+}\pi^-$ (BR ~ 0.3%)
  $\bar{B}^0 \to D^{*+}\pi^-\pi^0$ (BR ~ 1.5%)
  $\bar{B}^0 \to D^{*+}\pi^-\pi^+\pi^-$ (BR ~ 1.2%)
  $\bar{B}^0 \to D^{*+}\pi^-\pi^+\pi^-\pi^0$ (BR ~ 3.4%)
  ……
  followed by $D^{*+} \to D^0\pi_s^+$

$\bar{B}^0 \to D^{*+}\pi^-$
↳ $D^0\pi_s^+$
↳ $K^+\pi^-$



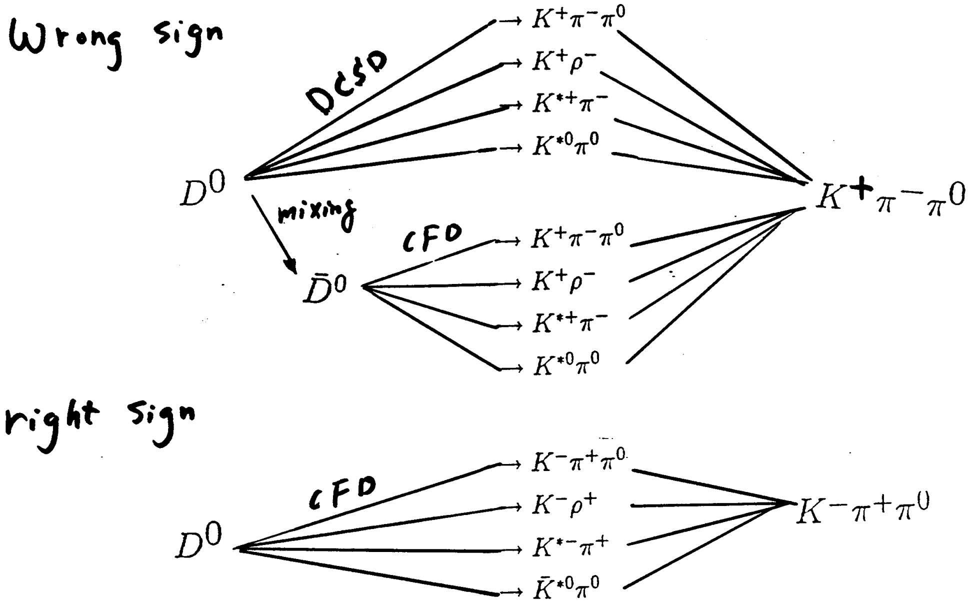
- Maybe possible to use hadronic B^{+-} decays:
  $B^- \to D^{*+}\pi^-\pi^-\pi^0$ (BR ~ 1.5%)
  $B^- \to D^{*+}\pi^+\pi^+$ (BR ~ 0.2%)
  ……
  followed by $D^{*+} \to D^0\pi_s^+$
- D^{*+} is moved out from the event vertex by the highly boosted B
- * B mass constrain can be used to reject background

- LHC: $> 10^{10}$ B produced $\Longrightarrow > 10^9$ $D^{*\pm} \to 10^7$?
  BF: $\sim 30\,fb^{-1}$ $\sim 10^7$ $D^{*\pm}$
-
- Similar ~~idea~~ case: $B_c^+ \longrightarrow B_s^0\pi^+(\rho^+)$…… B_s tagging

$K^+\pi^-\pi^+\pi^-$

Hadronic method: $D^0 \to K^+\pi^-\pi^0$

$D^{*+} \to D^0 \pi_s$


Wrong sign

DCSD

$D^0 \to K^+\pi^-\pi^0$

$D^0 \to K^+\rho^-$

$D^0 \to K^{*+}\pi^-$

$D^0 \to K^{*0}\pi^0$

mixing

$\bar{D}^0$

CFD

$\bar{D}^0 \to K^+\pi^-\pi^0$

$\bar{D}^0 \to K^+\rho^-$

$\bar{D}^0 \to K^{*+}\pi^-$

$\bar{D}^0 \to K^{*0}\pi^0$

$K^+\pi^-\pi^0$

right sign

CFD

$D^0 \to K^-\pi^+\pi^0$

$D^0 \to K^-\rho^+$

$D^0 \to K^{*-}\pi^+$

$D^0 \to \bar{K}^{*0}\pi^0$

$K^-\pi^+\pi^0$

$e^+e^- \xrightarrow{1\gamma} \psi''(3770) \rightarrow D^0\bar{D}^0 \rightarrow (K^-\pi^+)(K^-\pi^+)$

- $e_i(t) = e^{-im_it-\gamma_it/2}$ and $e_\pm(t) = (e_1(t) \pm e_2(t))/2$
- Time evolution for $|D^0\rangle$ or $|\bar{D}^0\rangle$
  $|D(t)\rangle = e_+(t)|D^0\rangle + e_-(t)|\bar{D}^0\rangle$
  $|\bar{D}(t)\rangle = e_-(t)|D^0\rangle + e_+(t)|\bar{D}^0\rangle$.
- For $D^0\bar{D}^0$ pair generated in the state $\underline{\underline{D^0\bar{D}^0 \ominus \bar{D}^0D^0}}$, time evolution: $|D(t)\bar{D}(t')\rangle - |\bar{D}(t)D(t')\rangle$
- Wrong sign amplitude $(K^-\pi^+)(K^-\pi^+)$:

$a = \langle K^-\pi^+|D^0\rangle$

$b = \langle K^-\pi^+|\bar{D}^0\rangle$
(DCSD)

CFD amp

DCSD amp

$$\mathcal{A}_w(t,t') = (e_+(t)e_-(t') - e_-(t)e_+(t'))(a^2 - b^2)$$

- Right sign amplitude $(K^-\pi^+)(K^+\pi^-)$:

$$\mathcal{A}_r(t,t') = (e_+(t)e_+(t') - e_-(t)e_-(t'))(a^2 - b^2)$$

- Wrong sign vs. right sign ratio R:

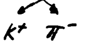
$$\mathrm{R} = \frac{\mathrm{N}(K^-\pi^+, K^-\pi^+) + \mathrm{N}(K^+\pi^-, K^+\pi^-)}{\mathrm{N}(K^-\pi^+, K^+\pi^-) + \mathrm{N}(K^+\pi^-, K^-\pi^+)} = \frac{\iint |\mathcal{A}_w(t,t')|^2\, dt\, dt'}{\iint |\mathcal{A}_r(t,t')|^2\, dt\, dt'}$$

- In taking the ratio, the amplitude term $(a^2 - b^2)$ drops out.
- Integrating over all times, one then obtains

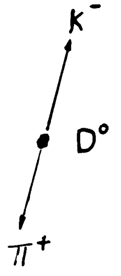
$$\mathrm{R} = (x^2 + y^2)/2 = \mathrm{R}_{\text{mixing}}$$

Comparison of e^+e^- experiments at $\psi''(3770)$ vs near $\Upsilon(4S)$

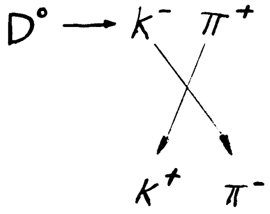
doubly misidentified $D^0 \to K^- \pi^+$ can be rejected kinematically ...



- At $\psi''(3770)$: D^0 is almost at rest



$P_K \simeq P_\pi$

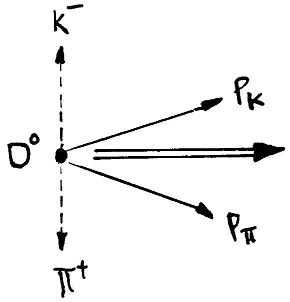


$M(K\pi) \equiv M(K \leftrightarrow \pi)$

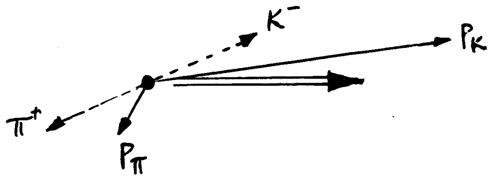
- Near $\Upsilon(4S)$: D^0 is highly boosted $\gamma_{D^0} \approx 2-3$

$\beta_{D^0} \approx 0.85 - 0.88$

$P_{D^0} \approx 3 - 5$ GeV



$P_K \simeq P_\pi$



$P_K \neq P_\pi$, $M(K\pi) \neq M(K \leftrightarrow \pi)$

in general

Hadronic Method : cont.

Use Quantum Statistics of the Production: $e^+e^- \to \psi''(3770) \to D^0\bar{D}^0$

- This is a good place for mixing search:

$$e^+e^- \to \psi'' \to D^0\bar{D}^0 \to (K^-\pi^+)(K^-\pi^+)$$

- $D^0\bar{D}^0$ is generated in a state of odd orbital angular momentum (L = 1) where DCSD contribution cancels out and only that of mixing survives —Yamamoto(1985), Bigi and Sanda (1986)
- The observation of $(K^-\pi^+)(K^-\pi^+)$ final state would be definite evidence for the existence of $D^0\bar{D}^0$ mixing

Ph.D Thesis CALT.

- Particle indentification is crucial
- Possible limitation: continuum background
- $\Longrightarrow$ τ-charm factory...
- One common misunderstanding about this method: DCSD is forbidden in the final state

$\downarrow$ allowed (with mixing)

method could become a feasible way to search for mixing (and CP violation).

2.1.3 *Method C —use quantum statistics of the production and decay processes*

This method is to search for dual identical two-body hadronic decays in $e^+e^- \to \Psi'' \to D^0\bar{D}^0$, such as $(K^-\pi^+)(K^-\pi^+)$, as was first suggested by Yamamoto in his Ph.D thesis [50]. The idea is that when $D^0\bar{D}^0$ pairs are generated in a state of odd orbital angular momentum (such as Ψ''), the DCSD contribution to identical two-body pseudo-scalar-vector ($D \to PV$) and pseudo-scalar-pseudo-scalar ($D \to PP$) hadronic decays (such as $(K^-\pi^+)(K^-\pi^+)$) cancels out, leaving only the contribution of mixing [50, 51, 52]. The essence of Yamamoto's original calculation for the $(K^-\pi^+)(K^-\pi^+)$ case is given below.

Let's define $e_i(t) = e^{-im_it-\gamma_it/2}$ $(i = 1,2)$ and $e_\pm(t) = (e_1(t) \pm e_2(t))/2$. A state that is purely $|D^0\rangle$ or $|\bar{D}^0\rangle$ at time $t = 0$ will evolve to $|D(t)\rangle$ or $|\bar{D}(t)\rangle$ at time t, with $|D(t)\rangle = e_+(t)|D^0\rangle + e_-(t)|\bar{D}^0\rangle$ and $|\bar{D}(t)\rangle = e_-(t)|D^0\rangle + e_+(t)|\bar{D}^0\rangle$. In $e^+e^- \to \Psi'' \to D^0\bar{D}^0$, the $D^0\bar{D}^0$ pair is generated in the state $D^0\bar{D}^0 - \bar{D}^0D^0$ as the relative orbital angular momentum of the pair $\mathcal{L} = 1$. Therefore, the time evolution of this state is given by $|D(t)\bar{D}(t')\rangle - |\bar{D}(t)D(t')\rangle$, where t (t') is the time of decay of the D $(\bar{D})$. Now the double-time amplitude $\mathcal{A}_w(t,t')$ that the left side decays to $K^-\pi^+$ at t and the right side decays to $K^-\pi^+$ at t', giving a wrong sign event $(K^-\pi^+)(K^-\pi^+)$, is given by:

$$\mathcal{A}_w(t,t') = (e_+(t)e_-(t') - e_-(t)e_+(t'))\underline{(a^2 - b^2)} \tag{9}$$

where $a = \langle K^-\pi^+|D^0\rangle$ is the amplitude of the Cabibbo favored decay $D^0 \to K^-\pi^+$, while $b = \langle K^-\pi^+|\bar{D}^0\rangle$ is the amplitude of DCSD $\bar{D}^0 \to K^-\pi^+$. Similarly, the double-time amplitude $\mathcal{A}_r(t,t')$ for the right sign event $(K^-\pi^+)(K^+\pi^-)$ is given by:

$$\mathcal{A}_r(t,t') = (e_+(t)e_+(t') - e_-(t)e_-(t'))\underline{(a^2 - b^2)} \tag{10}$$

drops out

One measures the wrong sign versus right sign ratio R, which is:

$$\mathrm{R} = \frac{\mathrm{N}(K^-\pi^+, K^-\pi^+) + \mathrm{N}(K^+\pi^-, K^+\pi^-)}{\mathrm{N}(K^-\pi^+, K^+\pi^-) + \mathrm{N}(K^+\pi^-, K^-\pi^+)} = \frac{\iint |\mathcal{A}_w(t,t')|^2\,dt\,dt'}{\iint |\mathcal{A}_r(t,t')|^2\,dt\,dt'} \tag{11}$$

Note in taking the ratio, the amplitude term $(a^2 - b^2)$ in Equations 9 and 10 drops out. Thus, clearly R does not depend on whether b is zero (no DCSD) or finite (with DCSD). Integrating over all times, one then obtains $\underline{\mathrm{R} = (x^2 + y^2)/2 = \mathrm{R}_{\mathrm{mixing}}}$, where x and y are defined as before.

The need to measure $y \equiv \frac{\Delta\gamma}{2\gamma}$

- The methods discussed so far only measure:

$$\mathrm{R}_{\mathrm{mixing}} = (\underline{x^2 + y^2})/2$$

- Long distance effects (SM physics) contribute to both x and y, and therectical calculations are plagued by large uncertainties
- One expects that New Physics does not affect the decays in a significant way thus does not contribute to y, but only to x
- If one can experimentally confirm that indeed $x >> y$, then one can claim New Physics, regardless of what theoretical calculations for long distance effects are.
- Otherwise, if it turns out $x \sim y$, then mostly likely we are seeing the Standard Model Physics
- Therefore, it is **important** to measure y in order to understand the size of x within the Standard Model

Use CP eigenstates to measure $y \equiv \frac{\Delta\gamma}{2\gamma}$

- $D^0 \to K^+K^-, \pi^+\pi^-$ are CP even final states $\Longrightarrow$ occur only through the CP even eigenstate D_1
- The decay time distribution is a pefect exponential distribution with the slope of γ_1 (assuming CP)
- One can also measure $\gamma \equiv (\gamma_2 + \gamma_1)/2$
- Thus one can measure the decay rate difference

$$y \equiv \frac{\Delta\gamma}{2\gamma}$$

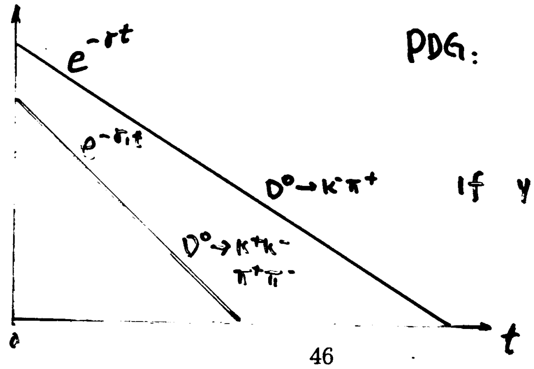
- together with $\mathrm{R}_{\text{mixing}} = \frac{1}{2}(x^2 + y^2)$, one can in effect measure

$$x \equiv \frac{\Delta m}{\gamma}$$

Exps. $R_{\text{mixing}} = \frac{1}{2}(x^2+y^2) \lesssim 0.5\% \Longrightarrow x, y \lesssim 9\%$

Use SCSD $D^0 \to \underline{\underline{K^+K^-}}, \underline{\underline{\pi^+\pi^-}}$ to measure $y \equiv \frac{\Delta\gamma}{2\gamma}$

- In the case of $D^0 \to K^+\pi^-$, mixing signature is a deviation from a perfect exponential time distribution
- In the case of $D^0 \to \underline{\underline{K^+K^-}}$..., mixing signature is a deviation of the slope from the average D^0 decay rate $\gamma \equiv (\gamma_2 + \gamma_1)/2$
- There is <u>no need to tag the D^0</u>, only need to determine the slope
- Use $D^0 \to K^-\pi^+$ to measure $\gamma \equiv (\gamma_2 + \gamma_1)/2$
- Observation of a non-zero $y \equiv \frac{\Delta\gamma}{2\gamma}$ would demonstrate mixing caused by the decay rate difference



D^0 mean life time

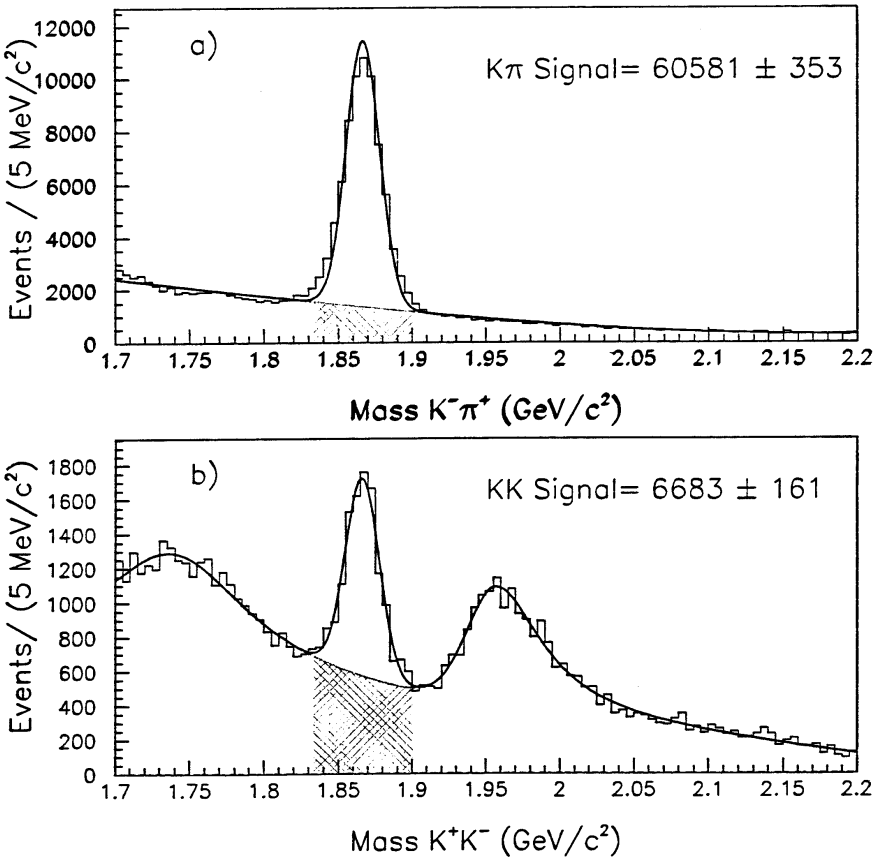
PDG: 0.415 ± 0.004 ps

1% with $D^0 \to K^-\pi^+$

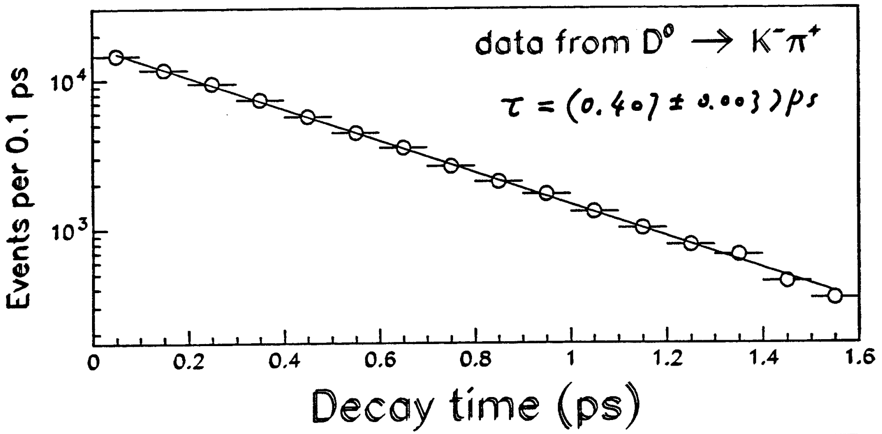
If $y \sim 1\% \Rightarrow R_{mixing} \sim 5 \times 10^{-5}$

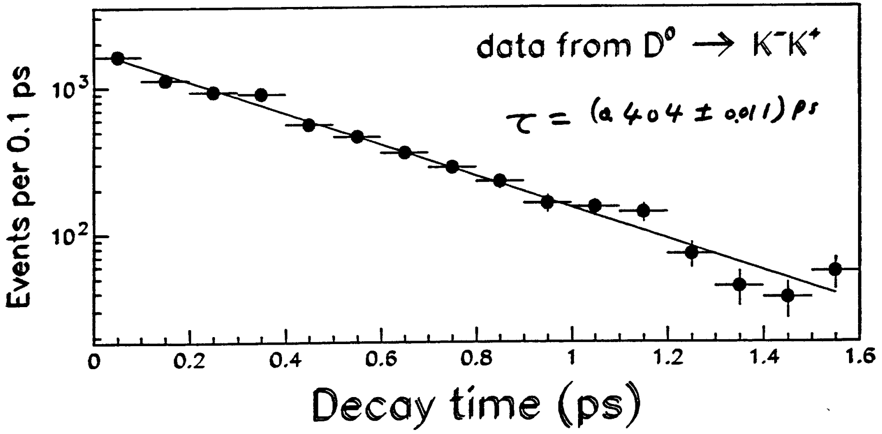
$R_{mixing} = \frac{1}{2}(x^2 + y^2)$

E791


a)
Kπ Signal= 60581 ± 353
Events / (5 MeV/c²)
0
2000
4000
6000
8000
10000
12000
1.7
1.75
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
2.2
Mass K⁻π⁺ (GeV/c²)
b)
KK Signal= 6683 ± 161
Events/ (5 MeV/c²)
0
200
400
600
800
1000
1200
1400
1600
1800
Mass K⁺K⁻ (GeV/c²)

E791





$\Longrightarrow \quad y = 0.005 \pm 0.015 \pm \text{syst.}$

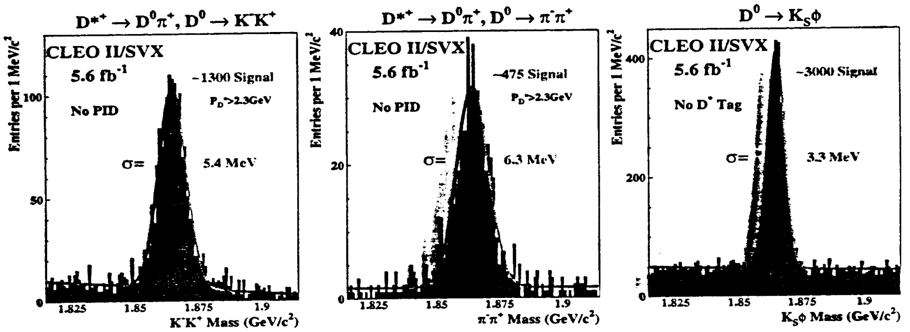
expected sensitivity:

$-0.042 < y < 0.058$ @ 90% C.L.

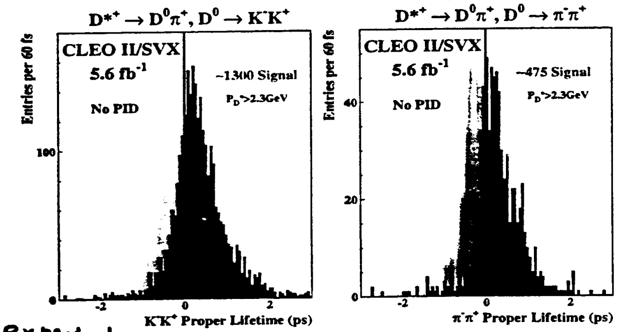
Lifetime Analysis CP$^+$, CP$^-$, $\overline{\text{CP}}$ $\Rightarrow y = \frac{\Delta\Gamma}{2\Gamma}$

- CP$^+$ = $K^+K^-, \pi^+\pi^-$
- CP$^-$ = $K_s^0\phi, K_s^0\omega, K_s^0\pi^0, K_s^0\rho^0, K_s^0\eta$
- $\overline{\text{CP}}$ = $K^-\pi^+$

D$^0 \to K^+K^-$ D$^0 \to \pi^+\pi^-$ D$^0 \to K_s^0\phi$



Mass (GeV/c^2)



CP$^-$ modes have 10× the statistics of CP$^+$ modes

expected sensitivity: $|y| < 0.003$

Proper Decay Time (ps)

CP+ & CP$^-$: → $|y| < 0.01$ expected

Fermilab-PUB-96/109-E
KSU-HEP-96-01

Search for $D^0\overline{D}^0$ Mixing in Semileptonic Decay Modes

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C. Darling,[18] K. Denisenko,[5] A. Fernandez,[11] P. Gagnon,[2] S. Gerzon,[14]
C. Gobel,[1] K. Gounder,[8] A. M. Halling,[5] G. Herrera,[4] G. Hurvits,[14] C. James,[5]
P. A. Kasper,[6] S. Kwan,[5] D. C. Langs,[10] J. Leslie,[2] B. Lundberg,[5] A. Manacero,[5]
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S. Radeztsky,[17] A. Rafatian,[8] N. W. Reay,[7] J. J. Reidy,[8] A. C. dos Reis,[1]
H. A. Rubin,[6] A. K. S. Santha,[3] A. F. S. Santoro,[1] A. J. Schwartz,[10] M. Sheaff,[17]
R. A. Sidwell,[7] A. J. Slaughter,[18] M. D. Sokoloff,[3] N. R. Stanton,[7] K. Stenson,[17]
K. Sugano,[2] D. J. Summers,[8] S. Takach,[18] K. Thorne,[5] A. K. Tripathi,[9]
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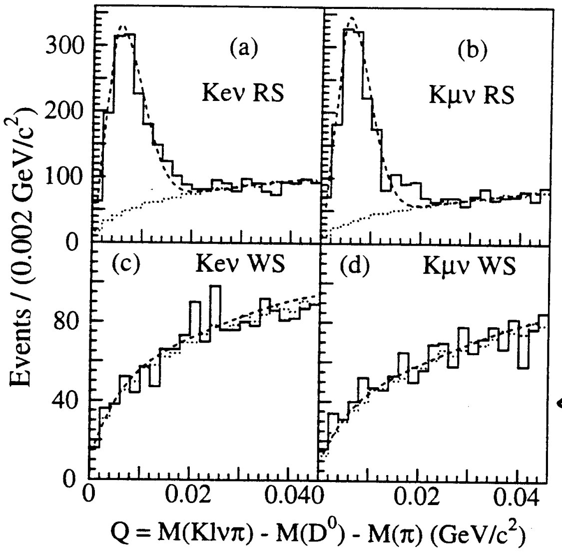
S. Yoshida,[7] R. Zaliznyak,[13] and C. Zhang[7]
(Fermilab E791 Collaboration)

Fermilab
E791



$D^{*+} \to D^0 \pi_s^+$
$\hookrightarrow K e \nu$

$R_{mixing} < 0.5\%$
@ 90% C.L.

⇐ background

Figure 1: The Q-value distributions for (a) $Ke\nu$ RS, (b) $K\mu\nu$ RS, (c) $Ke\nu$ WS, and (d) $K\mu\nu$ WS candidates. The solid line histograms show the data Q-value distributions, the dashed lines are the projections of the fit in Q-value, and the dotted lines show the Q-value distribution obtained from combining a D^0 from one event and π from another, normalized to the number of events with $Q > 0.025$ GeV/c^2 in the respective histograms.

Mixing Search: Semi-leptonic Methods

- Search for $D^0 \to \bar{D}^0 \to X l^- \nu$ No DCSD involved
- Need to tag the D^0
- Usually suffers from large background due to the missing ν
- One exception (at τcF): threshold kinematics constraints should provide clean signal (only one neutrino missing in the entire event):

$$e^+e^- \to D^- D^{*+} \to (K^+\pi^-\pi^-)(K^+l^-\nu)\pi_s^+$$

G. Gladding. 1989. Tau-Charm Workshop

Table IV

Rate Summary (1 year's run at L = 10^{33})

(a) $D^0 - \bar{D}^0$ Mixing

Reaction	Events (right sign)	r_D for 6 observed events
$\psi'' \to (K^-\pi^+)(K^-\pi^+)$	37500	1.6 ×10^{-4}
$\psi'' \to (K^-e^+\nu)(K^-e^+\nu)$	21600	7.4 ×10^{-5}
$\psi'' \to (K^-e^+\nu)(K^-\mu^+\nu)$	40000*	
$\psi'' \to (K^-\mu^+\nu)(K^-\mu^+\nu)$	20000*	
$D^{*+}D^- \to [\pi^+(K^+e^-\nu)(K^+\pi^-\pi^-)]$	19000	9.4 ×10^{-5}
$D^{*+}D^- \to [\pi^+(K^+\mu^-\nu)(K^+\pi^-\pi^-)]$	15000*	
$D^{*+}D^- \to [\pi^+(K^+e^-\nu)$(other D^- tag)]	15000*	
$D^{*+}D^- \to [\pi^+(K^+\mu^-\nu)$(other D^- tag)]	15000*	

(b) CP Violation

Reaction	Events	Comment
$D^{*0}\bar{D}^0 \to$ [(γ(semileptonic)][(CP eigenstate)] asymmetry measurement see Table I	6570	measures mixing-dependant CP violation asymmetry determined to 1.2%
$\psi'' \to$(semileptonic)(CP eigenstate) asymmetry measurement see Table II	34000*	measures magnitude of CP violating amplitude to 0.5%
$\psi'' \to$(CP eigenstate)(CP eigenstate) rate measurement see Table III	4000*	sensitive to phase of direct CP violating amplitude

* estimates based on scaling acceptances of similar processes

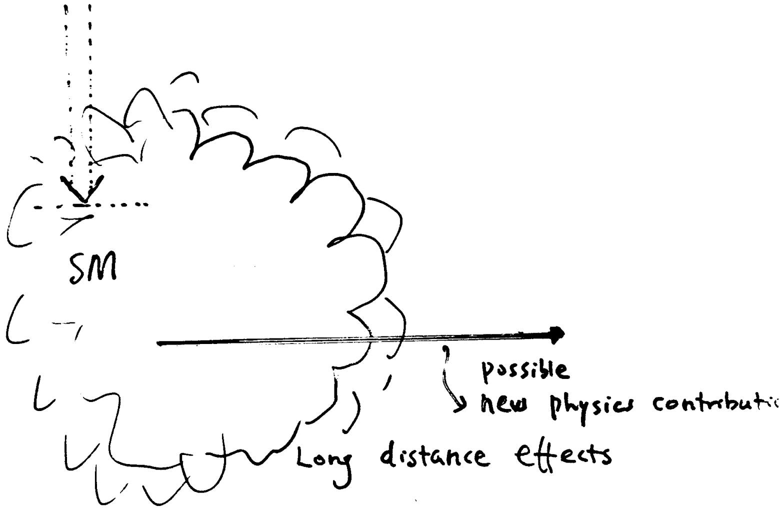
BaBar Physics book

Table 12-3. *Current experimental results on CP asymmetries for charged and neutral D mesons decays from E791 [11], E687 [12] and CLEO[13].*

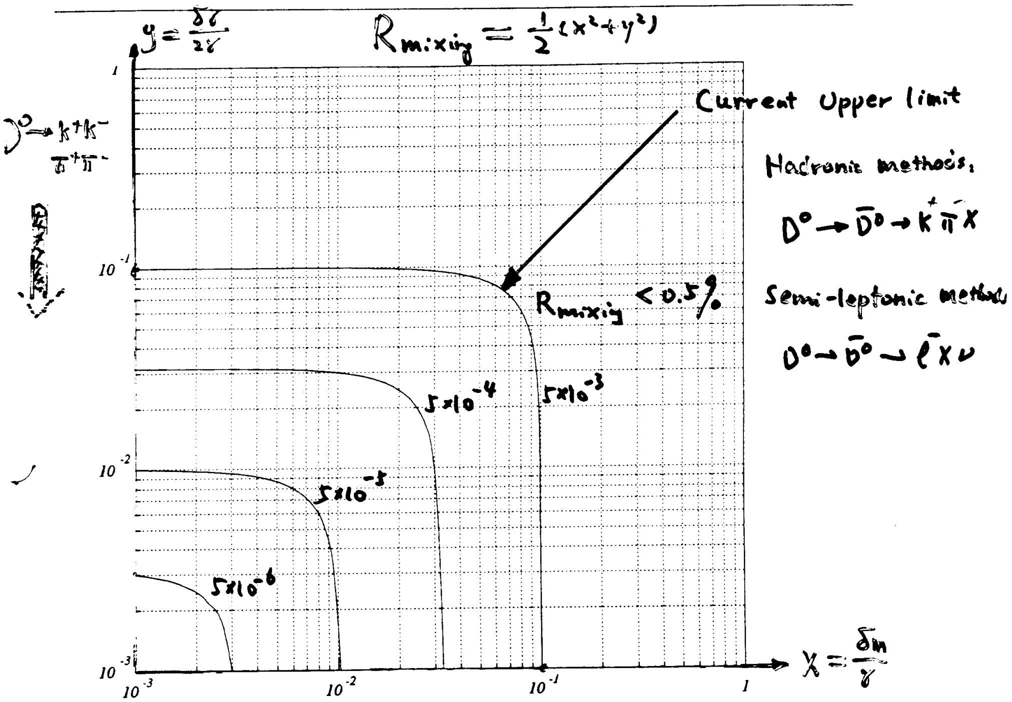
	Experiment	Mode	a_{CP}	90% CL Limits (%)
Charged	E791	$K^-K^+\pi^-$	-0.014±0.029	-6.2< a_{CP} <3.4
	E791	$\phi\pi^-$	-0.028±0.036	-8.7< a_{CP} <3.1
D mesons	E791	$\overline{K}^{*0}(892)K^+$	-0.010±0.050	-9.2< a_{CP} <7.2
	E791	$\pi^-\pi^-\pi^-$	-0.014±0.042	-8.6< a_{CP} <5.2
Neutral	CLEO	K^-K^+	+0.080±0.061	-2.2< a_{CP} <18
	E687	K^-K^+	+0.024±0.084	-11< a_{CP} <16
D mesons	CLEO	$K_s^0\phi$	-0.028±0.094	-18.2< a_{CP} < 12.6
	CLEO	$K_s^0\pi^0$	-0.018±0.030	-6.7< a_{CP} <3.1

BaBar: 30 fb^{-1} → sensitivity: $a_{cp} \sim 3 \times 10^{-3}$

$$a_{cp} = \frac{N(D^0 \to K^+K^-) - N(\bar{D}^0 \to K^+K^-)}{N(D^0 \to K^+K^-) + N(\bar{D}^0 \to K^+K^-)}$$

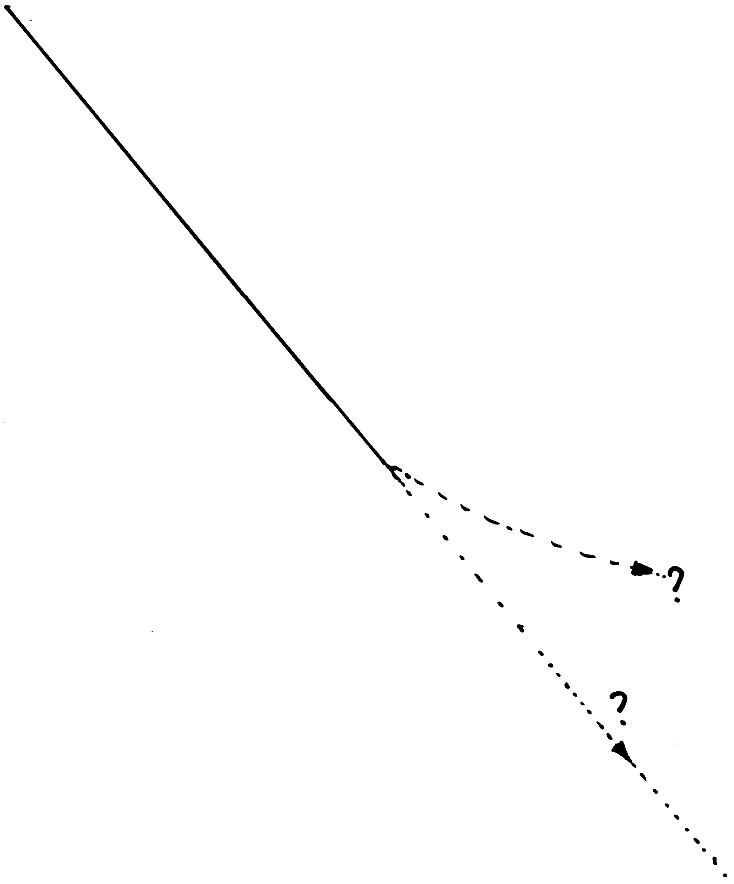

SM
possible
new physics contributi
Long distance effects

Future prospects of $D^0\bar{D}^0$ mixing search — a personal view



The quest to observe $D^0\bar{D}^0$ mixing is a program rather than a single effort

τcF F.T. BF TEVATRON, LHC ...



If history is a guide ...

The Quest for Charm Mixing