Charm Decays and Mixing:

Standard Model and Beyond

- 1ST programmatic look at QCD corrections for rare charm decays
- Update predictions for D-$\bar{D}$ Mixing and CP
- Identify processes which have open window for new physics and explain why they're important to explore
- Identify opportunities for τcF

Burdman, Golowich, JLH, Pakvasa

GIM Mechanism - Glashow, Iliopoulos, Maiani (1970)

[Introduced to suppress Kaon FCNC]

$$\mathcal{A}_{\text{penguin}} \sim \sum_{i=1}^{3} V_{iQ} V_{iq}^{*} F\left(m_i^2/m_W^2\right)$$

CKM is Unitary! $\sum_i V_{iQ} V_{iq}^{*} = 0$

$$\Rightarrow V_{1Q} V_{1q}^{*} = - V_{2Q} V_{2q}^{*} - V_{3Q} V_{3q}^{*}$$

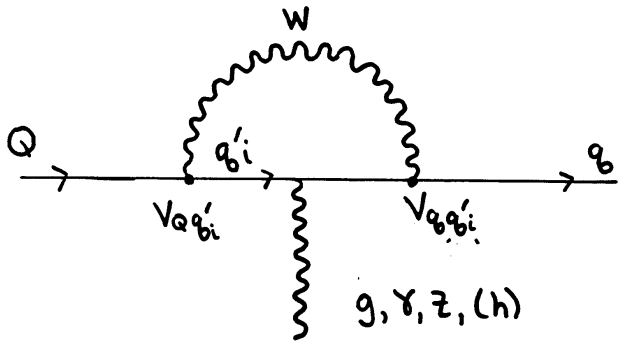
$$\mathcal{A}_{\text{penguin}} \sim V_{2Q} V_{2q}^{*} \left[F\left(m_2^2/m_W^2\right) - F\left(m_1^2/m_W^2\right)\right]$$

$$+ V_{3Q} V_{3q}^{*} \left[F\left(m_3^2/m_W^2\right) - F\left(m_1^2/m_W^2\right)\right]$$

$\rightarrow 0$ if quarks were degenerate

Size of FCNC related to size of internal quark mass splittings

Typical Flavor Changing Neutral Current Vertex Diagram (Penguin!)



$= -1/3\ Q$ u, c, t $= -1/3\ q_b$

$= +2/3 Q$ d, s, b $= +2/3 q_b$

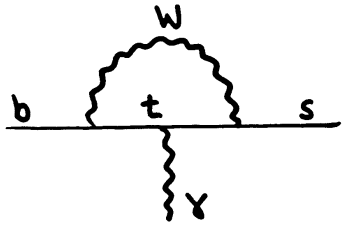
Typical Values of Branching Fractions

Meson / Quark	B
K	$10^{-10} - 10^{-8}$
D	$10^{-14} - 10^{-10}$
B	$10^{-8} - 10^{-4}$
t	$10^{-12} - 10^{-9}$

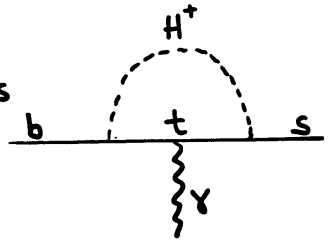
Small SM Rates in D System
⇒ Big window for new physics!

Loop-Level Processes are Sensitive to New Physics

SM:

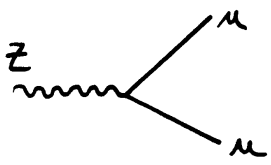


vs.
New Physics

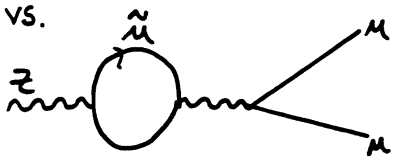


['Large' Rates in B System]

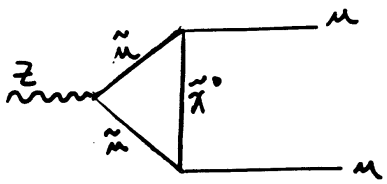
Compare to Precision EW:



vs.



+



JLH

FCNC Branching Fractions in Standard Model

Circa '96

Decay Mode	Experimental Limit	$B_{S.D.}$	$B_{L.D.}$
$D^0 \to \mu^+\mu^-$	$< 1.1 \times 10^{-5}$	$(1-20) \times 10^{-19}$	$< 3 \times 10^{-15}$
$D^0 \to e^+e^-$	$< 1.3 \times 10^{-4}$	even smaller !	
$D^0 \to \mu^\pm e^\mp$	$< 1.0 \times 10^{-4}$	0	0
$D^0 \to \gamma\gamma$	—	10^{-16}	$< 3 \times 10^{-9}$
$D \to X_u + \gamma$		1.4×10^{-17}	
$D^0 \to \rho^0\gamma$	$< 1.4 \times 10^{-4}$		$< 2 \times 10^{-5}$
$D^0 \to \phi^0\gamma$	$< 2.0 \times 10^{-4}$		$< 10^{-4}$
$D^+ \to \rho^+\gamma$	— CLEO		$< 2 \times 10^{-4}$
$D^+ \to \bar{K}^{*+}\gamma$	— prelim		3×10^{-7}
$D^0 \to \bar{K}^{*0}\gamma$	—		1.6×10^{-4}
$D \to X_u + \ell^+\ell^-$	E791 FNAL prelim	4×10^{-9}	
$D^0 \to \pi^0\mu\mu$	$< 1.7 \times 10^{-4}$		
$D^0 \to \bar{K}^0 ee/\mu\mu$	$< 17.0/2.5 \times 10^{-4}$		$< 2 \times 10^{-6\,8}$
$D^0 \to \rho^0 ee/\mu\mu$	$< 2.4/4.5 \times 10^{-4}$		
$D^+ \to \pi^+ ee/\mu\mu$	$< 250/2.6 \times 10^{-5}$ (6.6, 1.3)	few$\times 10^{-10}$	$< 10^{-8}$
$D^+ \to K^+ ee/\mu\mu$	$< 480/8.5 \times 10^{-5}$		$< 10^{-6\,8}$
$D^+ \to \rho^+\mu\mu$	$< 5.8 \times 10^{-4}$		
$D^0 \to X_u + \nu\bar{\nu}$		2.0×10^{-15}	
$D^0 \to \pi^0\nu\bar{\nu}$	—	4.9×10^{-16}	$< 6 \times 10^{-16}$
$D^0 \to \bar{K}^0\nu\bar{\nu}$	—		$< 10^{-12}$
$D^+ \to X_u + \nu\bar{\nu}$	—	4.5×10^{-15}	
$D^+ \to \pi^+\nu\bar{\nu}$	—	3.9×10^{-16}	$< 8 \times 10^{-16}$
$D^+ \to K^+\nu\bar{\nu}$	—		$< 10^{-14}$

No QCD!

Charm Radiative Decays

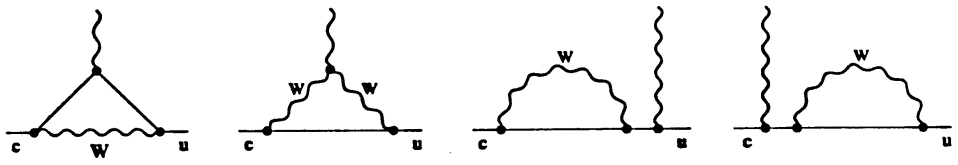


Figure 1: One-loop diagrams generating the $c \to u\gamma$ transition.

$$\mathcal{A} = \frac{e G_F}{4\sqrt{2}\pi^2} \sum_{\substack{i= \\ d,s,b}} \lambda_i F_2(x_i)\, \bar{u}\, \sigma_{\mu\nu} q^\nu m_c P_R\, c\, \epsilon^\mu$$

$$\lambda_i = V_{ci}^* V_{ui} \; ; \; x_i = \frac{m_{q_i}^2}{m_W^2}$$

$$\Gamma(c \to u\gamma) = \frac{\alpha G_F^2 m_c^5}{128\,\pi^4} \left[\lambda_s \left(F_2(x_s) - F_2(x_d) \right) + \lambda_b \left(F_2(x_b) - F_2(x_d) \right) \right]^2$$

Quark	F_2	$\lvert V_{ci}^* V_{ui} \rvert F_2$
d	1.6×10^{-9}	3.4×10^{-10}
s	2.9×10^{-7}	6.7×10^{-8}
b	3.3×10^{-4}	3.2×10^{-8}

$$B(c \to u\gamma) = \frac{\Gamma(c \to u\gamma)}{\Gamma(c \to X\ell\nu)} \qquad B(D^+ \to X\ell^+\nu) = 1.4 \times 10^{-17} \;\; !!$$

(data)

Large QCD Corrections!

Operator Product Expansion (technique pioneered Gilman + Wise '79)

$m_b < \mu < m_W$

$$\mathcal{H}_{eff}^{\Delta c=1} = \frac{4G_F}{\sqrt{2}} \left[\sum_{q=d,s,b} \lambda_q \left[C_1(\mu) Q_1^q(\mu) + C_2(\mu) Q_2^q(\mu) \right] + \sum_{i=3}^{8} C_i(\mu) Q_i(\mu) \right]$$

$\mu < m_b$

$$\mathcal{H}_{eff}^{\Delta c=1} = \frac{4G_F}{\sqrt{2}} \left[\sum_{q=d,s} \lambda_q \left[C_1(\mu) Q_1^q(\mu) + C_2(\mu) Q_2^q(\mu) \right] + \sum_{i=3}^{8} C_i'(\mu) Q_i'(\mu) \right]$$

$$\mathcal{M}^{LD}(c \to u\gamma) = \frac{4G_F}{\sqrt{2}} C_7^{eff}(\mu) \langle u\gamma | Q_7(\mu) | c \rangle \lambda_b$$

Complete set of operators governing $c \to u$ transitions

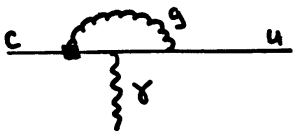
$$Q_1^q = (\bar{u}_\alpha \gamma_\mu P_L q_\beta)(\bar{q}_\beta \gamma^\mu P_L c_\alpha) \qquad Q_2^q = (\bar{u}_\alpha \gamma_\mu P_L q_\alpha)(\bar{q}_\beta \gamma^\mu P_L c_\beta)$$

$$Q_3 = (\bar{u}_\alpha \gamma_\mu P_L c_\alpha) \sum_q (\bar{q}_\beta \gamma^\mu P_L q_\beta) \qquad Q_4 = (\bar{u}_\alpha \gamma_\mu P_L c_\beta) \sum_q (\bar{q}_\beta \gamma^\mu P_L q_\alpha)$$

$$Q_5 = (\bar{u}_\alpha \gamma_\mu P_L c_\alpha) \sum_q (\bar{q}_\beta \gamma^\mu P_R q_\beta) \qquad Q_6 = (\bar{u}_\alpha \gamma_\mu P_L c_\beta) \sum_q (\bar{q}_\beta \gamma^\mu P_R q_\alpha)$$

$$Q_7 = \frac{e}{16\pi^2} m_c (\bar{u}_\alpha \sigma_{\mu\nu} P_R c_\alpha) F^{\mu\nu} \qquad Q_8 = \frac{g}{16\pi^2} m_c (\bar{u}_\alpha \sigma_{\mu\nu} T^a_{\alpha\beta} P_R c_\beta) G^{a\mu\nu}$$

Operator Mixing:



Grinstein, Springer, Wise – 2-loop operator mixing

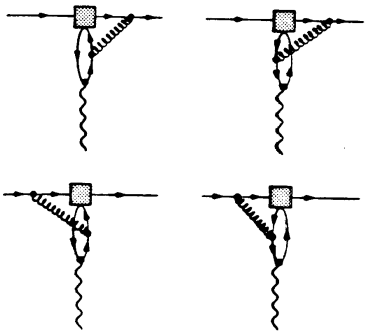


Fig. 1. Some two-loop Feynman diagrams contributing to the mixing of O_2 with O_7.

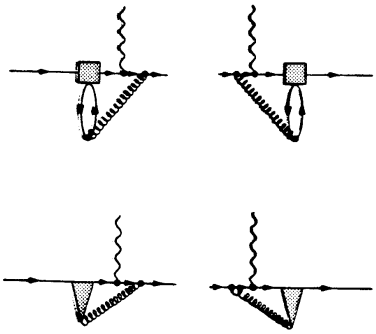
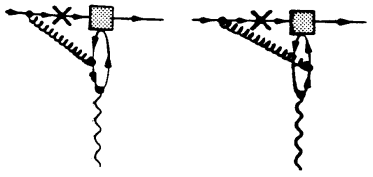


Fig. 2. Some two-loop Feynman diagrams contributing to the mixing of O_2 with O_7.

states is modeled by b-quark decay, only the tree level matrix element of the magnetic-moment-operator term $C_7(m_b)O_7$ is required to determine the rate for $B \to \gamma X_s$. The coefficient $C_7(m_b)$ is determined (via the renormalization group equations) by the anomalous dimension matrix for the renormalization of the operators O_1–O_7 and the initial values in eqs. (13).

In this paper we obtain a simple (but fairly accurate) analytic approximation to $C_7(\mu)$ by truncating the operator basis to O_1, O_2 and O_7. (A similar approximation was used in the effective hamiltonian for $\Delta S=2$ K^0–$\bar{K}^0$ mixing [9].) In a further publication we shall present a more complete treatment using the full anomalous dimension matrix and giving more details of the computation. The mixing of O_1 and O_2 is similar to the operator mixing in the weak $\Delta S=1$ nonleptonic decays and it gives [10,11] $\gamma_{11}=\gamma_{22}=-g^2/8\pi^2$, $\gamma_{12}=\gamma_{21}=3g^2/8\pi^2$. The renormalization of O_7 (including the factor of m_b) yields [6] $\gamma_{77}=2g^2/3\pi^2$. The mixing of operators O_1 and O_2 with O_7 arises at two loops from the Feynman diagrams in figs. 1–4. (Previous work on QCD



· Some two-loop Feynman diagrams contributing to the ·ng of O_2 with O_7.

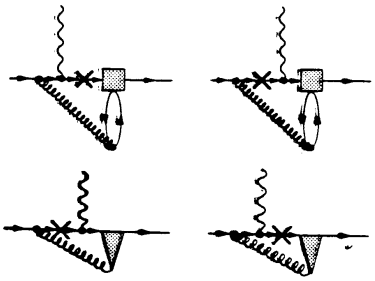


Fig. 4. Some two-loop Feynman diagrams contributing to the mixing of O_2 with O_7.

Wilson Coefficients

Calculate C_i at $\mu = m_W$ ⇒ matching conditions

At LL: $C_{1,3-6}^{(m_W)} = 0$ $\quad C_2(m_W) = -1$

$C_7^{q_b}(m_W) = -1/2\, F_2(x_b)\lambda_{q_b}$ $\quad C_8^{q_b}(m_W) = -1/2\, D(x_b)\lambda_{q_b}$

Use RGE to evolve to $\mu = m_b$ ⇒ $\mu = m_c$

$$\mu \frac{d}{d\mu} \mathcal{H} = 0 \Rightarrow \underbrace{\mu \frac{d}{d\mu}}_{\mu \frac{d\alpha_s}{d\mu}\frac{1}{d\alpha_s}} C_j(\mu) = \gamma_{ij}^T(g)\, C_i(\mu)$$

$\gamma_{ij}^T(g)$ → anomalous dimension matrix

$\mu \frac{d\alpha_s}{d\mu}$ → β-function!

$$-\frac{23}{3}\alpha_s \frac{dC_j(\mu)}{d\alpha_s} = \tilde{\gamma}_{ij} C_i(\mu) \qquad \tilde{\gamma}_{ij} = \frac{8\pi^2}{g_s^2}\gamma_{ij}$$

Perform similarity transformation:

$$-\frac{23}{3}\alpha_s \frac{d}{d\alpha_s} C_K = \underbrace{(V^{-1})_{K\ell}\, \tilde{\gamma}_{\ell m} V_{mn}}_{\text{Diagonalizes } \tilde{\gamma}\,!\, = (\tilde{\gamma}_{Diag}^T)_{Kn}\,\delta_{Kn}} C_n$$

$$\int_{m_b}^{m_W} \frac{dC_K}{C_K} = \frac{-3}{23}\int_{m_b}^{m_W} \frac{d\alpha_s}{\alpha_s}\left(\tilde{\gamma}_{Diag}^T\right)_{KK}$$

$$\boxed{C_i(m_b) = \eta^{-3/23}\, V_{ij}\, (\exp\, \tilde{\gamma}_{Diag}^T)_{jj}\, V_{jK}^{-1}\, C_K(m_W)} \qquad \underline{\text{LL}}\,!$$

$\eta = \alpha_s(m_b)/\alpha_s(m_W)$

RGE ⇒

$$C_7^{eff}(\mu) = U^4_{7i}(\mu, m_b)\, m(m_b)\, U^5_{ij}(m_b, m_W)\, C_j(m_W)$$

↑ quark matching matrix

$\mu = m_c$: $= 0.46\, C_7(m_W) + 0.13\, C_8(m_W) - 0.31\, C_2(m_W)$.

$C_7(m_W)$: -0.41×10^{-3} ; $C_8(m_W)$: -0.24×10^{-2} ; $C_2(m_W)$: -1

$$\underline{B(c \to u\gamma) = (4.2 - 7.9) \times 10^{-12}} \quad \text{for} \quad m_c \leq \mu \leq 2m_c$$

⇒ GIM power suppression replaced by QCD $\ln m_b/m_c$ contribution

But, $\lambda_b = -\lambda_d - \lambda_s$ suppression remains!

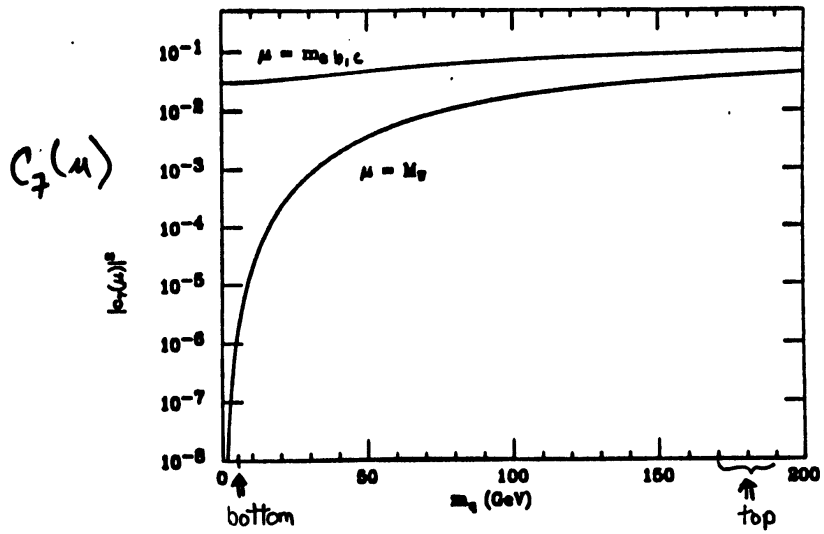


Figure 2. Dependence of c_7 on Intermediate-quark Mass

We now consider the case of radiative charm transitions. The $|\Delta c| = 1$ effective hamiltonian can be written as

$$H_{\rm eff}^{|\Delta c|=1} = \frac{-4G_F}{\sqrt{2}} \lambda_b \sum_{k=1}^{10} c_k(\mu) O_k(\mu)\,, \tag{18}$$

Leading 2-loop contributions

Greub etal

2-loop matrix element of $\mathcal{H}_{eff}^{\Delta c=1}$

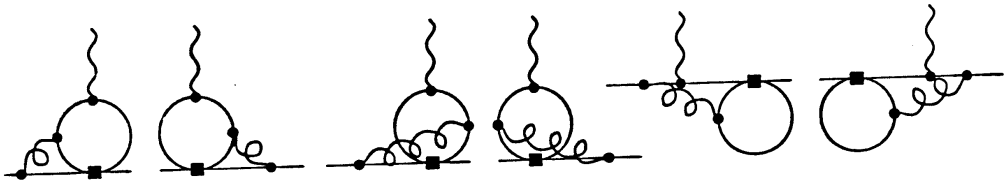


Figure 2: Diagrams contributing to the two-loop $c \to u\gamma$ matrix element of

$$\mathcal{M}(c\to u\gamma) = \frac{4G_F}{\sqrt{2}} A \langle u\gamma | Q_7(m_c) | c\rangle_{tree}$$

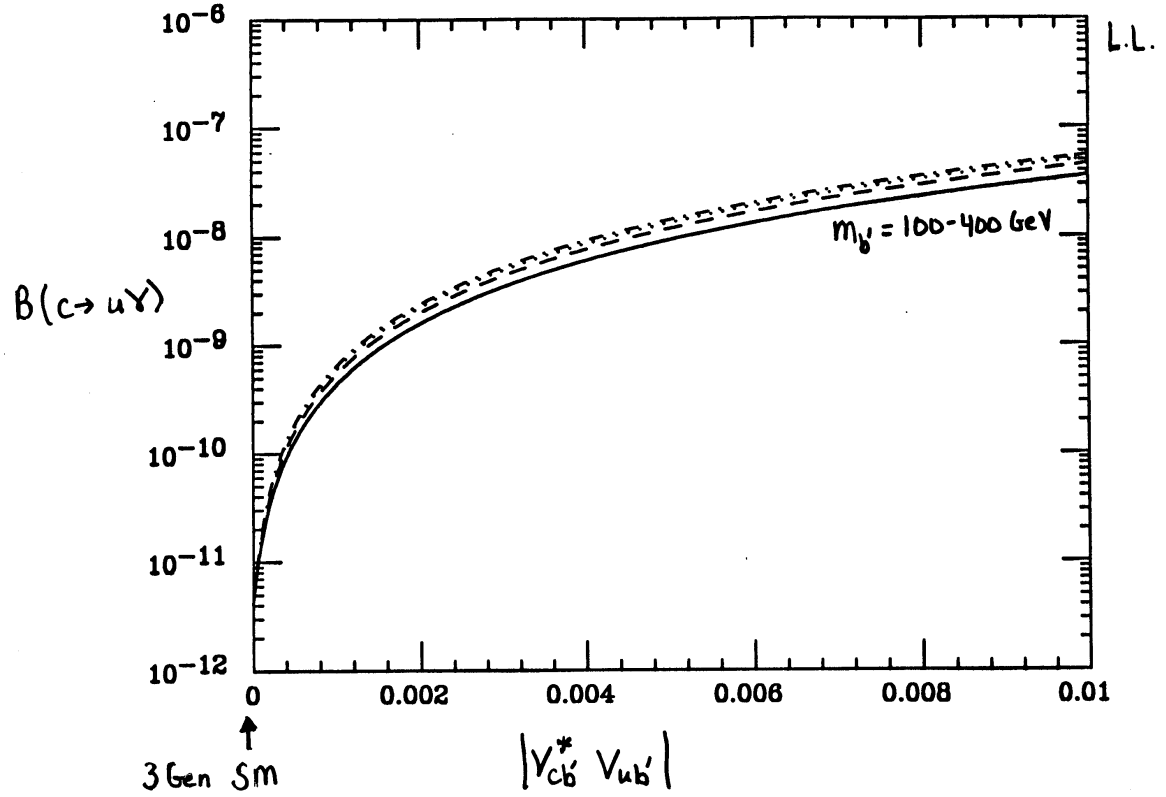
$$A = \frac{\alpha_s}{4\pi} V_{cs}^* V_{us} \underbrace{C_2(m_c)\left[f(m_s^2/m_c^2) - f(m_d^2/m_c^2)\right]}_{0.86} + \mathcal{O}(V_{cb}^* V_{ub})$$

$$\Rightarrow B(D\to X_u\gamma) = 5\times 10^{-8}\ !!!$$

1-Loop $\longrightarrow$ L.L. $\longrightarrow$ Leading 2-Loop

GIM suppressed $\longrightarrow$ CKM (λ_b) suppressed $\longrightarrow$ Cabibbo, m_s/m_c suppressed

New Physics Example : 4th Generation



Full NLO to $c \to u\gamma$ can now be computed!

Radiative Penguin - $b \to s\gamma$ at NLO!

NLO: matrix element
- QCD bremsstrahlung Corrections
  1-loop $\langle s\gamma g | \mathcal{O}_i | b \rangle$ — Ali, Greub, Pott
- Virtual Corrections
  2-loop $\langle s\gamma | \mathcal{O}_i | b \rangle$ — Greub, Hurth, Wyler

Wilson coeff
- $\mathcal{O}(\alpha_s)$ terms in matching conditions — Lin, Liu, Yao; Greub, Hurth
- $\mathcal{O}(\alpha_s^2)$ anomalous dimension matrix used in RGE — Chetyrkin, Misiak, Münz

E_γ Spectrum
- Consistent treatment of Fermi motion in HQET — Kagan, Neubert

Electroweak
- 2-loop EW Corrections — Czarnecki + Marciano

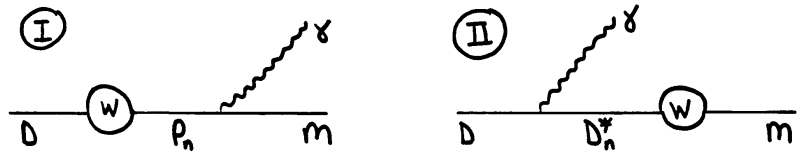
$$\Rightarrow \quad \boxed{\mathcal{B}(B \to X_s\gamma) = (3.29 \pm 0.33) \times 10^{-4}}$$

ALEPH: $\mathcal{B}(B \to X_s\gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}$

CLEO: $\mathcal{B}(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$

Exclusive Radiative Decays - Long Range Effects $c \to u\gamma$

- Pole Amplitudes - $c\bar{q}_1 \to q_2 \bar{q}_3 \gamma$



$$\mathcal{H}_W^{eff} = \frac{-G_F}{\sqrt{2}} \left[: a_1 (\bar{u}d')(\bar{s}'c) + a_2 (\bar{s}'d')(\bar{u}c) : \right]$$

Bauer Stech Wirbel

a_1: fit to data on $D \to \bar{K}\pi$; a_2: fit to data on $D \to \bar{K}\pi$

$$\mathcal{A}_I^{pc}(D \to m\gamma) = \sum_n \frac{h_{m\gamma P_n}}{m_D^2 - m_{P_n}^2} \langle P_n | \mathcal{H}_W^{eff} | D \rangle$$

$$\mathcal{A}_{II}^{pc}(D \to m\gamma) = \sum_n \frac{h_{D_n^* \gamma D}}{m_D^2 - m_{D_n^*}^2} \langle m | \mathcal{H}_W^{eff} | D_n^* \rangle$$

- Vector Meson Dominance - $c \to q_1 \underbrace{\bar{q}_2 q}_{\to \gamma} \gamma$

- Phenomenological Approach - use data $D \to mV$
- Theoretical Model - BSW

$$|\mathcal{A}_{VmD}|^2 = \frac{G_F^2 |V_{cq}^* V_{qu}|^2}{2 m_D K^2} a_i(m_c^2) f_x^2 I$$

$$\left(\frac{4\pi\alpha}{f_V^2}\right)\left[(m_D + m_V)^2 A_1^2(q_0^2) + \frac{4K^2 m_D^2 V^2(q_0^2)}{(m_D + m_V)^2}\right]$$

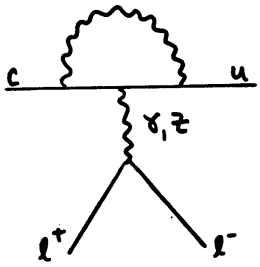
Long Distance Predictions

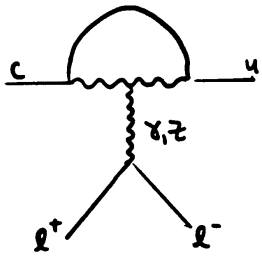
Table 11 Amplitude and Branching Fraction Predictions

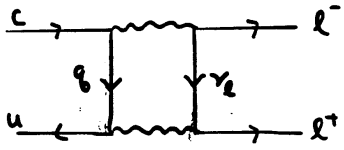
Mode	$\mathcal{A}^{pc}$			$\mathcal{A}^{pv}$	$B_{D\to M\gamma}$ (10^{-5})
	P-I	P-II	VMD	VMD	
$D_s^+\rho^+\gamma$	8.2	−1.9	±3.2	±2.8	6 − 38
$D^0\bar{K}^{*0}\gamma$	5.6	−5.9	±3.8	±(5.1 − 6.8)	7 − 12
$D_s^+b_1^+\gamma$	7.2	—	—	—	∼ 6.3
$D_s^+a_1^+\gamma$	1.2	—	—	—	∼ 0.2
$D_s^+a_2^+\gamma$	2.1	—	—	—	∼ 0.01
$D^+\rho^+\gamma$	1.3	−0.4	±1.6	±1.9	2 − 6
$D^+b_1^+\gamma$	1.2	—	—	—	∼ 3.5
$D^+a_1^+\gamma$	0.5	—	—	—	∼ 0.04
$D^+a_2^+\gamma$	3.4	—	—	—	∼ 0.03
$D_s^+K^{*+}\gamma$	2.8	−0.5	±0.9	±1.0	0.8 − 3
$D_s^+K_2^{*+}\gamma$	6.0	—	—	—	∼ 0.2
$D^0\rho^0\gamma$	0.5	−0.5	±(0.2 − 1.0)	±(0.6 − 1.0)	0.1 − 0.5
$D^0\omega^0\gamma$	0.6	−0.7	±0.6	±0.7	≃ 0.2
$D^0\phi^0\gamma$	0.7	−1.6	±(0.6 − 3.5)	±(0.9 − 2.1)	0.1 − 3.4
$D^+K^{*+}\gamma$	0.4	−0.1	±0.4	±0.4	0.1 − 0.3
$D^0K^{*0}\gamma$	0.2	−0.3	±0.2	±0.2	≃ 0.01

- Long range effects overwhelm short distance.
- Wide range of predictions
  - observation would 'select' calculational technique
  ⇒ extrapolate to $B \to \rho/\omega\,\gamma$ which will be used to determine $|V_{td}|/|V_{ts}|$

$D \to X_u \ell^+ \ell^-$







Computed to NLO

$$\mathcal{M} = \frac{\sqrt{2}\, G_F \alpha}{\pi} \left[C_9^{eff}\, \bar{u}_L \gamma_\mu c_L\, \bar{\ell} \gamma^\mu \ell + C_{10}\, \bar{u}_L \gamma_\mu c_L\, \bar{\ell} \gamma^\mu \gamma_5 \ell \right.$$
$$\left. - 2 C_7^{eff}\, m_c\, \bar{u}_L i \sigma_{\mu\nu} \frac{q_b^\nu}{q_b^2} c_R\, \bar{\ell} \gamma^\mu \ell \right]$$

q_b^2 = momentum transfer to $\ell^+\ell^-$ $\qquad (\hat{s} = q_b^2/m_c^2)$

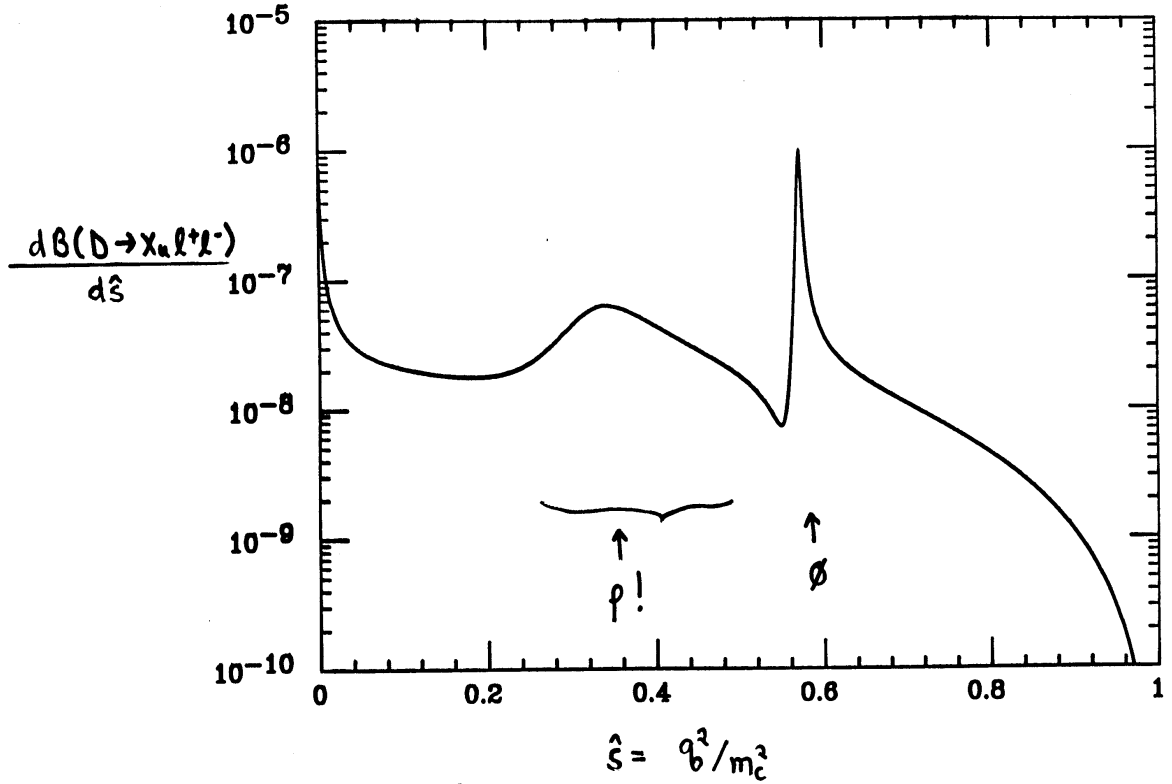
Long distance resonance contributions

$D \to X_u V$, $V \to \ell^+\ell^-$ $\qquad V = \phi, \rho$ $\qquad$ peaks at $q_b^2 = m_{\phi,\rho}^2$

$\Rightarrow$ Effective $(\bar{u}_L \gamma_\mu c_L)(\bar{\ell} \gamma^\mu \ell)$ interaction

$$C_9^{eff}(\mu) = C_9(\mu) + Y(\mu, \hat{s}) - \frac{3\pi}{\alpha^2 m_c^2} \sum_{\phi,\rho} \frac{m_{V_i}\, \Gamma(V_i \to \ell^+\ell^-)}{\left(\hat{s} - \frac{m_{V_i}^2}{m_c^2}\right) + \frac{i\, \Gamma_{V_i} m_{V_i}}{m_c^2}}$$

$Y(\mu, \hat{s})$: contribution from light-quarks



$B(D \to X_u l^+ l^-) = 2 \times 10^{-8}$

$l = e$ short distance

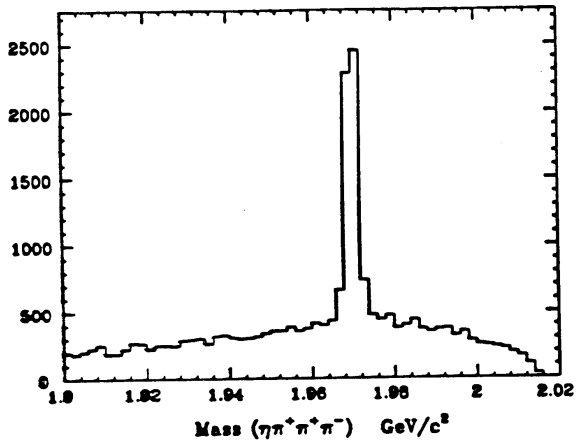


Fig. 3: Reconstruction[3] of the decay $D_s \to \eta\pi^+\pi^+\pi^-$ with $\eta \to \pi^+\pi^-\pi^0$ and $\pi^0 \to \gamma\gamma$.

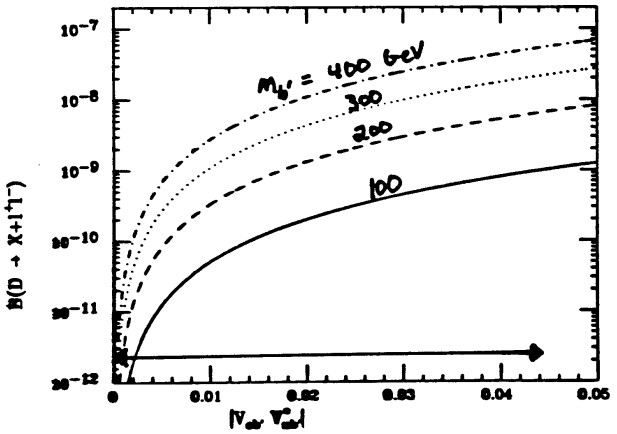
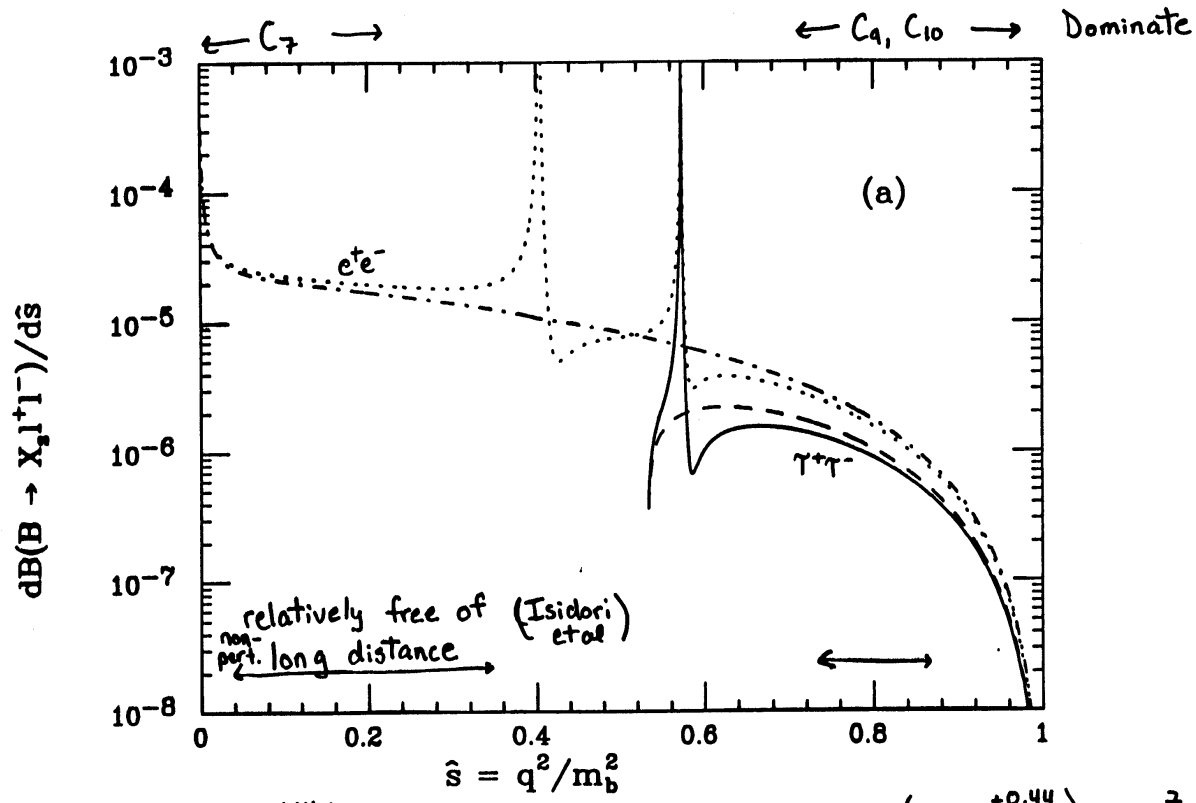


Fig. 4: Branching fraction for $D \to X + \ell^+\ell^-$ in the four generation SM as a function of the CKM mixing factor, with the solid, dashed, dotted, dash-dotted curve corresponding to $m_{b'}$ = 100, 200, 300, 400 GeV, respectively.



$B(B \to X_s e^+e^-) = (6.25^{+1.04}_{-0.93}) \times 10^{-6}$

$B(B \to X_s \tau\tau) = (3.24^{+0.44}_{-0.54}) \times 10^{-7}$

Kinematic Distributions - Requires high statistics!

- $m_{\ell^+\ell^-}$ Distribution (Lim, Morozumi, Sanda; Deshpande et al; Ali et al)

$$\frac{d\Gamma(B\to X_s\ell^+\ell^-)}{d\hat{s}} \sim (1-\hat{s})^2\left[\begin{array}{l}(|C_9^{eff}|^2+|C_{10}|^2)(1+2\hat{s})\\ +4|C_7|^2\frac{2+\hat{s}}{\hat{s}}+12\,Re(C_9^{eff}C_7)\end{array}\right]$$

- Forward-Backward Asymmetry of $\ell^+\ell^-$ angular distribution

$$A_{FB}=\frac{\int_0^1 dz\ d\Gamma/dz d\hat{s}-\int_{-1}^0 dz\ d\Gamma/dz d\hat{s}}{\int_0^1 dz\ d\Gamma/dz d\hat{s}+\int_{-1}^0 dz\ d\Gamma/dz d\hat{s}} \qquad \text{(Ali et al)}$$

$$=-3C_{10}\left[Re\,C_9^{eff}\hat{s}+2C_7\right]/d\Gamma/d\hat{s}$$

- ~~Tau~~ $\hat{\mu}$ Polarization Asymmetry in ~~$B\to X_s\tau^+\tau^-$~~ $D\to X_u\mu^+\mu^-$

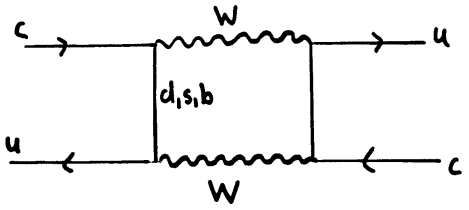
$$P_\tau=\frac{d\Gamma_{\lambda=-1}-d\Gamma_{\lambda=+1}}{d\Gamma_{\lambda=-1}+d\Gamma_{\lambda=+1}} \qquad \text{(JLH; Kruger, Sehgal)}$$

$$=-2C_{10}\left[Re\,C_9^{eff}F_1(\hat{s},m_\tau)+3C_7F_2(\hat{s},m_\tau)\right]/d\Gamma_\tau/d\hat{s}$$

D^0-$\bar{D}^0$ Mixing

$|D_L\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$

$|D_H\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$



Short-Distance - External momentum non-negligible

$$\mathcal{H}_{eff}^{\Delta c=2} = \frac{G_F \alpha}{\sqrt{2}\, 8\pi \sin^2\theta_W} \left[\begin{array}{l} |V_{cs} V_{us}^*|^2 \left[I_1\left(\frac{m_s^2}{m_W^2}, \frac{m_s^2}{m_c^2}\right) O_{LL} - m_c^2 I_2 O' \right] \\ + |V_{cb} V_{ub}^*|^2 \left[I_3\left(\frac{m_b^2}{m_W^2}, \frac{m_b^2}{m_c^2}\right) O_{LL} - m_c^2 I_4 O' \right] \end{array} \right]$$

$O_{LL} = \bar{u}\gamma_\mu(1-\gamma_5)c\ \bar{u}\gamma^\mu(1-\gamma_5)c$

$O' = \bar{u}(1+\gamma_5)c\ \bar{u}(1+\gamma_5)c$

$\Delta m_D = (4.5 \pm 0.5) \times 10^{-18}$ GeV

Experiment: E791 - FNAL

$$r_{mix} = \frac{1}{2\Gamma^2} \left| \frac{q}{p} \right|^2 \left[(\Delta m)^2 + \frac{(\Delta\Gamma)^2}{4} \right]$$

$< 0.85\%$ at 90% CL $\quad [\Delta m_D \lesssim 1.5 \times 10^{-13}$ GeV

Long Distance Update on Δm_D

- HQET $\Delta m_D \sim (1\text{-}2) \times 10^{-5}\,\Gamma$ (Georgi)
  $\simeq 10^{-17}$ GeV

  ? Reliability ? Large hadronic dynamical effects in m_c range

- 2-Particle Intermediate States (Donoghue et al)

  Use Data (!) on $D \to K^+\pi^-$

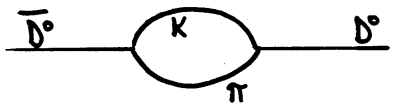
  $\Delta m_D = -(2.2 \pm 5.5) \times 10^{-16}$ GeV ← possibility of cancellation

- Pole Amplitudes (Golowich)

  $\Delta m_D = 5 \times 10^{-16}$ GeV

$$\boxed{\Delta m_{D\ \text{L.D.}} \sim 10^{-17} - 10^{-16}\ \text{GeV}}$$

Dispersive Example



$$\Delta m_D^{disp} \simeq \frac{1}{2\pi} \ln \frac{m_D^2}{\Lambda^2} \left[\Gamma(D^0 \to K^+K^-) + \Gamma(D^0 \to \pi^+\pi^-) - 2\left[\Gamma(D^0 \to K^-\pi^+)\,\Gamma(D^0 \to K^+\pi^-)\right]^{1/2} \right]$$

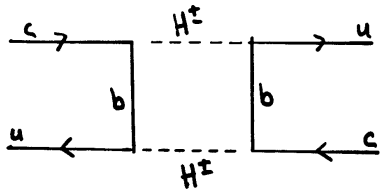
Multi-Higgs-Doublet-Models

• 2HDM

$$\begin{pmatrix}\phi_1^+\\ \phi_1^0\end{pmatrix}\quad\begin{pmatrix}\phi_2^+\\ \phi_2^0\end{pmatrix} \xrightarrow{\text{S.S.B.}} h^0, H^0, A^0, H^\pm$$

$$v_1^2 + v_2^2 = v_{sm}^2$$

$$\Rightarrow \tan\beta \equiv v_2/v_1$$

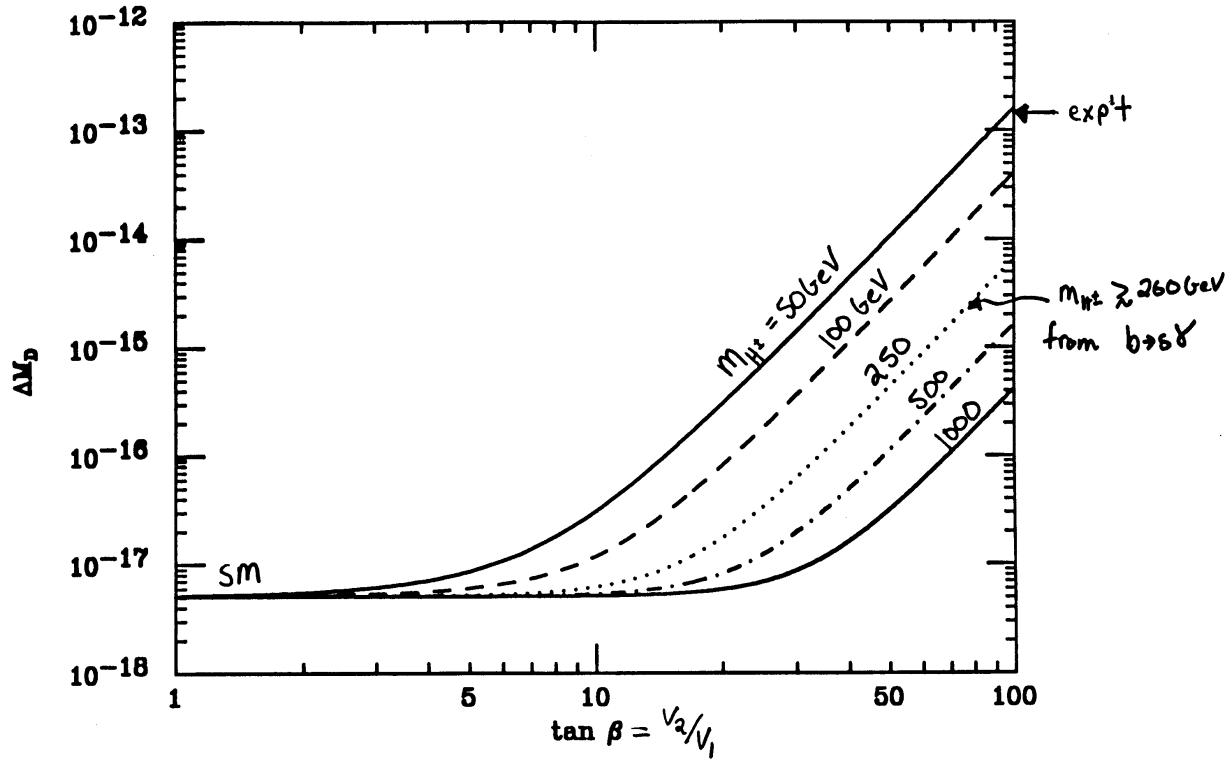


• No tree-level FCNC if each fermion type receives mass from only 1 doublet

$$\mathcal{L}_{cbH^+} = \frac{g}{2\sqrt{2}\,m_W}\left\{\frac{m_c}{\tan\beta}\,V_{cb}\,\bar{c}(1-\gamma_5)b + m_b\tan\beta\,V_{cb}\,\bar{c}(1+\gamma_5)b\right\}H^+ + \text{h.c.}$$

$$\Delta m_D = \frac{G_F^2 m_W^2 m_D}{6\pi^2} f_D^2 \eta \left|V_{cb}V_{ub}^*\right|^2 \left[\frac{m_c^2}{m_W^2 \tan^2\beta} F_1\left(m_c^2/m_{H^\pm}^2\right) + \frac{m_b^2\tan^2\beta}{m_W^2} F_2\left(m_b^2/m_{H^\pm}^2\right)\right]$$

Two-Higgs-Doublet-Model II



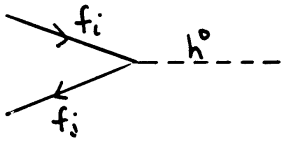
$B \to X \tau \bar{\nu}_\tau$ Constraints !

Flavor Changing Neutral Higgs

SM Higgs Coupling $\sim (\sqrt{2}\, G_F)^{1/2} m_f$

2HDM w/ FC Couplings

(Pakvasa + Sugawara; Hall + Weinberg; Cheng + Sher; Hou ...)

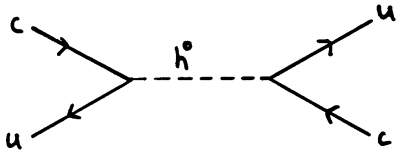


$\sim (\sqrt{2}\, G_F)^{1/2} \sqrt{m_i m_j}\, \Delta_{ij}$

Δ_{ij}: mixing factor

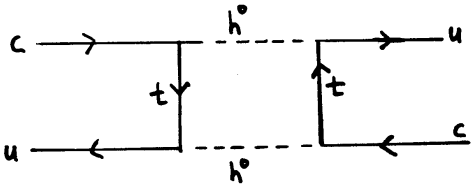
$\Rightarrow$ small effect for light fermions

Tree-level



$$\Delta m_D = \frac{4\sqrt{2}\, G_F\, m_u m_c\, \Delta^2\, f_D^2\, m_D}{3\, m_h^2}$$

box



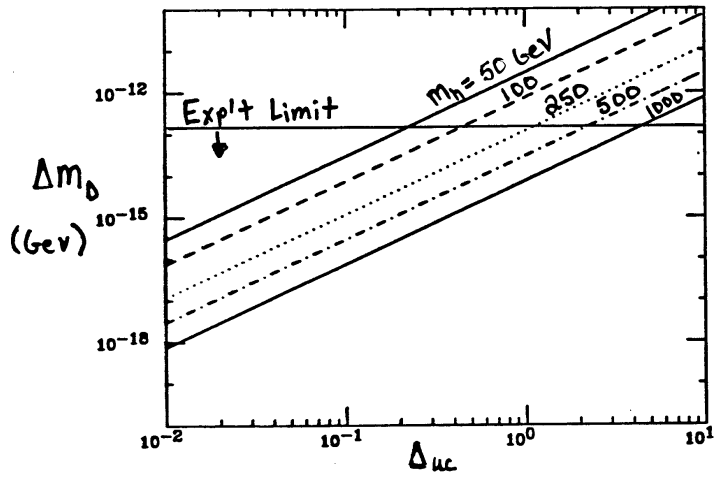
could be big!

$$\Delta m_D = \frac{2 G_F^2\, m_u m_c m_t^2\, \Delta^4 f_D^2\, m_D}{3\pi^2 m_h^2} \left[\frac{1}{(x-1)} - \frac{\ln x}{(x-1)^2} - \frac{1}{2(x-1)^3}\left(x^2 - 4x + 3 + 2\ln x\right) \right]$$

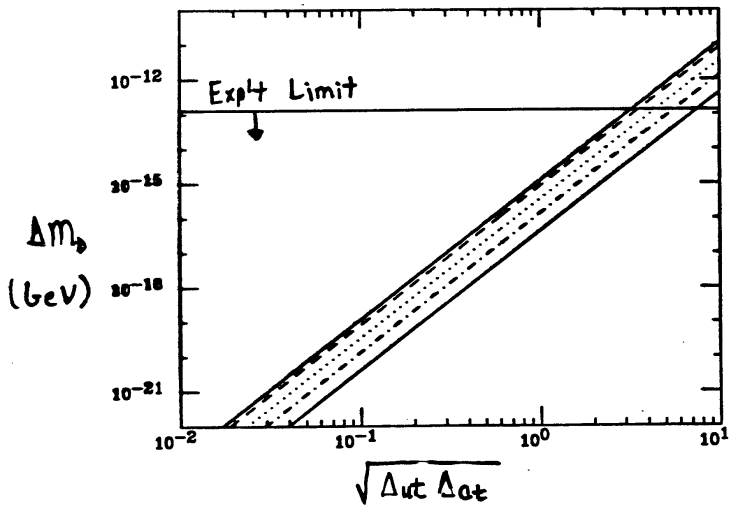
$x \equiv m_t^2/m_h^2$

Flavor Changing Neutral Higgs

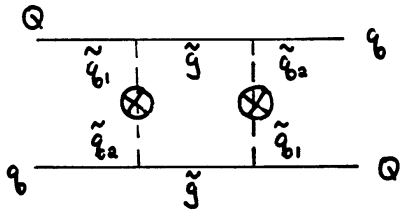
Tree-Level Contributions



Box Contributions



Supersymmetry

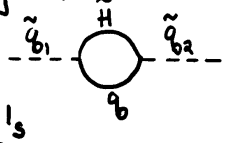


Gluino

All vertices $\sim \alpha_s$ ⇒ Potentially Lethal!

- MSSM – Super CKM = SM CKM
  ⇒ $\tilde{g}\tilde{q}_b q_b$ couplings flavor diagonal

  FCNC generated by $\tilde{q}_b$ mass mixing



  Invoke Super-GIM ⇒ degenerate $\tilde{q}_b$'s

Re Δm_K	$\dfrac{\Delta m^2_{\tilde{d}\tilde{s}}}{\tilde{m}_o^2}$	$\lesssim$	$1/30$	Ellis et al 1982
Im Δm_K			10^{-3}	
Δm_D	$\dfrac{\Delta m^2_{\tilde{u}\tilde{c}}}{\tilde{m}_o^2}$	$\lesssim$	10^{-2}	Burdman et al
Δm_{B_d}	– Similar...			

• Squark Alignment (Nir, Seiberg)

d-squark mass matrices aligned w/ d-quarks

→ Can satisfy all FCNC constraints w/o degenerate $\tilde{q}$'s

But, u-squarks/quarks are not aligned

⇒ Δm_D at exp't limit! **Prediction!**

• Decoupling SUSY (Dine, Kaplan, Nelson, Cohen, etal)

$$\mathcal{A}_{susy} \sim \frac{m_W^2}{m_{\tilde{g}}^2} F\left(m_{\tilde{q}_i}^2/m_{\tilde{g}}^2\right)$$

↑ SUSY Decoupling Effect

$$\text{Re } \Delta m_K \quad \frac{\theta_{12}^2}{m_{\tilde{q}}^2} \lesssim \left(\frac{1}{50\,\text{TeV}}\right)^2$$

$$\text{Im } \Delta m_K \quad \frac{\theta_{12}^2}{m_{\tilde{q}}^2} \lesssim \left(\frac{1}{500\,\text{TeV}}\right)^2$$

CP - Violation

(CP) in Mixing:

$$\frac{1-|q/p|^2}{1+|q/p|^2} = \frac{2\,\mathrm{Re}\,\epsilon_D}{1+|\epsilon_D|^2}$$

$$\simeq \frac{\mathrm{Im}\,\Gamma_{12} - \frac{\Delta\Gamma}{\Delta m}\,\mathrm{Im}\,m_{12}}{\Delta m_D\left[1 + \Delta\Gamma^2/4\Delta m^2\right]}$$

$$\simeq 0.05 - 0.1 \qquad \text{taking } \Delta m_D = 10^{-16}\ \text{GeV}$$

(CP) in interference between mixing and decay:

$$\tan\phi = \frac{\mathrm{Im}\,(q/p)}{\mathrm{Re}\,(q/p)} \simeq \frac{-\frac{\mathrm{Im}\,m_{12}}{\mathrm{Re}\,m_{12}} - \frac{\mathrm{Im}\,\Gamma_{12}}{4\,\mathrm{Re}\,\Gamma_{12}}}{1. + \frac{1}{2}\left[\frac{\mathrm{Im}\,m_{12}}{\mathrm{Re}\,m_{12}} - \frac{\mathrm{Im}\,\Gamma_{12}}{\mathrm{Re}\,\Gamma_{12}}\right]}$$

$$\simeq (5 - 10) \times 10^{-3}$$

can be $\simeq 0.1$ with new physics!

Direct CP Violation, or
CP Violation in Decay $(m^0, m^\pm)$

CPT : $\Gamma_{total} = \bar{\Gamma}_{total}$

CP : $\Gamma_{partial} = \bar{\Gamma}_{partial}$ $\quad [m \to f \text{ vs. } \bar{m} \to \bar{f}]$

~~CP~~ : $\Gamma_{partial} \neq \bar{\Gamma}_{partial}$

- requires interference between 2 phases

Assume 2 Amplitudes with $|A_1| \gg |A_2|$ [tree + penguin]

and $A_i = V_i \hat{A}_i \quad (i=1,2)$

(V_i: CKM elements)

$$\Gamma = \int dL \, |A_1 + A_2|^2$$

$$= \int dL \, |V_1 \hat{A}_1 + V_2 \hat{A}_2|^2$$

and $$\bar{\Gamma} = \int dL \, |V_1^\dagger \hat{A}_1 + V_2^\dagger \hat{A}_2|^2$$

Define CP Violation Asymmetry

$$a \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \simeq \frac{-2\,\mathrm{Im}(V_1 V_2^\dagger)}{\Gamma} \int \mathrm{Im}\,\hat{A}_1 \,\mathrm{Re}\,\hat{A}_2 \, dL$$

$\Rightarrow$ 2 phases - One odd under CP $\times$ One even under CP

SM: CKM phase; strong interaction, final state interaction

Conclusions

1) QCD corrections dramatically enhance rare decays!

- $D \to X_u \gamma$ increased 9 orders of magnitude but still below long distance
- $D \to X_u \ell\ell$ increased 10^4 - comparable to long distance

⇒ Measure $D \to X_u \gamma$ to sharpen non-perturbative calculational tools

Measure $D \to X_u \ell^+ \ell^-$ to look for new physics

2) D-$\bar{D}$ Mixing: Window wide open for new physics!
⇒ Probes models not constrained by $Q = -\frac{1}{3}$ FCNC!

3) (CP): Has yet to be explored in D system
Some window for new physics