Charm Decays and Mixing: Standard Model and Beyond

- 1[ST] programmatic look at QCD corrections for rare charm decays
- Update predictions for D-$\bar{D}$ Mixing and $\cancel{CP}$
- Identify processes which have open window for new physics and explain why they're important to explore
- Identify opportunities for τcF

Burdman, Golowich, JLH, Pakvasa

GIM Mechanism - Glashow, Iliopoulos, Maiani (1970)

[Introduced to suppress Kaon FCNC]

$$\mathcal{A}_{\text{penguin}} \sim \sum_{i=1}^{3} V_{iQ} V_{iq}^{*} \, F\left(m_i^2/m_W^2\right)$$

CKM is Unitary! $\sum_i V_{iQ} V_{iq}^{*} = 0$

$$\Rightarrow \quad V_{1Q} V_{1q}^{*} = - V_{2Q} V_{2q}^{*} - V_{3Q} V_{3q}^{*}$$

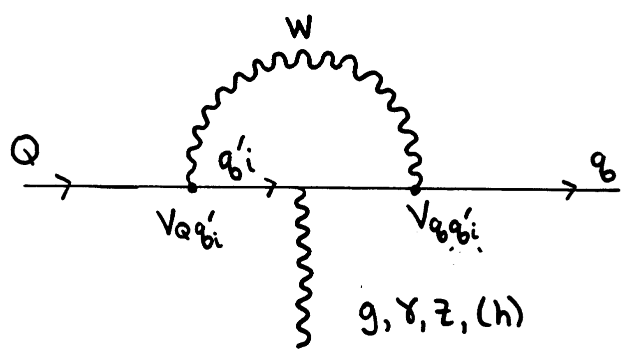
$$\mathcal{A}_{\text{penguin}} \sim V_{2Q} V_{2q}^{*} \left[F\left(m_2^2/m_W^2\right) - F\left(m_1^2/m_W^2\right) \right]$$

$$+ V_{3Q} V_{3q}^{*} \left[F\left(m_3^2/m_W^2\right) - F\left(m_1^2/m_W^2\right) \right]$$

$\rightarrow 0$ if quarks were degenerate

$$\boxed{\text{Size of FCNC related to size of internal quark mass splittings}}$$

Typical Flavor Changing Neutral Current Vertex Diagram (Penguin!)



$= -\frac{1}{3}Q$ $\quad$ u, c, t $\quad$ $= -\frac{1}{3}q_b$

$= +2/3\, Q$ $\qquad$ d, s, b $\qquad$ $= +2/3\, q_b$

Typical Values of Branching Fractions

Meson/Quark	B
K	$10^{-10} - 10^{-8}$
D	$10^{-18} - 10^{-10}$
B	$10^{-8} - 10^{-4}$
t	$10^{-12} - 10^{-9}$

c s,b u

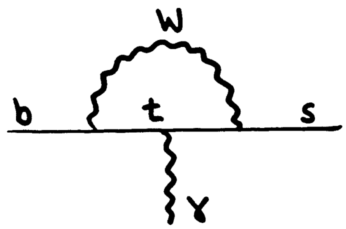
c s,b u

Small SM Rates in D System

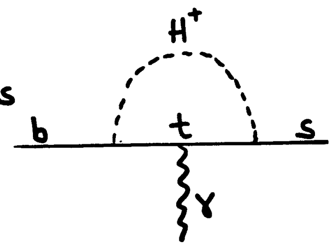
⇒ Big window for new physics!

Loop-Level Processes are Sensitive to New Physic

SM:

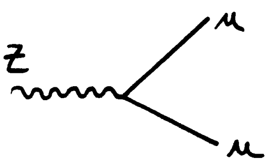


vs.
New Physics

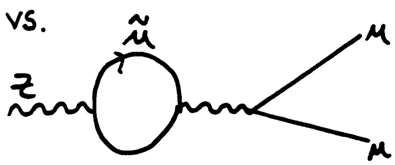


['Large' Rates in B System]

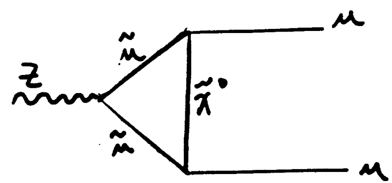
Compare to Precision EW:



vs.



+



JLH

FCNC Branching Fractions in Standard Model

Circa '96

Decay Mode	Experimental Limit	$B_{S.D.}$	$B_{L.D.}$
$D^0 \to \mu^+\mu^-$	$< 1.1 \times 10^{-5}$	$(1-20) \times 10^{-19}$	$< 3 \times 10^{-15}$
$D^0 \to e^+e^-$	$< 1.3 \times 10^{-4}$	even smaller !	
$D^0 \to \mu^\pm e^\mp$	$< 1.0 \times 10^{-4}$	0	0
$D^0 \to \gamma\gamma$	—	10^{-16}	$< 3 \times 10^{-9}$
$D \to X_u + \gamma$		1.4×10^{-17}	
$D^0 \to \rho^0\gamma$	$< 1.4 \times 10^{-4}$		$< 2 \times 10^{-5}$
$D^0 \to \phi^0\gamma$	$< 2.0 \times 10^{-4}$		$< 10^{-4}$
$D^+ \to \rho^+\gamma$	— CLEO		$< 2 \times 10^{-4}$
$D^+ \to \bar{K}^{*+}\gamma$	— prelim		3×10^{-7}
$D^0 \to \bar{K}^{*0}\gamma$	—		1.6×10^{-4}
$D \to X_u + \ell^+\ell^-$	E791 FNAL prelim	4×10^{-9}	
$D^0 \to \pi^0\mu\mu$	$< 1.7 \times 10^{-4}$		*
$D^0 \to \bar{K}^0 ee/\mu\mu$	$< 17.0/2.5 \times 10^{-4}$		$< 2 \times 10^{-\cancel{15}\,8}$
$D^0 \to \rho^0 ee/\mu\mu$	$< 2.4/4.5 \times 10^{-4}$		
$D^+ \to \pi^+ ee/\mu\mu$	$< \cancel{250}\,6.6/\cancel{4.6}\,1.8 \times 10^{-5}$	few$\times 10^{-10}$	$< 10^{-8}$
$D^+ \to K^+ ee/\mu\mu$	$< 480/8.5 \times 10^{-5}$		$< 10^{-\cancel{15}\,8}$
$D^+ \to \rho^+\mu\mu$	$< 5.8 \times 10^{-4}$		
$D^0 \to X_u + \nu\bar{\nu}$		2.0×10^{-15}	
$D^0 \to \pi^0\nu\bar{\nu}$	—	4.9×10^{-16}	$< 6 \times 10^{-16}$ *
$D^0 \to \bar{K}^0\nu\bar{\nu}$	—		$< 10^{-12}$
$D^+ \to X_u + \nu\bar{\nu}$	—	4.5×10^{-15}	
$D^+ \to \pi^+\nu\bar{\nu}$	—	3.9×10^{-16}	$< 8 \times 10^{-16}$
$D^+ \to K^+\nu\bar{\nu}$	—		$< 10^{-14}$

No QCD!

Charm Radiative Decays

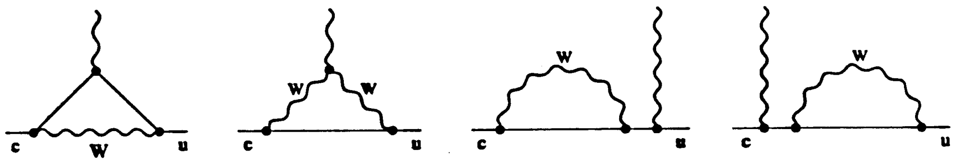


Figure 1: One-loop diagrams generating the $c \to u\gamma$ transition.

$$\mathcal{A} = \frac{e G_F}{4\sqrt{2}\pi^2} \sum_{i=d,s,b} \lambda_i F_2(x_i)\, \bar{u}\, \sigma_{\mu\nu} q^\nu m_c P_R\, c\, \epsilon^\mu$$

$$\lambda_i = V_{ci}^* V_{ui} \;;\; x_i = \frac{m_{q_i}^2}{m_W^2}$$

$$\Gamma(c \to u\gamma) = \frac{\alpha G_F^2 m_c^5}{128\pi^4} \Big[\lambda_s \left(F_2(x_s) - F_2(x_d) \right) + \lambda_b \left(F_2(x_b) - F_2(x_d) \right) \Big]^2$$

Quark	F_2	$\lvert V_{ci}^* V_{ui} \rvert F_2$
d	1.6×10^{-9}	3.4×10^{-10}
s	2.9×10^{-7}	6.7×10^{-8}
b	3.3×10^{-4}	3.2×10^{-8}

$$B(c \to u\gamma) = \frac{\Gamma(c \to u\gamma)}{\Gamma(c \to X\ell\nu)} \qquad B(D^+ \to X\ell^+\nu) = 1.4 \times 10^{-17} \quad !!$$

data

Large QCD Corrections!

Operator Product Expansion (technique pioneered: Gilman + Wise '79)

$m_b < \mu < m_W$

$$\mathcal{H}_{eff}^{\Delta c=1} = \frac{4G_F}{\sqrt{2}}\left[\sum_{q=d,s,b} \lambda_q \left[C_1(\mu)Q_1^q(\mu) + C_2(\mu)Q_2^q(\mu)\right] + \sum_{i=3}^{8} C_i(\mu)Q_i(\mu)\right]$$

$\mu < m_b$

$$\mathcal{H}_{eff}^{\Delta c=1} = \frac{4G_F}{\sqrt{2}}\left[\sum_{q=d,s} \lambda_q \left[C_1(\mu)Q_1^q(\mu) + C_2(\mu)Q_2^q(\mu)\right] + \sum_{i=3}^{8} C_i'(\mu)Q_i'(\mu)\right]$$

$$\mathcal{M}^{LD}(c\to u\gamma) = \frac{4G_F}{\sqrt{2}}\, C_7^{eff}(\mu)\langle u\gamma|\, Q_7(\mu)\,|c\rangle\, \lambda_b$$

Complete set of operators governing $c \to u$ transitions

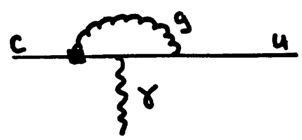
$Q_1^q = (\bar{u}_\alpha \gamma_\mu P_L q_\beta)(\bar{q}_\beta \gamma^\mu P_L c_\alpha)$ $\quad$ $Q_2^q = (\bar{u}_\alpha \gamma_\mu P_L q_\alpha)(\bar{q}_\beta \gamma^\mu P_L c_\beta)$

$Q_3 = (\bar{u}_\alpha \gamma_\mu P_L c_\alpha)\sum_q (\bar{q}_\beta \gamma^\mu P_L q_\beta)$ $\quad$ $Q_4 = (\bar{u}_\alpha \gamma_\mu P_L c_\beta)\sum_q (\bar{q}_\beta \gamma^\mu P_L q_\alpha)$

$Q_5 = (\bar{u}_\alpha \gamma_\mu P_L c_\alpha)\sum_q (\bar{q}_\beta \gamma^\mu P_R q_\beta)$ $\quad$ $Q_6 = (\bar{u}_\alpha \gamma_\mu P_L c_\beta)\sum_q (\bar{q}_\beta \gamma^\mu P_R q_\alpha)$

$Q_7 = \frac{e}{16\pi^2} m_c (\bar{u}_\alpha \sigma_{\mu\nu} P_R c_\alpha) F^{\mu\nu}$ $\quad$ $Q_8 = \frac{g}{16\pi^2} m_c (\bar{u}_\alpha \sigma_{\mu\nu} T^a_{\alpha\beta} P_R c_\beta) G^{a\mu\nu}$

Operator Mixing:



Grinstein, Springer, Wise – 2-loop operator mixing

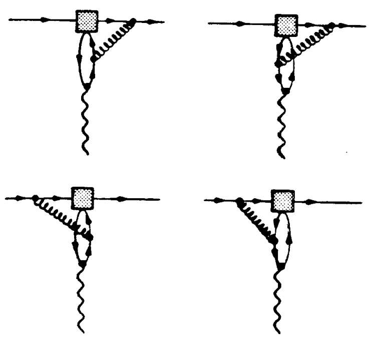


Fig. 1. Some two-loop Feynman diagrams contributing to the mixing of O_2 with O_7.

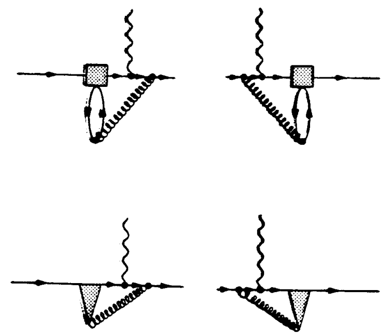
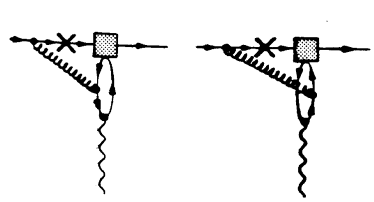


Fig. 2. Some two-loop Feynman diagrams contributing to the mixing of O_2 with O_7.

states is modeled by b-quark decay, only the tree level matrix element of the magnetic-moment-operator term $C_7(m_b)O_7$ is required to determine the rate for $B \to \gamma X_s$. The coefficient $C_7(m_b)$ is determined (via the renormalization group equations) by the anomalous dimension matrix for the renormalization of the operators O_1–O_7 and the initial values in eqs. (13).

In this paper we obtain a simple (but fairly accurate) analytic approximation to $C_7(\mu)$ by truncating the operator basis to O_1, O_2 and O_7. (A similar approximation was used in the effective hamiltonian for $\Delta S=2$ K^0–$\bar{K}^0$ mixing [9].) In a further publication we shall present a more complete treatment using the full anomalous dimension matrix and giving more details of the computation. The mixing of O_1 and O_2 is similar to the operator mixing in the weak $\Delta S=1$ nonleptonic decays and it gives [10,11] $\gamma_{11}=\gamma_{22}=-g^2/8\pi^2$, $\gamma_{12} = \gamma_{21} = 3g^2/8\pi^2$. The renormalization of O_7 (including the factor of m_b) yields [6] $\gamma_{77}=2g^2/3\pi^2$. The mixing of operators O_1 and O_2 with O_7 arises at two loops from the Feynman diagrams in figs. 1–4. (Previous work on QCD



. Some two-loop Feynman diagrams contributing to the of O_2 with O_7.

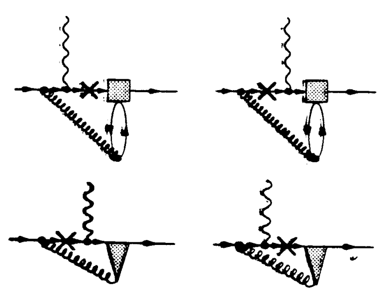


Fig. 4. Some two-loop Feynman diagrams contributing to the mixing of O_2 with O_7.

Wilson Coefficients

Calculate C_i at $\mu = m_W \Rightarrow$ matching conditions

At LL: $C_{1,3-6}^{(m_W)} = 0 \qquad C_2(m_W) = -1$

$$C_7^q(m_W) = -1/2\, F_2(x_q)\lambda_q \qquad C_8^q(m_W) = -1/2\, D(x_q)\lambda_q$$

Use RGE to evolve to $\mu = m_b \Rightarrow \mu = m_c$

$$\mu \frac{d}{d\mu} \mathcal{H} = 0 \Rightarrow \underbrace{\mu \frac{d}{d\mu}}_{\mu \frac{d\alpha_s}{d\mu} \frac{1}{d\alpha_s}} C_j(\mu) = \gamma_{ij}^T(g)\, C_i(\mu)$$

γ → anomalous dimension matrix

β-function!

$$-\frac{23}{3}\alpha_s \frac{dc_j(\mu)}{d\alpha_s} = \tilde{\gamma}_{ij}\, C_i(\mu) \qquad \tilde{\gamma}_{ij} = \frac{8\pi^2}{g_s^2}\gamma_{ij}$$

erform
imilarity
ransformation

$$-\frac{23}{3}\alpha_s \frac{d}{d\alpha_s} C_K = \underbrace{(V^{-1})_{K\ell}\, \tilde{\gamma}_{\ell m} V_{mn}}_{\text{Diagonalizes } \tilde{\gamma}\,!\, = (\tilde{\gamma}^T_{Diag})_{Kn}\delta_{Kn}} C_n$$

$$\int_{m_b}^{m_W} \frac{dc_K}{C_K} = \frac{-3}{23}\int_{m_b}^{m_W} \frac{d\alpha_s}{\alpha_s} (\tilde{\gamma}^T_{Diag})_{KK}$$

$$\boxed{c_i(m_b) = \eta^{-3/23} V_{ij} (\exp \tilde{\gamma}^T_{Diag})_{jj} V^{-1}_{jK} C_K(m_W)} \qquad \underline{\text{LL}}\,!$$

$\eta \to \alpha_s(m_b)/\alpha_s(m_W)$

RGE $\Rightarrow$

$$C_7^{eff}(\mu) = U^4_{7\ell}(\mu, m_b)\, \underbrace{M(m_b)}_{\text{quark matching matrix}}\, U^5_{\ell j}(m_b, m_W)\, C_j(m_W)$$

$$@\mu = m_c: \quad = 0.46\, \underbrace{C_7(m_W)}_{-0.41\times10^{-3}} + 0.13\, \underbrace{C_8(m_W)}_{-0.24\times10^{-2}} - 0.31\, \underbrace{C_2(m_W)}_{-1}.$$

$\underline{B(c\to u\gamma) = (4.2-7.9)\times10^{-12}}$ for $m_c \leq \mu \leq 2m_c$

$\Rightarrow$ GIM power suppression replaced by QCD $\ln m_b/m_c$ contribution

But, $\lambda_b = -\lambda_d - \lambda_s$ suppression remains!

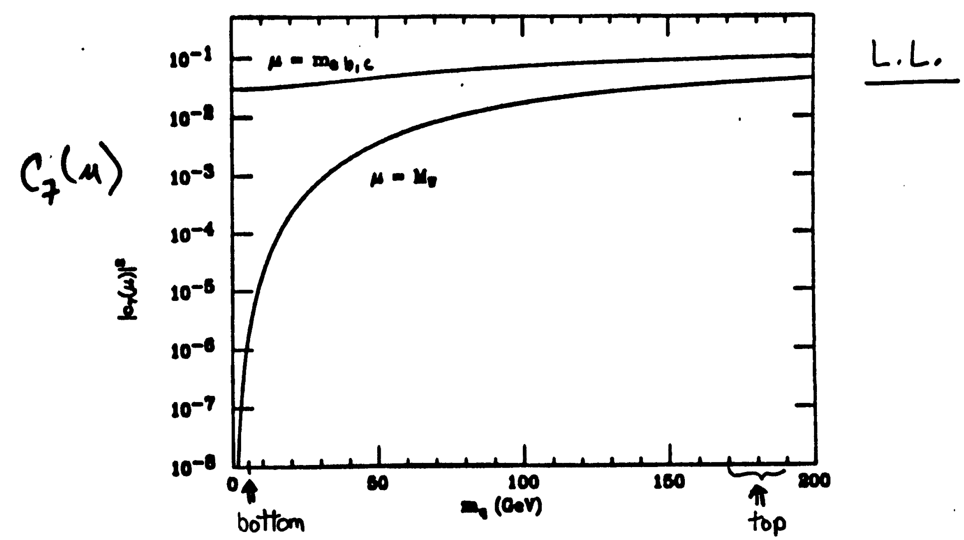


Figure 2. Dependence of c_7 on Intermediate-quark Mass

We now consider the case of radiative charm transitions. The $|\Delta c| = 1$ effective hamiltonian can be written as

$$H_{\rm eff}^{|\Delta c|=1} = \frac{-4G_F}{\sqrt{2}}\lambda_b \sum_{k=1}^{10} c_k(\mu)O_k(\mu)\,, \tag{18}$$

Leading 2-loop contributions

Greub et al

2-loop matrix element of $\mathcal{H}_{eff}^{\Delta c=1}$

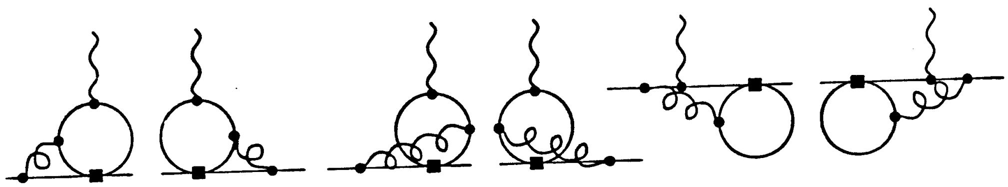


Figure 2: Diagrams contributing to the two-loop $c \to u\gamma$ matrix element of

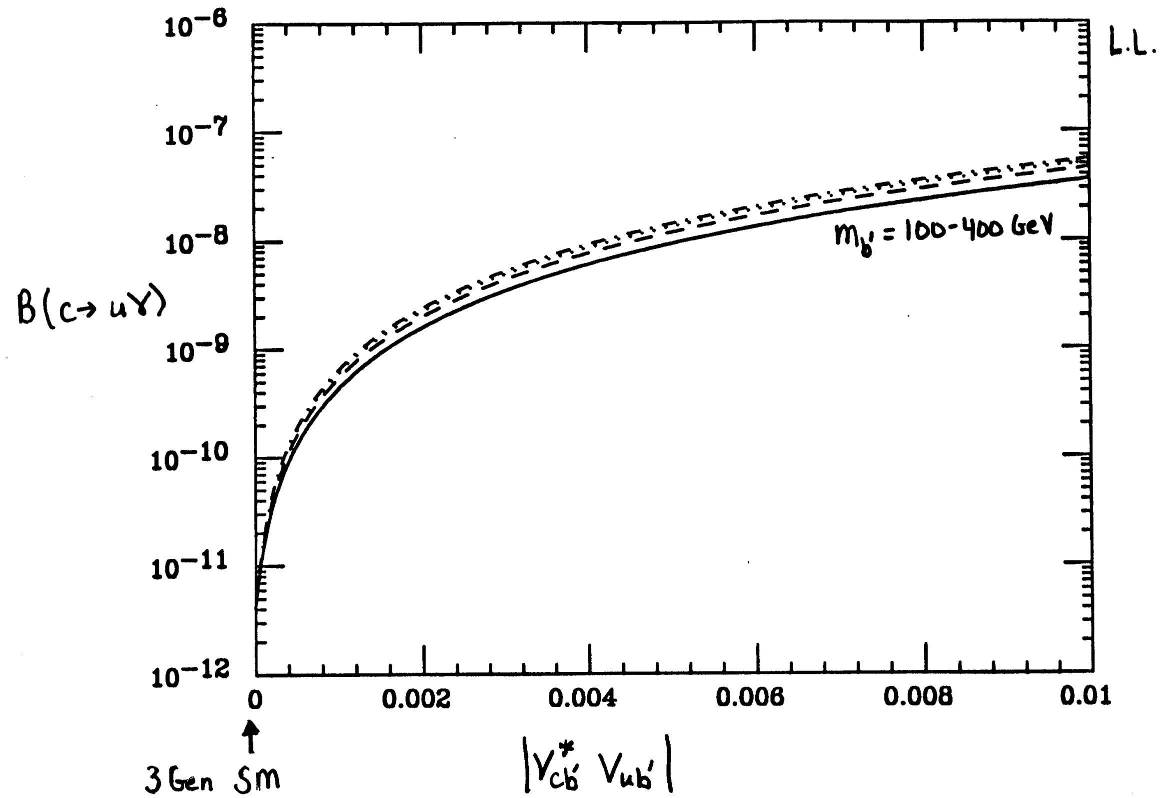
$$\mathcal{M}(c\to u\gamma) = \frac{4G_F}{\sqrt{2}} A \langle u\gamma | Q_7(m_c) | c\rangle_{tree}$$

$$A = \frac{\alpha_s}{4\pi} V_{cs}^* V_{us} \underbrace{C_2(m_c)\left[f(m_s^2/m_c^2) - f(m_d^2/m_c^2)\right]}_{0.86} + \mathcal{O}(V_{cb}^* V_{ub})$$

$$\Rightarrow B(D\to X_u\gamma) = 5\times 10^{-8}\ !!!$$

1-Loop	$\longrightarrow$	L.L.	$\longrightarrow$	Leading 2-Loop
GIM suppressed	$\longrightarrow$	CKM (λ_b) suppressed	$\longrightarrow$	Cabibbo, m_s/m_c suppressed

New Physics Example : 4^{th} Generation



Full NLO to $c \to u\gamma$ can now be computed!

Radiative Penguin - $b \to s\gamma$ at NLO!

NLO: matrix element - QCD bremsstrahlung corrections
1-loop $\langle s\gamma g | O_i | b \rangle$ Ali, Greub Pott

- Virtual Corrections Greub, Hurth, Wyler
2-loop $\langle s\gamma | O_i | b \rangle$

Wilson coeff - $O(\alpha_s)$ terms in matching conditions Lin, Liu, Yao Greub, Hurth

- $O(\alpha_s^2)$ anomalous dimension matrix used in RGE Chetyrkin, Misiak, Münz

E_γ Spectrum - Consistent treatment of Fermi motion in HQET Kagan, Neubert

Electroweak - 2-loop EW Corrections Czarnecki + Marciano

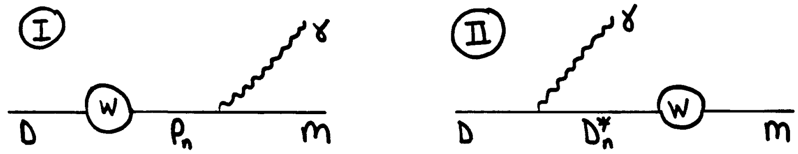
$$\Rightarrow \quad \boxed{B(B \to X_s\gamma) = (3.29 \pm 0.33) \times 10^{-4}}$$

ALEPH: $B(B \to X_s\gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}$

CLEO: $B(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$

Exclusive Radiative Decays - Long Range Effects $c \to u \gamma$

- Pole Amplitudes - $c\bar{q}_1 \to q_2 \bar{q}_3 \gamma$



$$\mathcal{H}_W^{eff} = \frac{-G_F}{\sqrt{2}} \left[: a_1 (\bar{u} d')(\bar{s}' c) + a_2 (\bar{s}' d')(\bar{u} c) : \right]$$

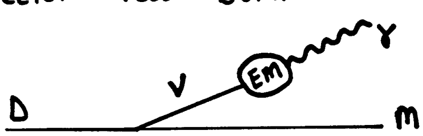
Bauer Stech Wirbel

fit to data on $D \to \bar{K}\pi$

$$\mathcal{A}_I^{pc}(D \to m\gamma) = \sum_n \frac{h_{m\gamma\rho_n}}{m_D^2 - m_{\rho_n}^2} \langle \rho_n | \mathcal{H}_W^{eff} | D \rangle$$

$$\mathcal{A}_{II}^{pc}(D \to m\gamma) = \sum_n \frac{h_{D_n^* \gamma D}}{m_D^2 - m_{D_n^*}^2} \langle m | \mathcal{H}_W^{eff} | D_n^* \rangle$$

- Vector Meson Dominance - $c \to q_1 \underbrace{\bar{q}_2 q}_{\to \gamma}$



- Phenomenological Approach - use data $D \to mV$
- Theoretical Model - BSW

$$|\mathcal{A}_{VMD}|^2 = \frac{G_F^2 |V_{cq}^* V_{qu}|^2}{2 m_D K^2} a_i(m_c^2) f_X^2 I$$

$$\left(\frac{4\pi\alpha}{f_V^2}\right)\left[(m_D + m_V)^2 A_1^2(q_0^2) + \frac{4K^2 m_D^2 V^2(q_0^2)}{(m_D + m_V)^2}\right]$$

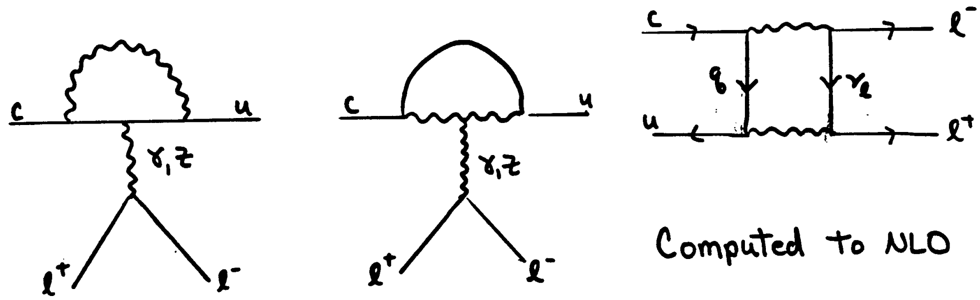
Long Distance Predictions

Table 11 Amplitude and Branching Fraction Predictions

Mode	$\mathcal{A}^{pc}$			$\mathcal{A}^{pv}$	$\mathcal{B}_{D\to M\gamma}$ (10^{-5})
	P-I	P-II	VMD	VMD	
$D_s^+\rho^+\gamma$	8.2	−1.9	±3.2	±2.8	6 − 38
$D^0\bar{K}^{*0}\gamma$	5.6	−5.9	±3.8	±(5.1 − 6.8)	7 − 12
$D_s^+b_1^+\gamma$	7.2	—	—	—	∼ 6.3
$D_s^+a_1^+\gamma$	1.2	—	—	—	∼ 0.2
$D_s^+a_2^+\gamma$	2.1	—	—	—	∼ 0.01
$D^+\rho^+\gamma$	1.3	−0.4	±1.6	±1.9	2 − 6
$D^+b_1^+\gamma$	1.2	—	—	—	∼ 3.5
$D^+a_1^+\gamma$	0.5	—	—	—	∼ 0.04
$D^+a_2^+\gamma$	3.4	—	—	—	∼ 0.03
$D_s^+K^{*+}\gamma$	2.8	−0.5	±0.9	±1.0	0.8 − 3
$D_s^+K_2^{*+}\gamma$	6.0	—	—	—	∼ 0.2
$D^0\rho^0\gamma$	0.5	−0.5	±(0.2 − 1.0)	±(0.6 − 1.0)	0.1 − 0.5
$D^0\omega^0\gamma$	0.6	−0.7	±0.6	±0.7	≃ 0.2
$D^0\phi^0\gamma$	0.7	−1.6	±(0.6 − 3.5)	±(0.9 − 2.1)	0.1 − 3.4
$D^+K^{*+}\gamma$	0.4	−0.1	±0.4	±0.4	0.1 − 0.3
$D^0K^{*0}\gamma$	0.2	−0.3	±0.2	±0.2	≃ 0.01

- Long range effects overwhelm short distance.
- Wide range of predictions
  - observation would 'select' calculational technique
  - ⇒ extrapolate to $B \to \rho/\omega\,\gamma$ which will be used to determine $|V_{td}|/|V_{ts}|$

$D \to X_u \ell^+ \ell^-$



Computed to NLO

$$\mathcal{M} = \frac{\sqrt{2}\, G_F \alpha}{\pi} \left[C_9^{eff}\, \bar{u}_L \gamma_\mu c_L \bar{\ell} \gamma^\mu \ell + C_{10}\, \bar{u}_L \gamma_\mu c_L \bar{\ell} \gamma^\mu \gamma_5 \ell - 2 C_7^{eff} m_c\, \bar{u}_L i \sigma_{\mu\nu} \frac{q^\nu}{q^2} c_R \bar{\ell} \gamma^\mu \ell \right]$$

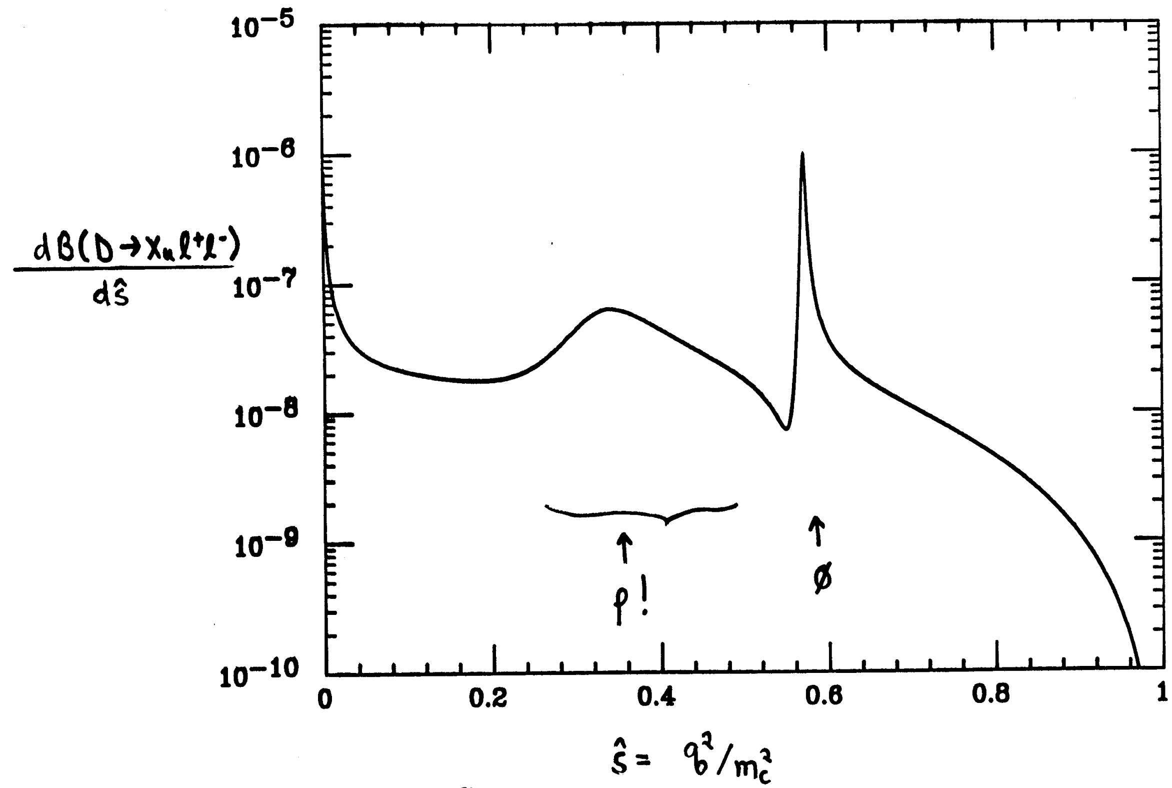
q^2 = momentum transfer to $\ell^+\ell^-$ $\qquad (\hat{s} = q^2/m_c^2)$

Long distance resonance contributions

$D \to X_u V$, $V \to \ell^+\ell^-$ $\qquad V = \phi, \rho \qquad$ peaks at $q^2 = m^2_{\phi,\rho}$

$\Rightarrow$ Effective $(\bar{u}_L \gamma_\mu c_L)(\bar{\ell} \gamma^\mu \ell)$ interaction

$$C_9^{eff}(\mu) = C_9(\mu) + \underbrace{Y(\mu, \hat{s})}_{\text{contribution from light-quarks}} - \frac{3\pi}{\alpha^2 m_c^2} \sum_{\phi,\rho} \frac{m_{V_i} \Gamma(V_i \to \ell^+\ell^-)}{\left(\hat{s} - \frac{m^2_{V_i}}{m_c^2}\right) + \frac{i \Gamma_{V_i} m_{V_i}}{m_c^2}}$$



$B(D \to X_u \ell^+\ell^-) = 2 \times 10^{-8}$

$\ell = e$ short distance

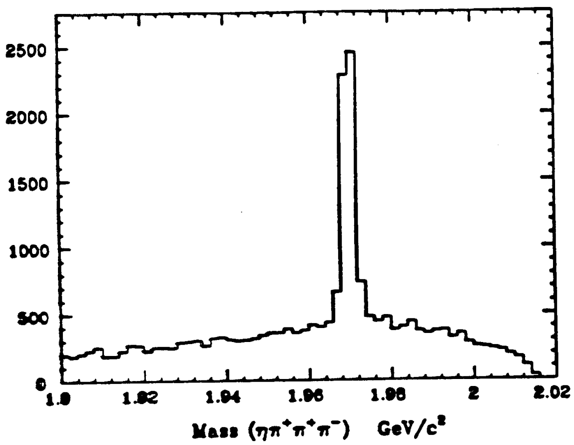
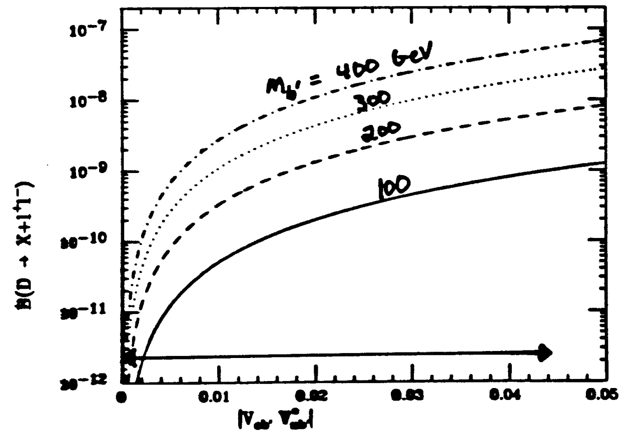


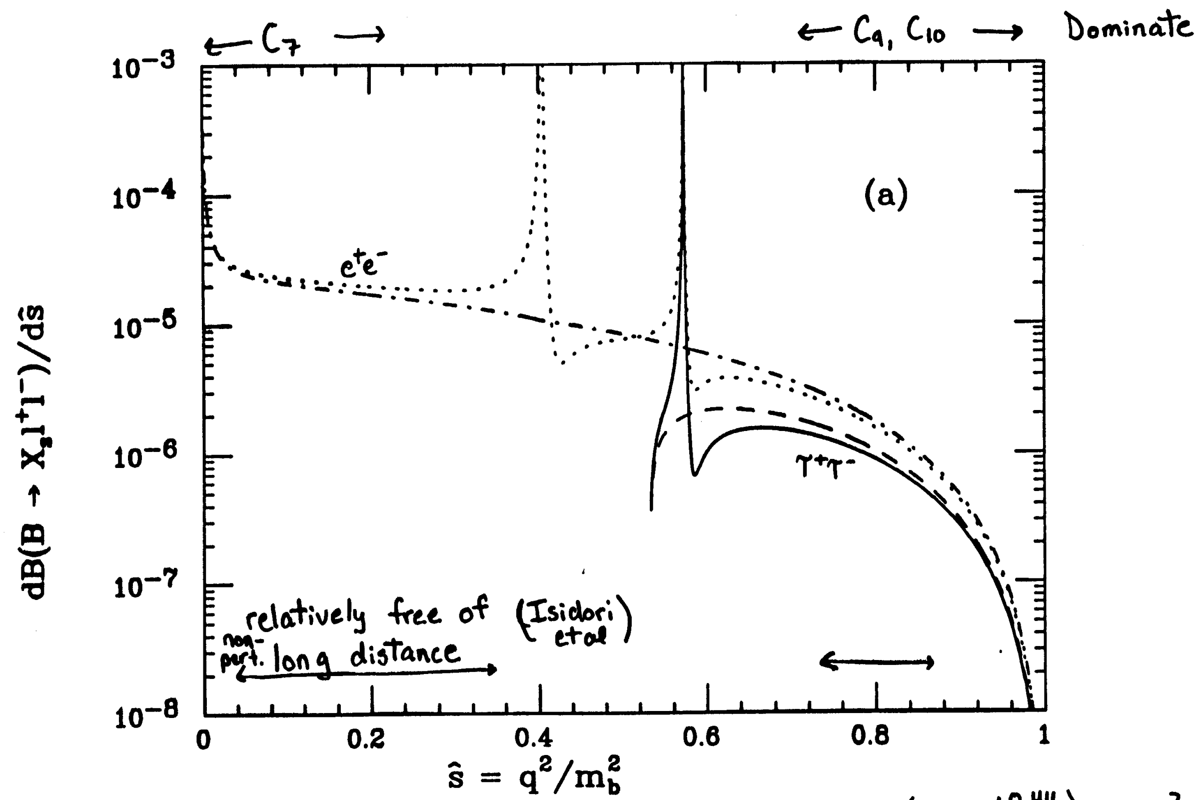
Fig. 3: Reconstruction[3] of the decay $D_s \to \eta\pi^+\pi^+\pi^-$ with $\eta \to \pi^+\pi^-\pi^0$ and $\pi^0 \to \gamma\gamma$.



$D \to \ell^+\ell^- + X$

Sans QCD

Fig. 4: Branching fraction for $D \to X + \ell^+\ell^-$ in the four generation SM as a function of the CKM mixing factor, with the solid, dashed, dotted, dash-dotted curve corresponding to $m_{b'} = 100, 200, 300, 400$ GeV, respectively.



$B(B \to X_s e^+e^-) = (6.25^{+1.04}_{-0.93}) \times 10^{-6}$

$B(B \to X_s \tau\tau) = (3.24^{+0.44}_{-0.54}) \times 10^{-7}$

Kinematic Distributions - Requires high statistics!

- $m_{\ell^+\ell^-}$ Distribution (Lim, Morozumi, Sanda; Deshpande etal; Ali etal)

$$\frac{d\Gamma(B\to X_s\ell^+\ell^-)}{d\hat{s}} \sim (1-\hat{s})^2\left[(|C_9^{eff}|^2+|C_{10}|^2)(1+2\hat{s}) + 4|C_7|^2\frac{2+\hat{s}}{\hat{s}} + 12\,\mathrm{Re}(C_9^{eff}C_7)\right]$$

- Forward-Backward Asymmetry of $\ell^+\ell^-$ angular distribution (Ali etal)

$$A_{FB} = \frac{\int_0^1 dz\, d\Gamma/dz\,d\hat{s} - \int_{-1}^0 dz\, d\Gamma/dz\,d\hat{s}}{\int_0^1 dz\, d\Gamma/dz\,d\hat{s} + \int_{-1}^0 dz\, d\Gamma/dz\,d\hat{s}}$$

$$= -3C_{10}\left[\mathrm{Re}\,C_9^{eff}\hat{s} + 2C_7\right] / d\Gamma/d\hat{s}$$

- ~~Tau~~ μ Polarization Asymmetry in ~~$B\to X_s\tau^+\tau^-$~~ - $B\to X_u\mu^+\mu^-$ (JLH; Kruger, Sehgal)

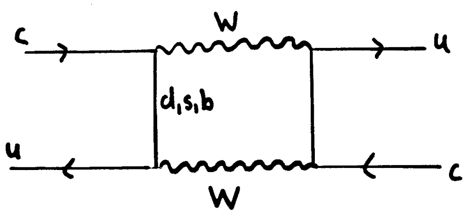
$$P_\tau = \frac{d\Gamma_{\lambda=-1} - d\Gamma_{\lambda=+1}}{d\Gamma_{\lambda=-1} + d\Gamma_{\lambda=+1}}$$

$$= -2C_{10}\left[\mathrm{Re}\,C_9^{eff}F_1(\hat{s},m_\tau) + 3C_7F_2(\hat{s},m_\tau)\right] / d\Gamma_\tau/d\hat{s}$$

$D^0 - \bar{D}^0$ Mixing

$|D_L\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$

$|D_H\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$



Short-Distance - External momentum non-negligible

$$\mathcal{H}_{eff}^{\Delta c=2} = \frac{G_F \alpha}{\sqrt{2}\, 8\pi x_w}\left[\begin{array}{l} |V_{cs}V_{us}^*|^2\left[I_1\left(\frac{m_s^2}{m_W^2}, \frac{m_s^2}{m_c^2}\right)O_{LL} - m_c^2 I_2 O'\right] \\ + |V_{cb}V_{ub}^*|^2\left[I_3\left(\frac{m_b^2}{m_W^2}, \frac{m_b^2}{m_c^2}\right)O_{LL} - m_c^2 I_4 O'\right] \end{array}\right]$$

$O_{LL} = \bar{u}\gamma_\mu(1-\gamma_5)c\ \bar{u}\gamma^\mu(1-\gamma_5)c$

$O' = \bar{u}(1+\gamma_5)c\ \bar{u}(1+\gamma_5)c$

$\Delta m_D = (4.5 \pm 0.5) \times 10^{-18}$ GeV

Experiment: E791 - FNAL

$$r_{mix} = \frac{1}{2\Gamma^2}\left|\frac{q}{p}\right|^2\left[(\Delta m)^2 + \frac{(\Delta\Gamma)^2}{4}\right]$$

$< 0.85\%$ at 90% CL $\quad [\Delta m_D \lesssim 1.5 \times 10^{-13}$ GeV

Long Distance Update on Δm_D

- HQET $\quad \Delta m_D \sim (1\text{-}2) \times 10^{-5}\, \Gamma \quad$ (Georgi)

$$\simeq 10^{-17} \text{ GeV}$$

? Reliability ? Large hadronic dynamical effects in m_c range

- 2-Particle Intermediate States (Donoghue et al)

Use Data (!) on $D \to K^+\pi^-$

$$\Delta m_D = -(2.2 \pm 5.5) \times 10^{-16} \text{ GeV}$$

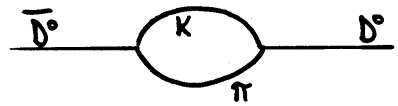
possibility of cancelation

- Pole Amplitudes (Golowich)

$$\Delta m_D = 5 \times 10^{-16} \text{ GeV}$$

$$\boxed{\Delta m_{D\,\text{L.D.}} \sim 10^{-17} - 10^{-16} \text{ GeV}}$$

Dispersive Example



$$\Delta m_D^{disp} \simeq \frac{1}{2\pi} \ln \frac{m_D^2}{\Lambda^2} \Big[\Gamma(D^0 \to K^+K^-) + \Gamma(D^0 \to \pi^+\pi^-) - 2\left[\Gamma(D^0 \to K^-\pi^+)\,\Gamma(D^0 \to K^+\pi^-)\right]^{1/2} \Big]$$

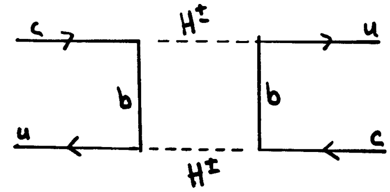
Multi-Higgs-Doublet-Models

• 2HDM

$$\begin{pmatrix}\phi_1^+\\ \phi_1^0\end{pmatrix} \quad \begin{pmatrix}\phi_2^+\\ \phi_2^0\end{pmatrix} \xRightarrow{\text{S.S.B.}} h^0,\, H^0,\, A^0,\, H^\pm$$

$$v_1^2 + v_2^2 = v_{sm}^2$$

$$\Rightarrow \tan\beta \equiv v_2/v_1$$

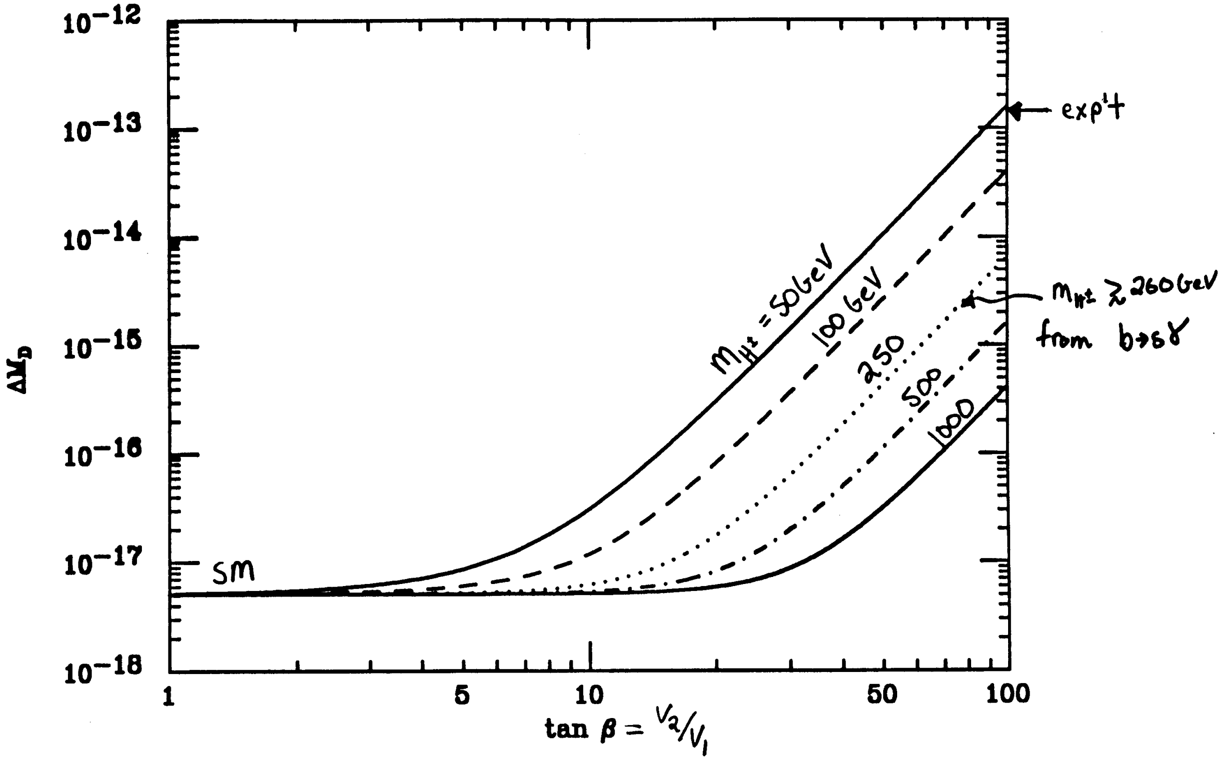


• No tree-level FCNC if each fermion type receives mass from only 1 doublet

$$\mathcal{L}_{cbH^+} = \frac{g}{2\sqrt{2}\, M_W}\left\{\frac{m_c}{\tan\beta} V_{cb}\, \bar{c}(1-\gamma_5)b + m_b \tan\beta\, V_{cb}\, \bar{c}(1+\gamma_5)b\right\} H^+ + \text{h.c.}$$

$$\Delta m_D = \frac{G_F^2 M_W^2 m_D}{6\pi^2} f_D^2 \eta \left|V_{cb} V_{ub}^*\right|^2 \left[\frac{m_c^2}{m_W^2 \tan^2\beta} F_1\left(m_c^2/m_{H^\pm}^2\right) + \frac{m_b^2 \tan^2\beta}{m_W^2} F_2\left(m_b^2/m_{H^\pm}^2\right)\right]$$

Two-Higgs-Doublet-Model II



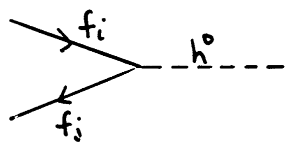
$B \to X \tau \bar{\nu}_\tau$ Constraints!

Flaver Changing Neutral Higgs

SM Higgs Coupling $\sim (\sqrt{2}\, G_F)^{1/2} m_f$

2HDM w/ FC Couplings

(Pakvasa + Sugawara
Hall + Weinberg
Cheng + Sher
Hou ...)

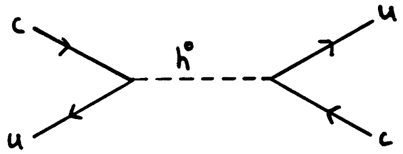


$\sim (\sqrt{2}\, G_F)^{1/2} \sqrt{m_i m_j}\, \Delta_{ij}$

mixing factor

$\Rightarrow$ small effect for light fermions

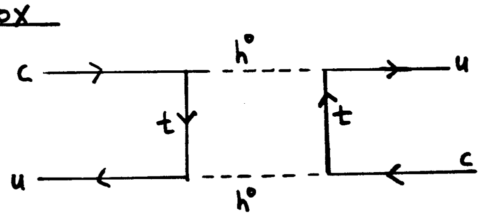
Tree-level



!

$$\Delta m_D = \frac{4\sqrt{2}\, G_F\, m_u m_c\, \Delta^2 f_D^2\, m_D}{3\, m_h^2}$$

box



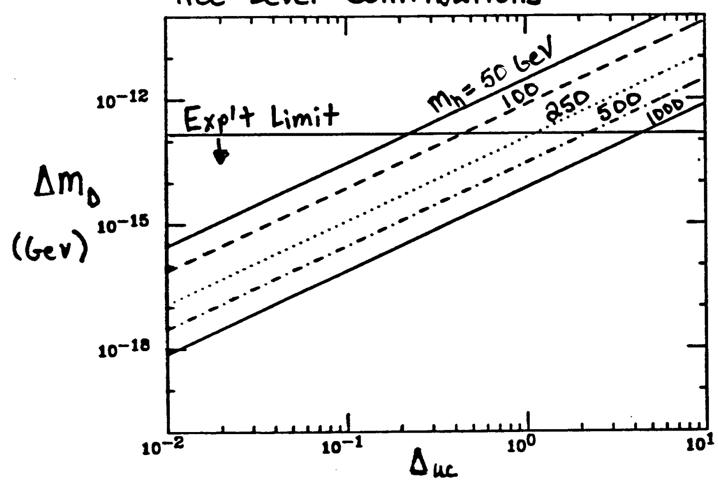
could be big!

$$\Delta m_D = \frac{2 G_F^2\, m_u m_c m_t^2\, \Delta^4 f_D^2\, m_D}{3\pi^2 m_h^2} \left[\frac{1}{(x-1)} - \frac{\ln x}{(x-1)^2} - \frac{1}{2(x-1)^3}\left(x^2 - 4x + 3 + 2\ln x\right) \right]$$

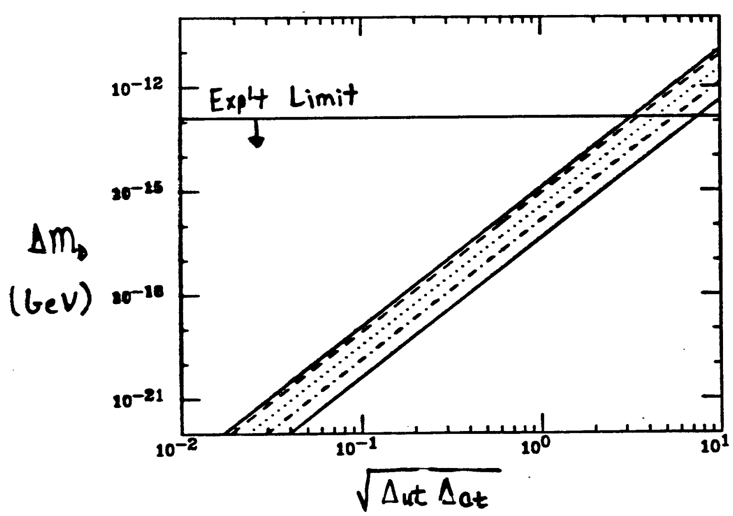
$$x \equiv m_t^2 / m_h^2$$

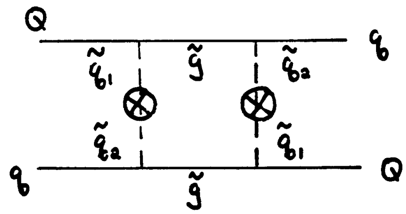
Flavor Changing Neutral Higgs

Tree-Level Contributions



Box Contributions





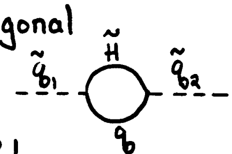
Supersymmetry

Gluino

All vertices $\sim \alpha_s \Rightarrow$ Potentially Lethal!

- MSSM - Super CKM = SM CKM
  $\Rightarrow \tilde{g}\tilde{q}q$ couplings flavor diagonal

FCNC generated by $\tilde{q}$ mass mixing



Invoke Super-GIM $\Rightarrow$ degenerate $\tilde{q}$'s

Re Δm_K	$\dfrac{\Delta m^2_{\tilde{d}\tilde{s}}}{\tilde{m}_0^2}$	$\lesssim$	$1/30$	Ellis et al 1982
Im Δm_K			10^{-3}	
Δm_D	$\dfrac{\Delta m^2_{\tilde{u}\tilde{c}}}{\tilde{m}_0^2}$	$\lesssim$	10^{-2}	Burdman et al
Δm_{B_d}	- Similar...			

• Squark Alignment (Nir, Seiberg)

d-squark mass matrices aligned w/ d-quarks

→ Can satisfy all FCNC constraints w/o degenerate $\tilde{q}$'s

But, u-squarks / quarks are not aligned

⇒ Δm_D at exp't limit! $\boxed{\text{Prediction!}}$

• Decoupling SUSY (Dine, Kaplan, Nelson, Cohen et al)

$$\mathcal{A}_{susy} \sim \underbrace{\frac{m_W^2}{m_{\tilde{g}}^2}}_{\text{SUSY Decoupling Effect}} F\left(m^2_{\tilde{q}_i}/m^2_{\tilde{g}}\right)$$

$$\text{Re } \Delta m_K \qquad \frac{\theta_{12}^2}{m_{\tilde{q}}^2} \lesssim \left(\frac{1}{50 \text{ TeV}}\right)^2$$

$$\text{Im } \Delta m_K \qquad \frac{\theta_{12}^2}{m_{\tilde{q}}^2} \lesssim \left(\frac{1}{500 \text{ TeV}}\right)^2$$

CP - Violation

$\cancel{CP}$ in Mixing:

$$\frac{1-|q/p|^2}{1+|q/p|^2} = \frac{2\,\mathrm{Re}\,\epsilon_D}{1+|\epsilon_D|^2}$$

$$\simeq \frac{\mathrm{Im}\,\Gamma_{12} - \frac{\Delta\Gamma}{\Delta m}\,\mathrm{Im}\,m_{12}}{\Delta m_D\left[1 + \Delta\Gamma^2/4\Delta m^2\right]}$$

$$\simeq 0.05 - 0.1 \qquad \text{taking } \Delta m_D = 10^{-16}\ \mathrm{GeV}$$

$\cancel{CP}$ in interference between mixing and decay:

$$\tan\phi = \frac{\mathrm{Im}\,(q/p)}{\mathrm{Re}\,(q/p)} \simeq \frac{-\frac{\mathrm{Im}\,m_{12}}{\mathrm{Re}\,m_{12}} - \frac{\mathrm{Im}\,\Gamma_{12}}{4\,\mathrm{Re}\,\Gamma_{12}}}{1. + \frac{1}{2}\left[\frac{\mathrm{Im}\,m_{12}}{\mathrm{Re}\,m_{12}} - \frac{\mathrm{Im}\,\Gamma_{12}}{\mathrm{Re}\,\Gamma_{12}}\right]}$$

$$\simeq (5 - 10) \times 10^{-3}$$

can be $\simeq 0.1$ with new physics!

Direct CP Violation, or

CP Violation in Decay $(m^0, m^\pm)$

CPT : $\Gamma_{total} = \bar{\Gamma}_{total}$

CP : $\Gamma_{partial} = \bar{\Gamma}_{partial}$ $[m \to f \text{ vs. } \bar{m} \to \bar{f}]$

$\not{CP}$: $\Gamma_{partial} \neq \bar{\Gamma}_{partial}$

- requires interference between 2 phases

Assume 2 Amplitudes with $|A_1| \gg |A_2|$ [tree + penguin]

and $A_i = V_i \hat{A}_i$ $(i=1,2)$

CKM elements

$$\Gamma = \int dL \, | A_1 + A_2 |^2$$
$$= \int dL \, | V_1 \hat{A}_1 + V_2 \hat{A}_2 |^2$$

and $$\bar{\Gamma} = \int dL \, | V_1^+ \hat{A}_1 + V_2^+ \hat{A}_2 |^2$$

Define CP Violation Asymmetry

$$a \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \simeq \frac{-2\,\mathrm{Im}(V_1 V_2^+)}{\Gamma} \int \mathrm{Im}\,\hat{A}_1 \,\mathrm{Re}\,\hat{A}_2 \, dL$$

$\Rightarrow$ 2 phases - $\underbrace{\text{One odd under CP}}_{\text{CKM phase}}$ & $\underbrace{\text{One even under CP}}_{\text{strong interaction, final state interaction}}$

SM: CKM phase; strong interaction, final state interaction

Conclusions

1) QCD corrections dramatically enhance rare decays!

   - $D \to X_u \gamma$ increased 9 orders of magnitude but still below long distance

   - $D \to X_u \ell \ell$ increased 10^4 - comparable to long distance

   ⇒ Measure $D \to X_u \gamma$ to sharpen non-perturbative calculational tools

   Measure $D \to X_u \ell^+ \ell^-$ to look for new physics

2) $D-\bar{D}$ Mixing: Window wide open for new physics!
   ⇒ Probes models not constrained by $Q=-1/3$ FCNC!

3) $\cancel{CP}$: Has yet to be explored in D system
   Some window for new physics