0-0

Lessons from Neutrino Mass Studies

J.E. Duboscq

The Ohio State University

Lessons from Neutrino Mass Studies

- Is it (still) worth looking at Neutrino Mass ?
- A Typical Analysis (CLEO $\tau \rightarrow 5\pi\nu$)
- Critical Thoughts
- Outlook

Caveat: I am a member of CLEO and did the 5π analysis

Astrophysical Hints on Neutrino Mass

- Stable ν left over from Big Bang:
  We exist $\rightarrow \Sigma m_\nu < 100 eV$ Cowsik, McClelland 1972
- BBN: Neutrinos at Freeze-out affect Light Elements
  Kawasaki, Steigman etal 1994
  - light and long lived ($M < 0.1 MeV$ $\tau > 0.01 s$)
  - very light and short lived :$M < 0.1 \times (\tau/10^{-2} s) MeV$
  - very massive and long lived ($M >> 50 MeV$)
  - massive and decay ($5 MeV < M$ $0.01 s < \tau < 40 s$)
  - Allowed Region Sensitive to Decay Model

- Dark Matter:
  Most of our Universe is missing. Neutrinos are a great place to hide.

Hints from SuperKamiokande

- SuperK claims to see expected ν_e, but missing a lot of ν_μ
- Explanation a) ν_μ oscillating into ν_τ, small Δm^2
- Explanation b) ν_μ oscillating into sterile ν_4, , small Δm^2
- (Explanation c) New Interaction?)

The existence of the BBN allowed window makes it imperative to closely examine accelerator based limits to distinguish a) and b).

PDG96: $m(\nu_\mu) < 170\ keV$

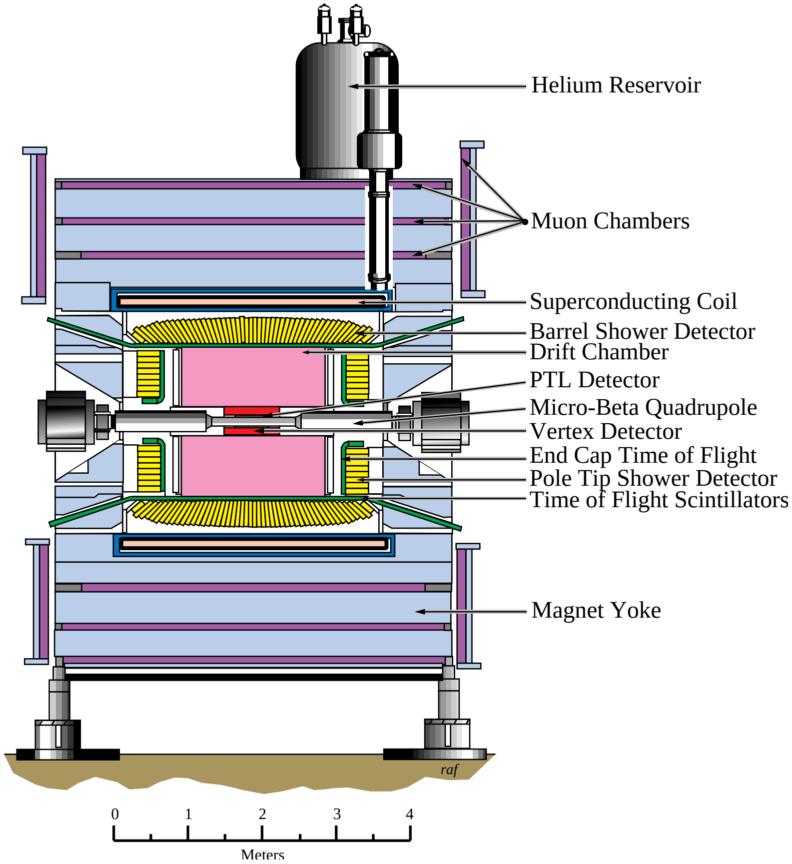
Other Accelerator Based Results

- ALEPH95 24 MeV from $\tau \rightarrow 5\pi(\pi^0)\nu_\tau$
- ALEPH98 22 MeV from $\tau \rightarrow 3\pi\nu_\tau$
  $\rightarrow 18MeV$
- OPAL97 29.9 MeV from 3π
- OPAL98 43.2 MeV from $\tau \rightarrow 5\pi\nu_\tau \rightarrow 27.6MeV$
- DELPHI97 33 MeV (62 MeV) from 3π
- OPAL95 74 MeV from $\tau \rightarrow 5\pi\nu_\tau$
- ARGUS92 31 MeV from 5π
- CLEO93 32.6 MeV from 5π, $3\pi 2\pi^0$

Is the Astrophysically Allowed Window Closing ?????

The CLEO II Detector /CESR

- e^+e^- at $E_{cm} = 10.58 GeV$
- τ's produced in pairs with $E_\tau = 5.29 GeV$ (+ ISR effects)
- 4.5×10^6 τ pairs on tape for CLEO II
- $\approx 8 \times 10^6$ τ pairs on tape for CLEO II.5

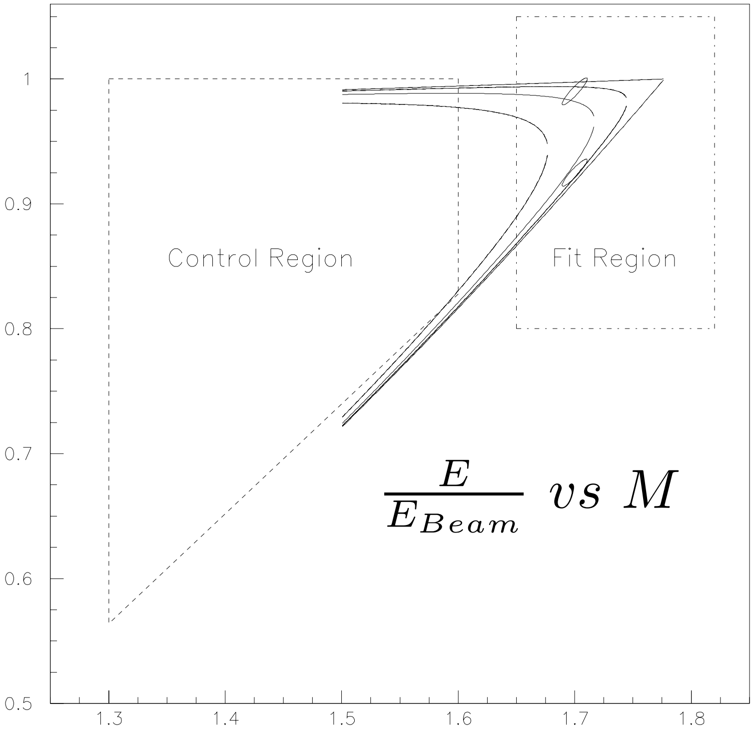


Two Dimensional Studies: Necessary Ingredients

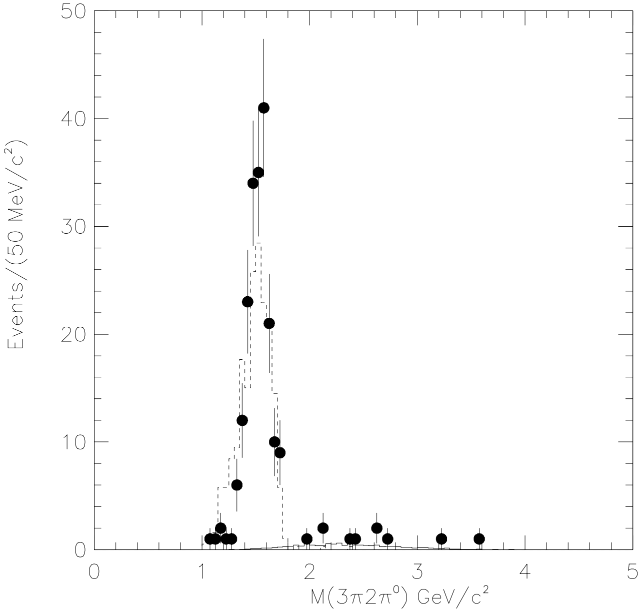
- Data
- Detector Smearing Modelling / Error propagation
- Background Estimation - τ and non-τ
- Spectral Function
- Likelihood Function
- Interpretation

The CLEO Analysis Method

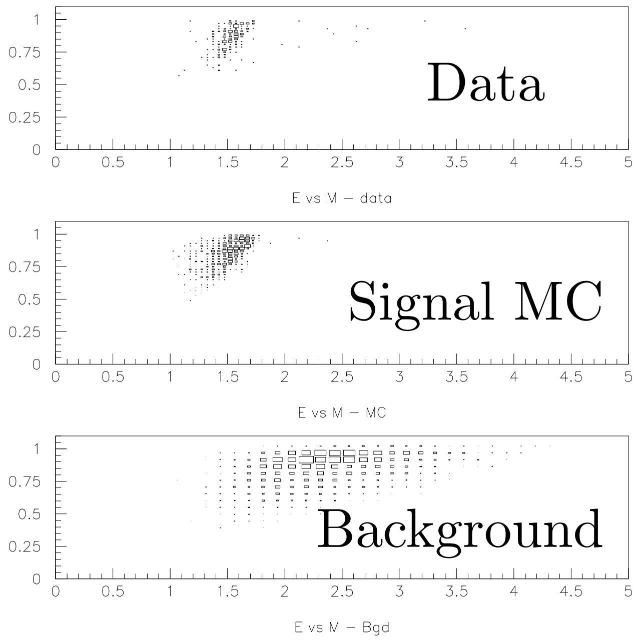
- Isolate CLEAN signals
  $\tau \to 3\pi 2\pi^0\nu, 5\pi\nu$
- Fit E/E_{Beam} *vs* M Spectrum
- Don't Forget:
  $\frac{d\Gamma}{dq^2} = \frac{d\Gamma}{dq^2}(M_{\nu_\tau})$
  → Poisson Error (Extended Likelihood)
- Include a Background Function



The Skim $\tau \rightarrow 3\pi 2\pi^0 \nu_\tau$

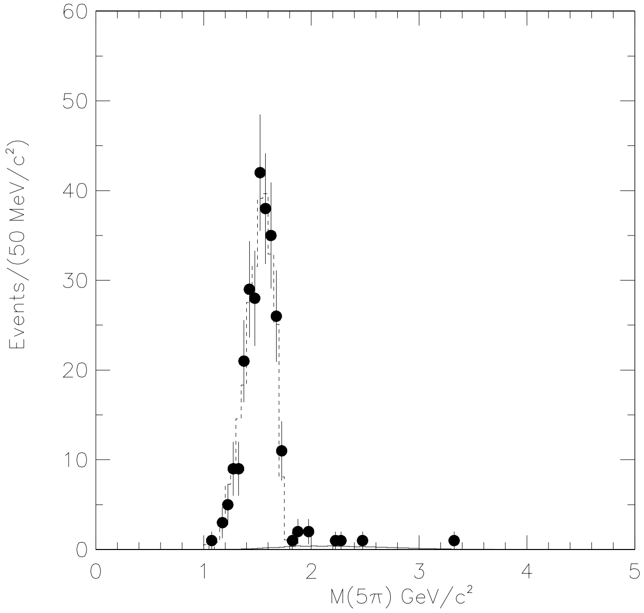


Hadronic Mass

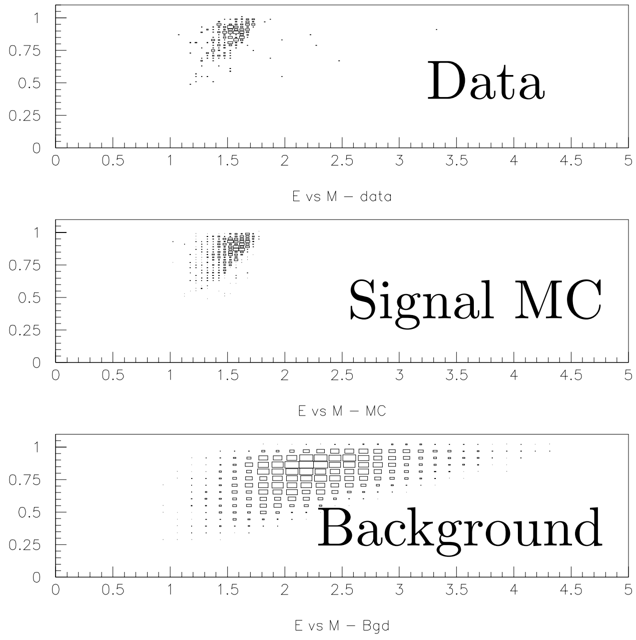


E vs M

The Skim $\tau \to 5\pi\nu_\tau$

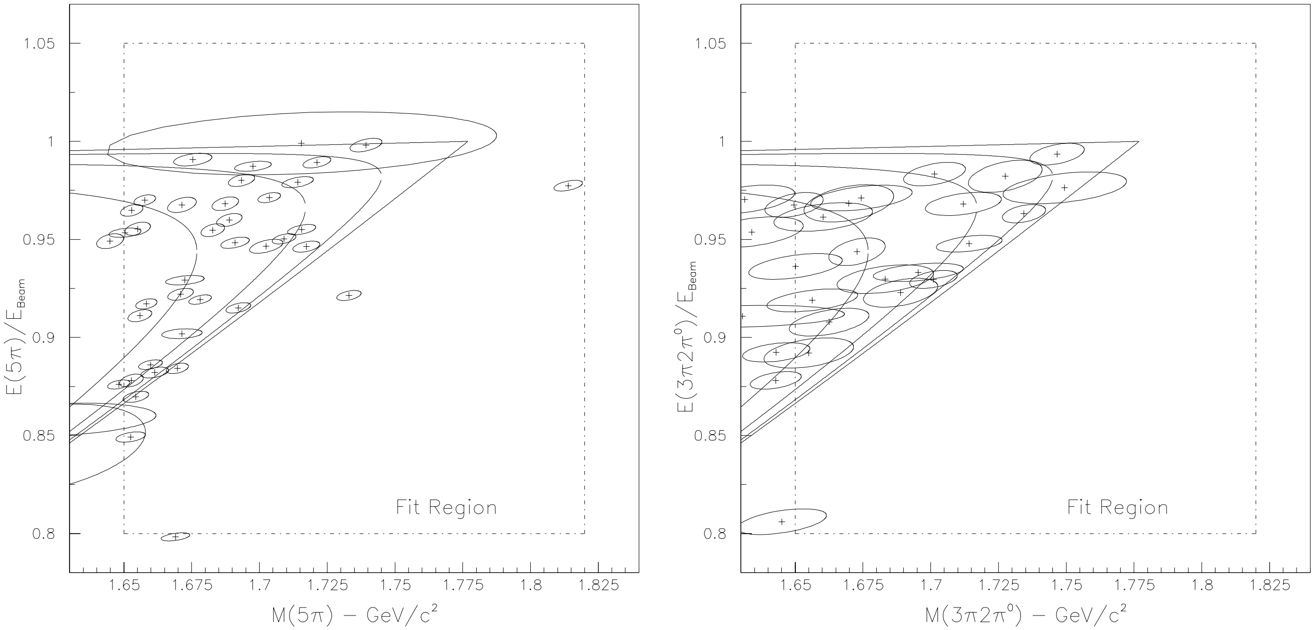


Hadronic Mass



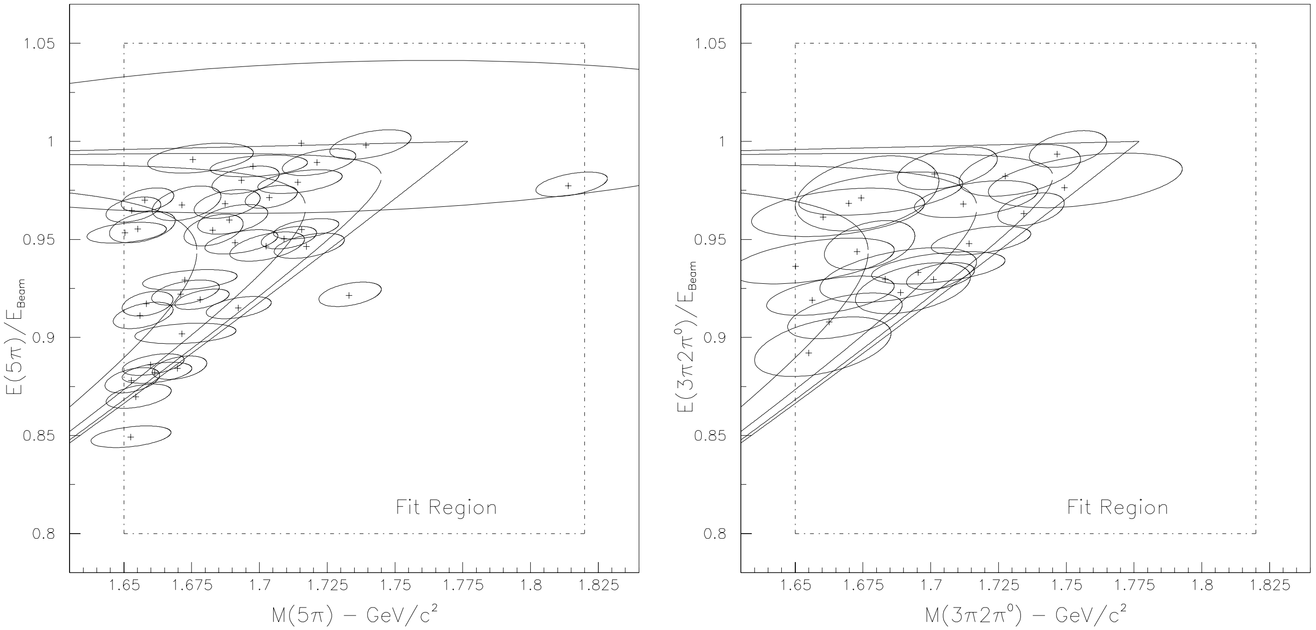
E vs M

The Fit Region Events



Error Circles are Uncorrected Propagated Tracking Errors

The Corrected Fit Region Events



Error Ellipses are effective 1 σ errors

The errors are calculated from scaled propagated tracking errors, and included non-Gaussian tails from the smearing function.

Event Breakdown

- Below Tau Mass: 197 ± 14 $3\pi2\pi^0$ Events MC expects 185 ± 7
- Below Tau Mass: 258 ± 16 5π Events MC expects 259 ± 8
- Fit Region: 18 ± 4 $3\pi2\pi^0$ Events MC expects 21 ± 2
- Fit Region: 35 ± 6 5π Events MC expects 33 ± 3
- Background Expected in Fit Region :
  0.4 ± 0.1 for $3\pi2\pi^0$ 0.3 ± 0.1 for 5π
- Tau Feed Across
  in $3\pi2\pi^0$ - 1 Event (of 21) is $3\pi\pi^0$
  in 5π - $<<$ 1% of Events are Feed Across

The Likelihood

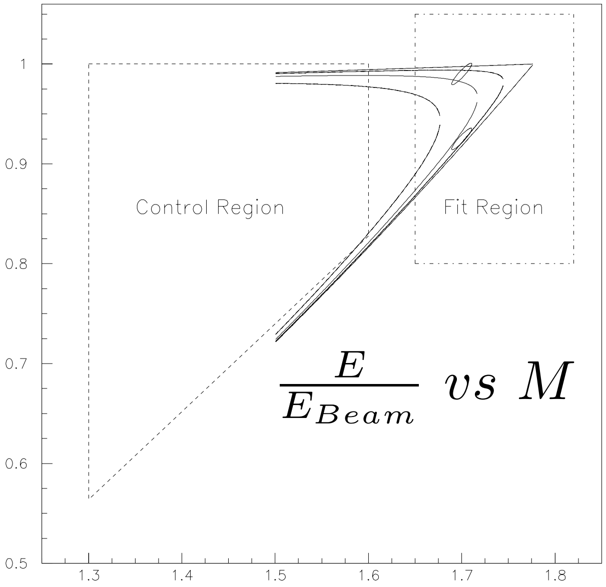
There are three components:

$$\mathcal{L}(M_{\nu_\tau}) = \mathcal{P}\left(N_{obs}, M_{\nu_\tau}\right) \prod_{Data} (\alpha \mathcal{L}_{Signal}(\tilde{X}_{data}, \sigma_{data}, M_{\nu_\tau}) \\ +(1-\alpha)\mathcal{L}_{BGD}(\tilde{X}_{data}, \sigma_{data}))$$

- $\mathcal{P}\left(N_{obs}, M_{\nu_\tau}\right)$ - Poisson Prob of seeing this Number of Events in Fit Region, given Number in Control Region
- $\mathcal{L}_{Signal}$ Signal Shape - Convolution of Physics and Detector
- $\mathcal{L}_{BGD}$ Background Shape - From $M > 1.8$ vs $3\pi 2\pi^0 (5\pi)$ Shape

We Use Novel Hybrid MC - Analytic Technique...

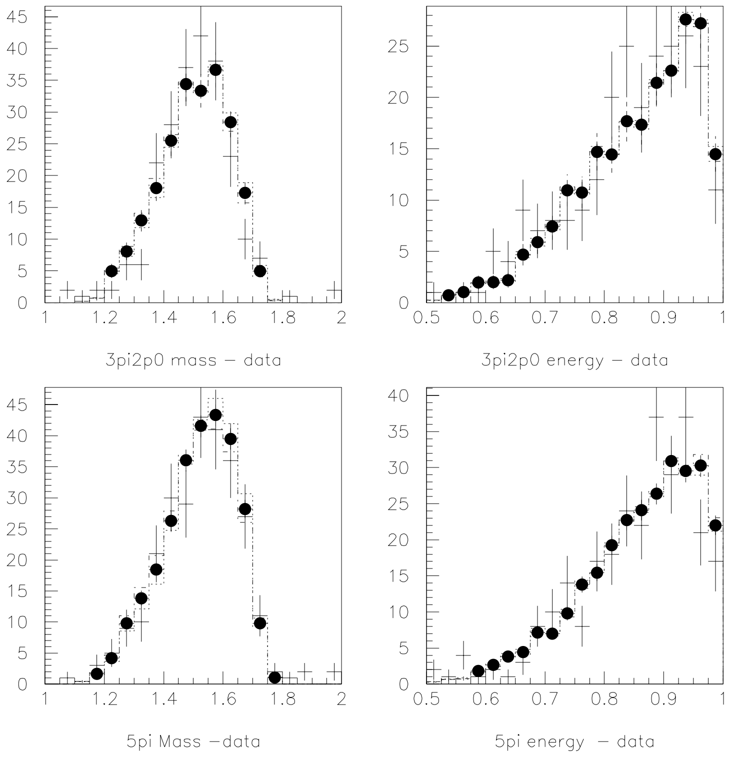
The Likelihood - The Poisson Term



- Refit Independent Sample tagged with $\tau \to \pi\nu$ to get Spectral Function
- Scale N_{Obs}^{CTL} to $< N^{Fit}(M_{\nu_\tau}) >$
- use Reweighted MC
- $\mathcal{W}(M, E|M_{\nu_\tau}) = (V - A) \times$ PhaseSpace $= f(M_{\nu_\tau})$

The Poisson term is a short-cut to avoid fitting over the whole (M,E) plane: it counts points that do not *by themselves* have any ν mass info.

The Spectral Function



The Spectral Functions used are from fits to π tag data

The Fits Excluded Region around τ mass

The Fits are a linear Comb. of two functions:

a) $e^+e^- \rightarrow 4\pi$ derived + soft extra pion theorems

b) $e^+e^- \rightarrow 6\pi$ derived (hard distribution)

The Likelihood - The Signal Term

- Usual Method: Convolution of Physics, Detector - loads of CPU
- Generate MC with $M_{\nu_\tau} = 0\,\mathrm{GeV}$ - takes care of physics
- Accept or reject MC events with skim job - takes care of ϵ
- Smear accepted raw MC event analytically with $G(X^{MC}_{gen}, \tilde{X}_{DATA})$ to the DATA point

$$\begin{aligned}\mathcal{L}_{Signal} &= \frac{\sum_{MC} G(\tilde{X}, X_{MC})\mathcal{W}(M_{\nu_\tau}, X_{MC})}{\mathcal{N}(M_{\nu_\tau})} \\ \mathcal{W}(M_{\nu_\tau}, X_{MX}) &= \frac{d\Gamma(M_{\nu_\tau}, X_{MC})}{d\Gamma(0, X_{MC})}\end{aligned}$$

The Smearing Function $G(X_{gen}^{MC}, \tilde{X}_{Data}, \tilde{\sigma}_{Data})$

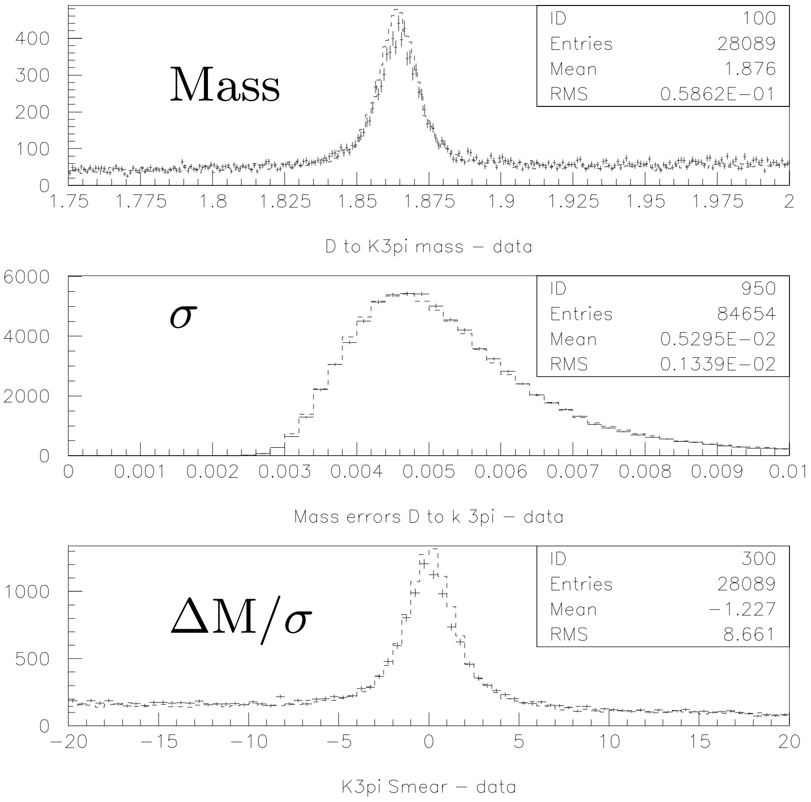
Smearing Functions gives the Probability that (smeared) data point $\tilde{X}_{Data}$ could have come from (unsmeared) MC point X_{gen}^{MC}.

Assume error $\tilde{\sigma}_{Data}$ on Data Point represents a measure of how big an area it could have come from.

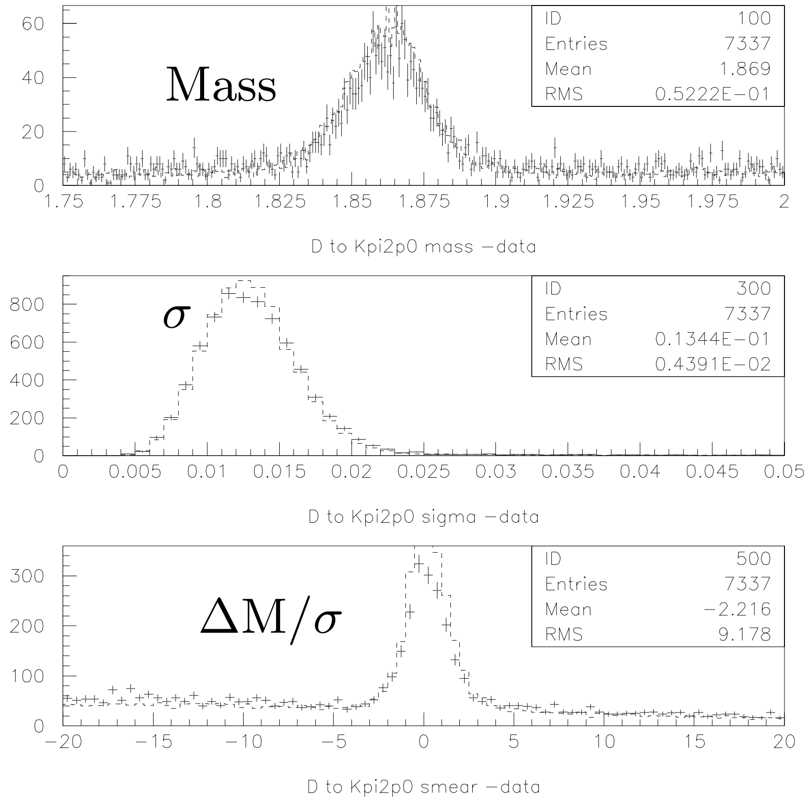
- CLEO detector Smearing is not Gaussian
- CLEO tracking errors are wrong
- CLEO tracking errors are wrong in MC and data in same way
- CLEO mass distributions for D^0 similar in Data and MC
- Use $\frac{\text{Generated}-\text{Reconstructed}}{\sigma}$ from MC

Smearing in D^0 Decays

$D^0 \rightarrow K3\pi$



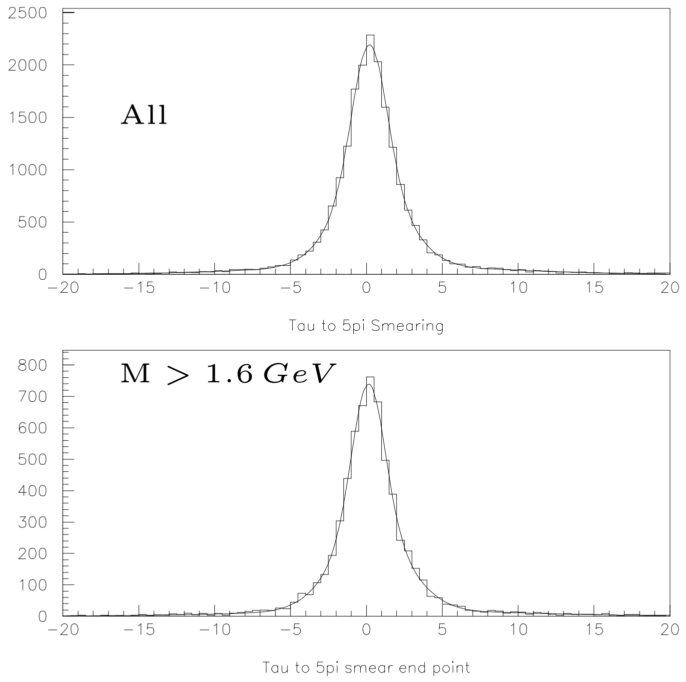
$D^0 \rightarrow K\pi 2\pi^0$



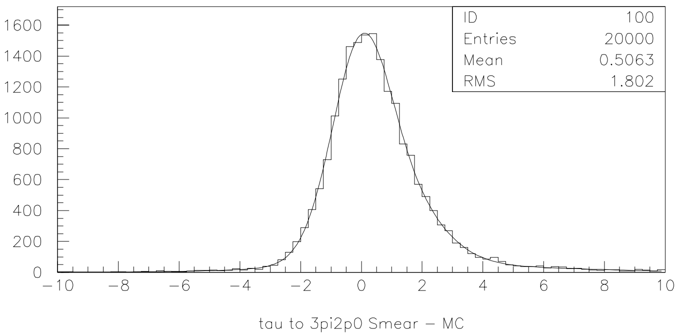
Smearing in MC τ Decays

Fit MC Smearing of Tau Decays to Sum of Three 2D Gaussians

5π



$3\pi 2\pi^0$



Smearing is Wider than a Simple Gaussian

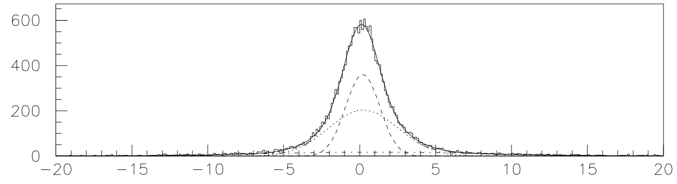
Smearing is offset

More Smearing in MC τ Decays

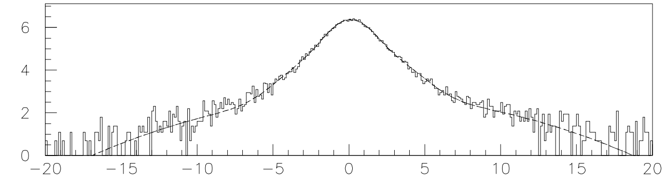
Fit MC Smearing of Tau Decays to Sum of Three 2D Gaussians

$$\frac{\text{M(Generated)}-\text{M(Reconstructed)}}{\sigma}$$

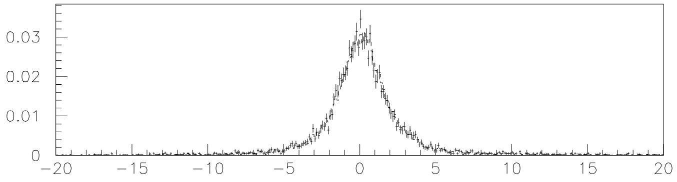
5π



5π Mass Smearing − MCz (Gen−Recon)/sigma

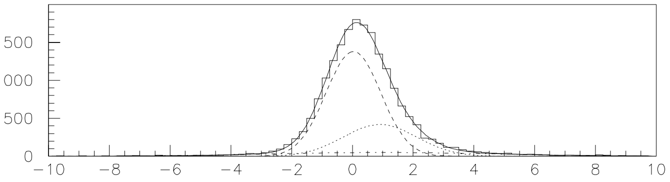


5π log smearing

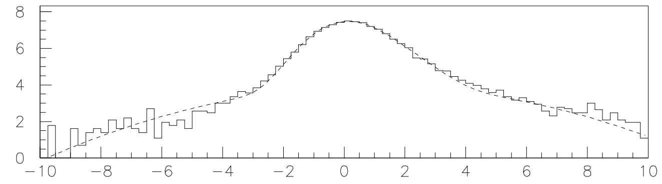


5π Mass Smearing above 1.6 GeV− MCz (Gen−Recon)/sigma

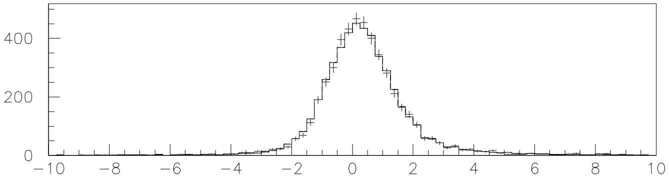
$3\pi 2\pi^0$



3π2π⁰ Mass Smearing − MC (Gen−Recon)/sigma

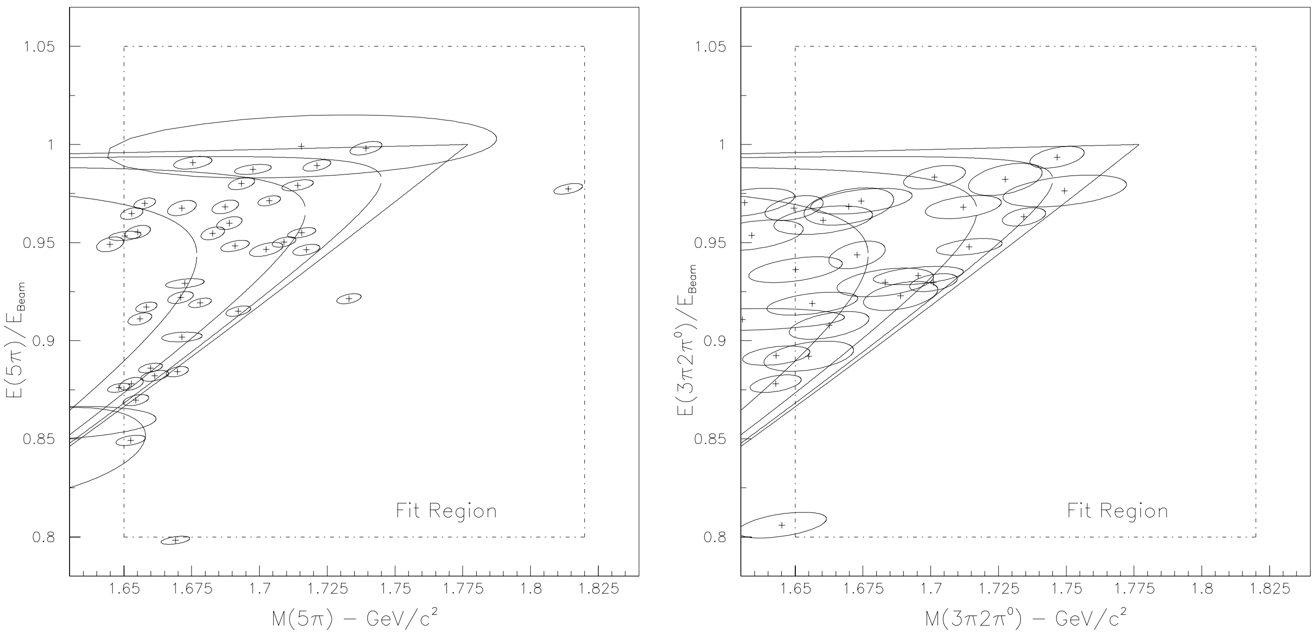


3π2π⁰ log smearing



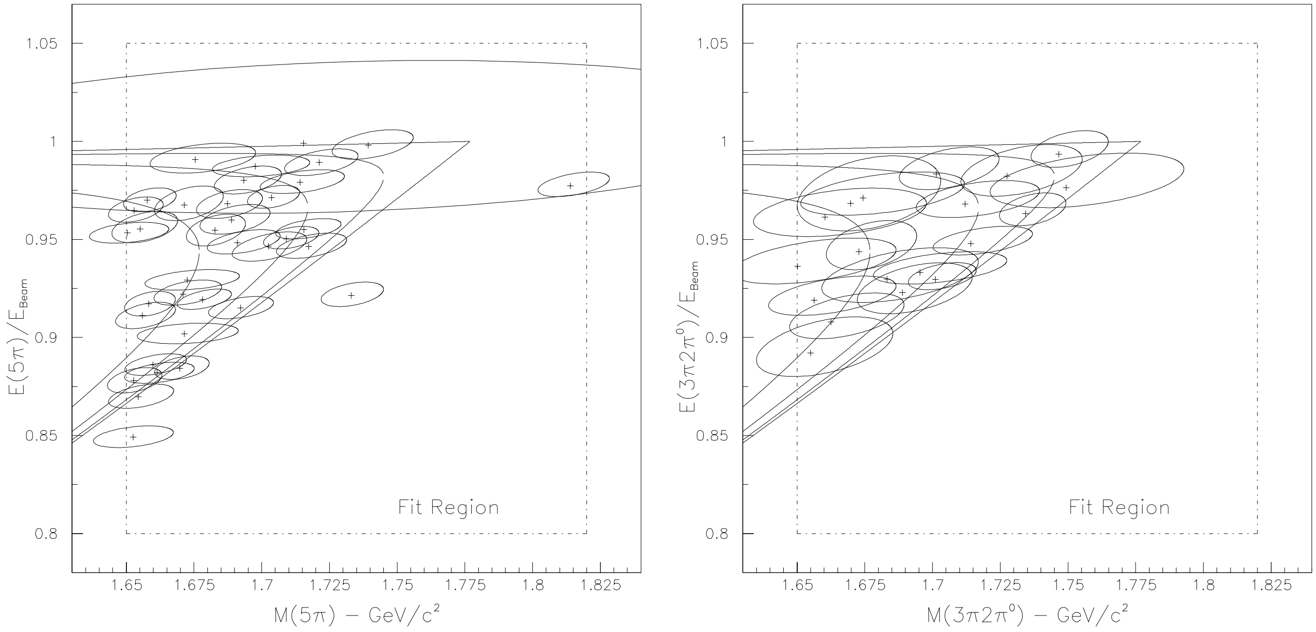
3π2π⁰ Smearing Gen above 1.6 GeV

The Fit Region Events



Error Circles are Uncorrected Propagated Tracking Errors

The Corrected Fit Region Events



Error Ellipses are effective 1 σ errors

The Likelihood

We now have the 3 important ingredients for the Likelihood:

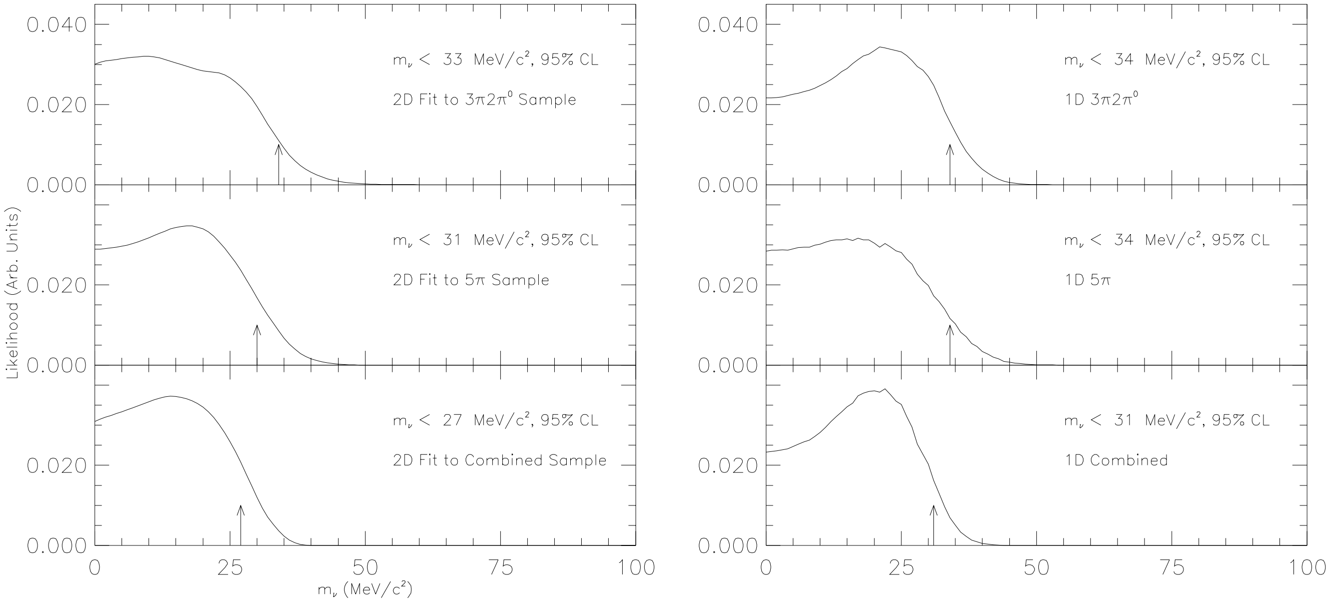
- The Physics - Spectral Function
- The Expected Number of Events near Endpoint (Poisson Term)
- The 2D Detector Smearing Function

Let's turn the crank

The Likelihood

Integrate Output Likelihood to 95th Percentile

$2DFit$ $1DFit$



How Lucky Are We ?

Throw Many MC experiments with Stats like ours and $M_{\nu_\tau} = 0 MeV$

Fit Type	Prob of Peak $\geq$ Data	Prob of Limit $\geq$ Data
5π 1 D	37 %	14 %
$3\pi 2\pi^0$ 1D	55 %	73 %
5π 2 D	41 %	35 %
$3\pi 2\pi^0$ 2D	59 %	74 %

We are neither extremely lucky nor unlucky

Therefore the likelihood is representative and meaningful

Systematic Error Estimation

- Follow Extremely Conservative LEP Technique
- Add in a Linear Systematic Error to our Raw Limit (27 MeV)

$$\Delta(m_{95}) = \sqrt{\Sigma(\tilde{m}_{95} - m_{95})^2}$$

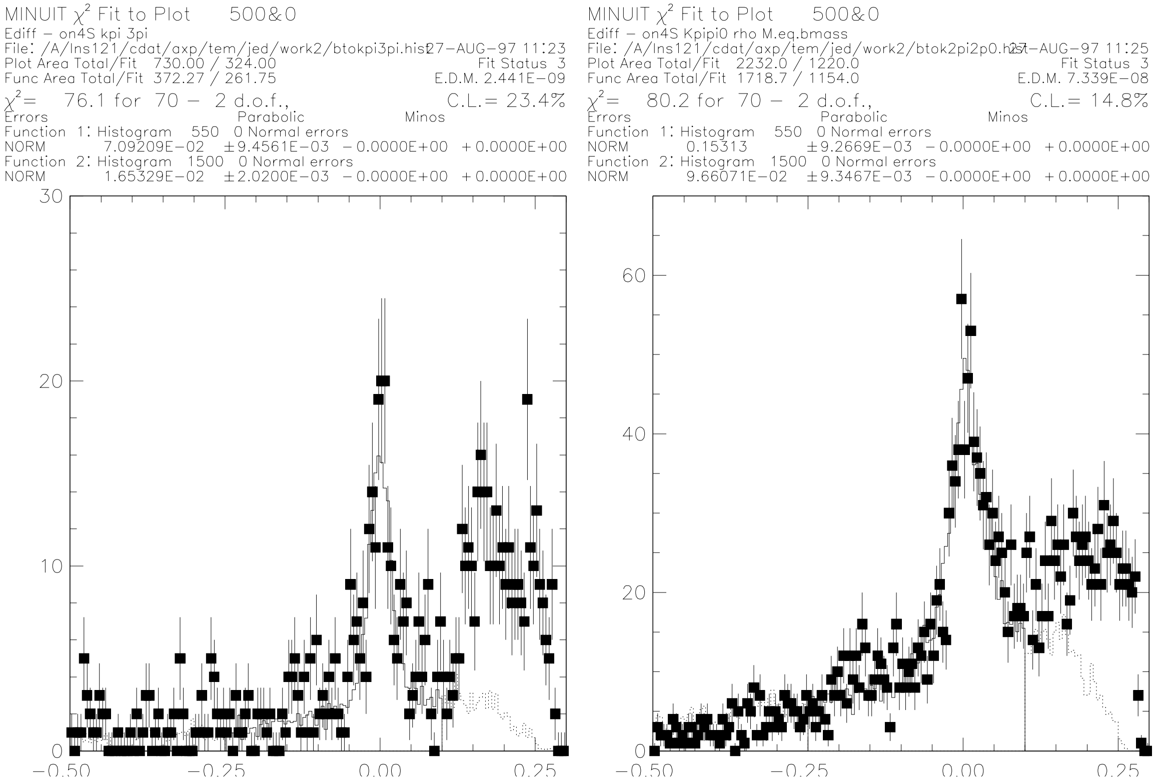
where
m_{95} : limit before change
$\tilde{m}_{95}$: limit after change

Systematic Error Sources - $5h$

Spectral Function	1.9 MeV
Mass Scale	1.5 MeV
Extra Unaccounted For Big Smearing Tails	1.4 MeV
Smearing Width	0.5 MeV
MC Stats	0.4 MeV
Smearing Component Offsets	0.4 MeV
Background Size	0.3 MeV
Energy Scale	0.2 MeV
Smearing Component Amounts	0.1 MeV
Total	3.1 MeV

Systematic Error Sources - Extra Smearing

Fit Data to MC + Gaussian in B decays for Energy Smearing



```
MINUIT χ² Fit to Plot      500&0
Ediff - on4S kpi 3pi
File: /A/Ins121/cdat/axp/tem/jed/work2/btokpi3pi.hist27-AUG-97 11:23
Plot Area Total/Fit  730.00 / 324.00                    Fit Status  3
Func Area Total/Fit  372.27 / 261.75                E.D.M. 2.441E-09
χ²=    76.1 for  70 -  2 d.o.f.,                      C.L.= 23.4%
Errors                          Parabolic                Minos
Function  1: Histogram   550   0 Normal errors
NORM          7.09209E-02   ±9.4561E-03  -0.0000E+00  +0.0000E+00
Function  2: Histogram  1500   0 Normal errors
NORM          1.65329E-02   ±2.0200E-03  -0.0000E+00  +0.0000E+00
```

```
MINUIT χ² Fit to Plot      500&0
Ediff - on4S Kpipi0 rho M.eq.bmass
File: /A/Ins121/cdat/axp/tem/jed/work2/btok2pi2p0.hist27-AUG-97 11:25
Plot Area Total/Fit   2232.0 / 1220.0                   Fit Status  3
Func Area Total/Fit   1718.7 / 1154.0               E.D.M. 7.339E-08
χ²=    80.2 for  70 -  2 d.o.f.,                      C.L.= 14.8%
Errors                          Parabolic                Minos
Function  1: Histogram   550   0 Normal errors
NORM            0.15313     ±9.2669E-03  -0.0000E+00  +0.0000E+00
Function  2: Histogram  1500   0 Normal errors
NORM          9.66071E-02   ±9.3467E-03  -0.0000E+00  +0.0000E+00
```

$E_{beam} - E_{Recon}\ (GeV)$

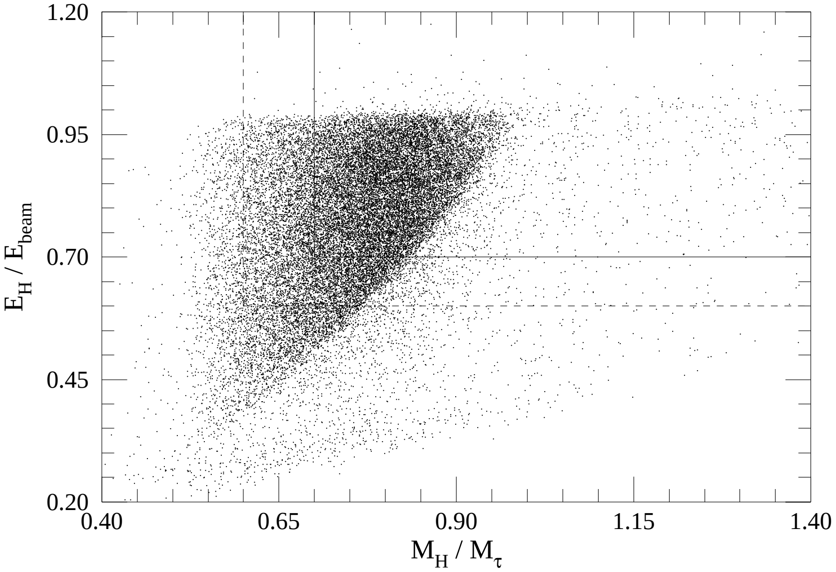
The $\tau \rightarrow 3\pi\pi^0\nu$ Analysis

- Events (mostly) far from Endpoint
- Large Branching Ratio
- Method similar to $5h$ analysis
- Big Fit Region → No Poisson Factor
- 1 Prong tag → significant background
- All Results Preliminary

Data Distribution for $\tau \rightarrow 3\pi\pi^0\nu_\tau$

Event Breakdown:

- $29K$ Total Events
- $17K$ in Fit Region
- 7% Tau Feed-down in Fit Region
- 3% $q\overline{q}$ Bgd in Fit Region



The $4h$ Spectral Function

Fit all 2π, 3π combinations, and 4π ($M \leq 1.6\ GeV$) to an extension of tauola model:

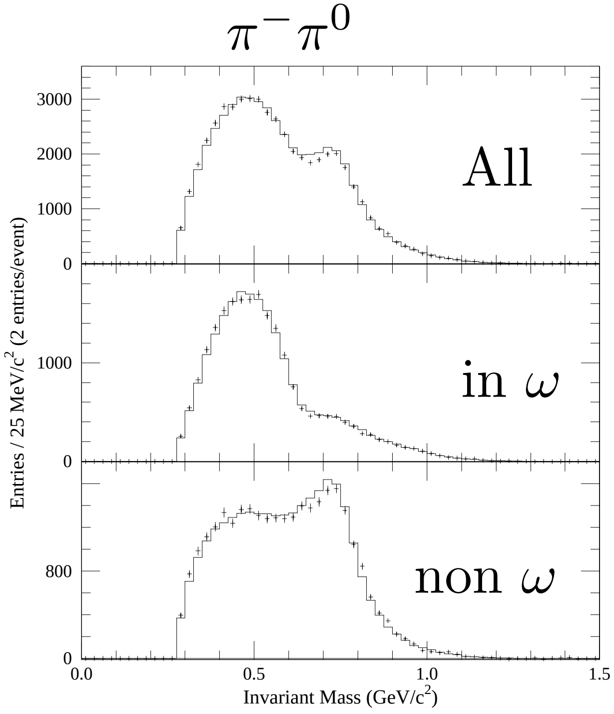
$$J = F^{\rho\pi\pi}(Q^2_{4\pi})\Sigma_{i=1,5}\mathcal{A}_i f^\rho_i(q_{2i}) + F^{\omega\pi}(Q^2_{4\pi})\Sigma_{i=1,2}\mathcal{A}_i f^\omega_i(q_{3i})$$

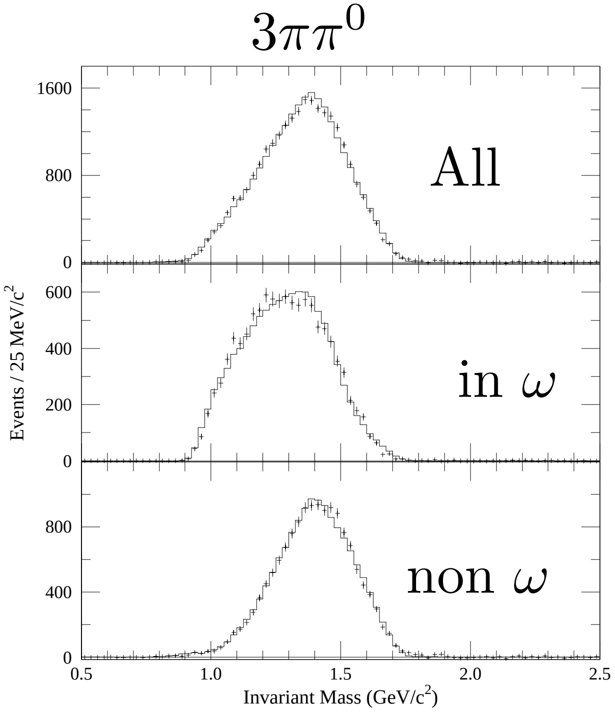
where

- $F^{\rho\pi\pi}$, $F^{\omega\pi}$,f^ρ_i are sums of BW for ρ, ρ',ρ'',
- $f^\omega_i(q_{3i})$ is BW for the ω

Fit is not claimed to correctly represent physics - just a model.

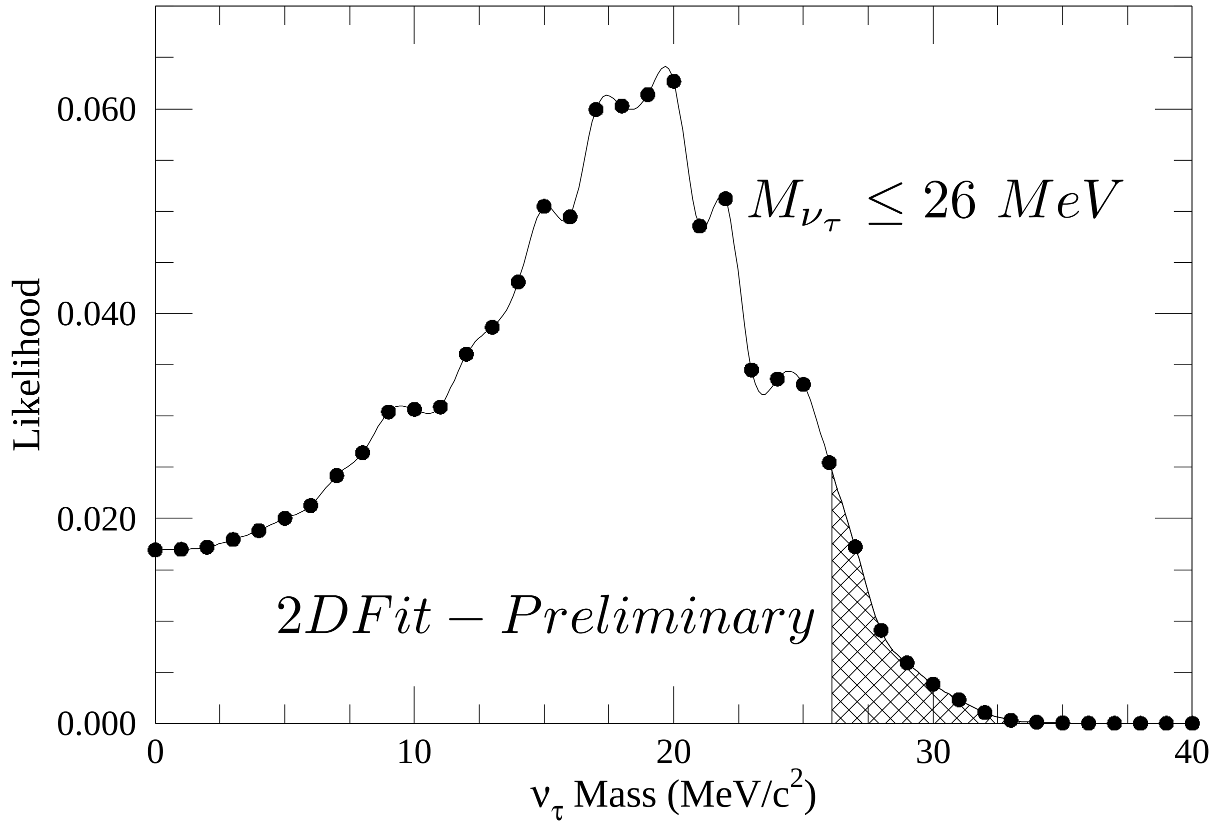
The $4h$ Spectral Function





The 4*h* Likelihood

Integrate Output Likelihood to 95th Percentile



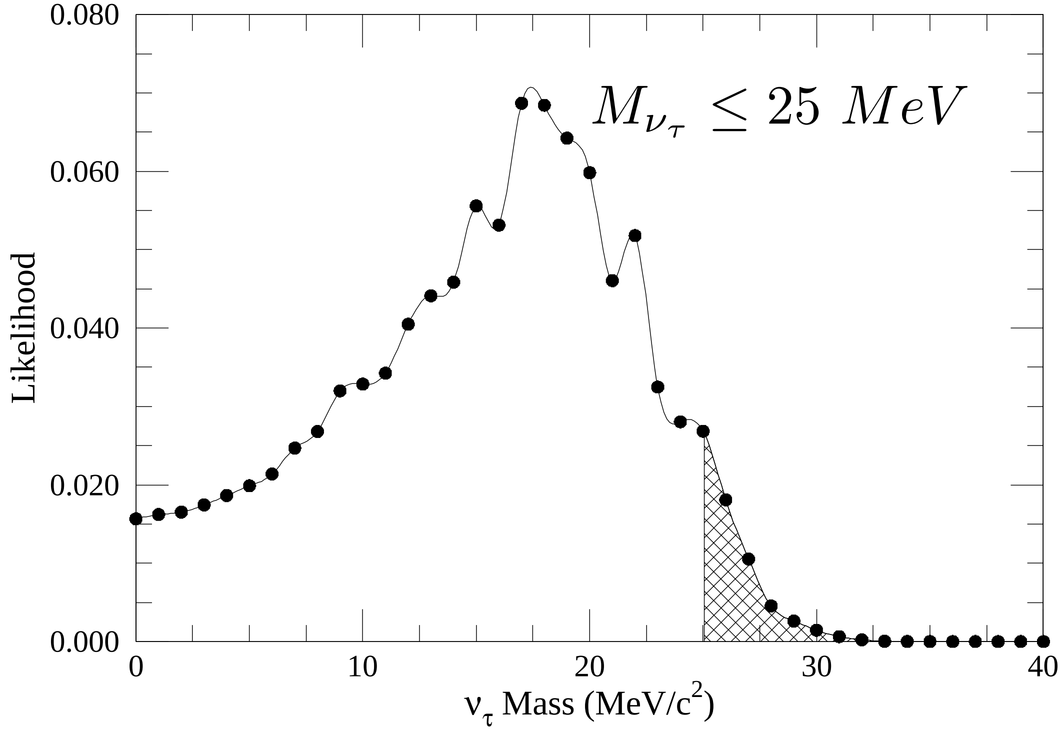
Systematic Error Sources - 4*h*

	4*h* Prelim
Spectral Function	1.2 MeV
Mass/Momentum Scale	2.3 MeV
Energy Scale	3.7 MeV
Smearing	0.4 MeV
Background Size	0.8 MeV
MC Stats	0.5 MeV
Total	5.1 MeV

NB: Error definitions are slightly different for 4*h* and 5*h*

Preliminary Combined $4h$ and $5h$ Likelihood

Integrate Output Likelihood to 95th Percentile



Systematic error not yet ready

Peaking away from 0 MeV not significant

The Final Numbers

- World's Largest Data Sample 29K $\tau \rightarrow 3h\pi^0$ events
  258 $\tau \rightarrow 5\pi\nu$ events 197 $\tau \rightarrow 3\pi 2\pi^0\nu$ events
- Accounted for $< \mathcal{N}_{Fit} >= f(M_{\nu_\tau})$
- Carefully accounted for Smearing Tails in E and M
- Raw 95% U.L. $5h$ $27\,MeV$ $4h$ Preliminary $26\,MeV$
- Total Systematic $5h$ $3\,MeV$ $4h$ $5\,MeV$
- 2D Final 95% U.L. $5h$ $M_{\nu_\tau} < 30\,MeV$ $4h$ Preliminary $31\,MeV$
- These limits *Not* unlikely for these data sets
  → Believable limit

$5h$ Published in Phys Lett B 431, 209 (1998) hep-ex/9803031

Mass limits and Statistics

	Events	$< \sigma_M >$	m_{95}
CLEO98	≈ 450	15 MeV	30 MeV
ALEPH98	55	15 MeV	22 MeV
ALEPH95	25	15 MeV	24 MeV
OPAL98	22	25 MeV ?	43.2 MeV
OPAL95	5	25 MeV	74 MeV
ARGUS92	19	10 MeV ?	31 MeV
CLEO93	113	10 MeV	32.6 MeV
ALEPH98	≈ 2900		22 MeV
DELPHI97	?		33 MeV (62 MeV)
OPAL97	≈ 2500		29.9 MeV

"Excellent Detectors": Hints of Undetected Bias

- OPAL98 43.2 MeV from $\tau \rightarrow 5\pi\nu_\tau$ $\hat{m} = 0 - 2MeV$
- ALEPH98 22 MeV from $\tau \rightarrow 3\pi\nu_\tau$ $\hat{m} = 0 - 5MeV$
- OPAL97 29.9 MeV from 3π $\hat{m} = 0MeV$
- DELPHI97 33 MeV (62 MeV) from 3π $\hat{m} = -12MeV$
- ALEPH95 24 MeV from $\tau \rightarrow 5\pi(\pi^0)\nu_\tau$ $\hat{m} = 0 - 3MeV$
- OPAL95 74 MeV from $\tau \rightarrow 5\pi\nu_\tau$ $\hat{m} = 0MeV$
- CLEO93 32.6 MeV from 5π, $3\pi2\pi^0$ $\hat{m} = 0MeV$
- ARGUS92 31 MeV from 5π $\hat{m} =?MeV$

"Excellent Detectors": Hints of Undetected Bias

- The Scatter in $\hat{m}$ is much smaller than the naively expected $< m_{95} > /1.64 = 18$ MeV
- There is a 4% chance that 7 (Gaussian) experiments could get $-\infty < \hat{m} < \ 5MeV$
- An experimental likelihood should not peak at the "true" (theorist's bias) value all the time
- Some combination of subtle systematic biases must exist

All we can do is point to possible sources of bias

The Backgrounds

- ARGUS92 Estimate background too small to matter - Throw away most massive event It's higher than the τ anyway
- CLEO93 Estimate background too small to matter
  - Reasonable from MC and data studies
- LEP: Ignore non tau backgrounds
  - Reasonable because of good cone separation from boost
- CLEO98 Include Background Function - background still small

The Smearing Functions

- ARGUS92 1D single Gaussian
- CLEO93 1D single Gaussian
- ALEPH95, ALEPH98 5π 2 D Gaussian + 5% flat tails to 10 σ
- OPAL95 2D Gaussian
- DELPHI97 3π $100 MeV^2$ Binned Fit
- OPAL98 2D Gaussian (check with Flat Tail or 2 Gaussians)
- ALEPH98 3π Sum of 2 2D Gaussians + 7 sigma flat tail
- CLEO98 5π Sum of 3 2D gaussians + 4th for systematics + Offsets

The Error Ellipses

- ARGUS92 Unscaled Propagated Tracking errors ?
- CLEO93 Propagated Tracking errors
- ALEPH95 , 98 5π Reconstructed Event Replication Method
- OPAL95 Reconstructed Event Replication Method
- DELPHI97 None (?)
- OPAL98 Reconstructed Event Replication Method
- ALEPH98 3π Average Sigmas for Binned Reconstructed MC
- CLEO98 Scaled Propagated Tracking errors w/ offsets in Smearing

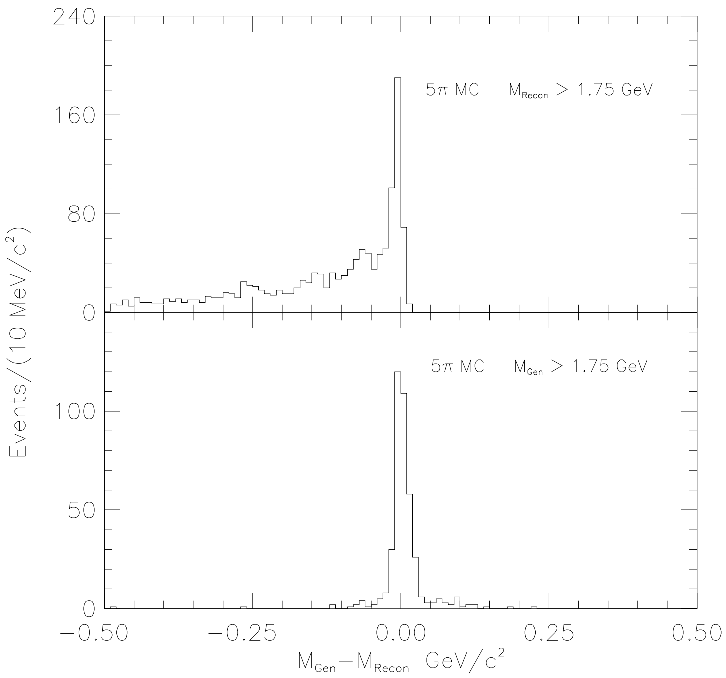
Reconstructed Event Replication Method

- Assume reconstructed event is “close” to true input event
- Take measured 3 momenta of reconstructed data event tracks $\{\mathbf{p_i}\}$.
- Feed into MC several thousand times.
- Fit resulting Reconstructed Energy vs Mass distribution

Any systematic reconstruction offset invalidates the method.

Reconstructed Event Replication Method

This works well most of the time, **except** at the edge of a distribution where events are dominated by upward fluctuations (even for “well-reconstructed” tracks) The neutrino mass sensitive region is at the edge of a falling distribution.



Can lead to underestimate of Dectector Smearing in the most sensitive part of fit region - but how much ???

Spectral Functions

The spectral function is effectively the physics (or mass distribution) before the V-A neutrino interaction + neutrino mass is put in.

For CLEO results - 5π shape = $e^+e^- \rightarrow 4\pi$ plus soft π theorems + free floating harder function

Other Results use ad hoc mixtures of ρ, $a1$, etc.

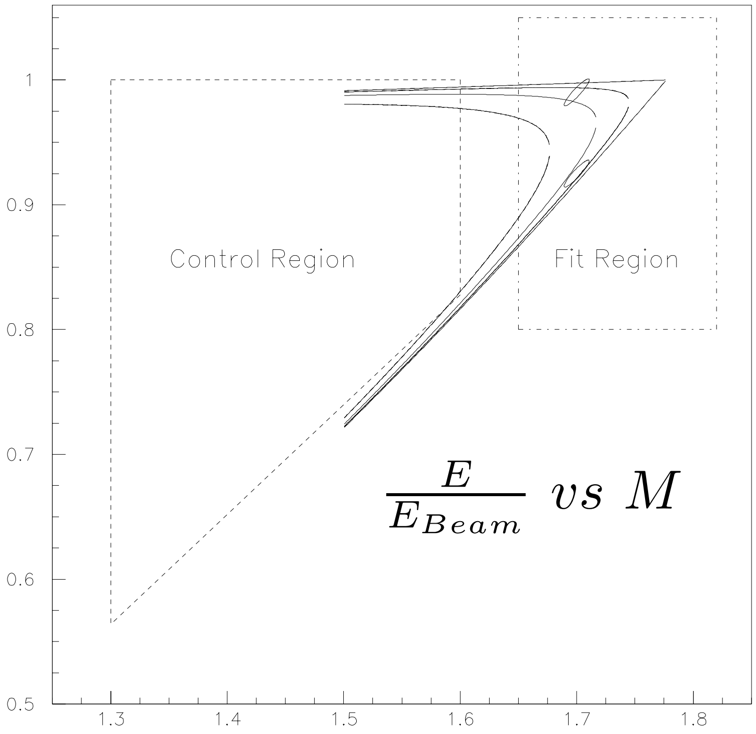
Wrong Spectral function can fake neutrino mass → best to use a motivated shape

The Energy Dependance

At fixed $\mathrm{M}_{\mathrm{hadron}}$

- the $\mathrm{E}_{\mathrm{hadron}}$ spectrum is only sensitive because of a sharp cutoff at the kinematic edge (Θ function)
- Far from the endpoint, kinematic edge is a steep function of m_ν

The smearing in Energy at low $\mathrm{M}_{\mathrm{hadron}}$ needs to be very well modeled.

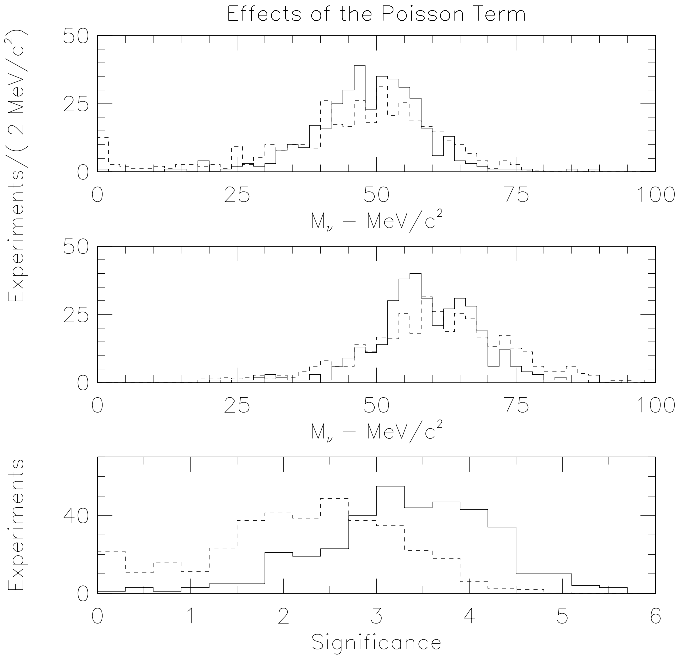


$\frac{E}{E_{Beam}}$ *vs* M

Likelihoods + Fit Regions

A fit over the whole mass/energy plane takes into account the expected number of events at high mass given the number at low mass.

A fit over a small endpoint region (ALEPH) is therefore less sensitive to spectral function variations, but has more inherent fluctuations or variance.



Comments on Maximum Likelihood

- The maximum likelihood method finds the value of m_ν that maximizes the probability of the observed data.
- If the observed data are very unlikely, the resulting best fit value is not meaningful.
- The probability of “too good” a limit decreases with statistics.

Upper Limits Are NOT What You Think They Are

The Interpretation of a 95th percentile as a statement about nature is not necessarily straightforward

Probability of a 27 MeV limit or lower

	25 events	450 events
$M_\nu^{input} = 0\ MeV$	3%	67%
$M_\nu^{input} = 50\ MeV$	$\approx 1\%$	$< 1\%$

Statistics, not lucky events, give discriminatory power

What is an Upper Limit ?

- The 95th Percentile of the Likelihood ?
- The value of ν input mass for which the data likelihood peak is at a larger value than 5% of all MC experiments peak values?

The answer to these two questions is not the same for low statistics.

What is the large statistics limit? It depends on mode and chosen spectral function.

Lessons for TCF

- Luminosity, Luminosity, Luminosity
- Two D limits have little advantage over 1D limits at TCF
- Luminosity, Luminosity, Luminosity
- Large tails of Smearing functions need to be carefully studied
- Luminosity, Luminosity, Luminosity
- Need small smearing
- Luminosity, Luminosity, Luminosity
- Other methods should not be underestimated
- Luminosity, Luminosity, Luminosity

Conclusions

- Using by far the world's largest data set, CLEO sets an upper limit of 30 MeV on the mass of the tau neutrino
- The world ensemble of neutrino mass limits shows some hints of bias
- Underestimated detector smearing effects are important bias sources
- "Lucky" limits do not have discriminatory power

So, what **do** we know about the ν_τ mass ?
Far less than we thought
There's plenty of room for TCF to make a real dent