

**FORBIDDEN AND RARE
 τ DECAYS**

**Richard Stroynowski
Southern Methodist University**

Tau-Charm Workshop, SLAC, March 6-8, 1999

Rare Decays

Allowed by the Standard Model but with very small rates.

Definition of what is rare changes with time.

- High multiplicity final states suppressed by Phase Space

$$- \tau \rightarrow 6\pi\nu_\tau, \tau \rightarrow 7\pi\nu_\tau$$

- Heavy final state suppressed by Phase Space

$$- \tau^- \rightarrow K^- K^+ K^- \nu_\tau$$

- Internal radiation

$$- \tau \rightarrow eee\nu_e\nu_\tau, \tau \rightarrow e\mu\mu\nu_e\nu_\tau, \tau \rightarrow eeeee\nu_e\nu_\tau \dots$$

- Second class currents

$$- \tau^- \rightarrow \pi^- \eta \nu_\tau$$

Such decays are allowed by specific mechanisms and the identification of decay rates and decay distributions permits a study of such mechanisms.

SELECTED BRANCHING FRACTIONS
FOR ALLOWED τ DECAYS

5 PRONGS	$\tau^- \rightarrow 3\pi^- 2\pi^+ \nu_\tau$	$(7.5 \pm 0.7) \cdot 10^{-4}$
"	$\tau^- \rightarrow 3\pi^- 2\pi^+ \pi^0 \nu_\tau$	$(2.2 \pm 0.5) \cdot 10^{-4}$
7 PRONGS	$\tau^- \rightarrow 4\pi^- 3\pi^+ \nu_\tau$	$< 2.4 \cdot 10^{-6}$
HEAVY FINAL STATE	$\tau^- \rightarrow \pi^- K^+ K^- \pi^0 \nu_\tau$	$(6.9 \pm 3.0) \cdot 10^{-4}$
	$\tau^- \rightarrow K^- K^+ K^- \nu_\tau$	$< 1.9 \cdot 10^{-4}$
SECOND CLASS	$\tau^- \rightarrow \pi^- \eta \nu_\tau$	$< 1.4 \cdot 10^{-4}$
QCD	$\tau^- \rightarrow \eta \pi^- \pi^+ \pi^- \nu_\tau$	$(3.4 \pm 0.8) \cdot 10^{-4}$
"	$\tau^- \rightarrow f_1(1260) \pi^- \nu_\tau$	$(1.9 \pm 0.7) \cdot 10^{-4}$
"	$\tau^- \rightarrow \phi \pi^- \nu_\tau$	$< 2.0 \cdot 10^{-4}$
INTERNAL CONVERSION*	$\tau^- \rightarrow e^- e^+ e^- \bar{\nu}_e \nu_\tau$	$(2.8 \pm 1.5) \cdot 10^{-5}$
	$\tau^- \rightarrow \mu^- e^+ e^- \bar{\nu}_e \nu_\tau$	$< 3.6 \cdot 10^{-5}$

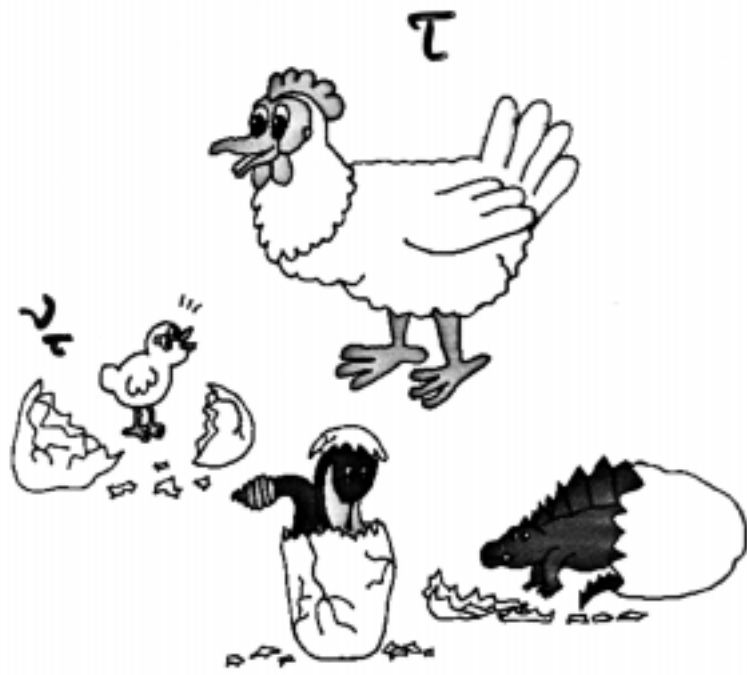
Rare Decays

Motivation for studies of rare decays depends on the particular channel:

- Decays suppressed by Phase Space are likely to provide best limits on ν_τ mass
- Rare Cabbibo suppressed decays are the cleanest environment to study strange meson resonances
- Decays suppressed by isospin and/or G-parity allow for studies of final state interactions, special aspects of QCD and of the mass difference between u and d quarks

All of these studies will be attempted at the B-factories
Within the present theoretical framework we do not expect major discoveries deriving from precision measurements of rare allowed decays but surprizes are always possible





Lepton Number/ Flavor Violation

Neutrinoless decays are a subset of decays that are not expected in the Standard Model which exhibit

- Lepton number violation
 - when lepton number is not conserved e.g., $\tau \rightarrow p\gamma$
- Lepton flavor violation
 - when lepton flavor is not conserved e.g., $\tau \rightarrow e\gamma$

One can consider other lepton number/ flavor violating decays that result in e.g., more than one neutrino : $\tau \rightarrow e\nu_e\nu_\tau$ but experimentally these searches are much more difficult.

Lepton Number/ Flavor Violation

- Searches date to the discovery of the muon
- Pontecorvo looked for $\mu \rightarrow e\gamma$ in 1948
- No experimental evidence ever found \rightarrow we assume conservation law in Standard Model
- BUT -There is no flavor conserving symmetry !!!

Actually - non-perturbative Bell-Jackiw anomaly allows for lepton and baryon number violation in the Standard Model via the tunneling transition between topologically distinct vacua (G. 't Hooft, PRL 37, 8 (1976)). It is suppressed by a factor

$$\exp(-4\pi \sin^2\theta_W/\alpha) \sim 10^{-170}$$

Not a problem at accelerator energies but may have effect in cosmological models (V. A. Kuzmin *et al.*, PLB 155, 36 (1976)).

Lepton Flavor Violation Beyond the SM

Lepton flavor violation is expected in many extensions of the Standard Model.

- GUTs
- Left-right symmetric models
- SUSY with broken R-parity
- Superstring theories
- Supergravity theories

Lepton flavor violation is induced through a presence of new couplings and/or new particles - additional gauge bosons, heavy neutral leptons etc.

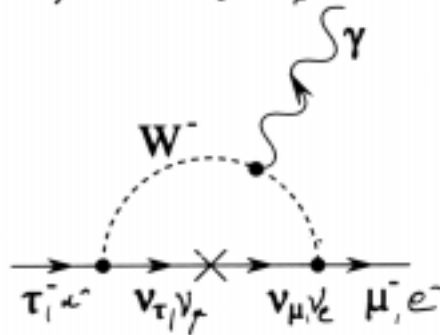
Particular interest for the τ :

- τ lepton is heavy \rightarrow many decay channels
- 3rd generation \rightarrow new coupling may be enhanced

Observation of Lepton Flavor violation \rightarrow New Physics beyond the Standard Model

Neutrino Oscillations

Effects of neutrino mixing on charged lepton number violation was pointed out by Bilenkii and Pontecorvo, PL 61B, 248 (1976) for the $\nu_e \rightarrow \nu_\mu$ oscillations.



$$\Gamma/\Gamma_{tot} \sim \frac{3\alpha}{32\pi} \sin^2(2\theta) \left(\frac{\Delta m_\nu^2}{m_W^2}\right)^2 =$$

$$= 5.5 \cdot 10^{-48} \sin^2(2\theta) \Delta m_\nu^2$$

Mass dependence of the couplings cancels out and the relation is the same for $\nu_\mu \rightarrow \nu_\tau$ mixing.

Massive Neutrino

There are two other effects introduced by a massive neutrino:

- The mass term in the Lagrangian will introduce right-handed currents leading to polarization effects
- The lepton masses will be mixed via a matrix analogous to CKM and that would introduce lepton flavor violation.

Since neutrino oscillation appears to be small, the corresponding CKM matrix is expected to be almost diagonal

Mass Dependent Couplings

Much larger effects of lepton flavor violation are expected in models with mass dependent couplings.

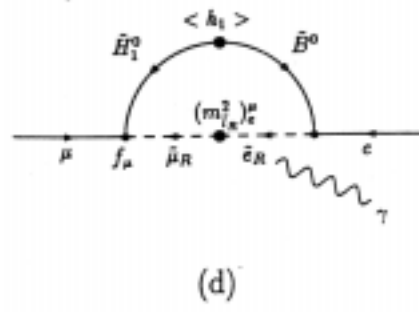
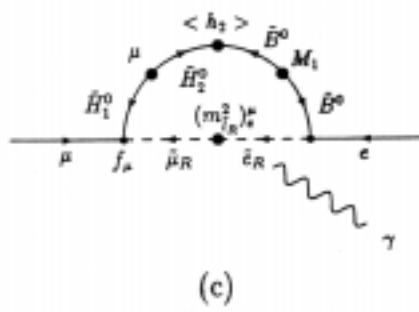
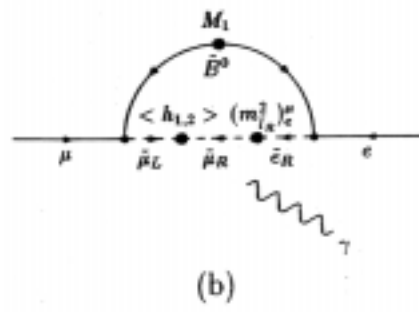
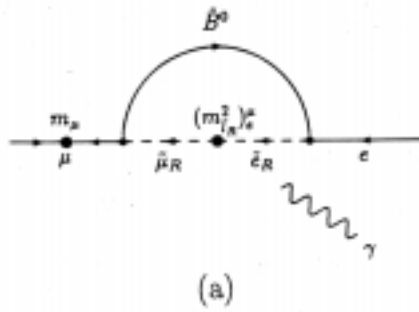
- Supersymmetric Models:
 - Barbieri and Hall - no righthanded ν
 - Hisano *et al.* - with righthanded ν and m_ν close to GUT scale
- Left-right symmetric models - Mohapatra *et al.*
- GUT and Superstring models - Ilakovac *et al.*, King and Oliveira,..
- E(6) models - Arnowitt *et al.*

In general these models enhance the lepton flavor violation for the τ as compared to μ by a factor

$$(m_\tau/m_\mu)^n \text{ or } (m_\tau/m_\mu)^n \cdot (m_\mu/m_e)^n$$

with n in the range 2 - 5. The choices of particular values for model parameters are harder to justify.

hep-ph/9605296 14 May 1996



Mass Dependent Couplings

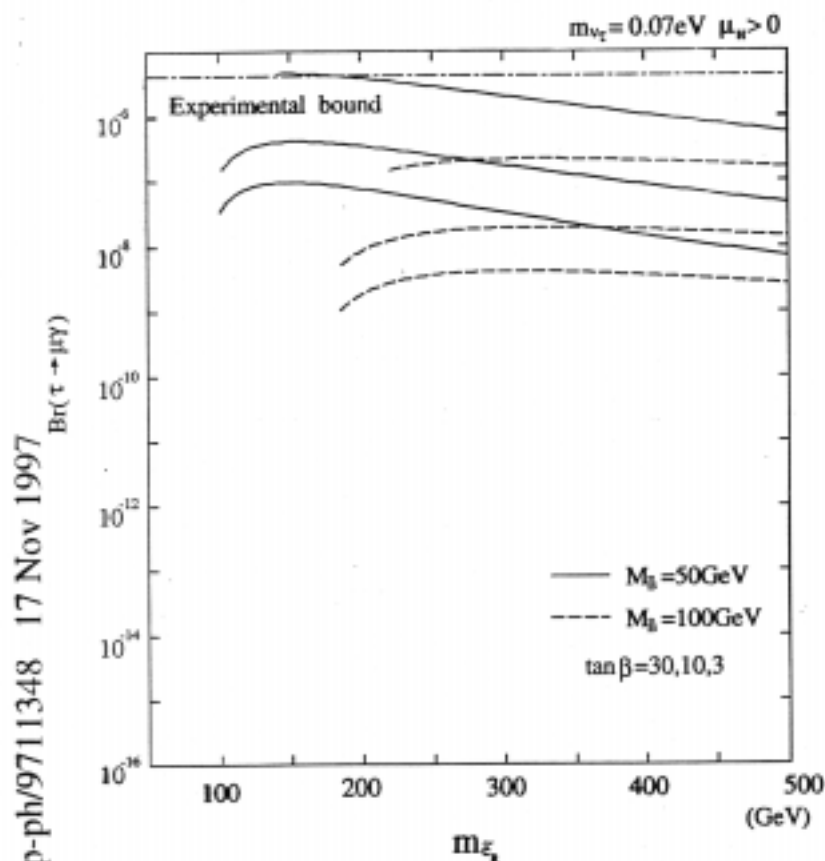
Violation rates closest to the experimentally accessible range for $\tau \rightarrow \mu\gamma$ decays are obtained by Hisano, Nomura and Yanagida (hep-ph/9711348 and hep-ph/9805367) within SUSY $SU(5)_{GUT}$ under assumption of large Yukawa couplings at the GUT scale and $\nu_\mu \rightarrow \nu_\tau$ mixing as explanation of Kamiokande results.

τ decays into lepton + one or two mesons have been calculated by Ilakovac *et al* (hep-ph/9608218) in the framework of the Standard Model with righthanded neutrino.

Overall theoretical expectations:

- Several of the calculated rates are within one or two orders of magnitude from present limits
- $\tau \rightarrow \mu\gamma$ has largest expected rates

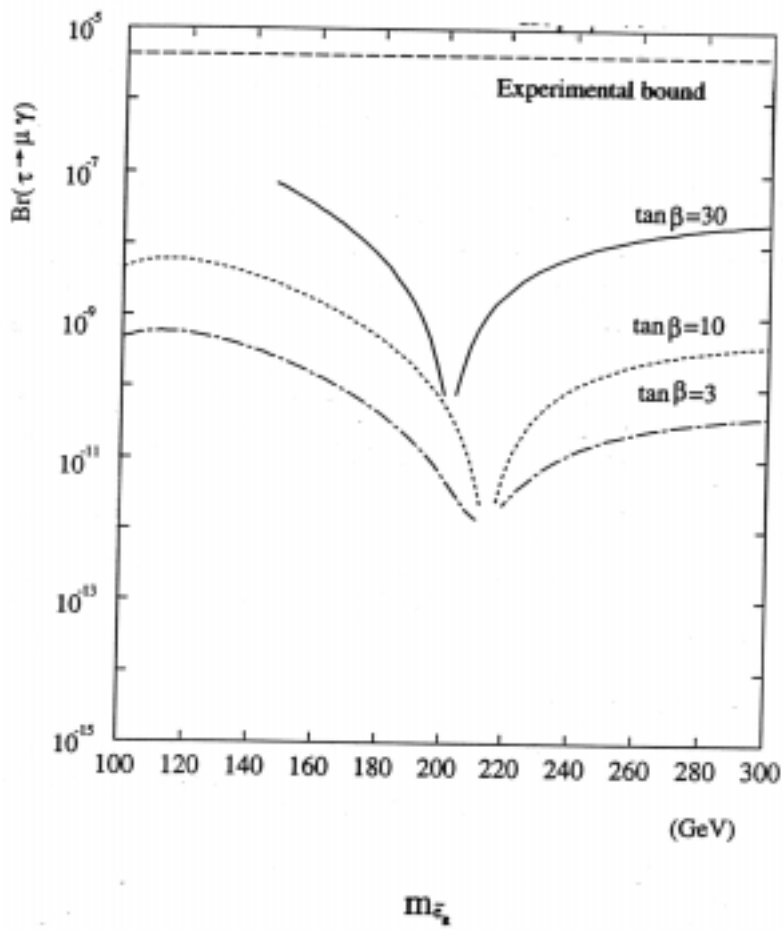
$\tau \rightarrow \mu \gamma$



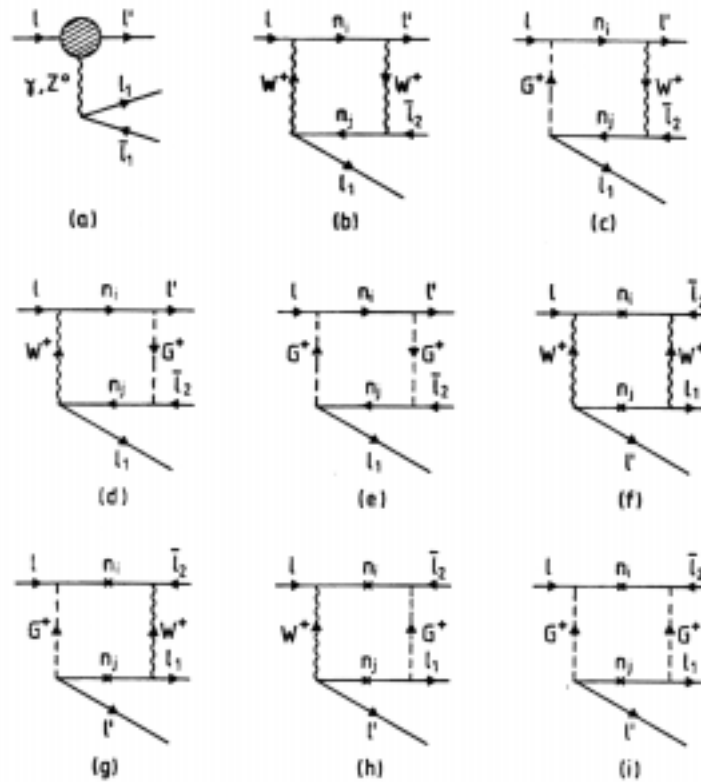
MASS OF RIGHT HANDED SELECTRON

Fig. 4

$B(\tau \rightarrow \mu \gamma)$



Feynman diagrams for $l \rightarrow l' l_1 l_2$ decays in models with neutral heavy leptons (GUTs, superstrings, L-R symmetric):

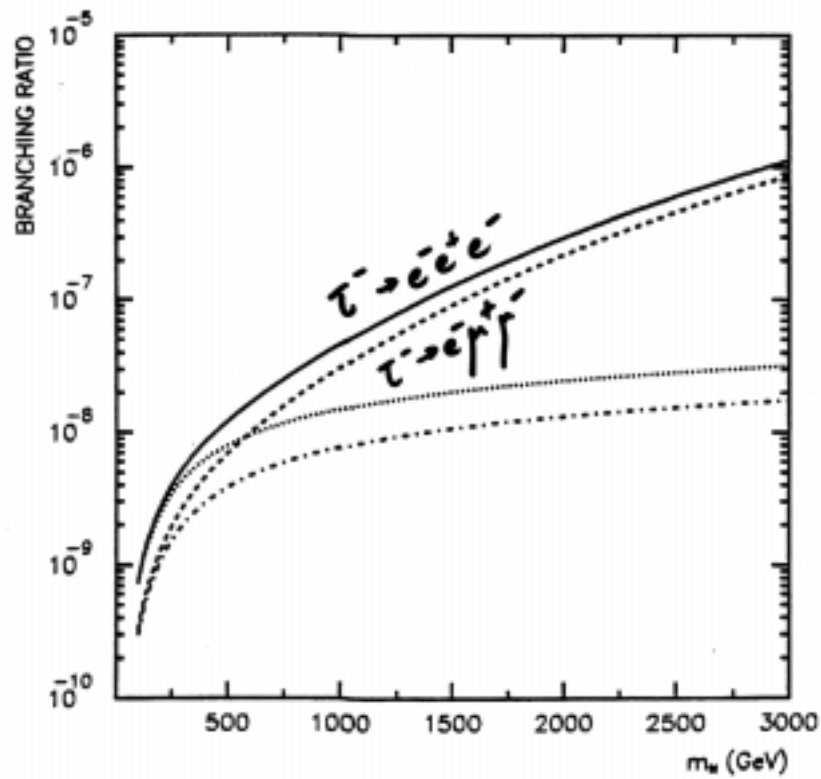


+ $(l_1 \rightarrow l')$

n_i — neutral heavy leptons (Majorana neutrinos)

G — Goldstone bosons

Neutrinoless τ decay branching fractions attainable in models
with neutral heavy leptons:



A. Ilakovac and A. Pilaftsis, Nucl. Phys. B **437**, 491 (1995).

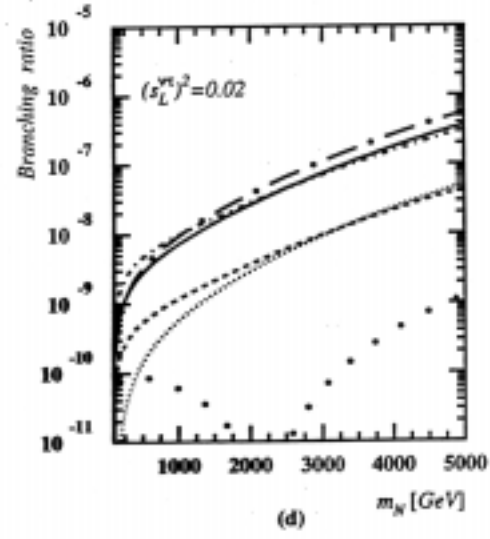
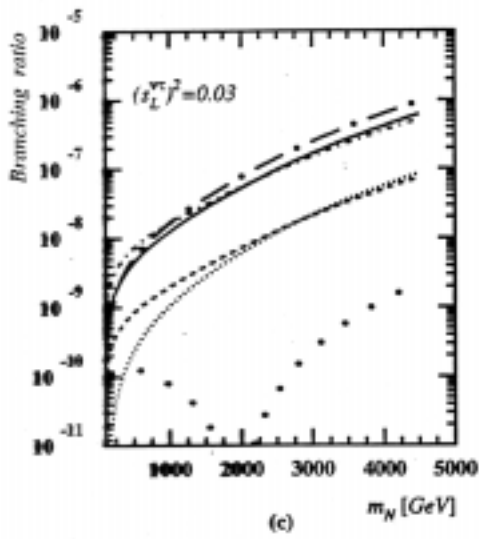
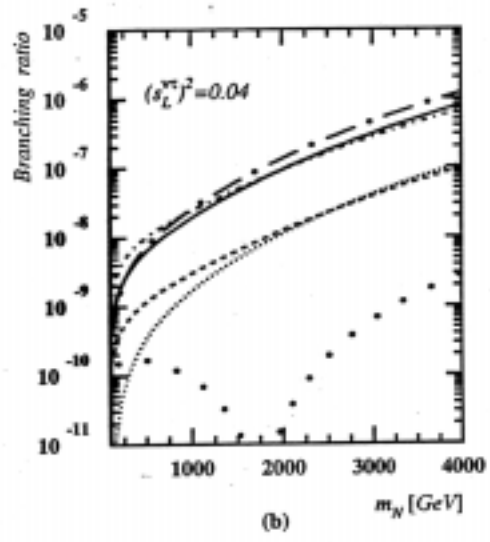
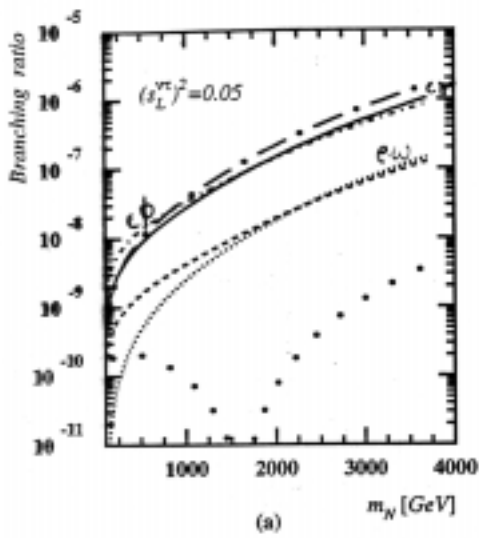


Fig. 2

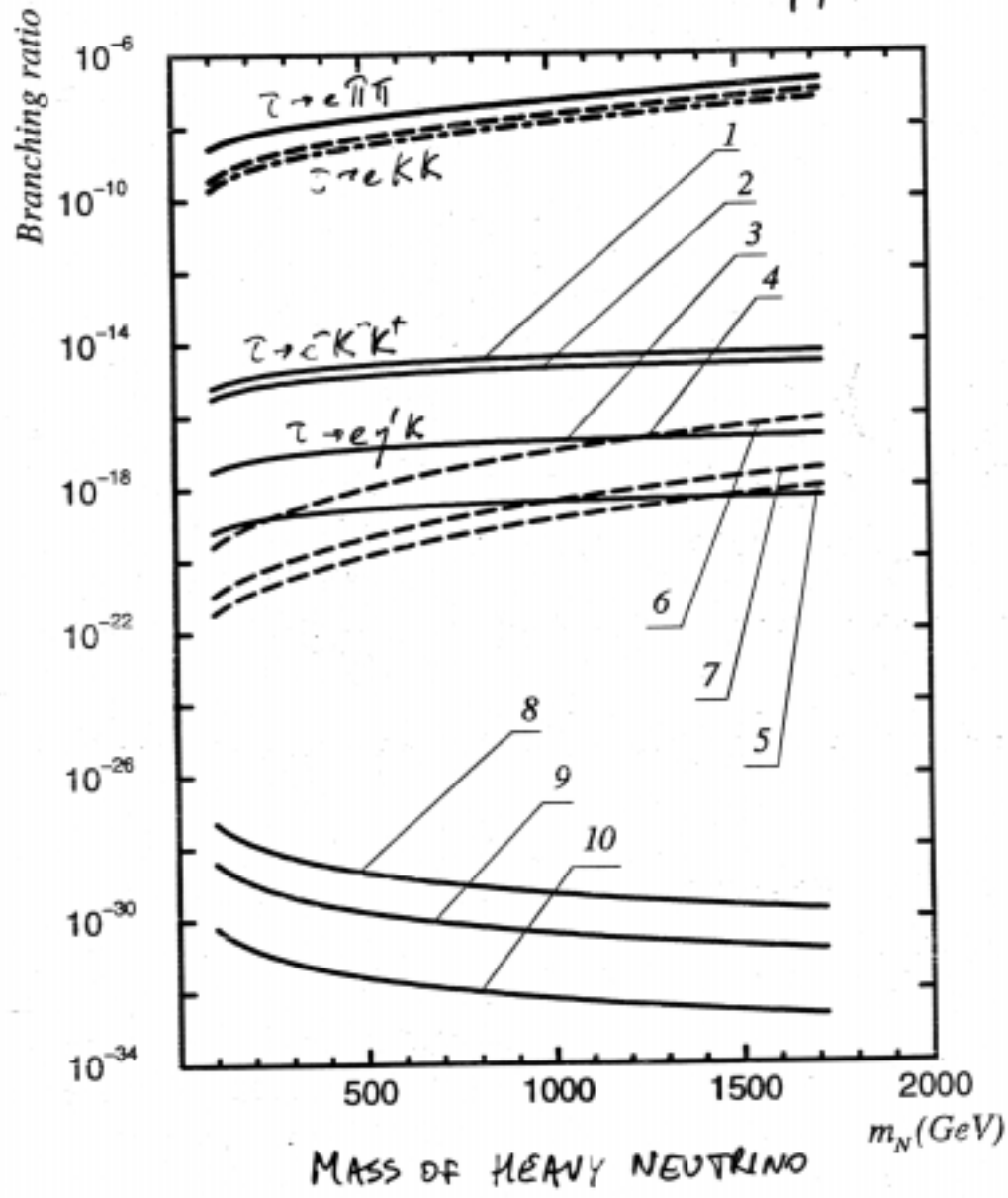
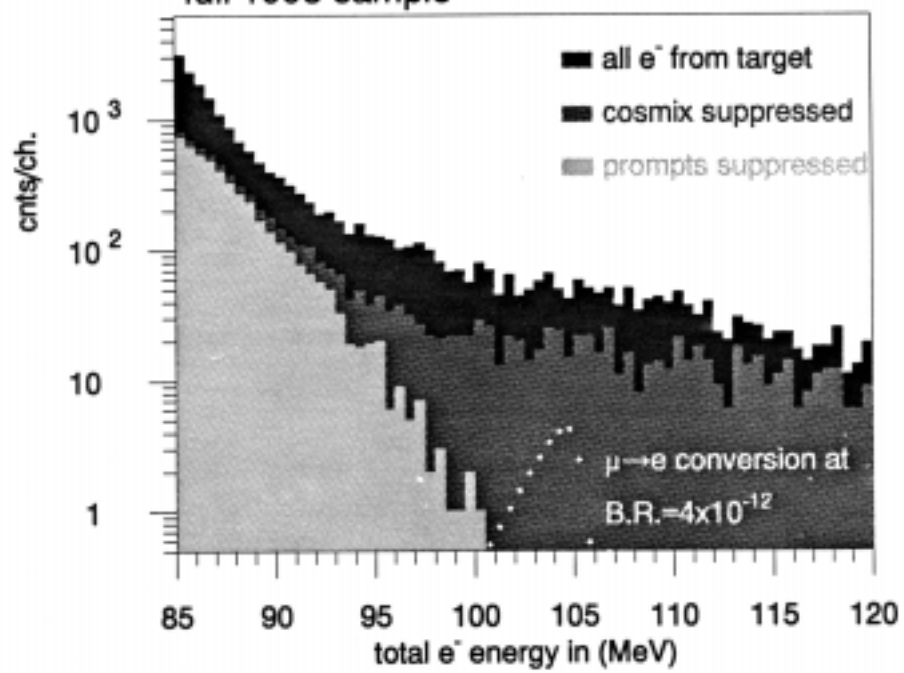


Fig. 2

$\mu^- \text{Ti} \rightarrow e^- \text{Ti}$
full 1993 sample



Experimental Searches

Numerically, limits are dominated by results from muon experiments

$$B(\mu \rightarrow e\gamma) < 4.9 \cdot 10^{-11} \quad \text{LAMPF}$$

$$B(\mu \rightarrow eee) < 1.0 \cdot 10^{-12} \quad \text{SINDRUM}$$

and by the μ to e transitions in nuclear targets

$$(A, Z) + \mu^- \rightarrow e^- + (A, Z)^*$$

SINDRUM II preliminary conversion results at Vancouver for the titanium ^{48}Ti target $R_{\mu e}^{\text{Ti}} < 6.1 \cdot 10^{-13}$

All muon results are about 1-2 orders of magnitude above the optimistic theoretical expectations. New experiment MECO, planned at BNL, hopes to reach the conversion rate of $R_{\mu e} \sim 10^{-16}$.

τ Results

Limits for neutrinoless τ decays are dominated by CLEO. Three papers published recently

- PRD 55, 3919 - $\tau \rightarrow \mu\gamma, e\gamma$
- PRL 79, 1221 - 10 channels with π^0 's and η 's
- PRD 57, 5903 - 28 channels with e, μ and 2 prongs

Search is "easy":

- $M_{inv}(signal) = m_\tau$
- $E_{tot}(signal) = E_{beam}$
- Direction of other τ in the event is known
- Presence of lepton among decay products helps to reject $q\bar{q}$ events
- Signal region defined by the E and M constraints
- Background small - extrapolated from sidebands

τ Analysis

There are tricky nuances which have to be considered in the analysis:

- Selection criteria must be established independently of the data to avoid subjective judgement based on fluctuations in the signal region.
- Optimization of the sizes of signal and sidebands regions is difficult when signal and background are small, correlated and/or unknown.

Optimization techniques using Monte Carlo:

- Minimize branching fraction for which the probability of discovery equals $1/2$ ($B_{1/2}$)
- Minimize average expected upper limit

In the case of very small background, the minimization of upper limit produces larger signal regions (greater selection efficiency) than minimization of $B_{1/2}$.

Minimization of \overline{UL}

Definition of upper limit at 90% CL for the branching fraction when there are n events observed (conservative):

$$B(\tau \rightarrow \dots) < UL \equiv \frac{\lambda_n}{2 \epsilon N_{\tau\tau}},$$

where λ_n is a solution of the equation

$$e^{-\lambda_n} \sum_{k=0}^n \frac{\lambda_n^k}{k!} = 0.1.$$

1. Define signal region. Estimate signal efficiency ϵ .
2. Interpolate background from sidebands. Obtain background estimate n_b .
3. Calculate average expected upper limit, \overline{UL} , as follows:

$$\overline{UL} = \frac{1}{2 \epsilon N_{\tau\tau}} [\lambda_0 P(0|n_b) + \lambda_1 P(1|n_b) + \dots],$$

where $P(k|n_b)$ is the Poisson probability of observing k events given the n_b mean value.

4. Select signal region which minimizes \overline{UL} .

Minimization of \overline{UL} typically produces larger signal regions than minimization of $B_{1/2}$. Use \overline{UL} .

Optimization of $B_{1/2}$

Definition of discovery: observed data are incompatible with background expectation at 99.9% confidence level (for normal distribution corresponds to $\sim 3.3\sigma$ deviation).

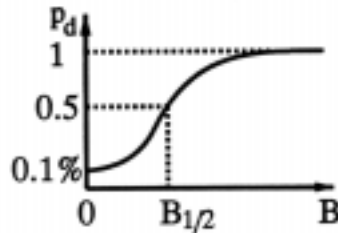
1. Define signal region. Estimate signal efficiency ϵ .
2. Interpolate background from sidebands. Obtain background estimate n_b .
3. Estimate discovery probability, p_d , assuming signal branching fraction B . For Poisson process

$$p_d = 1 - e^{-(n_s+n_b)} \sum_{k=0}^N \frac{(n_s+n_b)^k}{k!},$$

where n_s is the expected number of signal events and N is the smallest integer for which

$$e^{-n_b} \sum_{k=0}^N \frac{n_b^k}{k!} \geq 0.999.$$

For small values of B , $n_s = 2\epsilon BN_{\tau\tau}$.



4. Find $B_{1/2}$ for which $p_d = 1/2$.
5. Select signal region which minimizes $B_{1/2}$.

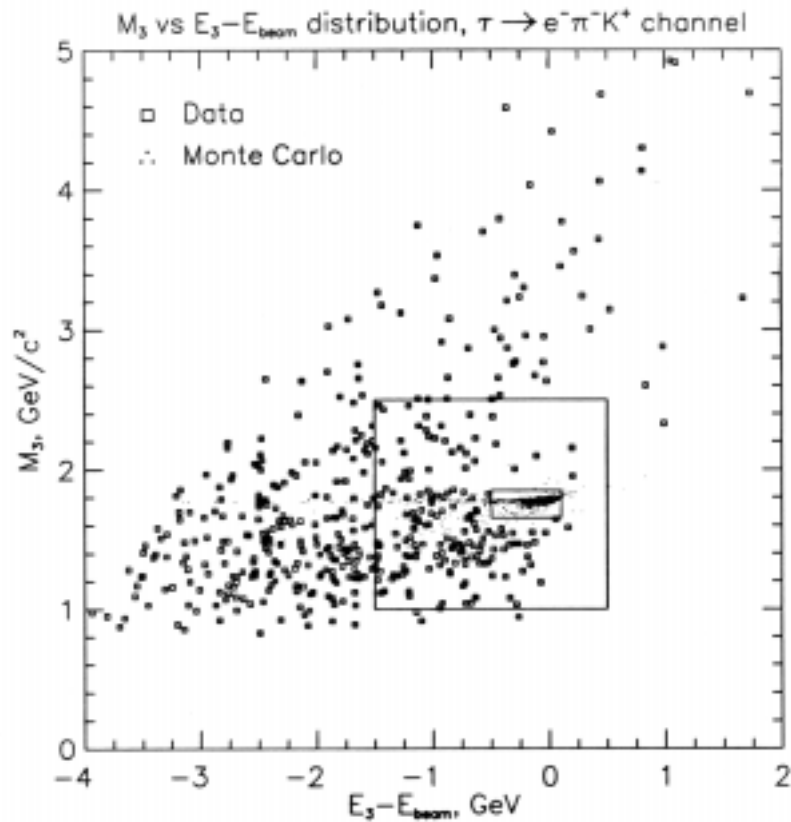
CLEO $\tau \rightarrow \mu\gamma$ ANALYSIS

DEPENDENCE ON STATISTICAL METHOD USED

Table 1. Upper limits at 90% CL for the $\tau \rightarrow \mu\gamma$ analysis of Ref. [13].

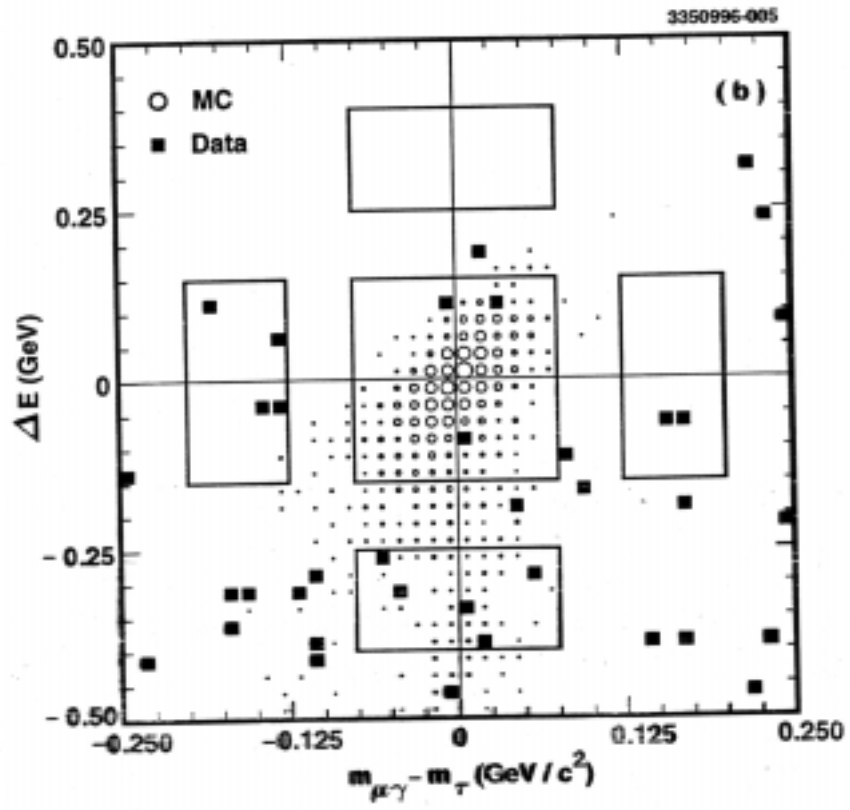
Method	Upper limit at 90% CL
Bayesian with flat prior	3.57
Bayesian $1/\sqrt{s+b}$	3.30
classical	1.18
classical, based on statistical significance	1.03
MC technique of Sec. 2.3.2	1.60
Feldman & Cousins	~ 2.5

Sideband Definition

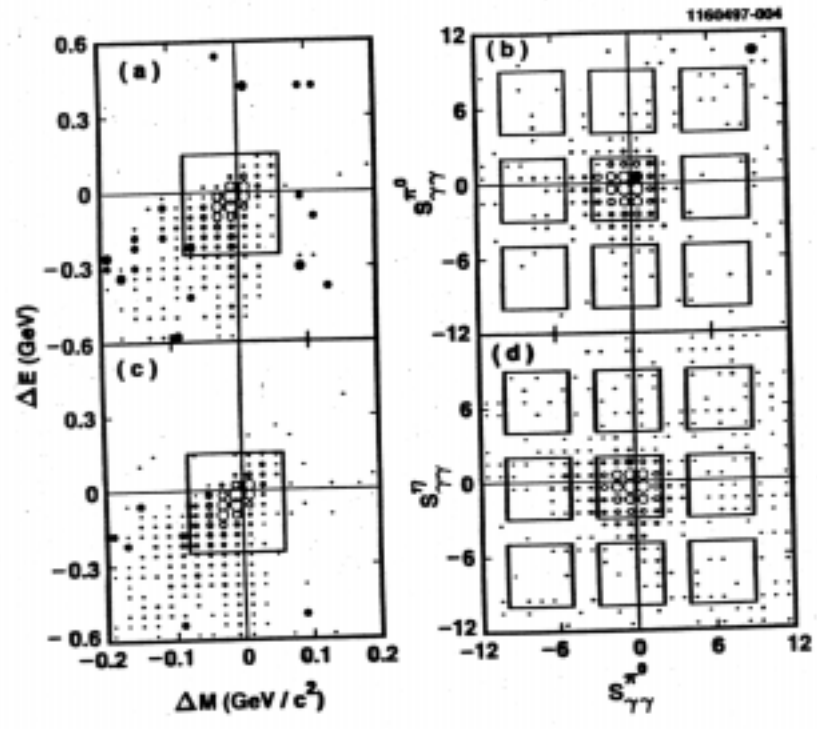


The outer sideband rectangle is defined by
 $-1.5 \text{ GeV} < E_3 - E_{beam} < 0.5 \text{ GeV}$ and
 $1.0 \text{ GeV}/c^2 < M_3 < 2.5 \text{ GeV}/c^2$.

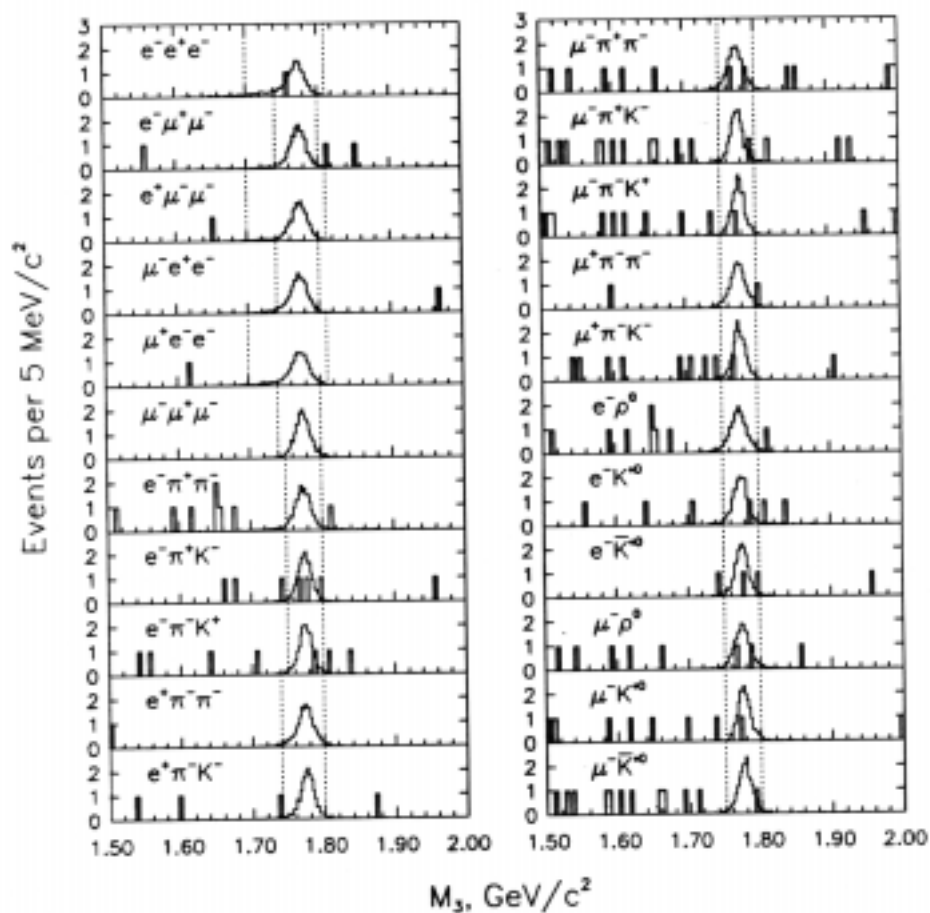
The inner (excluded) rectangle is defined by
 $-0.5 \text{ GeV} < E_3 - E_{beam} < 0.1 \text{ GeV}$ and
 $1.65 \text{ GeV}/c^2 < M_3 < 1.85 \text{ GeV}/c^2$.

$\tau \rightarrow \mu \gamma$ 

$$\tau \rightarrow \rho \pi^0 \pi$$



Inv. Mass of τ Decay Products



Signal MC: phase space. Expected signal shapes are shown with arbitrary normalization.

ν -less Decay Search Results

Channel	UL, 10^{-6}	Channel	UL, 10^{-6}
$e^- \gamma$	2.7	$\mu^- \gamma$	3.0
$e^- e^+ e^-$	2.9	$\mu^- \mu^+ \mu^-$	1.9
$e^- e^+ \mu^-$	1.7	$\mu^- \mu^+ e^-$	1.8
$e^- \mu^+ e^-$	1.5	$\mu^- e^+ \mu^-$	1.5
$e^- \pi^0$	3.7	$\mu^- \pi^0$	4.0
$e^- \eta$	8.2	$\mu^- \eta$	9.6
$e^- \rho^0$	2.0	$\mu^- \rho^0$	6.3
$e^- K^{*0}$	5.1	$\mu^- K^{*0}$	7.5
$e^- \bar{K}^{*0}$	7.4	$\mu^- \bar{K}^{*0}$	7.5
$e^- \phi$	6.9	$\mu^- \phi$	7.0
$e^- \pi^+ \pi^-$	2.2	$\mu^- \pi^+ \pi^-$	8.2
$e^- \pi^+ K^-$	6.4	$\mu^- \pi^+ K^-$	7.5
$e^- K^+ \pi^-$	3.8	$\mu^- K^+ \pi^-$	7.4
$e^- K^+ K^-$	6.0	$\mu^- K^+ K^-$	15
$e^+ \pi^- \pi^-$	1.9	$\mu^+ \pi^- \pi^-$	3.4
$e^+ \pi^- K^-$	2.1	$\mu^+ \pi^- K^-$	7.0
$e^+ K^- K^-$	3.8	$\mu^+ K^- K^-$	6.0
$e^- \pi^0 \pi^0$	6.5	$\mu^- \pi^0 \pi^0$	14
$e^- \pi^0 \eta$	24	$\mu^- \pi^0 \eta$	22
$e^- \eta \eta$	35	$\mu^- \eta \eta$	60

Other Searches

Channel	Upper Limit 10^{-6}	Experiment
$e^- K^0$	1300	Mark II
$\mu^- K^0$	1000	Mark II
$\bar{p} \gamma$	290	ARGUS*
$\bar{p} \pi$	660	ARGUS*
$\bar{p} \eta$	1300	ARGUS*
$\pi \pi^0$	370	ARGUS*

* CAN BE RELATED TO p DECAY

* VIOLATES ANGULAR MOMENTUM CONSERVATION
(ROTATIONAL SYMMETRY OF SPACE)

FUTURE

New machines/detectors will become operational in the next few years:

- BaBar
- KEK-B
- CLEO III
- TEVATRON: CDF, D0
- LHC: ATLAS, CMS

Can these experiments study lepton flavor violation ?

ATLAS study (Stroynowski+Serin): Search for $\tau \rightarrow \mu\gamma$

- High rate of τ production from W, Z, B, H decays.
Estimated rate for $q\bar{q} \rightarrow W \rightarrow \tau\nu_\tau$ is 1.5×10^9 events/year.
- Mass of the τ is the only constraint.
- Many backgrounds. The dominant one is due to radiative W production and decays
 $qq' \rightarrow \gamma W \rightarrow \gamma\mu\nu_\mu$ and $qq' \rightarrow \gamma\tau\nu_\tau \rightarrow \gamma\mu\nu_\mu\nu_\tau$.
- Conclusion: Search is background limited to a Branching Fraction limit of about $1. \times 10^{-6}$.

ATLAS Internal Note
PHYS-NO-114
21 October 1997

Study of lepton number violating decay $\tau \rightarrow \mu\gamma$ in
ATLAS.

L. Serin
Laboratoire de l'Accélérateur Linéaire,
IN2P3-CNRS et Université Paris-Sud
91405 Orsay Cédex, France.

R. Stroynowski
Southern Methodist University, Dallas, Tx 75275, USA.

SEARCH FOR LEPTON NUMBER VIOLATION AT LHC: $\tau \rightarrow \mu \gamma$

SIGNAL

$$q\bar{q} \rightarrow W \rightarrow \tau \nu_\tau$$

\swarrow
 $\mu \gamma$

$$\sigma = 14.8 \text{ nb} \Rightarrow \sim 1.5 \cdot 10^8 \text{ ev/yr at low luminosity}$$

VARIABLES $P_T(\tau)$, $P_T(\mu)$, $M(\mu\gamma)$, $R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$

CONSTRAINT $M(\mu\gamma) = m_\tau$

BACKGROUNDS

RADIATIVE

① $q\bar{q} \rightarrow W \rightarrow \mu \nu_\mu + \gamma$

② $q\bar{q} \rightarrow \gamma W \rightarrow \mu \nu_\mu$

③ $q\bar{q} \rightarrow W \rightarrow \tau \nu_\tau + \gamma$
 \swarrow
 $\mu \nu_\tau$

PYTHIA + ATFAST
GENERATION

$$4 \cdot 10^6 \times 10$$

$$1 \cdot 10^6$$

$$1 \cdot 10^6 \times 10$$

QCD jets

New Physics

RESULTS

BACKGROUND $\sim 24 \text{ ev/yr} \Rightarrow \text{LIMIT } \mathcal{B}(\tau \rightarrow \mu \gamma) < 3.3 \cdot 10^{-7}$
in 1 year at low luminosity
(factor of 10 better than CLEO)

UNRESOLVED QUESTION

RELATIVE NORMALIZATION OF BACKGROUND ① AND ② IN PYTHIA
SHOULD CROSS-CHECK WITH PHOTOS.

$M(\mu\gamma)$ RESOLUTION
BETTER THAN
AT CLEO.

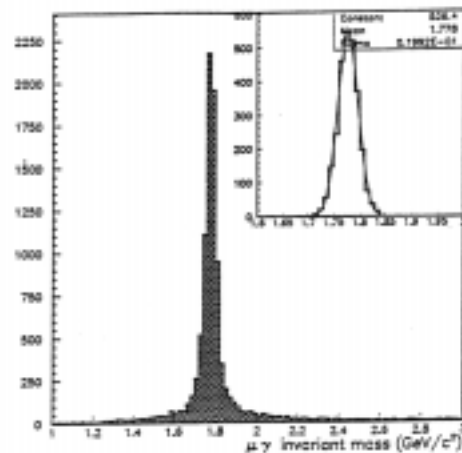


Figure 1: The muon-photon invariant mass for Monte Carlo signal events before applying the selection criteria.

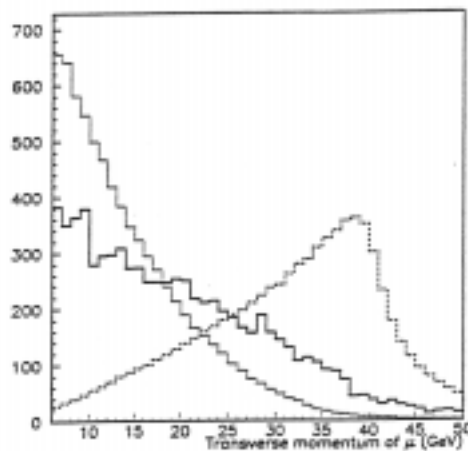
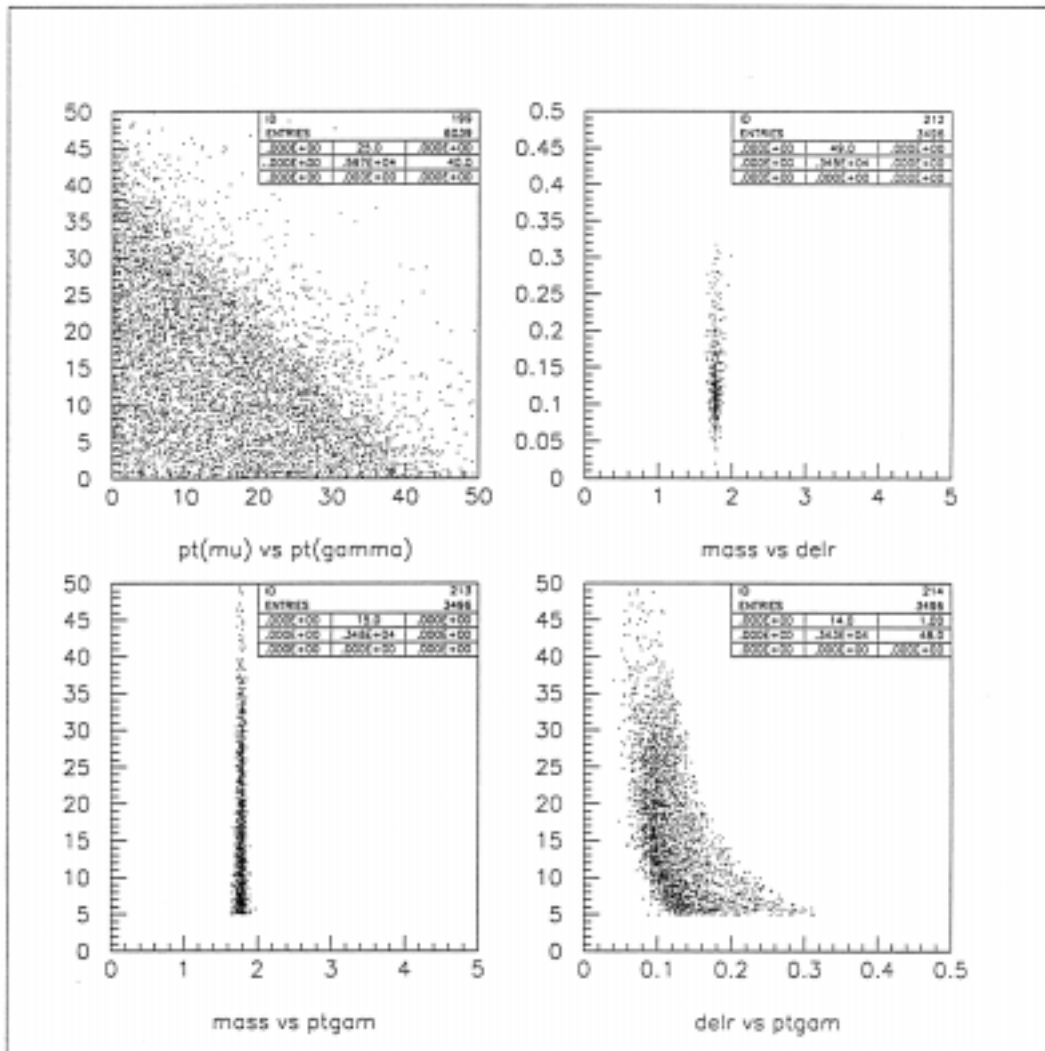
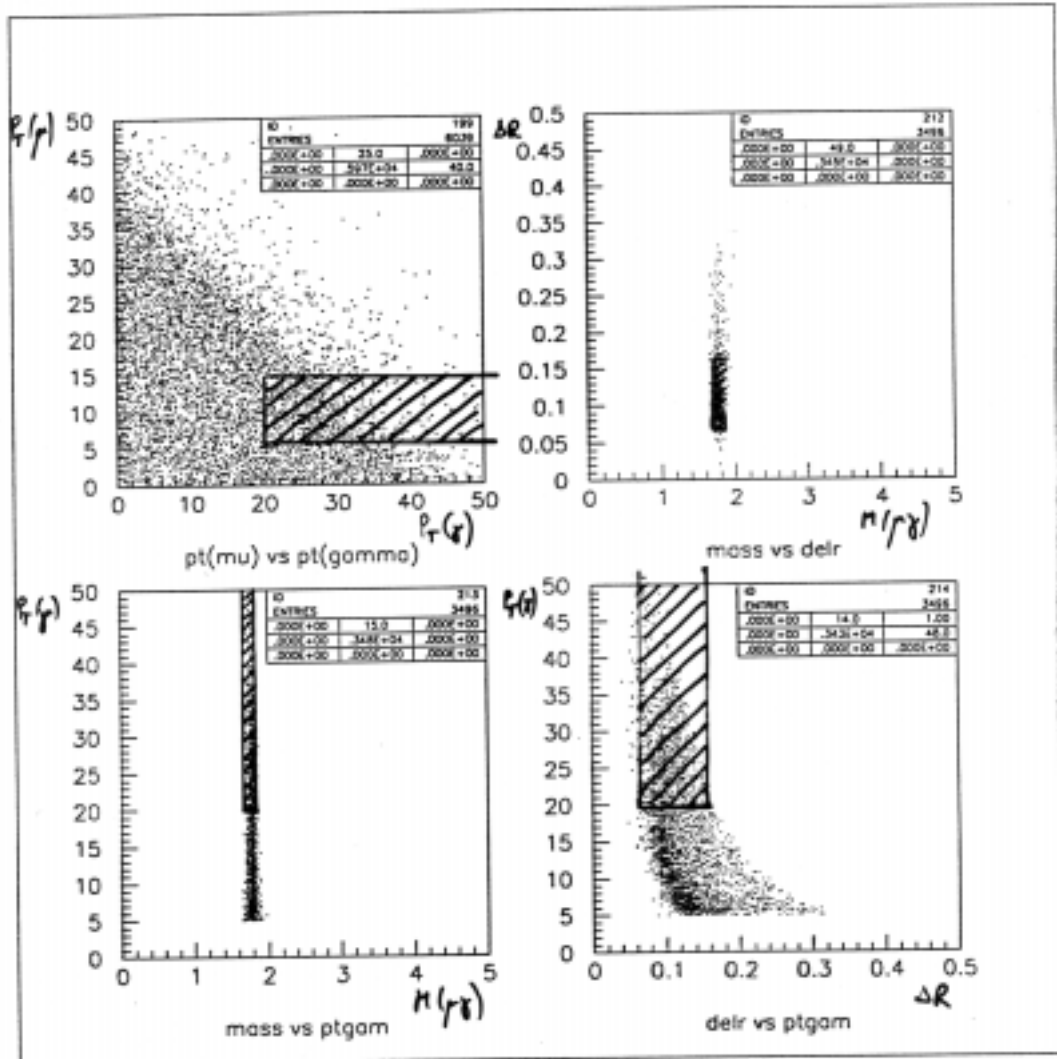


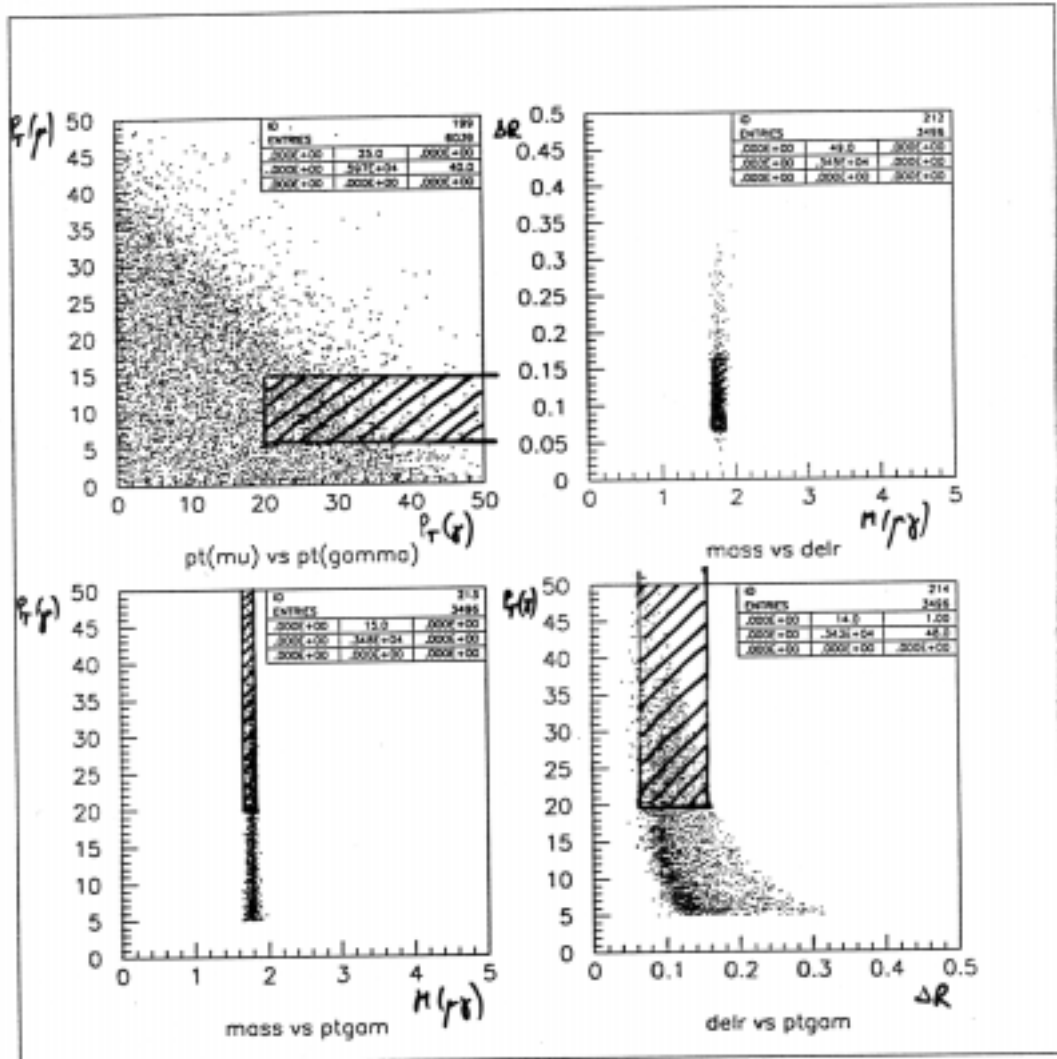
Figure 2: Transverse momentum spectrum of the muon for signal events (solid line) and the radiative backgrounds: W decay to a muon (dashed line) and W decay to a tau with subsequent tau decay to a muon (dotted line).

SIGNAL : $\tau \rightarrow \mu \gamma$

Fig.4

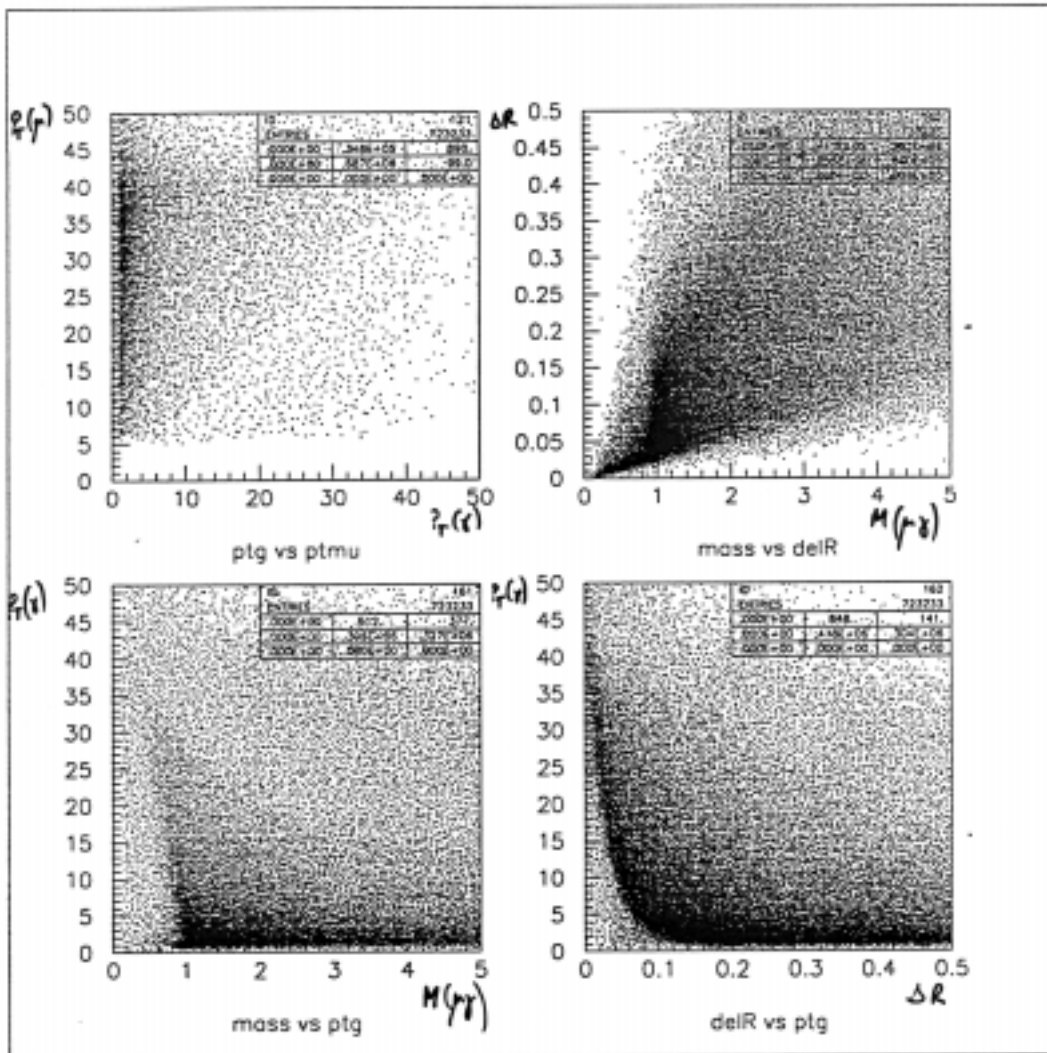
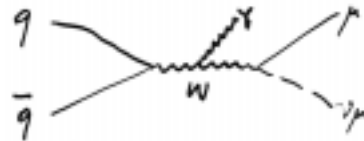
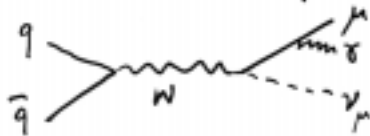






BACKGROUND
 $H \rightarrow \mu \nu \mu + \text{FSR}$

Fig. 6



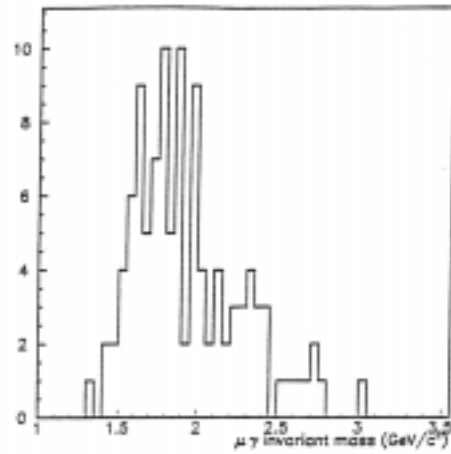
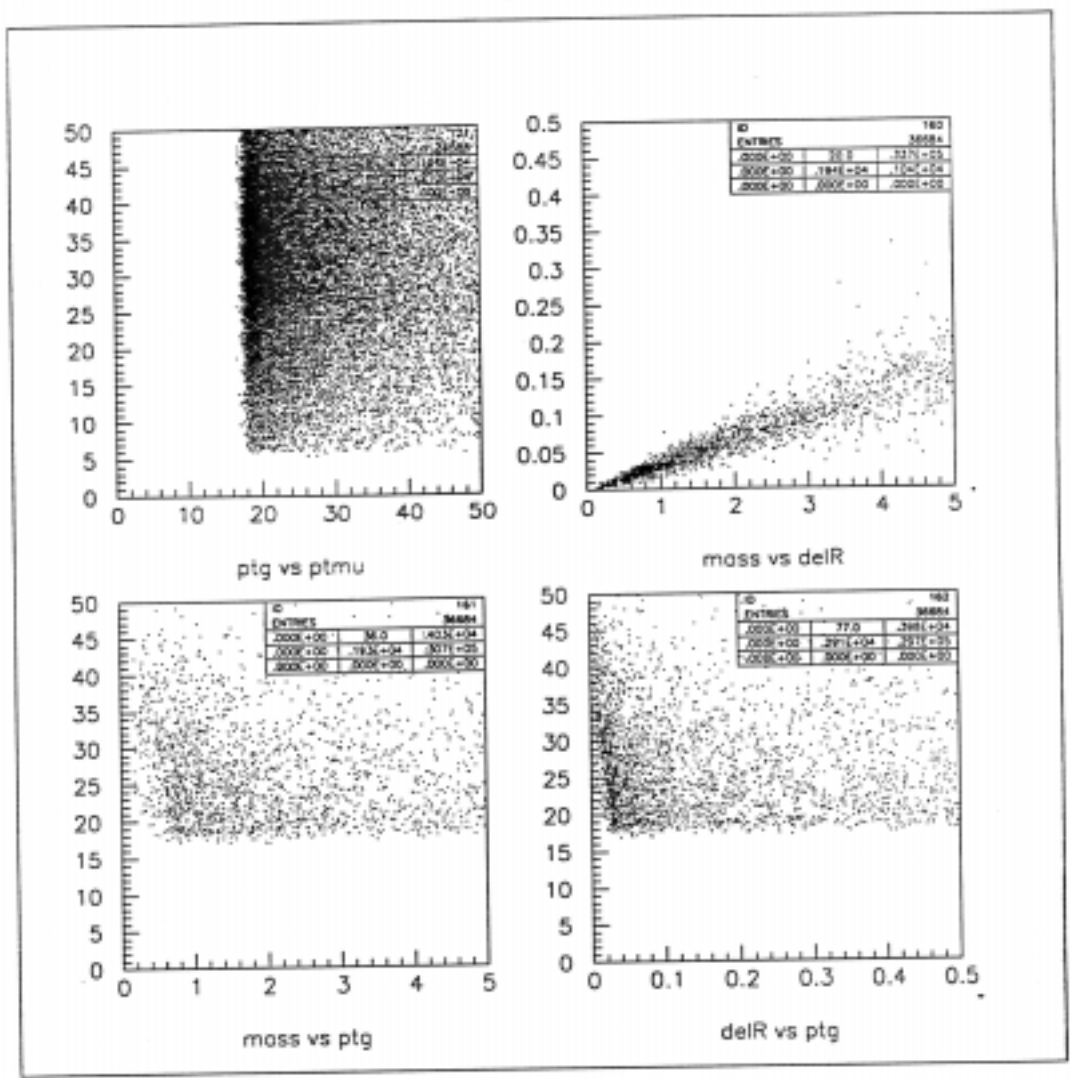
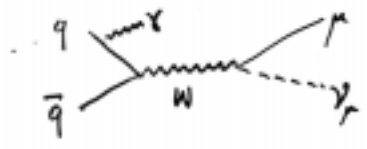


Figure 5: Invariant mass of μ and γ from radiative W decay to a τ after the selection cuts

Fig. 10

BACKGROUND - RADIATIVE W



BACKGROUND

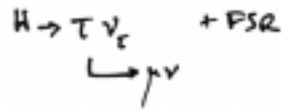
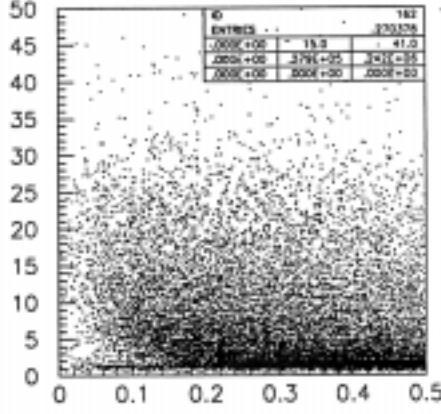
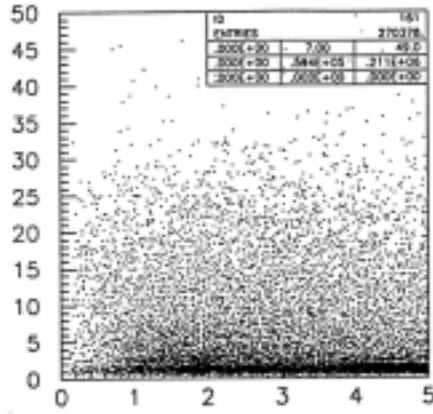
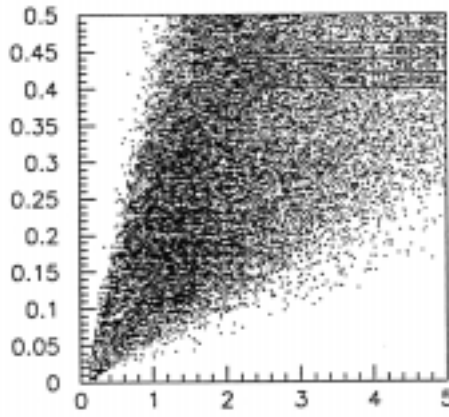
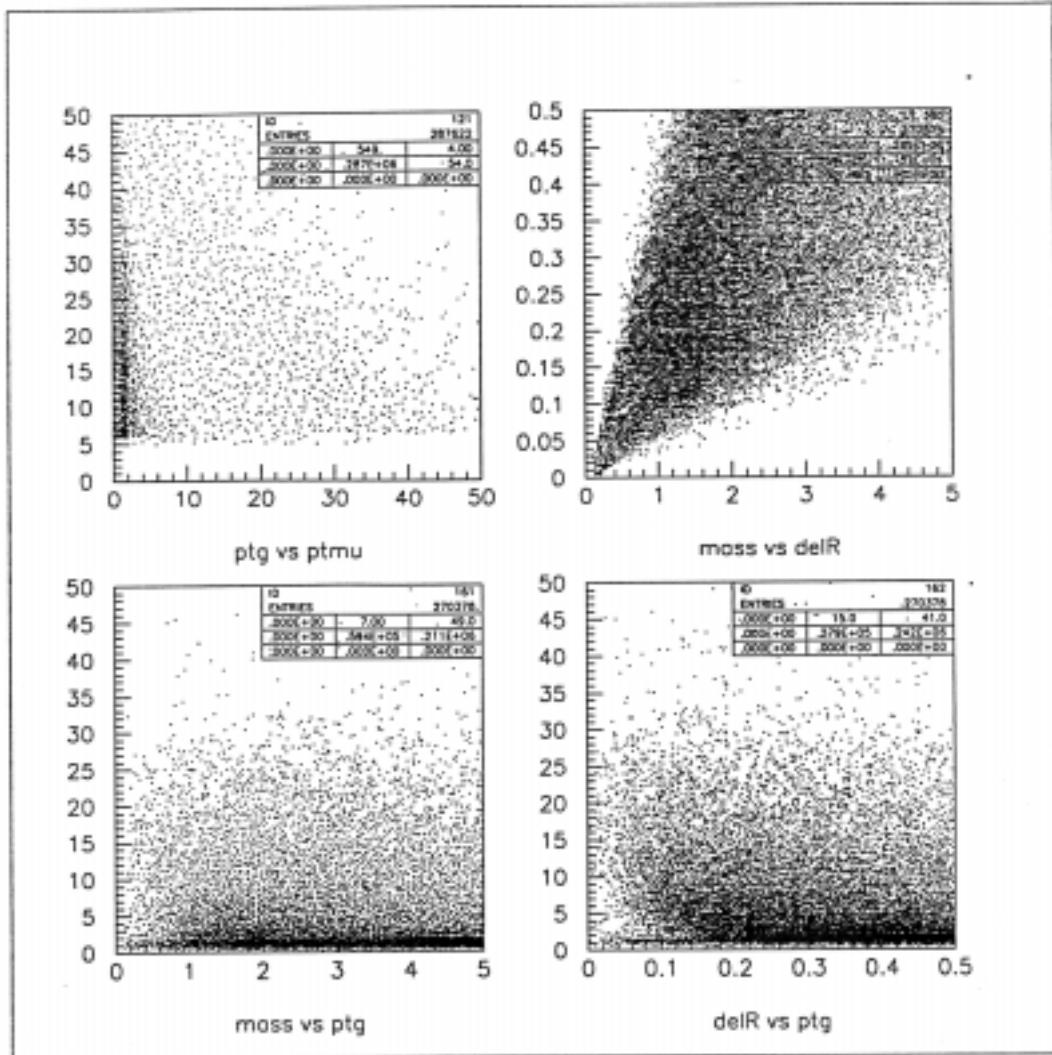


Fig. 13



Background Limits at B Factories

with Ilia Narsky

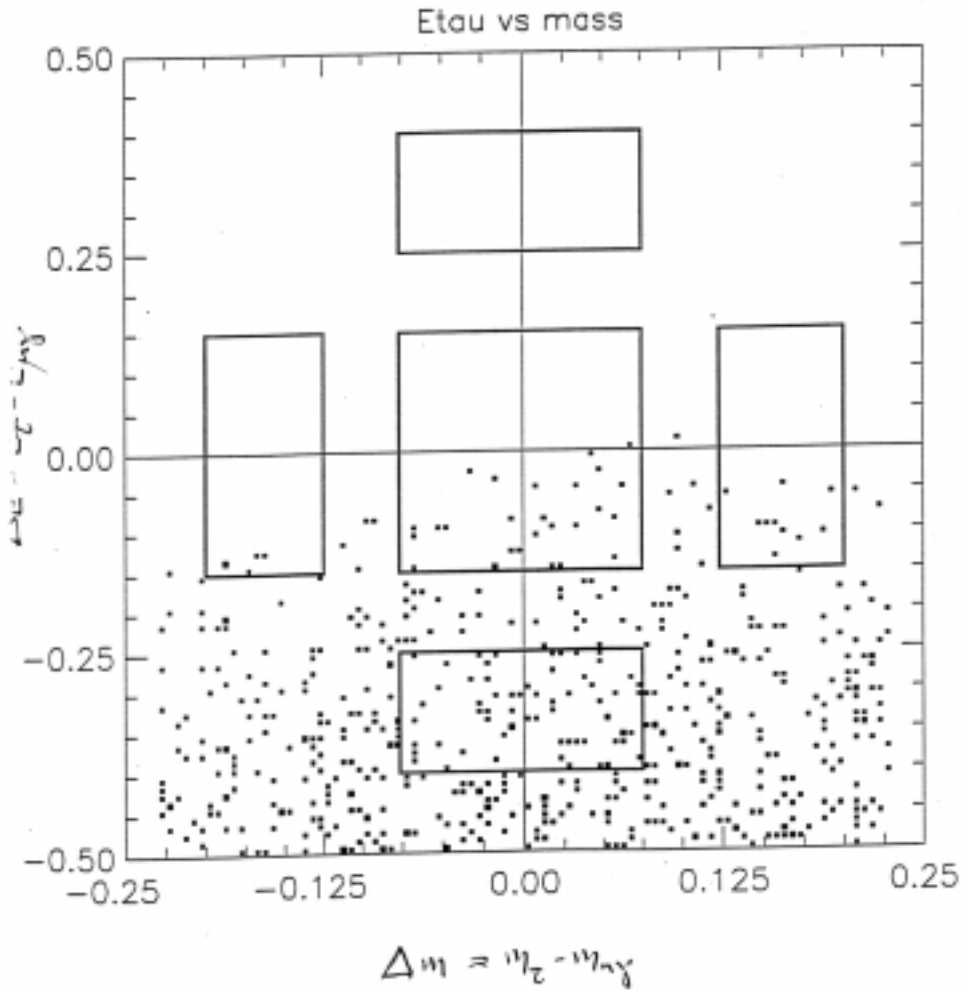
Background limitation is due to a radiative decay $\tau \rightarrow \mu\gamma\nu_\mu\nu_\tau$, where both neutrinos are almost at rest.

- Generated 2.0×10^7 events for the process $e^+e^- \rightarrow \tau + \tau^- \rightarrow (\mu\gamma)(e\nu_e\nu_\tau)$ using CLEO Monte Carlo and including detector effects at two cm energies: 10.57 GeV and 3.7 GeV.
- Performed CLEO-like search for $\tau \rightarrow \mu\gamma$.
- Large background at higher E_{cm} .
- Negligible background at low E_{cm} .

Conclusion: Expect irreducible background limitation at CLEO at the Branching Fraction level of about 10^{-7} .

$\tau \rightarrow \mu \gamma$

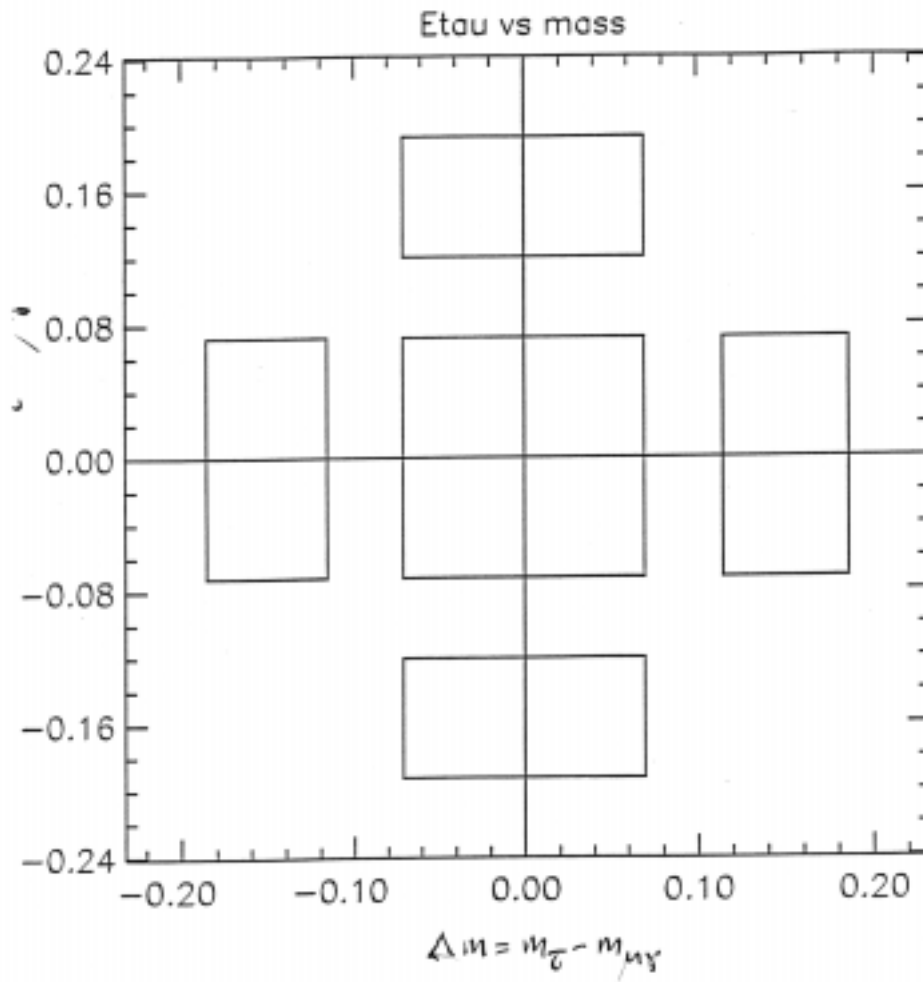
$E_{CM} = 10.58 \text{ GeV}$



$2 \cdot 10^7$ EVENTS

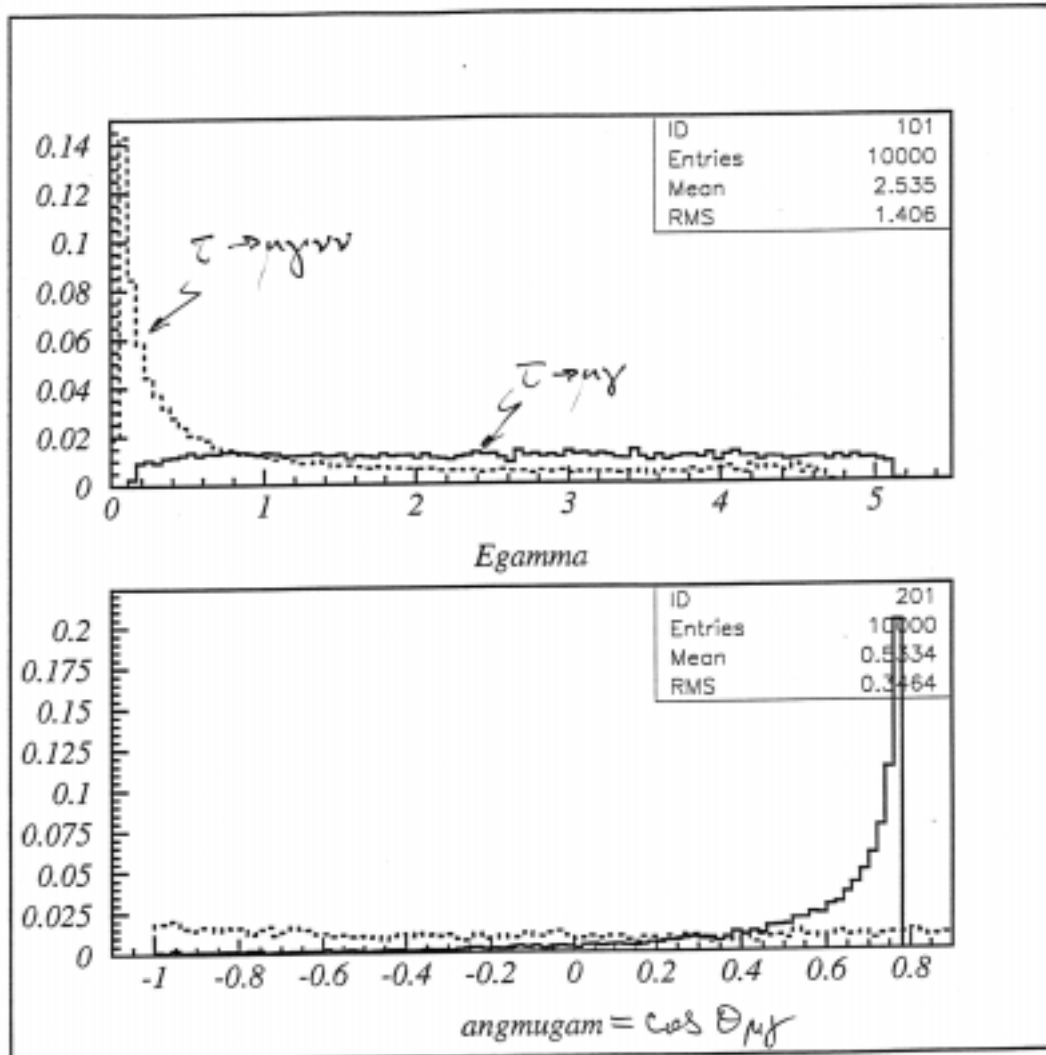
$\tau \rightarrow \mu \gamma$

$E_{CM} = 3.7 \text{ GeV}$

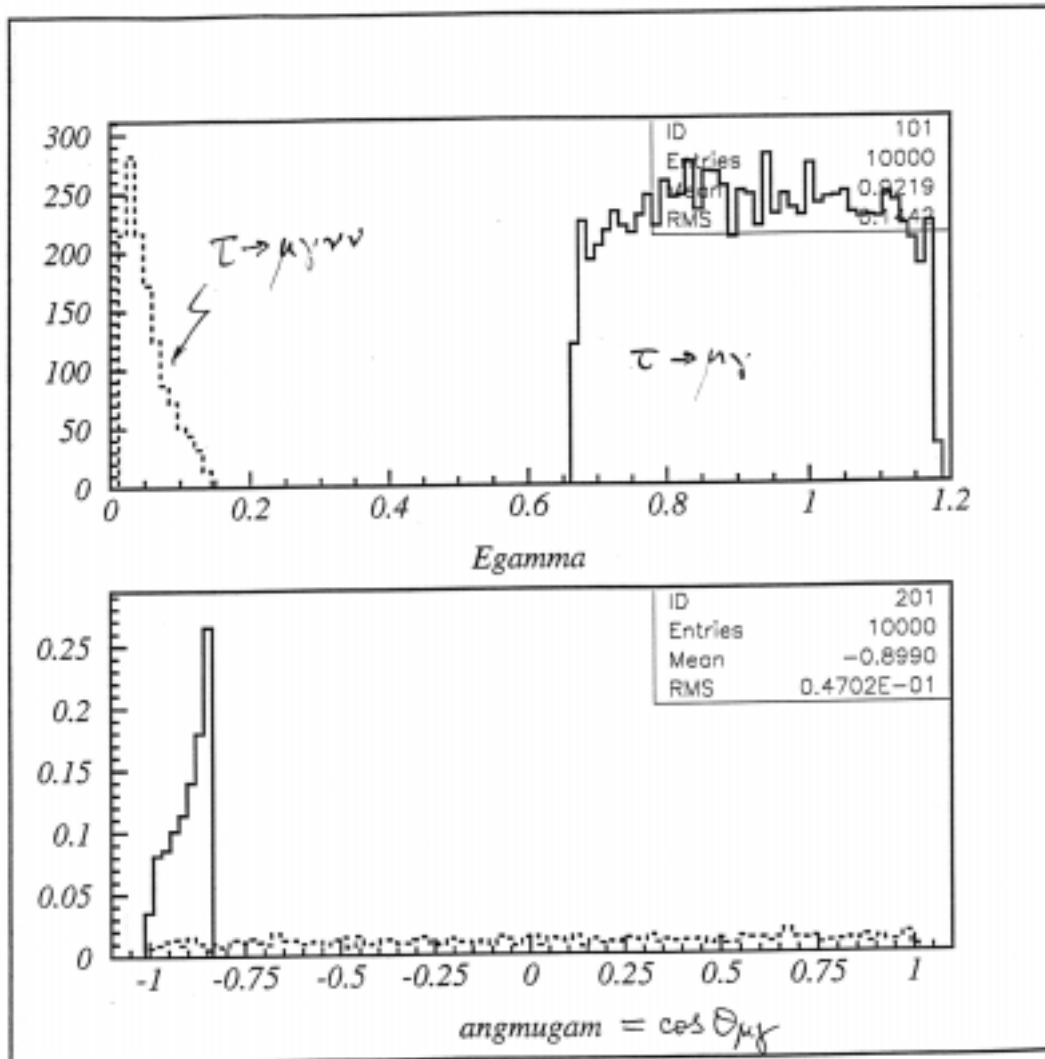


$2 \cdot 10^7$ EVENTS

$$E_{cn} = 10.58 \text{ GeV}$$



$E_{cn} = 3.7 \text{ GeV}$



Summary

Best limits in neutrinoless τ decays are in the range $1 - 3 \cdot 10^{-6}$. These limits are tantalizingly close to some of the rates expected in theoretical models. For these models, they are competitive with neutrinoless muon decay results.

We may expect new searches in the next few years:

- CLEO limits are obtained from a sample of $\sim 5 \cdot 10^6 \tau\tau$ pairs.
- CLEO 2.5 expects to have another sample of $5 \cdot 10^6 \tau\tau$ pairs available in Summer 1999.
- BaBar, BELLE and CLEO III expect $\sim 10^7$ pairs/year

Additional statistical power may come from projected improvements of signal selection efficiency.

No help is expected from LHC. W is a copious source of τ production, but the search for neutrinoless tau decays is limited by backgrounds and lack of "Energy" constraint. ATLAS study (L. Serin, R.S.) limits the discovery range at the LHC to about $1.0 \cdot 10^{-6}$.