Semi-hadronic Tau Decays

- Hadronic substructure: low-energy meson dynamics
- EW physics: h_{ν_τ}, CP violation
- $\tau^- \to \pi^- \pi^0 \nu_\tau$
- $\tau^- \to (K\pi)^- \nu_\tau$
- $\tau^- \to (3\pi)^- \nu_\tau$
- $\tau^- \to (K\pi\pi)^- \nu_\tau$
- $\tau^- \to \nu_\tau\ KK,\ KK\pi,\ K3\pi,\ KK\pi\pi$
- $\tau^- \to (4\pi)^- \nu_\tau$
- $\tau^- \to \eta X^- \nu_\tau$
- $\tau^- \to \nu_\tau\ (5\pi)^-,\ (6\pi)^-,\ (7\pi)^-$
- Inclusive hadron physics
- B factories vs τ-charm factories

Hadronic substructure

- All the tau decay branching fractions larger than 1% have been measured reasonably well;
  results are usually dominated by systematic errors
- Next step: hadronic substructure in tau decays as a clean probe of low energy meson dynamics

$\tau \to e\nu\nu$	$\approx 18\%$	Br, Michel Parameters
$\tau \to \mu\nu\nu$	$\approx 17\%$	Br, Michel Parameters
$\tau \to \pi\nu,\ K\nu$	$\approx 12\%$	Br
$\tau \to \pi\pi\nu$	$\approx 25\%$	Br, ρ Propagator
$\tau \to K\pi\nu$	$\approx 1.4\%$	Br, $K^\star$ Propagator
$\tau \to 3\pi\nu$	$\approx 18\%$	Br, a_1 Propagator, substructure
$\tau \to K\pi\pi\nu$	$\approx 0.8\%$	Br, K_1 Propagator, substructure
$\tau \to 4\pi\nu$	$\approx 5\%$	Br, ρ' Propagator, substructure
$\tau \to$ rare	$\approx 2\%$	5π, 6π, KK, $KK\pi$, $K3\pi$, $\eta\pi\pi$, $\eta 3\pi$

Hadronic substructure

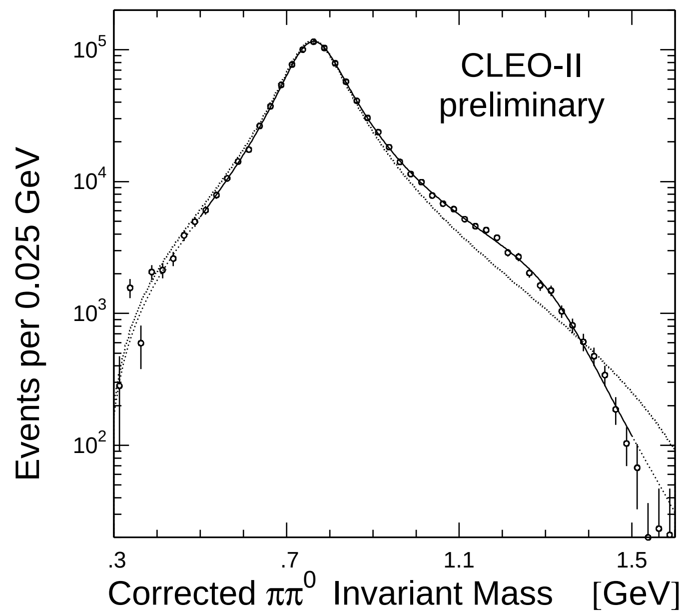
- Studying hadronic substructure is analogous, in tau physics, to measuring the leptonic Michel parameters (EW physics)

- Electroweak physics: *sexy* (to a drunken man);
  low energy meson dynamics: boring? *mysterious!*

- Hadronic dynamics as a tool for EW physics:
  spin analyzers for tau polarization; CP tests.
  Tag taus at hadron colliders via $\tau \to 3\pi\nu$.
  Precision $\Rightarrow$ good descrip of hadronic dynamics!

- All we have to understand hadronic dynamics are:
  - Chiral perturbation theory
  - QCD sum rules
  - QCD on the lattice
  - Lorentz inv, isospin, $SU(3)_f$, quark model, *etc.*
  - models inspired by S-matrix theory
  - the PDG catalog

$\tau^- \rightarrow \pi^-\pi^0\nu_\tau$



- Dynamics – Kuhn model:

$$F_\pi(q^2) \propto \frac{\left(BW_\rho(q^2) + \beta BW_{\rho'}(q^2) + \gamma BW_{\rho''} + \cdots\right)}{(1 + \beta + \gamma + \cdots)}$$

- Use Breit Wigners, normalized to $BW(q^2 = 0) = 1$, to extrapolate from chiral limit ($q^2 = 0$) to $q^2 = m_\rho^2$ and beyond, with constant coefficients β, γ;
ensure agreement with chiral limit with denomin.

- This seems terribly ad hoc and wrong to me,
but it works pretty well!

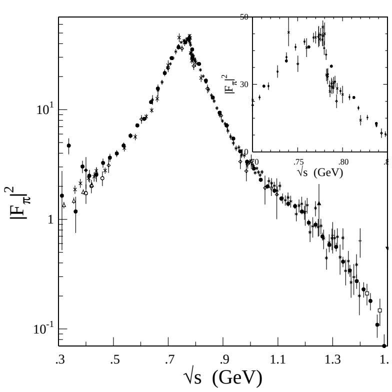
- Detailed analysis: complicated efficiency, unfold to correct for mass resolution bin migration, *etc.*

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$

- Scalar currents: $0^{+-} \rightarrow \pi^- \pi^0$, CVC violation: study $\rho \rightarrow \pi\pi$ pseudo-helicity angle distribution (or reconstruct τ rest frame, true helicity angle). Note: poor efficiency for $|\cos\theta_P| \simeq 1$.

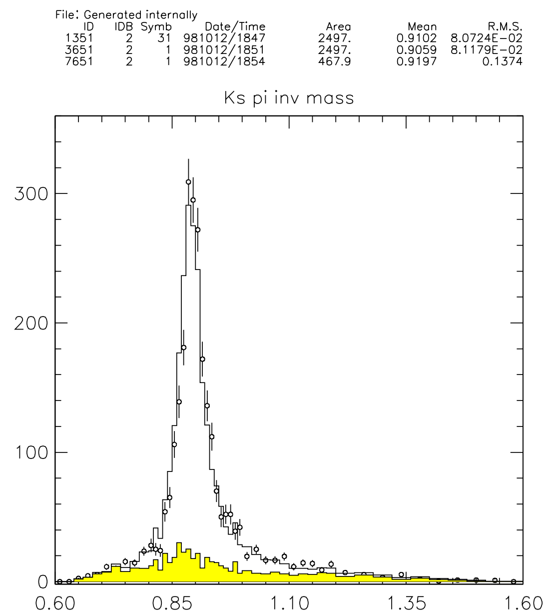
- BW and propagator form:
  - Mass dependent width , (q^2)
  - mass dependent mass $m(q^2)$, Kramers-Kronig
  - Blatt-Weisskopf barrier penetration factor, *etc.*
  - induced scalar currents:
    $(-g^{\mu\nu} + q^\mu q^\nu / q^2) \neq (-g^{\mu\nu} + q^\mu q^\nu / m_r^2)$

- Tests of CVC: total BR, differential $v_1(q^2)$



- Important ingredient in hadronic vacuum polarization contribution to $(g-2)_\mu$, $\alpha_{QED}(q^2)$

$\tau^- \to (K\pi)^- \nu_\tau$



- Dynamics: $K^*(892) + \beta K^{*\prime}(1410) + \gamma K^{*\prime\prime} + \cdots$, Kuhn-Finkemeier-Mirkes model
- Measure $V_{us} f_{K^*}$; test DMO strange sum rule
- Same issues wrt BW and propagator form
- Scalar currents: $K_0^*(1430) \to (K\pi)$ S-wave. Since $SU(3)_f$ is violated, contributions possible.
- Interference between vector and scalar, with relatively complex couplings, can give CP-violation: CLEO/Jessop.

$\tau^- \to (K\pi)^- \nu_\tau$

- CLEO mainly uses $K^0_S\pi^-$; ALEPH uses $K^-\pi^0$ and $K^0_L\pi^-$ as well.
  Background from fake K^0, $K^\pm \leftrightarrow \pi^\pm$.

- ALEPH sees some $K^{*\prime}(1410)$;
  CLEO does not, with much higher statistics (background!).

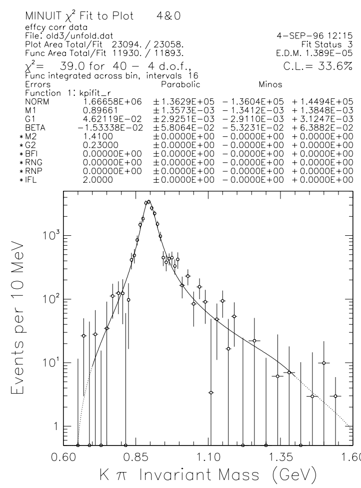
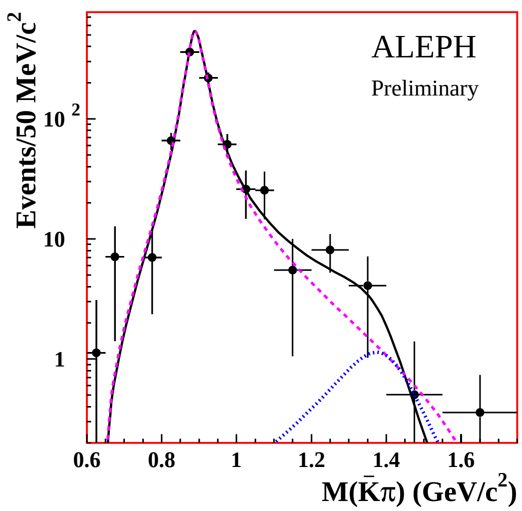


Figure 13: Fully corrected $M_{K_S\pi}$ spectrum (points with error bars), with the Kühn and Santamaria fit function overlaid (solid curve).



- CLEO sees $m(K^*)$ approx 5 MeV higher than PDG value! And we can't make it go away!

- Still insufficient statistics for $m(K\pi) > 1.1$ GeV!

$\tau \to 3\pi\nu$ – MOTIVATION

- Low energy hadron dynamics
  couplings of Scalars (S), Pseudoscalars (P), Vectors (V), and Axialvectors (A):
  - Due to G-Parity conservation in $\tau \to 3\pi\nu$:
    Study of the axial vector meson sector, a_1 and possible radial excitations
  - Due to the possible participation of scalar mesons in the subsequent decay of the axial vector meson:
    Study of the poorly understood scalar mesons
  - search for PCAC-violating $\tau \to \nu_\tau \pi'$
  - Lineshapes, form factors, thresholds, meson radii
- PV signed Tau neutrino helicity h_{ν_τ}
  (Kühn and Wagner 1984)
- Tau neutrino mass measurements
- Identifying τ Leptons at hadron machines

Complications with $\tau \to 3\pi\nu_\tau$, I

- Dominated by $a_1 \to \rho\pi$ S-wave

- Phase space integral over $\rho\pi$ S-wave is non-trivial.
  $\sqrt{q^2}$, $_{3\pi}(q^2)$ parameterized by Bowler in 1988.

- There's lots more than just $\rho\pi$ S-wave!
  $\rho'\pi$ S-wave; $\rho\pi$ D-wave; $\rho'\pi$ D-wave; $f_2(1275)\pi$ P-wave; $f_0(1285)\pi$ P-wave; and $\sigma(890)\pi$ P-wave.
  $\sqrt{q^2}$, $_{3\pi}(q^2)$ must be obtained
  from detailed study of Dalitz plot.

- The a_1 also decays to $K^*K \to KK\pi$,
  contributes to total , $_{a_1}(q^2)$ in BW.

- Isospin relates $a_1 \to \pi^-\pi^0\pi^0$ to $\pi^-\pi^+\pi^-$.
  Non-trivial relation, because of isoscalars.

- Bose symmetrization of identical pions.

- radially-excited a_1' meson?

COMPLICATIONS WITH $\tau \to 3\pi\nu_\tau$, II

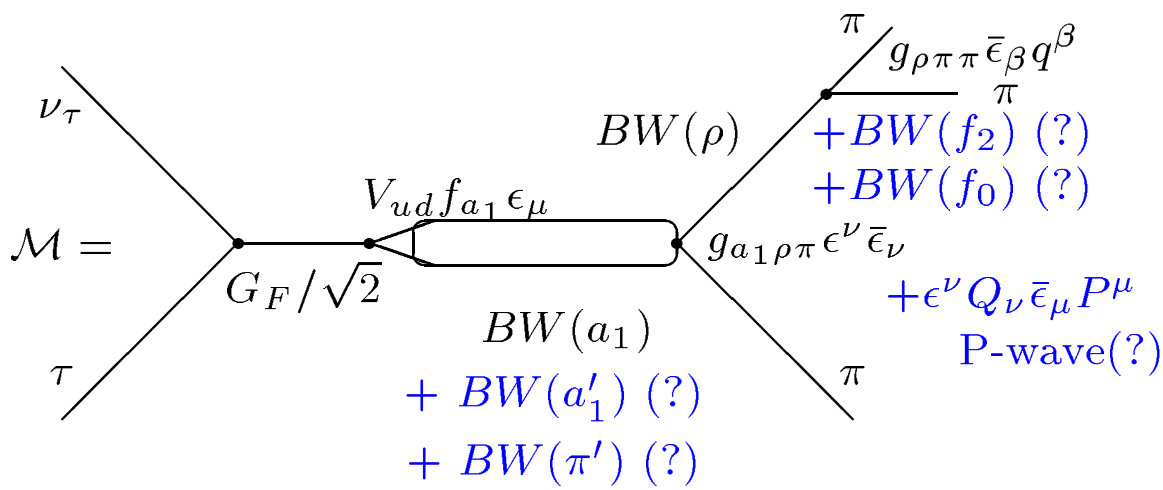
- There are two axial-vector ($J^P = 1^+$) states:
  $a_1(1260)$ in the $^3P_1$ octet, $J^{PG} = 1^{+-}$,
  couples to W as a "first-class" current;
  $b_1(1235)$ in the $^1P_1$ octet, $J^{PG} = 1^{++}$.
  doesn't couple to W ("second-class" current)
  except via isospin violation ($f_{b_1} \approx 0$).

- More on this, in $\tau \to 4\pi\nu_\tau$

- Might also be a scalar current, $\pi'^- \to (3\pi)^-$;
  forbidden by CVC.

- Vector current to $(3\pi)^-$ forbidden by Bose symmetry

$\tau \to 3\pi\nu$ (THEORY)



$q = p_{\pi_1} - p_{\pi_{2/3}} \quad Q = p_{\pi_1} + p_{\pi_{2/3}} - p_{\pi_{3/2}} \quad P = p_{\pi_1} + p_{\pi_2} + p_{\pi_3}$

$$|\mathcal{M}|^2 = \text{Lepton Tensor} \times \text{Hadron Tensor} = L_{\mu\nu} \times J^\mu J^{\star\nu} = (S_{\mu\nu} + i h_{\nu_\tau} A_{\mu\nu}) \times J^\mu J^{\star\nu}$$

- momentum transfer small in τ decays $\Longrightarrow$
  Resonance dominance $\Longrightarrow$ Models
- Conservation of G-Parity and Parity $\Longrightarrow$
  Meson X in $\tau \to X\nu \to 3\pi\nu$ has J^P: 0^- or 1^+
  ($P_\mu J^\mu_{0^-} \neq 0 \Longrightarrow 0^-$ suppressed)

$\tau \to 3\pi\nu$ (Theory) *cont.*

Lorentz structure of J_μ is well-defined:

$$\Longrightarrow J_\mu \;=\; \left(-g_{\mu\nu} + \frac{P_\mu P_\nu}{P^2}\right)\left[(p_{\pi_1} - p_{\pi_2})^\nu F_1 + (p_{\pi_1} - p_{\pi_3})^\nu F_2 + (p_{\pi_2} - p_{\pi_3})^\nu F_3\right] + P_\mu F_4$$

Form Factors F_i:

$$F_i \;=\; \text{Breit Wigner functions} \times \text{Angular momentum factors (S,P,D...-wave)} \times (?)$$

For example Kühn Santamaria (KS) Model:

$$F_i = BW(a_1) \cdot BW(\rho + \beta \cdot \rho') \times 1(\text{S-wave}) \times 1$$

Other Models:

- Isgur, Morningstar and Reader (IMR) Model
- Feindt (F) Model
- ...

$\tau \rightarrow 3\pi\nu$ (Theory) *cont.*

$$
\begin{aligned}
d,\,(\tau \rightarrow \nu_\tau 3\pi) &= \frac{G_F^2 V_{ud}^2}{2m_\tau}\left[L^{\mu\nu} J_\mu J_\nu^*\right] dLips \\
&= \frac{G_F^2 V_{ud}^2}{32\pi^2 m_\tau}\left(1+2\frac{s}{m_\tau^2}\right)\left(1-\frac{s}{m_\tau^2}\right) \times \\
& \quad |BW(s)|^2 \times \frac{,_{3\pi}(s)}{s} ds
\end{aligned}
$$

- determine $,_{3\pi}(s) = \int J_\mu J^{\star\mu} ds_1 ds_2$ by measuring Dalitz plot distribution s_1 and s_2 (plus angular momentum observables of production)
- determine $BW(s)$ by measuring invariant mass distribution of three pions

and/or

- determine Structure funct. W_X (model independent) by expanding $|\mathcal{M}|^2$ in a sum of 16 independent terms $|\mathcal{M}|^2 = L_{\mu\nu} \times J^\mu J^{\star\nu} = \sum_{X=1}^{16} L_X W_X$

$\tau \to 3\pi\nu$ (Theory) *cont.*

Tau Neutrino Helicity h_{ν_τ}

$$|\mathcal{M}|^2 = L_{\mu\nu} \times J^\mu J^{\star\nu} = (S_{\mu\nu} + i h_{\nu_\tau} A_{\mu\nu}) \times J^\mu J^{\star\nu}$$

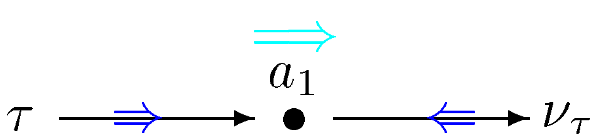
$\Longrightarrow$ Asymmetric part of Hadron tensor $J^\mu J^{\star\nu}$ needed

- At least three pseudoscalars in final state needed
- Interference term needed

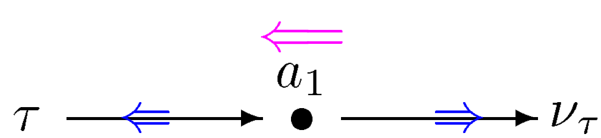
Two indentical Pions! ρ can be formed in two ways:

$$\begin{aligned} \tau^- &\rightarrow a_1^- \nu_\tau \\ &\quad \hookrightarrow \rho_1^0 \pi_2^- \\ &\qquad \hookrightarrow \pi_1^- \pi^+ \end{aligned} \qquad\qquad \begin{aligned} \tau^- &\rightarrow a_1^- \nu_\tau \\ &\quad \hookrightarrow \rho_2^0 \pi_1^- \\ &\qquad \hookrightarrow \pi_2^- \pi^+ \end{aligned}$$

$\Longrightarrow \Im(BW(\rho_1) \cdot BW(\rho_2)^\star)$ resolves the left- and right-handed part of the transverse polarization of the a_1:



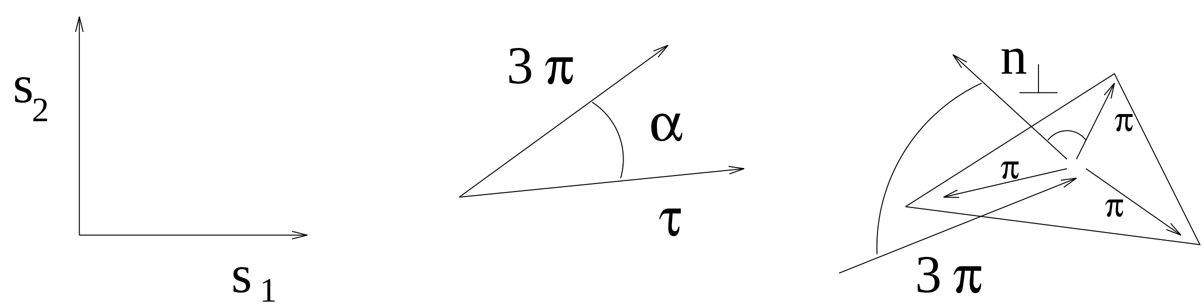
left handed ν_τ



right handed ν_τ

CLEO $\tau \to \pi^- \pi^0 \pi^0 \nu_\tau$

- 30800 $\tau^\mp \to \pi^\mp \pi^0 \pi^0 \nu$ events (all tag)
  14600 $\tau^\mp \to \pi^\mp \pi^0 \pi^0 \nu$ lepton tag events
- Substructure: determine hadronic current J^μ in context of a model, via Likelihood fit to Dalitz plot in full kinematic space, in bins of $m_{3\pi}$.
- Variables s, $s_1 = m^2(\pi^- \pi^0_1)$, $s_1 = m^2(\pi^- \pi^0_2)$; and angular observables ψ, β from production:



- overall resonance shape:
  determine $,_{3\pi}(s) = \int J_\mu J^{\star\mu} ds_1 ds_2$
  determine $BW(s)$
  χ^2 fit to three pion mass spectrum

CLEO $\tau \rightarrow \pi^- \pi^0 \pi^0 \nu_\tau$

Amplitudes in fit to 3π substructure:

- J_1^μ: s-wave $1^+ \rightarrow \rho\pi$
- J_2^μ: s-wave $1^+ \rightarrow \rho'\pi$
- J_3^μ: d-wave $1^+ \rightarrow \rho\pi$
- J_4^μ: d-wave $1^+ \rightarrow \rho'\pi$
- J_5^μ: p-wave $1^+ \rightarrow f_2(1275)\pi$
- J_6^μ: p-wave $1^+ \rightarrow f_0(400-1200)\pi$, denoted as $\sigma\pi$
- J_7^μ: p-wave amplitude of $1^+ \rightarrow f_0(1370)\pi$

mass and width for $f_0(1370)$ and $f_0(400-1200)$ (σ) according to Törnqvist's UQM

$$m_{f_0(1370)} = 1.186 \text{ GeV/c}^2; \quad ,_{f_0(1370)} = 0.350 \text{ GeV};$$
$$m_\sigma = 0.860 \text{ GeV/c}^2; \quad ,_\sigma = 0.880 \text{ GeV}$$

$$A^\mu = \sum_{i=1}^{i=7} \beta_i \times J_i^\mu \times F_i$$

$F_i = e^{-0.5 R^2 p_i^{\star 2}}$; nominal fit with $R = 0 \Longrightarrow F_i = 1$

CLEO $\tau \to \pi^- \pi^0 \pi^0 \nu_\tau$

$s_3 \;=\; m^2_{\pi^0_1+\pi^0_2}$ distr.

solid line:

fit result

a) $m_{3\pi}$: 0.6 – 0.9

b) $m_{3\pi}$: 0.9 – 1.0

c) $m_{3\pi}$: 1.0 – 1.1

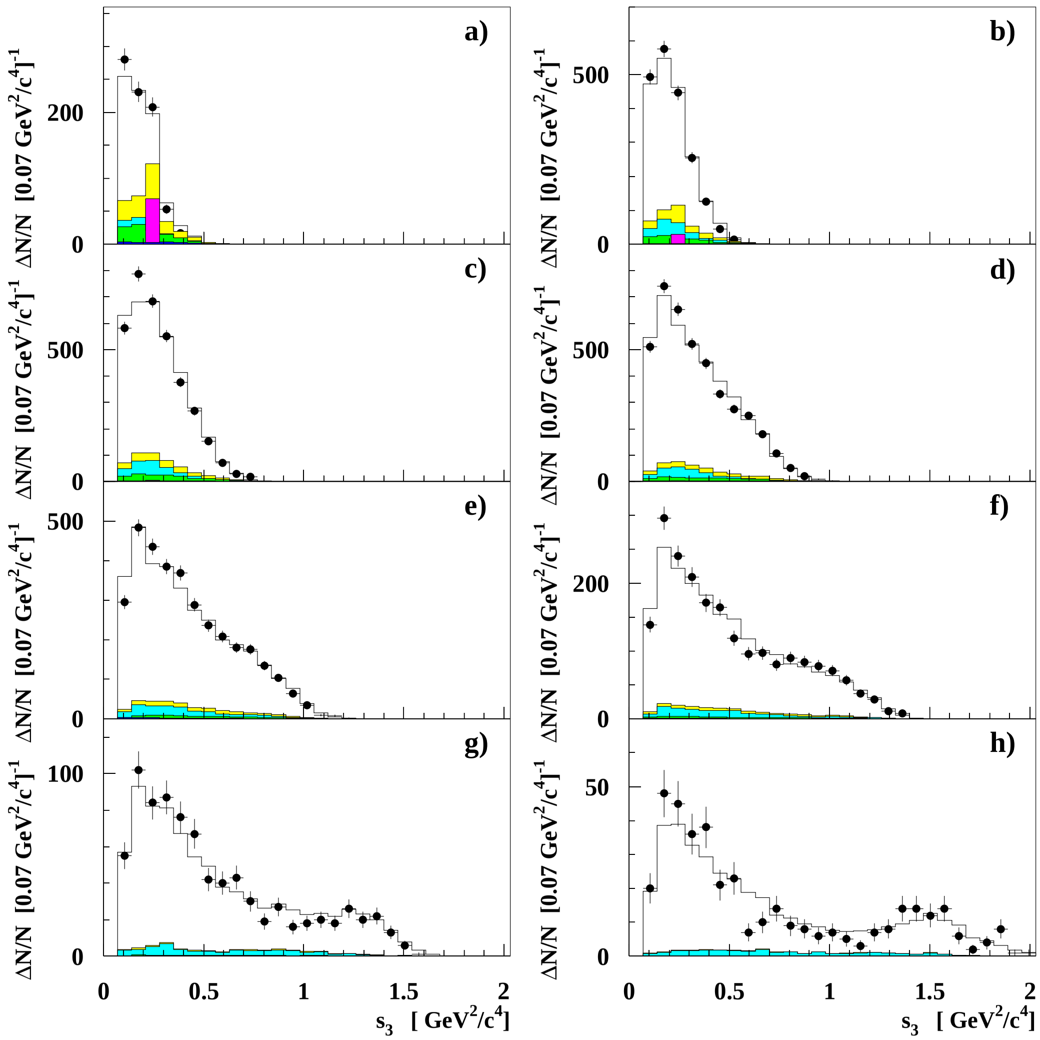
d) $m_{3\pi}$: 1.1 – 1.2

e) $m_{3\pi}$: 1.2 – 1.3

f) $m_{3\pi}$: 1.3 – 1.4

g) $m_{3\pi}$: 1.4 – 1.5

h) $m_{3\pi}$: 1.5 – 1.8

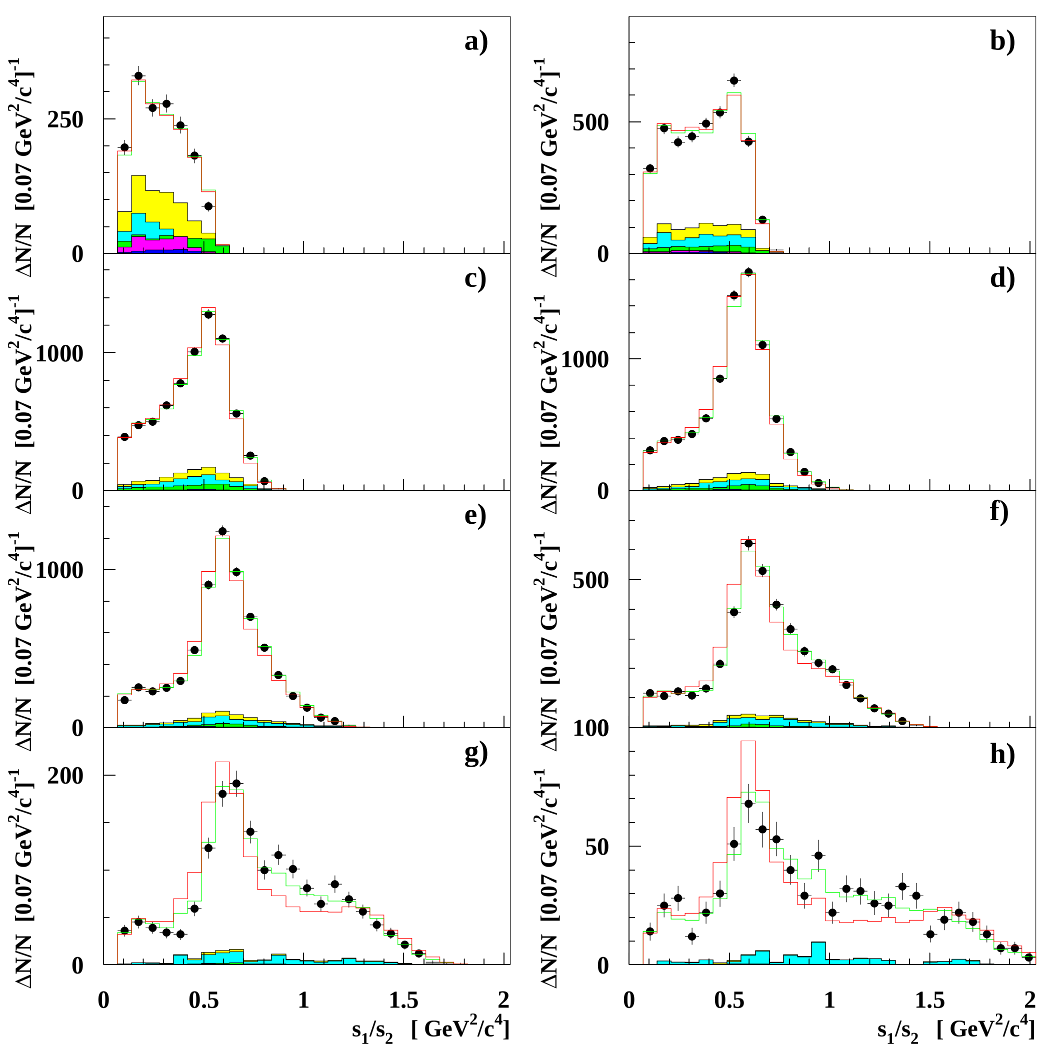


$\Longrightarrow$good fits ($< 3\sigma$) in all $m_{3\pi}$ bins!

CLEO $\tau \to \pi^-\pi^0\pi^0\nu_\tau$

$s_1 = m^2_{\pi^- + \pi^0_1}$
and
$s_2 = m^2_{\pi^- + \pi^0_2}$
distr.
solid line:
fit result

a) $m_{3\pi}$: $0.6 - 0.9$
b) $m_{3\pi}$: $0.9 - 1.0$
c) $m_{3\pi}$: $1.0 - 1.1$
d) $m_{3\pi}$: $1.1 - 1.2$
e) $m_{3\pi}$: $1.2 - 1.3$
f) $m_{3\pi}$: $1.3 - 1.4$
g) $m_{3\pi}$: $1.4 - 1.5$
h) $m_{3\pi}$: $1.5 - 1.8$



$\Longrightarrow$good fits ($< 3\sigma$) in all $m_{3\pi}$ bins!

CLEO $\tau \to \pi^-\pi^0\pi^0\nu_\tau$

		Significance	$\mathcal{B}$ fraction(%)
ρ	s-wave		69.4
$\rho(1370)$	s-wave	1.4σ	$0.30 \pm 0.64 \pm 0.17$
ρ	d-wave	5.0σ	$0.36 \pm 0.17 \pm 0.06$
$\rho(1370)$	d-wave	3.1σ	$0.43 \pm 0.28 \pm 0.06$
$f_2(1275)$	p-wave	4.2σ	$0.14 \pm 0.06 \pm 0.02$
σ	p-wave	8.2σ	$16.18 \pm 3.85 \pm 1.28$
$f_0(1186)$	p-wave	5.4σ	$4.29 \pm 2.29 \pm 0.73$

- $\rho\pi$ s-wave with $\mathcal{B} \approx 70\%$ dominant as expected
- with the exception of $\rho'\pi$ s-wave all amplitudes significant
- isoscalars contribute with $\mathcal{B} \approx 20\%$ to 3π hadronic current; especially σ cannot be neglected
- couplings constant over $m_{3\pi}$; (decoupling ρ' s- and d-wave)
- ρ' shows up more strongly in d-wave than s-wave

$$\mathcal{B}(\tau \to \pi'\nu \to \rho\pi\nu \to 3\pi\nu) < 1.0 \times 10^{-4} \text{ at } 90\% \text{ CL}$$
$$\mathcal{B}(\tau \to \pi'\nu \to \sigma\pi\nu \to 3\pi\nu) < 1.9 \times 10^{-4} \text{ at } 90\% \text{ CL}$$
$$h_{\nu_\tau} = -1.02 \pm 0.13 \pm 0.01 \pm 0.03 \text{ (SM } h_{\nu_\tau} = \text{-1)}$$

CLEO $\tau \to \pi^-\pi^+\pi^-\nu_\tau$

$\approx 80000\ \tau^\mp \to \pi^\mp\pi^\mp\pi^\pm\nu$ events

due to isoscalars neutral differs from charged pion mode:

$$|0,0\rangle = \frac{1}{\sqrt{3}}|1,+1\rangle|1,-1\rangle - \frac{1}{\sqrt{3}}|1,0\rangle|1,0\rangle + \frac{1}{\sqrt{3}}|1,-1\rangle|1,+1\rangle$$

s_1/s_2 distr.

solid line:

isopsin predict.

as measured in

neutral mode

a) $m_{3\pi}$: $0.6 - 0.9$

b) $m_{3\pi}$: $0.9 - 1.0$

c) $m_{3\pi}$: $1.0 - 1.1$

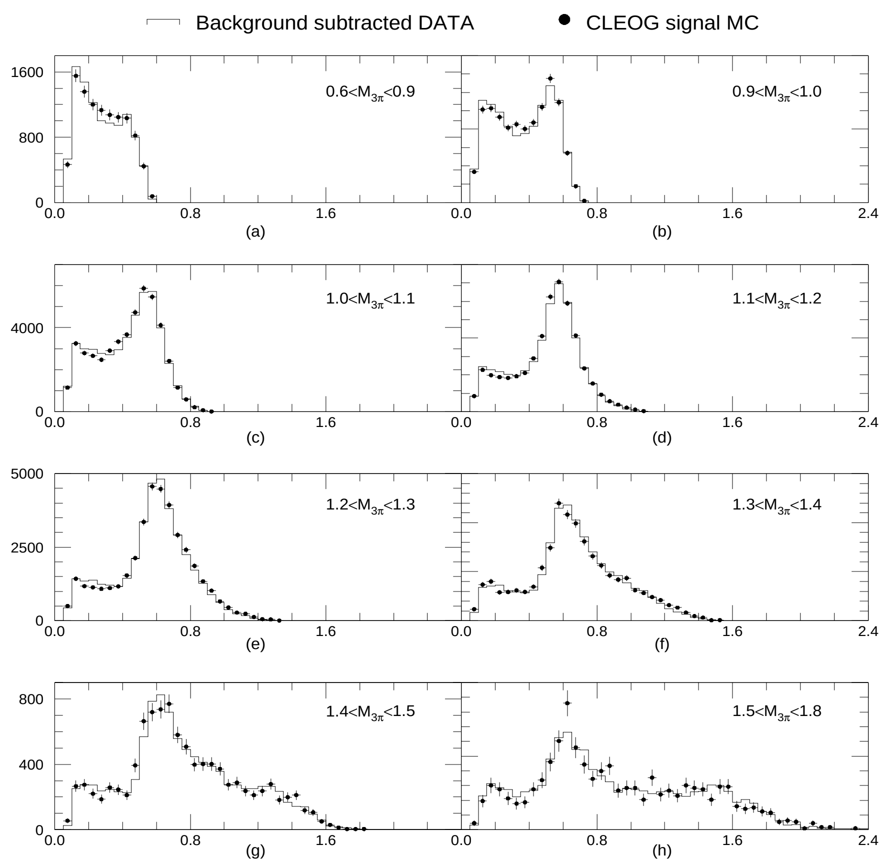
d) $m_{3\pi}$: $1.1 - 1.2$

e) $m_{3\pi}$: $1.2 - 1.3$

f) $m_{3\pi}$: $1.3 - 1.4$

g) $m_{3\pi}$: $1.4 - 1.5$

h) $m_{3\pi}$: $1.5 - 1.8$



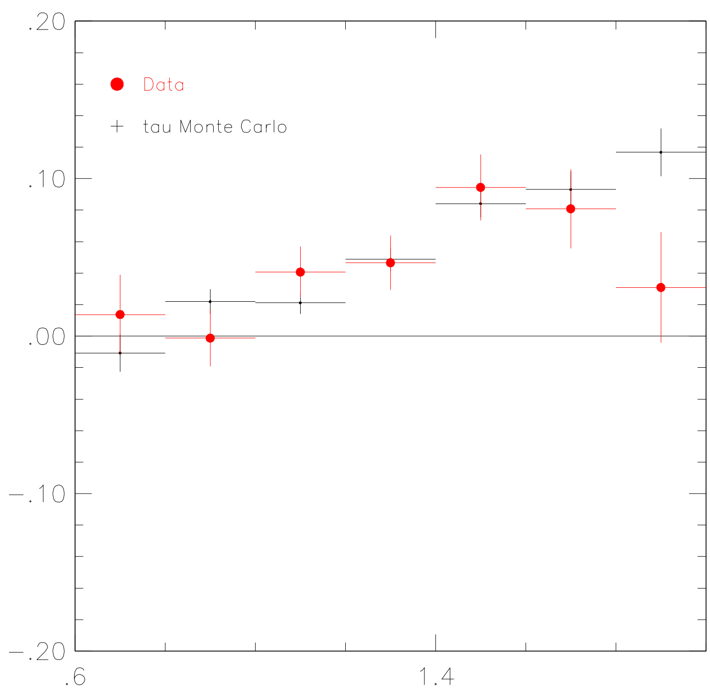
$\Longrightarrow$ charged mode in good agreement with neutral mode

CLEO $\tau \rightarrow \pi^-\pi^+\pi^-\nu_\tau$

The Asymmetry function $\frac{a(x,m^2_{3\pi})}{\cos\psi} = h_{\nu_\tau} A(m^2_{3\pi})$ plotted versus the mass of the 3π system, where

$$h_{\nu_\tau} = -\frac{2 g_V g_A}{(g_V^2 + g_A^2)}$$

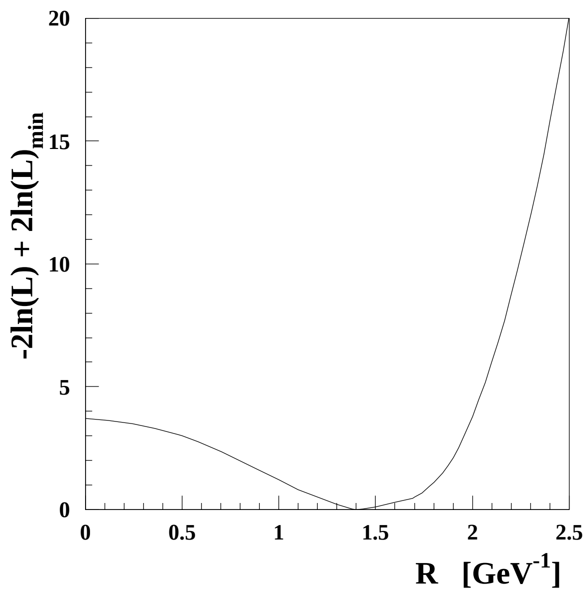
$$a(x, m^2_{3\pi}) = \left(\hat{\mathbf{p}}^{\mathrm{lab}}_{3\pi} \cdot \left[\hat{\mathbf{p}}^{\mathrm{a1}}_{\pi_1^-} \times \hat{\mathbf{p}}^{\mathrm{a1}}_{\pi^+}\right]\right) sign(s_1 - s_2).$$



CLEO $\tau \to \pi^- \pi^0 \pi^0 \nu_\tau$

fits to substructure with varying meson radius R in form factor F

satisfactory goodness of fit for $0 \leq R \leq 2$ GeV^{-1}



best fit with $R = 1.4$ GeV^{-1}

$\Longrightarrow$ meson size of ≈ 0.7 fm

CLEO $\tau \to 3\pi\nu_\tau$

Three pion mass spectrum:

$$B(s) = B_{a_1}(s) + \epsilon \cdot B_{a_1'}(s) \quad = \quad \frac{1}{s - m^2_{a_1}(s) + i m_{0\,a_1} ,\, ^{a_1}_{tot}(s)} + \frac{\epsilon}{s - m^2_{0\,a_1'} + i m_{0\,a_1'} ,\, ^{a_1'}_{tot}(s)}$$

Running mass $m^2(s)$:

$$m^2(s) = m_0^2 + \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{m_0 ,\, _{tot}(s/)}{(s - s/)} ds/$$

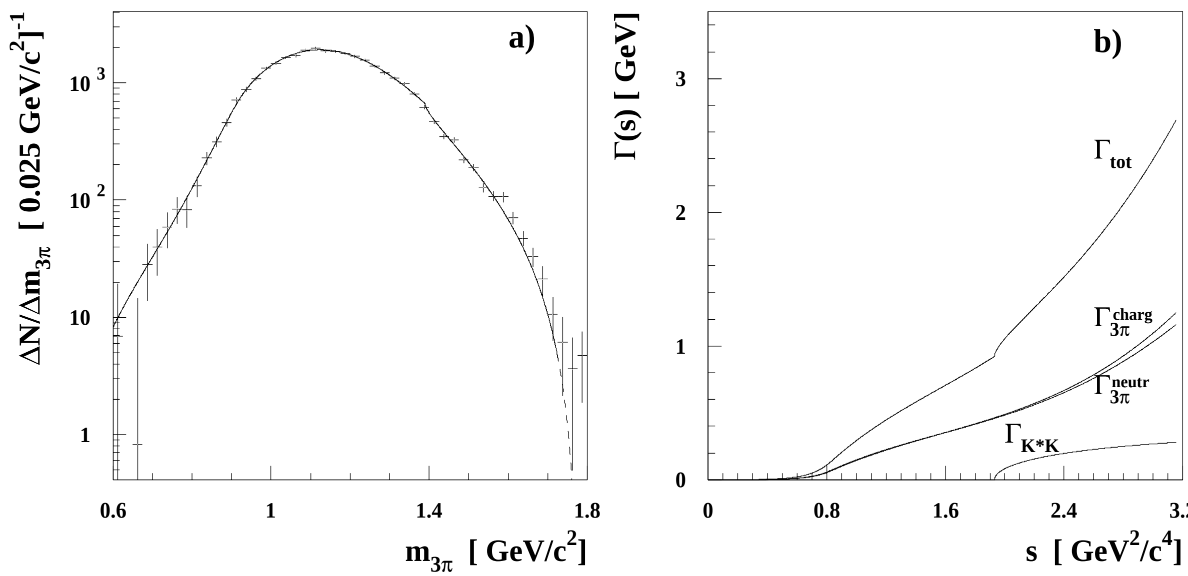
Total width $,\, _{tot}(s)$:

$$,\, _{tot}(s) = ,\, _{2\pi^0\pi^-}(s) + ,\, _{2\pi^-\pi^+}(s) + ,\, _{K^\star K}(s) + ,\, _{f_0(980)\pi}(s)$$

CLEO $\tau \rightarrow 3\pi\nu_\tau$

constant/running mass, $K^\star K$, $f_0(980)\pi$, meson radius R

- good fits: constant/running mass, with and without $f_0(980)\pi$, $0 \leq R \leq 2$ GeV^{-1}
- $K^\star K$ threshold needed for good fit
- best value for R: $1.2 \leq R \leq 1.4$ GeV^{-1}



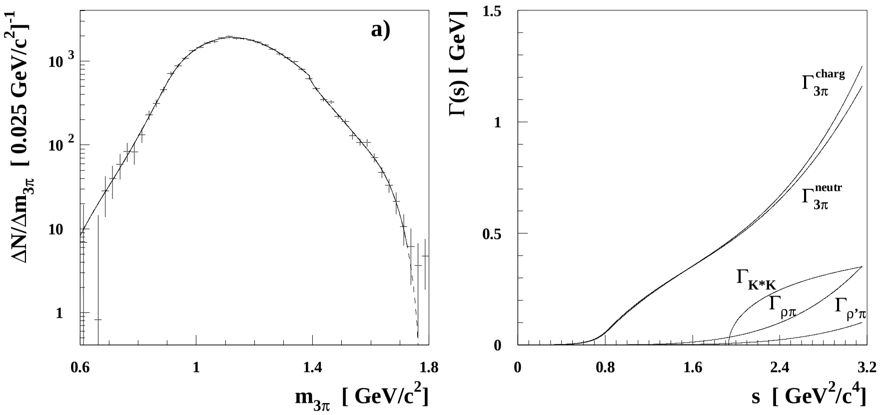
nominal fit: constant mass, $K^\star K$ threshold included, no $f_0(980)\pi$ threshold, $R = 0$

$$m_{a_1} = 1.331 \pm 0.010 \pm 0.003 \;,\; {}_{a_1} = 0.814 \pm 0.036 \pm 0.013$$

$$\mathcal{B}(a_1 \rightarrow K^\star K) = (3.3 \pm 0.5 \pm 0.1)\%$$

small excess of data at high $m_{3\pi}$ values $\Longrightarrow a_1'$?

CLEO $\tau \to 3\pi\nu_\tau$; FIT INCLUDING a_1'



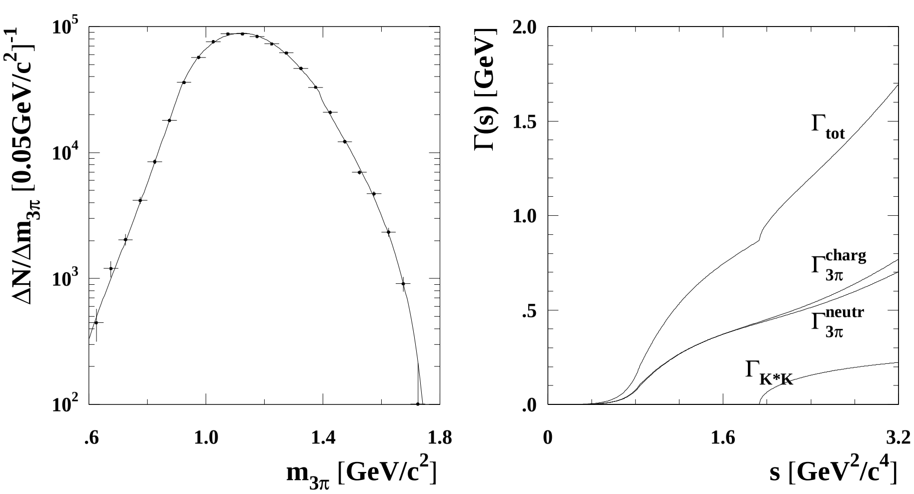
Improvement of fit yields significance of 2.9σ for a_1'

$$\mathcal{B}(\tau \to a_1'\nu) = (1.6 \pm 1.1 \pm 0.3 \pm 0.7) \times 10^{-4}$$

$\Longrightarrow$ more statistics needed to conclusively state if a_1' participates or not participates in $\tau \to 3\pi\nu$

$$\tau^\mp \to \pi^\mp\pi^\mp\pi^\pm\nu \text{ mode}$$

Fit with constant mass and $K^\star K$ threshold



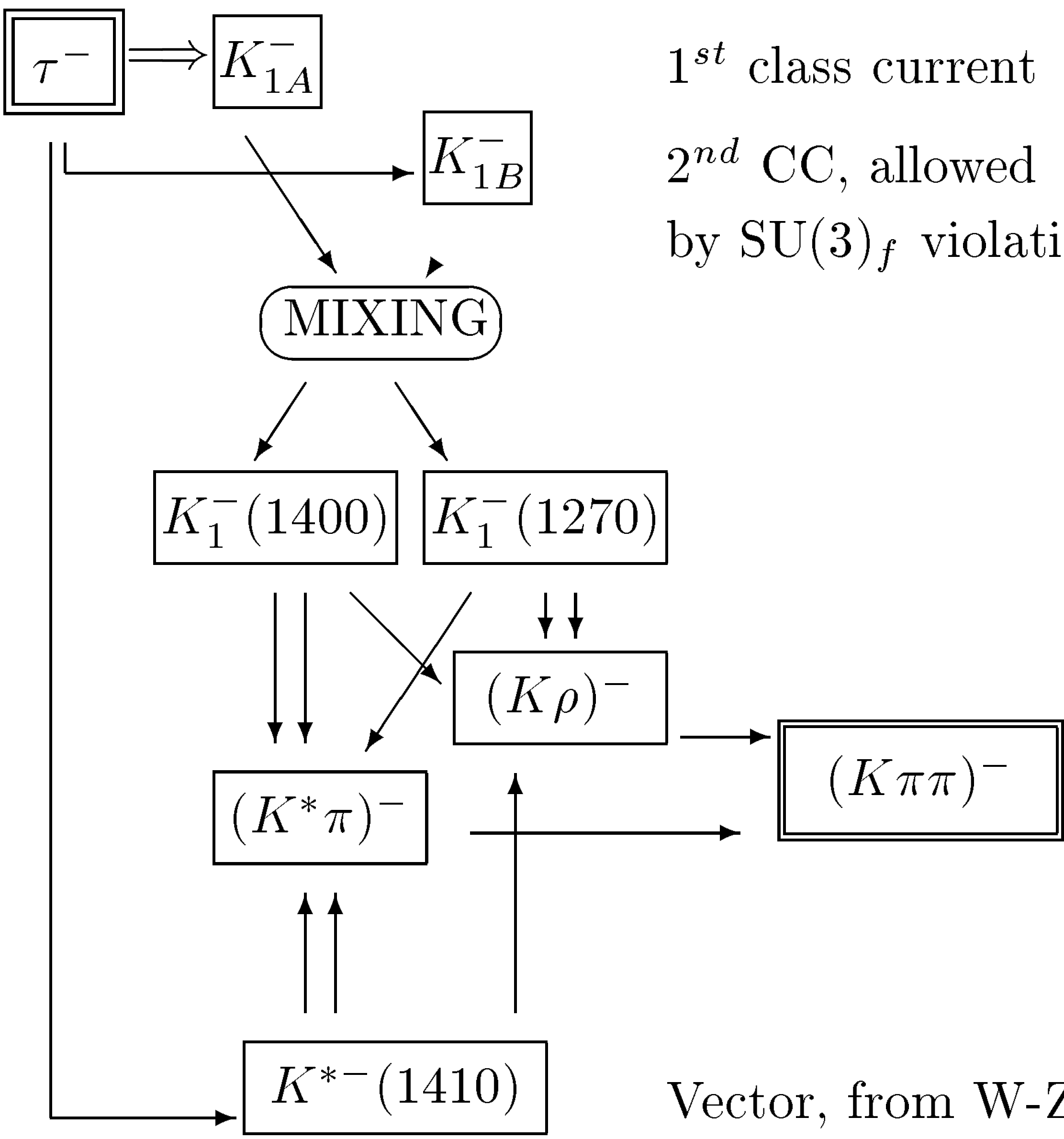
CLEO $\tau \to 3\pi\nu_\tau$ – Summary

- high statistics in the 3π channel is permitting:
  - detailed studies of the hadronic substructure
  - precision measurements of PV signed ν_τ helicity
  - significant contrib. other than $a_1 \to \rho\pi$
- model-independent structure function analyses provide:
  - measurements of h_{ν_τ}
  - limits on non axial vector
  - clean tests of models
- model-dependent fits to full kinematic distrib. gives:
  - significant signals for isoscalars f_0, f_2, and σ
  - evidence for a_1'(?)
  - evidence for $K^\star K$ threshold
  - limits on PCAC-violating π'
- good model (charged/neutral) especially at $m_{3\pi} \lesssim m_\tau$:
  - useful for $\tau \to 3\pi\nu$ detection at hadron colliders
  - essential for extraction of m_{ν_τ}
- but sill many open questions:
  - a_1 lineshape: running/constant mass, thresholds??
  - a_1': How much?? How does it decay??
  - substructure: couplings?? mass dependence??

$\tau^- \to (K\pi\pi)^- \nu_\tau$

- final states $K_S^0\pi^-\pi^0\nu_\tau$, $K^-\pi^+\pi^-\nu_\tau$, or $K^-\pi^0\pi^0\nu_\tau$
- There are two axial-vector ($J^P = 1^+$) states:
  K_a in the ${}^3P_1$ octet, strange partner of the $a_1(1260)$;
  K_b in the ${}^1P_1$ octet, strange partner of the $b_1(1235)$.
  K_b couples to W as SU(3)-violating "second-class" current.
- Both both decay to $K\pi\pi$ via $K^*\pi$ and $K\rho$
  (other final states (*e.g.*, $K\omega$) have been observed);
  mix into the physical mesons $K_1(1270)$, $K_1(1400)$
- So, we have SU(3)-violation, and mixing.
- Can get $K\pi\pi$ from vector current via Wess-Zumino:
  $K^{*\prime} \to (K^*\pi, K\rho) \to K\pi\pi$;
  expected to be numerically small. Ignored for now.

$\tau^- \rightarrow (K\pi\pi)^- \nu_\tau$



1^{st} class current

2^{nd} CC, allowed by $SU(3)_f$ violation

$K^0_S\pi^-\pi^0$

$K^0_L\pi^-\pi^0$

$K^-\pi^+\pi^-$

$K^-\pi^0\pi^0$

Vector, from W-Z anomaly

Parameterizations of K_1 dynamics

- parameterize the couplings of the K_1 mesons to the W following Suzuki:

$$|W\rangle \rightarrow f_{K_1}(|K_a\rangle - \delta\,|K_b\rangle)$$

where δ is the SU(3)/PCAC violation parameter, to be determined phenomenologically.

- The K_a and K_b then decay via the strong interaction to a vector and a pseudoscalar.
By C invariance and SU(3), the couplings of the K_a and K_b to the 1^- octet and the 0^- octet are:

$$H_{int}^{(a)} = \frac{f_a}{2}(K\rho - K^*\pi + K\phi_8 - K^*\eta_8)K_a$$

$$H_{int}^{(b)} = \frac{f_b}{2\sqrt{5}}(3(K\rho + K^*\pi) - (K\phi_8 - K^*\eta_8))K_b$$

where f_a and f_b are couplings to be determined phenomenologically.

- The K_a and K_b mix into physical K_{1a} and K_{1b}:

$$\begin{aligned} |K_{1a}\rangle &= \cos\theta\,|K_a\rangle - \sin\theta\,|K_b\rangle \\ |K_{1b}\rangle &= \cos\theta\,|K_b\rangle + \sin\theta\,|K_a\rangle \end{aligned}$$

  where θ is the mixing angle, a parameter to be determined phenomenologically.

- We identify the K_{1b} with the $K_1(1270)$ and the K_{1a} with the $K_1(1400)$.

- Notational shorthand:

$$g_a \equiv \frac{f_a}{2}; \qquad g_b \equiv \frac{3f_b}{2\sqrt{5}}; \qquad c \equiv \cos\theta; \qquad s \equiv \sin\theta$$

- For the $K\pi\pi$ final state, the relevant couplings are:

$$|K_a\rangle \rightarrow g_a(|K\rho\rangle - |K^*\pi\rangle); \qquad |K_b\rangle \rightarrow g_b(|K\rho\rangle + |K^*\pi\rangle)$$

$$\begin{aligned} \langle K\rho|K_a\rangle &= g_a; \qquad \langle K\rho|K_b\rangle = g_b; \\ \langle K^*\pi|K_a\rangle &= -g_a; \qquad \langle K^*\pi|K_b\rangle = g_b. \end{aligned}$$

- The mass eigenstates K_{1a} and K_{1b} propagate with $BW(K_{1a})$, $BW(K_{1b})$

Parameterizations of K_1 dynamics, II

- The couplings:

$$
\begin{aligned}
\left\langle K^*\pi|H|K_{1a}\right\rangle \langle K_{1a}|H|0\rangle &= f_{K_1}(c+\delta s)(-cg_a - sg_b)BW(K_{1a}) \\
\langle K\rho|H|K_{1a}\rangle \langle K_{1a}|H|0\rangle &= f_{K_1}(c+\delta s)(cg_a - sg_b)BW(K_{1a}) \\
\left\langle K^*\pi|H|K_{1b}\right\rangle \left\langle K_{1b}|H|0\right\rangle &= f_{K_1}(s-\delta c)(cg_b - sg_a)BW(K_{1b}) \\
\left\langle K\rho|H|K_{1b}\right\rangle \left\langle K_{1b}|H|0\right\rangle &= f_{K_1}(s-\delta c)(cg_b + sg_a)BW(K_{1b})
\end{aligned}
$$

- Fitting for those four amplitudes (assumed relatively real) is equivalent to fitting for the parameters:
  δ (SU(3)-breaking weak current),
  $f_{K_1}g_a$ (overall coupling),
  g_b/g_a (relative coupling of ${}^1P_1$, ${}^3P_1$ to 1^-, 0^- octets),
  the K_1 mixing angle $c = \cos\theta$, $s = \sin\theta$

The mixing parameters

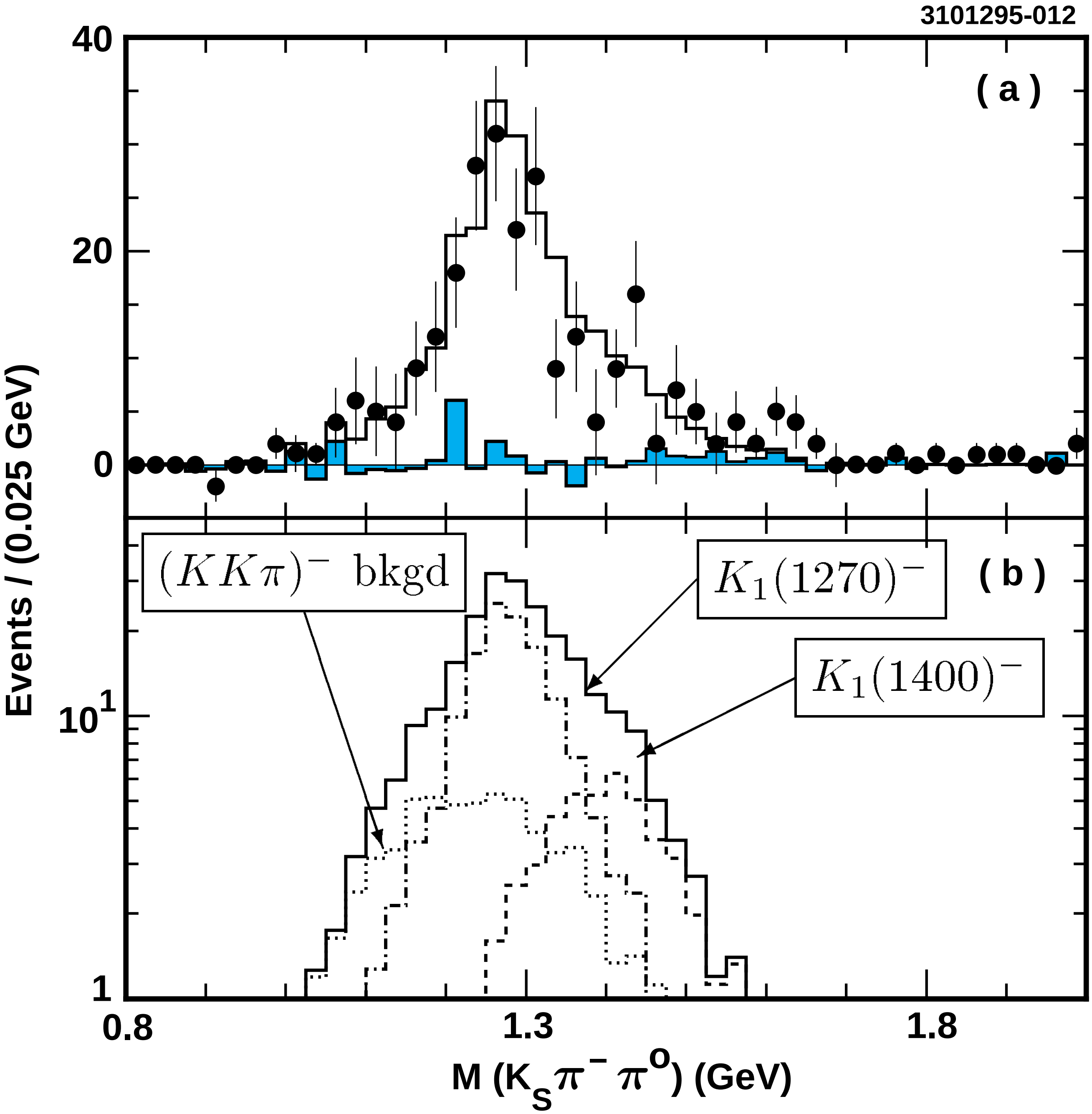
- Masses, and decay rates from PDG96, for:
  $, (K_{1a} \to K^*\pi)$, $, (K_{1a} \to K\rho)$,
  $, (K_{1b} \to K^*\pi)$, $, (K_{1b} \to K\rho)$
- We then get (all in GeV):

$$
\begin{aligned}
, (K_{1a} \to K^*\pi) &= (0.1636 \pm 0.0161) &= 0.00864 \times (-cg_a - sg_b)^2 \\
, (K_{1a} \to K\rho) &= (0.0052 \pm 0.0052) &= 0.00631 \times (cg_a - sg_b)^2 \\
, (K_{1b} \to K^*\pi) &= (0.0144 \pm 0.0055) &= 0.00764 \times (-sg_a + cg_b)^2 \\
, (K_{1b} \to K\rho) &= (0.0378 \pm 0.0100) &= 0.00161 \times (sg_a + sg_b)^2
\end{aligned}
$$

- Note that $K_{1b} \to K\rho$ is close to threshold; important to get $BW(\rho)$ right (to be done).
- Fit for f_a, x, and θ. 4-fold ambiguity in the minimum:

Soln	f_a (GeV)	x	θ (degrees)	$P(\chi^2)$
1	$4.7 \pm 0.x$	1.25 ± 0.00	42 ± 2	59%
2	$7.9 \pm 0.x$	0.44 ± 0.00	48 ± 2	59%
3	$6.1 \pm 0.x$	0.83 ± 0.00	31 ± 2	59%
4	$6.9 \pm 0.x$	0.66 ± 0.00	59 ± 2	59%

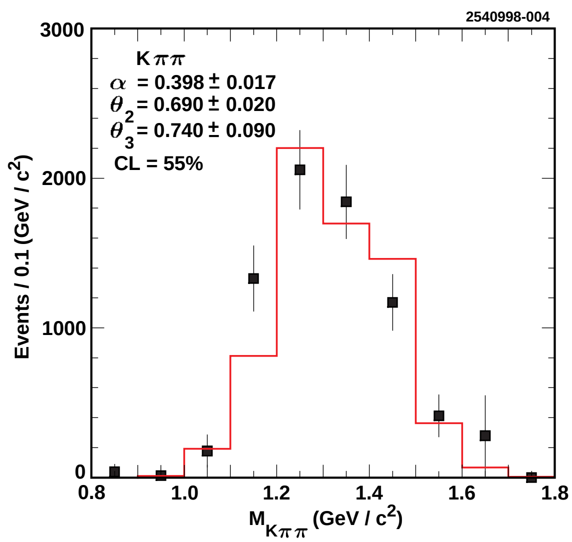
CLEO $K_S^0\pi^-\pi^0$

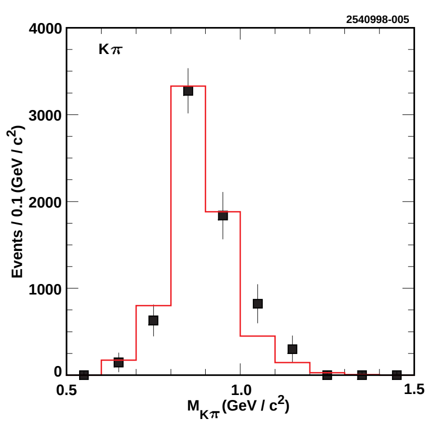


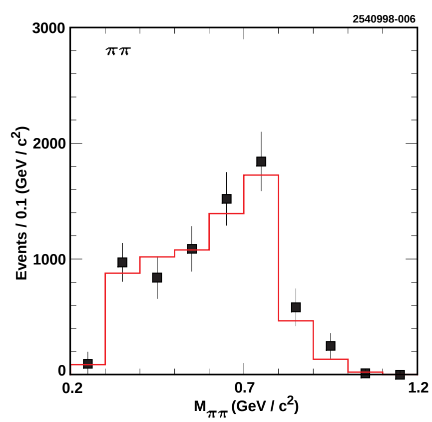
- Find K by $K_S^0 \to \pi^+\pi^-$. More efficient than doing dE/dx and TOF π^-/K^- separation at CLEOII.

CLEO $K^-\pi^+\pi^-$

- Statistical extraction of $(K^-\pi^+\pi^-)/(\pi^-\pi^+\pi^-)$
- Fit for interfering $K_1(1270) + \beta K_1(1430)$
- Fit for $K^*\pi$ and $K\rho$ in projection

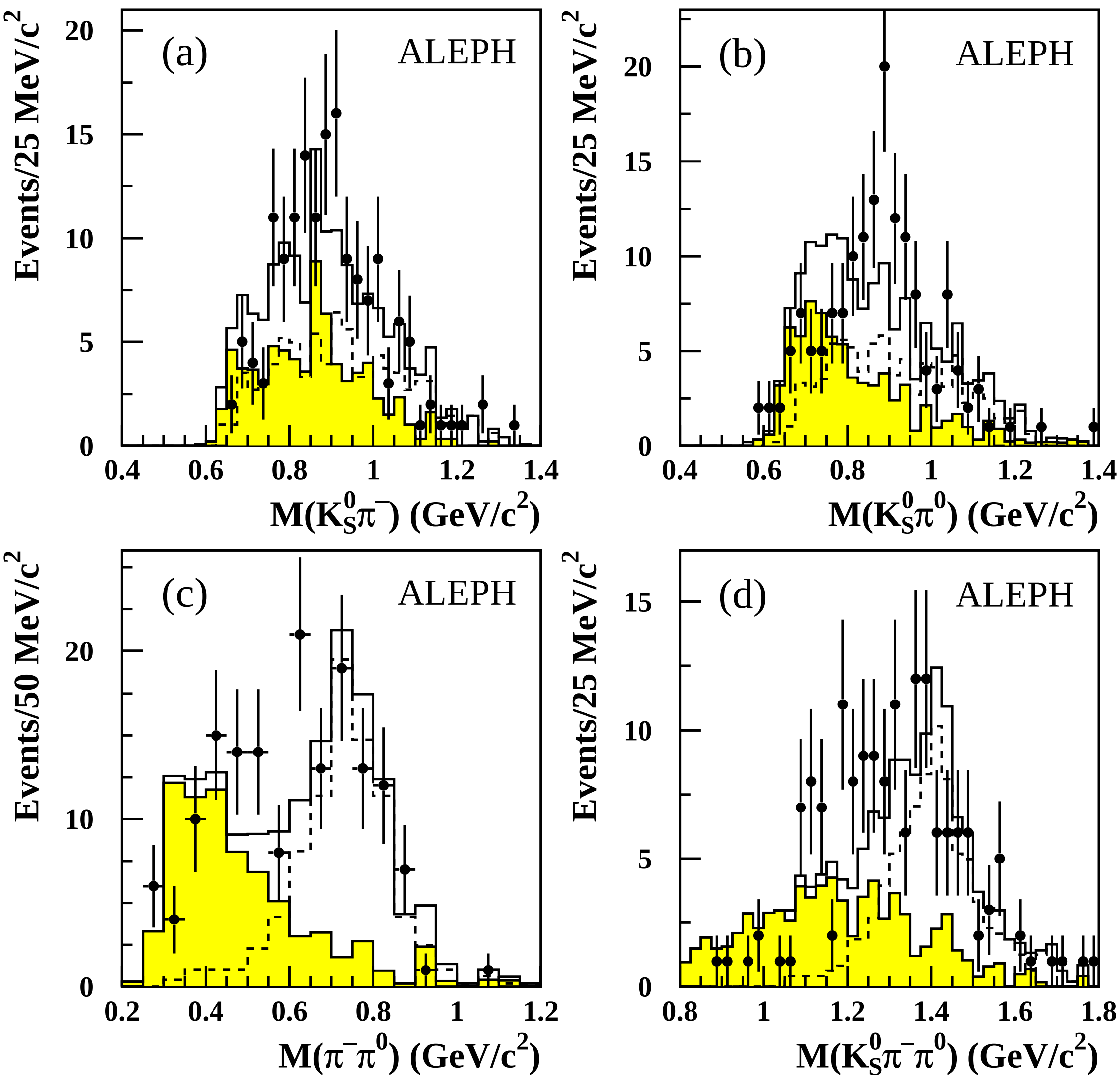






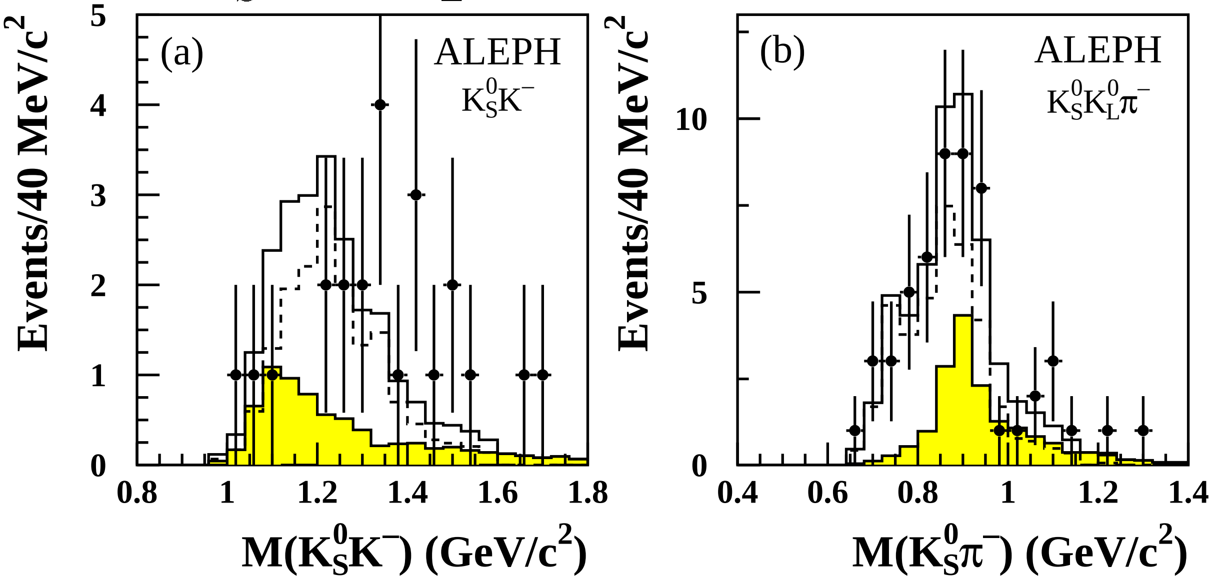
ALEPH $K^-\pi^+\pi^-$

- Signals in $K_S^0\pi^-\pi^0$, $K^-\pi^+\pi^-$
- Simple model fits, no interpretation yet

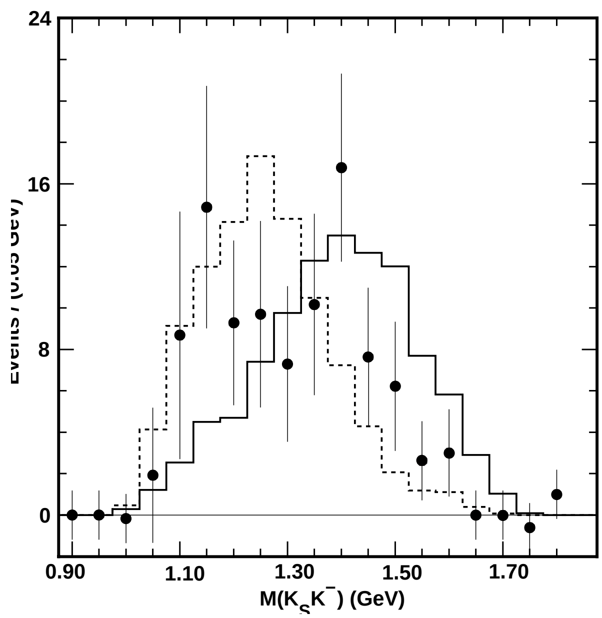


$\tau^- \to K^- K^0 \nu_\tau$

- ALEPH uses K^0_S and K^0_L

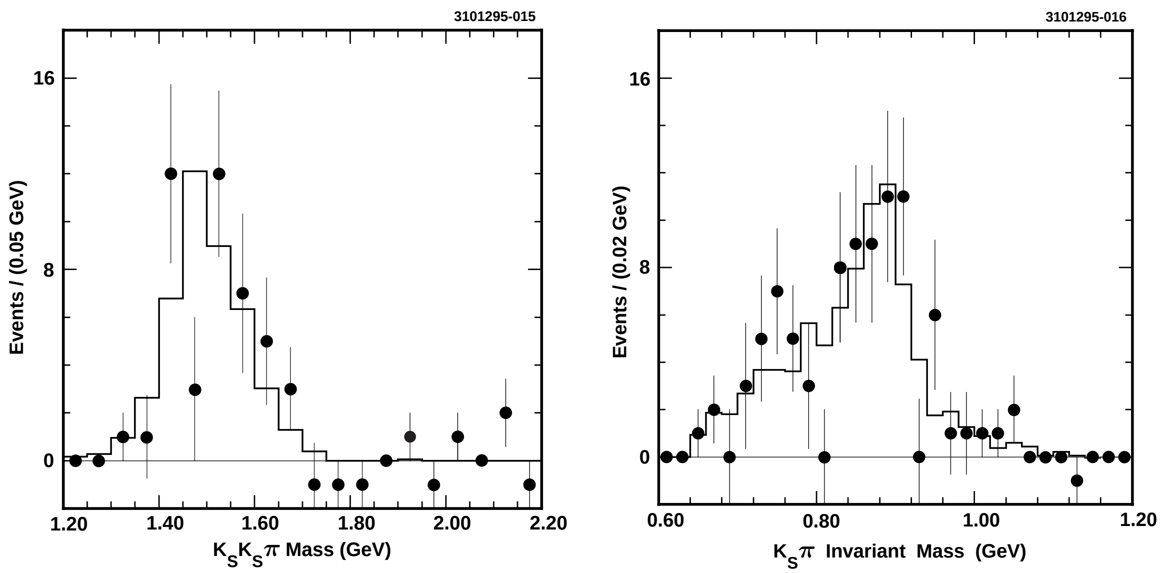


- CLEO spectrum consistent with $\rho \to KK$; ALEPH spectrum is harder.



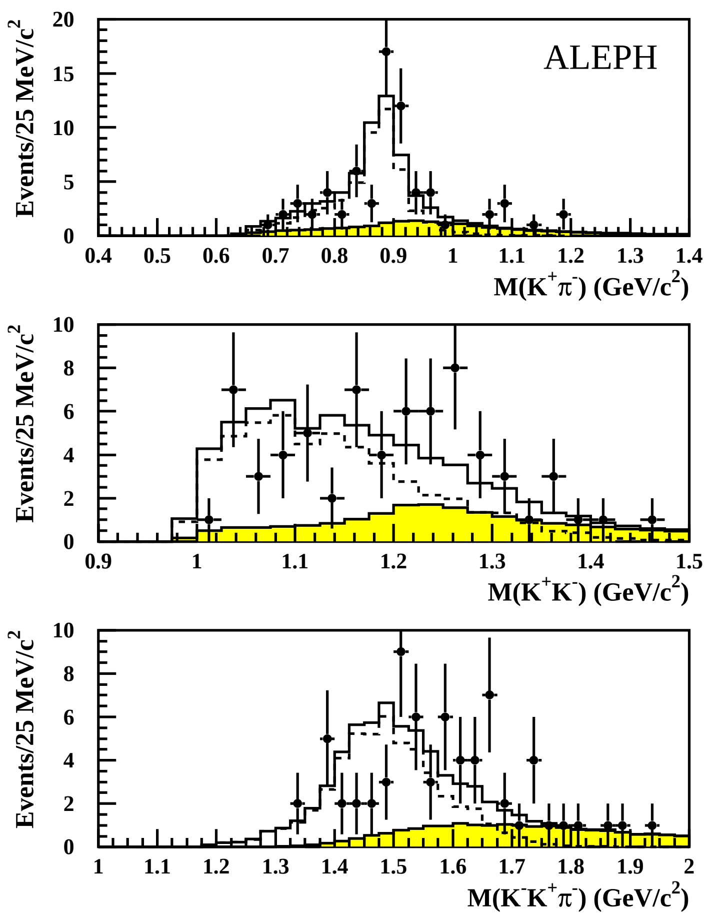
CLEO $\tau^- \to K^0_S K^0_S \pi \nu_\tau$

- CLEO sees ~200 events;
  $\mathcal{B}(\tau^- \to K^0_S K^0_S \pi^- \nu_\tau) = (3.5 \pm 0.4) \times 10^{-4}$
- Interpretation in terms of $K^0 \bar{K}^0 \pi^- \nu_\tau$ is problematical!
  (Depends on intermediate quantum states)
- Good candidate for m_{ν_τ} limit,
  especially since $m(KK\pi)$ resolution is good.
  BUT: no very high mass candidates seen yet.



$\tau^- \to K3\pi,\ KK,\ KK\pi,\ KK\pi\pi,\ \cdots$

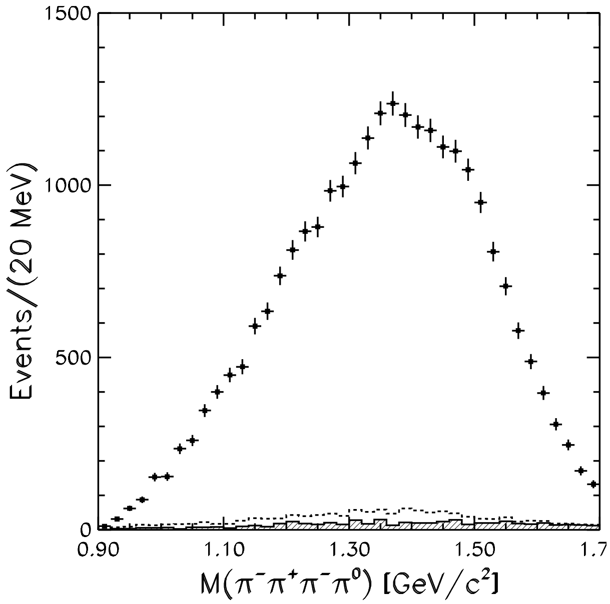
- ALEPH gave a kaon blitzkrieg at TAU98: signals (and fits to models!) in many modes: $K^0\pi^-\pi^0$, $K^-\pi^+\pi^-$, $K^-\pi^+\pi^-\pi^0$, K^-K^0, $K^-K^+\pi^-$, $K^-K^+\pi^-\pi^0$, $K^-K^0\pi^0$, $K^0K^0\pi^-$.

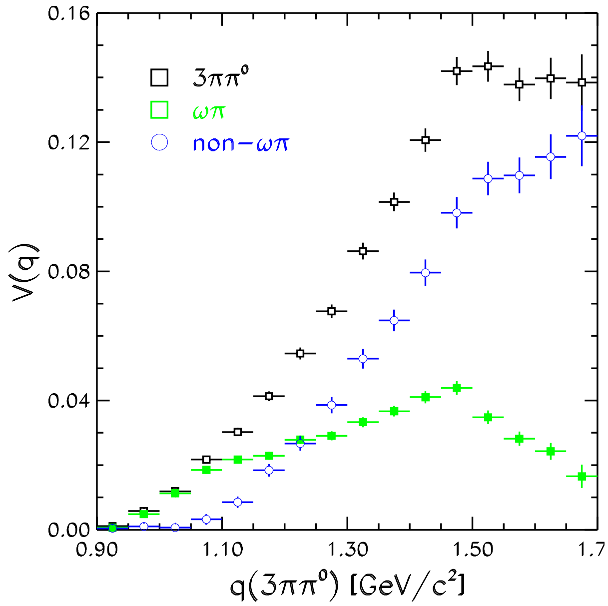


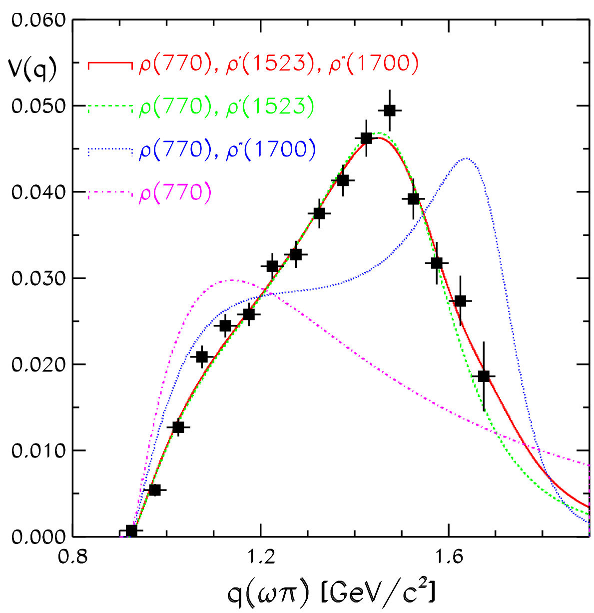
$\tau^- \to 4\pi\nu_\tau$

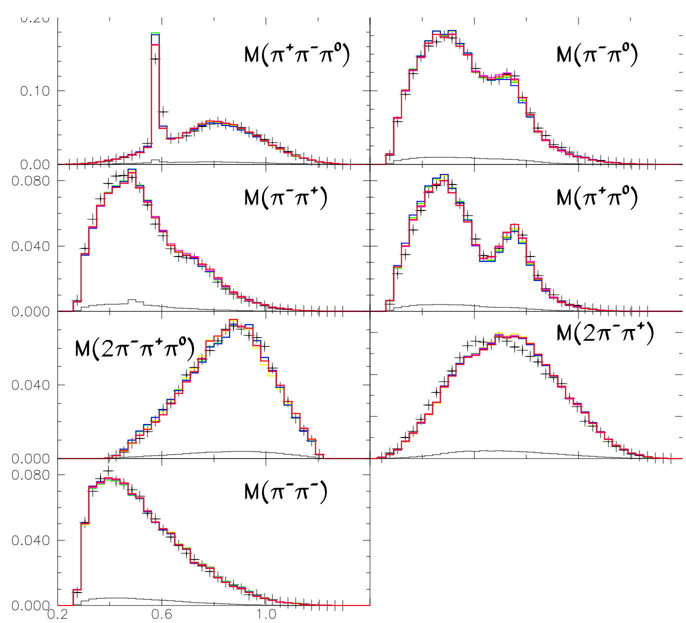
- Must know spectral function for m_{ν_τ} measurements (CLEO 1999: $m_{\nu_\tau} < 28$ MeV/c^2, 95% CL; 4 MeV model-dependence syst error dominates!)
- CVC tests, comparing $e^+e^- \to 2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$ to $\tau^- \to \nu_\tau 2\pi^-\pi^+\pi^0$, $\pi^-3\pi^0$
- Dominated by $\rho' \to \rho\pi\pi$, $\pi\omega$
- ρ', ρ'' parameters of interest
- Even the simplest models are already complicated!
- $\rho' \to a_1\pi$ seen in e^+e^-;
  CLEO sees no significant evidence for it!
- Search for second-class currents: $\tau \to \nu_\tau b_1$

CLEO $\tau^- \rightarrow 4\pi\nu_\tau$









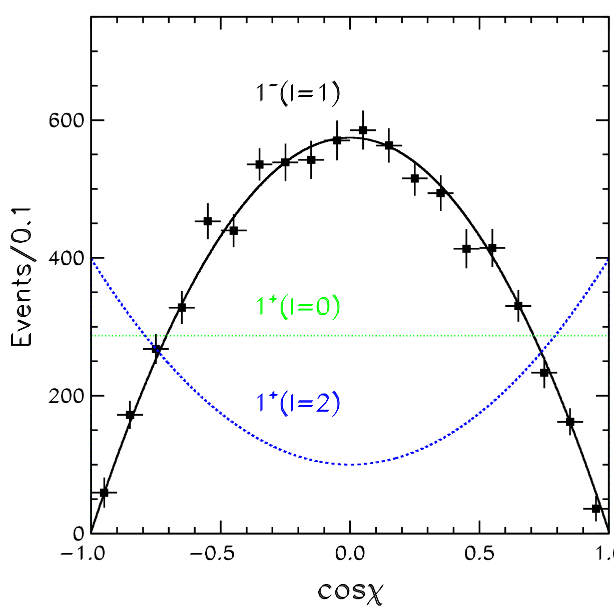
Second-class currents

- There are two axial-vector ($J^P = 1^+$) states:
  $a_1(1260)$ in the ${}^3P_1$ octet, $J^{PG} = 1^{+-}$,
  couples to W as a "first-class" current;
  $b_1(1235)$ in the ${}^1P_1$ octet, $J^{PG} = 1^{++}$.
  doesn't couple to W ("second-class" current)
  except via isospin (G-parity) violation ($f_{b_1} \approx 0$).
- $a_1 \to \rho\pi$ (S-wave) $\to 3\pi$ dominant;
  $\rho' \to \omega\pi$ (P-wave) $\to 4\pi$ dominant;
  $b_1 \to \omega\pi$ (S-wave) $\to 4\pi$ dominant.
- The difference in G-parity for these states
  is is reflected in the different expected polarization of
  the vector meson, and thus the angular distribution
  $\cos\chi = \hat{n}_\perp^\omega \cdot \hat{p}_{\pi_4}$

Second-class currents

J^P	L	$F(\cos\chi)$
1^-	1	$1-\cos^2\chi$
1^+	0	1
1^+	2	$1+3\cos^2\chi$
0^-	1	$\cos^2\chi$

- CLEO sees no evidence of $b_1 \to \omega\pi$:



$\tau^- \to \nu_\tau \eta (n\pi)^-$

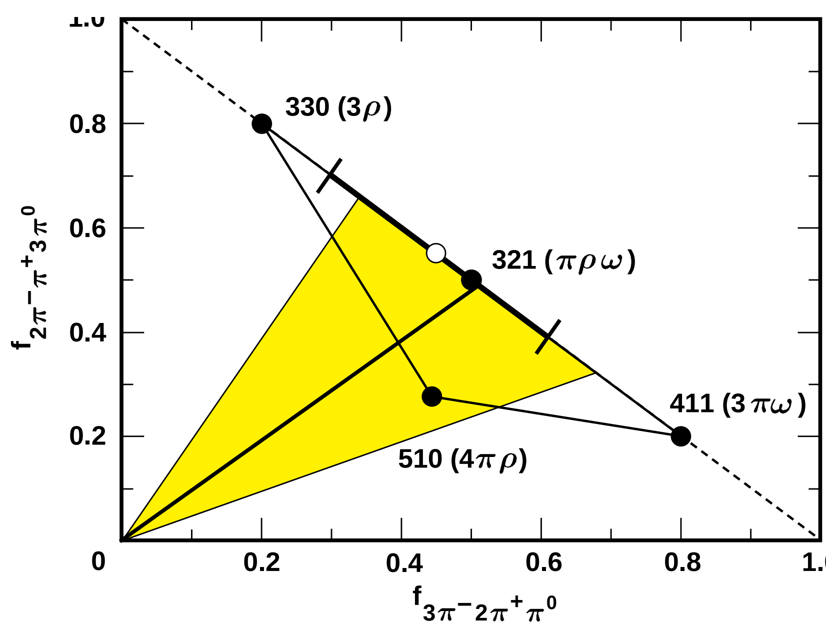
- $\tau^- \to \nu_\tau \eta \pi^-$ is forbidden by G-parity; $\mathcal{B}(\nu_\tau \eta \pi^-) < 1.4 \times 10^{-4}$ at 95% CL
- G-parity (isospin) is violated; this decay will be seen at some level
- $SU(3)_f$-violating $\tau^- \to \nu_\tau \eta K^-$ is seen: $\mathcal{B}(\nu_\tau \eta K^-) = (2.6 \pm 0.5) \times 10^{-4}$.
- $\tau^- \to \nu_\tau \eta \pi^- \pi^0$ proceeds via the W-Z chiral anomaly; $\mathcal{B}(\nu_\tau \eta \pi^- \pi^0) = (1.7 \pm 0.3) \times 10^{-3}$.
- W-Z Lorentz structure has not been definitively established.
- CLEO sees $\tau^- \to \nu_\tau \eta (3\pi)^-$:
  $\mathcal{B}(\nu_\tau \eta \pi^- \pi^+ \pi^-) = (3.4 \pm 0.8) \times 10^{-4}$.
  $\mathcal{B}(\nu_\tau \eta \pi^- \pi^0 \pi^0) = (1.4 \pm 0.6) \times 10^{-4}$.
- Rich substructure! Only beginning to be explored. Eg, $f_1\pi$, $f_1 \to a_0\pi$, $a_0 \to \eta\pi$

5π, 6π, 7π

- Small BRs:

$$\begin{aligned}
\mathcal{B}(\nu_\tau 2\pi^-\pi^+2\pi^0) &= (5.3 \pm 0.4) \times 10^{-3} \\
\mathcal{B}(\nu_\tau 3\pi^-2\pi^+) &= (7.5 \pm 0.7) \times 10^{-4} \\
\mathcal{B}(\nu_\tau 2\pi^-\pi^+3\pi^0) &= (2.9 \pm 0.7) \times 10^{-4} \\
\mathcal{B}(\nu_\tau 3\pi^-2\pi^+\pi^0) &= (2.2 \pm 0.5) \times 10^{-4} \\
\mathcal{B}(\nu_\tau 3\pi^-2\pi^+2\pi^0) &< 1.1 \times 10^{-4} \\
\mathcal{B}(\nu_\tau 7\pi^\pm) &< 2.4 \times 10^{-6}.
\end{aligned}$$

- Very complex sub-structure
- First step: enumerate isospin content; eg, $(3\pi^-2\pi^+\pi^0)/(6\pi)$ vs $(2\pi^-\pi^+3\pi^0)/(6\pi)$:

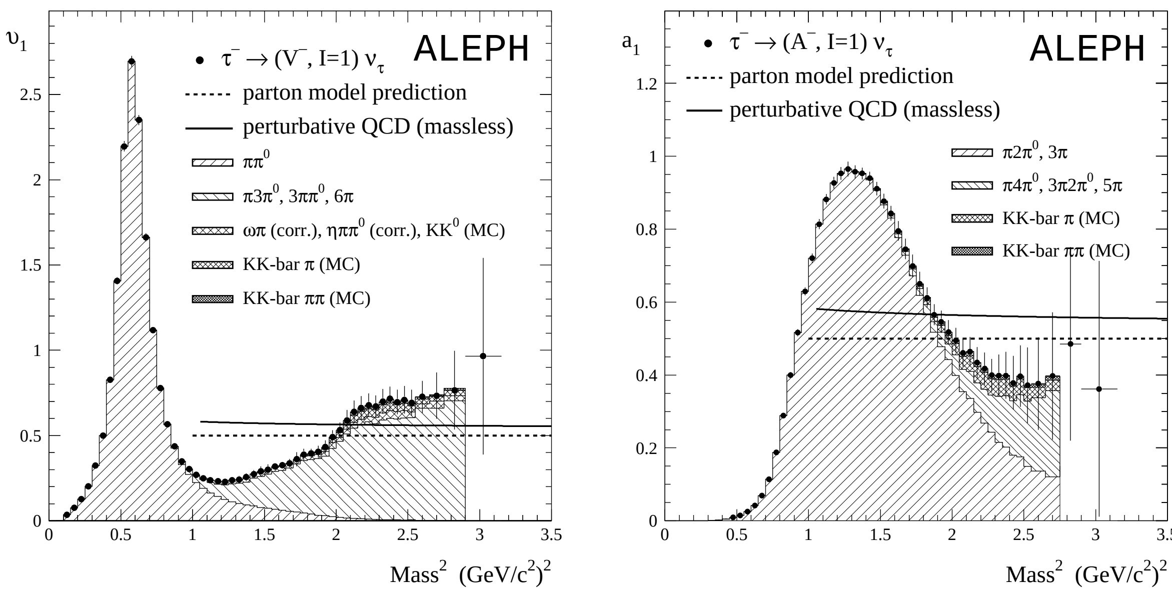


INCLUSIVE HADRONIC PHYSICS

- V, A spectral functions $\Rightarrow$ α_S, chiral condensates

$$R_{kl}^{v/a} = \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{1}{N_{v/a}} \frac{dN_{v/a}}{ds},$$

$$R_{kl}^{v/a} = \frac{3}{2} V_{ud}^2 S_{EW} (1 + \delta_{pert} + \delta_{mass}^{v/a} + \delta_{NP}^{v/a}),$$



- V^s, A^s spectral functions $\Rightarrow$ m_s

$$\delta_{mass}^s \simeq -8 \frac{\bar{m}_s^2}{m_\tau^2} \left[1 + \frac{16}{3}\frac{\alpha_S}{\pi} + \mathcal{O}\left(\frac{\alpha_S}{\pi}\right)^2\right],$$

- tests of CVC in inclusive rate

- improvements on hadronic vacuum polarization contribution to $(g-2)_\mu$, $\alpha_{QED}(q^2)$

Inclusive sum rules

- First Weinberg sum rule:

$$\frac{1}{4\pi^2}\int_0^\infty ds\,(v_1(s)-a_1(s)) = f_\pi^2$$

- Second Weinberg sum rule:

$$\frac{1}{4\pi^2}\int_0^\infty ds\cdot s\,(v_1(s)-a_1(s)) = 0$$

- Das-Mathur-Okubo sum rule:

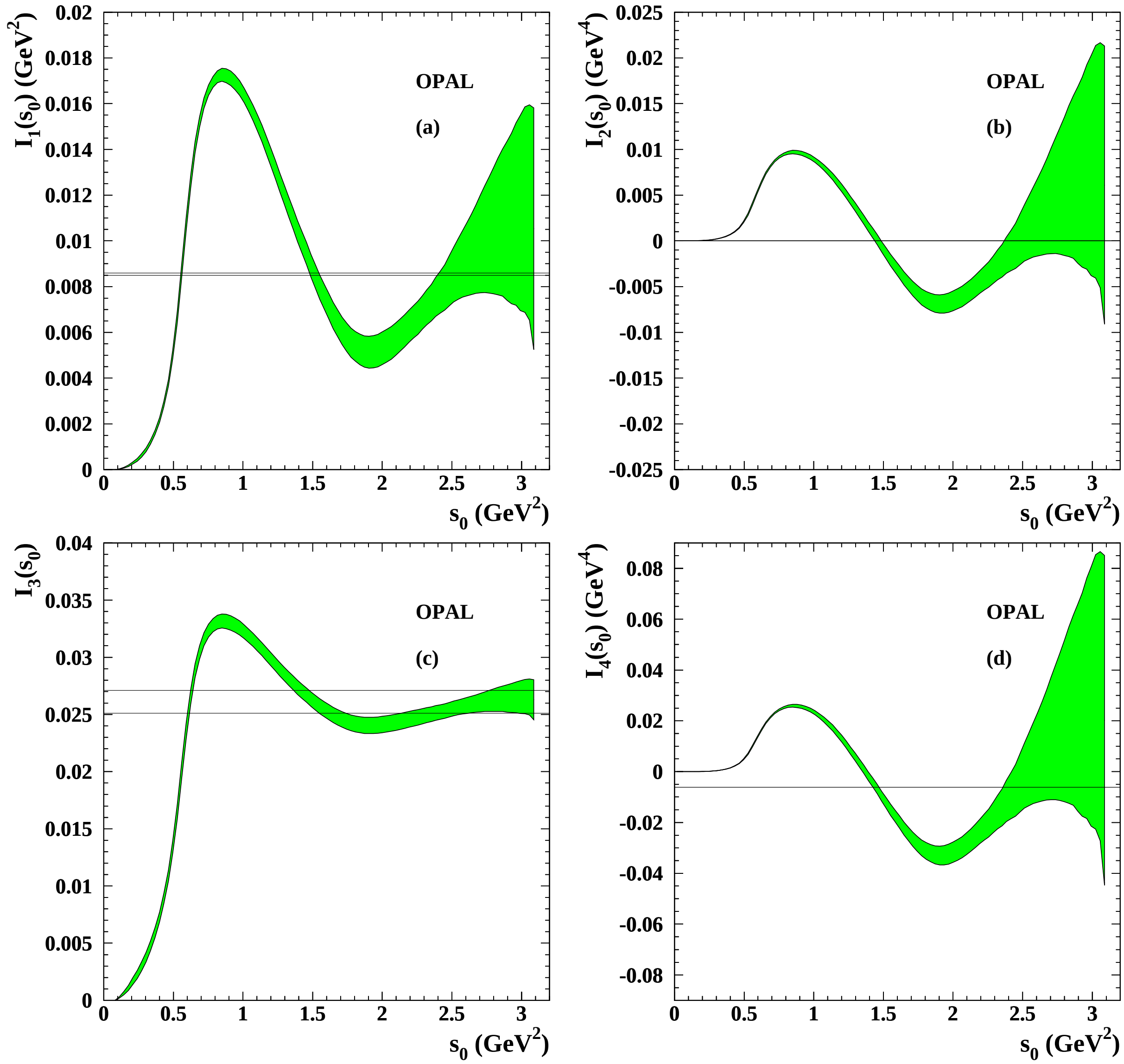
$$\frac{1}{4\pi^2}\int_0^\infty \frac{ds}{s}\,(v_1(s)-a_1(s)) = f_\pi^2\frac{\left\langle r_\pi^2\right\rangle}{3} - F_A$$

- Isospin-violating sum rule:

$$\frac{1}{4\pi^2}\int_0^\infty ds \qquad s\ln\frac{s}{\Lambda^2}\,(v_1(s)-a_1(s)) =$$

$$-\frac{16\pi^2 f_\pi^2}{3\alpha}\left(m_{\pi^\pm}^2 - m_{\pi^0}^2\right).$$

Inclusive sum rules - OPAL



- Note inevitable lack of statistical precision near $s_0 = m_\tau^2$;
  just where you need it most!

Conclusions on hadronic structure in tau decays

- There's LOTS OF IT
- there are many unresolved questions, even in low multiplicity final states
- All the open questions require:
  - lots more statistics;
  - excellent π/K separation;
  - tight control of backgrounds.
- if the *mystery* of low energy meson dynamics appeals to you, there is lots to do!

Issues re E_{cm} for tau physics

- Efficiency at 10.6 GeV B-factory:
  - The boost of the hadronic system means that soft pions will not get absorbed by the beam-pipe.
  - Soft π^0's will not get lost in the calorimeter under the background from secondary hadronic showers.
  - The tracks all go in approx the same direction, so acceptance (eg, n particles into $|\cos\theta| < 0.9$) is 0.9^1 rather than $(0.9)^n$.
- Cutting into dynamics:
  - At B-F, acceptance cuts (on min p_t and max $|\cos\theta|$) maximum polar angle) $\Rightarrow$ big loss of efficiency, BUT, don't cut into phase space of the decay (*e.g.*, acceptance is reasonably uniform accross 3pi Dalitz plot).
  - At the low energies of a τcF, the acceptance cuts into the dynamics (corners of DP).

This may severely limit the attainable systematic errors.

Issues re E_{cm} for tau physics, II

- multiple-scattering:
  Higher momentum tracks will not be severely multiple-scattered. Measurement errors are worse since σ_p/p goes like p, but multiple scattering gets better since p is larger, and that will dominate at both energies. So mass resolution is better at high energy.

- Particle ID: $K\pi$ separation:
  A τcF definitely needs good K/π separation in order to compete with this generation of semileptonic decay analyses. It should not be too difficult, with, eg, precision TOF. The main problem is that low-momentum kaons range out and are thus lost, cutting into the dynamics.

- K_S^0, K_L^0 efficiency, background:
  - Higher momentum K_S^0's are easier to separate from background using seperated-vertex cuts; reconstruction efficiency *might* be worse.
  - Higher momentum K_L^0's can be tagged more efficiently in instrumented flux returns (like BaBar's).

Issues re E_{cm} for tau physics, III

- Lepton ID:
  At low momentum, electrons and muons are harder to distinguish from pions using conventional techniques (E/p and muon walls) so precision TOF and/or Cerenkov, and finely-segmented muon rangeout system, is of course required.

- Displaced τ vertices:
  The boost means that the tau decay vertex is displaced, which can help in a variety of analyses, especially those which hope to use that info to estimate the tau direction (which won't work very well, even at B-Factories). I don't have much faith in the utility of using the separated vertices to distinguish tau pairs from hadronic background. There are better and easier ways.

Issues re E_{cm} for tau physics, IV

- $q\bar{q}$ backgrounds:
  - At τcF, events in which both taus decay semi-hadronically have a severe combinatoric background from hadronic events; to do precision physics, you need to tag using leptons (requiring good lepton id) or monochomatic pion. There's no such problem at B-Factories.
  - Background from qqbar events at B-Factories means that only leptonic-tagged events are useful for high-multiplicity semi-leptonic decays; but that problem is also at tcF (I'm not sure whether it is better or worse!).
- Polarization:
  At threshold, the taus are polarized along the beam. That *might* prove useful for, polarization-dependent measurements like Michel params or analysis of 3pinu. Might.
- For the study of semi-hadronic tau decays, I see no particular difference between threshold and 3.67 GeV...