Scalar Glueonium in J/ψ Radiative Decay to Two Pseudoscalars

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Tau-Charm Factory Workshop,
SLAC
March, 1999

Scalar (& Tensor?) Glueonium in Amplitude Analyses of J/ψ Radiative Decay to Two Pseudoscalars

{applies to ψ' also}

1) Why do we expect glueonium states to exist?
2) Why do we need amplitude analyses to find them? {=> need for very large data samples}
3) If you found $J^{PC}=0^{++}$ glueonium, would you know it?
4) Exptal. evidence so far (non J/ψ radiative)
5) Why is J/ψ (or ψ') radiative decay the best place to look? (not totally objective!)
6) Outline of Mark III analysis & results on $J/\psi \to \gamma\pi^+\pi^-$, γK^+K^-, $\gamma K^0_S K^0_S$ (+ remarks on $\gamma\eta\eta$)

   Interpretation?
7) Why is the TCF necessary to this effort?
8) Two (seemingly) mundane questions whose answers also require the TCF

Why do we expect gluonium states to exist?

(1) In the context of QCD:

"... meson states would appear that act as if they were made of gluons rather than $q\bar{q}$ pairs."

Such states would result in

"... a sequence of <u>extra</u> SU(3) singlet meson states." {I=0}

<u>Fritzsch & Gell-mann</u> ICHEP, Batavia (1972) Proc. v.2, p.135

(2) Non-Abelian nature of QCD $\mathcal{L}$ => ∃ point-like couplings of 3 & 4 gluons

=> possible existance of bound states

(3) <u>Lattice Gauge Calcus:</u> → spectrum of such states;

Lowest Mass $J^{PC}=0^{++}$ state at $\sim 1.6^{+\text{Weingarten}}_{-\text{Michael}}$ GeV

with Weingarten ≃ Michael ≅ 0.1 GeV

{ $J^{PC}=2^{++}$ at ~ 2.3 GeV ± ? }

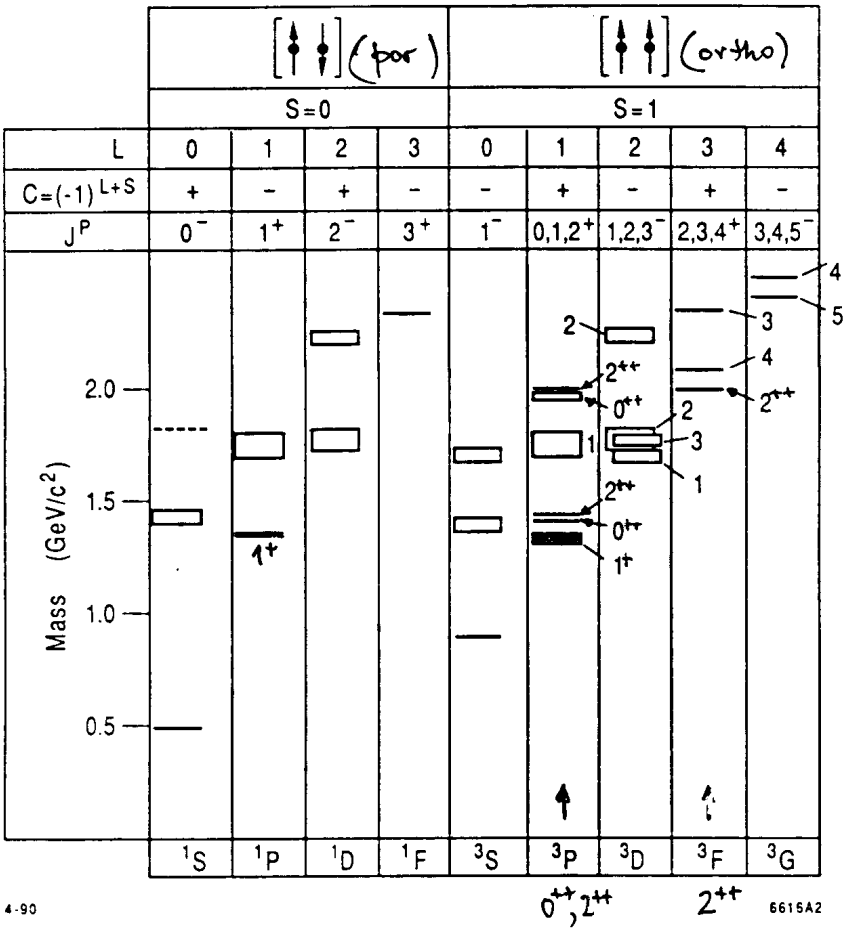
Why do we need amplitude analyses ?

Take a look at the strange meson sector.

a) don't expect extra states
   i.e. good qualitative agreement with quark model expectations

b) $\Rightarrow$ if o.k., can use quark model as "template" in search for extra states.

Strange Meson Level Scheme

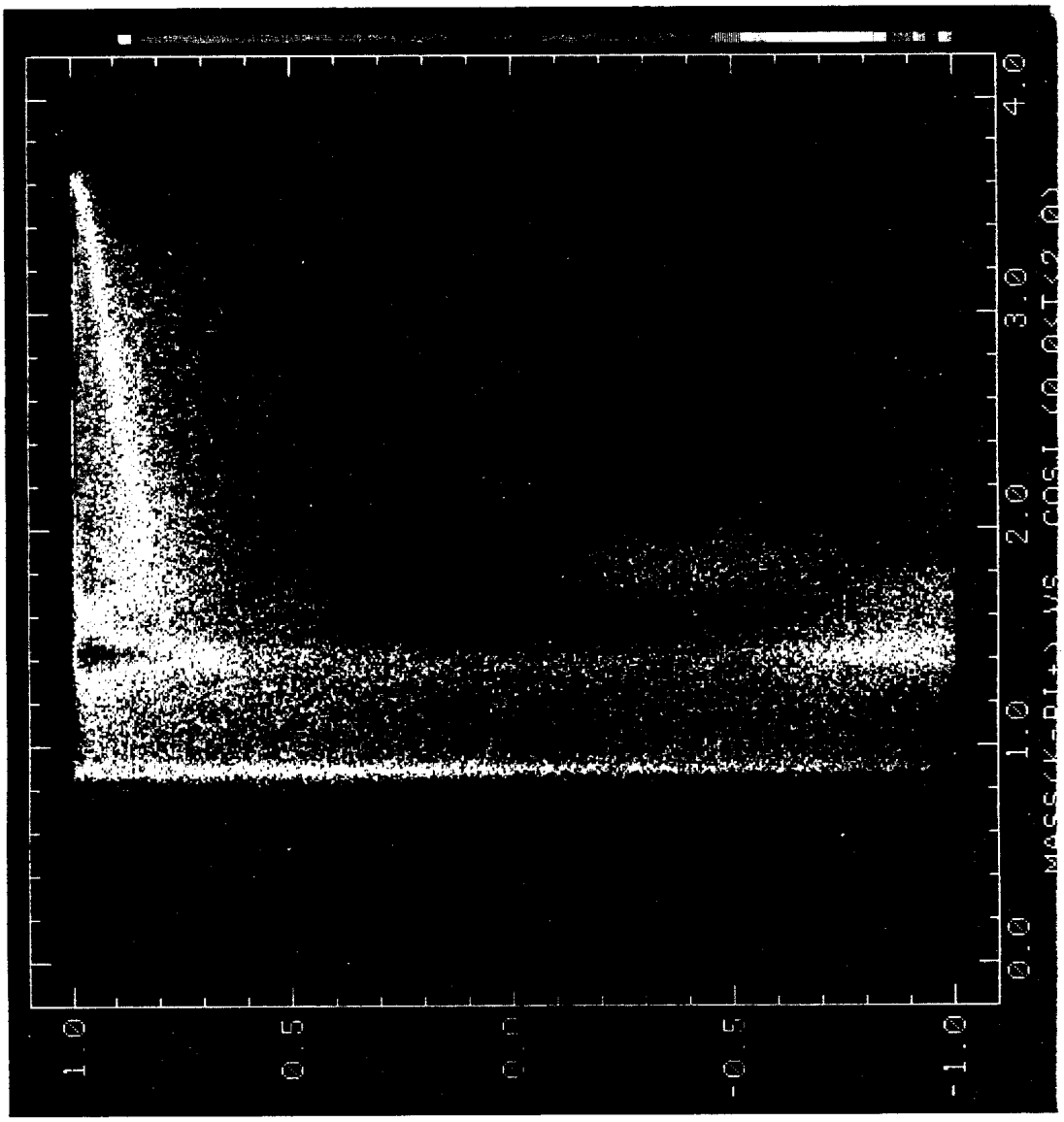


1) qualitatively similar to atomic He

2) no extra states ; approx. mass degeneracy

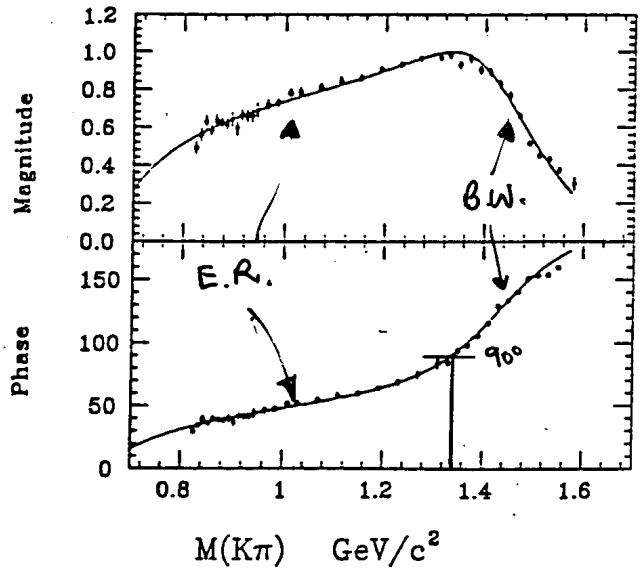
3) 0^{++} only in ^3P col. ; 2^{++} in ^3P & ^3F

4) $J^P = 1^+$ ground states mix ; $K_1(1270)$ & $K_1(1400)$ are mixtures of K_{1a} & K_{1b}


1.0
0.5
0.0
-0.5
-1.0
0.0
1.0
2.0
3.0
4.0

Kπ S-wave solution (s)

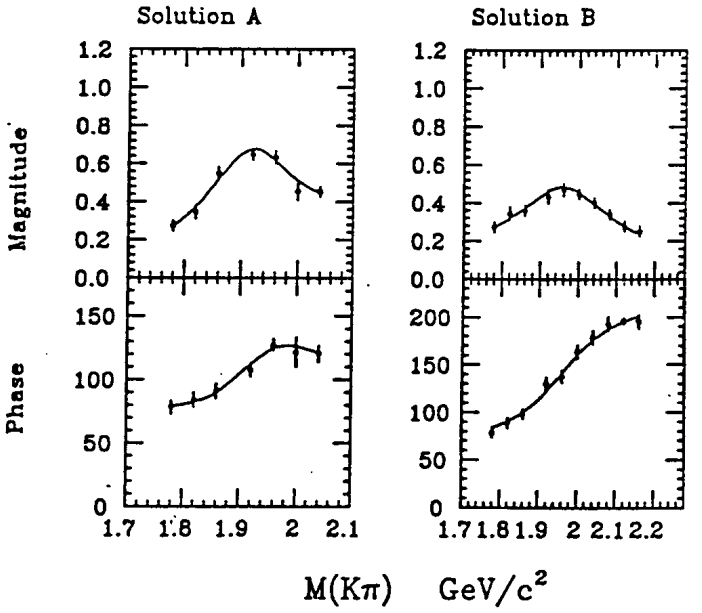
$\kappa(1350)$



M
1412±4±5
Γ
294±10±21

Elasticity
~1

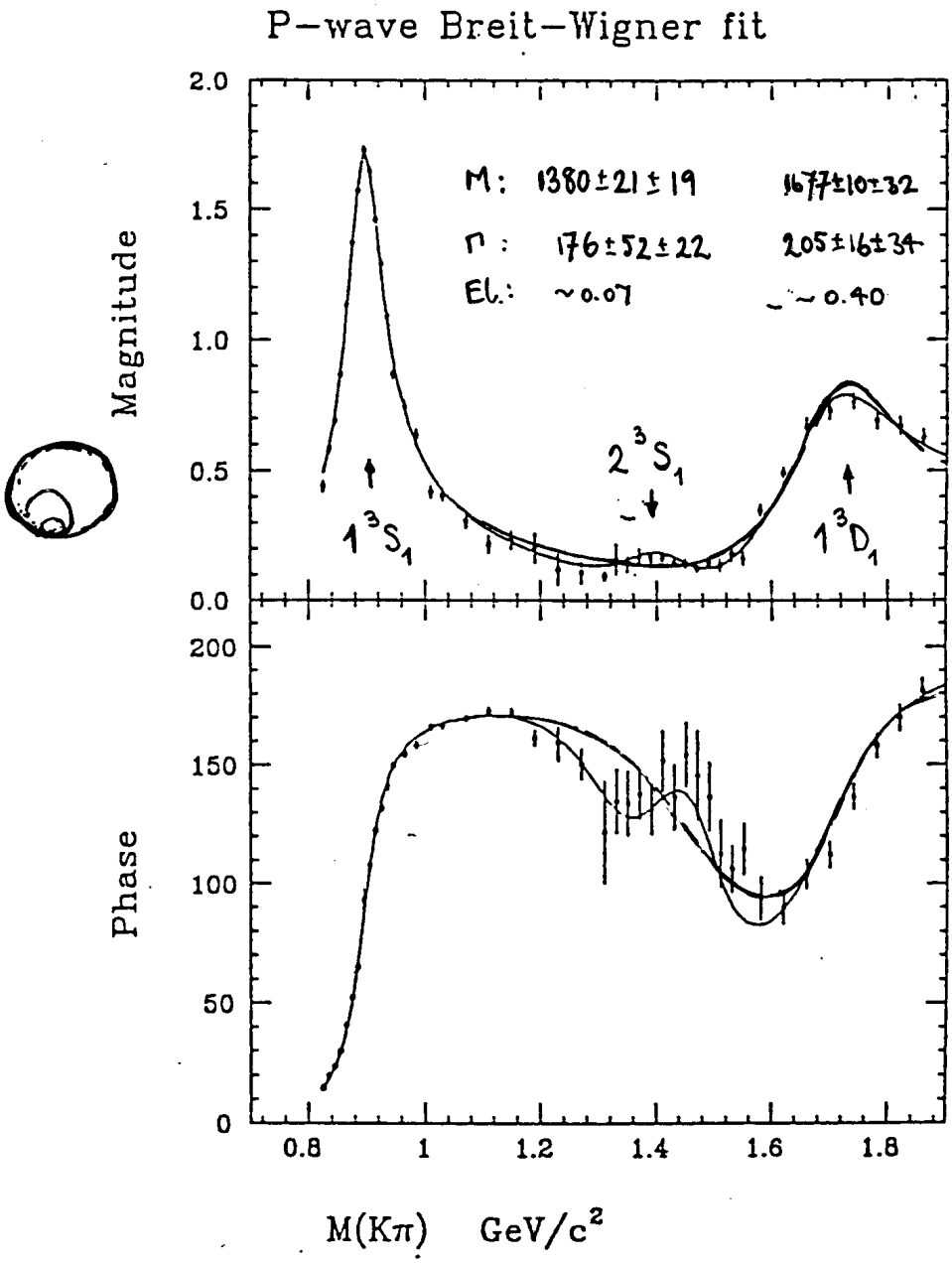
0^+ $K^*(1950)$ [Radial excitation]



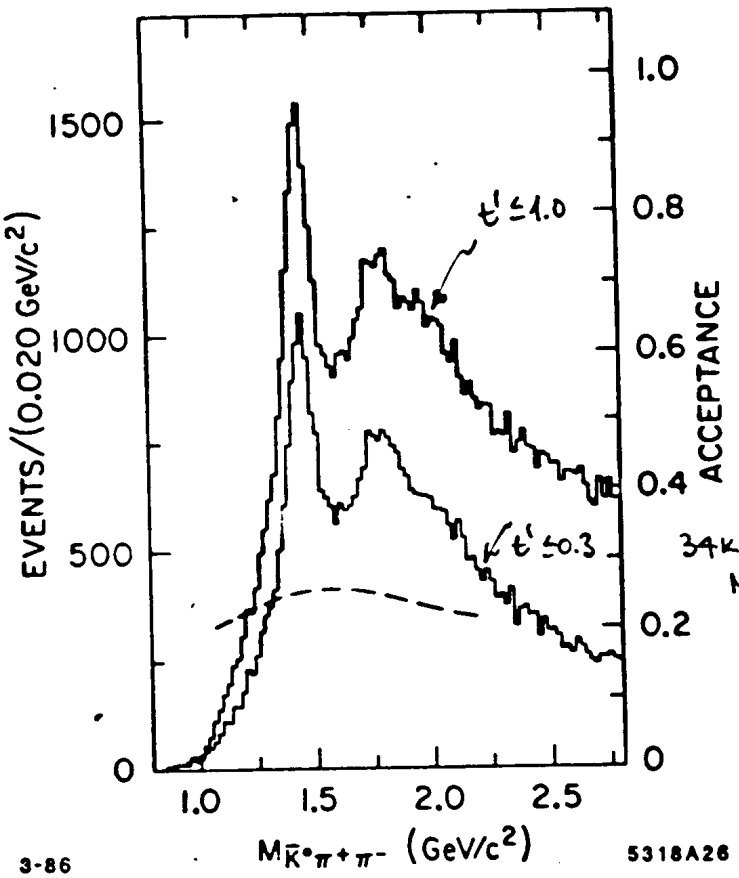
M
1913±5±20
Γ
152±12±79

M
195[illegible]
Γ
232±15±22

Elasticity
~0.45


P−wave Breit−Wigner fit
Magnitude
Phase
M: 1380±21±19 1677±10±32
Γ: 176±52±22 205±16±34
El.: ~0.07 ~0.40
1^3S_1
2^3S_1
1^3D_1
M(Kπ) GeV/c²

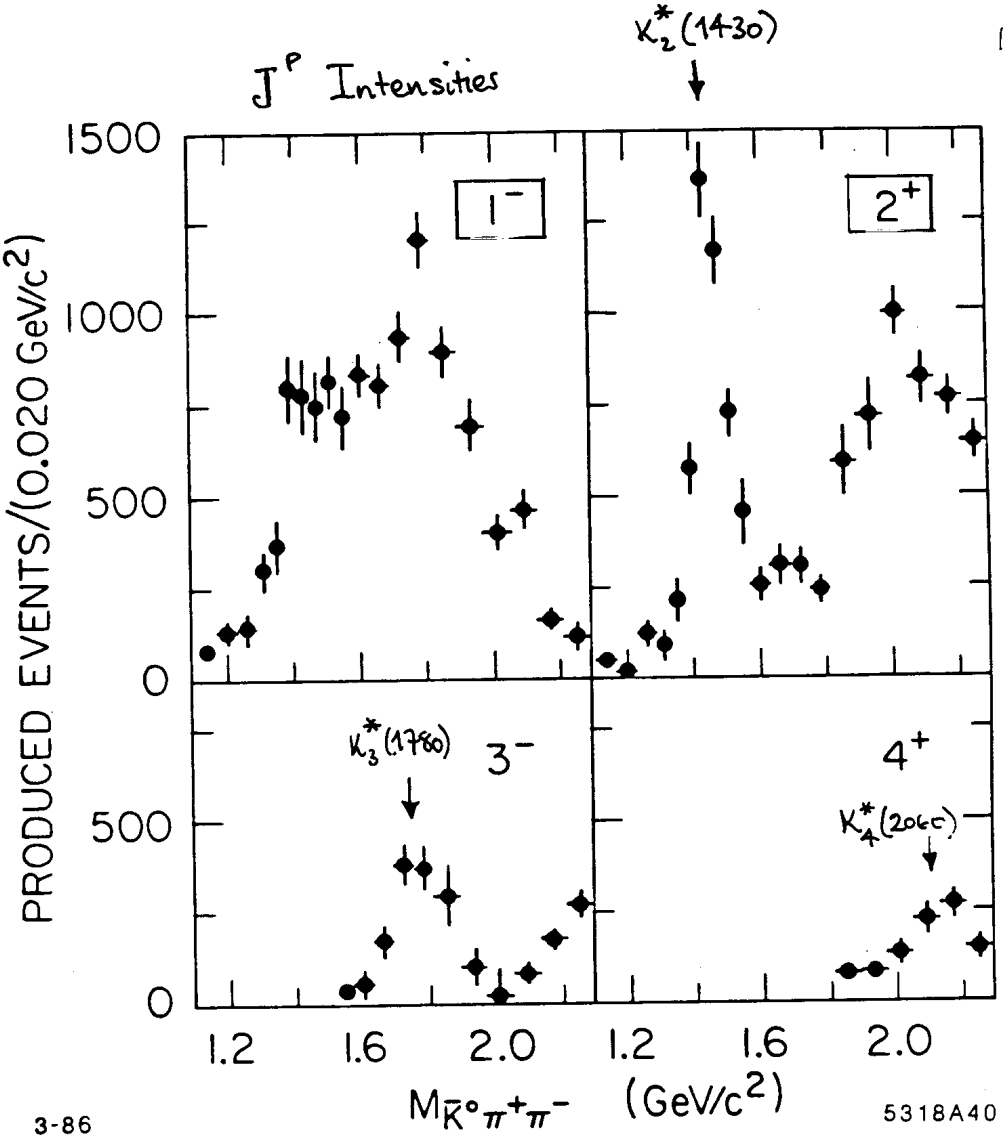
$K^- p \to \bar{K}^0 \pi^+ \pi^- n$



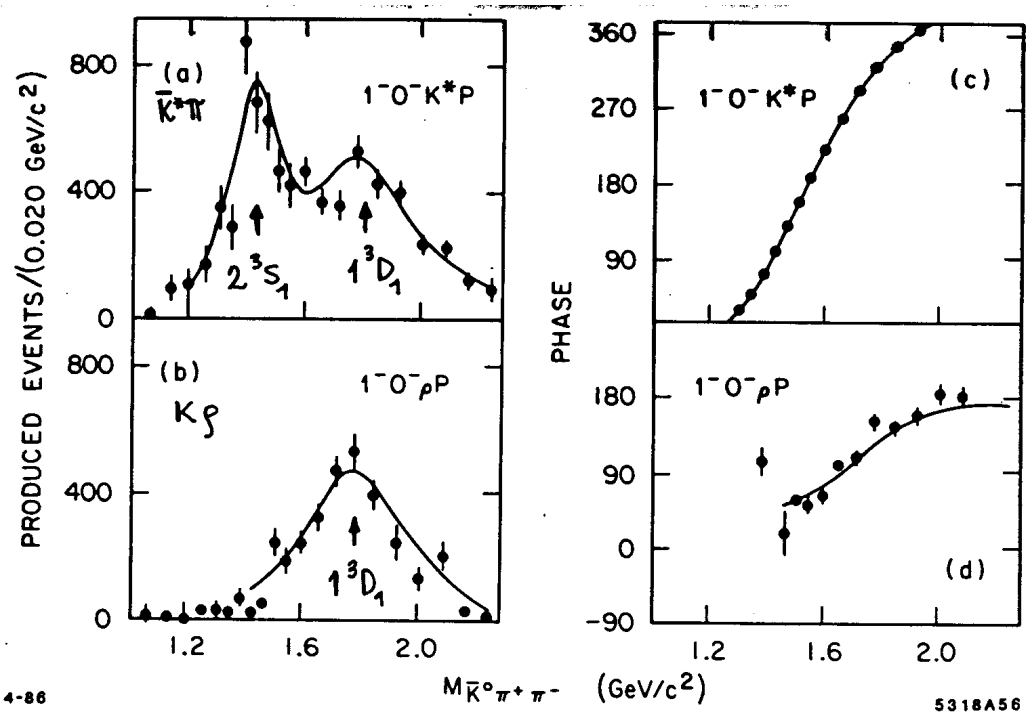
looks simple, but:

a) ~ 2/3 spectrum resonant

b) complicated amplitude structure



Using SLAC - LBL PWA program



M_1 : $1420 \pm 7 \pm 10$ $\quad \Gamma_1$: $240 \pm 18 \pm 12$

from $(K\pi)$ $\quad 1380 \pm 21 \pm 19 \quad 176 \pm 52 \pm 22$

M_2 : $1735 \pm 10 \pm 20 \quad \Gamma_2$: $423 \pm 18 \pm 20$

from $(K\pi)$ $\quad 1677 \pm 10 \pm 32 \quad 205 \pm 16 \pm 34$

Lower mass state: (i) broad t- dependence.
(ii) $\sim 90°$ phase diff w.r.t. $K_2^*(1430)$

$\rightleftharpoons$ weak coupling to $(K\pi)$

If you found $J^{PC} = 0^{++}$ gg, would you know it?

1) What are the quark model expectations?

For isoscalars & $J^{PC} = 0^{++}$:

ground state { $n\bar{n} - s\bar{s}$: ~ 1400 MeV : $f_0(1370)$
$s\bar{s}$: ~ 1500 MeV (?)

first radial excitations { from strange sector, expect ~ 500 MeV higher in mass } — no candidates as yet.

Adding gluonium, expect 5 states below 2 GeV; have $f_0(1370)$ & radial excitations at $M \gtrsim 1.9$ GeV.

— Might have something at ~ 1.5 GeV;
LASS + Bf(1370) => mainly $s\bar{s}$

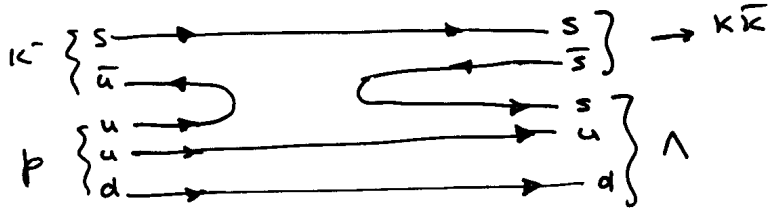
— Crystal Barrel has stray signal at ~1500, but $Bf(K\bar{K})/Bf(\pi\pi)$ small
— prob. not the same state as LASS "state".
— interpret as mainly gluonium

2) What are the properties? Mixing of gluonium with $q\bar{q}$ states may obscure.

Experimental Evidence [non J/ψ]

Fixed Target — Peripheral Production

① $K^- p \to \Lambda K \bar{K}$ (LASS)

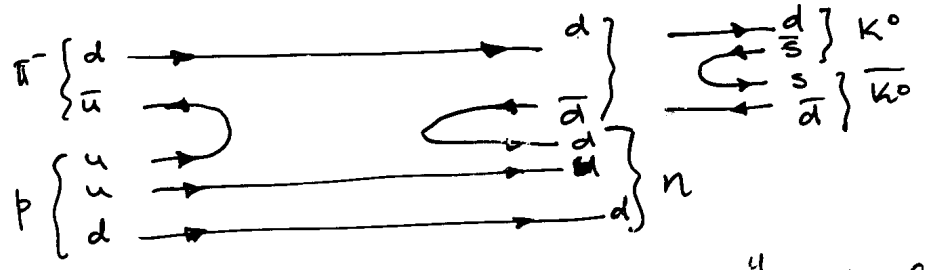


(i) no obvious source of glue

(ii) weak evidence for 0^{++} at ~ 1.5 GeV

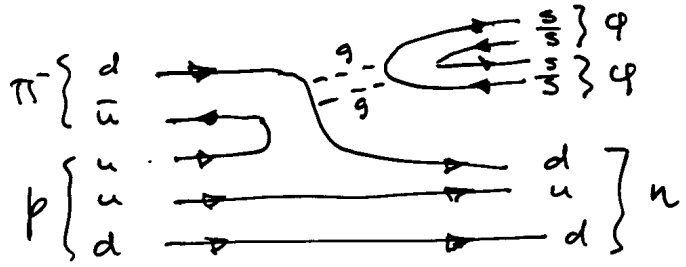
(iii) no evidence for radial excitation at $\gtrsim 1.9$ GeV + nothing in between

② $\pi^- p \to K^0_S K^0_S n$



Lindenbaum et al claimed "glueball" in S wave; amplitude extracted looks like a dog's breakfast! Not believable.

③ $\varphi\varphi$ production (Lindenbaum et al)



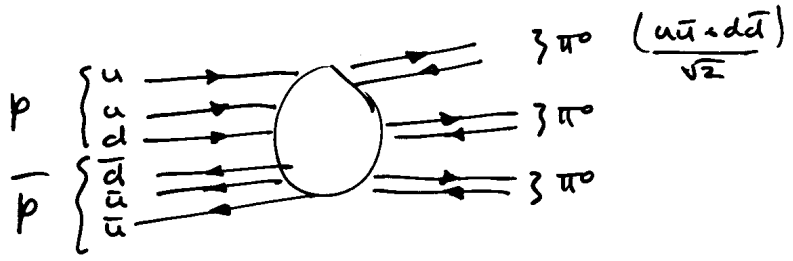
- $\varphi\varphi$ coupled via gg
- PWA finds 3 states with $J^{PC} = 2^{++}$ at $\sim$ 2.01, 2.30, 2.34 GeV
- quark model: expect 2 states in this region viz. $\begin{cases} 3P & 1^{st} \text{ radial excitation} \\ 3F & \text{ground state} \end{cases}$

$\Rightarrow$ perhaps 1 extra state

⊛ OBELIX claims only one state in:

$p\bar{p} \to \varphi\varphi$ cm energy scan in this region.

$\bar{p}p$ Annihilation — Crystal Barrel @ LEAR



— observe $f_0(1500)$ $\left\{\begin{array}{l} M = 1500 \pm 10 \text{ MeV} \\ \Gamma = 112 \pm 10 \text{ MeV} \end{array}\right\}$

$\frac{\Gamma(2\pi)}{\Gamma} \sim 45\%$, $\frac{\Gamma(K\bar{K})}{\Gamma(4\pi)} \sim 20\%$

$\frac{\Gamma(\pi\pi)}{\Gamma(\eta\eta')} \sim 4.5$

— seems real

But:

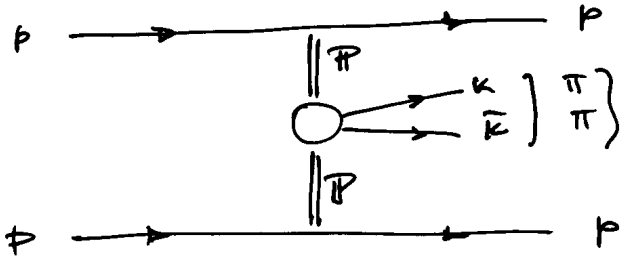
a) no compelling reason to think that $q\bar{q}$ source (same q & $\bar{q}$ in and out)

b) still need $f_2(1565$ to fit Dalitz Plot

$\left\{\begin{array}{l} \text{a) is touted as glue candidate} \\ \text{b) is not touted at all.} \end{array}\right\}$

c) no strong signal in $J/\psi \to \left\{\begin{array}{l} \gamma K\bar{K} \\ \gamma\pi\pi \end{array}\right\}$

Fixed Target, Central Production @ high Incident Beam Energy



Hope is that dominantly Pomeron - Pomeron scattering.

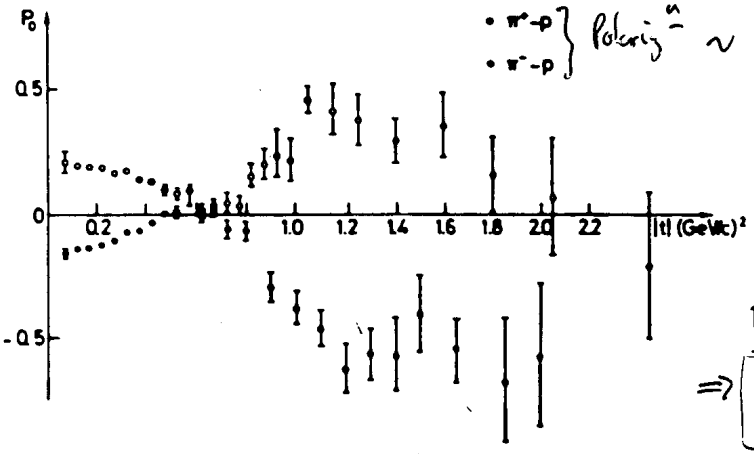
— what is a Pomeron? { $q\bar{q}$ colour singlet? }

⇒ source of gluons
ie provides "gluon rich" initial state.

— is it just the shadow of everything else due to unitarity? ($\mathrm{Im} f^{(0)}_{nf} \sim \sigma_T$)

— is it a Regge trajectory which is ~flat with intercept $(1+\varepsilon)$ contributing dominantly to $\mathrm{Im}\, f_{nf}(0)$ so that $\sigma_T \sim \left(\frac{s}{s_0}\right)^{\varepsilon}$
for pp, $\bar{p}p$, $\pi^{\pm}p$ etc. (ε used to be 0)
Why does it behave as though $J^{PC} = 0^{++}$?

CERN-ORSAY-PISA 6.0 GeV/c



$\left.\begin{matrix}\bullet\ \pi^+-p \\ \bullet\ \pi^--p\end{matrix}\right\}$ Polariz$^{\underline{n}}$ $\sim \mathrm{Im}\left(f_{nf} f_f^*\right)$

$f_{nf} = i\mathbb{P}$

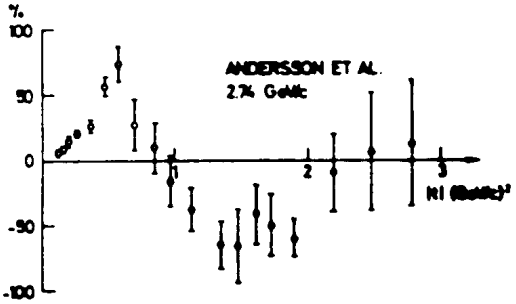
$f_f \cong \rho$

$\sim \sqrt{t}\, i e^{\frac{i\pi\alpha_\rho}{2}} \sin\frac{\pi\alpha_\rho}{2}$

$\pi^\pm p \to \mathbb{P} \pm \rho$

$\Rightarrow \boxed{\mathrm{Pol}^{\underline{n}} \sim \pm\sqrt{t}\, \sin^2\frac{\pi\alpha_\rho}{2}}$

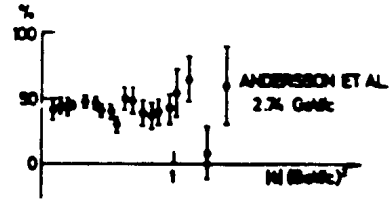
K⁻P POLARIZATION



$-\rho + A_2 \to e^{i\pi\alpha_\rho}$

$\Rightarrow \mathrm{Pol}^{\underline{n}} \sim \sqrt{t}\, \cos\pi\alpha_\rho$

K⁺-P
POLARIZATION

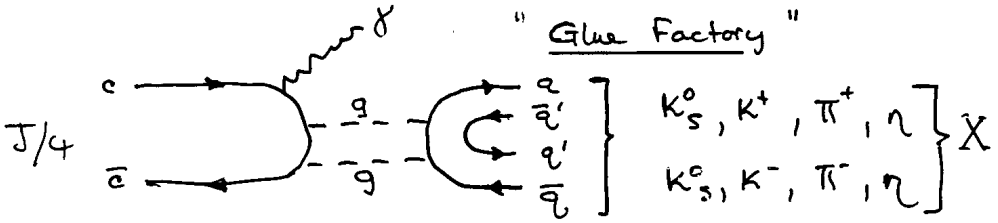


$\rho + A_2 \to 1$

$\mathrm{Pol}^{\underline{n}} \sim \sqrt{t}$

① Introduction

The decay $J/\psi \to \gamma P\bar{P}$ ($P \equiv$ pseudoscalar meson) is considered to proceed as follows:



(i) gg coupling $\Rightarrow$ X should be isoscalar

(ii) $C_X = +1 \Rightarrow (-1)^{J_X} = +1 \Rightarrow J_X$ is even

$\Rightarrow P_X$ is +

i.e. X is $I=0$, $J^{PC} = (\text{even})^{++}$

(iii) for $M_X \lesssim 2\,\text{GeV}$, ignore $J^{PC} = 4^{++}$

i.e. analyze in terms of $J^{PC} = 0^{++}$ and 2^{++}

Generalities for $J/\psi \to \gamma\, P\bar{P}$ ($P \equiv$ pseudoscalar mesons)

(i) $P\bar{P}$ system has $C = +1$ $\{J/\psi \to \gamma \ldots\}$

(ii) $C = (-1)^L \Rightarrow P\bar{P}$ system has even L
ie positive Parity

It follows that if $J/\psi \to \gamma\, X$, $X \to P\bar{P}$

then $\boxed{X \text{ has } J^{PC} = (\text{even})^{++}}$

Isospin ~ 0 $\left\{ \text{eg } \frac{BF(\gamma\pi^0)}{BF(\gamma\eta)} \sim 5\% \right\}$

States considered

{Merging of γ's, Undetected soft γ}

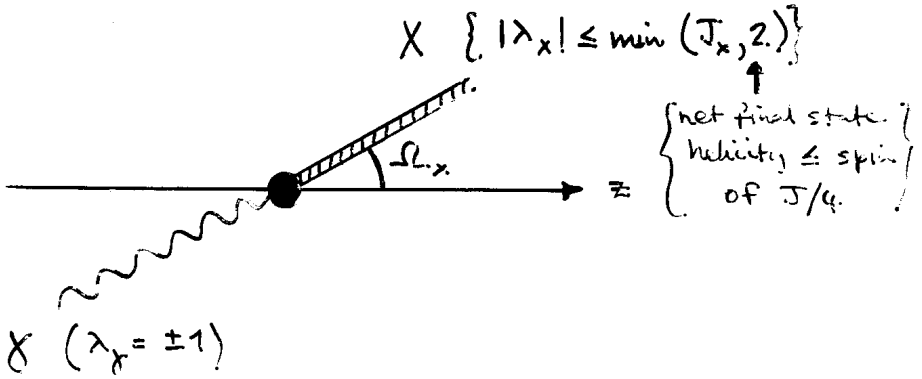
quies feedthrough from:

$\gamma\, K^0_S\, K^0_S$	$\cancel{\pi^0\, K^0_S\, K^0_S}$ $(C=+1)$
$\gamma\, K^+\, K^-$	$\pi^0\, K^+\, K^-$ $(K^*\bar{K} + \bar{K}^* K)$
$\gamma\, \pi^+\, \pi^-$	$\pi^0\, \pi^+\, \pi^-$ $(\rho\pi)$
$\gamma\, \pi^0\, \pi^0$	$\cancel{\pi^0\, \pi^0\, \pi^0}$ $(C=+1)$
$\gamma\, \eta\, \eta$	$\cancel{\pi^0\, \eta\, \eta}$ $(C=+1)$

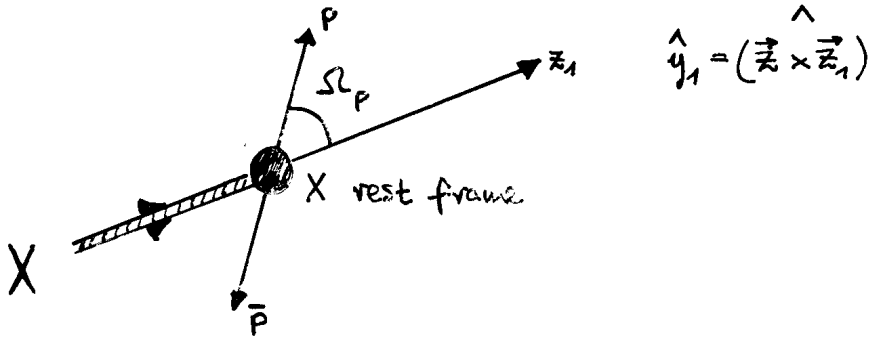
statistics too low for Amp. Analysis

Helicity Formalism

J/ψ helicity = ±1



* After acceptance correction, final state angular dist^{n} does not depend on φ_X

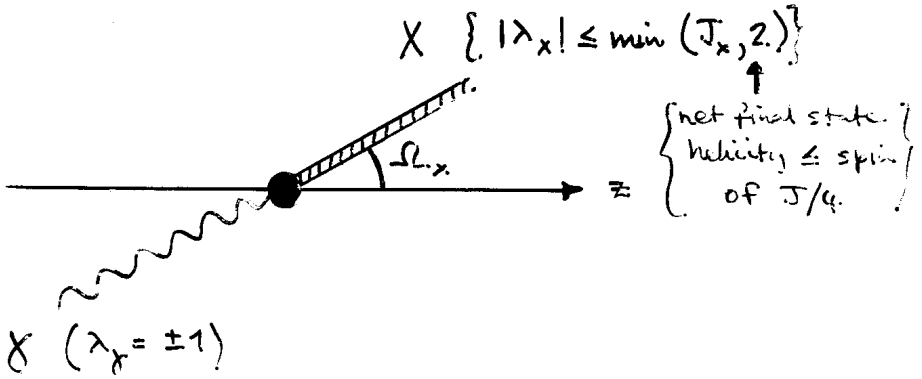


For each mass interval, m_X, the joint decay angular dist^{n} is specified in terms of $\theta_X, \theta_p, \varphi_p$

* For ϕ or ω recoil; need 2 more angles describing recoil decay

Helicity Formalism

J/ψ helicity = ±1



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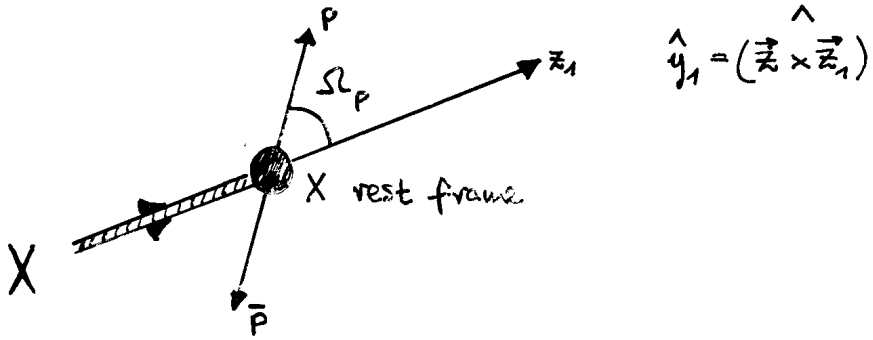


For each mass interval, m_X, the joint decay angular dist^{n} is specified in terms of $\theta_X, \theta_p, \varphi_p$

* For ϕ or ω recoil; need 2 more angles describing recoil decay

The matrix element is given by

$$\mathbf{M} = \underbrace{A^{1}_{\lambda_X,\lambda_\gamma}\sqrt{\frac{3}{4\pi}}\mathbf{D}^{1*}_{\lambda_\psi,\lambda_X-\lambda_\gamma}(\Omega_X)}_{J/\psi\to\gamma X}\underbrace{A^{J_X}_{0,0}\sqrt{\frac{2J_X+1}{4\pi}}\mathbf{D}^{J_X*}_{\lambda_X,0}(\Omega^*_p)}_{X\to P\bar{P}}.$$

and the distribution resulting is

$$4\pi\frac{dN}{dm\,d\Omega_X\,d\Omega^*_p} = \frac{3}{4}\sum_{\lambda_\psi,\lambda_\gamma}\left|\sum_{J_X,\lambda_X} a^{\lambda_\gamma}_{J_X,\lambda_X}\sqrt{\frac{2J_X+1}{4\pi}}\mathbf{D}^{1*}_{\lambda_\psi,\lambda_X-\lambda_\gamma}(\Omega_X)\mathbf{D}^{J_X*}_{\lambda_X,0}(\Omega^*_p)\right|^2$$

with

$$a^{\lambda_\gamma}_{J_X,\lambda_X} = A^{1}_{\lambda_X,\lambda_\gamma}A^{J_X}_{0,0}\sqrt{PQ}\,.$$

Parity conservation implies

$$\underline{a^{-\lambda_\gamma}_{J_X,-\lambda_X} = a^{\lambda_\gamma}_{J_X,\lambda_X}} \quad \Rightarrow \text{not}^{n}\ a_{J,\lambda}$$

The distribution takes the form, in general:

$$4\pi\frac{dN}{dm\,d\Omega_X\,d\Omega^*_p} = \sum_{j,l,m,n}\boxed{\mathbf{T}^{j}_{l,m,n}}\mathbf{Y}_{j,m}(\Omega_X)\mathbf{Y}_{l,n}(\Omega^*_p),$$

and the coeffts. are obtained from

$$T^{j}_{l,m,n} = \int_{\Omega_X,\Omega^*_p}\mathbf{Y}^*_{j,m}(\Omega_X)\mathbf{Y}^*_{l,n}(\Omega^*_p)\frac{4\pi dN}{dm\,d\Omega_X\,d\Omega^*_p}d\Omega_X d\Omega^*_p$$

$$\cong \sum_{i=1}^{All\ Events} 4\pi\mathbf{Y}^*_{j,m}(\Omega^i_X)\mathbf{Y}^*_{l,m}(\Omega^{*i}_p);$$

For this reason the $T^{j}_{l,m,n}$ are called the moments of the angular dist^{n}.

In the present case :

since no dependence on ϕ_X

$$4\pi \frac{i\ dN}{dm\ d\Omega_X\ d\Omega_p^*} = \sum_{j,l,m,n} T^j_{l,m,n} W_{j,m}(\theta_X, \widetilde{\phi_X = 0}) Y^*_{l,n}(\Omega_p^*),$$

with

$$T^j_{l,m,n} = (-1)^m \frac{3}{4} \sum_{\lambda_\psi,\lambda_\gamma} \sum_{J_X,\lambda_X} \sum_{J'_X,\lambda'_X} a^{\lambda_\gamma}_{J_X,\lambda_X} a^{\lambda_\gamma *}_{J'_X,\lambda'_X} \sqrt{\frac{2J_X+1}{2j+1}} \sqrt{\frac{2J'_X+1}{2l+1}}$$

$$\times \langle 1, -\lambda_\psi; 1, \lambda_\psi | j, 0 \rangle \langle 1, -\underline{\lambda_X + \lambda_\gamma}; 1, \underline{\lambda'_X - \lambda_\gamma} | j, \underline{m} \rangle \Rightarrow m = \lambda'_X - \lambda_X$$

$n = \lambda'_X - \lambda_X \Leftarrow$ $\times \langle J_X, \underline{-\lambda_X}; J'_X, \underline{\lambda'_X} | l, \underline{n} \rangle \langle J_X, 0; J'_X, 0 | l, 0 \rangle,$

Since $m = n = \lambda'_X - \lambda_X$, write $T^{j,m}_{l,m} \equiv T^j_{l,m,m}$

Parity conservation $\rightarrow$ $T^{j,m}_{l,m} = T^{j,-m}_{l,-m}$ & choose $\lambda_\gamma = +1$

$$T^{00}_{0,0} = |a_{0,0}|^2 + |a_{2,0}|^2 + |a_{2,1}|^2 + |a_{2,2}|^2$$

$$T^{20}_{0,0} = \frac{\sqrt{5}}{10}[|a_{0,0}|^2 + |a_{2,0}|^2 - 2|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$T^{00}_{2,0} = \frac{\sqrt{5}}{5}[2\sqrt{5}Re(a_{0,0}a^*_{2,0}) + \frac{5}{7}(2|a_{2,0}|^2 + |a_{2,1}|^2 - 2|a_{2,2}|^2)]$$

$$T^{20}_{2,0} = \frac{1}{5}[\sqrt{5}Re(a_{0,0}a^*_{2,0}) + \frac{5}{7}(|a_{2,0}|^2 - |a_{2,1}|^2 - |a_{2,2}|^2)]$$

$$T^{21}_{2,1} = -\frac{\sqrt{3}}{10}[\sqrt{5}Re(a_{0,0}a^*_{2,1}) + \frac{5}{7}(Re(a_{2,0}a^*_{2,1}) - \sqrt{6}Re(a_{2,1}a^*_{2,2}))]$$

$$T^{22}_{2,2} = \frac{\sqrt{6}}{10}[\sqrt{5}Re(a_{0,0}a^*_{2,2}) - \frac{10}{7}(Re(a_{2,0}a^*_{2,2})]$$

$$T^{00}_{4,0} = \frac{1}{7}[6|a_{2,0}|^2 - 4|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$T^{20}_{4,0} = \frac{\sqrt{5}}{70}[6|a_{2,0}|^2 + 8|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$T^{21}_{4,1} = -\frac{\sqrt{3}}{14}[\sqrt{6}Re(a_{2,0}a^*_{2,1}) + Re(a_{2,1}a^*_{2,2})]$$

$$T^{22}_{4,2} = \frac{3\sqrt{2}}{14}Re(a_{2,0}a^*_{2,2}).$$

i.e. have 10 moments in terms of 4 amplitudes

$a_{0,0}$, $a_{2,0}$, $a_{2,1}$, a_{22}. { one may be chosen real $\Rightarrow$ 7 parameters?

The angular distⁿ can then be written.

$$4\pi \frac{dN}{dm\, d\Omega_X\, d\Omega_p^*} = \sum_{j,l,(m\geq 0)} (2-\delta_{m0})\, T_{l,m}^{j,m} Re\left(Y_{j,m}(\theta_X) Y_{l,m}^*(\Omega_p^*)\right), \qquad (m \geq 0)$$

Taking account of the acceptance, A, the Observed Moments are given by:

$$\int_{\Omega_X, \Omega_p^*, \Delta m} Re\left(Y_{j',m'}^*(\theta_X) Y_{l',m'}(\Omega_p^*)\right) \boxed{A(\Omega_X, \Omega_p^*, m)} \frac{4\pi dN}{d\Omega_X\, d\Omega_p^*\, dm} d\Omega_X d\Omega_p^* dm$$

$$= \sum_{j,l,(m\geq 0)} \int_{\Omega_X, \Omega_p^*, \Delta m} Re\left(Y_{j',m'}^*(\theta_X) Y_{l',m'}(\Omega_p^*)\right) \boxed{A(\Omega_X, \Omega_p^*, m)}$$
$$\times (2-\delta_{m0}) \boxed{T_{l,m}^{j,m}} Re\left(Y_{j,m}(\theta_X) Y_{l,m}^*(\Omega_p^*)\right) d\Omega_X d\Omega_p^* dm.$$

The observed moment (L.H.S above) is then.

$$N_{l',m'}^{j',m'} \equiv 4\pi \sum_{i=1}^{N_{observed}} Re(Y_{j',m'}^*(\theta_X^i) Y_{l',m'}(\Omega_p^{*i}));$$

and we have.

$$N_\mu = \sum_{\nu=1}^{10} C_{\mu,\nu} T_\nu, \qquad \text{where} \quad \begin{cases} N_\mu = N_{l',m'}^{j'} \text{(observed)} \\ T_\nu = T_{l,m}^{j} \quad \text{(true.)} \end{cases}$$

with

$$C_{\mu,\nu} = \int_{\Omega_X, \Omega_p^*} Re\left(Y^*(\theta_X) Y(\Omega_p^*)\right)_\mu \boxed{A(\Omega_X, \Omega_p^*, m)} (2-\delta_{m0}) \qquad \text{the Acceptance Matrix.}$$
$$\times Re\left(Y(\theta_X) Y^*(\Omega_p^*)\right)_\nu d\Omega_X d\Omega_p^* dm.$$

$C_{\mu\nu}$ is obtained by Monte Carlo:

$$C_{\mu,\nu} = \frac{16\pi^2}{N_{generated}} \sum_{i=1}^{\boxed{N_{accepted}}} Re\left(Y^*(\theta_X^i) Y(\Omega_p^{*i})\right)_\mu (2-\delta_{m0})\, Re\left(Y(\theta_X^i) Y^*(\Omega_p^{*i})\right)_\nu$$

where the summation is over the accepted M.C. events.

The covariance matrix corresponding to the observed moments is:

$$O_{\mu,\nu} = 16\pi^2 \sum_{i=1}^{N_{observed}} Re\left(Y^*(\theta_X^i)Y(\Omega_p^{*i})\right)_\mu Re\left(Y^*(\theta_X^i)Y(\Omega_p^{*i})\right)_\nu ;$$

The true moments are then obtained from the observed moments by:

$$T_\mu = \sum_{\nu=1}^{10} C^{-1}_{\mu,\nu} N_\nu ,$$

and the corresponding covariance matrix is:

$$V = C^{-1} O (C^{-1})^+$$

The amplitudes are then obtained by minimizing the χ^2 given by:

$$\chi^2 = \sum_{\mu,\nu=1}^{10} \left(\overbrace{\sum_{\sigma=1}^{10} C^{-1}_{\mu\sigma} N_\sigma}^{\text{measured true moments}} - \underbrace{T_\mu(a)}_{\equiv} \right) V^{-1} \left(\overbrace{\sum_{\sigma=1}^{10} C^{-1}_{\nu\sigma} N_\sigma}^{\text{measured true moments}} - \underbrace{T_\nu(a)}_{\equiv} \right)$$

true moments expressed in terms of amplitudes $a_{L,M}$

This is carried out separately in each mass bin using MINUIT (MINOS errors); many random starting points are tried in each bin to ensure that there are no missed solutions.

② Data & Analysis Procedures

{ SLAC Pubs. 5378, 5669 ; SLAC Report 386 }

(i) Perform separate amplitude analyses for:

$J/\psi \to \gamma K^0_S K^0_S$ (582 evts.)

$\to \gamma K^+ K^-$ (1755 evts.)

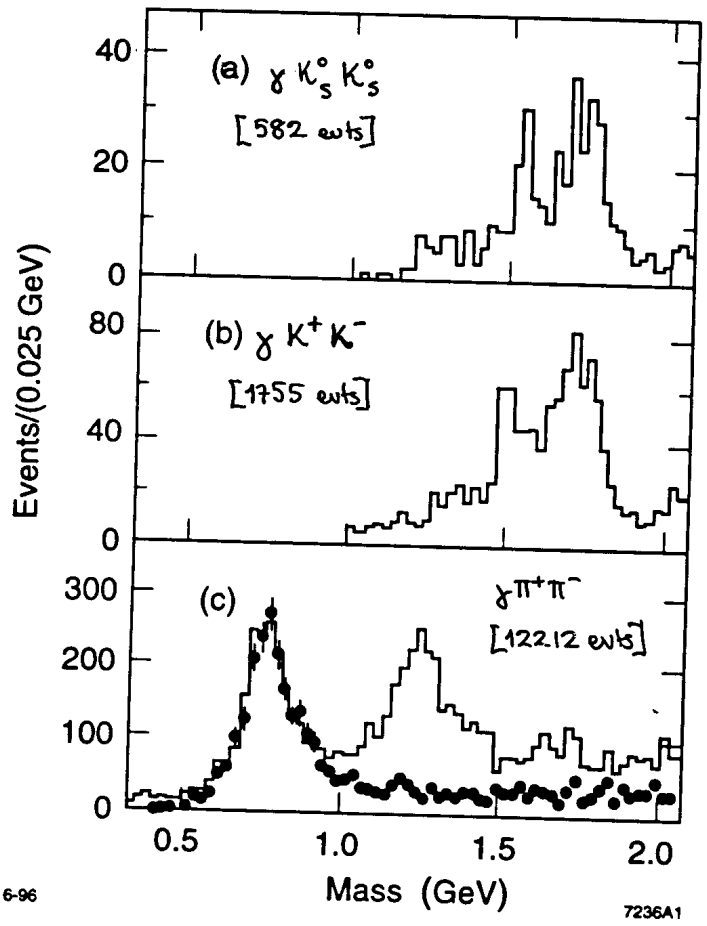
$\to \gamma \pi^+ \pi^-$ (12212 evts)

from 5.8×10^6 J/ψ evts.

(ii) The data are analyzed independently in each mass bin i.e. no continuity assumptions imposed.

(iii) For each bin, the joint production and decay angular distribution is expressed in terms of 3 angles θ_x, θ_p & φ_p.

$J/\psi \rightarrow$



ϕ feed through from $J/\psi \rightarrow \pi^+\pi^-\pi^0$ Monte Carlo {SLAC-PUB 5674}

(iv) The distribution is as follows:

$$\frac{dN}{dM_X \, d\cos\theta_X \, d\Omega_P} =$$

$$\frac{1}{2} \sum_{\substack{j\,\ell \\ (m\geq 0)}} T^{\ell m}_{j\,m}(M_X) * (2-\delta_{m0}) \, \mathrm{Re}\left\{ Y_{\ell m}(\theta_X, \phi_X = 0) \, Y^*_{jm}(\Omega_P) \right\}$$

where the $T^{\ell m}_{jm}$ are the unnormalized moments

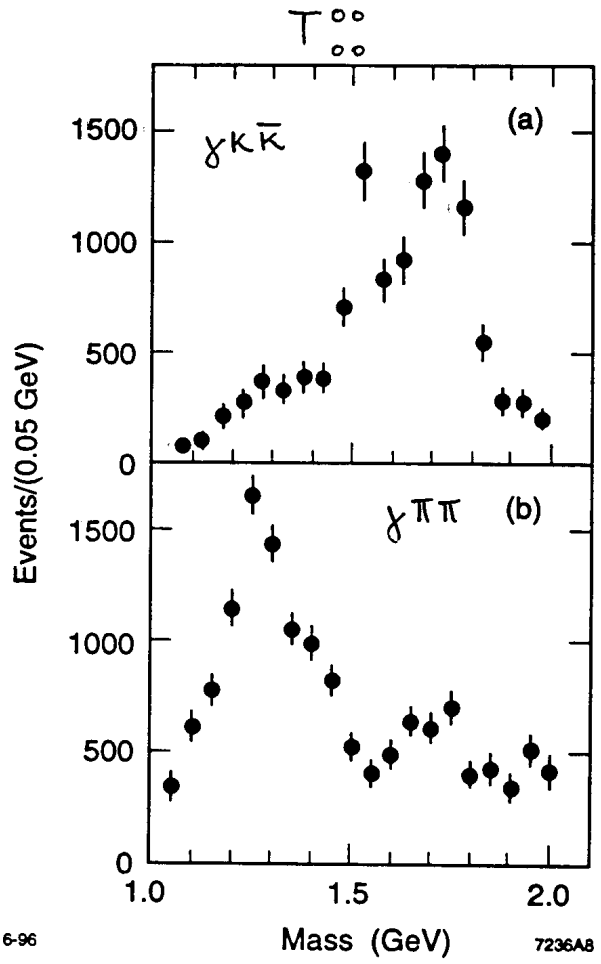
(v) The acceptance-corrected moments & covariance matrix are obtained in each bin in a model-independent way.

For $\gamma\pi^+\pi^-$, feedthrough from $\pi^+\pi^-\pi^0$ must be subtracted.

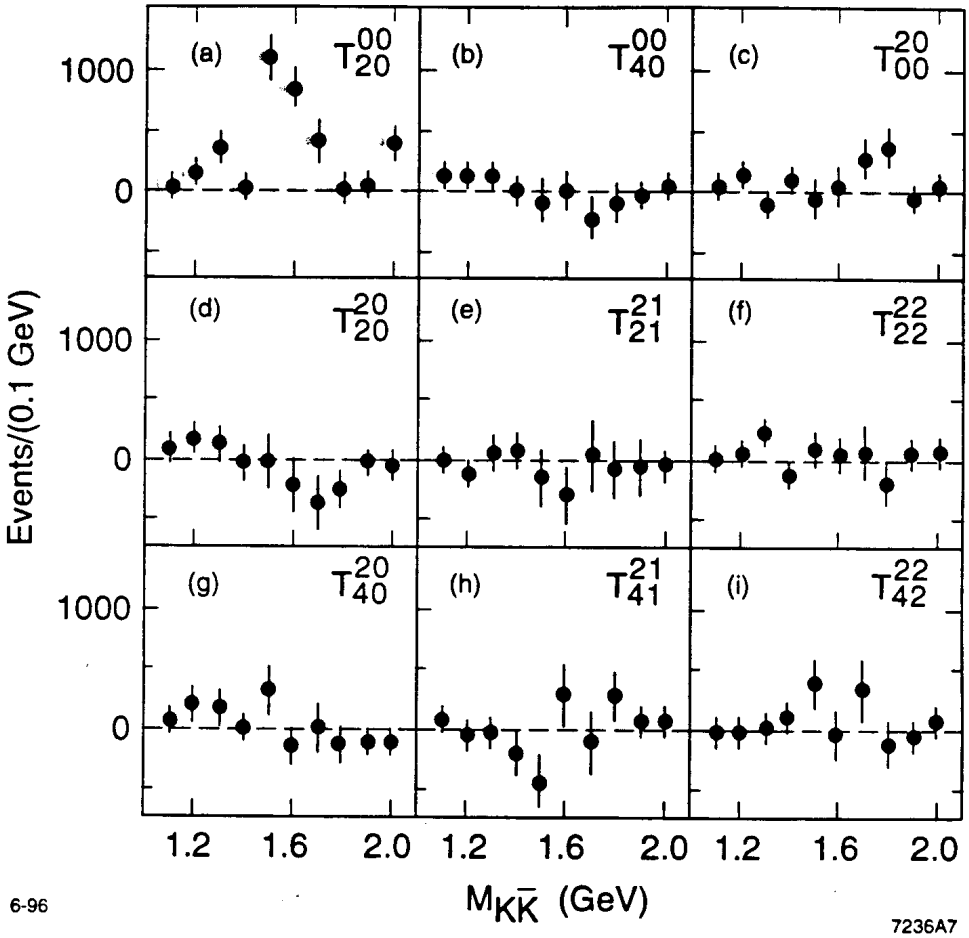
(vi) For $J^{PC} = 0^{++}$ & 2^{++}, 10 moments are obtained in each mass bin. The underlying amplitudes, $a_{j,\lambda}$, where j is spin & λ is helicity, are:

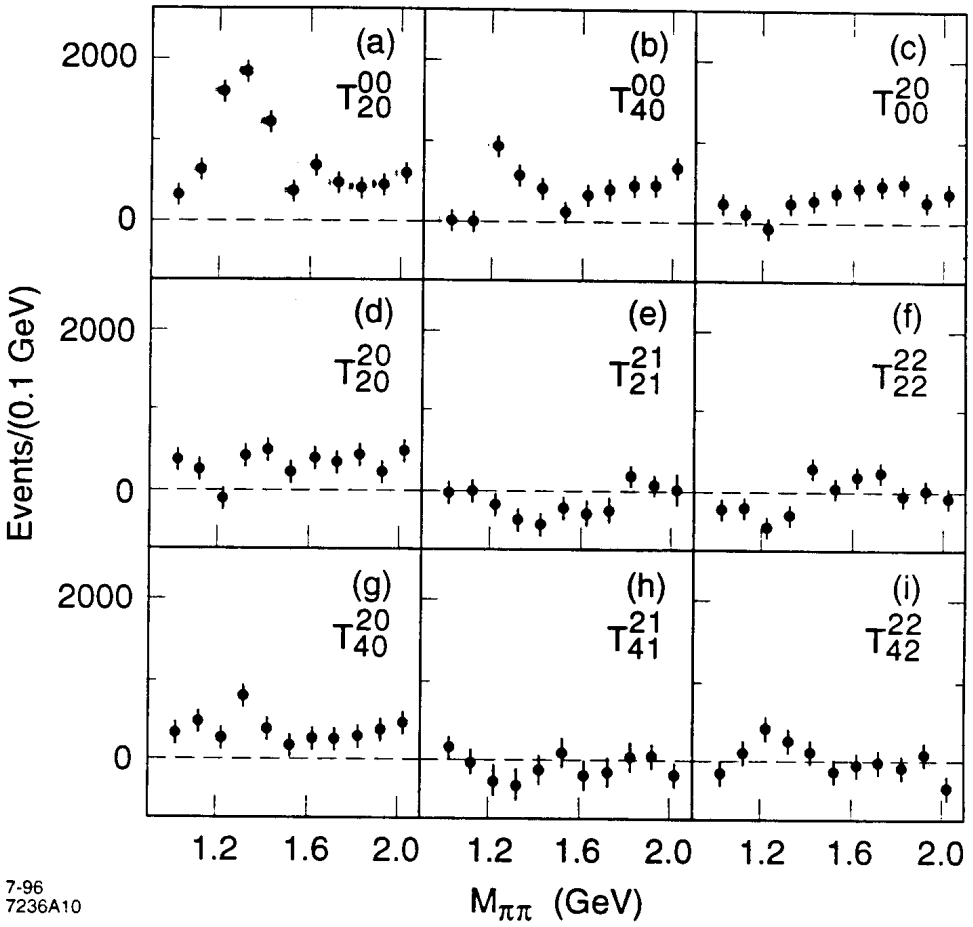
J^{PC}	Amplitudes	#Parameters
0^{++}	$a_{0,0}$	2
2^{++}	$a_{2,0}$ (chosen real)	1
	$a_{2,1}$, $a_{2,2}$	4
		7 in total

(*) Parity: $a_{2,-\lambda} = a_{2,\lambda}$



(*) After all corrections for acceptance & isospin C-G coefffts.


(a) T_{20}^{00}
(b) T_{40}^{00}
(c) T_{00}^{20}
(d) T_{20}^{20}
(e) T_{21}^{21}
(f) T_{22}^{22}
(g) T_{40}^{20}
(h) T_{41}^{21}
(i) T_{42}^{22}
1000
0
Events/(0.1 GeV)
1.2
1.6
2.0
$M_{K\bar{K}}$ (GeV)
6-96
7236A7


(a) T_{20}^{00}
(b) T_{40}^{00}
(c) T_{00}^{20}
(d) T_{20}^{20}
(e) T_{21}^{21}
(f) T_{22}^{22}
(g) T_{40}^{20}
(h) T_{41}^{21}
(i) T_{42}^{22}
2000
0
1.2
1.6
2.0
Events/(0.1 GeV)
$M_{\pi\pi}$ (GeV)

7-96
7236A10

$$T^{0\,0}_{0,0} = |a_{0,0}|^2 + \underline{|a_{2,0}|^2 + |a_{2,1}|^2 + |a_{2,2}|^2}$$

$$T^{2\,0}_{0,0} = \frac{\sqrt{5}}{10}[|a_{0,0}|^2 + |a_{2,0}|^2 - 2|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$T^{0\,0}_{2,0} = \frac{\sqrt{5}}{5}[2\sqrt{5}Re(a_{0,0}a^*_{2,0}) + \frac{5}{7}(2|a_{2,0}|^2 + |a_{2,1}|^2 - 2|a_{2,2}|^2)]$$

$$T^{2\,0}_{2,0} = \frac{1}{5}[\sqrt{5}Re(a_{0,0}a^*_{2,0}) + \frac{5}{7}(|a_{2,0}|^2 - |a_{2,1}|^2 - |a_{2,2}|^2)]$$

$$T^{2\,4}_{2,1} = -\frac{\sqrt{3}}{10}[\sqrt{5}Re(a_{0,0}a^*_{2,1}) + \frac{5}{7}(Re(a_{2,0}a^*_{2,1}) - \sqrt{6}Re(a_{2,1}a^*_{2,2}))]$$

$$T^{2\,2}_{2,2} = \frac{\sqrt{6}}{10}[\sqrt{5}Re(a_{0,0}a^*_{2,2}) - \frac{10}{7}(Re(a_{2,0}a^*_{2,2})]$$

$$T^{0\,0}_{4,0} = \frac{1}{7}[6|a_{2,0}|^2 - 4|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$T^{2\,0}_{4,0} = \frac{\sqrt{5}}{70}[6|a_{2,0}|^2 + 8|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$T^{2\,4}_{4,1} = -\frac{\sqrt{3}}{14}[\sqrt{6}Re(a_{2,0}a^*_{2,1}) + Re(a_{2,1}a^*_{2,2})]$$

$$T^{2\,2}_{4,2} = \frac{3\sqrt{2}}{14}Re(a_{2,0}a^*_{2,2}).$$

(iv) Do we really need $J^{PC} = 0^{++}$?

Angular distribution in $\cos\theta_p$ is not flat.

In 1.7 GeV region, Crystal Ball ($\eta\eta$)

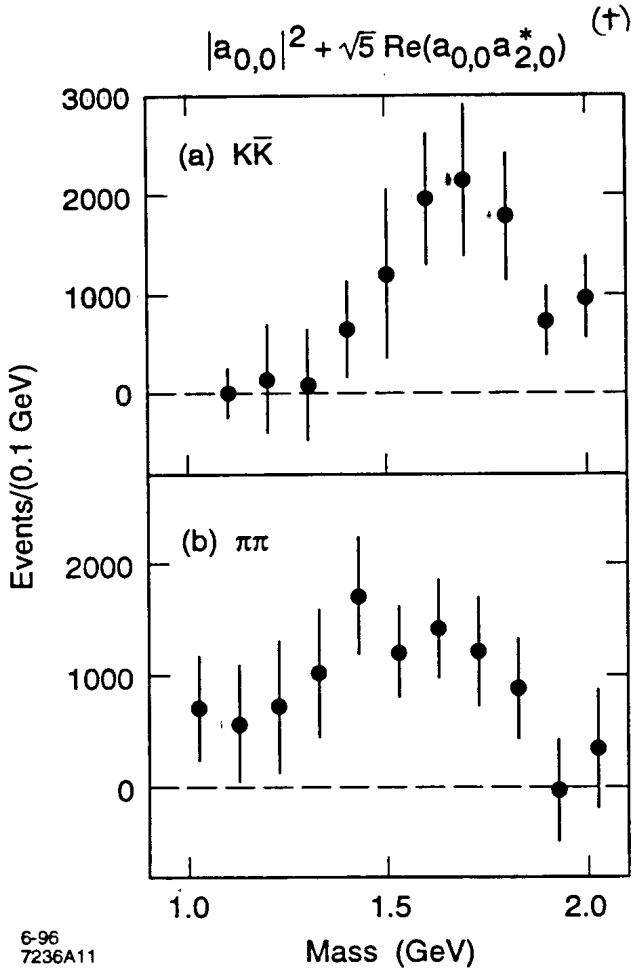
and Mark III asked is $J^{PC} = 0^{++}$ or 2^{++}

$\Rightarrow$ 2^{++} preferred.

Answer: YES

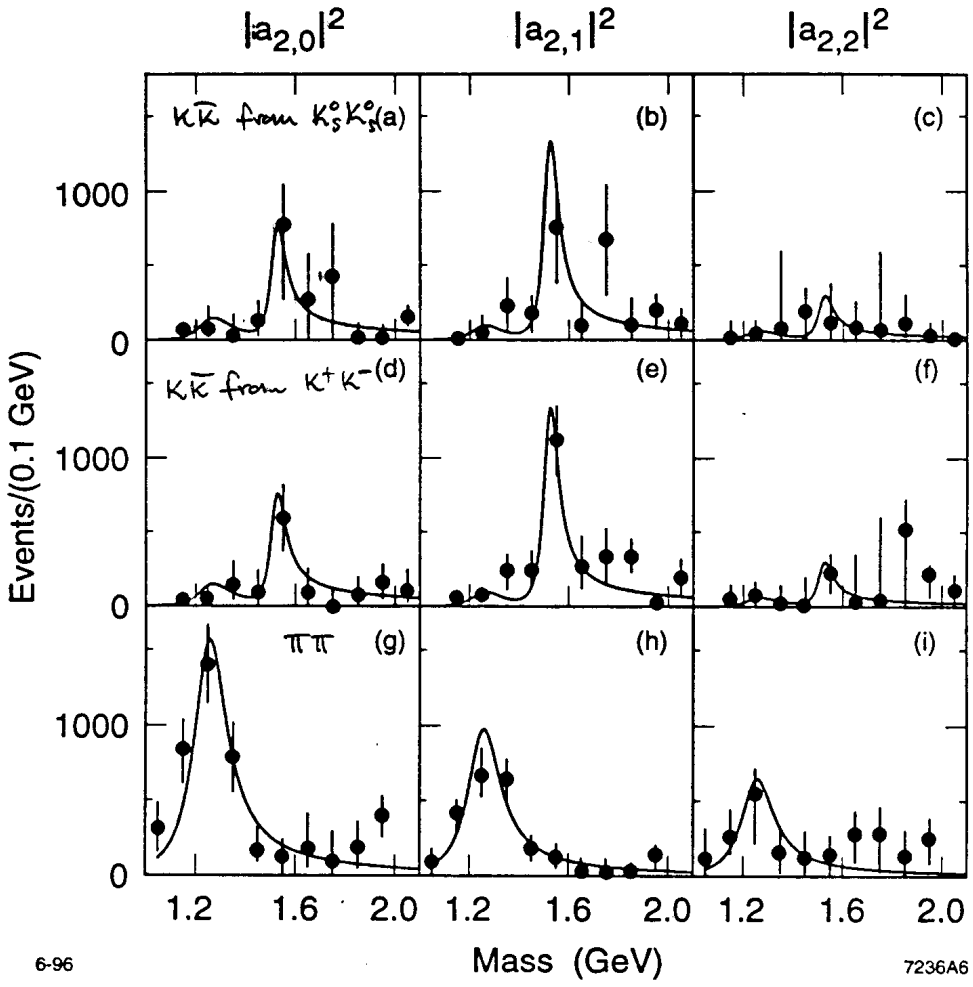
(v) Fitting Procedure

- χ^2 minimization (3 d.of.) in each mass bin
- multiple random starts in the parameter space (~20-30)
- acceptable fit has confidence level $> 1\%$
- errors estimated by MINOS search
- ambiguities: when exist, 2 solutions
  central value is mean
  errors are extrema of errors



(†) $\frac{2}{3}\left(T^{00}_{00} + \sqrt{5}\,T^{40}_{00}\right) + \frac{\sqrt{5}}{3}\left(T^{00}_{20} + \sqrt{5}\,T^{20}_{20}\right)$

$- \frac{4}{3}\left(T^{00}_{40} + \sqrt{5}\,T^{20}_{40}\right)$


$|a_{2,0}|^2$
$|a_{2,1}|^2$
$|a_{2,2}|^2$
$K\bar{K}$ from $K^0_S K^0_S$(a)
(b)
(c)
$K\bar{K}$ from K^+K^-(d)
(e)
(f)
$\pi\pi$ (g)
(h)
(i)
1000
0
Events/(0.1 GeV)
1.2
1.6
2.0
Mass (GeV)
6-96
7236A6

③ Results

$J^{PC} = 2^{++}$

(i) $K\bar{K}$: $f_2'(1525)$

(ii) $\pi\pi$: $f_2(1270)$

(iii) No clear signal at 1.7 GeV ; cannot rule out possibility of small signal

(iv) $\pi\pi$ seems to increase slowly toward 2 GeV ; possible 2^{++} state just above 2 GeV ?

(v) fits :

$K\bar{K}$: $f_2'(1525) + f_2(1270)$ [PDG values]

$\pi\pi$: $f_2(1270)$ [$f_2'(1525)$ rejected when included]

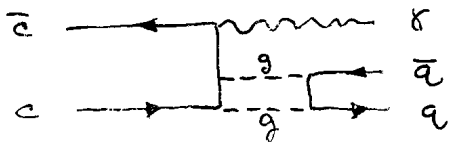
(vi) Are the results sensible ?

Branching Ratio	Value from fits	PDG Value
$\frac{f_2(1270) \to K\bar{K}}{f_2(1270) \to \pi\pi}$	$0.155^{+0.044}_{-0.041}$	0.054 ± 0.006
$\frac{f_2'(1525) \to \pi\pi}{f_2'(1525) \to K\bar{K}}$	~ 0	0.009 ± 0.002
$\frac{f_2'(1525) \to \eta\eta}{f_2'(1525) \to K\bar{K}}$	$0.215^{+0.181}_{-0.159}$	0.116 ± 0.004

Helicity amplitude ratio squared		$f_2(1270) \to \pi\pi$	$f_2'(1525) \to K\bar{K}$
$\left\|\frac{a_{21}}{a_{20}}\right\|^2$	measured	$0.58^{+0.12}_{-0.10}$	$1.66^{+1.10}_{-0.57}$
	predicted(*)	0.58 [0.5]	0.77 [0.8]
$\left\|\frac{a_{22}}{a_{20}}\right\|^2$	measured	$0.39^{+0.11}_{-0.11}$	$0.28^{+0.36}_{-0.21}$
	predicted(*)	0.29 [0.3]	0.49 [0.52]

previous Mk.III & DM2 analyses → ~0

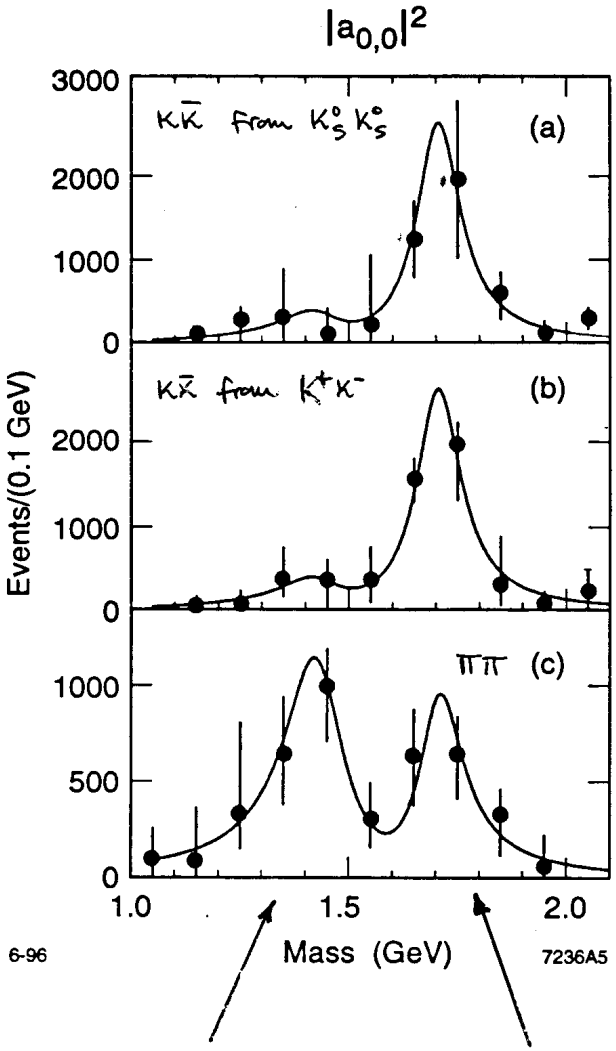
* Krammer, PL 74B (1978) 361 {imaginary}
[.] Körner et al., NPB 229 (1983) 115 {total}



(vii) J/ψ Branching Fractions

$$Bf\left(J/\psi \to \gamma f_2(1270)\right) = 15.6 \pm 1.4 \quad * 10^{-4}$$
(PDG: $13.8 \pm 1.4 \quad * 10^{-4}$)

$$Bf\left(J/\psi \to \gamma f_2'(1525)\right) = 4.0^{+0.8}_{-0.7} \quad * 10^{-4}$$
(PDG: $6.3 \pm 1.0 \quad * 10^{-4}$)



M (MeV) : 1429^{+43}_{-37} 1704^{+16}_{-23}

Γ (MeV) : 169^{+111}_{-46} 124^{+52}_{-44}

$J^{PC} = 0^{++}$

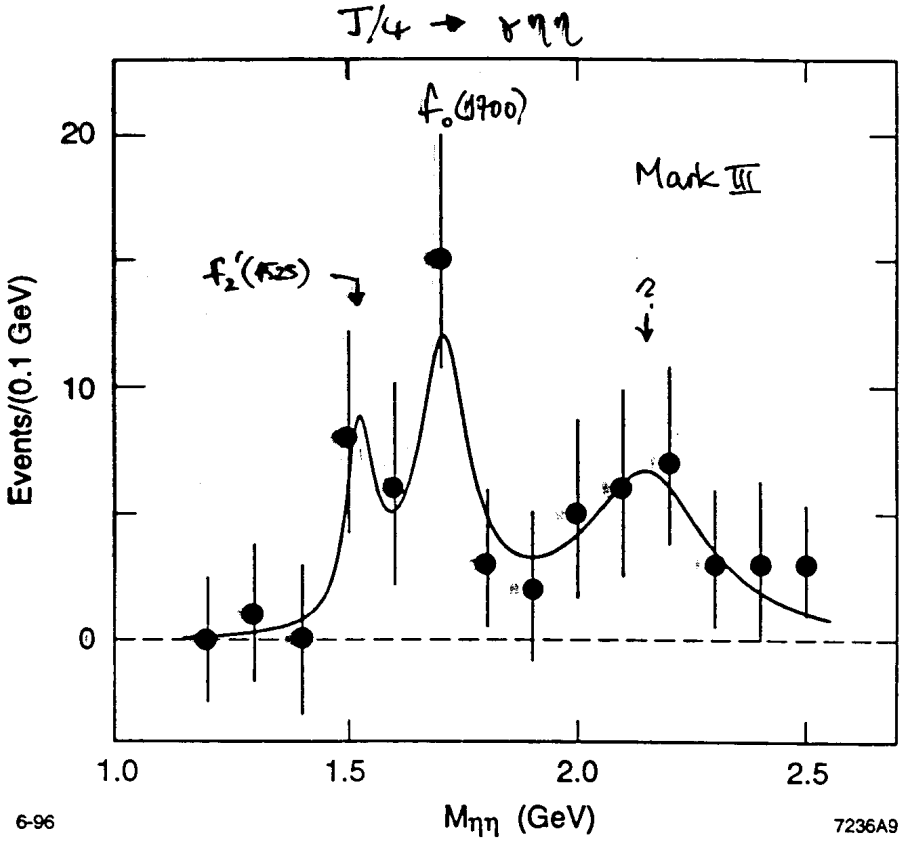
$K\bar{K}$: peak at ~ 1.7 GeV ; small shoulder at ~ 1.4 GeV

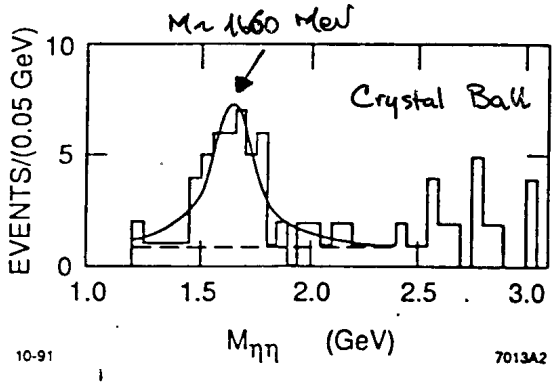
$\pi\pi$: peak at ~ 1.4 GeV ; smaller peak at ~ 1.7 GeV

Fit the 3 distrs. simultaneously with 2 coherent BW's [fitted rel. phase $\sim 0°$] $\left\{ \frac{\chi^2}{ndof} = \frac{6.5}{21} \right\}$

Results:

Measurement	$f_0(1430)$	$f_0(1700)$
Mass (MeV)	1429^{+43}_{-37}	1704^{+16}_{-23}
Width (MeV)	169^{+111}_{-76}	124^{+52}_{-44}
Branching Ratios	$\frac{K\bar{K}}{\pi\pi} : 0.149^{+0.215}_{-0.133}$	$\frac{\pi\pi}{K\bar{K}} : 0.268^{+0.169}_{-0.120}$ $\frac{\eta\eta}{K\bar{K}} : 0.194^{+0.094}_{-0.097}$
Branching Fraction $J/\psi \to \gamma R$: $R \to \pi\pi + K\bar{K}$	$4.3^{+2.7}_{-1.7} * 10^{-4}$	$9.5^{+2.5}_{-2.0} * 10^{-4}$
$J/\psi \to \gamma R$: $R \to \pi\pi + K\bar{K} + \eta\eta$	$4.3^{+2.7}_{-1.7} * 10^{-4}$	$10.9^{+2.6}_{-2.1} * 10^{-4}$


J/ψ → γηη
f₀(1700)
Mark III
f₂'(1525)
?
Events/(0.1 GeV)
20
10
0
1.0
1.5
2.0
2.5
Mηη (GeV)
6-96
7236A9


M≈ 1660 MeV
Crystal Ball
EVENTS/(0.05 GeV)
10
5
0
1.0
1.5
2.0
2.5
3.0
Mηη (GeV)
10-91
7013A2

Summary

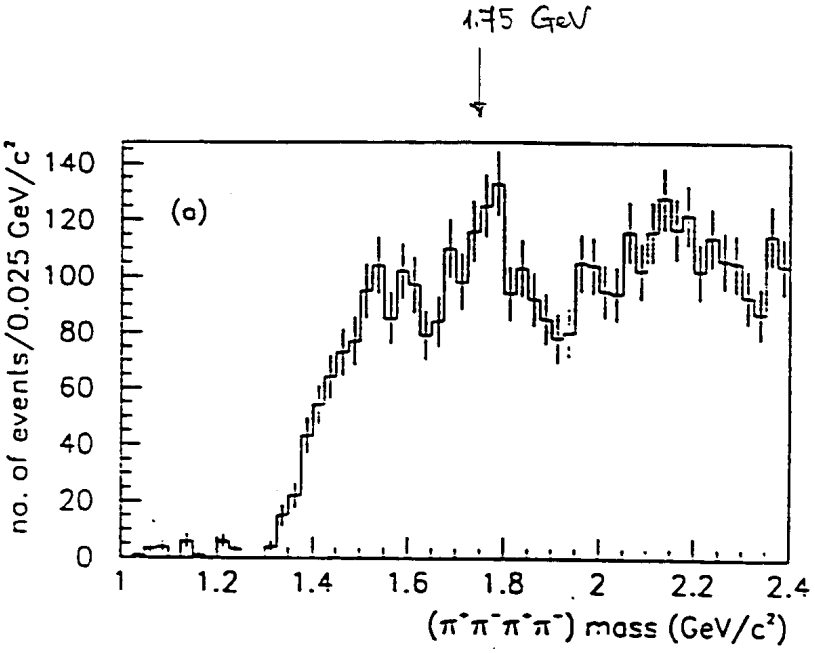
Final State (X)	Bf $\{J/\psi \to \gamma X\}$
$f_2(1270)$	$15.6 \pm 1.4 \quad * 10^{-4}$ $\langle \|D\|^2 \rangle = 5.2 \pm 0.5 \quad * 10^{-4}$
$f_2'(1525)$	$4.0 \,^{+0.8}_{-0.7} \quad * 10^{-4}$ $\langle \|D\|^2 \rangle = 1.3 \pm 0.3 \quad * 10^{-4}$
$f_0(1430) \to p\bar{p}$	$4.3 \,^{+2.7}_{-1.7} \quad * 10^{-4}$
$f_0(1700) \to p\bar{p}$	$10.9 \,^{+2.6}_{-2.1} \quad * 10^{-4}$
$f_0(1750) \to 4\pi$ (*)	$10.9 \pm 1.3 \quad * 10^{-4}$

$f_2(1270)$, $f_2'(1525)$, $a_2(1320)$ & $K_2^*(1430)$ form the ~ ideally mixed ground state $J^{PC} = 2^{++}$ nonet.

What are the $f_0(1430)$ & $f_0(1700)$?

(*) Bugg et al PLB 353 (1995) 378

$J/\psi \to \gamma \pi^+\pi^-\pi^+\pi^-$ [Mark III]



$M(4\pi)$ GeV/c^2

⑤ Summary

In $J/\psi \to \gamma p\bar{p}$: { The results obtained separately from $K^0_S K^0_S$ & K^+K^- agree well }

(i) The $f_2(1270)$ & $f_2'(1525)$ have masses, widths & branching ratios consistent with PDG values (including $\eta\eta$)

(ii) The relative magnitudes of the $f_2(1270)$ and $f_2'(1525)$ helicity amplitudes agree with predictions (Krammer)

(iii) Two $J^{PC} = 0^{++}$ states are observed as follows:

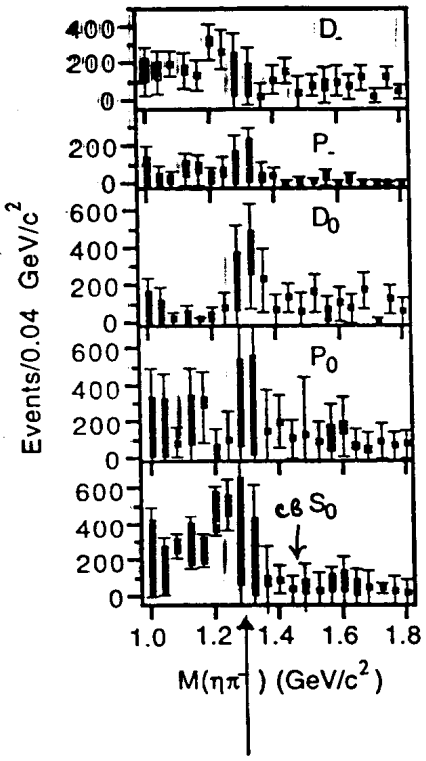
	$f_0(1430)$	$f_0(1700)$
Mass:	$1429\ ^{+43}_{-37}$	$1704\ ^{+16}_{-23}$
Width:	$169\ ^{+111}_{-76}$	$124\ ^{+52}_{-44}$
Br :	$\frac{K\bar{K}}{\pi\pi} = 0.15\ ^{+0.22}_{-0.13}$	$\frac{\pi\pi}{K\bar{K}} = 0.27\ ^{+0.12}_{-0.12}$
		$\frac{\eta\eta}{K\bar{K}} = 0.19 \pm 0.10$

(iv)

X	$BF[J/\psi \to \gamma X] * 10^4$	
$f_2(1270)$	15.6 ± 1.4	; $\langle \lvert D \rvert^2 \rangle \to 5.2 \pm 0.5$
$f_2'(1525)$	$4.0\ ^{+0.8}_{-0.7}$	; $\langle \lvert D \rvert^2 \rangle \to 1.3 \pm 0.3$
$f_0(1430) \to p\bar{p}$	$4.3\ ^{+2.7}_{-1.9}$	
$f_0(1700) \to p\bar{p}$	$10.9\ ^{+2.6}_{-2.1}$	⊛ No evidence for Crystal Barrel $f_0(1500)$
$+4\pi$	$21.8\ ^{+3.0}_{-2.5}$	

$\pi^- p \to \pi^- \eta p$ @ 18 GeV/c

[BNL 852]



Is this due to resonant behaviour at ~ 1.3 GeV ?

[cf. $K\pi$ S-wave]

Interpretation

1) The $J^{PC}=0^{++}$ $f_0(1700)$ is composed mainly of the lowest mass gluonium state; it contains a significant amount of $u\bar{u}$ and $d\bar{d}$, but very little $s\bar{s}$.

2) The $J^{PC}=0^{++}$ $f_0(1430)$ corresponds mainly to the $u\bar{u}/d\bar{d}$ member of the ground state $^3P_0$ $q\bar{q}$ nonet; it contains some $s\bar{s}$ & a significant admixture of the gluonium ground state.

3) The ground state $J^{PC}=0^{++}$ nonet is made up as follows: {mixing angle θ_0}

$$K^*_0(1410),\ f_0'(1520)\ (\text{MASS etc}),\ \left\{\begin{matrix} a_0(1450)\ ? \\ \kappa_0(1300)\ ? \end{matrix}\right\},\ f_0(1430)$$

$a_0(1450)$: $\tan^2\theta_0 > 0 \Rightarrow m[f_0(1430)] < 1400$ MeV
ideal mixing $\Rightarrow \sim 1120$ MeV

$\kappa_0(1300)$: $\tan^2\theta_0 > 0 \Rightarrow m[f_0(1430)] < 1457$ MeV
ideal mixing $\Rightarrow$ 1290 MeV

In both cases, suggest that mixing of $q\bar{q}$ f_0 state with gluonium increases the observed mass

Clearly, it would be of help to resolve the question of the mass of the a_0

4) Amsler & Close have proposed that the $f_0(1500)$ found by Crystal Barrel might be composed mainly of the ground state glueball. However, it does not appear to be produced strongly in J/ψ radiative decay to $p\bar{p}$. Also, its small $K\bar{K}/\pi\pi$ b.r. would seem to argue against its being the mainly $s\bar{s}$ quarkonium state.

One possibility is that it is neither of the above, but is e.g. a $4q$ quark state. Not very satisfactory.

5) What does Lattice Gauge Theory have to say?

Weingarten et al

Use the GF11 Parallel Computer at IBM Research, N.Y.

Obtain:

① $m_G = 1710 \pm 63$ MeV

{ UKQCD – Wuppertal: $m_G = 1625 \pm 92$ MeV }

$\Rightarrow \langle m_G \rangle = 1693 \pm 60$ MeV

② $\Gamma_G \{\Sigma' P \bar{P}\} = 108 \pm 29$ MeV

Using $\dfrac{\Gamma_G \{\Sigma' P \bar{P}\}}{\Gamma_G(\text{tot})} \sim 1/2$

$\Rightarrow \Gamma_G(\text{tot}) \cong 220 \pm 60$ MeV

③ $\Gamma(\pi\pi) : \Gamma(K\bar{K}) = 10^{+25}_{-9}$ %

$\Gamma(\eta\eta) : \Gamma(K\bar{K}) = 30 \pm 10$ %

$\Gamma(\eta\eta') : \Gamma(K\bar{K}) = 8 \pm 5$ %

④ $m(s\bar{s})$ quarkonium $\sim$ 200 MeV lower in mass than m_G

{ Amsler & Close suggest $f_0(1710)$ is mainly $s\bar{s}$ quarkonium }

(5) "To leading order in the valence approx^{n}, with valence quark annihilation turned off, corresponding isotriplet and isosinglet states composed of u and d quarks will be degenerate"
{ Weingarten : NPB(Proc. Supp.) 53 (1997) 232 }

J^{PC}	I=1	I=0
0^{++}	$a_0(1480)$	$f_0(1430)$
1^{++}	$a_1(1260)$	$f_1(1285)$
1^{+-}	$b_1(1235)$	$h_1(1170)$
1^{--}	$\rho(770)$	$\omega(782)$
2^{++}	$a_2(1320)$	$f_2(1275)$
3^{--}	$\rho_3(1690)$	$\omega_3(1670)$
4^{++}	$a_4(2040)$	$f_4(2050)$

(6) In this same refer., a glueball – quarkonium mixing scheme is proposed ; using observed masses for $|n\bar{n}\rangle$, $|s\bar{s}\rangle$ & $|gg\rangle$ (main components) of 1390, 1500 & 1710 MeV W. gets :

$$m_G = 1635 \text{ MeV}, \quad m_{s\bar{s}} = 1516 \text{ MeV}$$

$$|1710\rangle = 0.87\,|gg\rangle + 0.34\,|s\bar{s}\rangle + 0.36\,|n\bar{n}\rangle$$
$$|1500\rangle = -0.19\,|gg\rangle + 0.90\,|s\bar{s}\rangle - 0.40\,|n\bar{n}\rangle$$
$$|1390\rangle = -0.46\,|gg\rangle + 0.28\,|s\bar{s}\rangle + 0.84\,|n\bar{n}\rangle$$

Then :

a) to the extent that J/ψ radiative decay projects $|gg\rangle$, the prediction would be

$f_0(1710) : f_0(1390) : f_0(1500) =$

$4 : 1 : 0.2$ (o.k.)

b) $f_0(1500) \to K\bar{K}$ is suppressed because of destructive intfce. between $|s\bar{s}\rangle$ and $|n\bar{n}\rangle$ (?)

c) $f_0(1710)$ is mostly $|gg\rangle$

$f_0(1500)$ is mostly $|s\bar{s}\rangle$

$f_0(1390)$ is mostly $|n\bar{n}\rangle$

seems to be ~ consistent with data from many different sources

Why is the TCF necessary?

Statistics .....

① To study $J/\psi \to \gamma \pi\pi$, $K\bar{K}$
need $\gtrsim 10^8$ evts. in sample
[Weijun: would take ~6yrs of BES]

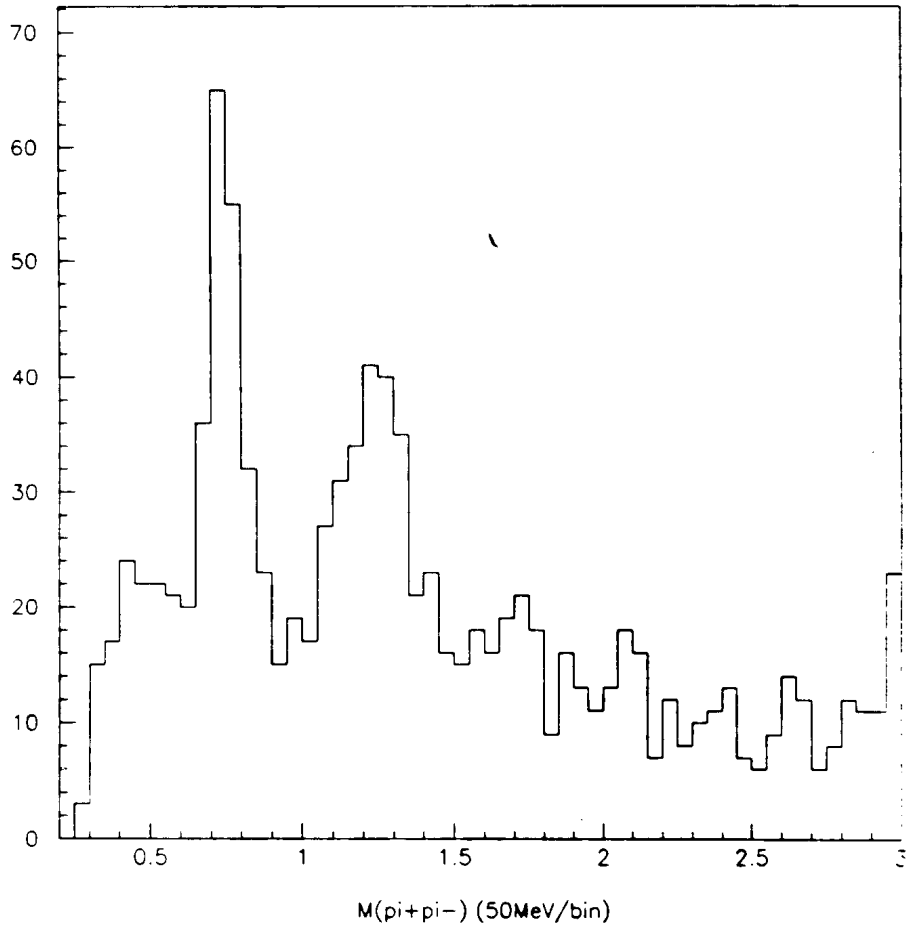
② To study $J/\psi \to \gamma\eta$, $\eta\eta'$,
need at least 10^9 evts

③ Such samples make same studies with recoil ϕ or ω possible
(2 more angles — analysis in 5 dim. for each mass bin; but ω has bkgnd problems)

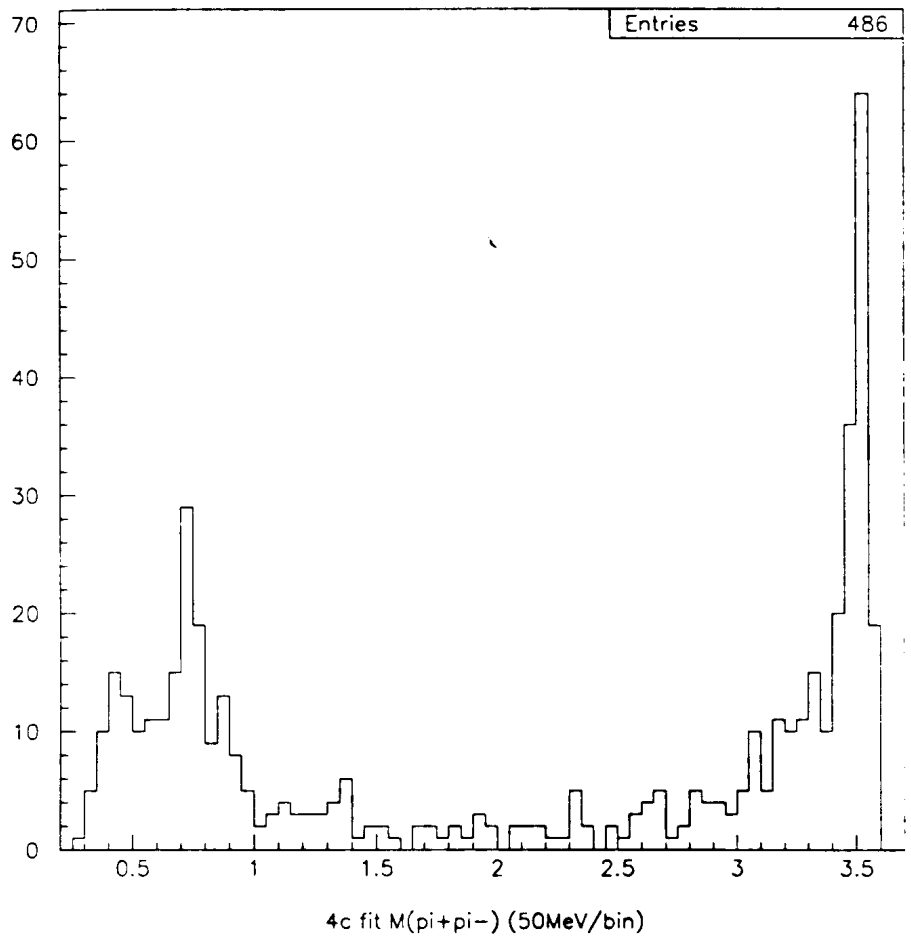
④ Can extend analyses above 2GeV & include $J^{PC} = 4^{++}$ if necessary.

⑤ Can perform sim. analyses with ψ';
with ϕ, ω recoil, go up to ~2.5 GeV.

⑥ ψ' previews.

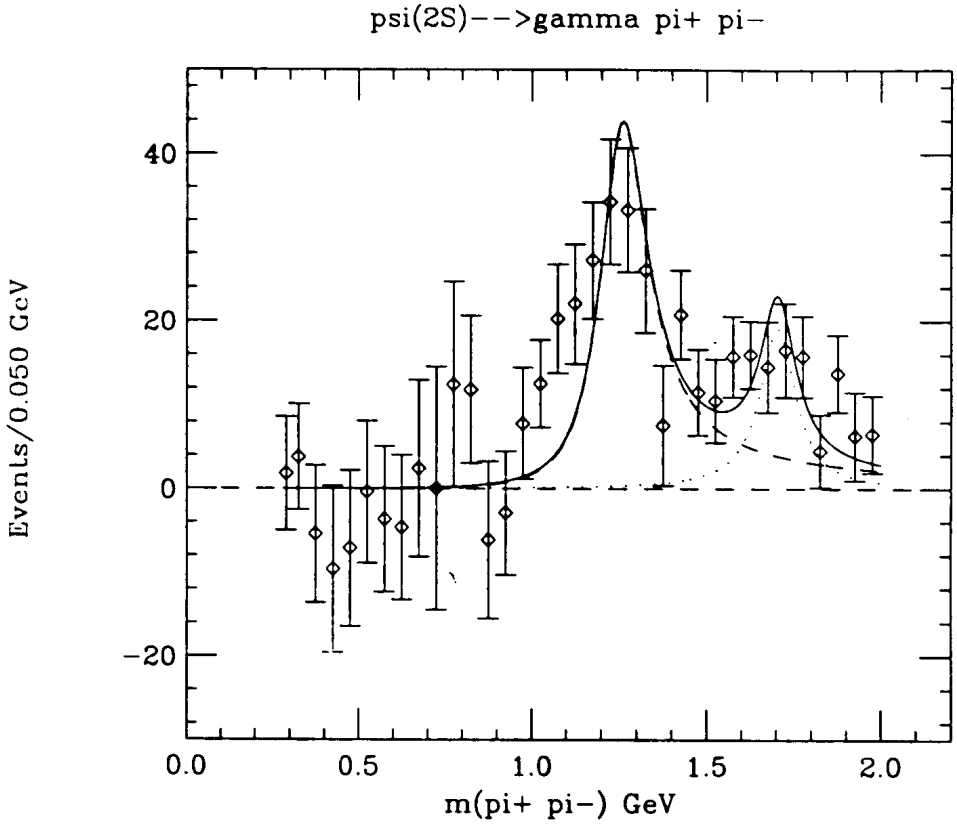


Final $m(\pi^+\pi^-)$ distn from $\psi' \to J/\psi \pi^+\pi^-$



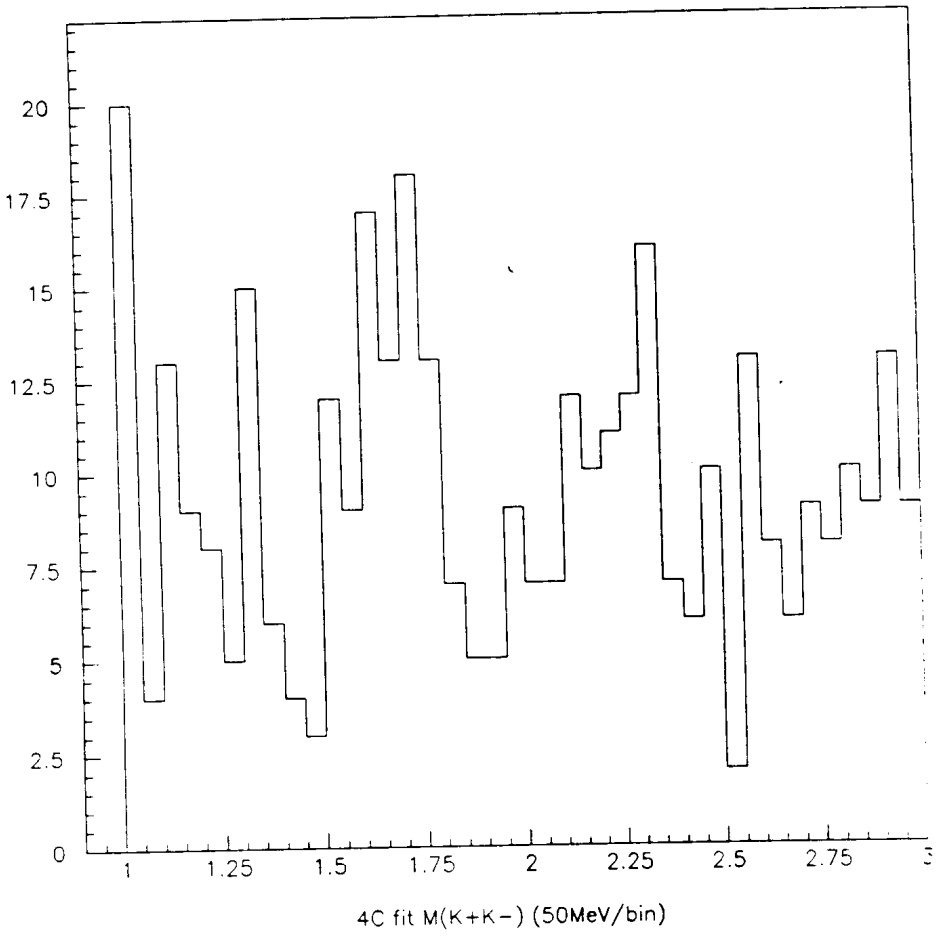
$M(\pi^+\pi^-)$

$\gamma\pi^+\pi^-$ from τ threshold data sample



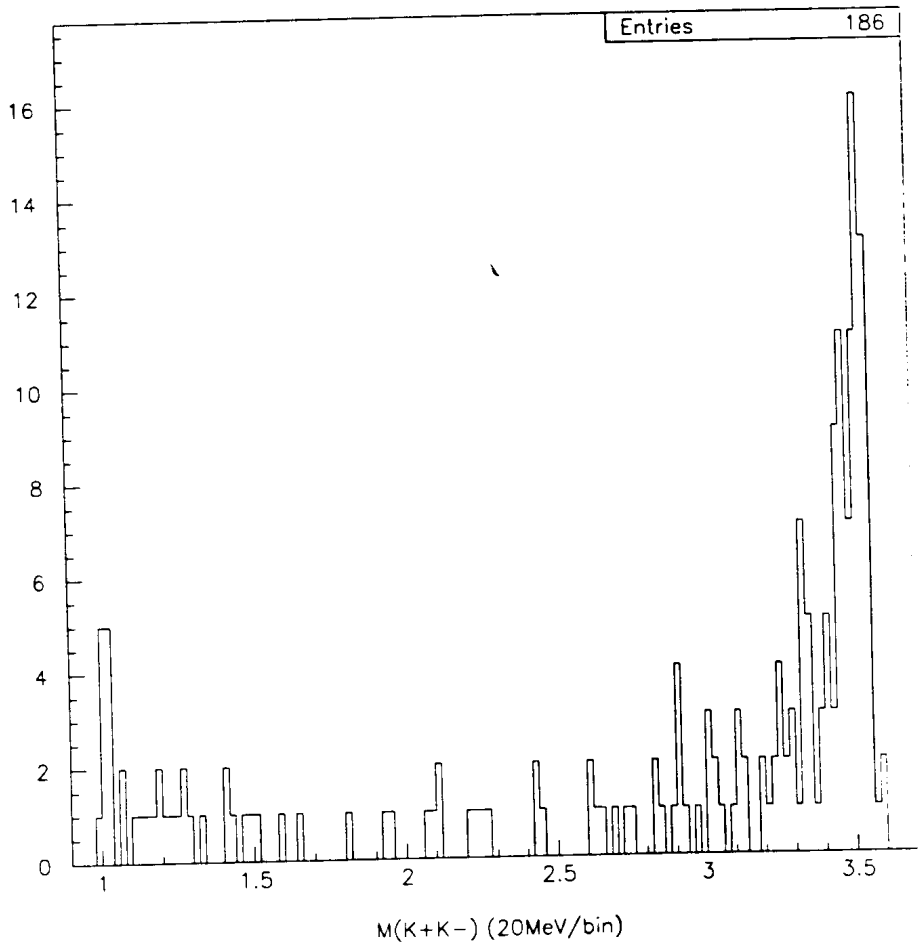
{ψ' − τ scaled threshold} data

~ 50 K evts wt ⇒ ~ 10^9 ψ'

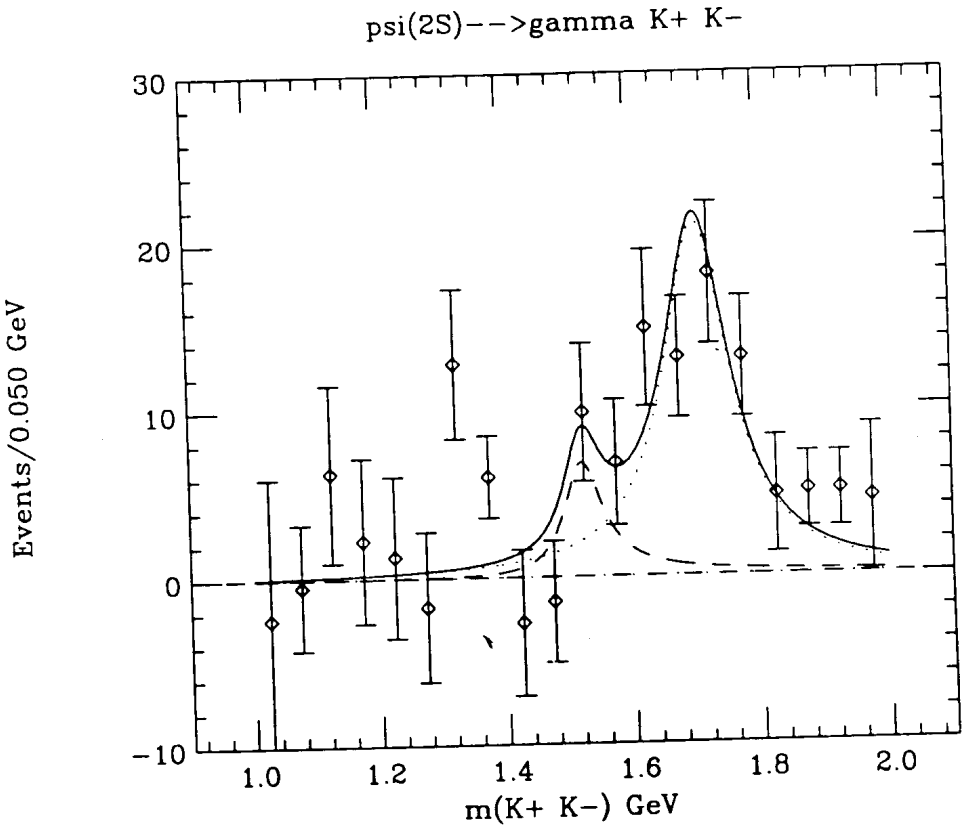


$M(K^+K^-)$

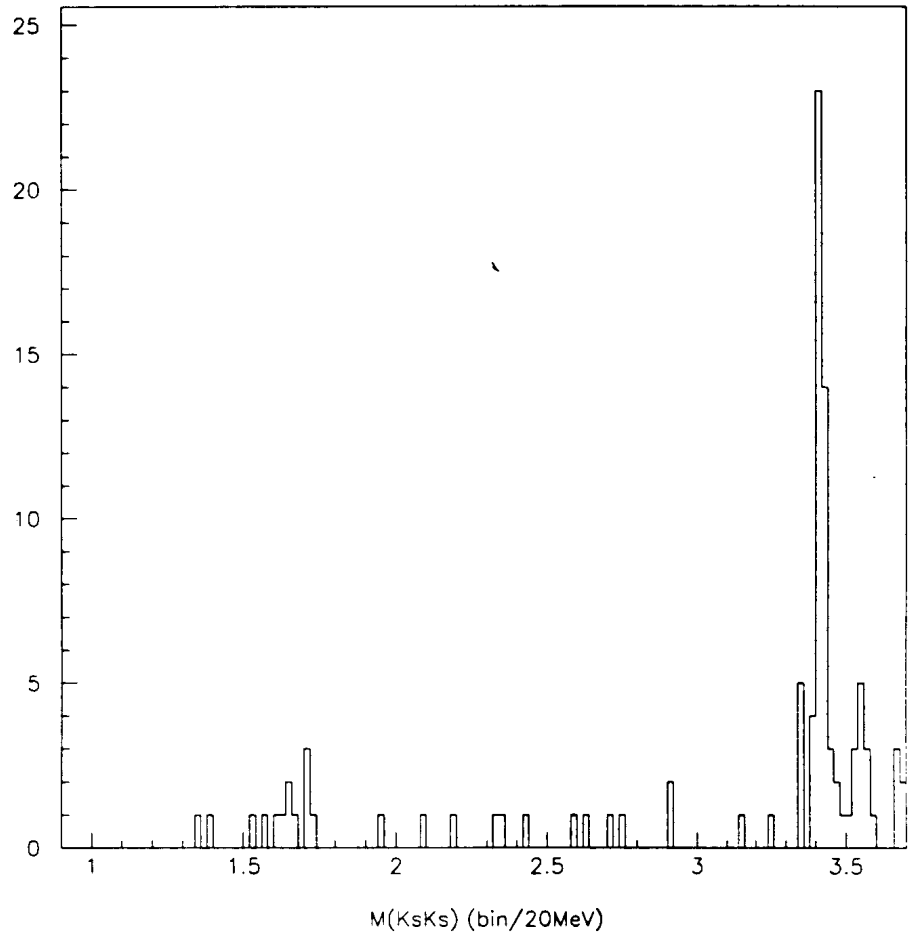
$M_{K^+K^-}$ from $\psi' \to \gamma K^+K^-$



γK^+K^- from τ threshold data



[ψ' − τ scaled threshold] data



$M(K^0_S K^0_S)$

$\psi' \to \gamma K^0_S K^0_S$

Two (Seemingly) Mundane Questions

① Is $J/\psi \to \pi^+\pi^-\pi^0$ $\{K^+K^-\pi^0\}$ understood ?

② Does $\psi' \to \pi^+\pi^-\pi^0$ $\{K^+K^-\pi^0\}$ follow suit ?

Remember :

$$\frac{Bf(\psi' \to X)}{Bf(J/\psi \to X)} \cong \frac{Bf(\psi' \to e^+e^-)}{Bf(J/\psi \to e^+e^-)} \cong 14\,\%$$

{ Applequist & Politzer PRL 34 (1975) 43 }

Evidence for $\rho(1600)$ from the Decay $J/\psi \to \pi^-\pi^+\pi^0$ (Mark III)

Mark III: Typical e^+e^- collider detector

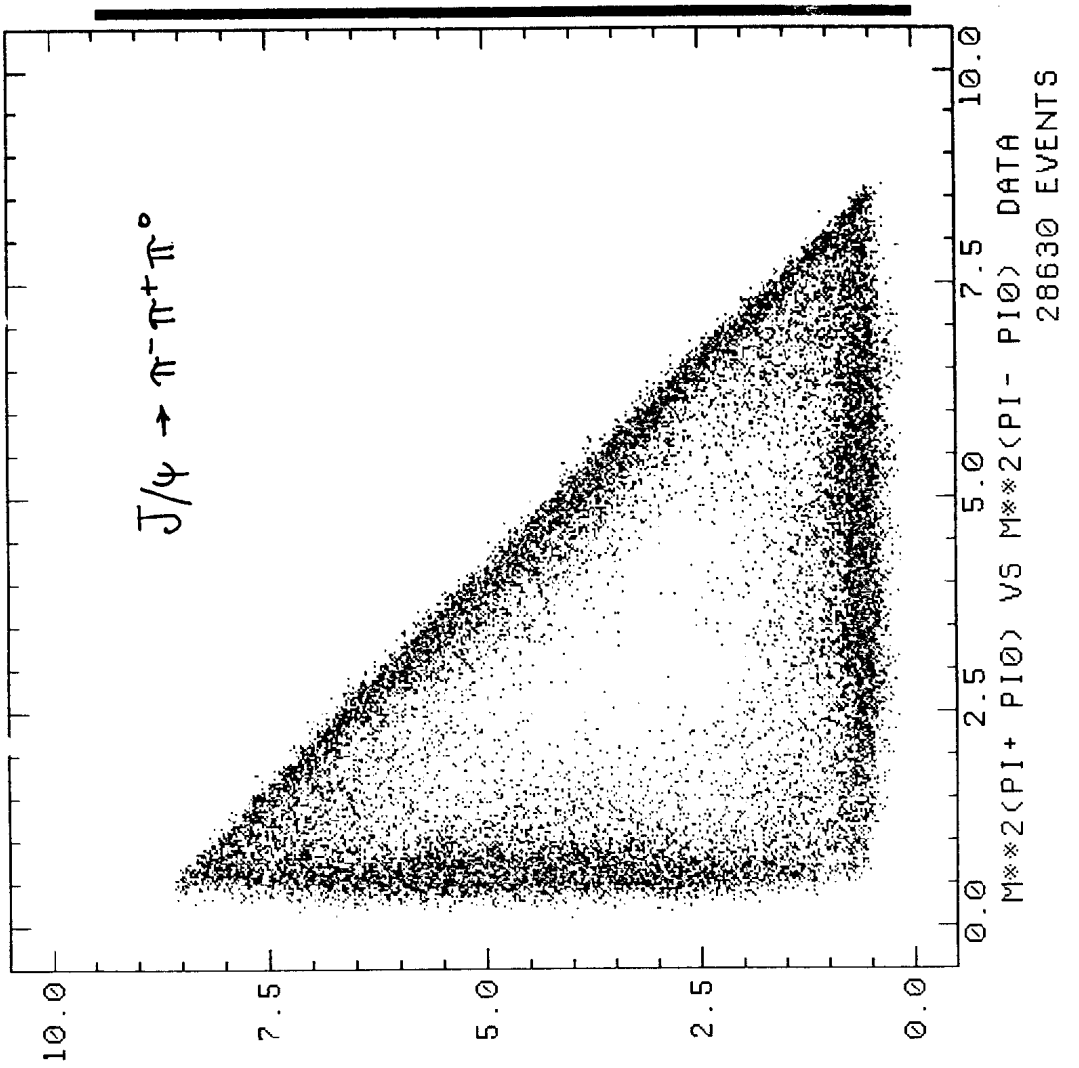
- essentially cylindrical geometry
- TOF & μ detection
- charged particle track reconstruction over $\sim 0.85 * 4\pi$
- γ detection over $\sim 0.95 * 4\pi$ (end-cap shower cntrs.)

For the events corresponding to

$$J/\psi \to \pi^-\pi^+\pi^0 \quad (\pi^0 \to \gamma\gamma)$$

- all final state tracks are detected & reconstructed
- the events are subjected to 5C kinematic fits
- events having $E_\gamma \geq 60$ MeV for each photon are retained.

→ data sample $\sim$ 28,000 events


J/ψ → π⁻π⁺π°
M**2(PI+ PI0) VS M**2(PI- PI0) DATA
28630 EVENTS
0.0
2.5
5.0
7.5
10.0

The Amplitude Describing $J/\psi \to \rho\pi \to \pi^+\pi^-\pi^0$ [SLAC PUB 5674]

(i) The J/ψ is produced with helicity ± 1.

(ii) For each helicity configuration, the properly-symmetrized amplitude describing decay via $\rho\pi$ takes the form:

$$\alpha_i = A^0_i + A^+_i + A^-_i$$

where i denotes the helicity & the superscript denotes the charge of the ρ; in this regard it is important to specify the convention adopted in defining the decay angles of each ρ i.e. which π is to be used to define the angles. The proper choice is given by

$$\alpha_i = A^0_i(\pi^-,\pi^+) + A^+_i(\pi^+,\pi^0) + A^-_i(\pi^0,\pi^-)$$

where the first π in the bracket is the one which is used to define the angles; this ensures Bose symmetry w.r.t. the final amplitude.

The intensity distribution is then given by

$$I = \left(|\alpha_1|^2 + |\alpha_{-1}|^2\right) \cdot d(\text{LIPS})$$

$$= \left(\left|A_1^0 + A_1^+ + A_1^-\right|^2 + \left|A_{-1}^0 + A_{-1}^+ + A_{-1}^-\right|^2\right) \cdot d(\text{LIPS})$$

(iii) Using e.g. the helicity formalism, it can be shown that

$$A_1 = \frac{B \sin\theta_1}{\sqrt{2}} \left\{ \cos\varphi_1 + i \cos\theta \sin\varphi_1 \right\} e^{i\varphi}$$

$$A_{-1} = \frac{B \sin\theta_1}{\sqrt{2}} \left\{ \cos\varphi_1 - i \cos\theta \sin\varphi_1 \right\} e^{-i\varphi}$$

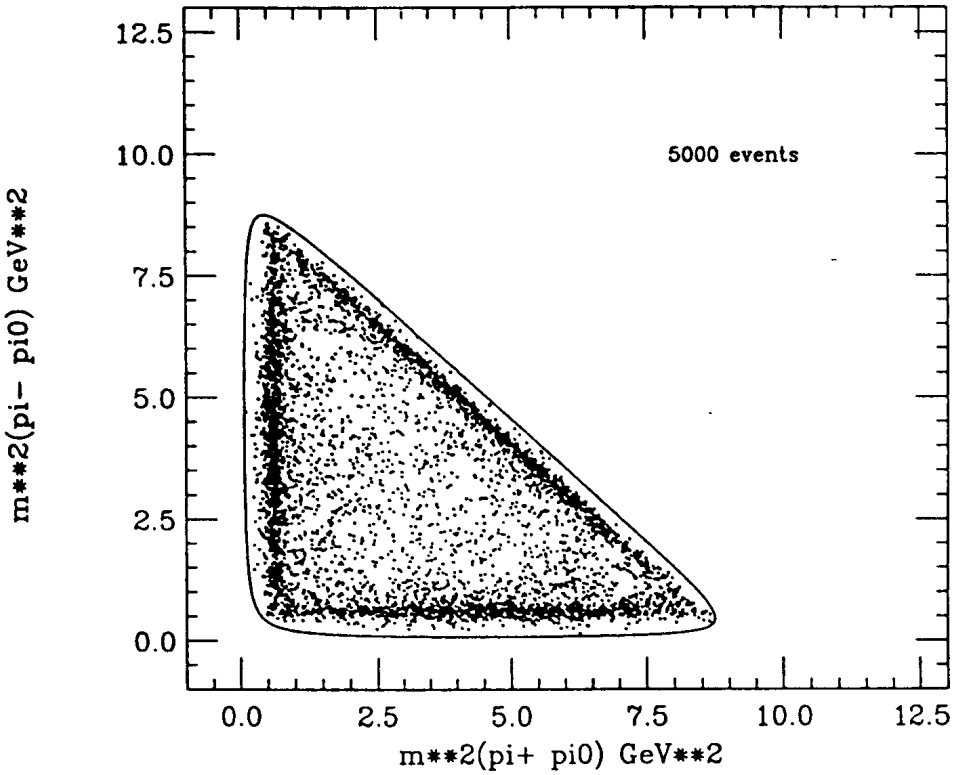
where (θ, φ) are the ρ - polar & azimuthal angles in the lab frame.

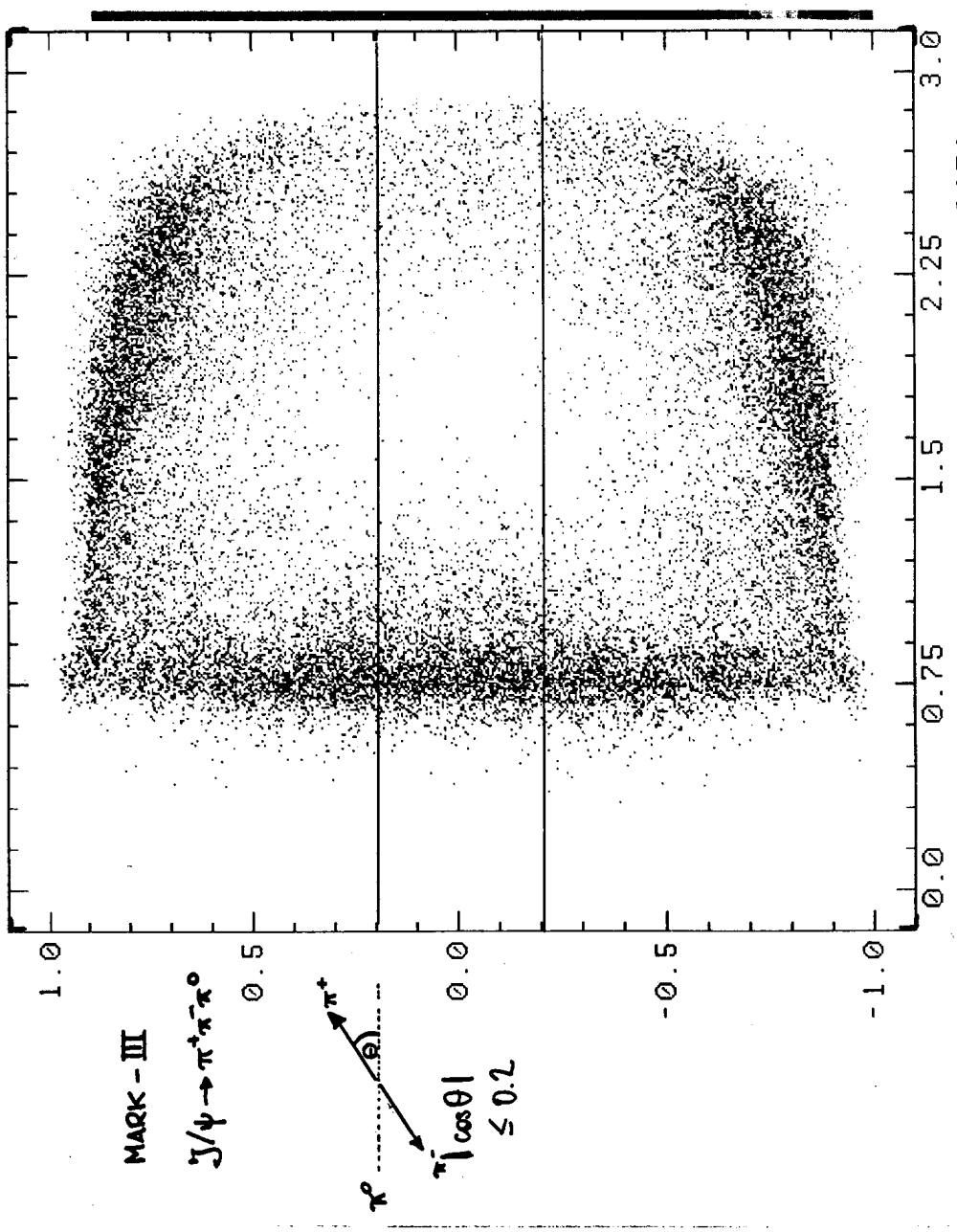
(θ_1, φ_1) are the decay π polar & azimuth. angles in the ρ rest frame.

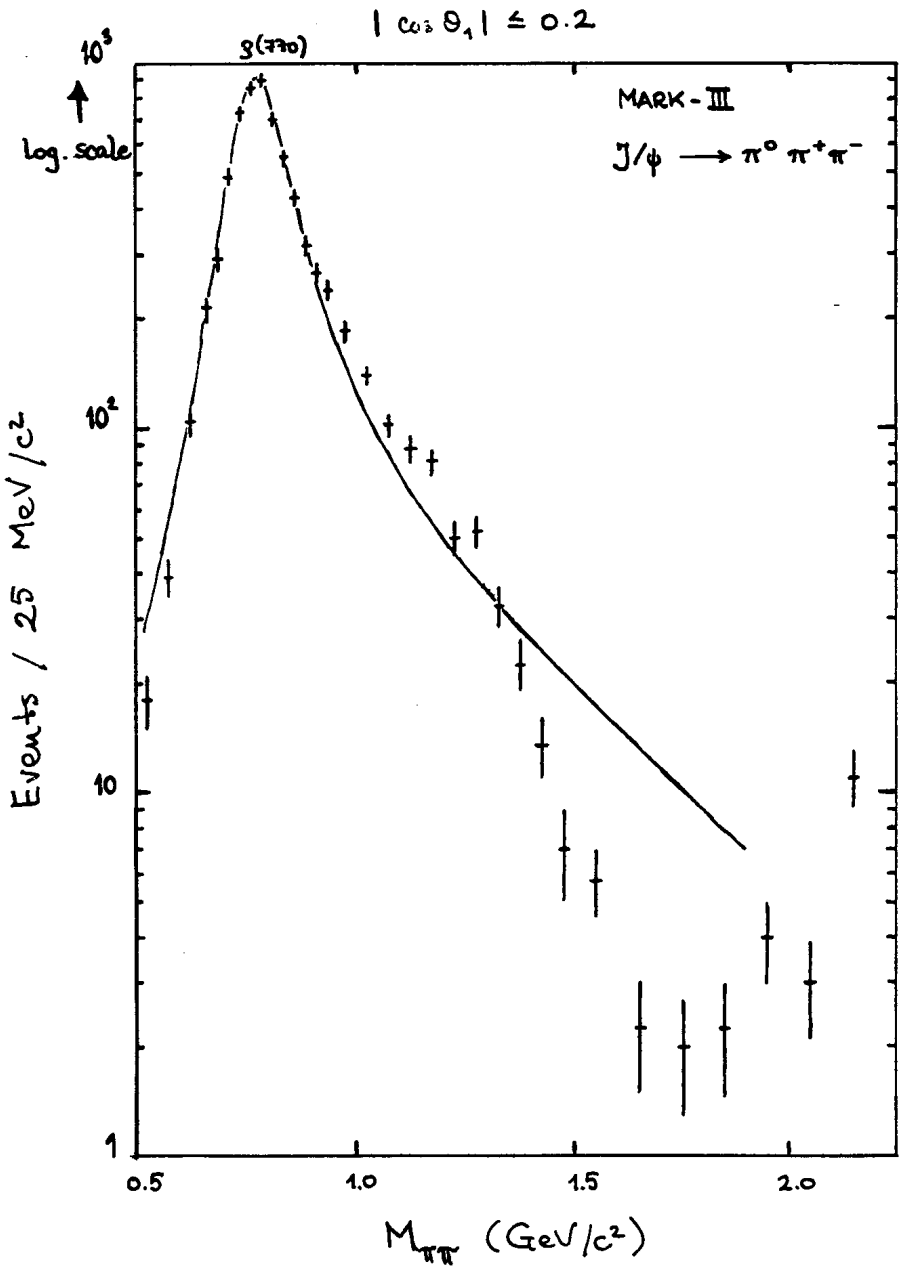
$$B = \frac{C \cdot p \left(\frac{q}{q_0}\right) \sqrt{\frac{1 + (q_0 R)^2}{1 + (q R)^2}}}{m_\rho^2 - m^2 - i m_\rho \Gamma(m)}$$

* Fits give $R = 0$

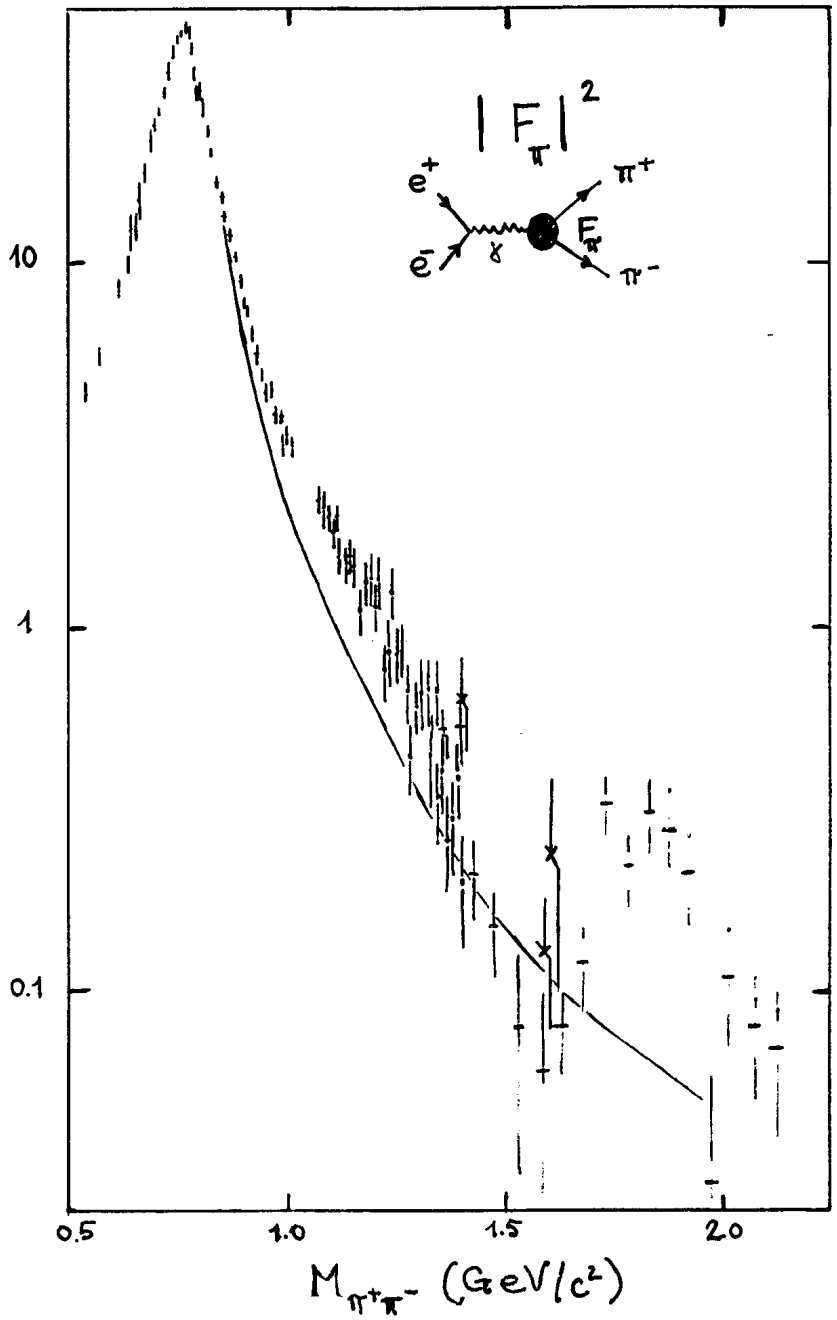
$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^3 \left(\frac{m_\rho}{m}\right) \left\{ \frac{1 + (q_0 R)^2}{1 + (q R)^2} \right\}$$


5000 events
m**2(pi- pi0) GeV**2
m**2(pi+ pi0) GeV**2
0.0
2.5
5.0
7.5
10.0
12.5


MARK - III
J/ψ → π⁺π⁻π°
|cosθ| ≤ 0.2
θ
π⁺
π⁻
π°
1.0
0.5
0.0
-0.5
-1.0
0.0
0.75
1.5
2.25
3.0
M(PI+ PI-) VS COS(THETA)
DATA



| cos θ₁ | ≤ 0.2
ρ(770)
MARK - III
J/ψ ⟶ π⁰ π⁺ π⁻
log. scale
10³
10²
10
1
Events / 25 MeV/c²
0.5
1.0
1.5
2.0
M_ππ (GeV/c²)


$|F_\pi|^2$
e^+
e^-
γ
F_π
π^+
π^-
10
1
0.1
0.5
1.0
1.5
2.0
$M_{\pi^+\pi^-}$ (GeV/c^2)

Modify B as follows:

$$B = B[\rho(770)] + c_1 e^{i\varphi_1} B[\rho_1(1300)] + c_2 e^{i\varphi_2} B[\rho_2(X)]$$

where c_1 c_2 φ_1 & φ_2 are the same for each $\pi\pi$ charge configuration.

Perform a max. $\mathcal{L}$ fit with 9 parameters:

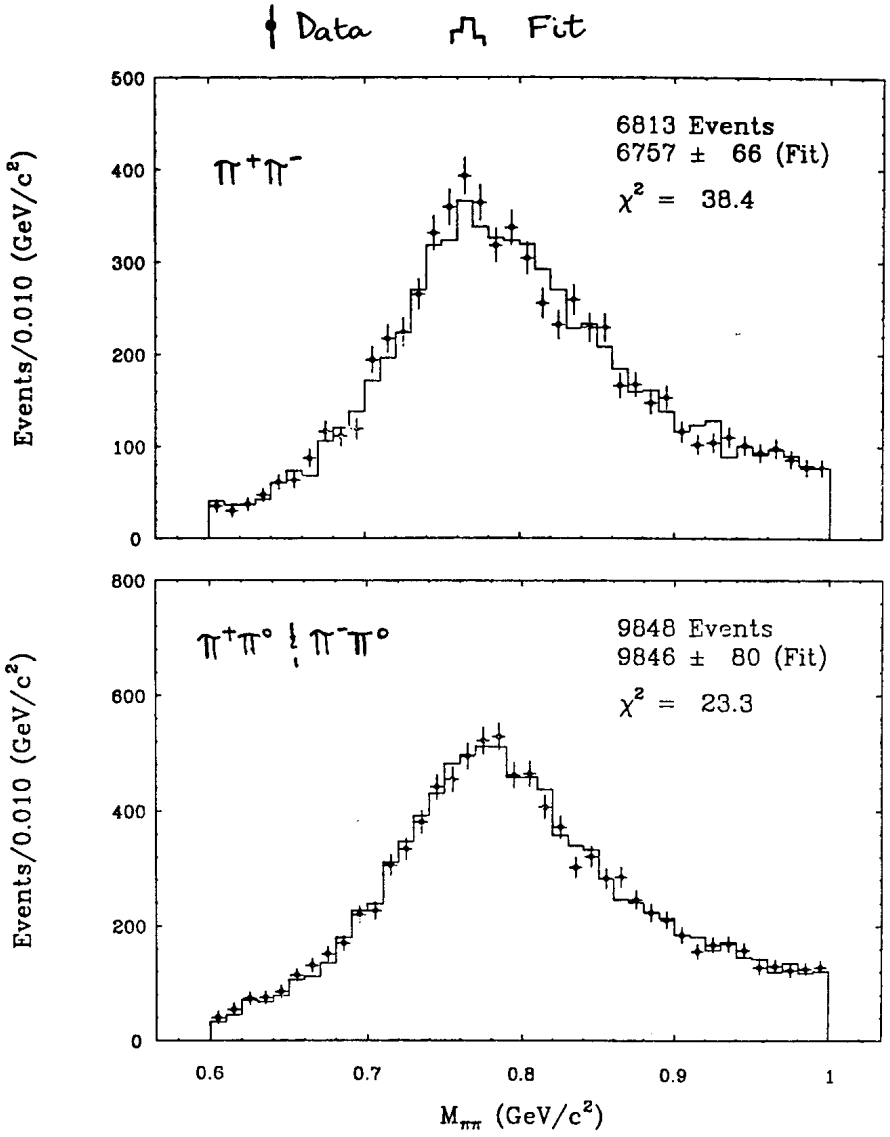
M_{ρ^0} , Γ_ρ , $\delta \equiv M_{\rho^\pm} - M_{\rho^0}$

M_{ρ_2} , Γ_{ρ_2}

c_1 , c_2 , φ_1 , φ_2

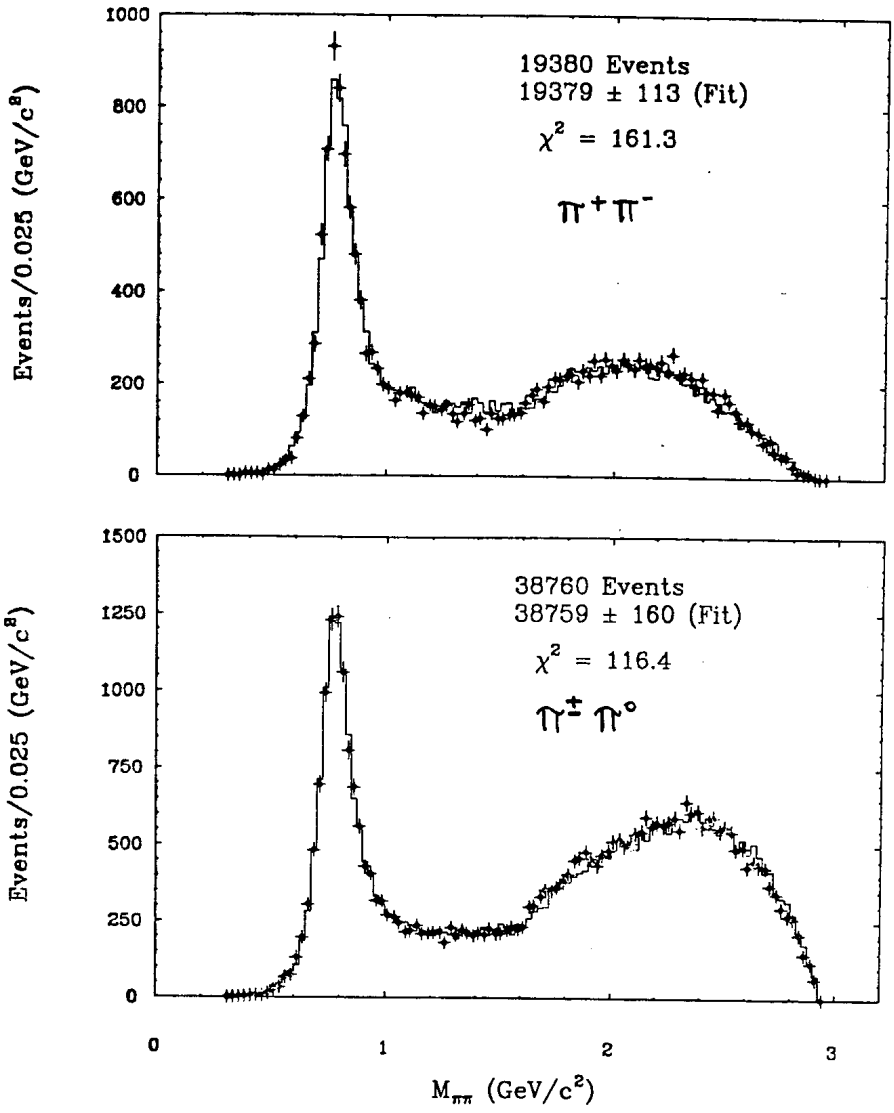
(*) The radius in the usual Blatt-Weisskopf Barrier factor is set to 0.

$R > 0$ results in a poor description of the ρ line-shape.

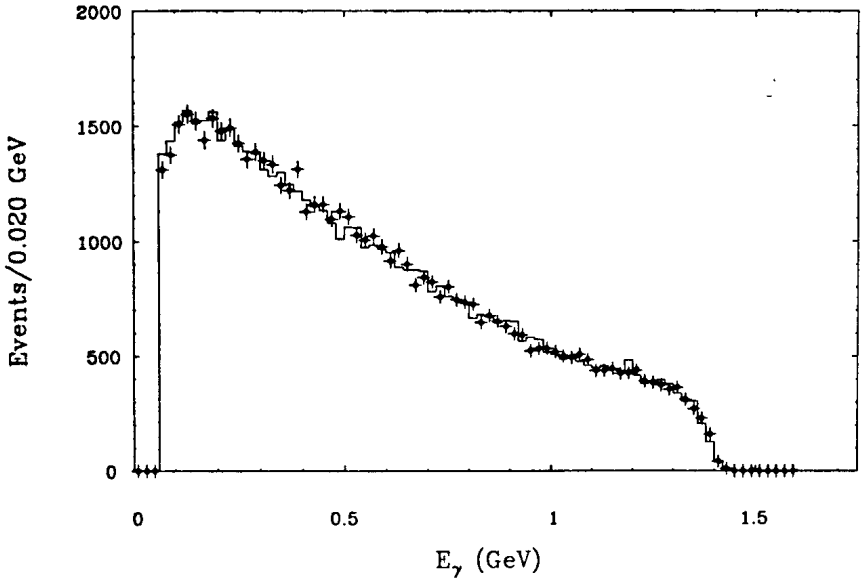


$R = 0$: $M_{\rho^0} = 776.2 \pm 1.3$, $\Gamma_\rho = 150.6 \pm 2.4$; $M_{\rho^\pm} - M_{\rho^0} = 1.6 \pm 1.5$

Barkov et al (F_π)	775.9 ± 1.1	150.5 ± 3.0	[R = 0]
Hyams et al ($\pi\pi$ PWA)	778 ± 2	152 ± 2	[R = 4.5 GeV^{-1}]

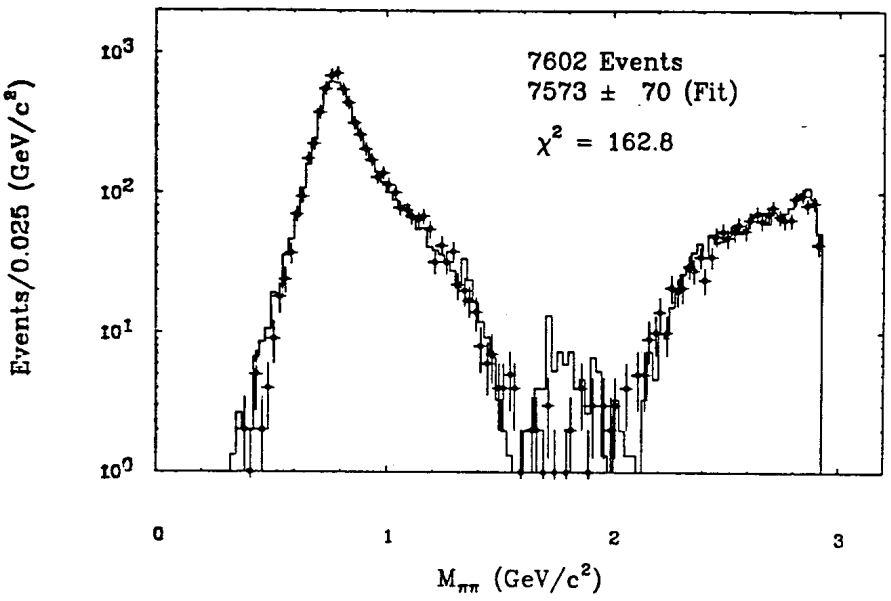

19380 Events
19379 ± 113 (Fit)
χ^2 = 161.3
$\pi^+\pi^-$
38760 Events
38759 ± 160 (Fit)
χ^2 = 116.4
$\pi^\pm\pi^\circ$
Events/0.025 (GeV/c^2)
Events/0.025 (GeV/c^2)
$M_{\pi\pi}$ (GeV/c^2)

E_γ (Lab. Frame) for $\pi^0 \to \gamma\gamma$



$|\cos\theta_1| \leq 0.2$

$\pi^+\pi^-$ & $\pi^\pm\pi^0$ combined



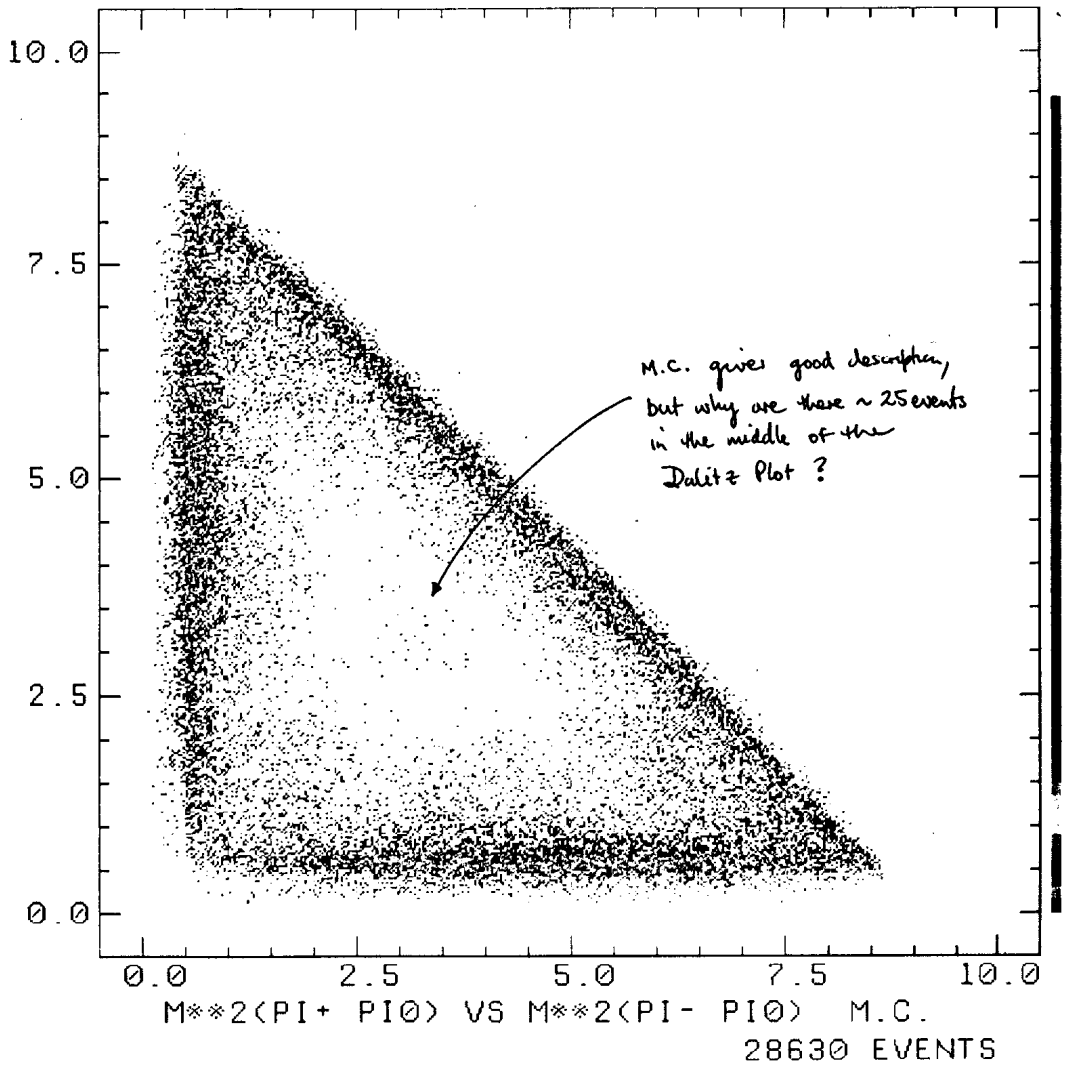
$M_{\rho_2} = 1600 \pm 28$ MeV/c², $\Gamma_{\rho_2} = 383 \pm 25$ MeV/c²

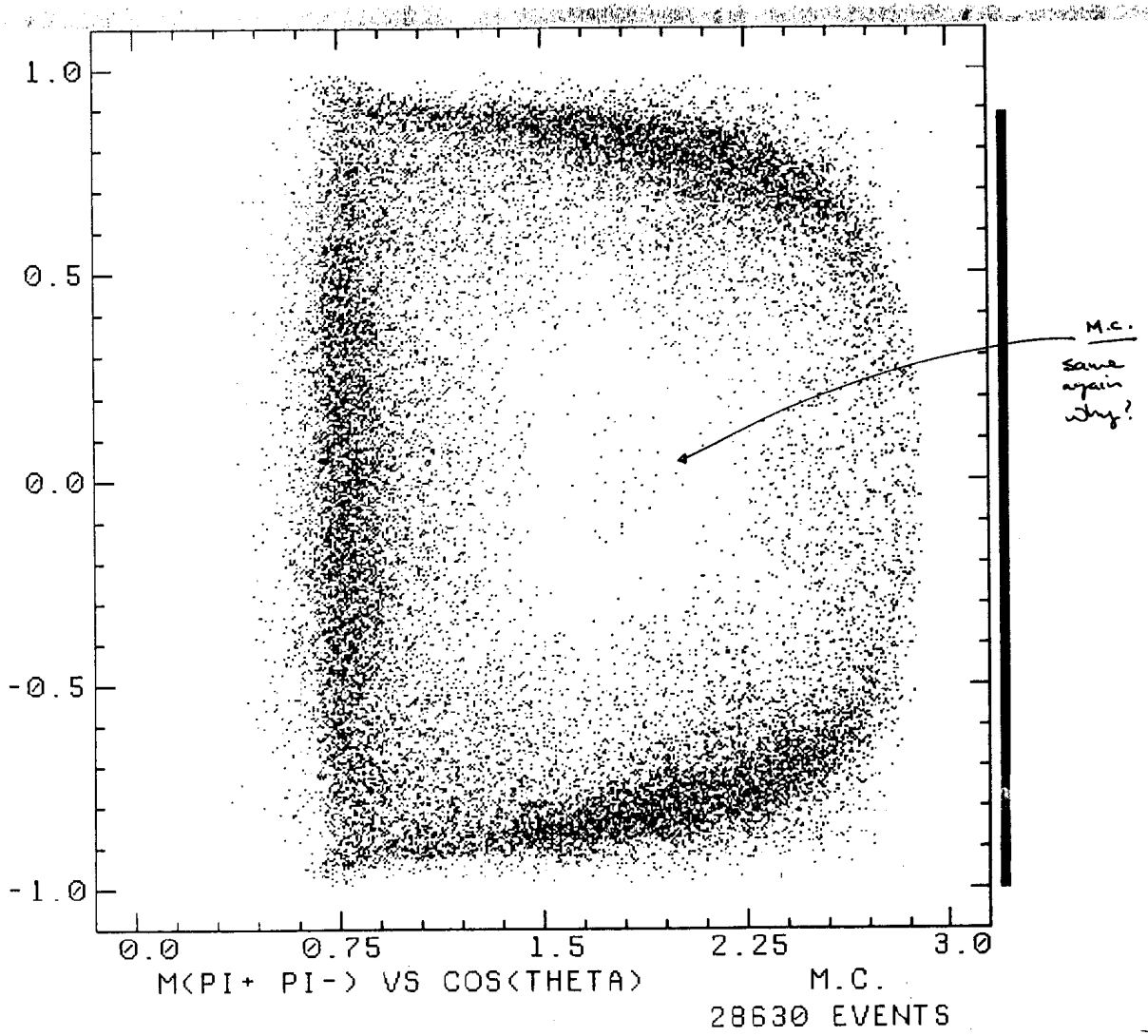
$A_\rho : A_{\rho_1} : A_{\rho_2} = 1 : 0.031 \pm 0.016 : 0.508 \pm 0.017$

$\Rightarrow$ (?) $x_\rho : x_{\rho_1} : x_{\rho_2} = 1 : \boxed{\sim 10^{-3}} : \sim 0.25$

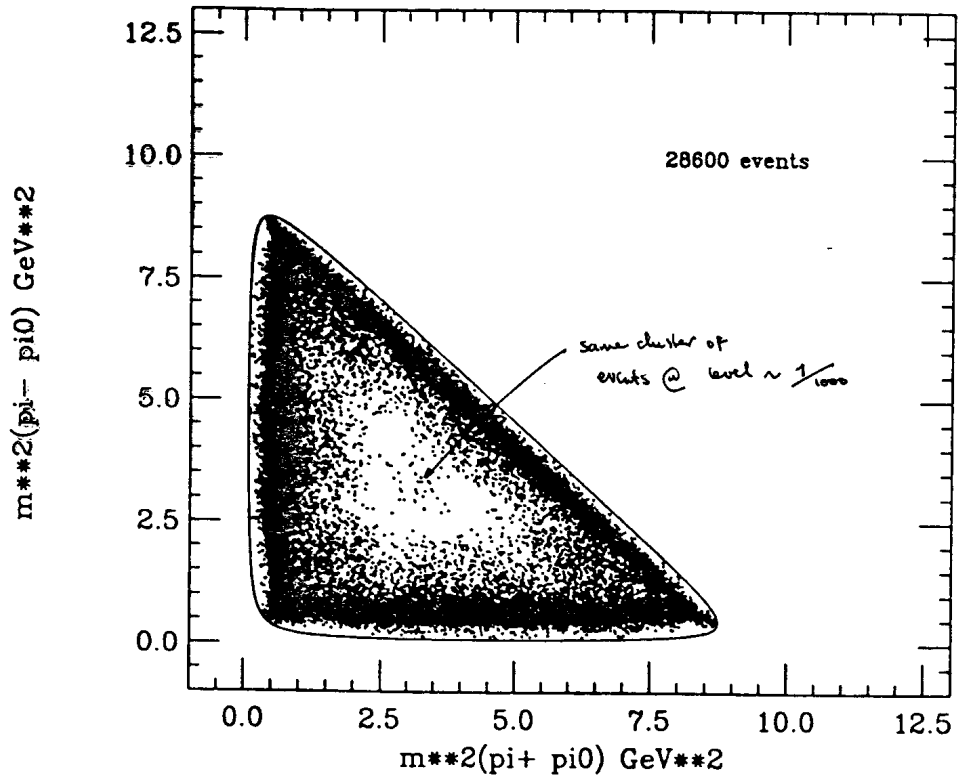
$q\bar{q}$ $\quad 1^3S_1 : 2^3S_1 : 1^3D_1$

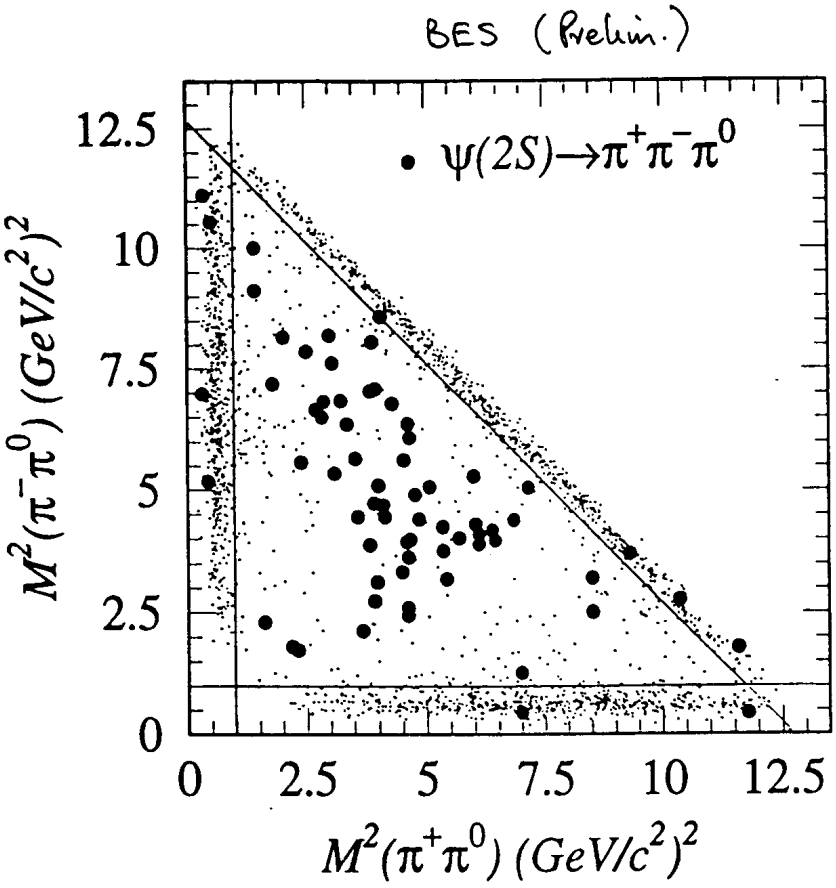
$\varphi_1 = (200 \pm 42)^\circ$; $\varphi_2 = (-120 \pm 8)^\circ$


10.0
7.5
5.0
2.5
0.0
0.0
2.5
5.0
7.5
10.0
M.C. gives good description, but why are there ~ 25 events in the middle of the Dalitz Plot ?
M**2(PI+ PI0) VS M**2(PI- PI0) M.C.
28630 EVENTS


1.0
0.5
0.0
-0.5
-1.0
0.0
0.75
1.5
2.25
3.0
M(PI+ PI-) VS COS(THETA)
M.C.
28630 EVENTS
M.C.
same again why?

$J/\psi \to \pi^+\pi^-\pi^0$ (M.C.)

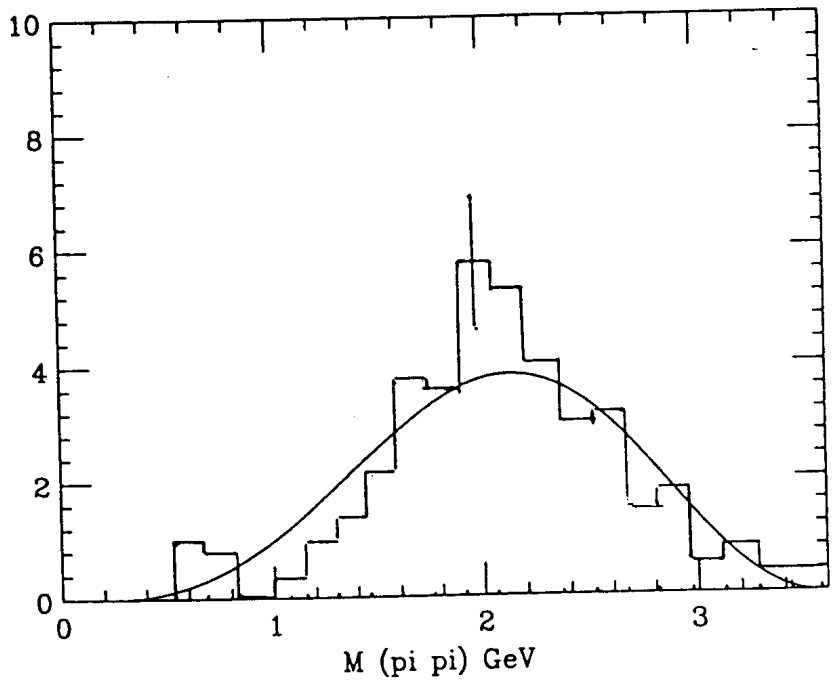




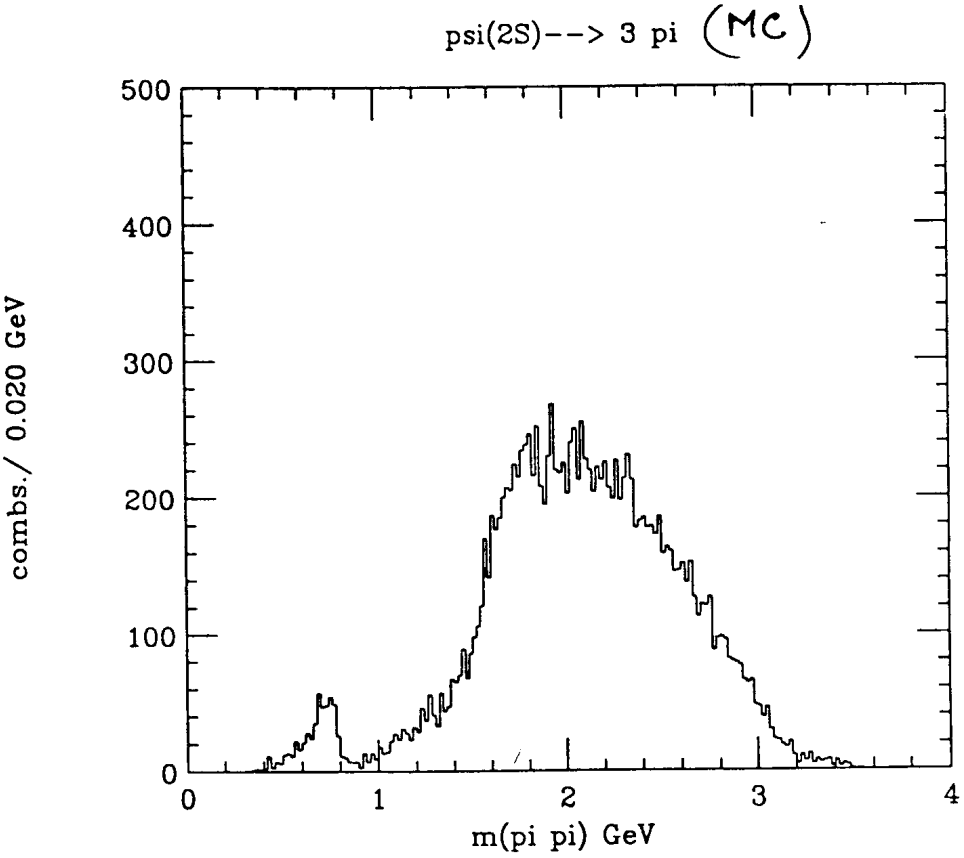
Very different from $J/\psi \to 3\pi$;
similar behaviour observed for
$K\bar{K}\pi$.

BES (Prelim.)

psi(2S)--> 3 pi

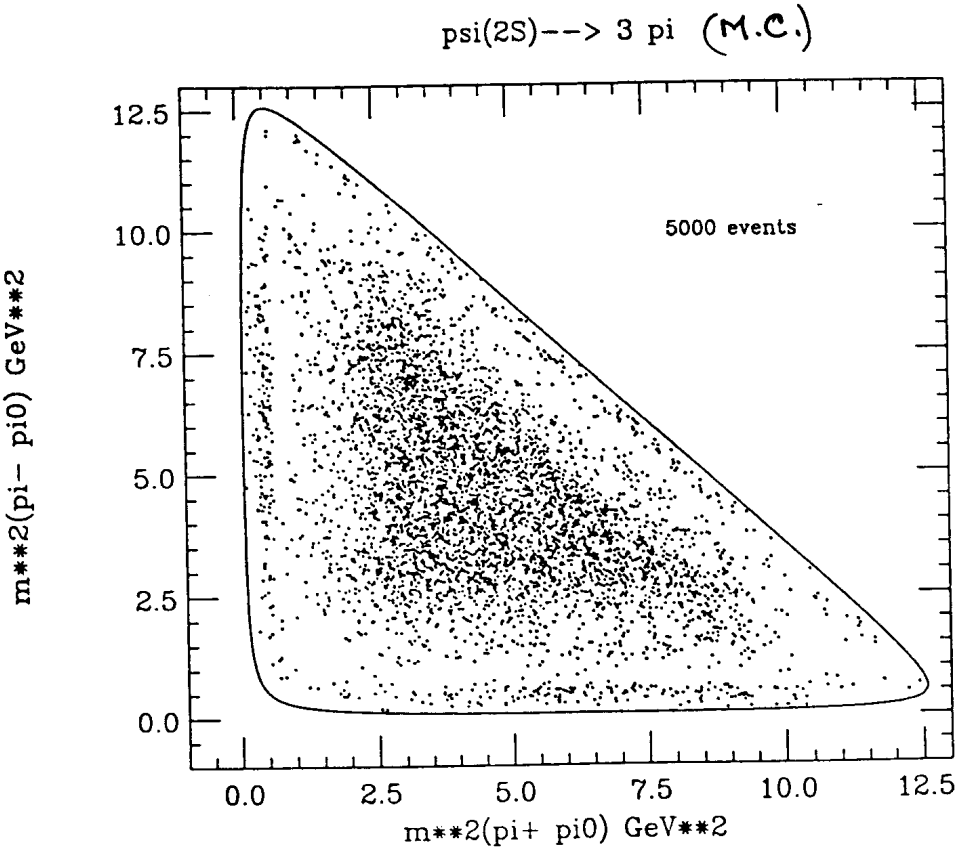


— $\eta' \to 3\pi$ (non-isobar ie. like $\omega \to 3\pi$)



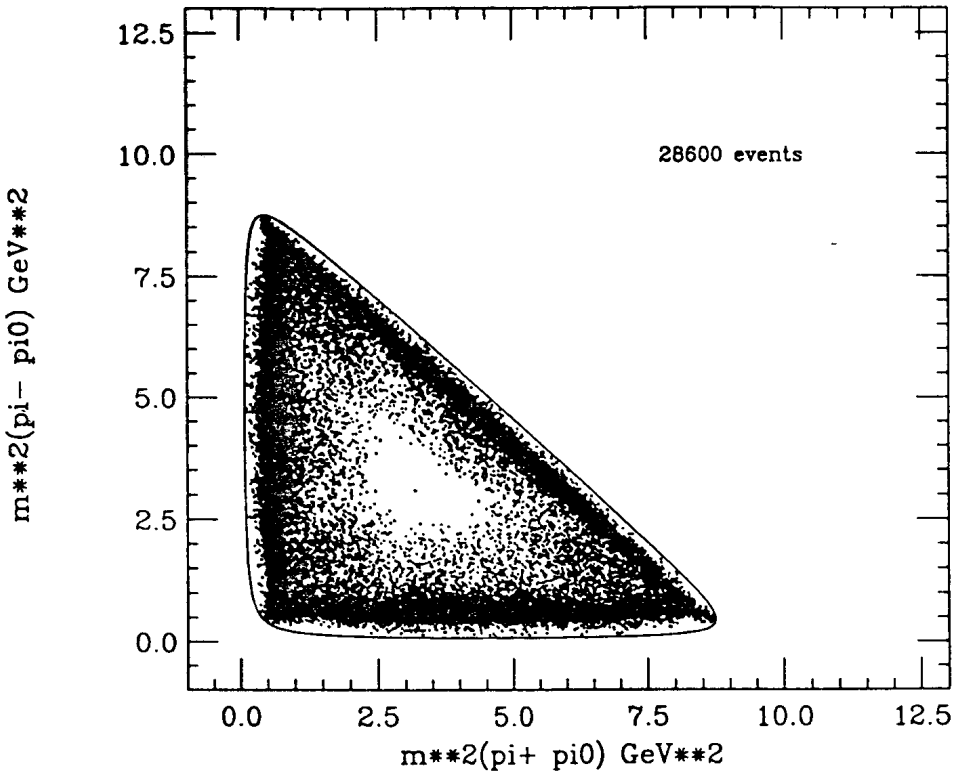
$\rho + \rho'(1600, 383) + 3\pi$ (non-isobar)

Same model as for $J/\psi \to 3\pi$, but with addition of decay to 3π with no $\pi\pi$ isobar structure (i.e. like $\omega \to 3\pi$)



$\rho + \rho'(1600, 383) + 3\pi$ (non-isobar)

$J/\psi \to 3\pi$ (M.C.)



$\rho + \rho'(1600, 383) + 3\pi$ (non-isobar)

↑

include this in previous description of $J/\psi \to 3\pi$; can remove cluster of events in centre of plot (almost!)

i.e. describe $J/\psi \to \psi(2S)$ decay to 3π with same model, but with individual contributions having different strengths.

$J/\psi \; \& \; \psi' \rightarrow \pi^+\pi^-\pi^0$

(4th March, 1998)

Contribution	$B(J/\psi)$	$B(\psi')$ (*) (very Crude)
All	$(134 \pm 10) \times 10^{-4}$	$\sim 1.6 \times 10^{-4}$
$\sum_{i=1}^{3}{}' \rho_i(770)\pi$	$(169 \pm 13) \times 10^{-4}$	$\sim 0.35 \times 10^{-4}$
Note: $3 * (\rho\pi)$ [i.e. no interference]	$(134 \pm 10) \times 10^{-4}$	
$\sum_{i=1}^{3} \rho_i'(1600)\pi$	$(11.9 \pm 1.2) \times 10^{-4}$	$\sim 0.32 \times 10^{-4}$
$(\pi^+\pi^-\pi^0)_{\text{non-quasi 2body}}$ (i.e. like $\omega \to 3\pi$)	$(2.25 \pm 0.23) \times 10^{-4}$	$\sim 1.7 \times 10^{-4}$

(*) based on assumption of ~600 evts. after acceptance correction

$$B(m_{\pi\pi}) = A_\rho e^{i\varphi_\rho} BW_\rho(m_{\pi\pi}) + A_{\rho'} e^{i\varphi_{\rho'}} BW_{\rho'}(m_{\pi\pi}) + A_{3\pi} e^{i\varphi_{3\pi}} B_{\omega\to 3\pi}(m_{\pi\pi})$$

Parameter	J/ψ	ψ'
A_ρ	1.0	0.6
φ_ρ	0.°	0.°
$A_{\rho'}$	0.508	1.0
$\varphi_{\rho'}$	$-120°$	$120°$
$A_{3\pi}$	0.12	1.0
$\varphi_{3\pi}$	$-95°$	$-45°$