Scalar Gluonium in J/ψ Radiative

Decay to Two Pseudoscalars

W. Dunwoodie (SLAC)

Tau-Charm Factory Workshop,

SLAC

March, 1999

Scalar (& Tensor?) Gluonium in Amplitude Analyses of J/ψ Radiative Decay to Two Pseudoscalars

{applies to ψ′ also}

1) Why do we expect gluonium states to exist?
2) Why do we need amplitude analyses to find them? {⇒ need for very large data samples}
3) If you found $J^{PC} = 0^{++}$ gluonium, would you know it?
4) Exptal. evidence so far (non J/ψ radiative)
5) Why is J/ψ (or ψ′) radiative decay the best place to look? (not totally objective!)
6) Outline of Mark III analysis & results on $J/\psi \to \gamma \pi^+\pi^-$, γK^+K^-, $\gamma K^0_S K^0_S$ (+ remarks on $\gamma\eta\eta$)

   Interpretation?
7) Why is the TCF necessary to this effort?
8) Two (seemingly) mundane questions whose answers also require the TCF

Why do we expect gluonium states to exist?

① In the context of QCD :

"... meson states would appear that act as if they were made of gluons rather than $q\bar{q}$ pairs."

Such states would result in

"... a sequence of extra SU(3) singlet meson states." $\{I=0\}$

Fritzsch & Gell-mann   ICHEP, Batavia (1972) Proc. v.2, p.135

② Non-Abelian nature of QCD $\mathcal{L} \Rightarrow \exists$ point-like couplings of 3 & 4 gluons

$\Rightarrow$ possible existance of bound states

③ Lattice Gauge Calcns: $\rightarrow$ spectrum of such states;

Lowest Mass $J^{PC} = 0^{++}$ state at $\sim 1.6 \, {}^{+\text{Weingarten}}_{-\text{Michael}}$ GeV

with Weingarten $\simeq$ Michael $\cong$ 0.1 GeV

$\{ J^{PC} = 2^{++}$ at ~ 2.3 GeV $\pm$? $\}$

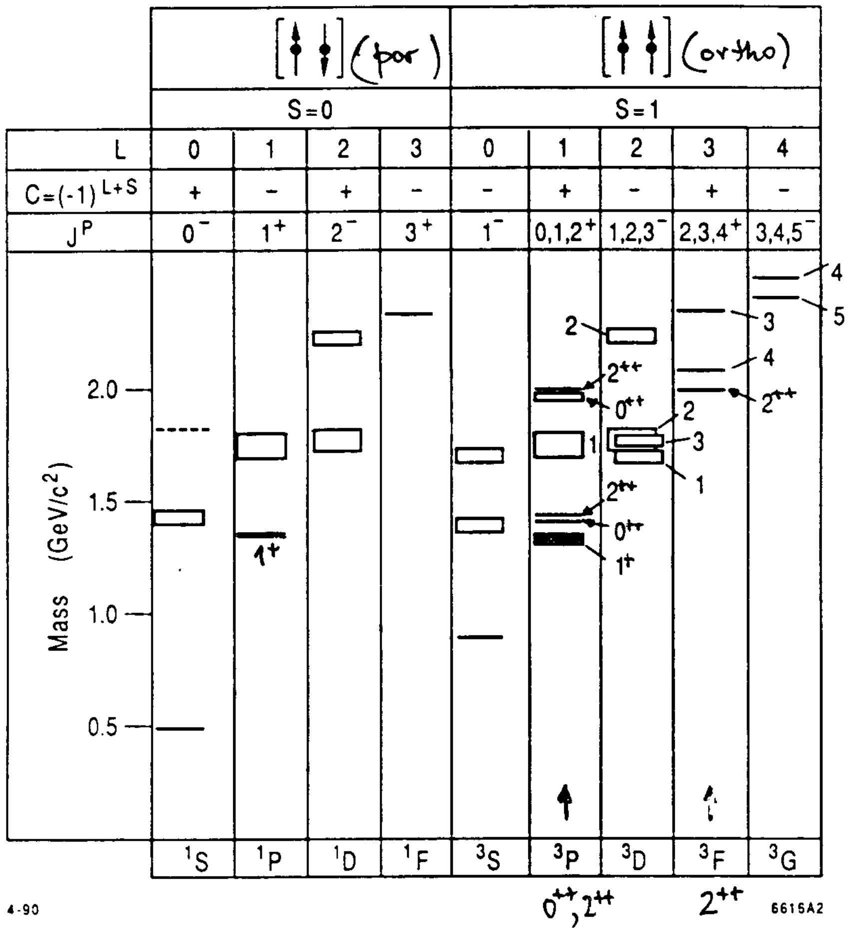
Why do we need amplitude analyses ?

Take a look at the strange meson sector.

a) don't expect extra states
i.e. good qualitative agreement
with quark model expectations

b) ⇒ if o.k., can use quark model
as "template" in search for
extra states.

Strange Meson Level Scheme



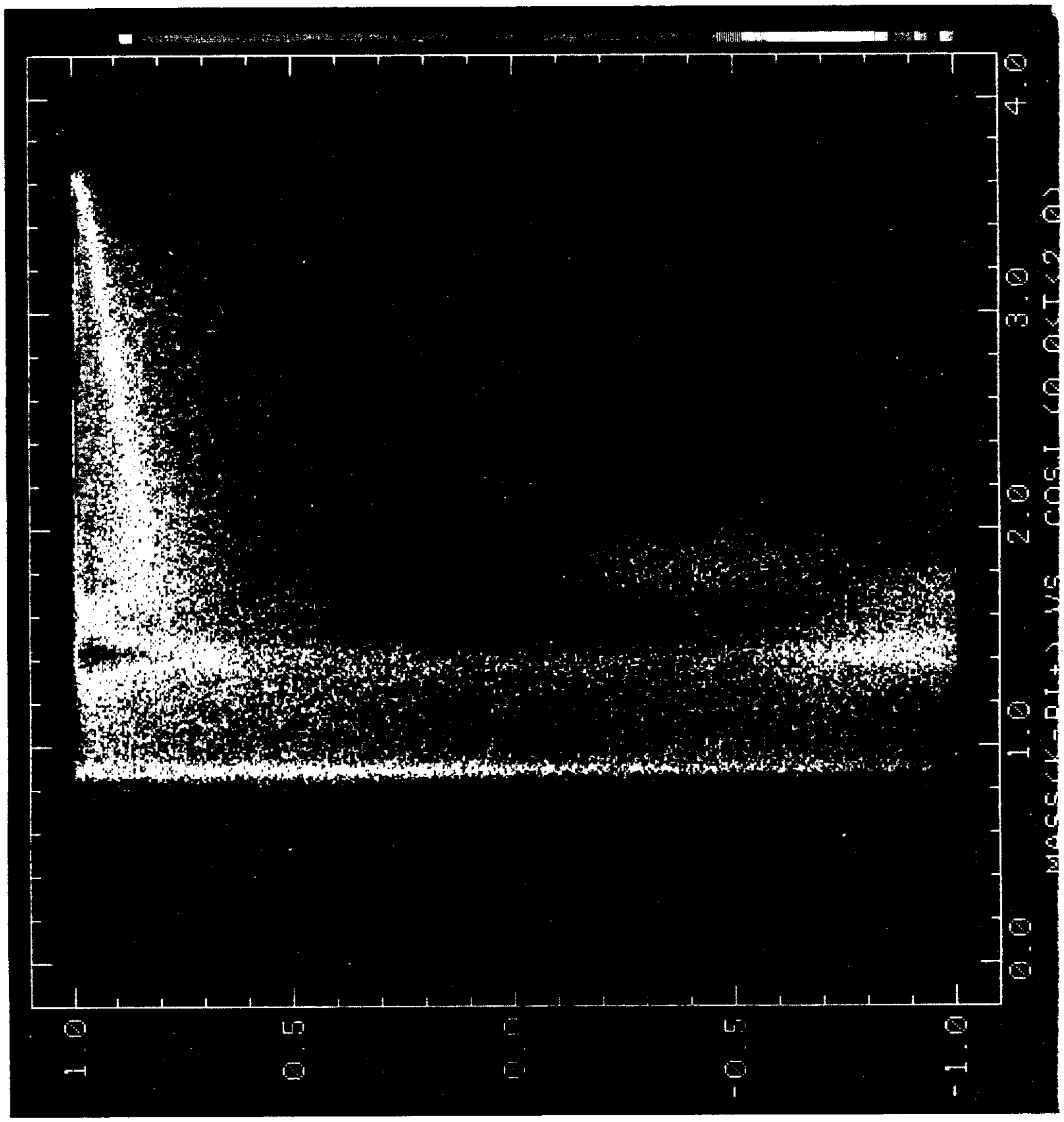
1) qualitatively similar to atomic He

2) no extra states ; approx. mass degeneracy

3) 0^{++} only in ^3P col. ; 2^{++} in ^3P & ^3F

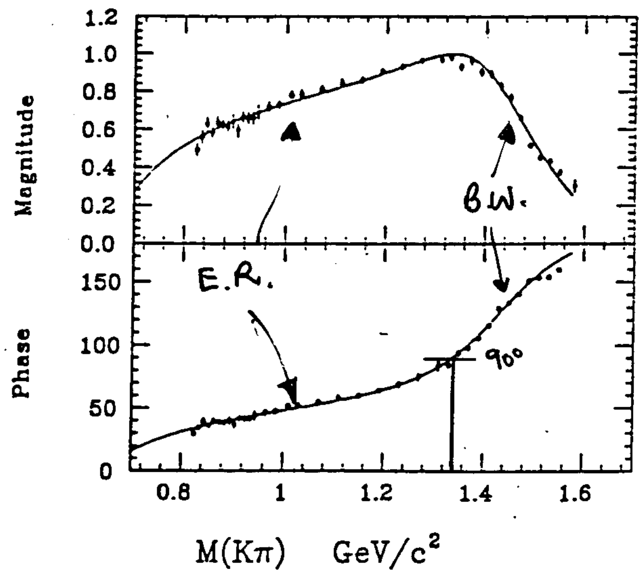
4) $J^P = 1^+$ ground states mix ; $K_1(1270)$ & $K_1(1400)$ are mixtures of K_{1a} & K_{1b}

6 1



Kπ S-wave solution (s)

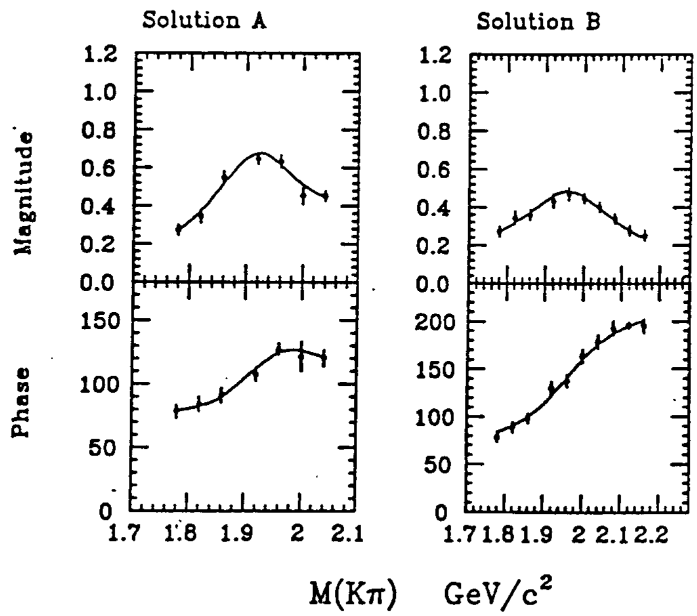
κ(1350)



M
1412±4±5
Γ
294±10±21
Elasticity
~1

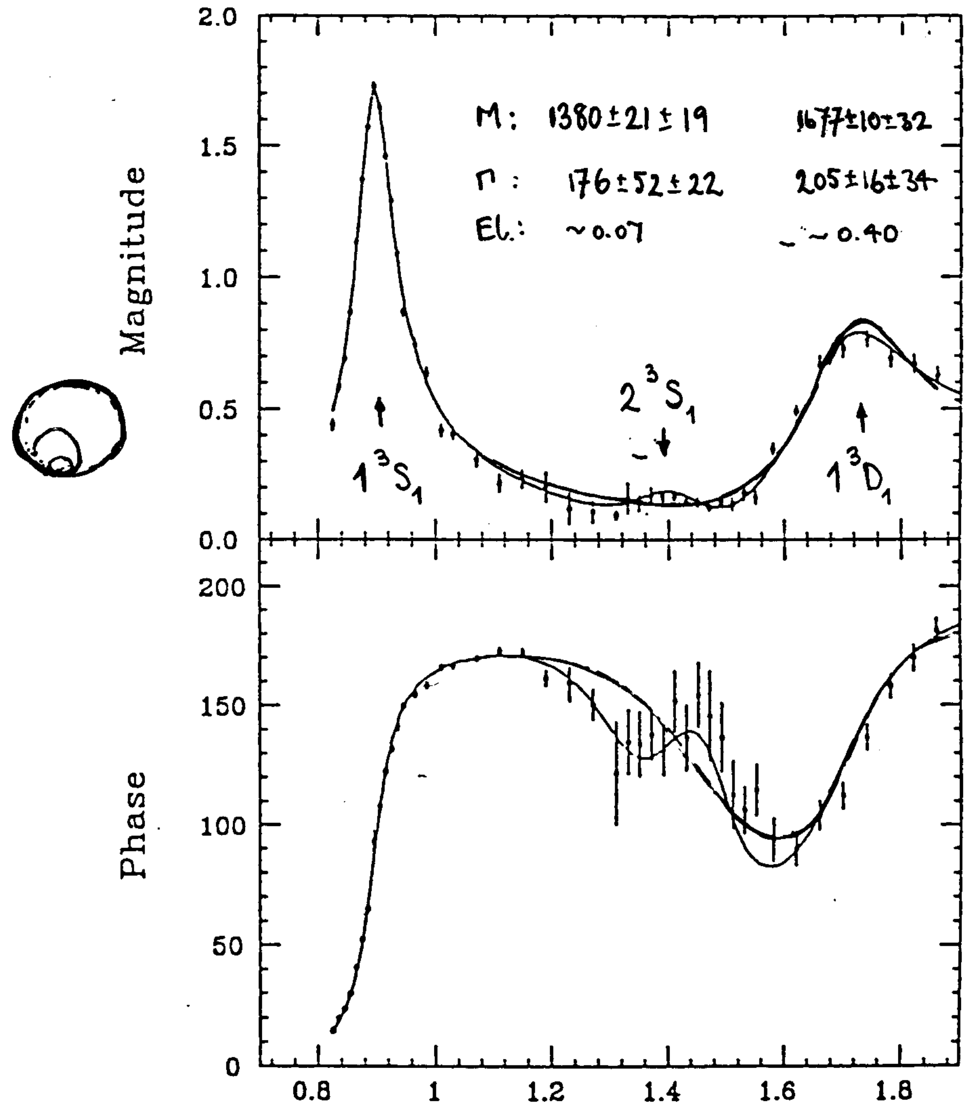
0^+ $K^*(1950)$ [radial excitation]

M
1913±5±20
Γ
152±12±79



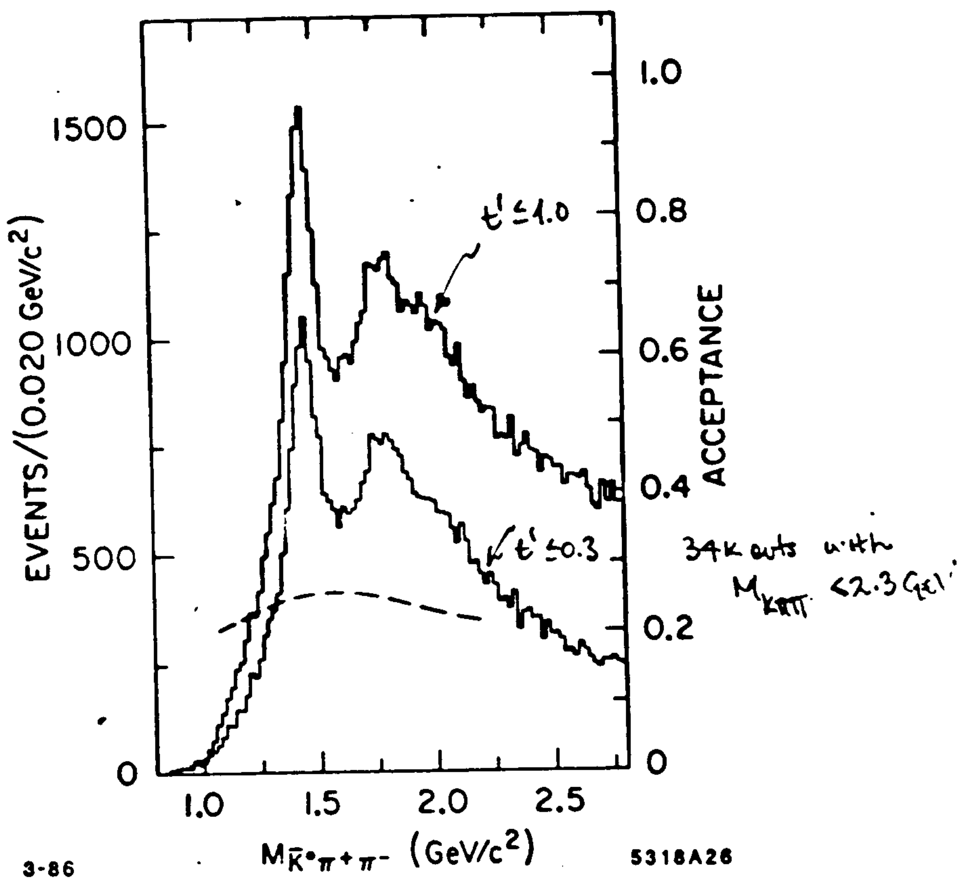
M
195[illegible]
Γ
232±15±22
Elasticity
~0.45

P-wave Breit-Wigner fit



M(Kπ) GeV/c^2

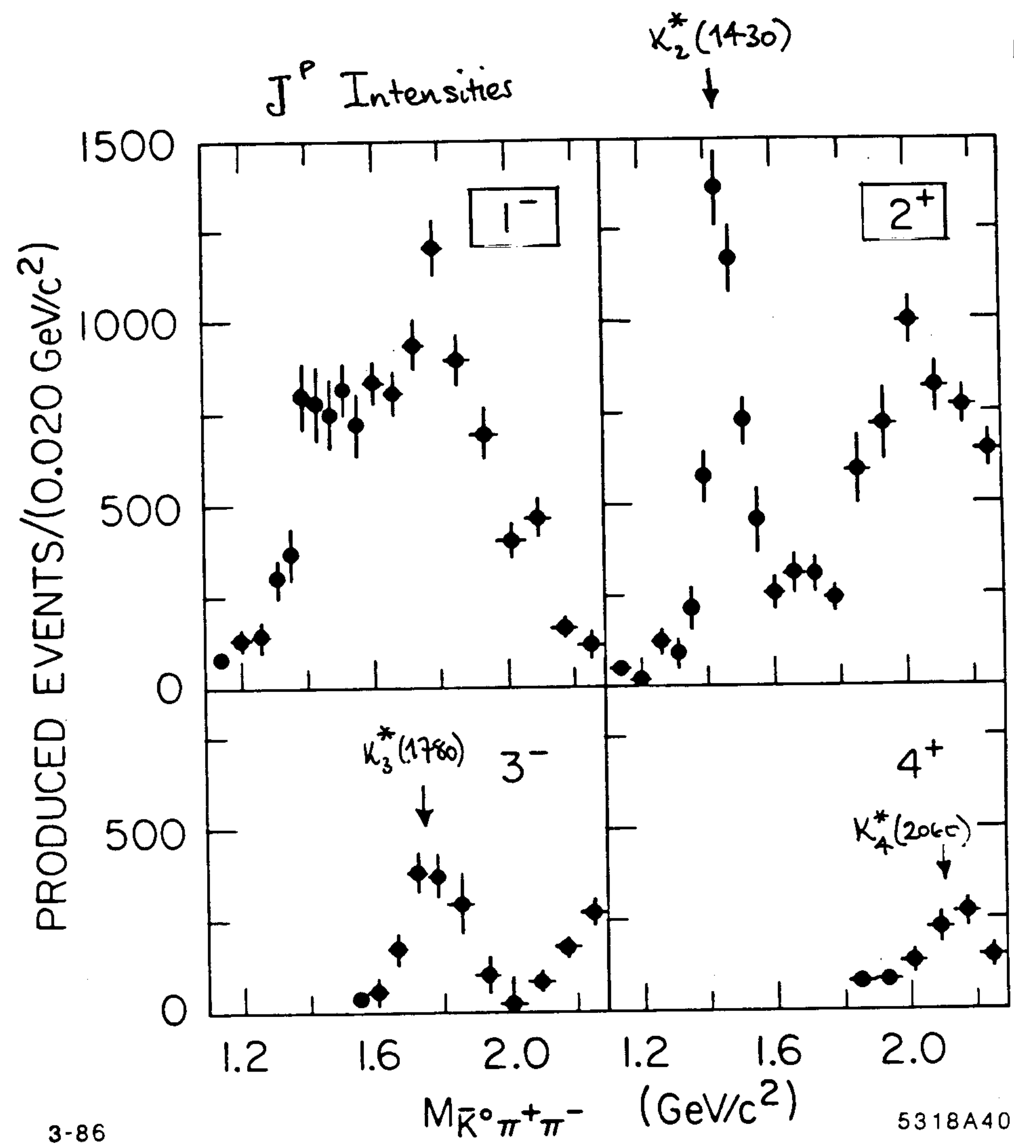
$K^- p \to \bar{K}^0 \pi^+ \pi^- n$



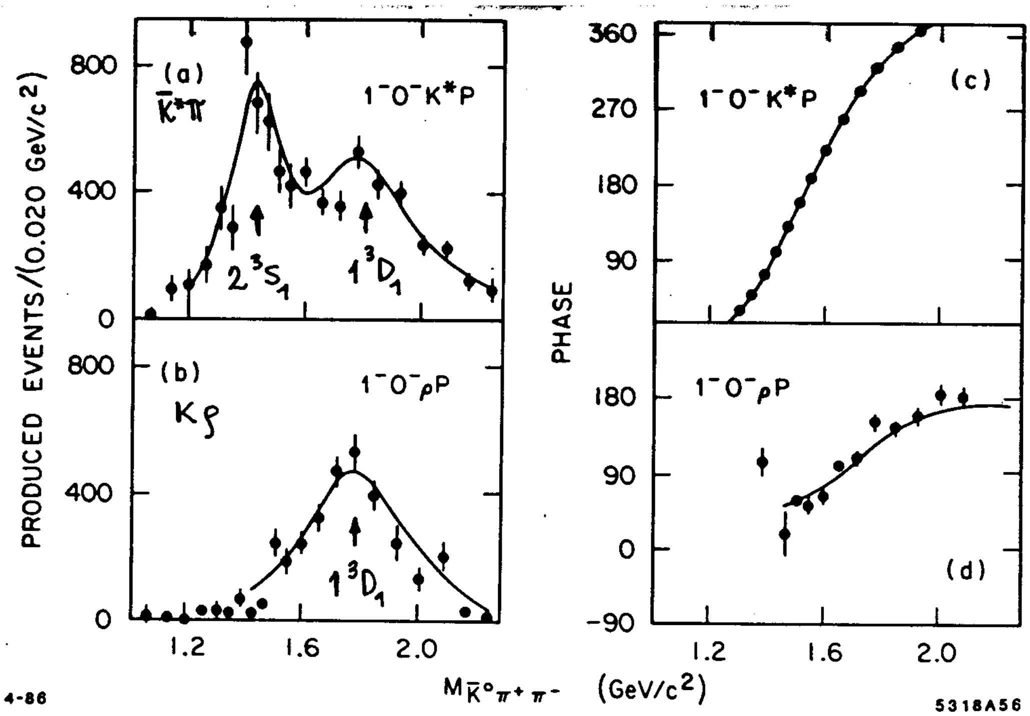
looks simple, but:

a) ~ 2/3 spectrum resonant

b) complicated amplitude structure



Using SLAC - LBL PWA program



M_1 :	$1420 \pm 7 \pm 10$	Γ_1 :	$240 \pm 18 \pm 12$
from $(K\pi)$	$1380 \pm 21 \pm 19$		$176 \pm 52 \pm 22$
M_2 :	$1735 \pm 10 \pm 20$	Γ_2 :	$423 \pm 18 \pm 20$
from $(K\pi)$	$1677 \pm 10 \pm 32$		$205 \pm 16 \pm 34$

<u>Lower mass state</u>: (i) broad. t- dependence.
(ii) $\sim 90°$ phase diff w.r.t. $K_2^*(1430)$

$\Rightarrow$ weak coupling to $(K\pi)$

If you found $J^{PC} = 0^{++}$ gg, would you know it?

1) What are the quark model expectations?
For isoscalars & $J^{PC} = 0^{++}$:

ground state { $n\bar{n} - s\bar{s}$: $\sim$ 1400 MeV : $f_0(1370)$
$s\bar{s}$: $\sim$ 1500 MeV (?) }

first radial excitations { From strange sector, expect $\sim$ 500 MeV higher in mass }
— no candidates as yet.

Adding glueball, expect 5 states below 2 GeV; have $f_0(1370)$ & radial excitations at $M \gtrsim 1.9$ GeV.

— Might have something at $\sim$ 1.5 GeV; LASS + Bf(1370) => mainly $s\bar{s}$

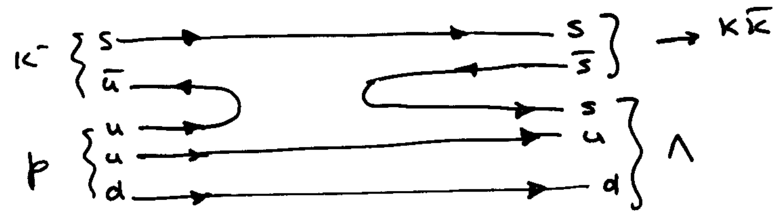
— Crystal Barrel has strong signal at $\sim$ 1500, but $Bf(K\bar{K}) / Bf(\pi\pi)$ small
  — prob. not the same state as LASS "state".
  — interpret as mainly glueball

2) What are the properties? Mixing of glueball with $q\bar{q}$ states may obscure.

Experimental Evidence [non J/ψ]

Fixed Target — Peripheral Production

① $K^- p \to \Lambda K \bar{K}$ (LASS)

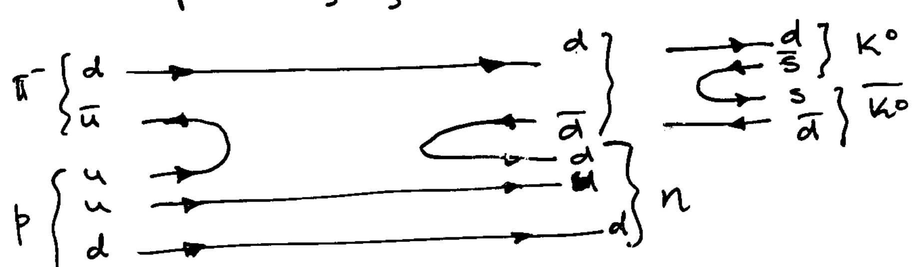


(i) no obvious source of glue

(ii) weak evidence for 0^{++} at ~ 1.5 GeV

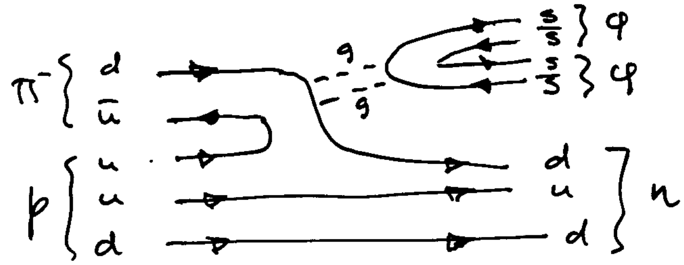
(iii) no evidence for radial excitation at $\gtrsim 1.9$ GeV + nothing in between

② $\pi^- p \to K^0_S K^0_S n$



Lindenbaum et al claimed "glueball" in S wave; amplitude extracted looks like a dog's breakfast! Not believable.

(3) $\varphi\varphi$ production (Lindenbaum et al)



— $\varphi\varphi$ coupled via gg

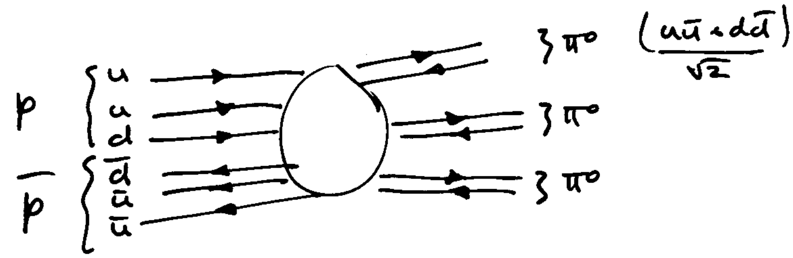
— PWA finds 3 states with $J^{PC} = 2^{++}$
at $\sim$ 2.01, 2.30, 2.34 GeV

— quark model: expect 2 states in this region viz.
$\begin{cases} 3P & 1^{st} \text{ radial excitation} \\ 3F & \text{ground state} \end{cases}$

$\Rightarrow$ perhaps 1 extra state

(*) OBELIX claims only one state in
$p\bar{p} \rightarrow \varphi\varphi$ c.m. energy scan in this region.

$\bar{p}p$ Annihilation – Crystal Barrel @ LEAR



– observe f_0 (1500) $\left\{ \begin{matrix} M = 1500 \pm 10 \text{ MeV} \\ \Gamma = 112 \pm 10 \text{ MeV} \end{matrix} \right\}$

$\frac{\Gamma(2\pi)}{\Gamma} \sim 45\%$, $\frac{\Gamma(K\bar{K})}{\Gamma(\pi\pi)} \sim 20\%$

$\frac{\Gamma(\pi\pi)}{\Gamma(\eta\eta)} \sim 4.5$

– seems real

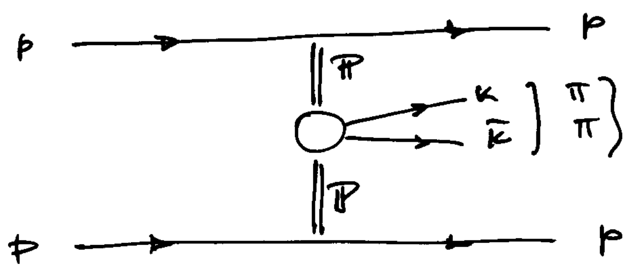
But: a) no compelling reason to think that $g\bar{g}$ source (same q & $\bar{q}$ in and out)

b) still used f_2 (1565 to fit Dalitz Plot

$\left\{ \begin{matrix} \text{a) is touted as glue candidate} \\ \text{b) is not touted at all.} \end{matrix} \right\}$

c) no strong signal in $J/\psi \to \left\{ \begin{matrix} \gamma K\bar{K} \\ \gamma \pi\pi \end{matrix} \right\}$

<u>Fixed Target, Central Production @ High Incident Beam Energy</u>



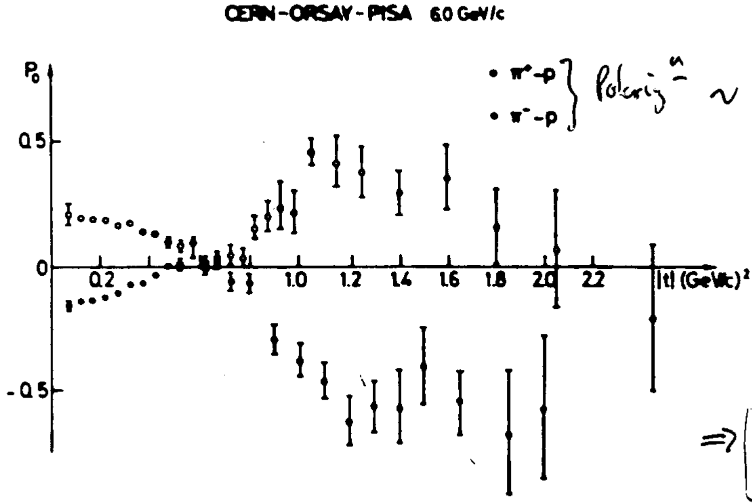
Hope is that dominantly Pomeron - Pomeron scattering.

— what is a Pomeron? { qq colour singlet? }

=> source of gluons
ie provides "gluon rich" initial state.

— is it just the shadow of everything else due to unitarity? ($\text{Im} f_{el}(0) \sim \sigma_T$)

— is it a Regge trajectory which is ~flat with intercept $(1+\varepsilon)$ contributing dominantly to $\text{Im}\, f_{el}(0)$ so that $\sigma_T \sim \left(\frac{s}{s_0}\right)^{\varepsilon}$ for $pp, \bar{p}p, \pi^{\pm}p$ etc. (ε used to be 0)
why does it behave as though $J^{PC} = 0^{++}$?



Polariz^n $\sim \mathrm{Im}\left(f_{nf}\, f_f^*\right)$

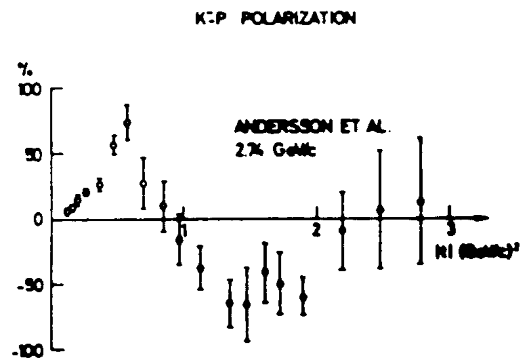
$f_{nf} = i\mathbb{P}$

$f_f \cong \rho$

$\sim \sqrt{t}\, i\, e^{\frac{i\pi\alpha_\rho}{2}} \sin\frac{\pi\alpha_\rho}{2}$

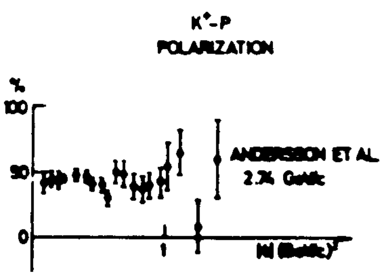
$\pi^\pm p \to \mathbb{P} \pm \rho$

$\Rightarrow \boxed{P_0|^{\pm} \sim \mp\sqrt{t}\, \sin^2\frac{\pi\alpha_\rho}{2}}$



$-\rho + A_2 \to e^{i\pi\alpha_\rho}$

$\Rightarrow P_0|^{\pm} \sim \sqrt{t}\, \cos\pi\alpha_\rho$

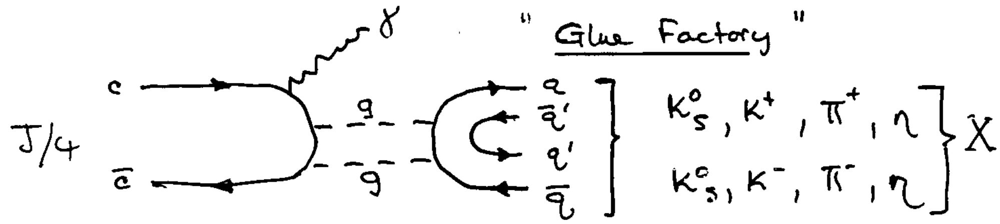


$\rho + A_2 \to 1$

$P_0|^{\pm} \sim \sqrt{t}$

① Introduction

The decay $J/\psi \to \gamma P\bar{P}$ ($P \equiv$ pseudoscalar meson)
is considered to proceed as follows:



(i) gg coupling $\Rightarrow$ X should be isoscalar

(ii) $C_X = +1 \Rightarrow (-1)^{J_X} = +1 \Rightarrow J_X$ is even
$\Rightarrow P_X$ is $+$

$$\boxed{\text{i.e. } X \text{ is } I=0,\; J^{PC} = (\text{even})^{++}}$$

(iii) for $M_X \lesssim 2\,\text{GeV}$, ignore $J^{PC} = 4^{++}$

$$\boxed{\text{i.e. analyze in terms of } J^{PC} = 0^{++} \text{ and } 2^{++}}$$

Generalities for $J/\psi \to \gamma\, P\bar{P}$ ($P \equiv$ pseudoscalar mesons)

(i) $P\bar{P}$ system has $C = +1$ $\{J/\psi \to \gamma \ldots\}$

(ii) $C = (-1)^L \Rightarrow$ $P\bar{P}$ system has even L
ie positive Parity

It follows that if $J/\psi \to \gamma\, X$
$\quad\quad\quad\quad\quad\quad\quad\quad \hookrightarrow P\bar{P}$

then $\boxed{X \text{ has } J^{PC} = (\text{even})^{++}}$

Isospin ~ 0 $\quad \left\{ \text{eg } \dfrac{BF(\gamma\pi^0)}{BF(\gamma\eta)} \sim 5\% \right.$

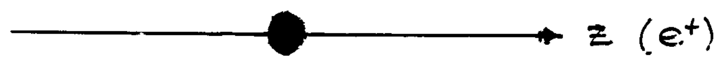
States considered

{Merging of γ's, Undetected soft γ's}
gives feedthrough from:

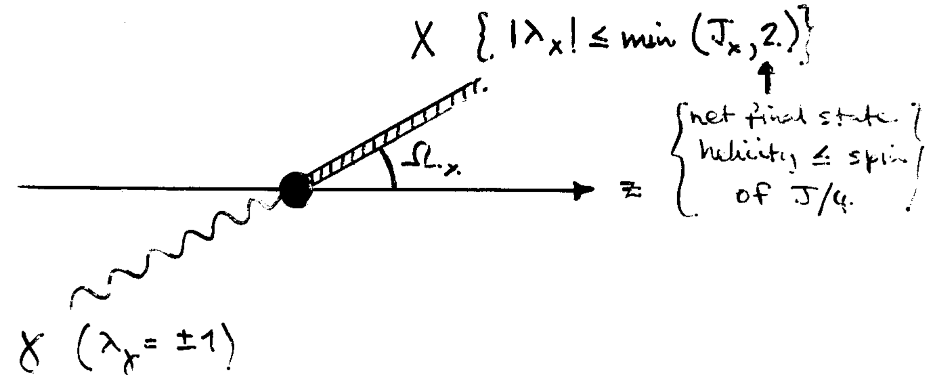
$\gamma\, K^0_S\, K^0_S$	$\cancel{\pi^0\, K^0_S\, K^0_S}$ ($C=+1$)
$\gamma\, K^+\, K^-$	$\pi^0\, K^+ K^-$ ($K^*\bar{K} + \bar{K}^*K$)
$\gamma\, \pi^+\, \pi^-$	$\pi^0\, \pi^+ \pi^-$ ($\rho\pi$)
$\gamma\, \pi^0\, \pi^0$	$\cancel{\pi^0\, \pi^0\, \pi^0}$ ($C=+1$)
$\gamma\, \eta\, \eta$	$\cancel{\pi^0\, \eta\, \eta}$ ($C \neq +1$)

($\gamma\,\pi^0\,\pi^0$ and $\gamma\,\eta\,\eta$: statistics too low for Amp. Analysis)

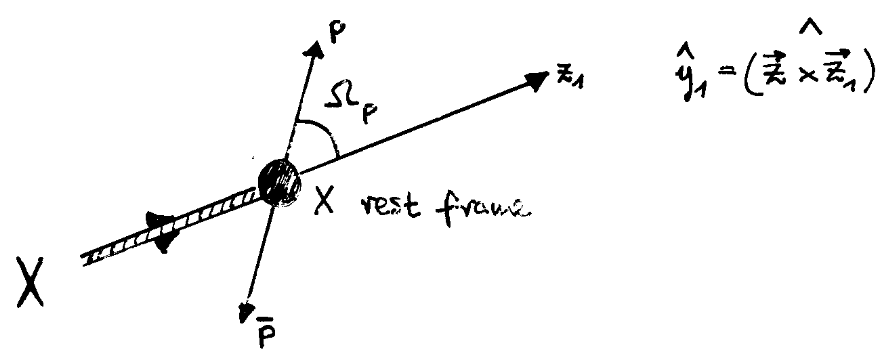
Helicity Formalism



J/ψ helicity $= \pm 1$



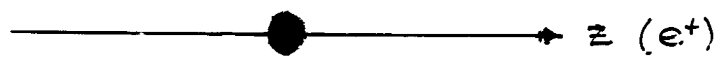
(*) After acceptance correction, final state angular dist^{n} does not depend on φ_X



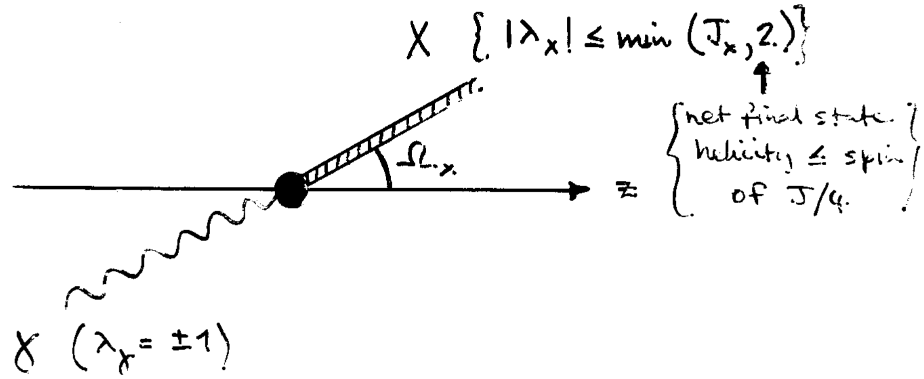
For each mass interval, m_X, the joint decay angular dist^{n} is specified in terms of $\theta_X, \theta_p, \varphi_p$

* For φ or ω recoil; need 2 more angles describing recoil decay

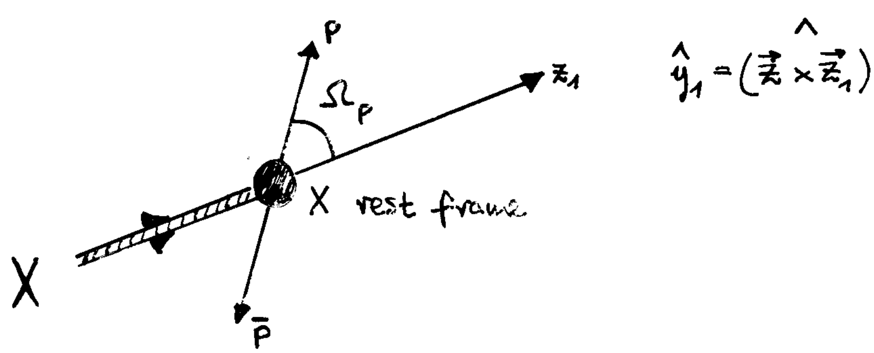
Helicity Formalism



J/ψ helicity $= \pm 1$



(*) After acceptance correction, final state angular distⁿ does not depend on φ_X



For each mass interval, m_X, the joint decay angular distⁿ is specified in terms of $\theta_X, \theta_p, \varphi_p$

* For φ or ω recoil; need 2 more angles describing recoil decay

The matrix element is given by

$$\mathbf{M} = \underbrace{A^1_{\lambda_X,\lambda_\gamma}\sqrt{\frac{3}{4\pi}}\mathbf{D}^{1*}_{\lambda_\psi,\lambda_X-\lambda_\gamma}(\Omega_X)}_{J/\psi \to \gamma X}\underbrace{A^{J_X}_{0,0}\sqrt{\frac{2\mathrm{J_X}+1}{4\pi}}\mathbf{D}^{J_X*}_{\lambda_X,0}(\Omega^*_p)}_{X \to p\bar{p}}.$$

and the distribution resulting is

$$4\pi\frac{dN}{dm\ d\Omega_X\ d\Omega^*_p} = \frac{3}{4}\sum_{\lambda_\psi,\lambda_\gamma}\left|\sum_{J_X,\lambda_X} a^{\lambda_\gamma}_{J_X,\lambda_X}\sqrt{\frac{2\mathrm{J_X}+1}{4\pi}}\mathbf{D}^{1*}_{\lambda_\psi,\lambda_X-\lambda_\gamma}(\Omega_X)\mathbf{D}^{J_X*}_{\lambda_X,0}(\Omega^*_p)\right|^2$$

with

$$a^{\lambda_\gamma}_{J_X,\lambda_X} = A^1_{\lambda_X,\lambda_\gamma}A^{J_X}_{0,0}\sqrt{PQ}\,.$$

Parity conservation implies

$$\underline{a^{-\lambda_\gamma}_{J_X,-\lambda_X} = a^{\lambda_\gamma}_{J_X,\lambda_X}} \qquad \Rightarrow \text{not}^{\underline{n}} \ a_{J,\lambda}$$

The distribution takes the form, in general:

$$4\pi\frac{dN}{dm\ d\Omega_X\ d\Omega^*_p} = \sum_{\mathrm{j,l,m,n}}\boxed{\mathbf{T}^{\mathrm{j}}_{\mathrm{l,m,n}}}\mathbf{Y}_{\mathrm{j,m}}(\Omega_X)\mathbf{Y}_{\mathrm{l,n}}(\Omega^*_p),$$

and the coeffts. are obtained from

$$\mathrm{T}^{j}_{l,m,n} = \int_{\Omega_X,\Omega^*_p}\mathbf{Y}^*_{\mathrm{j,m}}(\Omega_X)\mathbf{Y}^*_{\mathrm{l,n}}(\Omega^*_p)\frac{4\pi dN}{dm\ d\Omega_X\ d\Omega^*_p}d\Omega_X d\Omega^*_p$$

$$\cong \sum_{i=1}^{All\ Events} 4\pi\mathbf{Y}^*_{\mathrm{j,m}}(\Omega^i_X)\mathbf{Y}^*_{\mathrm{l,m}}(\Omega^{*i}_p);$$

For this reason the $\mathrm{T}^{j}_{l,m,n}$ are called the moments of the angular dist$^{\underline{n}}$.

In the present case :

since no dependence on φ_X

$$4\pi \frac{1}{dm} \frac{dN}{d\Omega_X\, d\Omega_\psi^*} = \sum_{j,l,m,n} \mathbf{T}^{j}_{l,m,n} \mathbf{Y}_{j,m}(\theta_X, \underbrace{\phi_X = 0}) \mathbf{Y}^*_{l,n}(\Omega^*_\psi),$$

with

$$\mathbf{T}^{j}_{l,m,n} = (-1)^{m} \frac{3}{4} \sum_{\lambda_\psi,\lambda_\gamma} \sum_{J_X,\lambda_X} \sum_{J'_X,\lambda'_X} a^{\lambda_\gamma}_{J_X,\lambda_X} a^{\lambda_\gamma *}_{J'_X,\lambda'_X} \sqrt{\frac{2J_X+1}{2j+1}} \sqrt{\frac{2J'_X+1}{2l+1}}$$

$$\times \left\langle 1, -\lambda_\psi; 1, \lambda_\psi | j, 0 \right\rangle \left\langle 1, \underbrace{-\lambda_X + \lambda_\gamma}; 1, \underbrace{\lambda'_X - \lambda_\gamma} | j, \underbrace{m} \right\rangle \quad \Rightarrow m = \lambda'_X - \lambda_X$$

$$n = \lambda'_X - \lambda_X \Leftarrow \quad \times \left\langle J_X, \underbrace{-\lambda_X}; J'_X, \underbrace{\lambda'_X} | l, \underbrace{n} \right\rangle \left\langle J_X, 0; J'_X, 0 | l, 0 \right\rangle,$$

Since $m = n = \lambda'_X - \lambda_X$, write $T^{j,m}_{l,m} \equiv T^{j}_{l,m,m}$

Parity cons^n $\rightarrow$ $T^{j,m}_{l,m} = T^{j,-m}_{l,-m}$ & choose $\lambda_\gamma = +1$

$$\boxed{\begin{aligned}
\mathbf{T}^{00}_{0,0} &= |a_{0,0}|^2 + |a_{2,0}|^2 + |a_{2,1}|^2 + |a_{2,2}|^2 \\
\mathbf{T}^{20}_{0,0} &= \frac{\sqrt{5}}{10}[|a_{0,0}|^2 + |a_{2,0}|^2 - 2|a_{2,1}|^2 + |a_{2,2}|^2] \\
\mathbf{T}^{00}_{2,0} &= \frac{\sqrt{5}}{5}[2\sqrt{5}Re(a_{0,0}a^*_{2,0}) + \frac{5}{7}(2|a_{2,0}|^2 + |a_{2,1}|^2 - 2|a_{2,2}|^2)] \\
\mathbf{T}^{20}_{2,0} &= \frac{1}{5}[\sqrt{5}Re(a_{0,0}a^*_{2,0}) + \frac{5}{7}(|a_{2,0}|^2 - |a_{2,1}|^2 - |a_{2,2}|^2)] \\
\mathbf{T}^{21}_{2,1} &= -\frac{\sqrt{3}}{10}[\sqrt{5}Re(a_{0,0}a^*_{2,1}) + \frac{5}{7}(Re(a_{2,0}a^*_{2,1}) - \sqrt{6}Re(a_{2,1}a^*_{2,2}))] \\
\mathbf{T}^{22}_{2,2} &= \frac{\sqrt{6}}{10}[\sqrt{5}Re(a_{0,0}a^*_{2,2}) - \frac{10}{7}(Re(a_{2,0}a^*_{2,2})] \\
\mathbf{T}^{00}_{4,0} &= \frac{1}{7}[6|a_{2,0}|^2 - 4|a_{2,1}|^2 + |a_{2,2}|^2] \\
\mathbf{T}^{20}_{4,0} &= \frac{\sqrt{5}}{70}[6|a_{2,0}|^2 + 8|a_{2,1}|^2 + |a_{2,2}|^2] \\
\mathbf{T}^{21}_{4,1} &= -\frac{\sqrt{3}}{14}[\sqrt{6}Re(a_{2,0}a^*_{2,1}) + Re(a_{2,1}a^*_{2,2})] \\
\mathbf{T}^{22}_{4,2} &= \frac{3\sqrt{2}}{14}Re(a_{2,0}a^*_{2,2}).
\end{aligned}}$$

i.e. have 10 moments in terms of 4 amplitudes $a_{0,0}$, $a_{2,0}$, $a_{2,1}$, a_{22}. $\{$ one may be chosen real $\Rightarrow$ 7 parameters$\}$

The angular dist^{n} can then be written

$$4\pi \frac{dN}{dm\; d\Omega_X\; d\Omega_p^*} = \sum_{\mathrm{j,l,(m \geq 0)}} (2 - \delta_{\mathrm{m0}})\, \mathbf{T}_{\mathrm{l,m}}^{\mathrm{j\,m}} Re\left(\mathbf{Y}_{\mathrm{j,m}}(\theta_X)\mathbf{Y}^*_{\mathrm{l,m}}(\Omega_p^*)\right), \qquad (m \geq 0)$$

Taking account of the acceptance, A, the observed moments are given by:

$$\int_{\Omega_X,\Omega_p^*,\Delta m} Re\left(\mathbf{Y}^*_{\mathrm{j',m'}}(\theta_X)\mathbf{Y}_{\mathrm{l',m'}}(\Omega_p^*)\right)\boxed{A(\Omega_X,\Omega_p^*,m)}\frac{4\pi dN}{d\Omega_X\; d\Omega_p^*\; dm} d\Omega_X d\Omega_p^* dm$$

$$= \sum_{\mathrm{j,l,(m \geq 0)}} \int_{\Omega_X,\Omega_p^*,\Delta m} Re\left(\mathbf{Y}^*_{\mathrm{j',m'}}(\theta_X)\mathbf{Y}_{\mathrm{l',m'}}(\Omega_p^*)\right)\boxed{A(\Omega_X,\Omega_p^*,m)}$$

$$\times\, (2 - \delta_{\mathrm{m0}})\boxed{\mathbf{T}_{\mathrm{l,m}}^{\mathrm{j\,m}}} Re\left(\mathbf{Y}_{\mathrm{j,m}}(\theta_X)\mathbf{Y}^*_{\mathrm{l,m}}(\Omega_p^*)\right) d\Omega_X d\Omega_p^* dm.$$

The observed moment (LHS above) is then

$$\mathbf{N}_{\mathrm{l',m'}}^{\mathrm{j'\,m'}} \equiv 4\pi \sum_{i=1}^{N_{observed}} Re(\mathbf{Y}^*_{\mathrm{j',m'}}(\theta_X^i)\mathbf{Y}_{\mathrm{l',m'}}(\Omega_p^{*i}));$$

and we have

$$\mathbf{N}_\mu = \sum_{\nu=1}^{10} C_{\mu,\nu}\mathbf{T}_\nu, \qquad \text{where} \begin{cases} \mathbf{N}_\mu = \mathbf{N}_{\mathrm{l',m'}}^{\mathrm{j'}} \text{ (observed)} \\ \mathbf{T}_\nu = \mathbf{T}_{\mathrm{l,m}}^{\mathrm{j}} \text{ (true.)} \end{cases}$$

with

$$\mathbf{C}_{\mu,\nu} = \int_{\Omega_X,\Omega_p^*} Re\left(\mathbf{Y}^*(\theta_X)\mathbf{Y}(\Omega_p^*)\right)_\mu \boxed{A(\Omega_X,\Omega_p^*,m)}(2 - \delta_{\mathrm{m0}})$$

$$\times\, Re\left(\mathbf{Y}(\theta_X)\mathbf{Y}^*(\Omega_p^*)\right)_\nu d\Omega_X d\Omega_p^* dm.$$

the Acceptance Matrix.

$C_{\mu\nu}$ is obtained by Monte Carlo:

$$\mathbf{C}_{\mu,\nu} = \frac{16\pi^2}{N_{generated}} \sum_{i=1}^{\boxed{N_{accepted}}} Re\left(\mathbf{Y}^*(\theta_X^i)\mathbf{Y}(\Omega_p^{*i})\right)_\mu (2 - \delta_{\mathrm{m0}})\, Re\left(\mathbf{Y}(\theta_X^i)\mathbf{Y}^*(\Omega_p^{*i})\right)_\nu$$

where the summation is over the accepted M.C. evts [illegible]

The covariance matrix corresponding to the observed moments is:

$$\mathbf{O}_{\mu,\nu} = 16\pi^2 \sum_{i=1}^{N_{observed}} Re\left(\mathbf{Y}^*(\theta_X^i)\mathbf{Y}(\Omega_p^{*i})\right)_\mu Re\left(\mathbf{Y}^*(\theta_X^i)\mathbf{Y}(\Omega_p^{*i})\right)_\nu ;$$

The true moments are then obtained from the observed moments by:

$$\mathbf{T}_\mu = \sum_{\nu=1}^{10} \mathbf{C}_{\mu,\nu}^{-1}\mathbf{N}_\nu,$$

and the corresponding covariance matrix is:

$$\mathbf{V} = \mathbf{C}^{-1}\mathbf{O}(\mathbf{C}^{-1})^+$$

The amplitudes are then obtained by minimizing the χ^2 given by:

measured true moments

$$\chi^2 = \sum_{\mu,\nu=1}^{10}\left(\overbrace{\sum_{\sigma=1}^{10}\mathbf{C}_{\mu\sigma}^{-1}\mathbf{N}_\sigma} - \underbrace{\mathbf{T}_\mu(\underline{\underline{\underline{a}}})}\right)\mathbf{V}^{-1}\left(\overbrace{\sum_{\sigma=1}^{10}\mathbf{C}_{\nu\sigma}^{-1}\mathbf{N}_\sigma} - \underbrace{\mathbf{T}_\nu(\underline{\underline{\underline{a}}})}\right)$$

true moments expressed in terms of amplitudes $a_{L,M}$

This is carried out separately in each mass bin using MINUIT (MINOS errors); many random starting points are tried in each bin to ensure that there are no missed solutions.

② Data & Analysis Procedures

{ SLAC Pubs. 5378, 5669 ; SLAC Report 386 }

(i) Perform separate amplitude analyses for :

$J/\psi \to \gamma K^0_S K^0_S$ (582 evts.)

$\to \gamma K^+ K^-$ (1755 evts.)

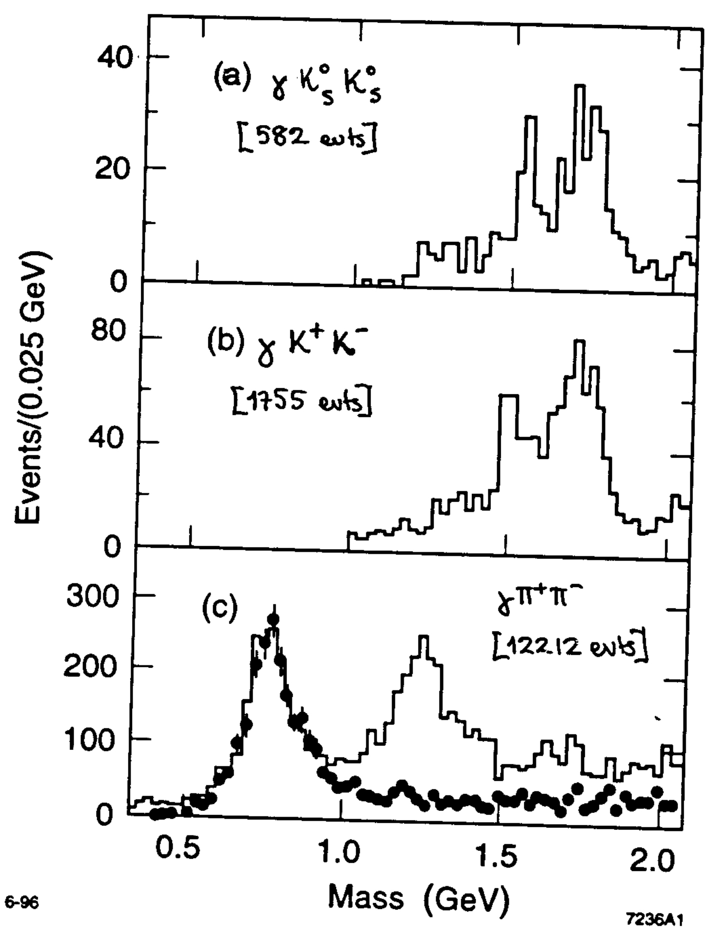
$\to \gamma \pi^+ \pi^-$ (12212 evts)

from 5.8×10^6 J/ψ evts.

(ii) The data are analyzed independently in each mass bin i.e. no continuity assumptions imposed.

(iii) For each bin, the joint production and decay angular distribution is expressed in terms of 3 angles θ_X, θ_P & φ_P.

$J/\psi \rightarrow$



⸸ feed through from $J/\psi \rightarrow \pi^+ \pi^- \pi^0$ Monte Carlo {SLAC-PUB 5674}

(iv) The distribution is as follows:

$$\frac{dN}{dM_X \, d\cos\theta_X \, d\Omega_P} =$$

$$\frac{1}{2} \sum_{\substack{j\,\ell \\ (m \geq 0)}} T^{\ell m}_{jm}(M_X) * (2-\delta_{m0}) \, \mathrm{Re}\left\{ Y_{\ell m}(\theta_X, \varphi_X{=}0)\, Y^*_{jm}(\Omega_P) \right\}$$

where the $T^{\ell m}_{jm}$ are the unnormalized moments

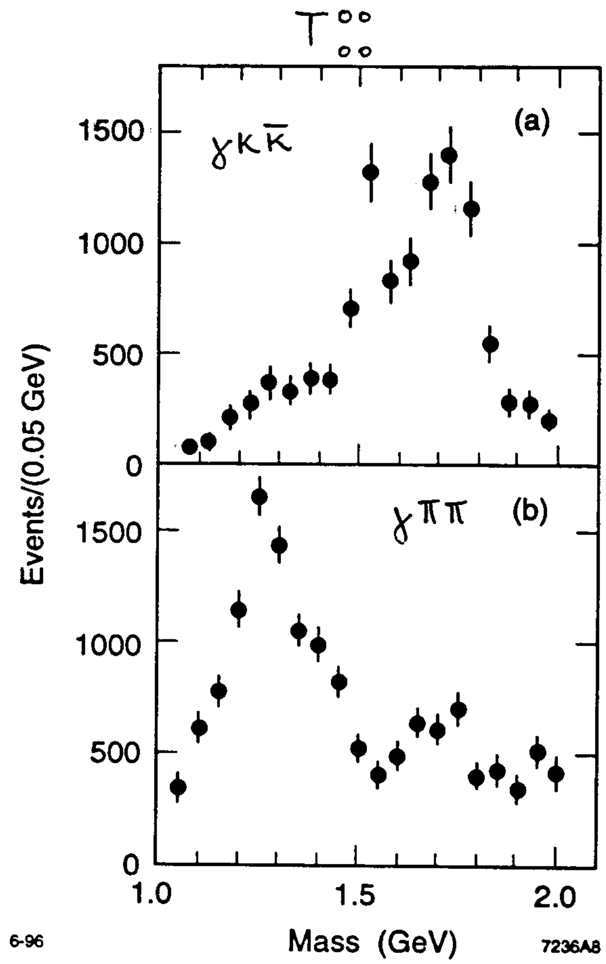
(v) The acceptance-corrected moments & covariance matrix are obtained in each bin in a model-independent way.

For $\gamma\pi^+\pi^-$, feedthrough from $\pi^+\pi^-\pi^0$ must be subtracted.

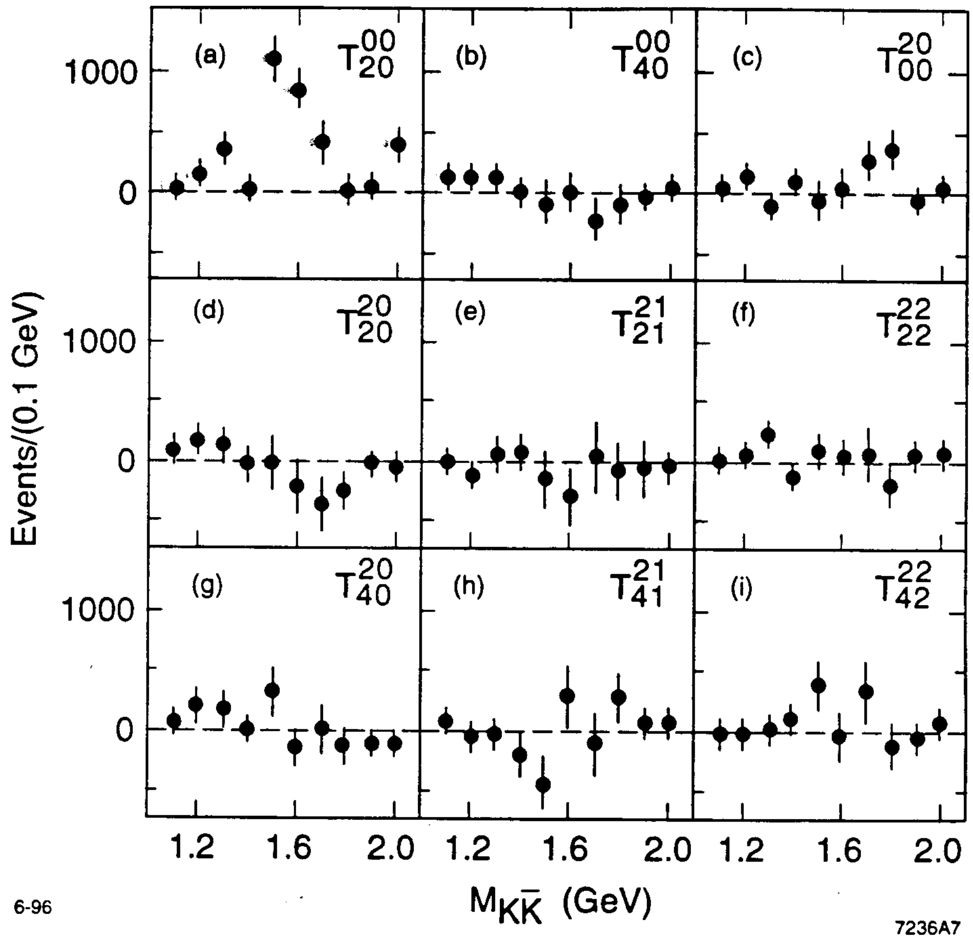
(vi) For $J^{PC} = 0^{++}$ & 2^{++}, 10 moments are obtained in each mass bin. The underlying amplitudes, $a_{j,\lambda}$, where j is spin & λ is helicity, are:

J^{PC}	Amplitudes	# Parameters
0^{++}	$a_{0,0}$	2
2^{++}	$a_{2,0}$ (chosen real)	1
	$a_{2,1}$, $a_{2,2}$	4
		7 in total

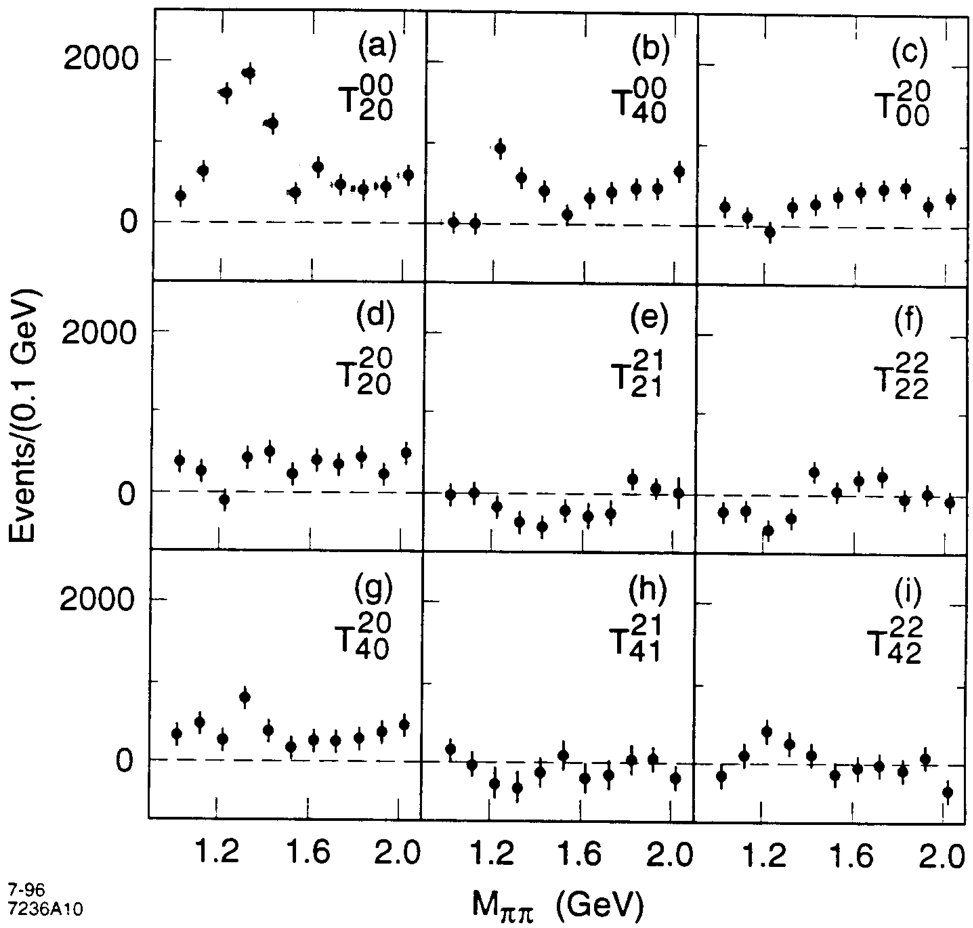
(*) Parity: $a_{2,-\lambda} = a_{2,\lambda}$



(*) After all corrections for acceptance & isospin C-G coeffts.



(a)
T^{00}_{20}
(b)
T^{00}_{40}
(c)
T^{20}_{00}
(d)
T^{20}_{20}
(e)
T^{21}_{21}
(f)
T^{22}_{22}
(g)
T^{20}_{40}
(h)
T^{21}_{41}
(i)
T^{22}_{42}
1000
0
Events/(0.1 GeV)
1.2
1.6
2.0
$M_{K\bar{K}}$ (GeV)
6-96
7236A7



2000
0
Events/(0.1 GeV)
(a) T_{20}^{00}
(b) T_{40}^{00}
(c) T_{00}^{20}
(d) T_{20}^{20}
(e) T_{21}^{21}
(f) T_{22}^{22}
(g) T_{40}^{20}
(h) T_{41}^{21}
(i) T_{42}^{22}
1.2
1.6
2.0
$M_{\pi\pi}$ (GeV)
7-96
7236A10

$$\mathbf{T}^{00}_{0,0} = |a_{0,0}|^2 + |a_{2,0}|^2 + |a_{2,1}|^2 + |a_{2,2}|^2$$

$$\mathbf{T}^{20}_{0,0} = \frac{\sqrt{5}}{10}[|a_{0,0}|^2 + |a_{2,0}|^2 - 2|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$\mathbf{T}^{00}_{2,0} = \frac{\sqrt{5}}{5}[2\sqrt{5}Re(a_{0,0}a^*_{2,0}) + \frac{5}{7}(2|a_{2,0}|^2 + |a_{2,1}|^2 - 2|a_{2,2}|^2)]$$

$$\mathbf{T}^{20}_{2,0} = \frac{1}{5}[\sqrt{5}Re(a_{0,0}a^*_{2,0}) + \frac{5}{7}(|a_{2,0}|^2 - |a_{2,1}|^2 - |a_{2,2}|^2)]$$

$$\mathbf{T}^{21}_{2,1} = -\frac{\sqrt{3}}{10}[\sqrt{5}Re(a_{0,0}a^*_{2,1}) + \frac{5}{7}(Re(a_{2,0}a^*_{2,1}) - \sqrt{6}Re(a_{2,1}a^*_{2,2}))]$$

$$\mathbf{T}^{22}_{2,2} = \frac{\sqrt{6}}{10}[\sqrt{5}Re(a_{0,0}a^*_{2,2}) - \frac{10}{7}(Re(a_{2,0}a^*_{2,2})]$$

$$\mathbf{T}^{00}_{4,0} = \frac{1}{7}[6|a_{2,0}|^2 - 4|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$\mathbf{T}^{20}_{4,0} = \frac{\sqrt{5}}{70}[6|a_{2,0}|^2 + 8|a_{2,1}|^2 + |a_{2,2}|^2]$$

$$\mathbf{T}^{21}_{4,1} = -\frac{\sqrt{3}}{14}[\sqrt{6}Re(a_{2,0}a^*_{2,1}) + Re(a_{2,1}a^*_{2,2})]$$

$$\mathbf{T}^{22}_{4,2} = \frac{3\sqrt{2}}{14}Re(a_{2,0}a^*_{2,2}).$$

(iv) Do we really need $J^{PC} = 0^{++}$?

Angular distribution in $\cos\Theta_p$ is not flat.

In 1.7 GeV region, Crystal Ball ($\eta\eta$) and Mark III asked is $J^{PC} = 0^{++}$ or 2^{++}

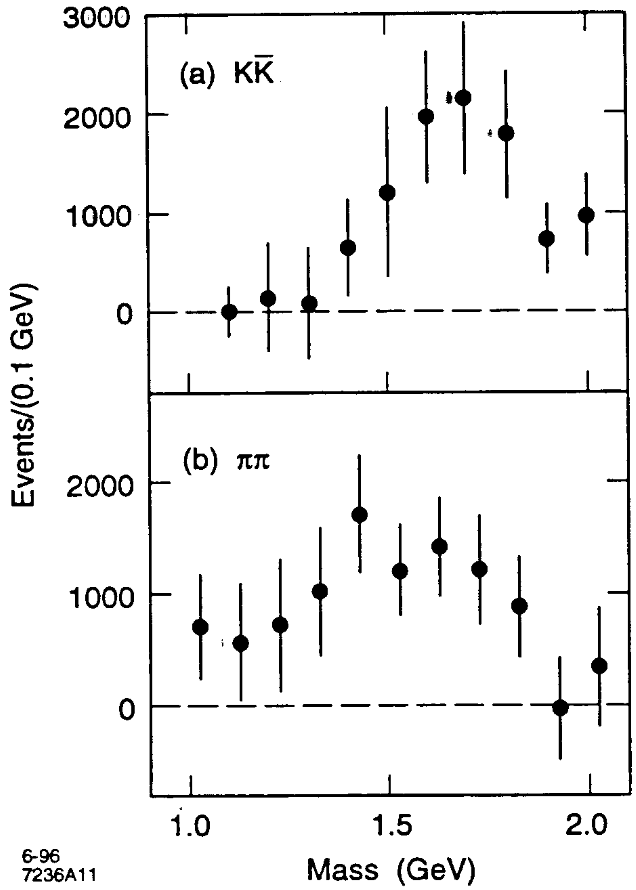
$\Rightarrow$ 2^{++} preferred.

Answer: YES

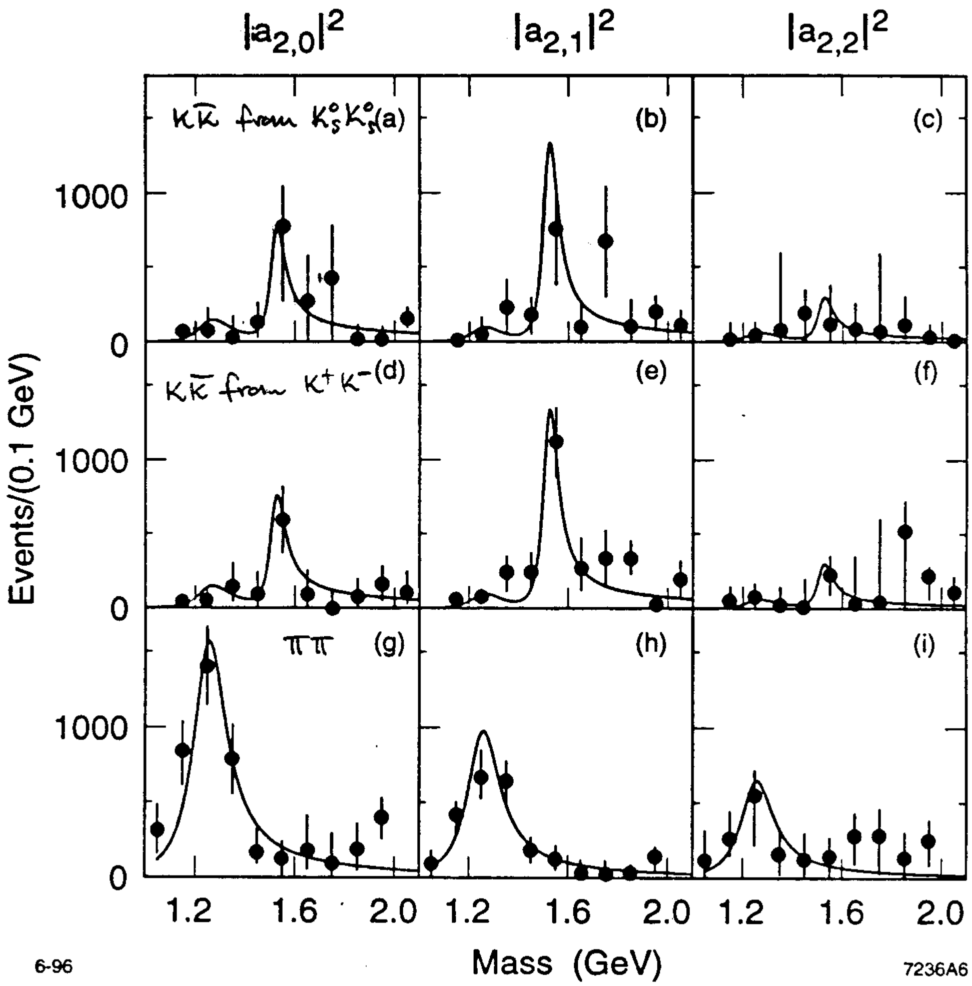
(v) Fitting Procedure

- χ^2 minimization (3 d.o.f.) in each mass bin
- multiple random starts in the parameter space (~20-30)
- acceptable fit has confidence level > 1%
- errors estimated by MINOS search
- ambiguities: when exist, 2 solutions
  central value is mean
  errors are extrema of errors

$|a_{0,0}|^2 + \sqrt{5}\,\mathrm{Re}(a_{0,0}a^*_{2,0})$ (+)



(+) $$\frac{2}{3}\left(T^{00}_{00} + \sqrt{5}\,T^{40}_{00}\right) + \frac{\sqrt{5}}{3}\left(T^{00}_{20} + \sqrt{5}\,T^{20}_{20}\right) - \frac{4}{3}\left(T^{00}_{40} + \sqrt{5}\,T^{20}_{40}\right)$$


$|a_{2,0}|^2$
$|a_{2,1}|^2$
$|a_{2,2}|^2$
KK̄ from $K^0_S K^0_S$ (a)
(b)
(c)
KK̄ from K^+K^- (d)
(e)
(f)
ππ (g)
(h)
(i)
1000
0
Events/(0.1 GeV)
1.2
1.6
2.0
Mass (GeV)
6-96
7236A6

③ Results

$J^{PC} = 2^{++}$

(i) $K\bar{K}$: $f_2'(1525)$

(ii) $\pi\pi$: $f_2(1270)$

(iii) No clear signal at 1.7 GeV ; cannot rule out possibility of small signal

(iv) $\pi\pi$ seems to increase slowly toward 2 GeV ; possible 2^{++} state just above 2 GeV ?

(v) fits :

$K\bar{K}$: $f_2'(1525) + f_2(1270)$ [PDG values]

$\pi\pi$: $f_2(1270)$ [$f_2'(1525)$ rejected when included]

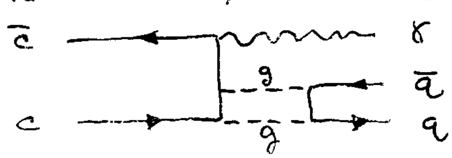
(vi) Are the results sensible?

Branching Ratio	Value from fits	PDG Value
$\frac{f_2(1270) \to K\bar{K}}{f_2(1270) \to \pi\pi}$	$0.155^{+0.044}_{-0.041}$	0.054 ± 0.006
$\frac{f_2'(1525) \to \pi\pi}{f_2'(1525) \to K\bar{K}}$	~ 0	0.009 ± 0.002
$\frac{f_2'(1525) \to \eta\eta}{f_2'(1525) \to K\bar{K}}$	$0.215^{+0.181}_{-0.159}$	0.116 ± 0.004

Helicity amplitude ratio squared		$f_2(1270) \to \pi\pi$	$f_2'(1525) \to K\bar{K}$
$\left\lvert \frac{a_{21}}{a_{20}} \right\rvert^2$	measured	$0.58^{+0.12}_{-0.10}$	$1.66^{+1.10}_{-0.57}$
	predicted$^{(*)}$	0.58 [0.57]	0.77 [0.81]
$\left\lvert \frac{a_{22}}{a_{20}} \right\rvert^2$	measured	$0.39^{+0.11}_{-0.11}$	$0.28^{+0.36}_{-0.21}$
	predicted$^{(*)}$	0.29 [0.30]	0.49 [0.52]

previous Mk.III & DM2 analyses → ~0

* Krammer, PL 74B (1978) 361 {imaginary}
[ ] Körner et al., NPB 229 (1983) 115 {total}



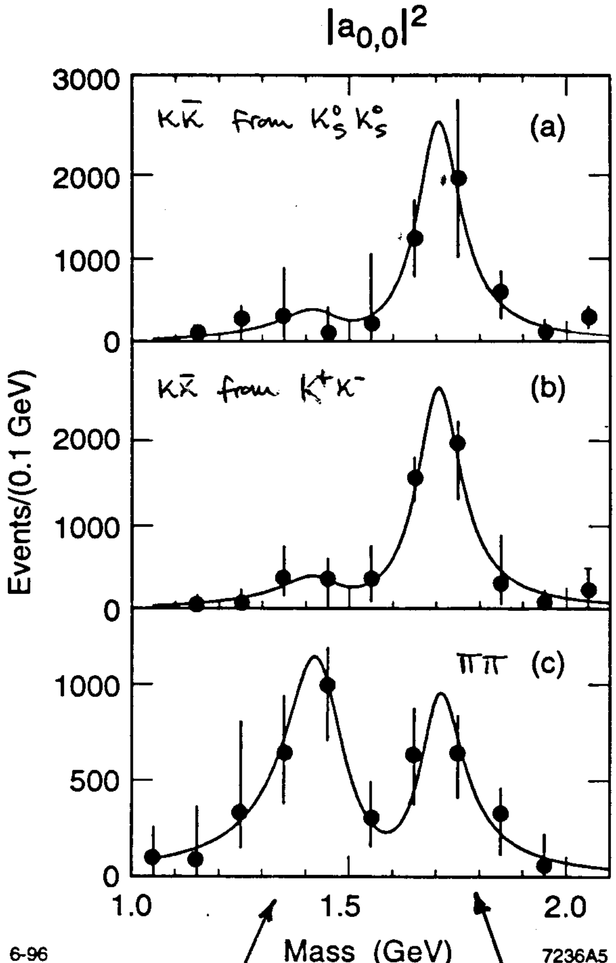
(vii) J/ψ Branching Fractions

$$Bf\,(J/\psi \to \gamma f_2(1270)) = 15.6 \pm 1.4 \quad * 10^{-4}$$

(PDG: $13.8 \pm 1.4 \quad * 10^{-4}$)

$$Bf\,(J/\psi \to \gamma f_2'(1525)) = 4.0^{+0.8}_{-0.7} \quad * 10^{-4}$$

(PDG: $6.3 \pm 1.0 \quad * 10^{-4}$)



M (MeV) : 1429^{+43}_{-37} 1704^{+16}_{-23}

Γ (MeV) : 169^{+111}_{-46} 124^{+52}_{-44}

$J^{PC} = 0^{++}$

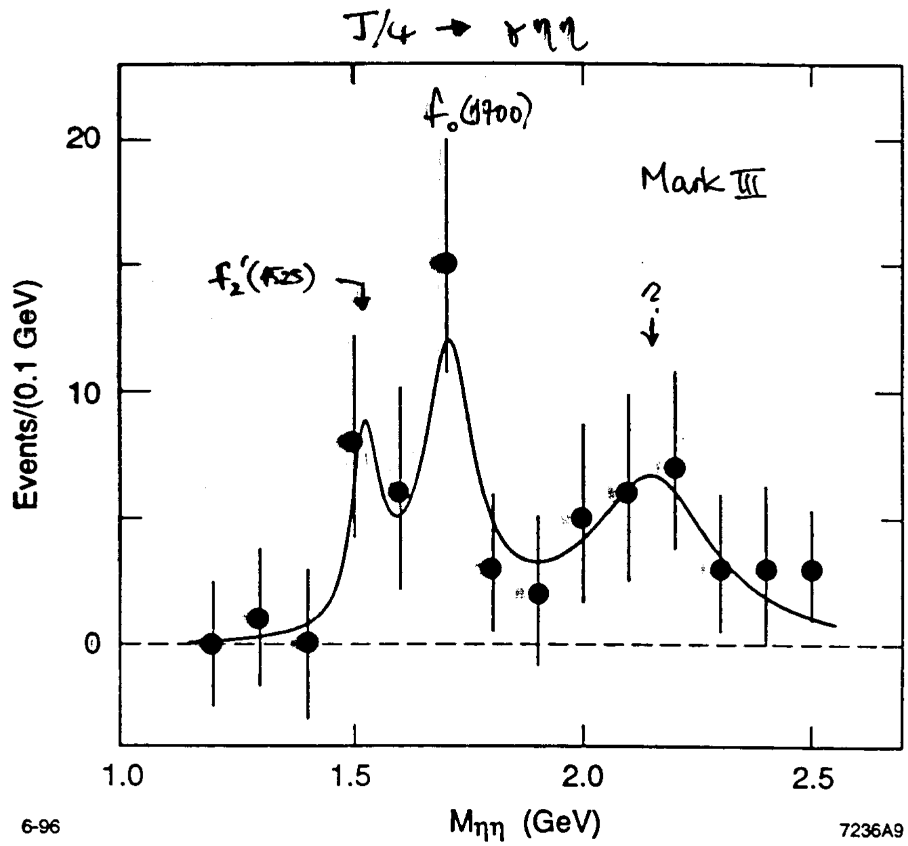
$K\bar{K}$: peak at ~1.7 GeV ; small shoulder at ~1.4 GeV

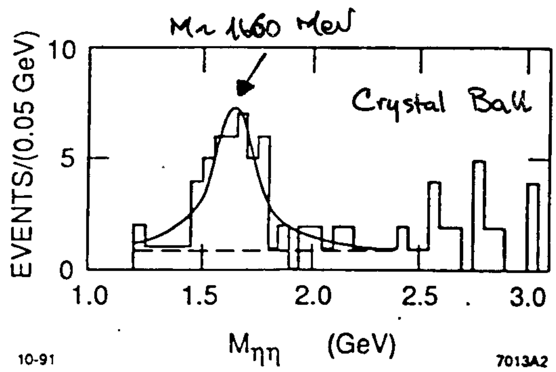
$\pi\pi$: peak at ~1.4 GeV ; smaller peak at ~1.7 GeV

Fit the 3 distns. simultaneously with 2 coherent BW's [fitted rel. phase ~ 0°] $\frac{\chi^2}{\text{ndof}} = \frac{6.5}{2.1}$

Results:

Measurement	$f_0(1430)$	$f_0(1700)$
Mass (MeV)	1429^{+43}_{-37}	1704^{+16}_{-23}
Width (MeV)	169^{+111}_{-76}	124^{+52}_{-44}
Branching Ratios	$\frac{K\bar{K}}{\pi\pi}$: $0.149^{+0.215}_{-0.133}$	$\frac{\pi\pi}{K\bar{K}}$: $0.268^{+0.169}_{-0.120}$ $\frac{\eta\eta}{K\bar{K}}$: $0.194^{+0.094}_{-0.097}$
Branching Fraction $J/\psi \to \gamma R$: $R \to \pi\pi + K\bar{K}$	$4.3^{+2.7}_{-1.7} * 10^{-4}$	$9.5^{+2.5}_{-2.0} * 10^{-4}$
$J/\psi \to \gamma R$: $R \to \pi\pi + K\bar{K} + \eta\eta$	$4.3^{+2.7}_{-1.7} * 10^{-4}$	$10.9^{+2.6}_{-2.1} * 10^{-4}$


J/ψ → γηη
f₀(1700)
Mark III
f₂'(1525)
?
20
10
0
Events/(0.1 GeV)
1.0
1.5
2.0
2.5
Mηη (GeV)
6-96
7236A9


M ~ 1660 MeV
Crystal Ball
EVENTS/(0.05 GeV)
10
5
0
1.0
1.5
2.0
2.5
3.0
Mηη (GeV)
10-91
7013A2

Summary

Final State (X)	$Bf\{J/\psi \to \gamma X\}$
$f_2(1270)$	$15.6 \pm 1.4 \quad * 10^{-4}$ $\langle \lvert D \rvert^2 \rangle = 5.2 \pm 0.5 \quad * 10^{-4}$
$f_2'(1525)$	$4.0 \begin{smallmatrix} +0.8 \\ -0.7 \end{smallmatrix} \quad * 10^{-4}$ $\langle \lvert D \rvert^2 \rangle = 1.3 \pm 0.3 * 10^{-4}$
$f_0(1430) \to p\bar{p}$	$4.3 \begin{smallmatrix} +2.7 \\ -1.7 \end{smallmatrix} \quad * 10^{-4}$
$f_0(1700) \to p\bar{p}$	$10.9 \begin{smallmatrix} +2.6 \\ -2.1 \end{smallmatrix} \quad * 10^{-4}$
$f_0(1750) \to 4\pi$ (*)	$10.9 \pm 1.3 \quad * 10^{-4}$

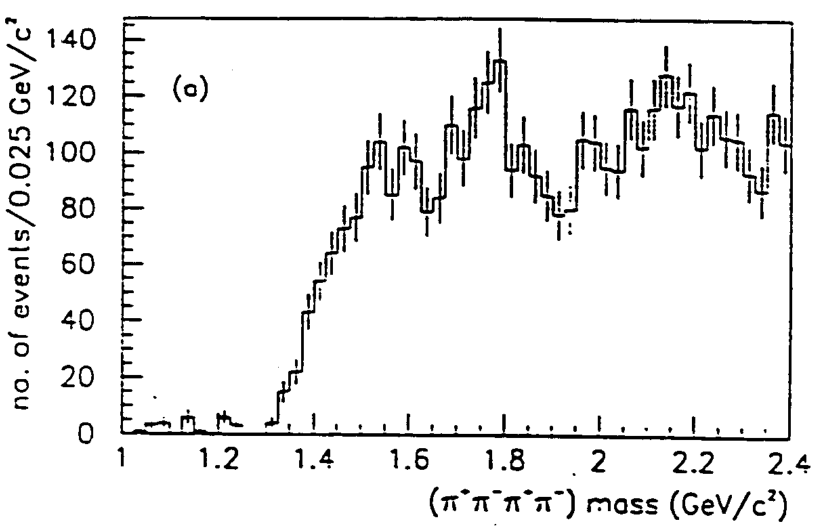
$f_2(1270)$, $f_2'(1525)$, $a_2(1320)$ & $K_2^*(1430)$ form the ~ ideally mixed ground state $J^{PC} = 2^{++}$ nonet.

What are the $f_0(1430)$ & $f_0(1700)$?

(*) Bugg et al PLB 353 (1995) 378

$J/\psi \to \gamma\, \pi^+\pi^-\pi^+\pi^-$ [Mark III]

1.75 GeV



M (4π) GeV/c²

(5) Summary

In $J/\psi \to \gamma P\bar{P}$: { The results obtained separately from $K^0_S K^0_S$ & K^+K^- agree well }

(i) The $f_2(1270)$ & $f_2'(1525)$ have masses, widths & branching ratios consistent with PDG values (including $\eta\eta$)

(ii) The relative magnitudes of the $f_2(1270)$ and $f_2'(1525)$ helicity amplitudes agree with predictions (Krammer)

(iii) Two $J^{PC} = 0^{++}$ states are observed as follows:

	$f_0(1430)$	$f_0(1700)$
Mass:	1429^{+43}_{-37}	1704^{+16}_{-23}
Width:	169^{+111}_{-76}	124^{+52}_{-44}
Br :	$\frac{K\bar{K}}{\pi\pi} = 0.15^{+0.22}_{-0.13}$	$\frac{\pi\pi}{K\bar{K}} = 0.27^{+0.17}_{-0.12}$
		$\frac{\eta\eta}{K\bar{K}} = 0.19 \pm 0.10$

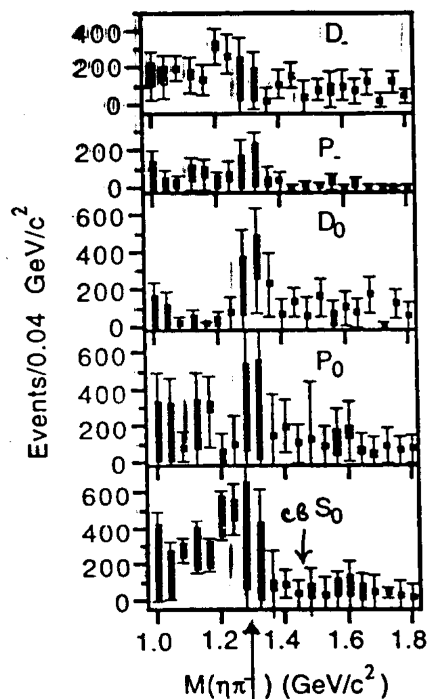
(iv)

X	$BF[J/\psi \to \gamma X] * 10^4$	
$f_2(1270)$	15.6 ± 1.4	; $\langle \lvert D \rvert^2 \rangle \to 5.2 \pm 0.5$
$f_2'(1525)$	$4.0^{+0.8}_{-0.7}$	; $\langle \lvert D \rvert^2 \rangle \to 1.3 \pm 0.3$
$f_0(1430) \to P\bar{P}$	$4.3^{+2.7}_{-1.9}$	
$f_0(1700) \to P\bar{P}$	$10.9^{+2.6}_{-2.1}$	
$+4\pi$	$21.8^{+3.0}_{-2.5}$	

(*) No evidence for Crystal Barrel $f_0(1500)$

$\pi^- p \to \pi^- \eta p$ @ 18 GeV/c

[BNL 852]



Is this due to resonant behaviour at ~ 1.3 GeV ?

[cf. $K\pi$ S-wave]

Interpretation

1) The $J^{PC}=0^{++}$ $f_0(1700)$ is composed mainly of the lowest mass gluonium state; it contains a significant amount of $u\bar{u}$ and $d\bar{d}$, but very little $s\bar{s}$.

2) The $J^{PC}=0^{++}$ $f_0(1430)$ corresponds mainly to the $u\bar{u}/d\bar{d}$ member of the ground state $^3P_0$ $q\bar{q}$ nonet; it contains some $s\bar{s}$ & a significant admixture of the gluonium ground state.

3) The ground state $J^{PC}=0^{++}$ nonet is made up as follows: {mixing angle θ_0}

$$K^*_0(1410),\ \underset{\text{(LASS etc)}}{f_0'(1520)},\ \left\{\begin{matrix} a_0(1450)\,? \\ a_0(1300)\,? \end{matrix}\right\},\ f_0(1430)$$

$a_0(1450)$: $\tan^2\theta_0 > 0 \Rightarrow m[f_0(1430)] < 1400$ MeV
ideal mixing $\Rightarrow \sim 1120$ MeV

$a_0(1300)$: $\tan^2\theta_0 > 0 \Rightarrow m[f_0(1430)] < 1457$ MeV
ideal mixing $\Rightarrow$ 1290 MeV

In both cases, suggests that mixing of $q\bar{q}$ to state with gluonium increases the observed mass

Clearly, it would be of help to resolve the question of the mass of the a_0

4) Amsler & Close have proposed that the $f_0(1500)$ found by Crystal Barrel might be composed mainly of the ground state glueball. However, it does not appear to be produced strongly in J/ψ radiative decay to $\rho\bar{\rho}$. Also, its small $K\bar{K}/\pi\pi$ b.r. would seem to argue against its being the mainly $s\bar{s}$ quarkonium state.

One possibility is that it is neither of the above, but is e.g. a 4 quark state. Not very satisfactory.

5) What does Lattice Gauge Theory have to say?

Weingarten et al

Use the GF11 Parallel Computer at IBM Research, N.Y.

Obtain:

(1) $m_G = 1710 \pm 63$ MeV

{ UKQCD - Wuppertal: $m_G = 1625 \pm 92$ MeV }

$\Rightarrow \quad \langle m_G \rangle = 1693 \pm 60$ MeV

(2) $\Gamma_G \{ \Sigma' P\bar{P} \} = 108 \pm 29$ MeV

Using $\dfrac{\Gamma_G \{ \Sigma' P\bar{P} \}}{\Gamma_G \text{ (tot)}} \sim 1/2$

$\Rightarrow \quad \Gamma_G(\text{tot}) \cong 220 \pm 60$ MeV

(3) $\Gamma(\pi\pi) : \Gamma(K\bar{K}) = 10^{+25}_{-9}$ %

$\Gamma(\eta\eta) : \Gamma(K\bar{K}) = 30 \pm 10$ %

$\Gamma(\eta\eta') : \Gamma(K\bar{K}) = 8 \pm 5$ %

(4) $m(s\bar{s})$ quarkonium ~ 200 MeV lower in mass than m_G

{ Amsler & Close suggest $f_0(1710)$ is mainly $s\bar{s}$ quarkonium }

(5) "To leading order in the valence approx^{n}, with valence quark annihilation turned off, corresponding isotriplet and isosinglet states composed of u and d quarks will be degenerate"
{Weingarten: NPB (Proc. Supp.) 53 (1997) 232}

J^{PC}	$I=1$	$I=0$
0^{++}	a_0 (1480)	f_0 (1430)
1^{++}	a_1 (1260)	f_1 (1285)
1^{+-}	b_1 (1235)	h_1 (1170)
1^{--}	ρ (770)	ω (782)
2^{++}	a_2 (1320)	f_2 (1275)
3^{--}	ρ_3 (1690)	ω_3 (1670)
4^{++}	a_4 (2040)	f_4 (2050)

(6) In this same refce., a glueball – quarkonium mixing scheme is proposed; using observed masses for $|n\bar{n}\rangle$, $|s\bar{s}\rangle$ & $|gg\rangle$ (main components) of 1390, 1500 & 1710 MeV W. gets:

$$m_G = 1635 \text{ MeV}, \quad m_{s\bar{s}} = 1516 \text{ MeV}$$

$$|1710\rangle = 0.87\,|gg\rangle + 0.34\,|s\bar{s}\rangle + 0.36\,|n\bar{n}\rangle$$

$$|1500\rangle = -0.19\,|gg\rangle + 0.90\,|s\bar{s}\rangle - 0.40\,|n\bar{n}\rangle$$

$$|1390\rangle = -0.46\,|gg\rangle + 0.28\,|s\bar{s}\rangle + 0.84\,|n\bar{n}\rangle$$

Then :

a) to the extent that J/ψ radiative decay projects $|gg\rangle$, the prediction would be

$$f_0(1710) : f_0(1390) : f_0(1500) =$$
$$4 : 1 : 0.2$$ (o.k.)

b) $f_0(1500) \to K\bar{K}$ is suppressed because of destructive intfce. between $|s\bar{s}\rangle$ and $|n\bar{n}\rangle$ (?)

c) $f_0(1710)$ is mostly $|gg\rangle$
$f_0(1500)$ is mostly $|s\bar{s}\rangle$
$f_0(1390)$ is mostly $|n\bar{n}\rangle$
seems to be a consistent with data from many different sources

Why is the TCF necessary?

Statistics .....

① To study $J/\psi \to \gamma\pi\pi$, $K\bar{K}$
need $\gtrsim 10^8$ evts. in sample
[Weiguo: would take ~6yrs of BES]

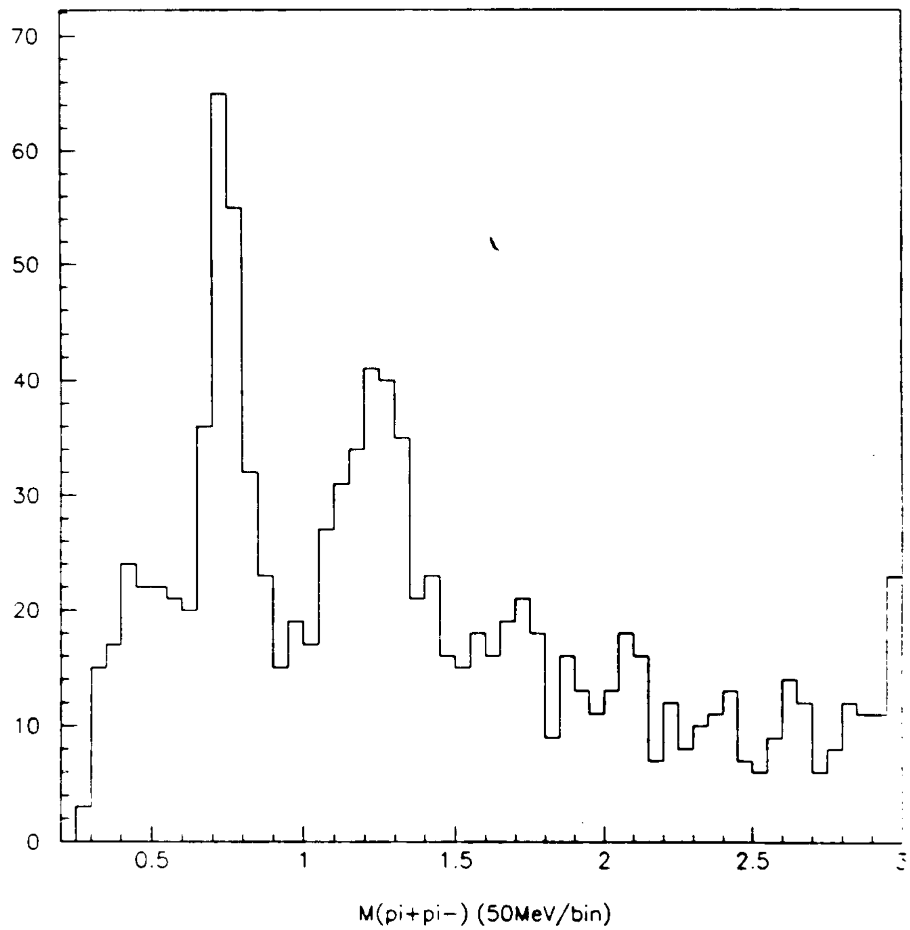
② To study $J/\psi \to \gamma\eta\eta$, $\eta\eta'$,
need at least 10^9 evts

③ Such samples make same studies with recoil ϕ or ω possible
(2 more angles — analysis in 5 dim. for each mass bin; but ω has bkgnd problems)

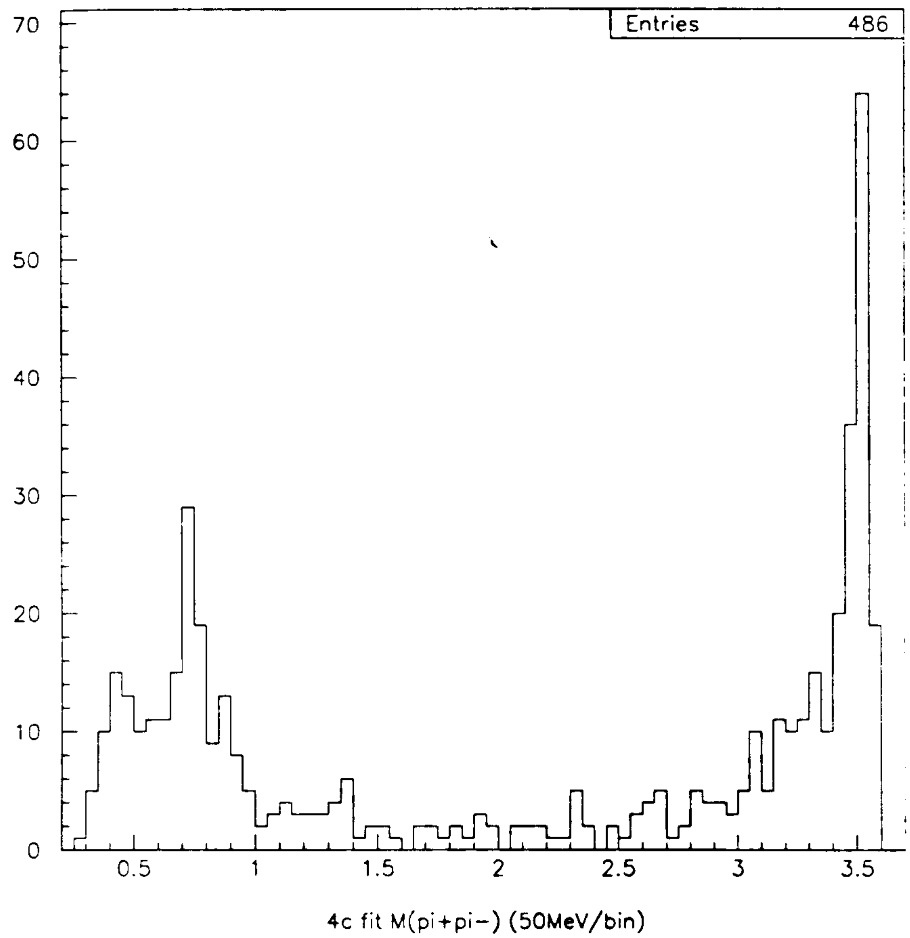
④ Can extend analyses above 2GeV & include $J^{PC} = 4^{++}$ if necessary.

⑤ Can perform sim. analyses with ψ'; with ϕ, ω recoil, go up to ~2.5 GeV.

(*) ψ' preview.

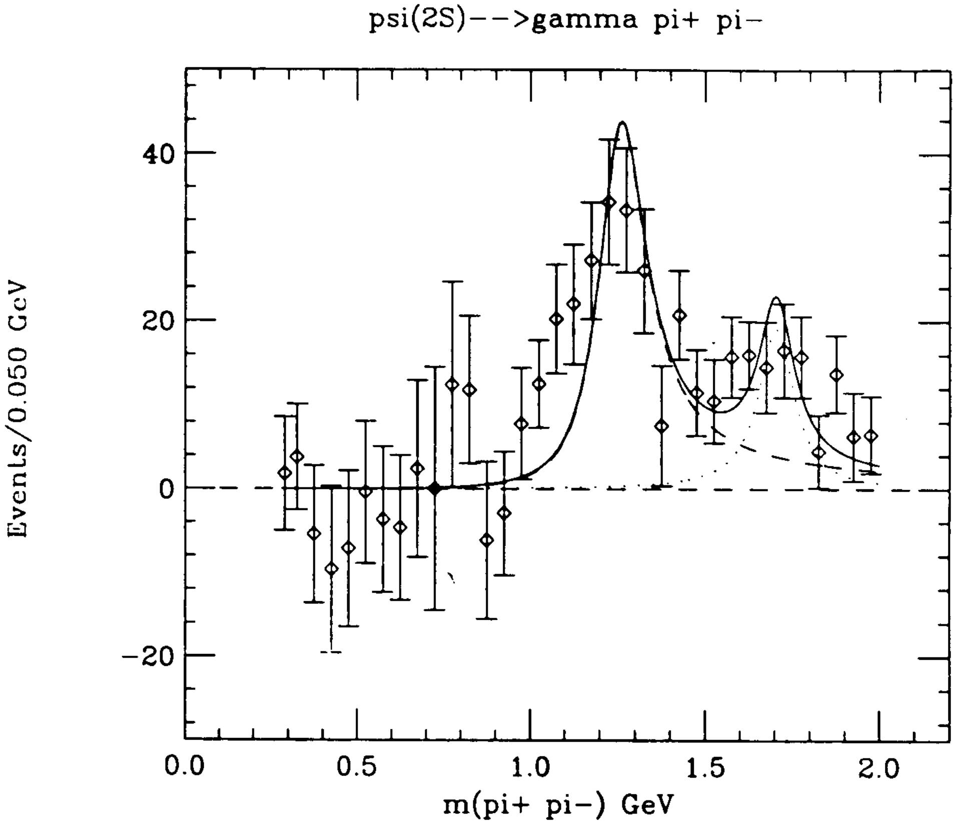


Final $m(\pi^+\pi^-)$ distn from $\psi' \to \gamma\pi^+\pi^-$



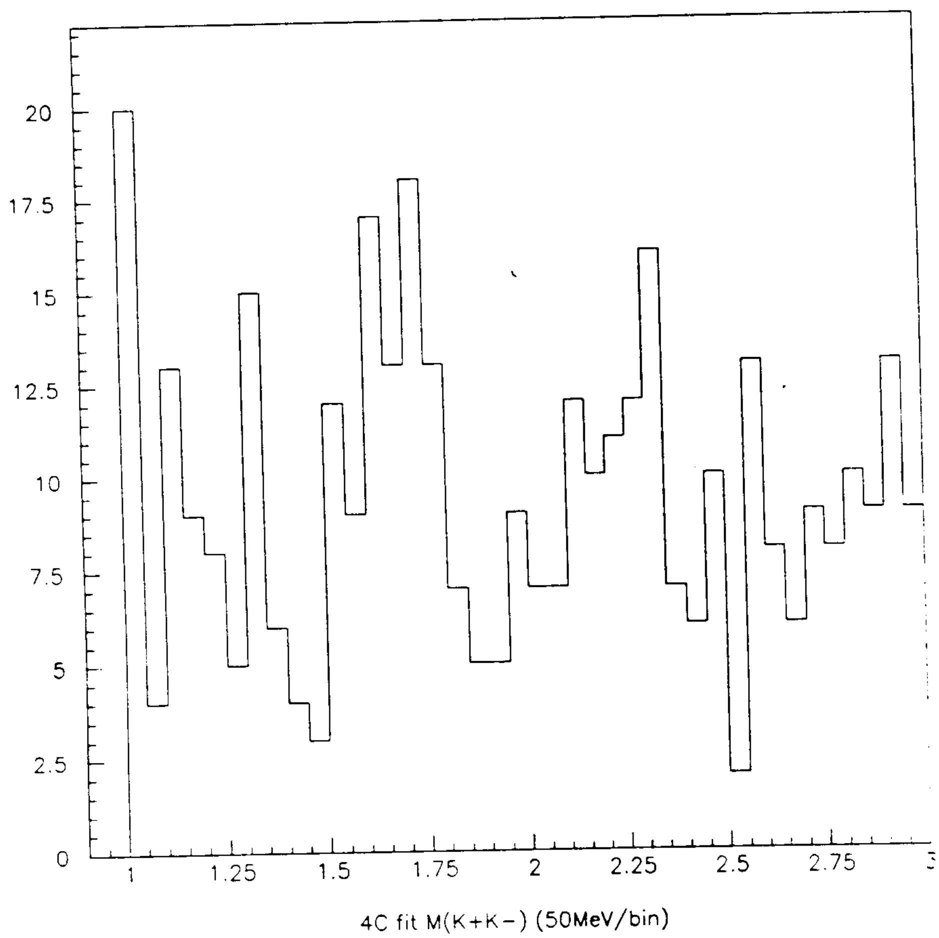
$M(\pi^+\pi^-)$

$\gamma\pi^+\pi^-$ from τ threshold data sample



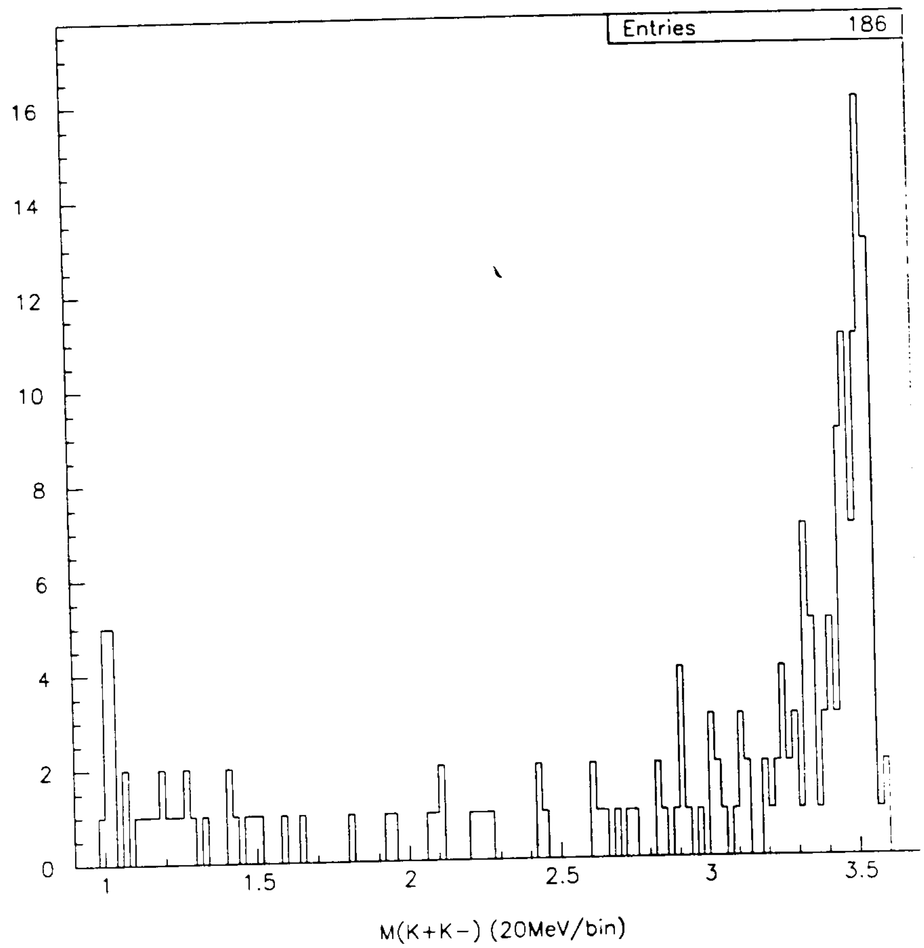
[ψ' − $\overset{\text{scaled}}{\tau \text{ threshold}}$] data

~50K evts net ⇒ ~ 10^9 ψ'

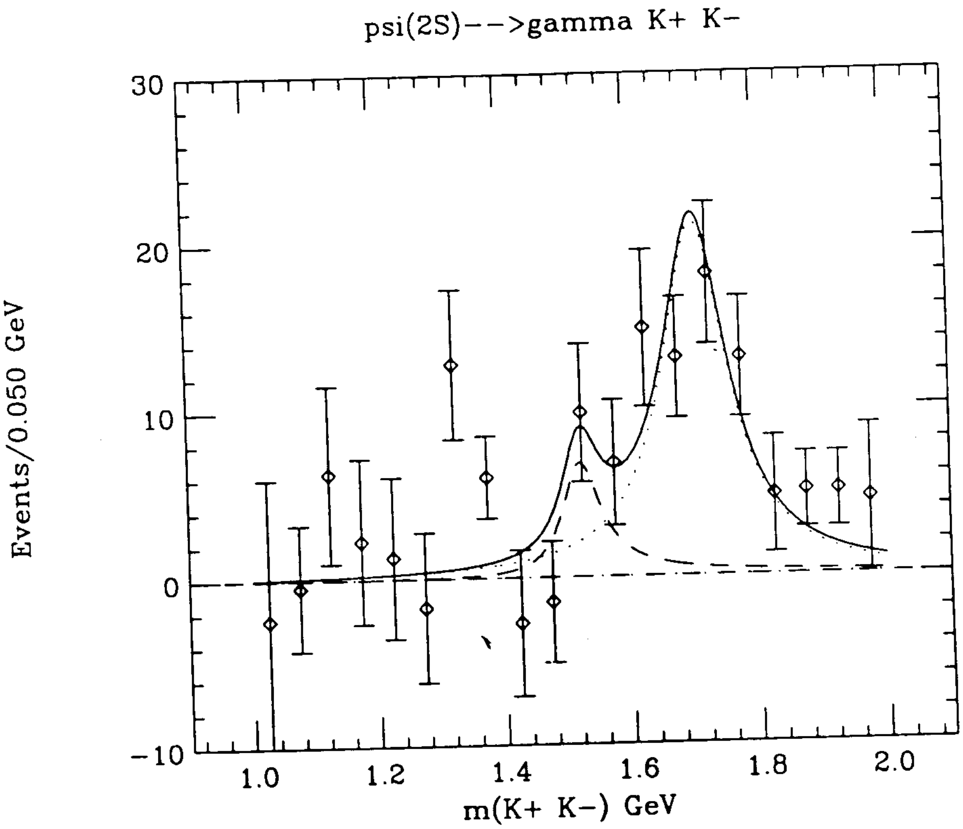


$M(K^+K^-)$

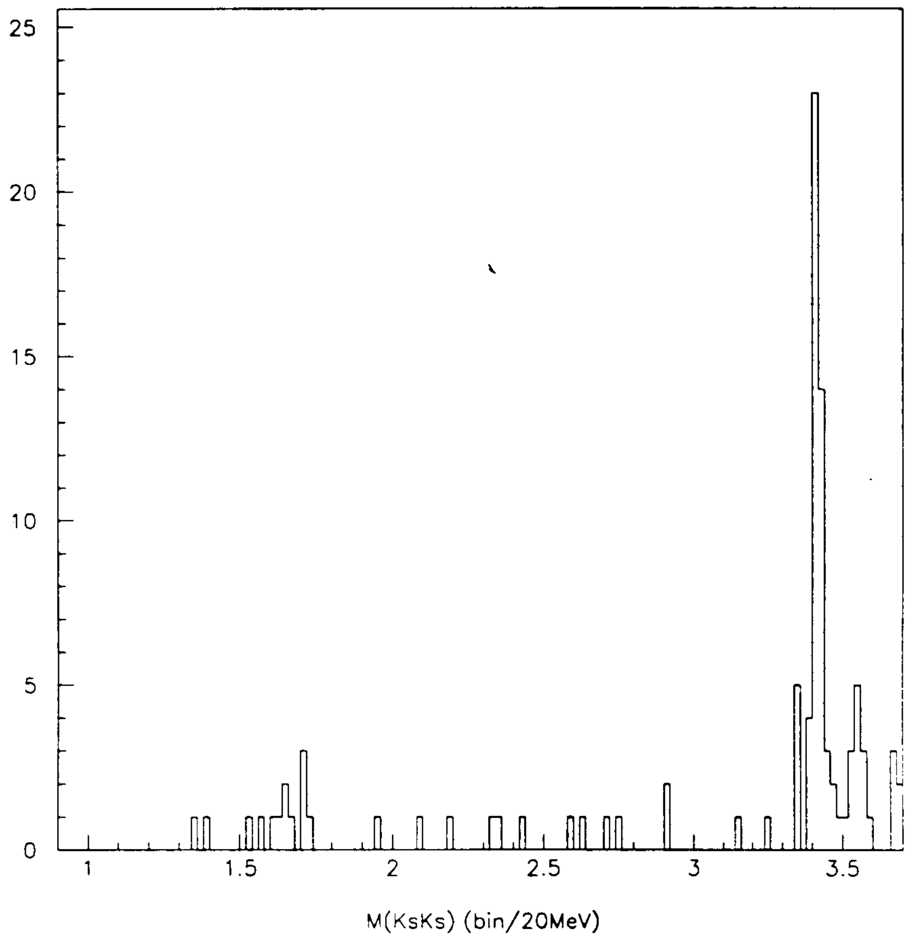
$M_{K^+K^-}$ from $\psi' \to \gamma K^+K^-$



γK^+K^- from τ threshold data



[ψ' - τ scaled threshold] data



$M\ (K^0_S\ K^0_S)$

$\psi' \rightarrow \gamma K^0_S K^0_S$

Two (Seemingly) Mundane Questions

1. Is $J/\psi \to \pi^+\pi^-\pi^0$ $\{K^+K^-\pi^0\}$ understood ?

2. Does $\psi' \to \pi^+\pi^-\pi^0$ $\{K^+K^-\pi^0\}$ follow suit ?

Remember :

$$\frac{Bf(\psi' \to X)}{Bf(J/\psi \to X)} \hat{\simeq} \frac{Bf(\psi' \to e^+e^-)}{Bf(J/\psi \to e^+e^-)} \simeq 14\%$$

{ Applequist & Politzer PRL 34 (1975) 43 }

Evidence for $\rho(1600)$ from the Decay $J/\psi \to \pi^- \pi^+ \pi^0$

(Mark III)

Mark III : Typical e^+e^- collider detector

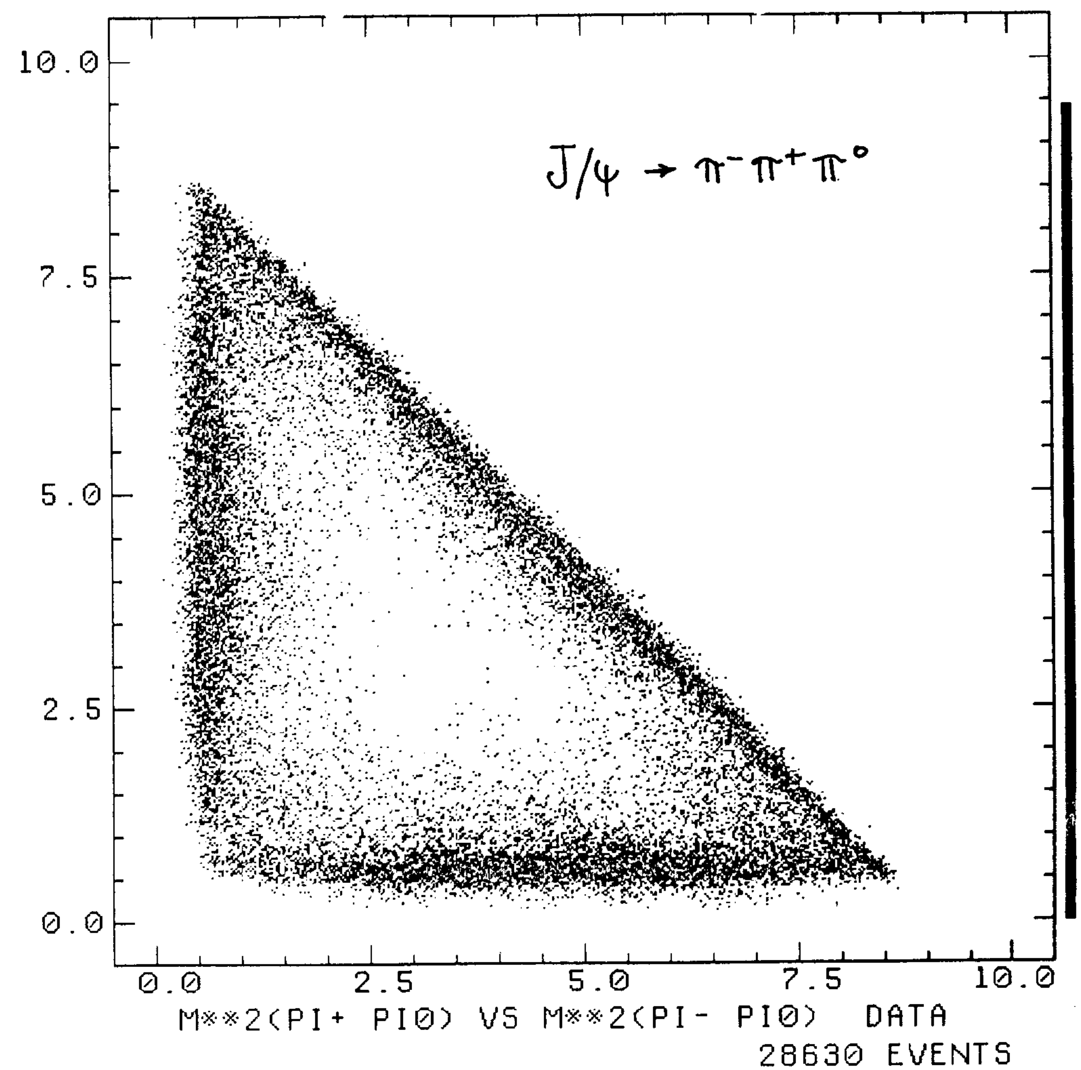
- essentially cylindrical geometry
- TOF & μ detection
- charged particle track reconstruction over $\sim 0.85 * 4\pi$
- γ detection over $\sim 0.95 * 4\pi$ (end-cap shower cntrs.)

For the events corresponding to

$$J/\psi \to \pi^- \pi^+ \pi^0$$
$$\hookrightarrow \gamma\gamma$$

- all final state tracks are detected & reconstructed
- the events are subjected to 5C kinematic fits
- events having $E_\gamma \geq 60$ MeV for each photon are retained.

→ data sample $\sim$ 28,000 events


$J/\psi \to \pi^- \pi^+ \pi^0$
10.0
7.5
5.0
2.5
0.0
0.0
2.5
5.0
7.5
10.0
M**2(PI+ PI0) VS M**2(PI- PI0) DATA
28630 EVENTS

The Amplitude Describing $J/\psi \to \rho\pi \to \pi^+\pi^-\pi^0$
[SLAC PUB 5674]

(i) The J/ψ is produced with helicity ± 1.

(ii) For each helicity configuration, the properly-symmetrized amplitude describing decay via $\rho\pi$ takes the form:

$$\boxed{\alpha_i = A_i^0 + A_i^+ + A_i^-}$$

where i denotes the helicity & the superscript denotes the charge of the ρ; in this regard it is important to specify the convention adopted in defining the decay angles of each ρ i.e. which π is to be used to define the angles. The proper choice is given by

$$\boxed{\alpha_i = A_i^0(\pi^-,\pi^+) + A_i^+(\pi^+,\pi^0) + A_i^-(\pi^0,\pi^-)}$$

where the first π in the bracket is the one which is used to define the angles; this ensures Bose symmetry w.r.t. the final amplitude.

The intensity distribution is then given by

$$I = \left(|\alpha_1|^2 + |\alpha_{-1}|^2\right) \cdot d(\text{LIPS})$$

$$= \left(|A_1^0 + A_1^+ + A_1^-|^2 + |A_{-1}^0 + A_{-1}^+ + A_{-1}^-|^2\right) \cdot d(\text{LIPS})$$

(iii) Using e.g. the helicity formalism, it can be shown that

$$\boxed{\begin{aligned} A_1 &= \frac{B \sin\theta_1}{\sqrt{2}} \left\{ \cos\varphi_1 + i \cos\theta \sin\varphi_1 \right\} e^{i\varphi} \\ \text{\&} \quad A_{-1} &= \frac{B \sin\theta_1}{\sqrt{2}} \left\{ \cos\varphi_1 - i \cos\theta \sin\varphi_1 \right\} e^{-i\varphi} \end{aligned}}$$

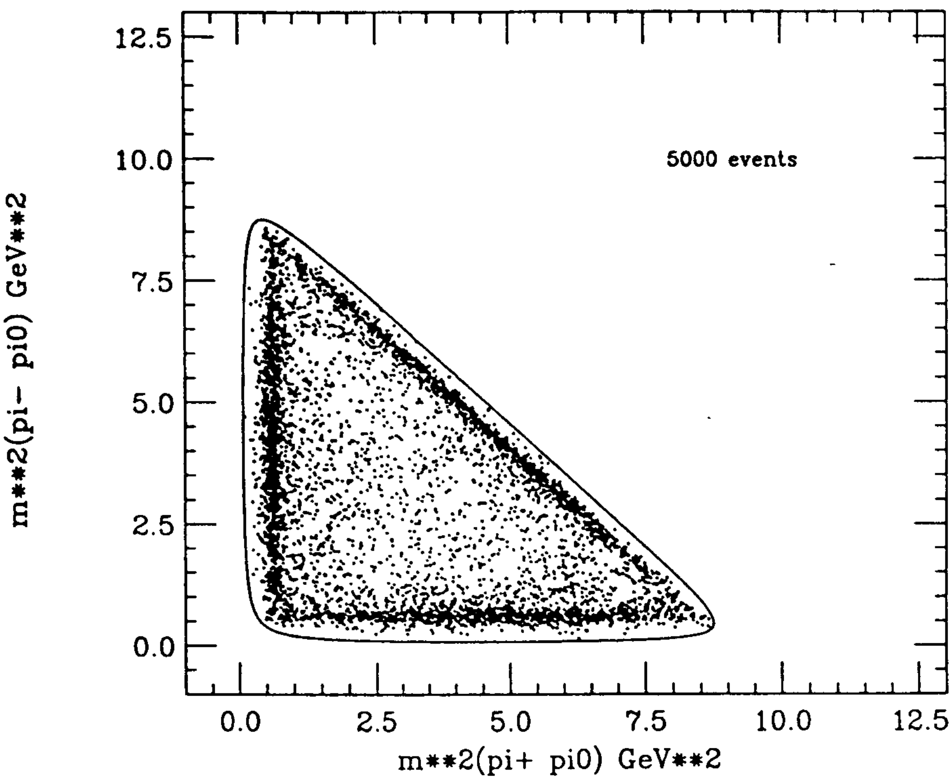
where (θ, φ) are the ρ polar & azimuthal angles in the lab frame.

(θ_1, φ_1) are the decay π polar & azimuthal angles in the ρ rest frame.

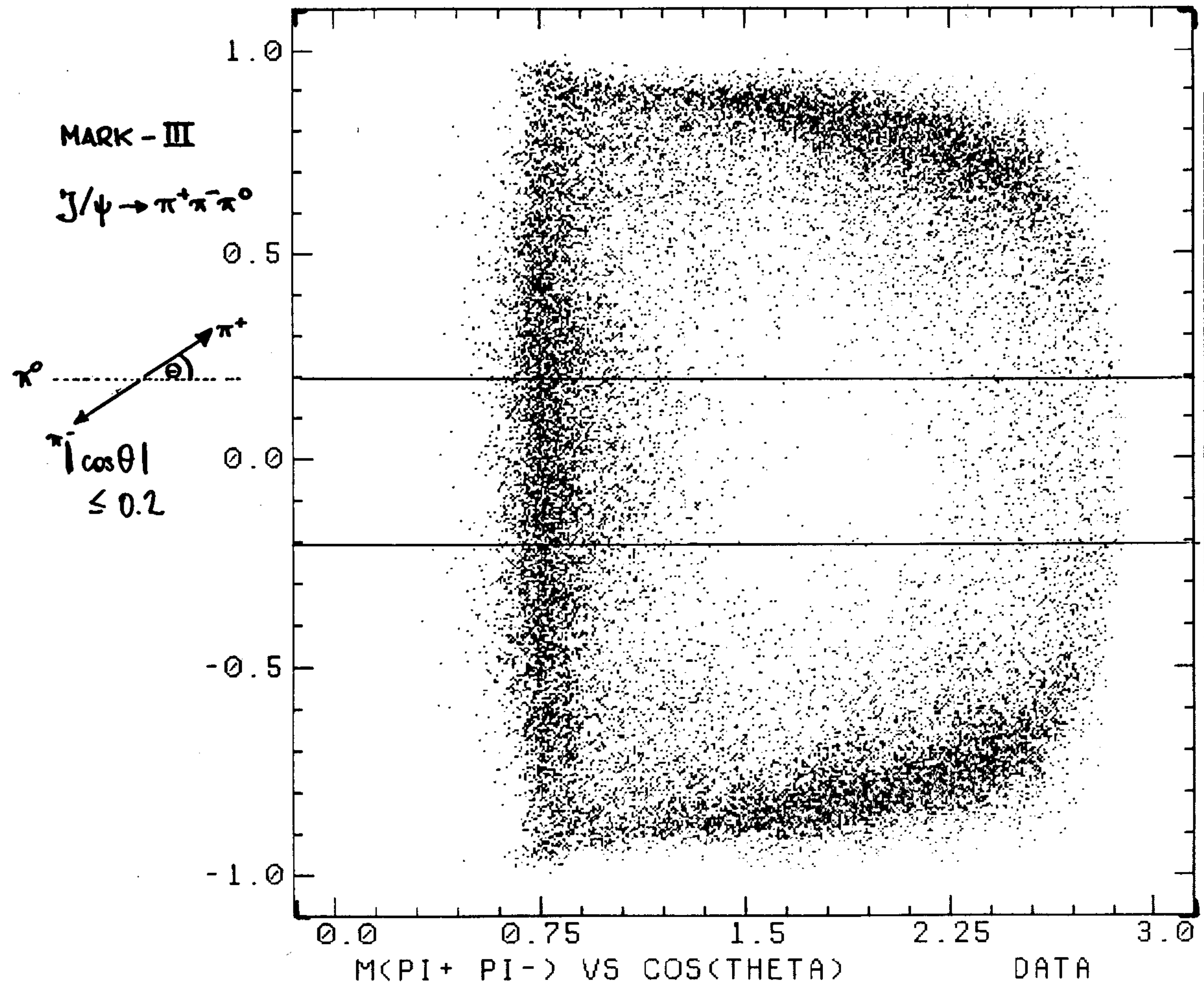
$$\text{\&} \quad B = \frac{C \cdot p \left(\frac{q}{q_0}\right) \sqrt{\frac{1 + (q_0 R)^2}{1 + (q R)^2}}}{m_\rho^2 - m^2 - i m_\rho \Gamma(m)}$$

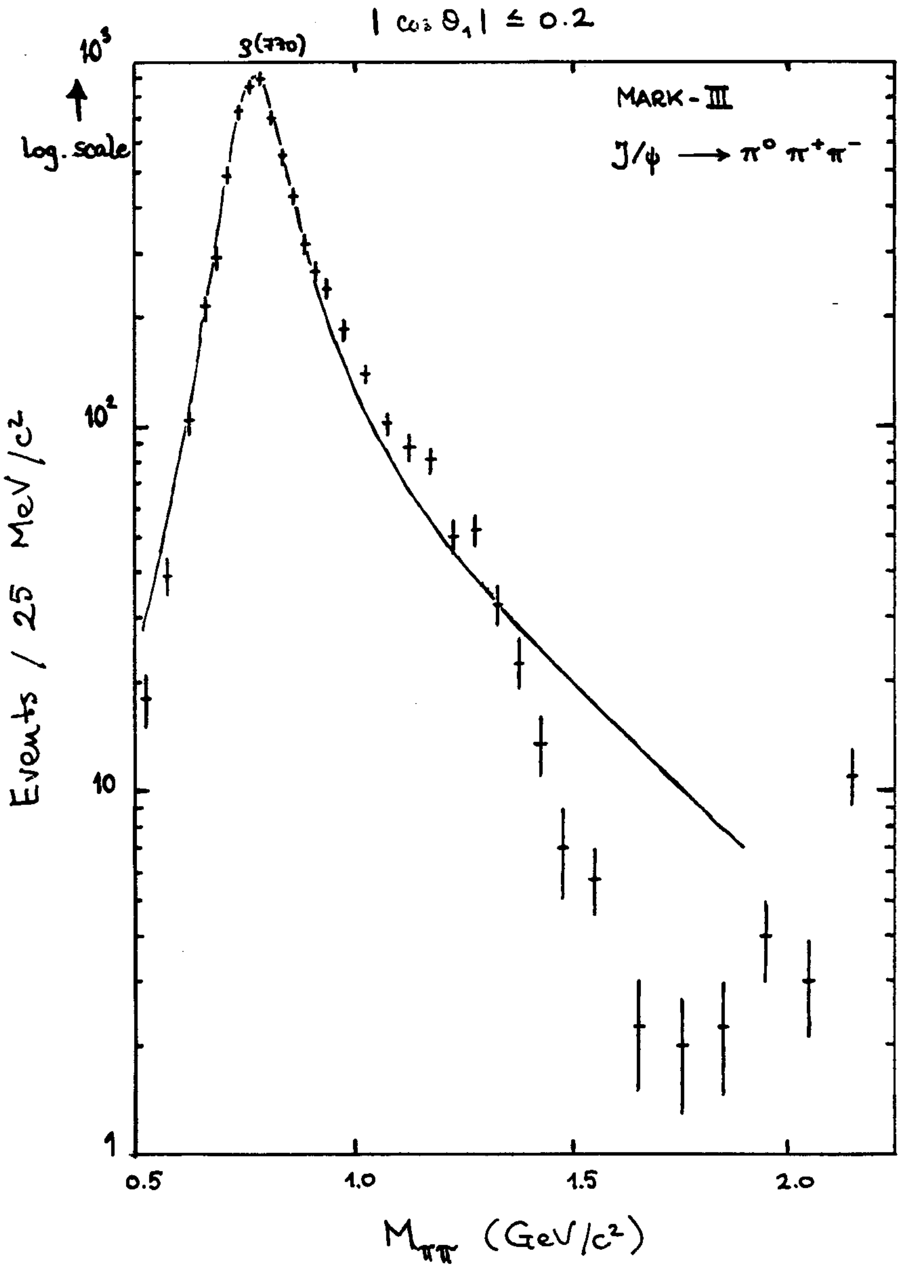
* Fits give $R = 0$

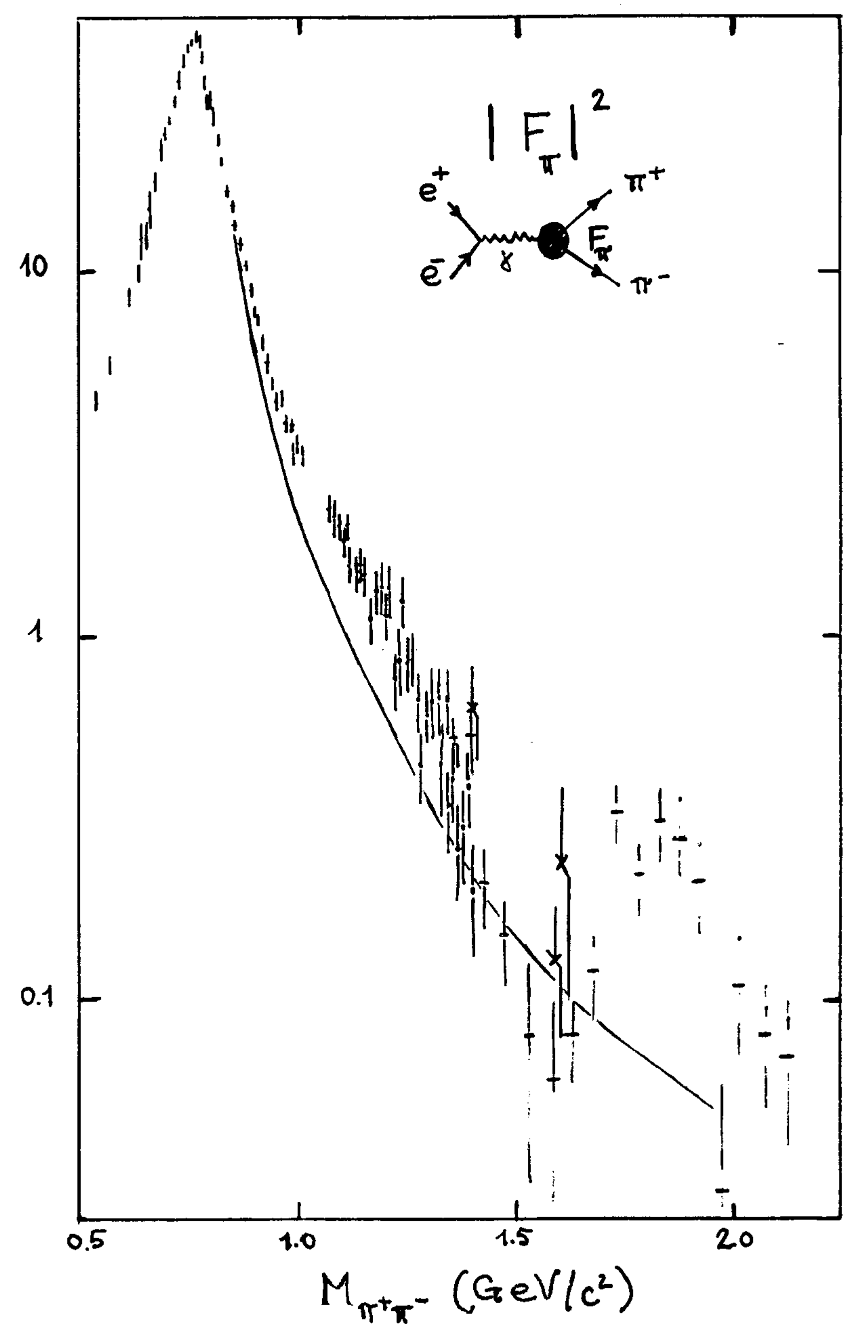
$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^3 \left(\frac{m_\rho}{m}\right) \left\{ \frac{1 + (q_0 R)^2}{1 + (q R)^2} \right\}$$



5000 events
m**2(pi- pi0) GeV**2
m**2(pi+ pi0) GeV**2
0.0
2.5
5.0
7.5
10.0
12.5


MARK - III
J/ψ → π⁺π⁻π⁰
π⁰
π⁺
π⁻
θ
|cos θ| ≤ 0.2
1.0
0.5
0.0
-0.5
-1.0
0.0
0.75
1.5
2.25
3.0
M(PI+ PI-) VS COS(THETA)
DATA


| cos θ₁ | ≤ 0.2
ρ(770)
10³
log. scale
MARK-III
J/ψ → π⁰ π⁺ π⁻
Events / 25 MeV/c²
10²
10
1
0.5
1.0
1.5
2.0
M_ππ (GeV/c²)


$|F_\pi|^2$
e^+
e^-
γ
F_π
π^+
π^-
10
1
0.1
0.5
1.0
1.5
2.0
$M_{\pi^+\pi^-}$ (GeV/c^2)

Modify B as follows:

$$B = B[\rho(770)] + c_1 e^{i\varphi_1} B[\rho_1(1300)] + c_2 e^{i\varphi_2} B[\rho_2(X)]$$

where c_1 c_2 φ_1 & φ_2 are the same for each $\pi\pi$ charge configuration.

Perform a max. $\mathcal{L}$ fit with 9 parameters:

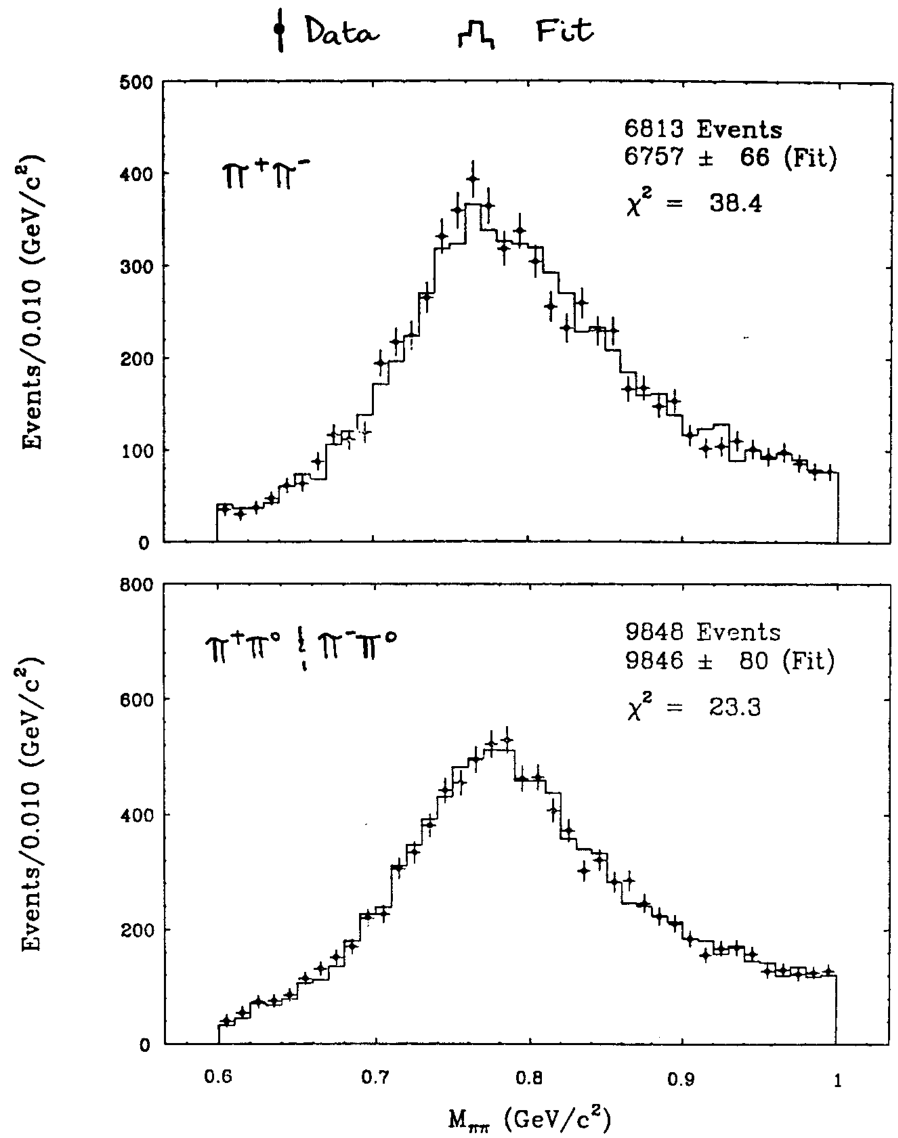
M_{ρ^0} , Γ_ρ , $\delta \equiv M_{\rho^\pm} - M_{\rho^0}$

M_{ρ_2} , Γ_{ρ_2}

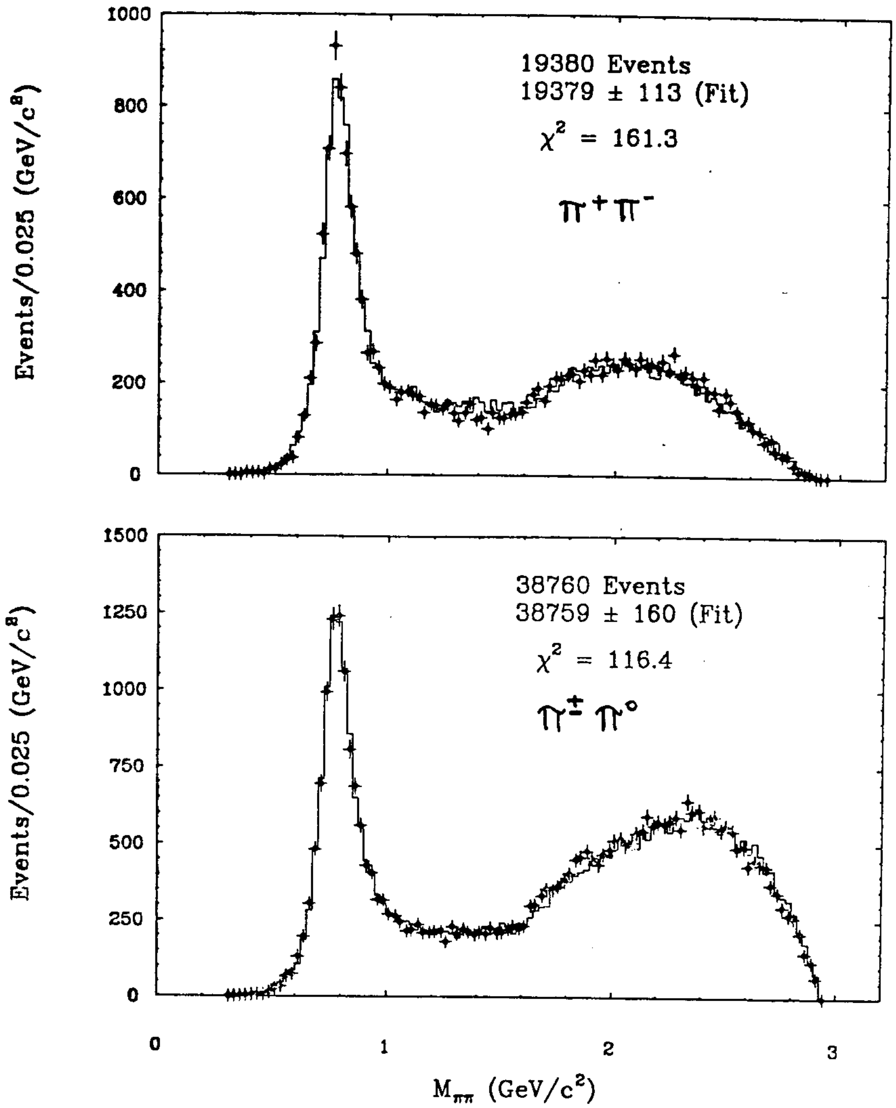
c_1 , c_2 , φ_1 , φ_2

(*) The radius in the usual Blatt-Weisskopf Barrier Factor is set to 0.

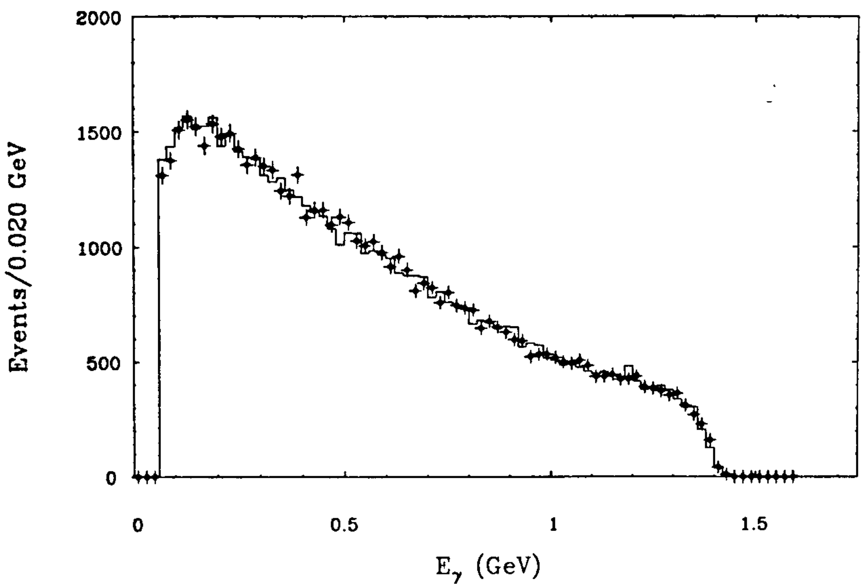
$R > 0$ results in a poor description of the ρ line-shape.



R = 0 :	$M_{\rho^0} = 776.2 \pm 1.3$,	$\Gamma_\rho = 150.6 \pm 2.4$;	$M_{\rho^\pm} - M_{\rho^0} = 1.6 \pm 1.5$
Barkov et al (F_π)	775.9 ± 1.1	150.5 ± 3.0	[R = 0]
Hyams et al ($\pi\pi$ PWA)	778 ± 2	152 ± 2	[$R = 4.5$ GeV^{-1}]

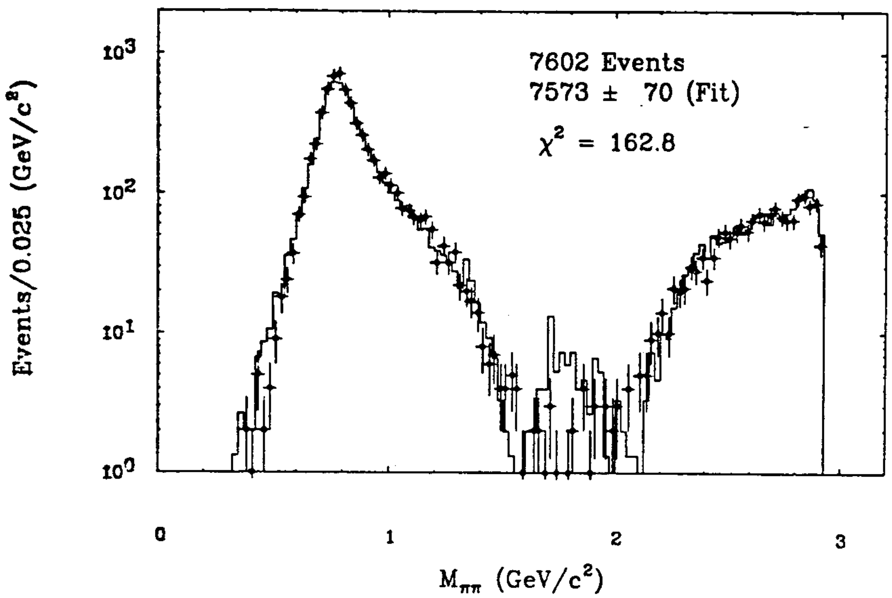

1000
800
600
400
200
0
Events/0.025 (GeV/c²)
19380 Events
19379 ± 113 (Fit)
χ² = 161.3
π⁺π⁻
1500
1250
1000
750
500
250
0
Events/0.025 (GeV/c²)
38760 Events
38759 ± 160 (Fit)
χ² = 116.4
π± π°
0
1
2
3
Mππ (GeV/c²)

E_γ (Lab. Frame) for $\pi^0 \to \gamma\gamma$



$|\cos\theta_1| \le 0.2$

$\pi^+\pi^-$ & $\pi^\pm\pi^0$ combined



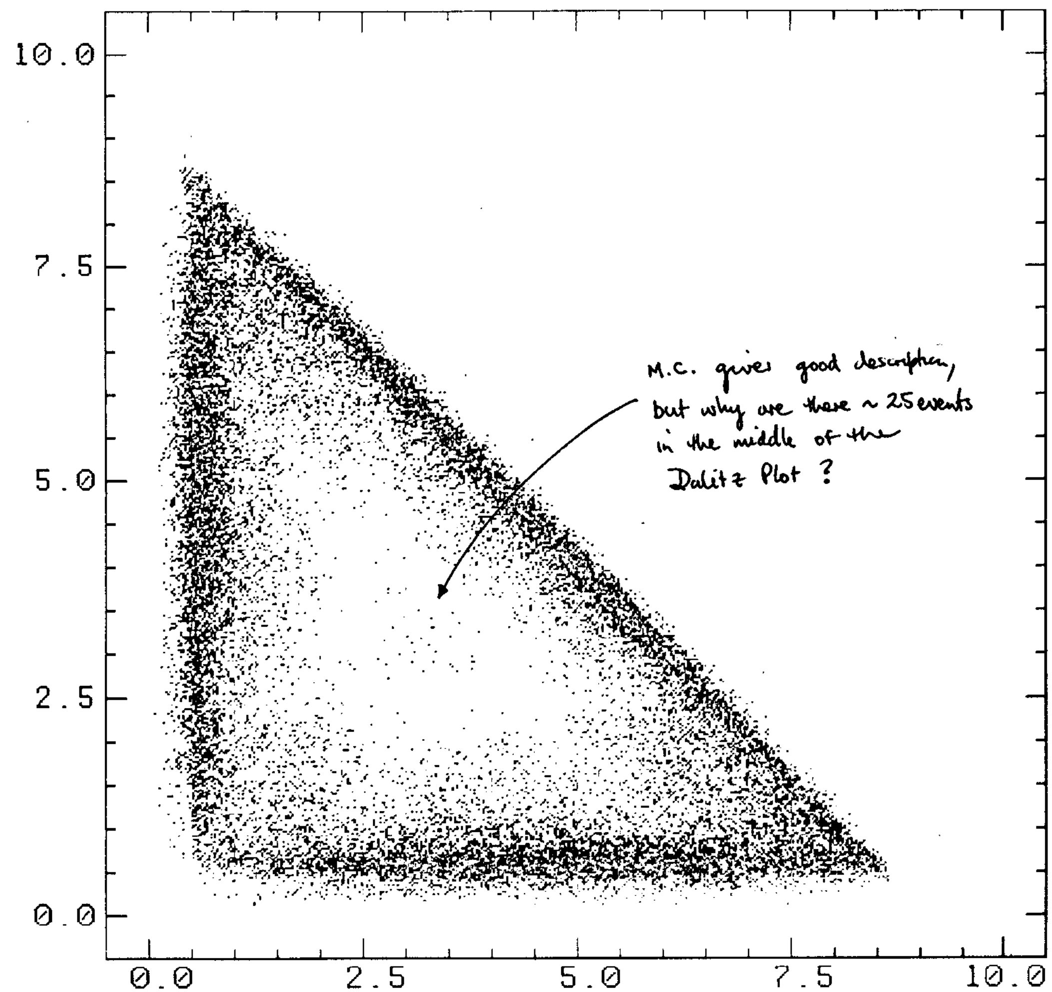
$M_{\rho_2} = 1600 \pm 28$ MeV/c², $\Gamma_{\rho_2} = 383 \pm 25$ MeV/c²

$A_\rho : A_{\rho_1} : A_{\rho_2} = 1 : 0.031 \pm 0.016 : 0.508 \pm 0.017$

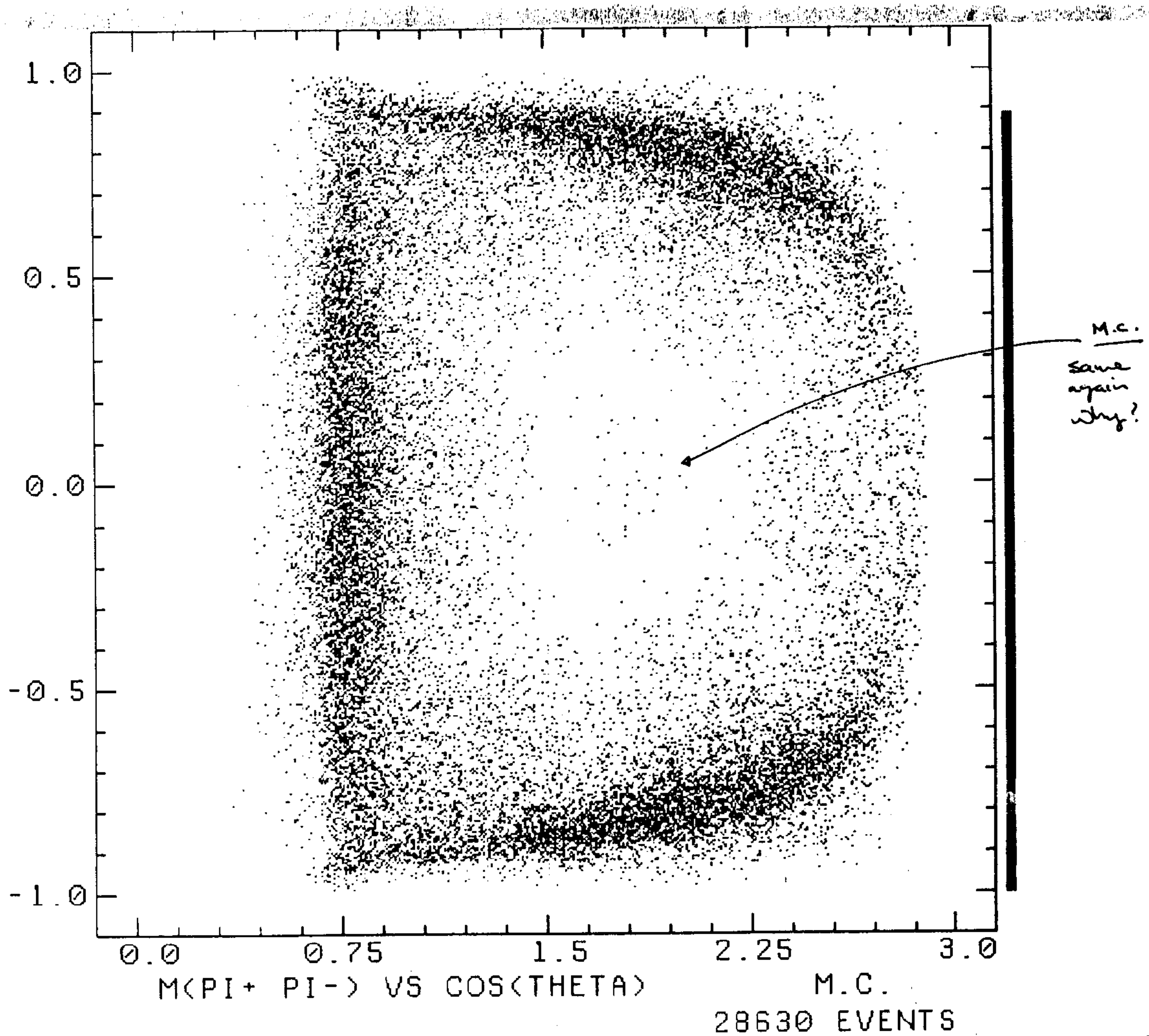
$\Rightarrow (?)\ x_\rho : x_{\rho_1} : x_{\rho_2} = 1 : \boxed{\sim 10^{-3}} : \sim 0.25$

$q\bar{q}$ $\quad 1^3S_1 : 2^3S_1 : 1^3D_1$

$\varphi_1 = (200 \pm 42)^\circ \quad ; \quad \varphi_2 = (-120 \pm 8)^\circ$

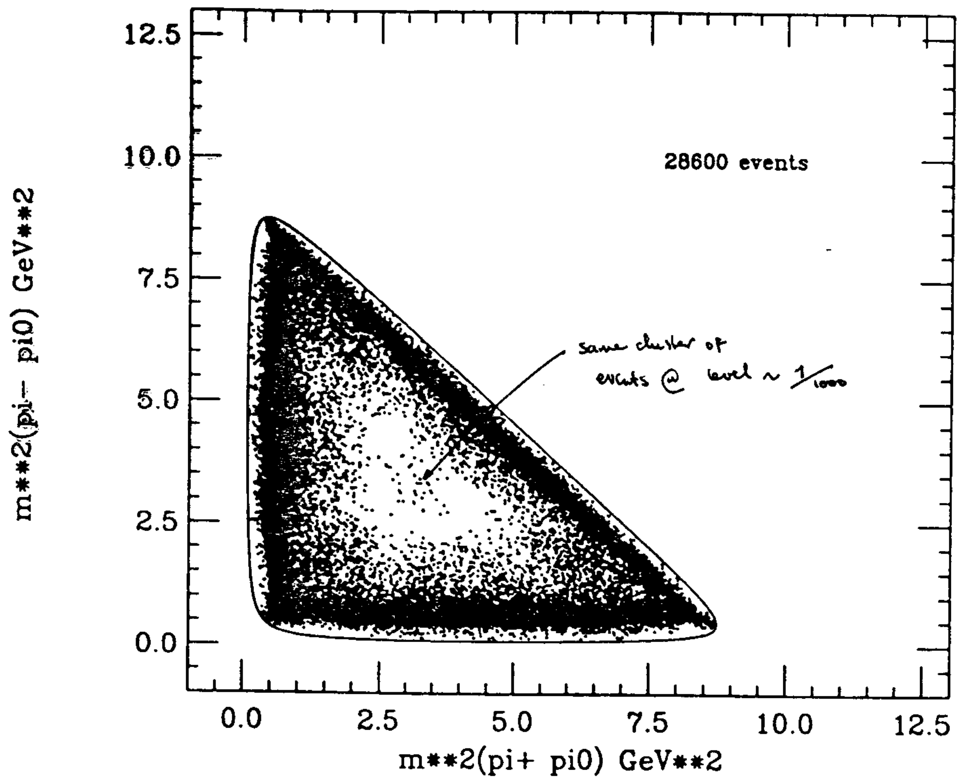


M**2(PI+ PI0) VS M**2(PI- PI0) M.C.
28630 EVENTS

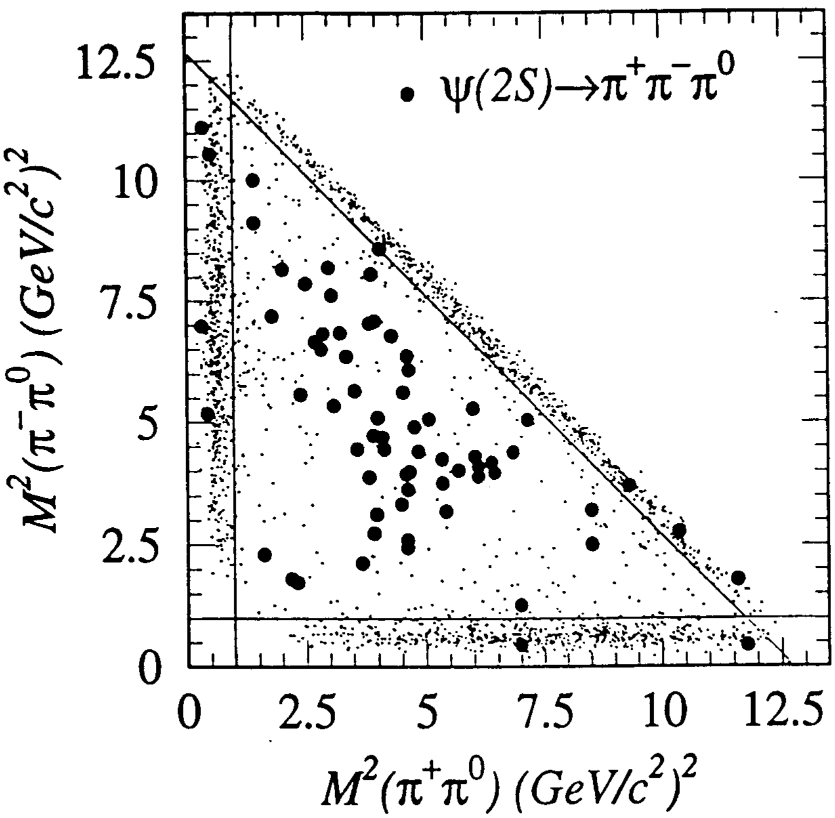


M(PI+ PI-) VS COS(THETA)
M.C.
28630 EVENTS

$J/\psi \to \pi^+ \pi^- \pi^0$ (M.C.)

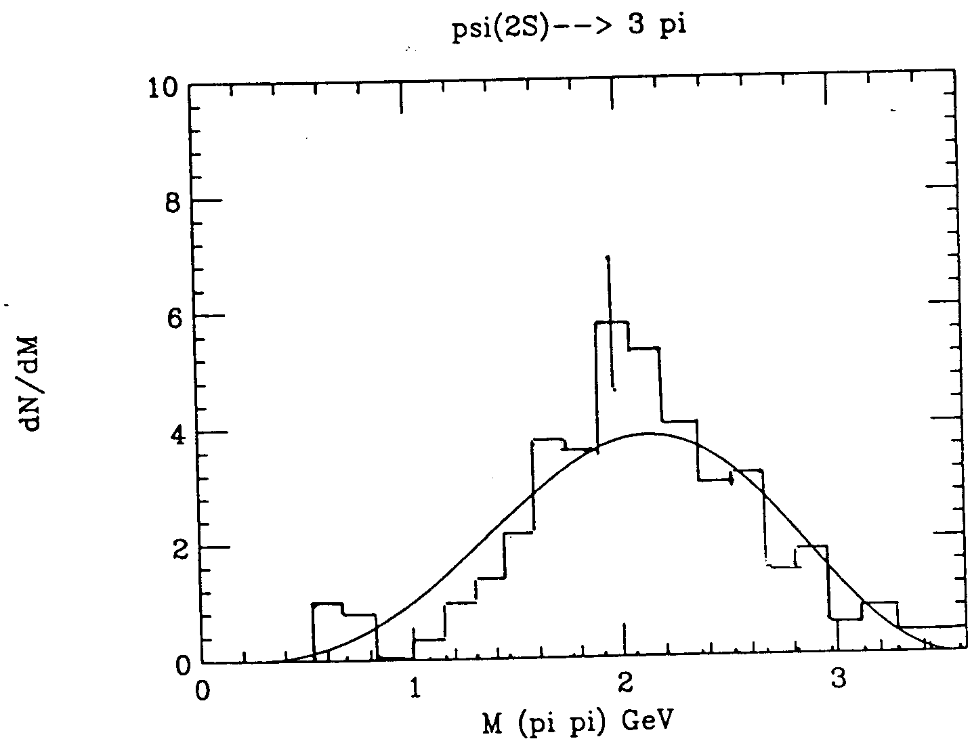


BES (Prelim.)



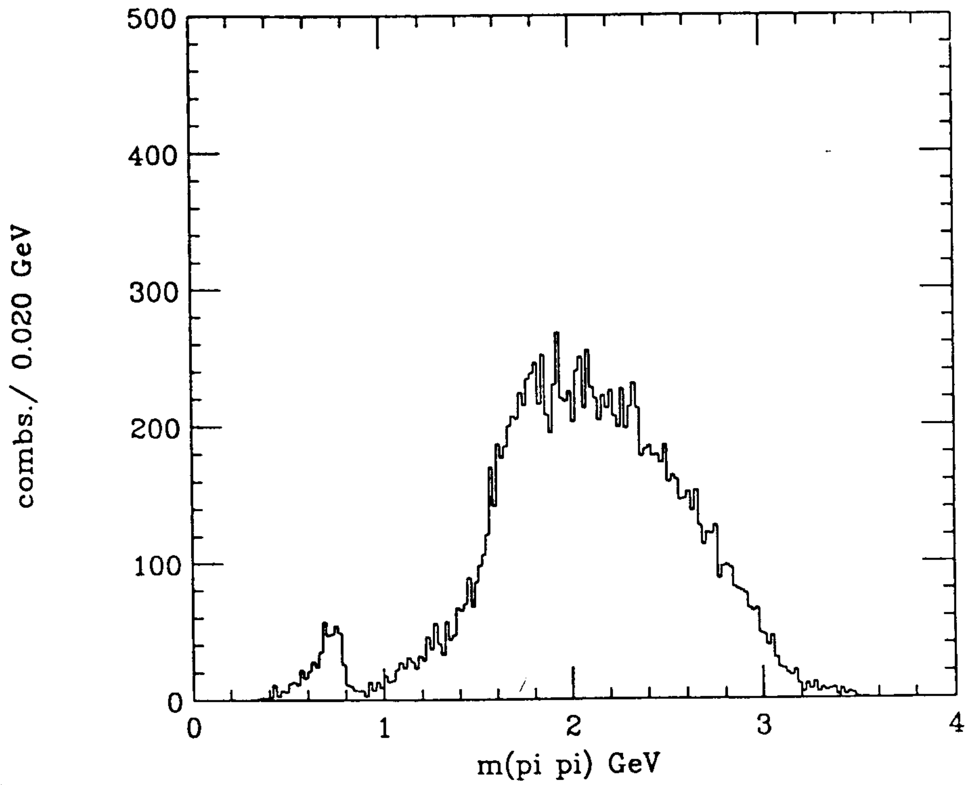
Very different from $J/\psi \to 3\pi$;
similar behaviour observed for
$K\bar{K}\pi$.

BES (Prelim.)



— $\psi' \to 3\pi$ (non-isobar ie. like $\omega \to 3\pi$)

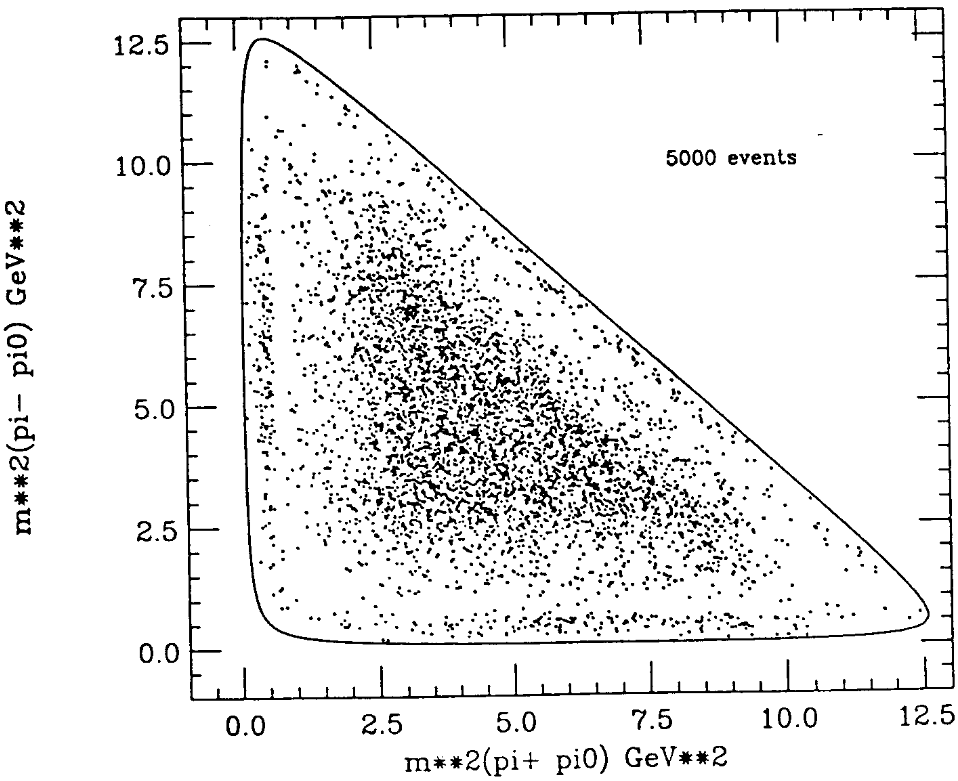
psi(2S)--> 3 pi (MC)



$\rho + \rho'(1600, 383) + 3\pi$ (non-isobar)

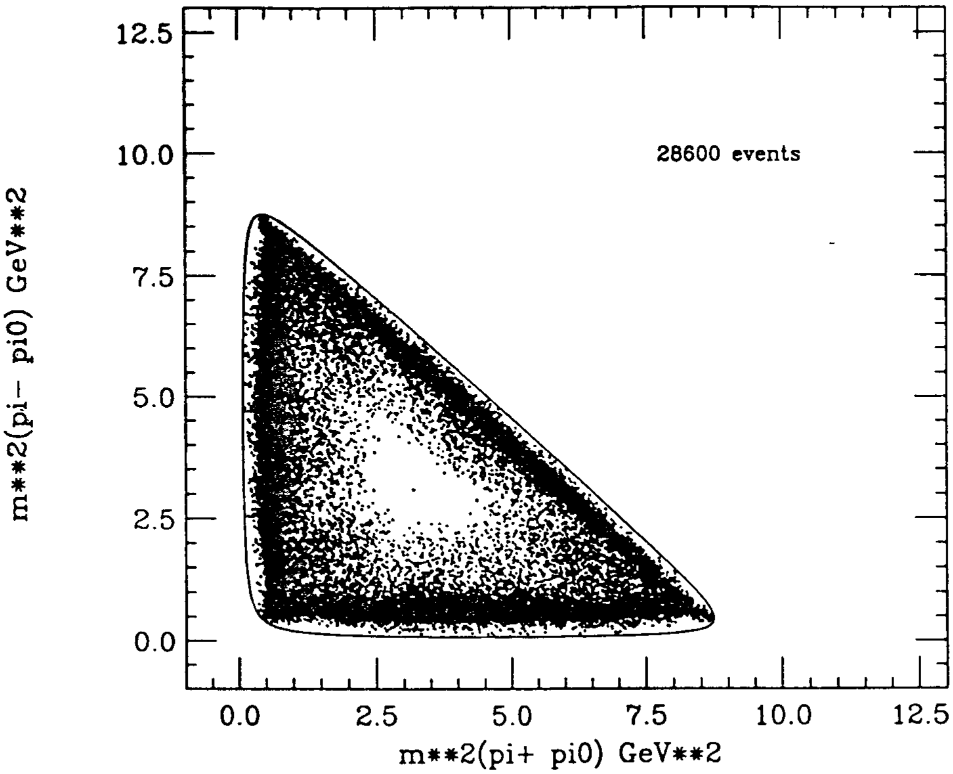
Same model as for $J/\psi \to 3\pi$, but with addition of decay to 3π with no $\pi\pi$ isobar structure (i.e. like $\omega \to 3\pi$)

psi(2S)--> 3 pi (M.C.)



ρ + ρ' (1600, 383) + 3π (non-isobar)

$J/\psi \to 3\pi$ (M.C.)



ρ + ρ' (1600, 383) + 3π (non-isobar)

include this in previous description of $J/\psi \to 3\pi$; can remove cluster of events in centre of plot (almost!)

i.e. describe J/ψ & $\psi(2S)$ decay to 3π with same model, but with individual contributions having different strength.

J/ψ & ψ' $\rightarrow$ $\pi^+\pi^-\pi^0$

(4th March, 1998)

Contribution	$B(J/\psi)$	$B(\psi')$ (*) (very crude)
All	$(134 \pm 10) \times 10^{-4}$	$\sim 1.6 \times 10^{-4}$
$\sum_{i=1}^{3} \rho_i(770)\pi$ Note: $3 * (\rho\pi)$ [i.e. no interference]	$(169 \pm 13) \times 10^{-4}$ $(134 \pm 10) \times 10^{-4}$	$\sim 0.35 \times 10^{-4}$
$\sum_{i=1}^{3} \rho_i'(1600)\pi$	$(11.9 \pm 1.2) \times 10^{-4}$	$\sim 0.32 \times 10^{-4}$
$(\pi^+\pi^-\pi^0)_{\text{non-quasi 2 body}}$ (i.e. like $\omega \to 3\pi$)	$(2.25 \pm 0.23) \times 10^{-4}$	$\sim 1.7 \times 10^{-4}$

(*) based on assumption of ~600 evts. after acceptance correction

$$B(m_{\pi\pi}) = A_\rho e^{i\varphi_\rho} BW_\rho(m_{\pi\pi}) + A_{\rho'} e^{i\varphi_{\rho'}} BW_{\rho'}(m_{\pi\pi}) + A_{3\pi} e^{i\varphi_{3\pi}} B_{\omega\to 3\pi}(m_{\pi\pi})$$

Parameter	J/ψ	ψ'
A_ρ	1.0	0.6
φ_ρ	0.°	0.°
$A_{\rho'}$	0.508	1.0
$\varphi_{\rho'}$	−120°	120°
$A_{3\pi}$	0.12	1.0
$\varphi_{3\pi}$	−95°	−45°