Study of Ring Aberration in the Focusing DIRC Prototype

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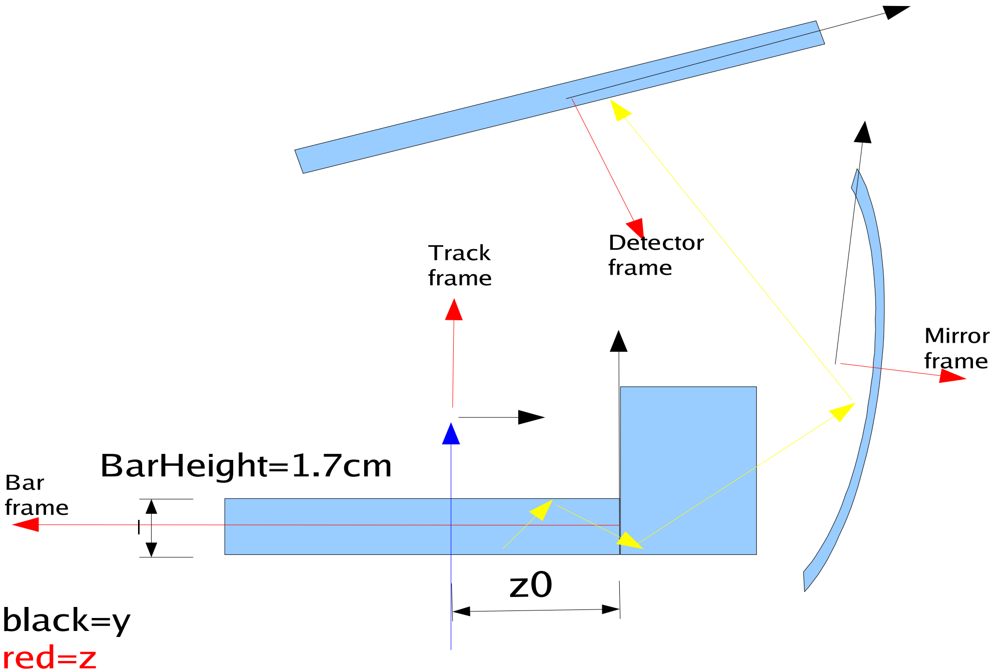
Purpose: Try to understand the origin and quantify the aberration of Cherenkov rings.

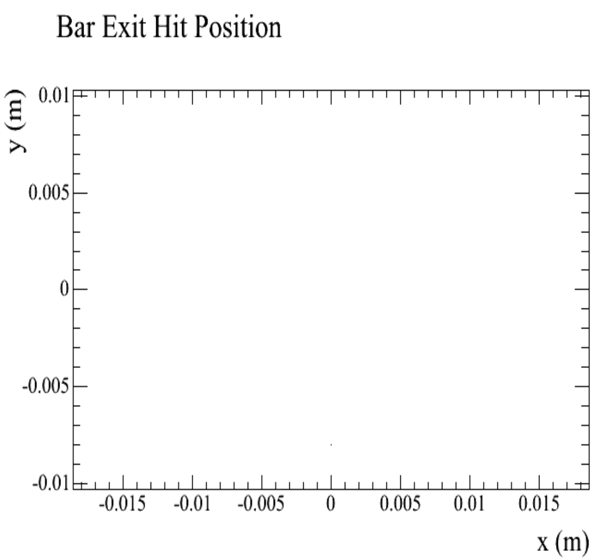
- Consider ray tracing a ~1/4 ring produced by track perpendicular to the bar. Assume θ=822mrad and $\pi/2<\phi<\pi*3.9/5$.
- How does the detected ring look like as we move the track further and further away from the end of the bar?

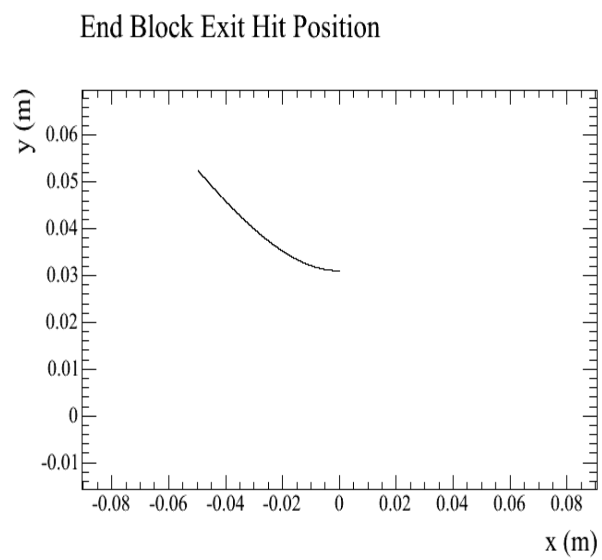


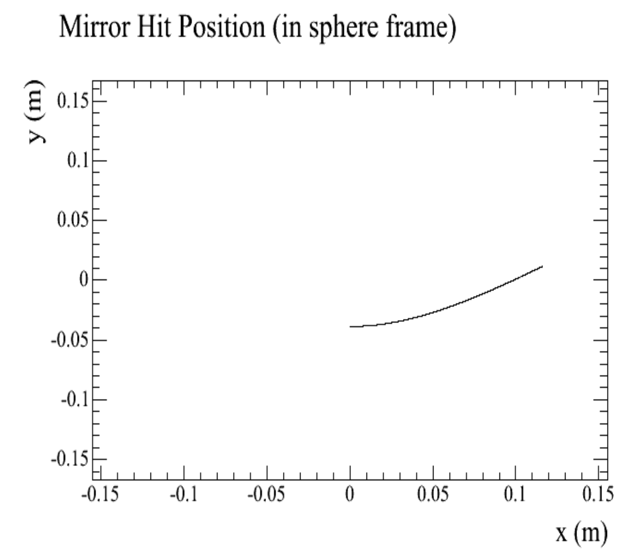
black=y
red=z
x : for bar and track frame it is out of the plane for mirror and detector frame it is into plane (Note the flip)

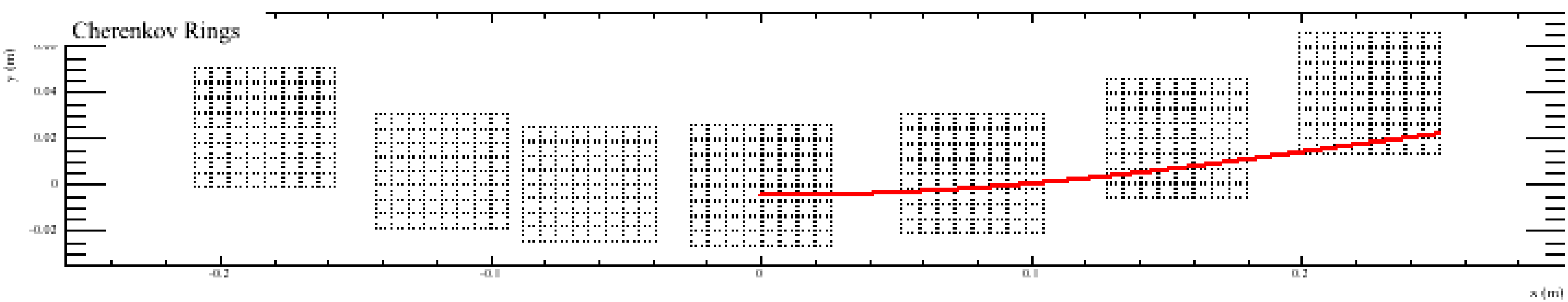
In the following plots:
Bar Position is bar frame,
EndBlock Position is in bar frame,
Mirror Position is in sphere frame,
Cherenkov Rings is in detector frame.
Also, only hits which make it into detector plane are plotted, all other hits are left out from all plots.

z0=0BarHeight

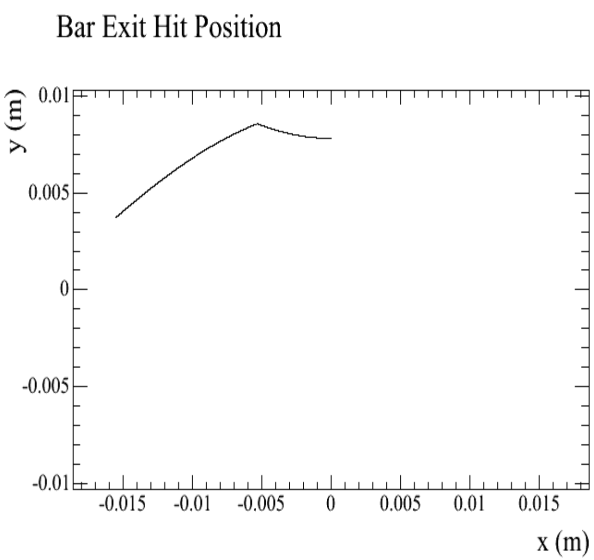


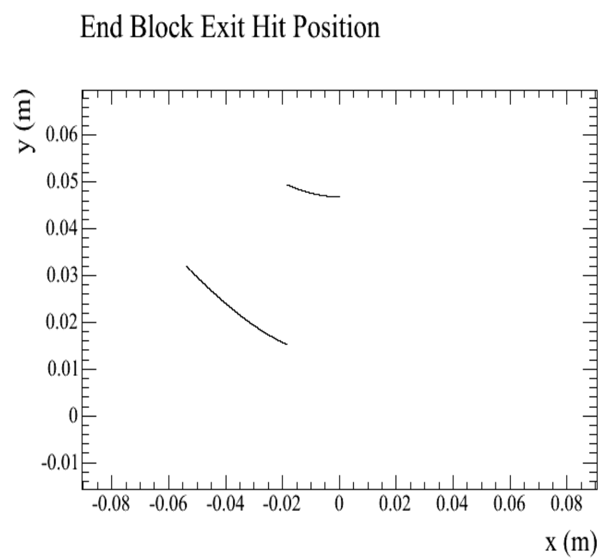


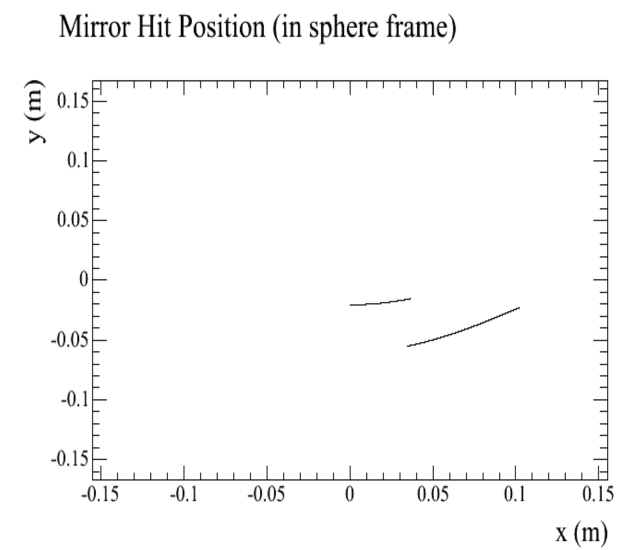


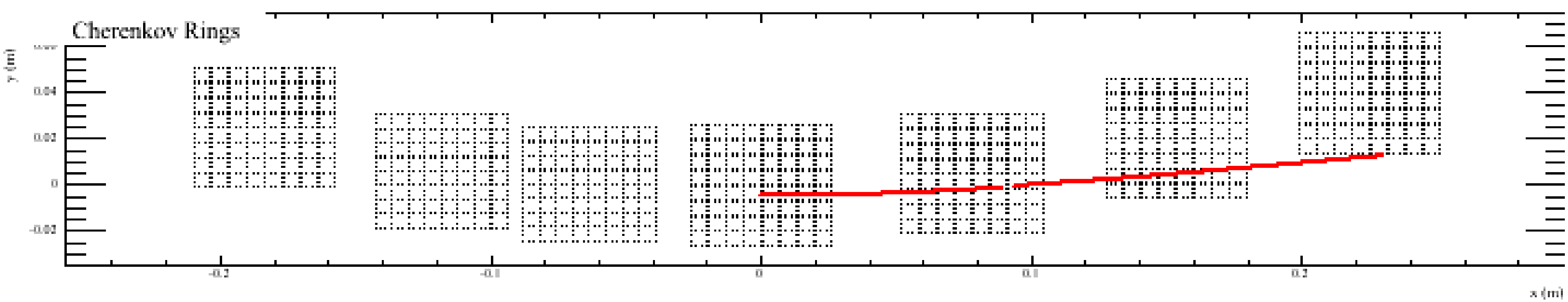


z0=1BarHeight

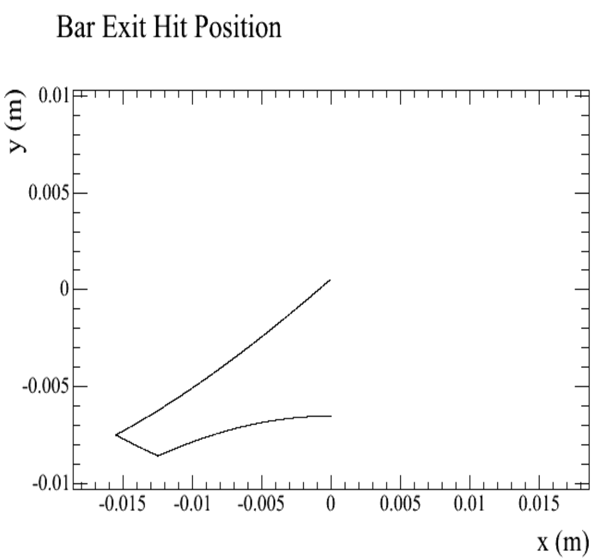


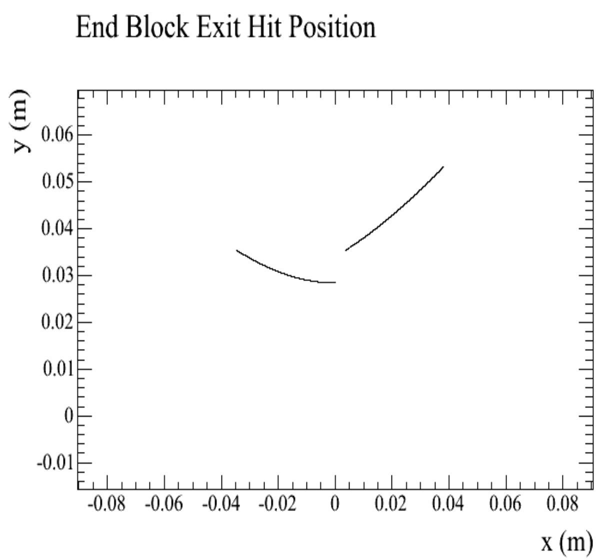


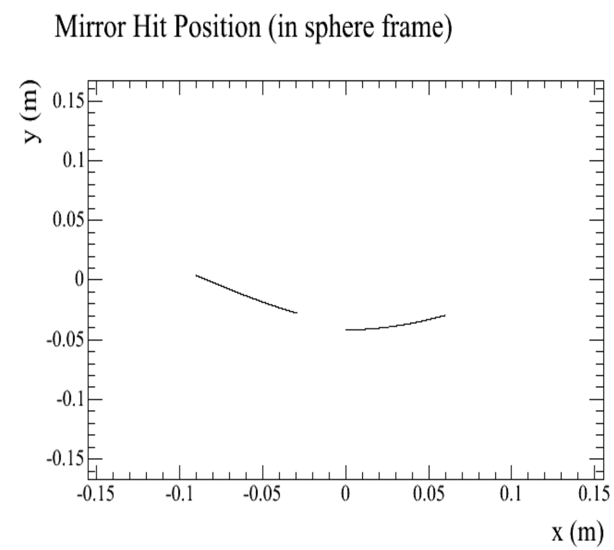


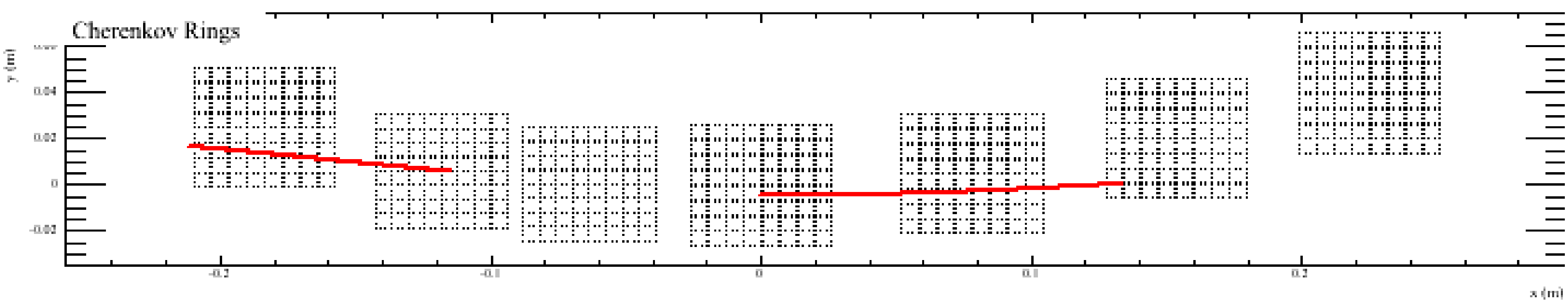


z0=2BarHeight

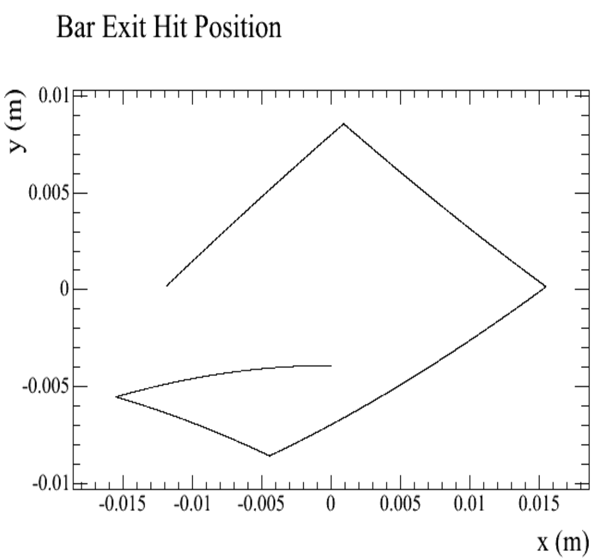


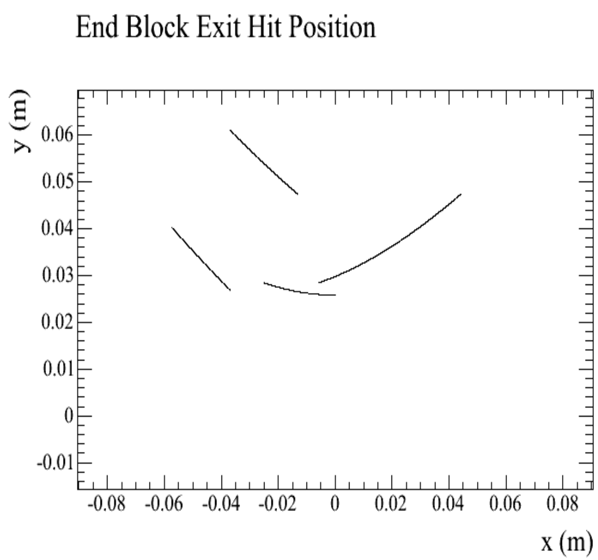


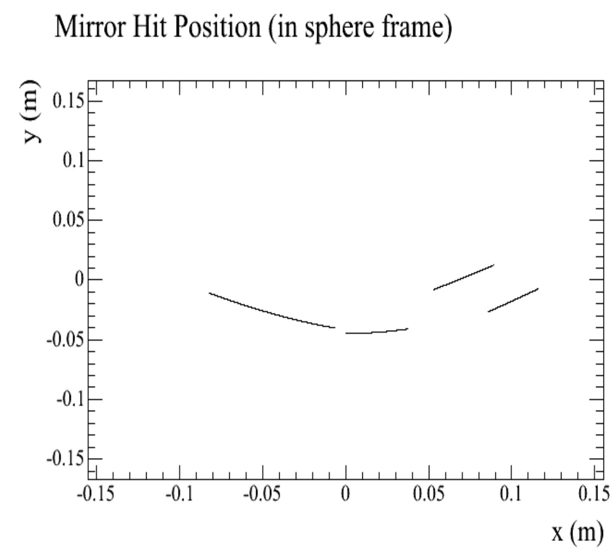


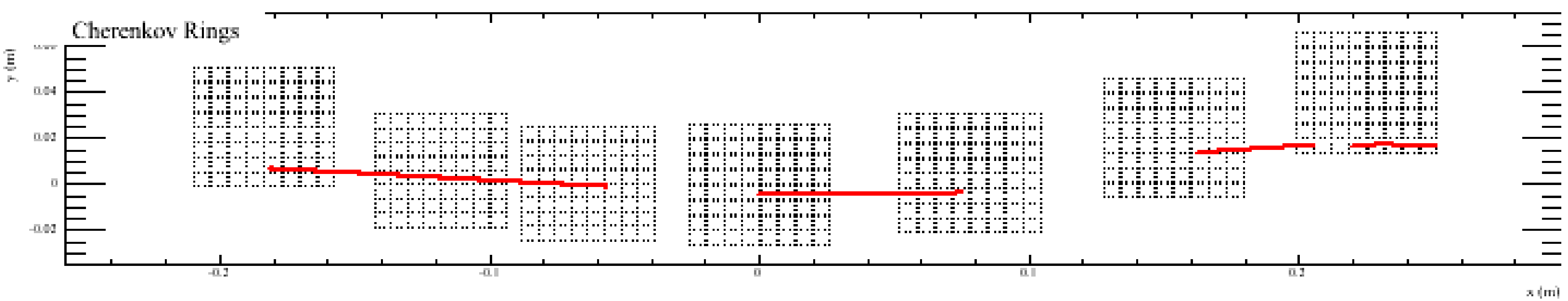


z0=4BarHeight

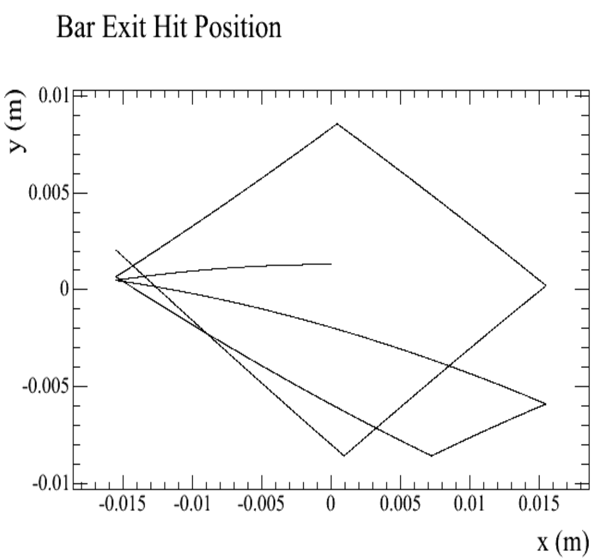


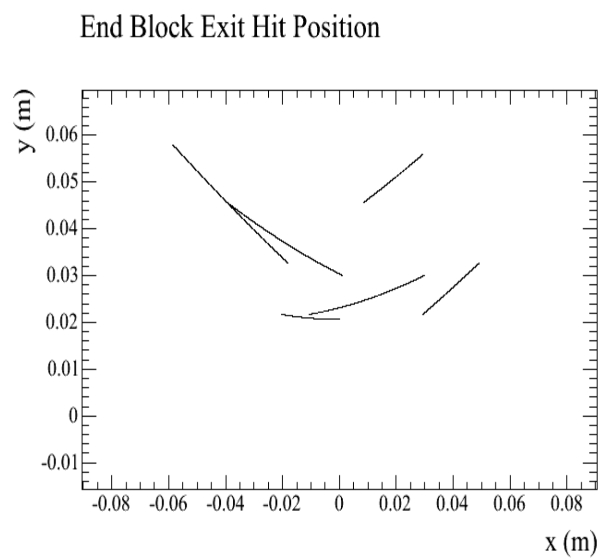


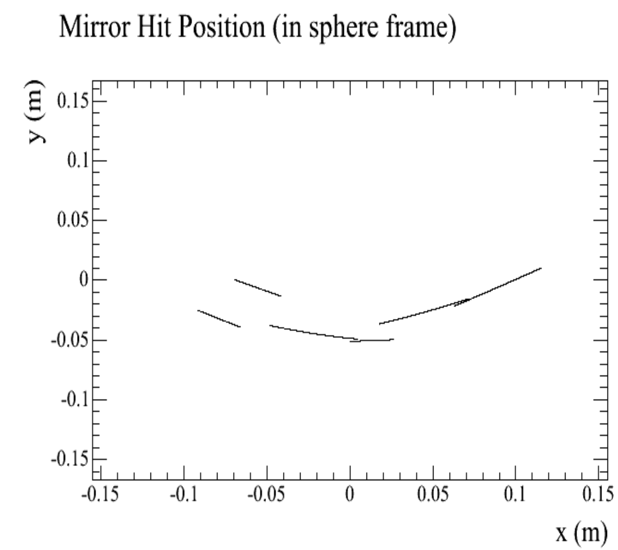


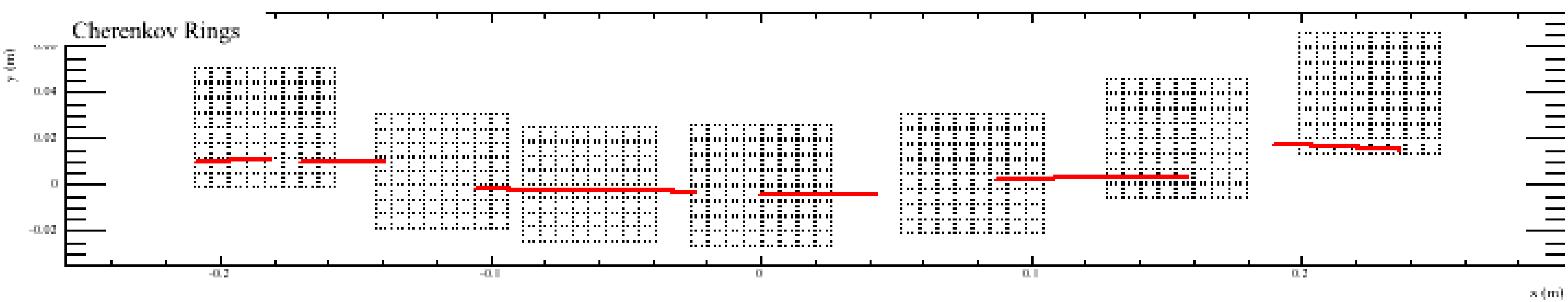


z0=8BarHeight

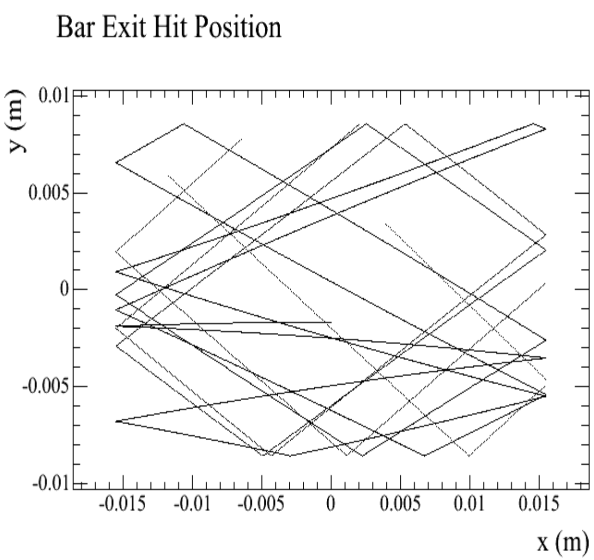


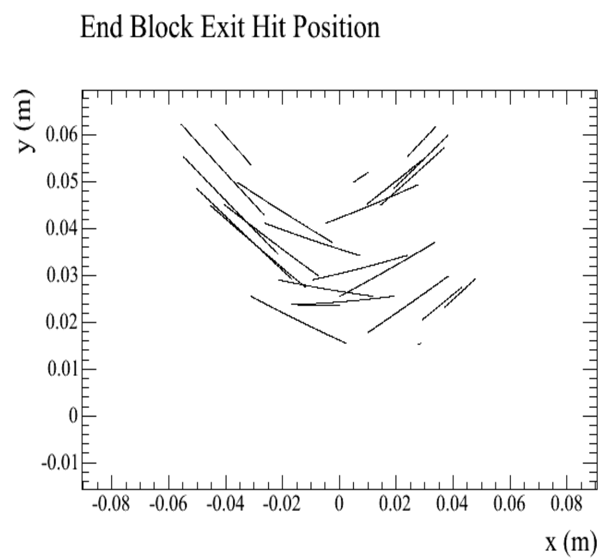


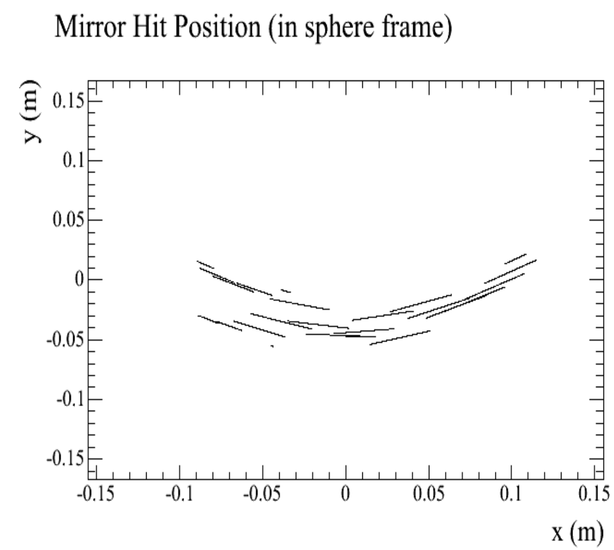


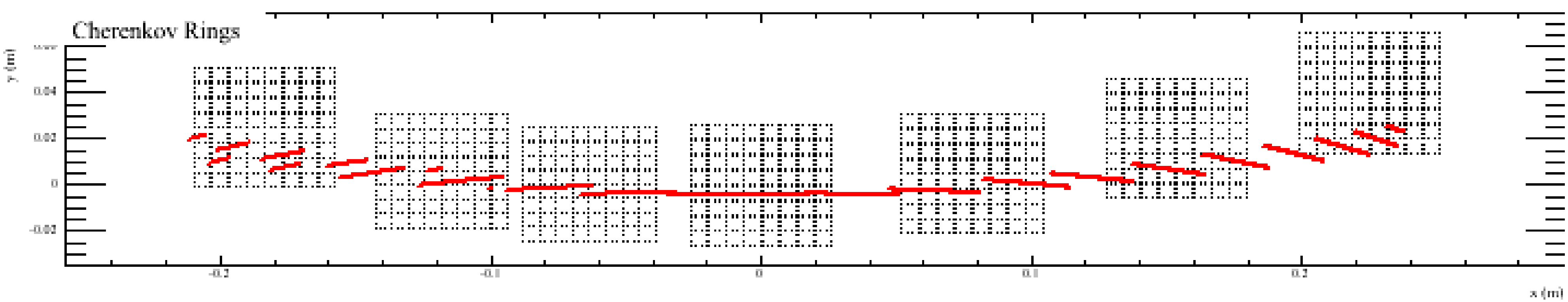


z0=32BarHeight

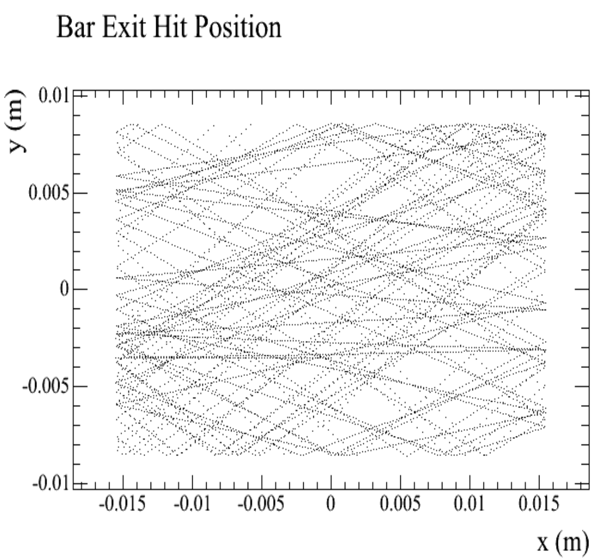


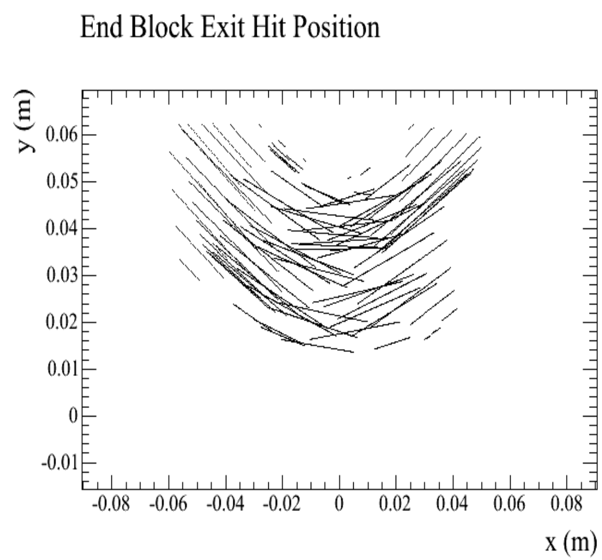


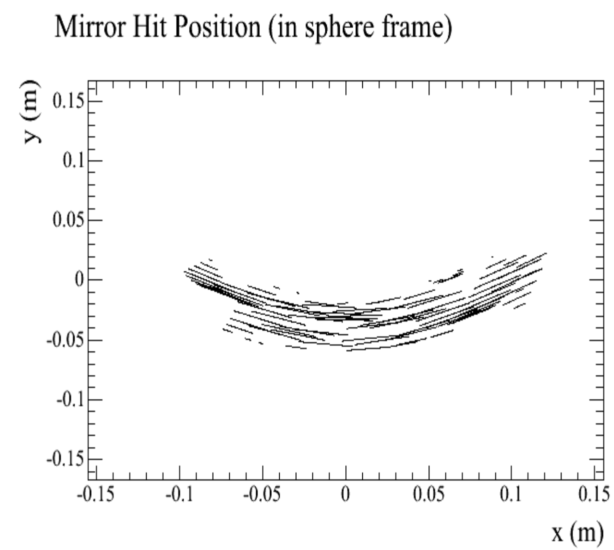


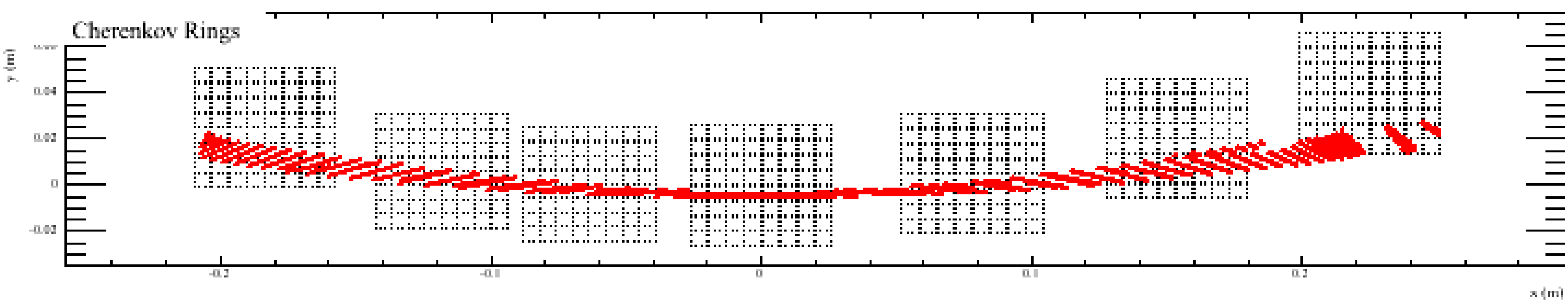


z0=128BarHeight

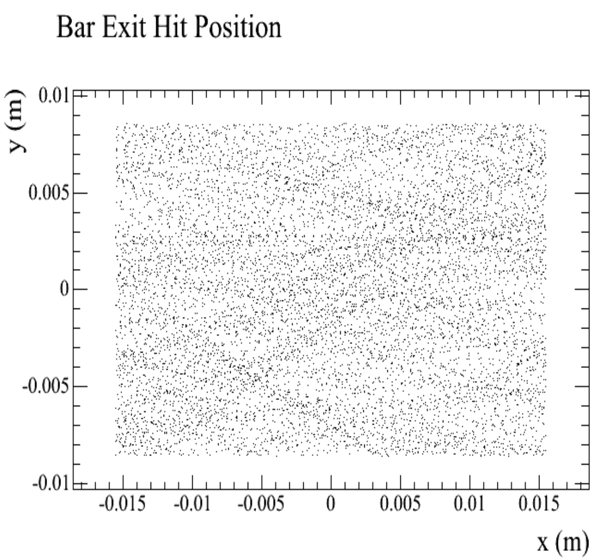


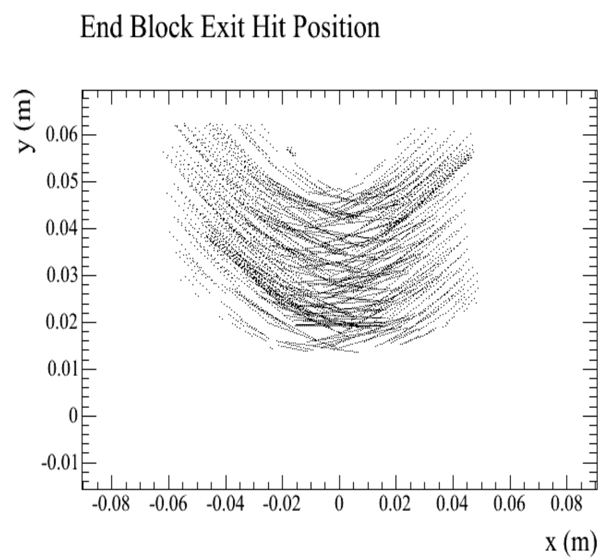


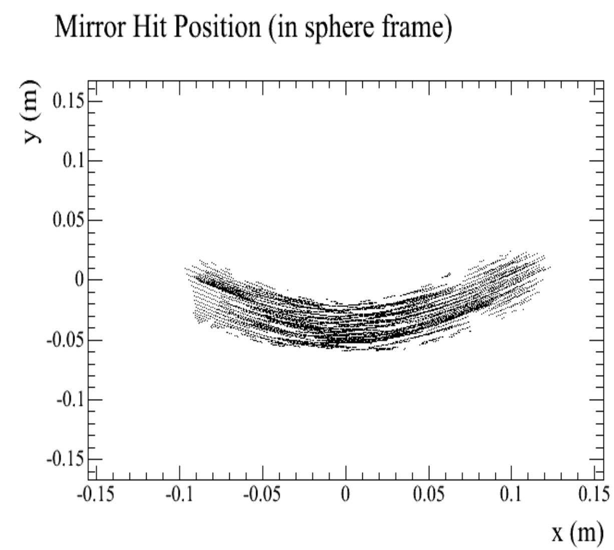


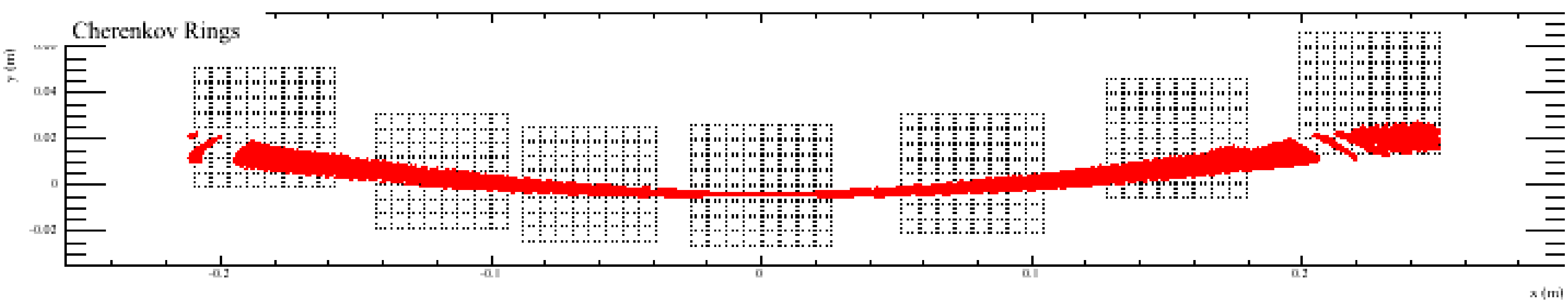


z0=0BarHeight Indirect

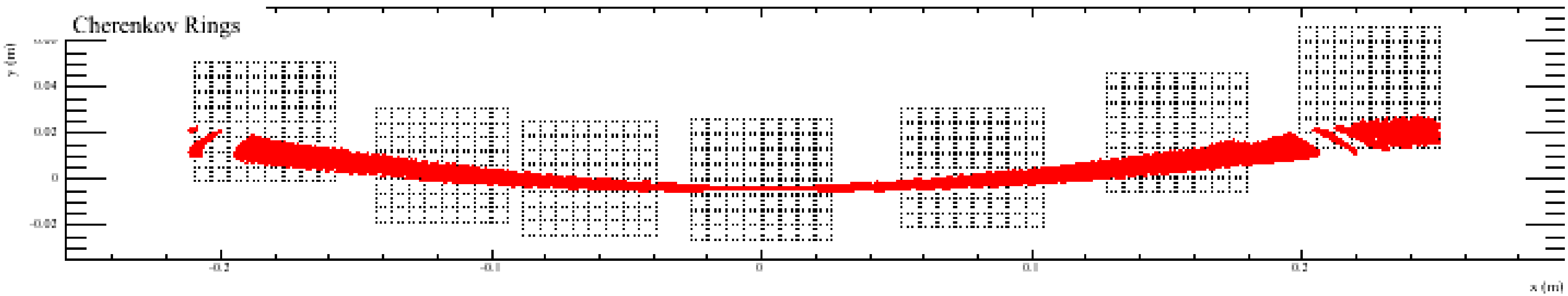


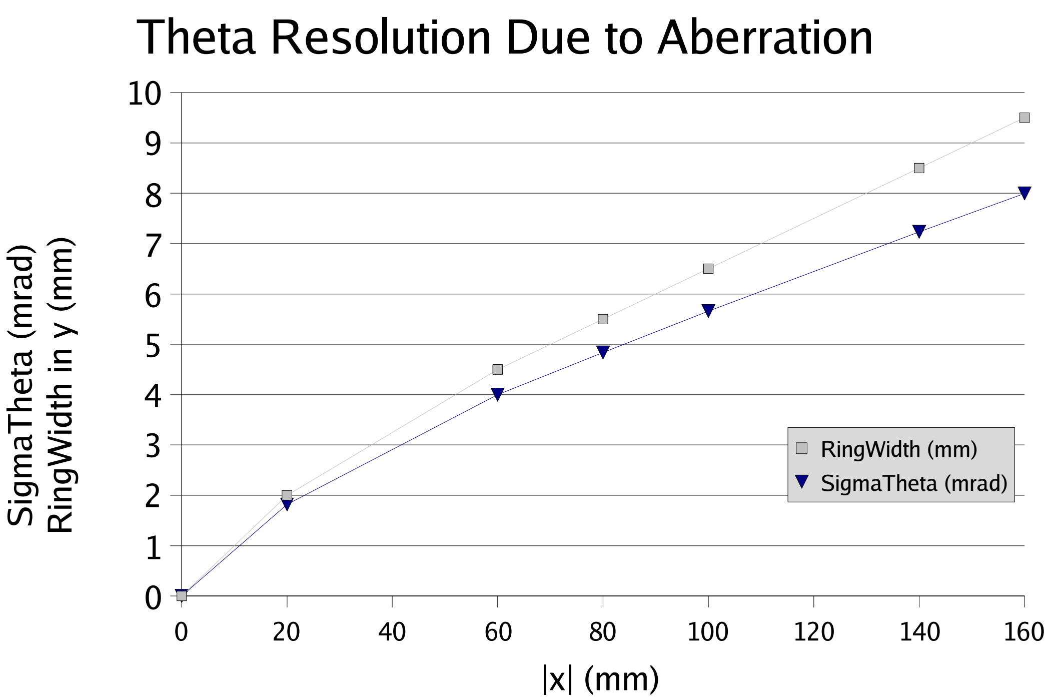




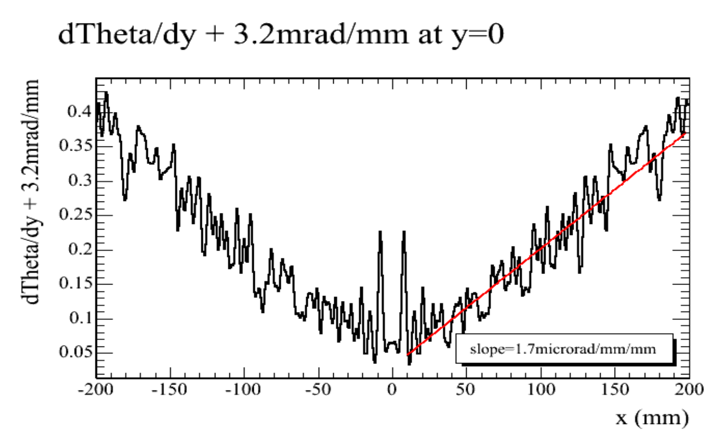


Use the ring from z0=0BarHeight Indirect to quantify the aberration





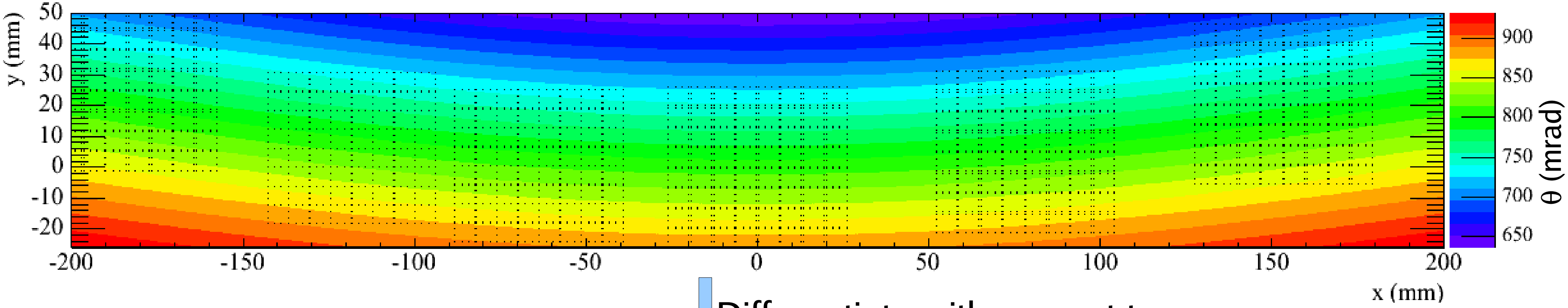
These Theta resolutions have been corrected for curvature of the ring:
σ_θ=(RingWidth in y) *|dθ/dy|/sqrt(12)
where dθ/dy is a function of x and y on the detector plane:



Determination of dTheta/dy

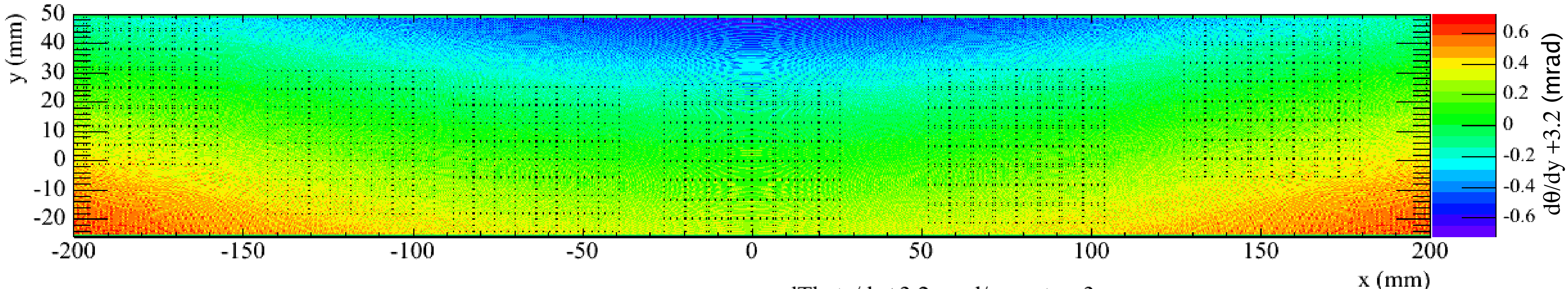
I have generated rays from the bottom center of the bar and determined theta for every point on the detector plane:

Theta



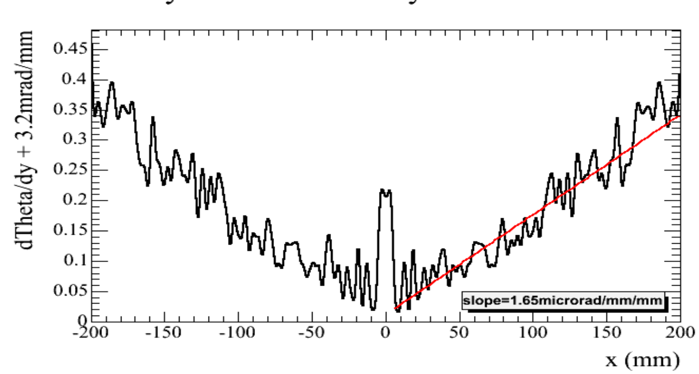
Differentiate with respect to y.

dTheta/dy + 3.2mrad/mm



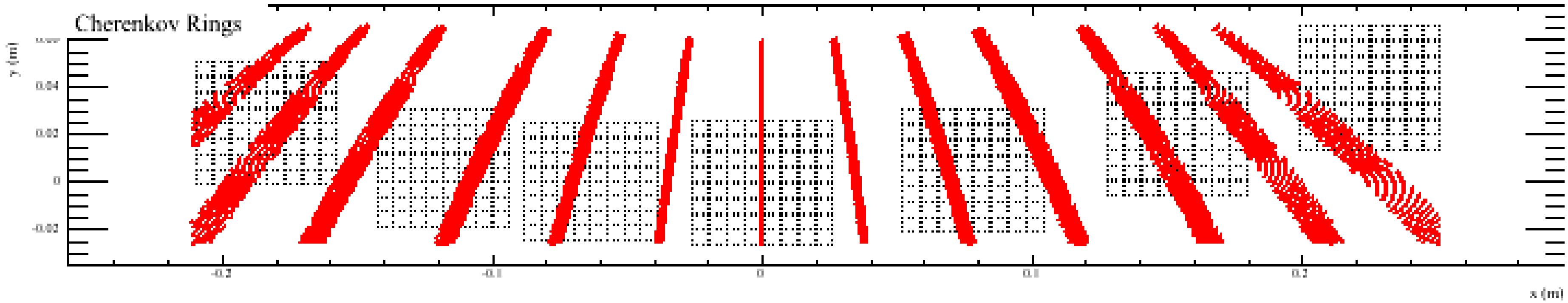
Technically I should use a dTheta/dy which also depends on y since the ring covers -4mm<y<6mm, however dTheta/dy does not depend too heavily on y. I have used dTheta/dy at y=0 in the calculations.

dTheta/dy+3.2mrad/mm at y=3mm



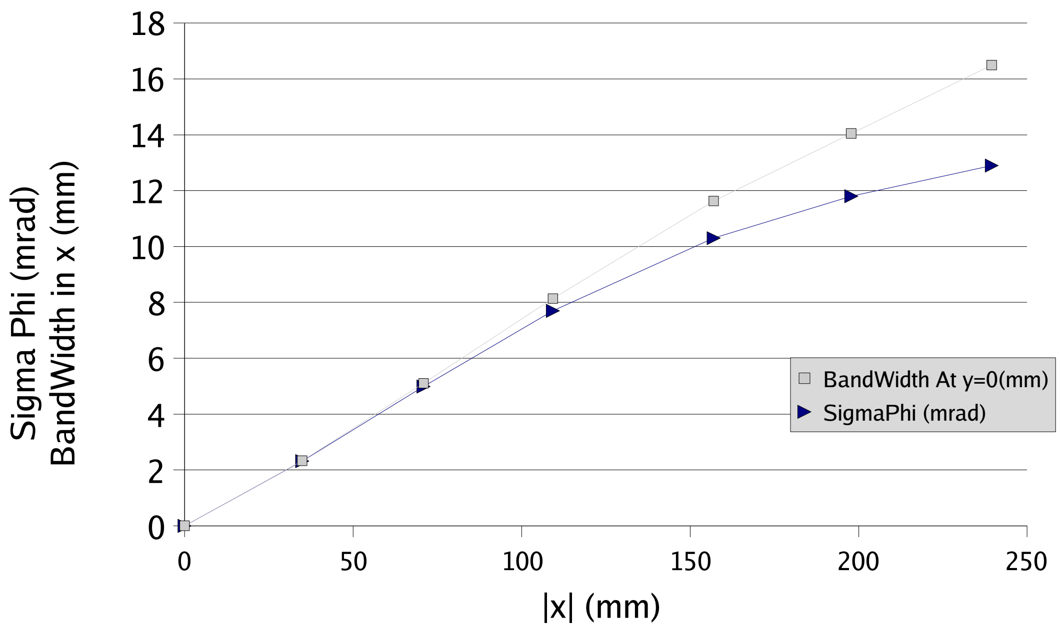
Ignore roughness, this is a problem with bining in the above 2D plot.

Now consider the aberration in the phi coordinate:



These are bands of constant phi, $\Delta\phi$=117mrad between each band. These bands where produced by generating discrete values of phi and scanning theta. Rays where originated at the bottom center of the bar propagated backward and then forward. There is no overlap between left and right bands,13 values of phi were produced.

Phi Resolution Due to Aberration

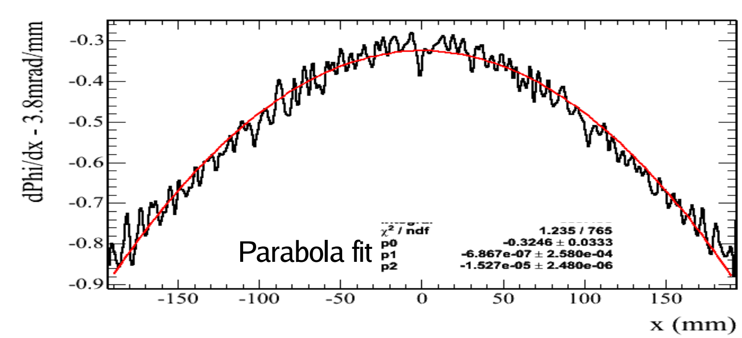


These Phi resolutions have been calculated as:

σ_ϕ=(BandWidth in x) *|dϕ/dx|/sqrt(12)

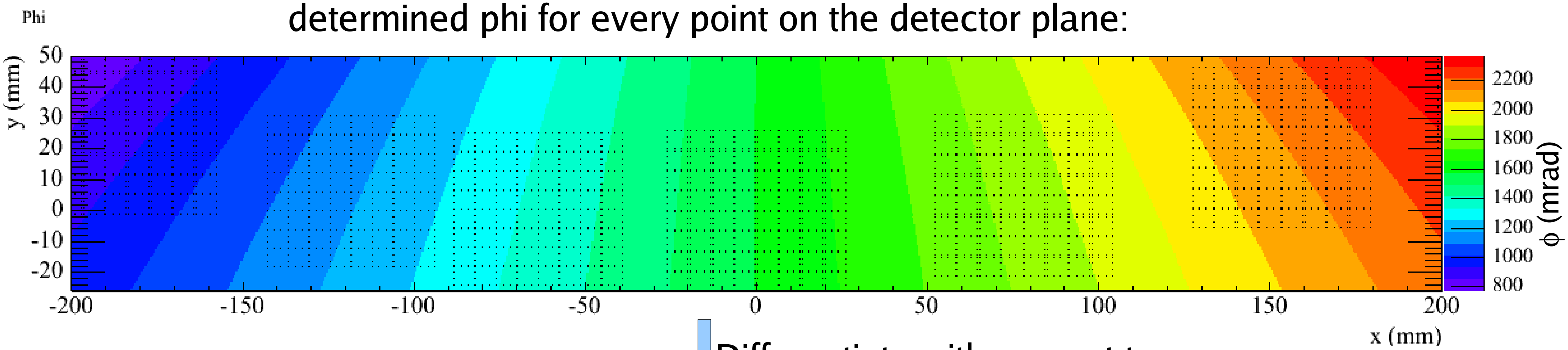
where dϕ/dx is a function of x and y on the detector plane:

dPhi/dx-3.8mrad/mm at y=0mm

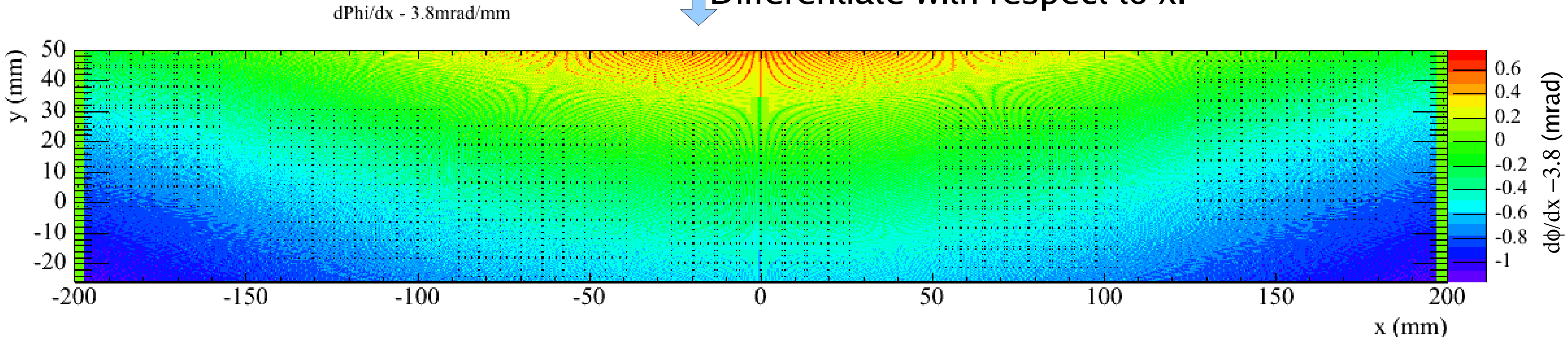


Determination of dPhi/dx

I have generated rays from the bottom center of the bar and determined phi for every point on the detector plane:



Differentiate with respect to x.



In the phi calculations one need not worry about the variation of dphi/dx in y because the band widths can all be determined at a single value of y.

Propagation of θ and ϕ errors into the path length error

The pathlength of photons in the bar is measured as

$L = z0_{eff} / k_z$

where k_z in turn is measured as

$k_z = sqrt(1-k_x^2-k_y^2)$

In terms of θ and ϕ

(Assuming perpendicular tracks):

$k_y = \cos(\theta)$ and $k_x = \sin(\theta)\cos(\phi)$

So,

$k_z = \sin(\theta)\sin(\phi)$

Therefore

$\sigma_L / L = \sigma_{kz} / k_z$

$\sim sqrt((\cot(\theta)\sigma_\theta)^2 + (\cot(\phi)\sigma_\phi)^2)$

For reference:

$\theta = .822$ rad.

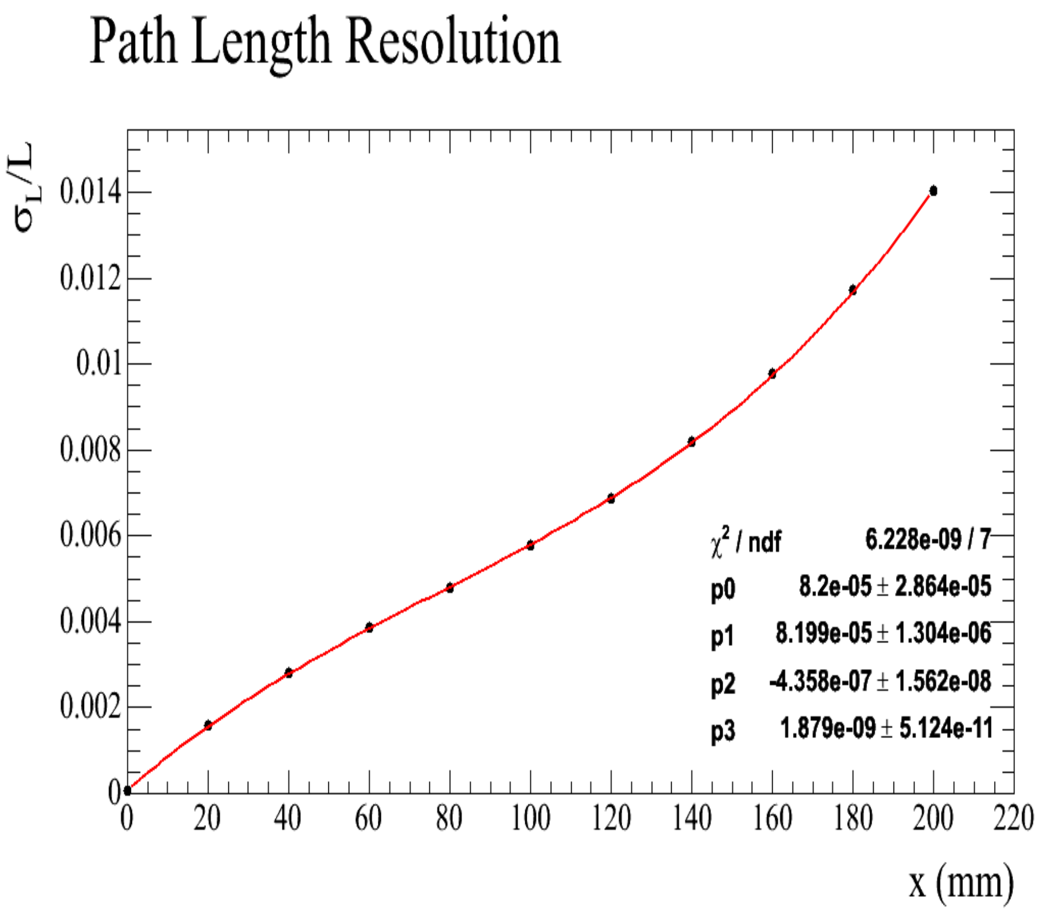
$\phi(x) = 1.5708+.003435*x$ rad,

$\sigma_\theta(x) \sim .063+.092*x-5.2e-4*x^2+1.6e-6*x^3$ mrad,

$\sigma_\phi(x) \sim -.082+.07*x-5.2e-6*x^2-1.8e-7*x^3$ mrad,

(x in mm)

The formulas apply to our Cherenkov ring only.



Conclusions

- This aberration comes about through a rotation of the ring pieces by the mirror, notice that all ring pieces are still not rotated when they hit the mirror. The amount of rotation depends on the number of bounces inside the bar but the width of the ring on the detector plane does not.
- The theta resolution increases up to **8mrad** in slot6.
- The phi resolution increases up to **11mrad** in slot6.
- These aberrations induce an error in the path length of the photon of up to **1.2%** in slot6.