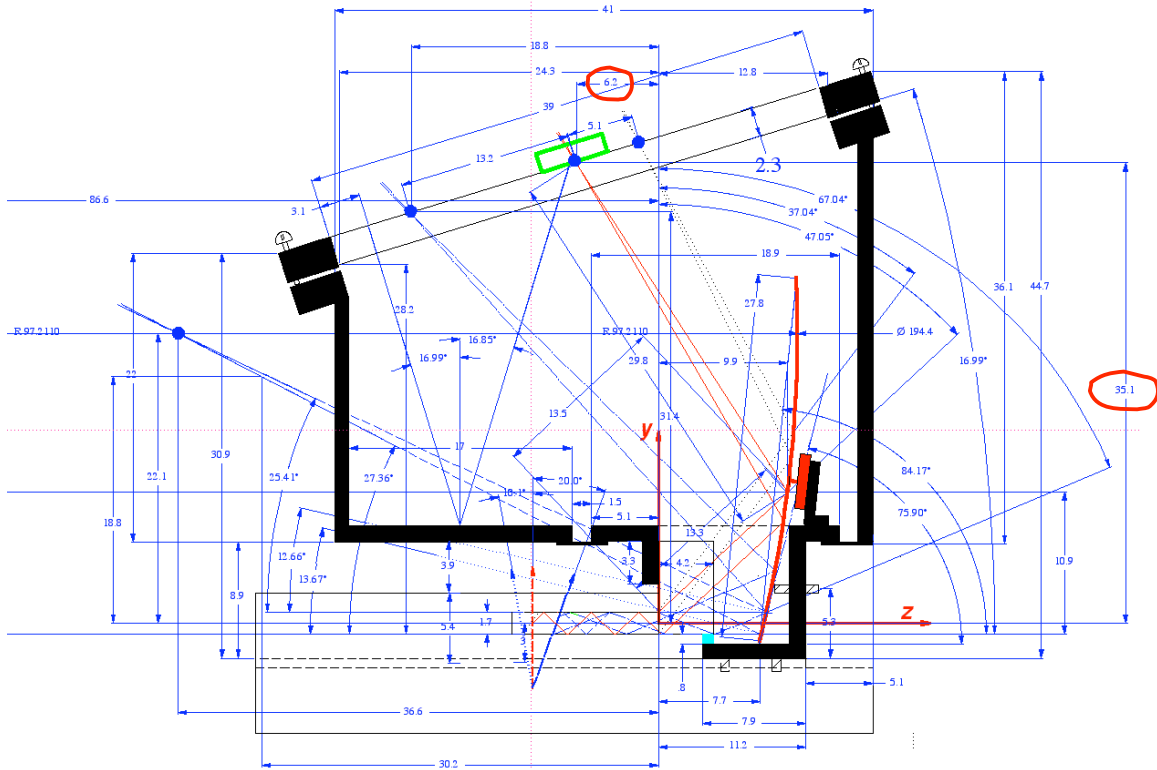


The initial look at the FDIRC Optical Design

J. Va'vra

Focusing DIRC Prototype optics:



- **Keep at least initially a similar geometry as in the FDIRC prototype, which has a size chosen so that a pixel size of 6x6 mm provides the same “pixel resolution” as the BaBar DIRC** (FDIRC prototype has a pixel resolution of $\sigma = 6 \text{ mm}/\sqrt{12} \sim 6 \text{ mrad}$).

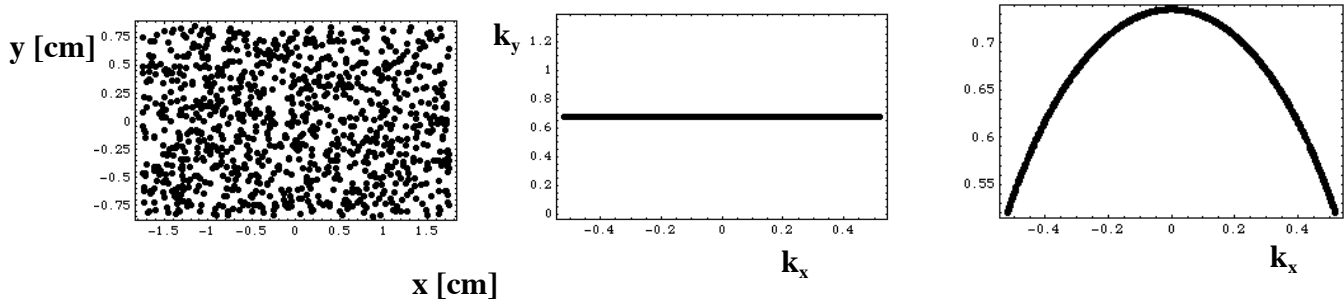
The point of this study:

- Transform the FDIRC prototype geometry to **Mathematica**. **Work out a geometry in 3D**.
- Do not use Optica (it is too unclear to me).
- Assume at the moment that the space has a constant refraction index of quartz (no glue, or oil, etc.).
- Use the most general parameterization of the quadratic mirror surface.
- **Study a flat detector surface & spherical, cylindrical and parabolic mirror surfaces.**
- **Find the most general detector surface in 3D for the final FDIRC.**

Photon x-y distribution at the end of the bar (z = 0):

Photon starting point in the middle of a bar:

```
nrefr=1.47;
barw = 3.5; barh = 1.7; barl = 144*2.54;
x0 = 0; y0 = (0+Random[]*barh-barh/2); z0 = -barl/2;
zbarstart = -barl; zbarend = 0;
Theta = 90/(180/Pi); Phi = 90/(180/Pi);
Thetac = 47.3/(180/Pi); Phic = (180+Random[]*2*45-45)/(180/Pi);
```



Assume a general quadratic shape for the mirror:

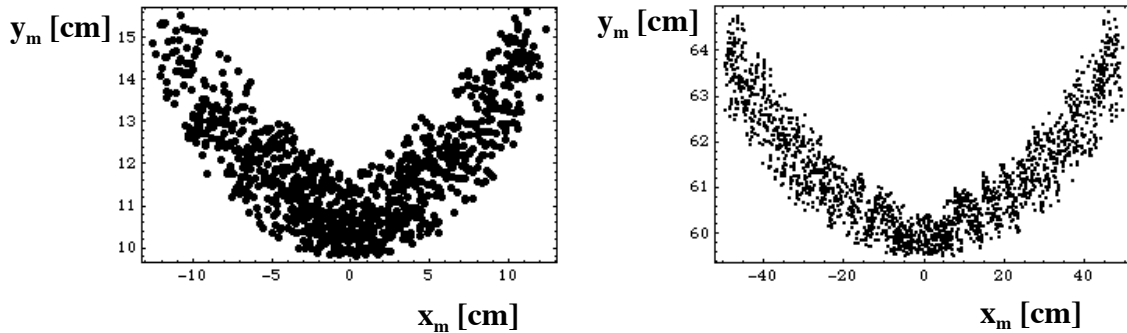
A general form of 3D **quadratic** shape is:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$$

Intersection of photons and mirror:

$xm0 = 0; ym0 = 23.0; zm0 = -86.6; r = 2*(49.5);$

$zm0 = -86.6 + 60; r = 2*(49.5);$ - mirror moved along z by 60cm



Note: Based on the Vellum program of the FDIRC prototype, expect for $x_m \sim -8$ to 8 , $y_m \sim 8$ - 11 in the bar coordinate system. The periodicity in this image can be observed if one moves the mirror a bit further along the z axis.

Intersection of photons and FDIRC detector plane:

a) Spherical mirror & flat detector plane:

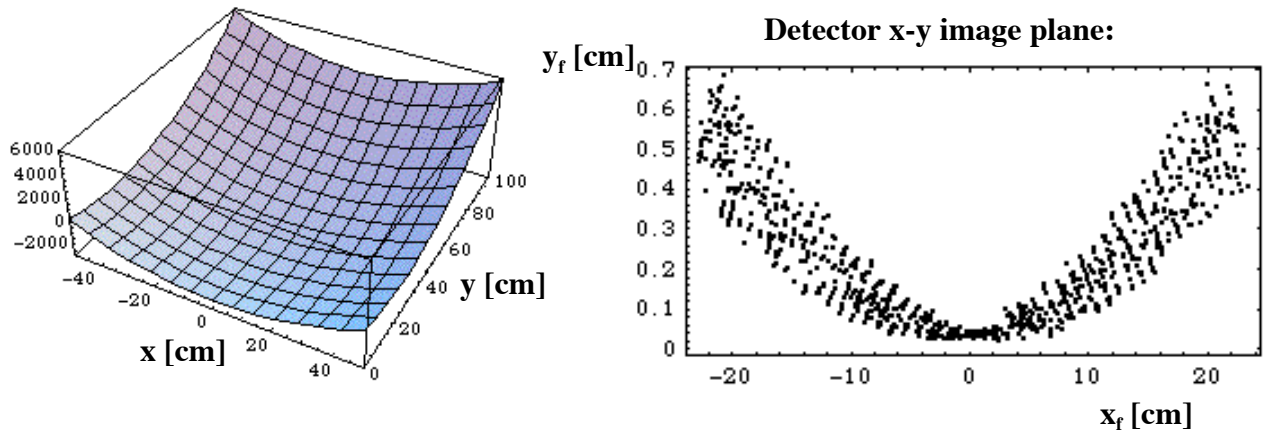
$$(x - x_{m0})^2 + (y - y_{m0})^2 + (z - z_{m0})^2 - r^2 = 0$$

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$$

$x_{m0} = 0; y_{m0} = 23.0; z_{m0} = -86.6; r = 2*(49.5)$; ← CRID mirror parameters

$a_{11} = 1; a_{22} = 1; a_{33} = 1; a_{12} = 0; a_{13} = 0; a_{23} = 0.0; a_{14} = -x_{m0}; a_{24} = -y_{m0}; a_{34} = -z_{m0}; a_{44} = x_{m0}*x_{m0} + y_{m0}*y_{m0} + z_{m0}*z_{m0} - r*r$;

For the FDIRC prototype geometry with a **spherical mirror** one gets a familiar image (in the bar coordinate system, and after a subtraction of a constant of 35.55 from y_f):



Note: Based on the Vellum program of the FDIRC prototype, expect for $x_f \sim -20$ to 20 , $y_f \sim 35$, $z_f \sim -(3-6)$ in the bar coordinate system. The calculation is not that far from the expected numbers.

b) Circular cylindrical mirror & flat detector plane:

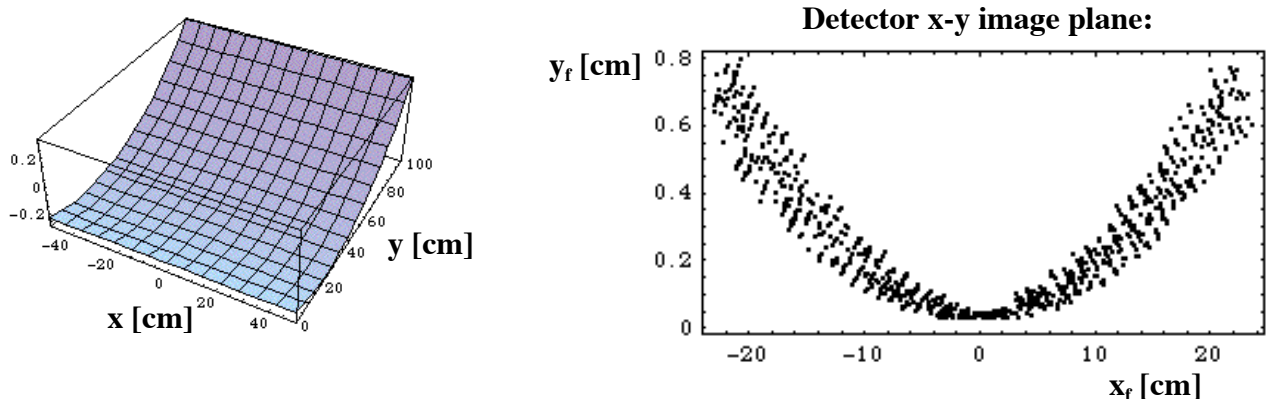
$$(y - y_{m0}/a)^2 + (z - z_{m0}/b)^2 - 1 = 0 \text{ (axis along the x-axis)}$$

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$$

$x_{m0} = 0; y_{m0} = 23.0; z_{m0} = -86.6; r = 2*(49.5); a = r; b = r$;

$a_{11} = 0; a_{22} = 1/(a*a); a_{33} = 1/(b*b); a_{12} = 0; a_{13} = 0; a_{23} = 0.0; a_{14} = 0; a_{24} = -y_{m0}/(a*a); a_{34} = -z_{m0}/(b*b); a_{44} = y_{m0}^2/a^2 + z_{m0}^2/b^2 - 1$;

If the FDIRC prototype would have a **circular cylindrical mirror** and similar dimensions, one would get this image (in a bar coordinate system):



The resolution loss seems to be smaller compared to the spherical mirror, perhaps by a factor of two.

c) Parabolic cylindrical mirror & flat detector plane:

$(y - y_{m0})^2 + b(z - z_{m0}) = 0$ (axis along the x-axis)

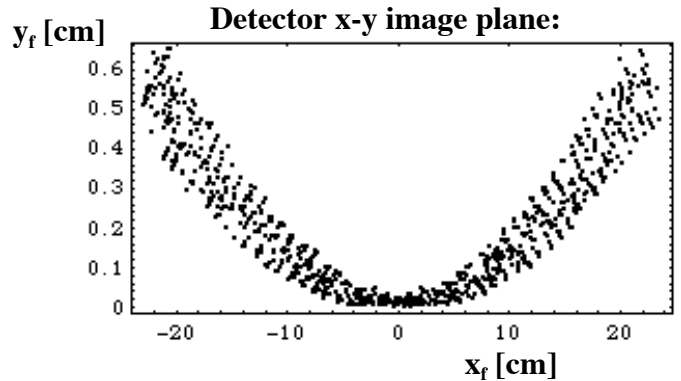
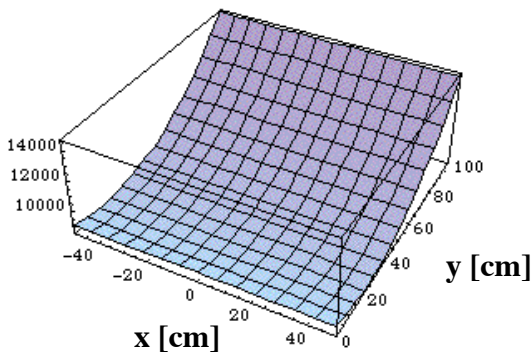
$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$

$x_{m0} = 0; y_{m0} = 23.0; z_{m0} = -86.6; r = 2*(49.5);$

$b = r;$

$a_{11} = 0; a_{22} = 1; a_{33} = 0; a_{12} = 0; a_{13} = 0; a_{23} = 0.0; a_{14} = 0; a_{24} = -y_{m0}; a_{34} = b/2; a_{44} = y_{m0}^2 - b*z_{m0};$

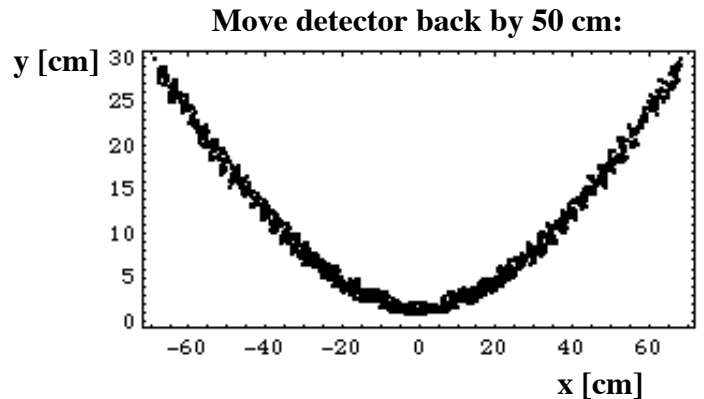
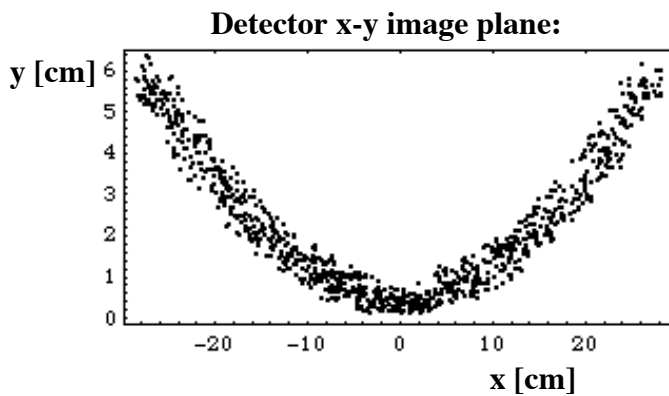
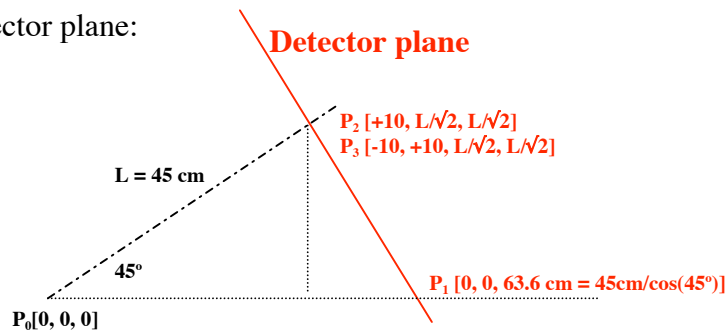
For the FDIRC prototype geometry with a **parabolic cylindrical mirror** one gets this image (in a bar coordinate system):



Similar result to that of the circular cylindrical mirror.

d) Flat detector plane, no mirror:

3 points defining the detector plane:

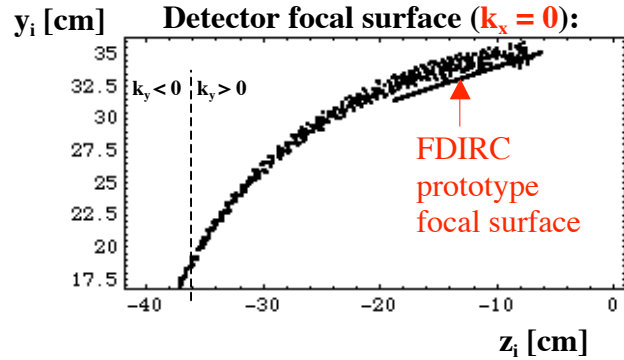
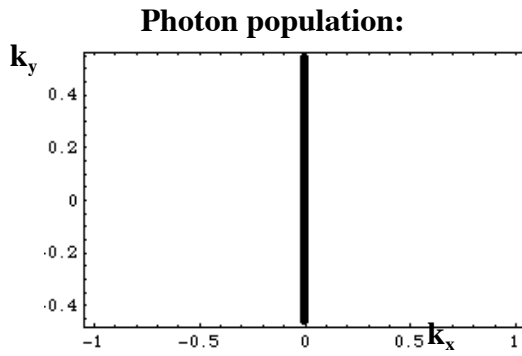


Indeed, there is a periodic kaleidoscopic pattern even without a mirror. It is created by the square bar. To see it better, move the detector plane back by 50 cm. Since the wiggles originate from the bar, i.e., the mirror has nothing to do with it, it cannot be easily fixed.

Find a detector focal surface in 3D

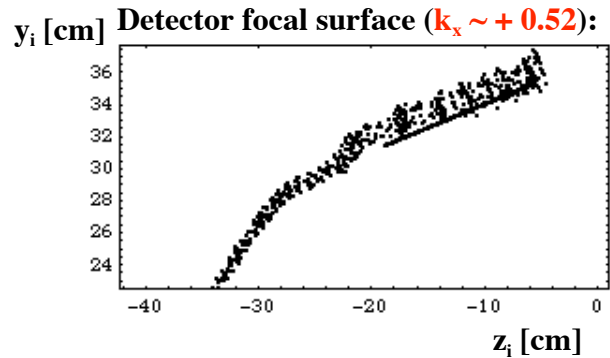
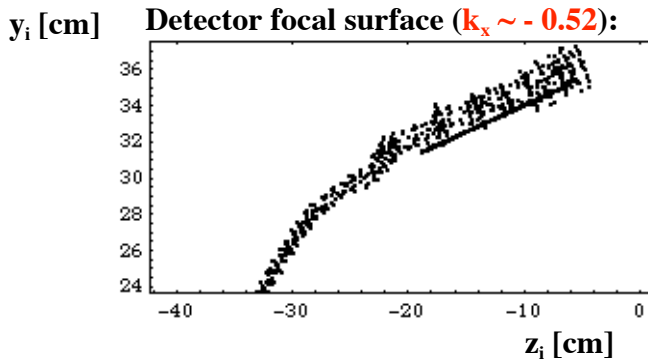
a) Spherical mirror:

zbarstart = - barl; zbarend = 0;
 Theta = (60 + Random[]*2*34 - 40)/(180/Pi); Phi = 90/(180/Pi);
 Thetac = 47.3/(180/Pi); Phic = 180/(180/Pi); Vary Phic
 xm0 = 0; ym0 = 23.0; zm0 = -86.6; r = 2*(48.6); **Choose r as one has in the FDIRC prototype Vellum study**
 a11 = 1; a22 = 1; a33 = 1; a12 = 0; a13 = 0; a23 = 0.0; a14 = - xm0; a24 = -ym0; a34 = - zm0; a44 = xm0*xm0 + ym0*ym0 + zm0*zm0 - r*r;

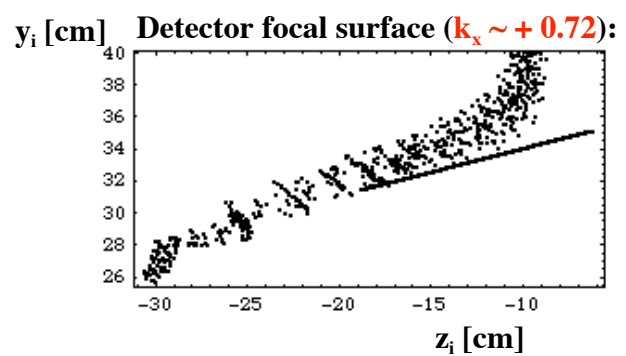
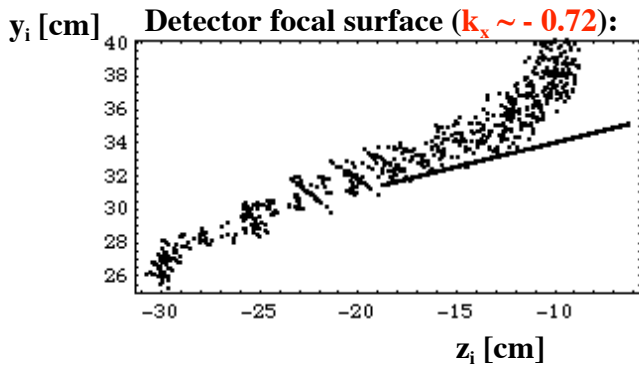


In the region, where the FDIRC prototype works ($z \sim -6$ cm), the calculated focal plane is close to a straight line. So our solution with a flat window was OK. Our results would be perfect for $k_x = 0$ of the detector plane would have zig-zag shape.

Phic = 135/(180/Pi); or Phic = 225/(180/Pi);



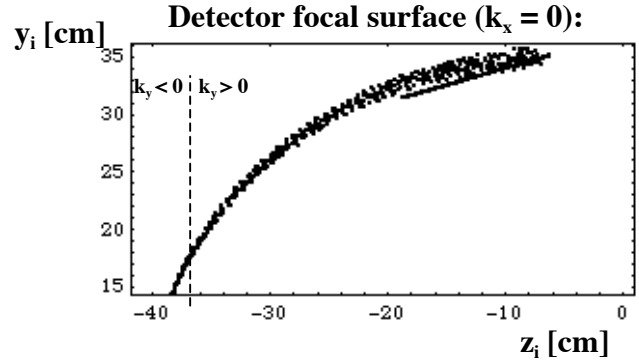
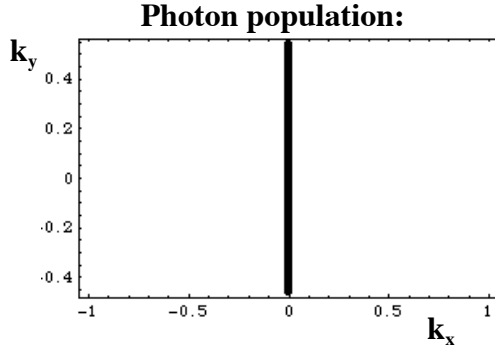
Phic = 110/(180/Pi); or Phic = 250/(180/Pi);



The detector surface shape is changing as a function of k_x , mainly as k_y is approaching 0, which corresponds to photons going parallel to z axis.

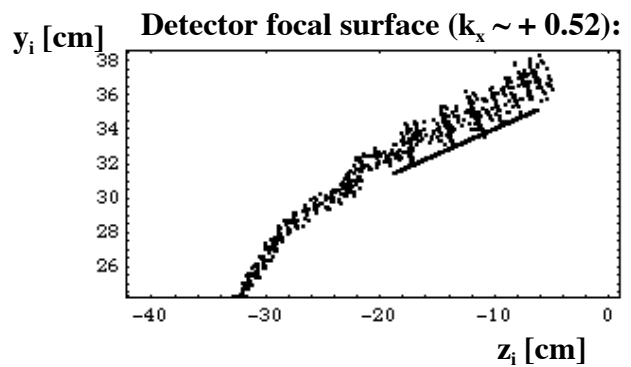
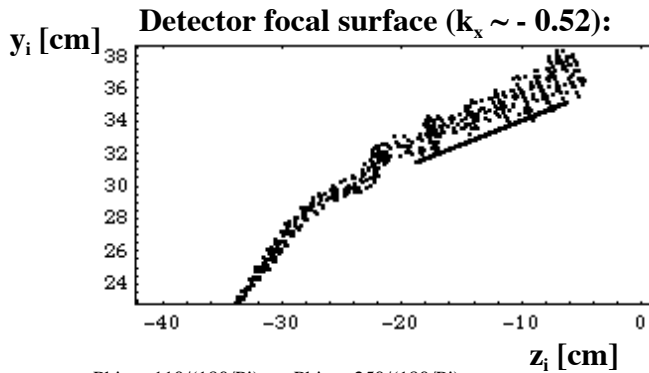
b) Circular cylindrical mirror:

zbarstart = - barl; zbarend = 0;
 Theta = (60 + Random[]*2*30 - 40)/(180/Pi); Phi = 90/(180/Pi);
 Thetac = 47.3/(180/Pi); Phic = 180/(180/Pi); Vary Phic
 xm0 = 0; ym0 = 23.0; zm0 = -86.6; r = 2*(48.6); **Choose r as one has in the FDIRC prototype Vellum study**
 a11 = 1; a22 = 1; a33 = 1; a12 = 0; a13 = 0; a23 = 0.0; a14 = - xm0; a24 = -ym0; a34 = - zm0; a44 = xm0*xm0 + ym0*ym0 + zm0*zm0 - r*r;

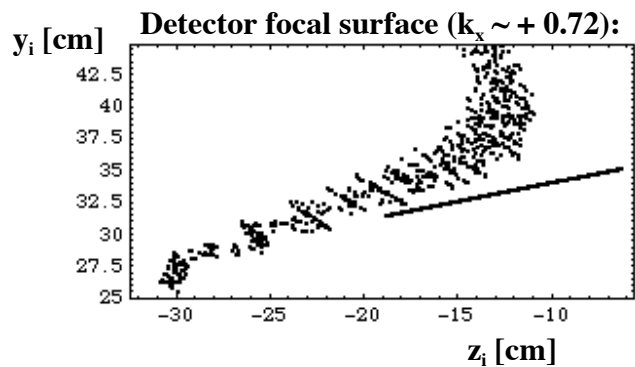
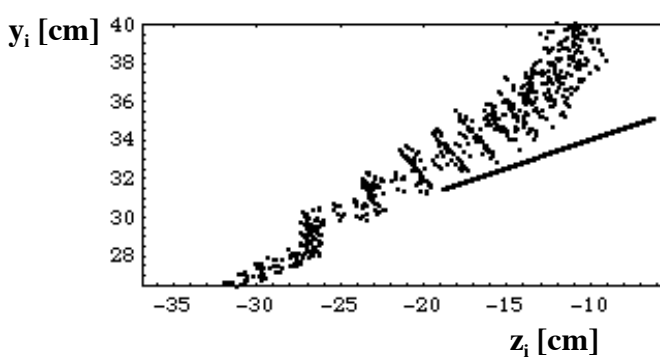


In the region, where the FDIRC prototype works ($z \sim -6$ cm), the calculated focal plane is close to a straight line. So our solution with a flat window was OK.

Phic = 135/(180/Pi); or Phic = 225/(180/Pi);



Phic = 110/(180/Pi); or Phic = 250/(180/Pi);



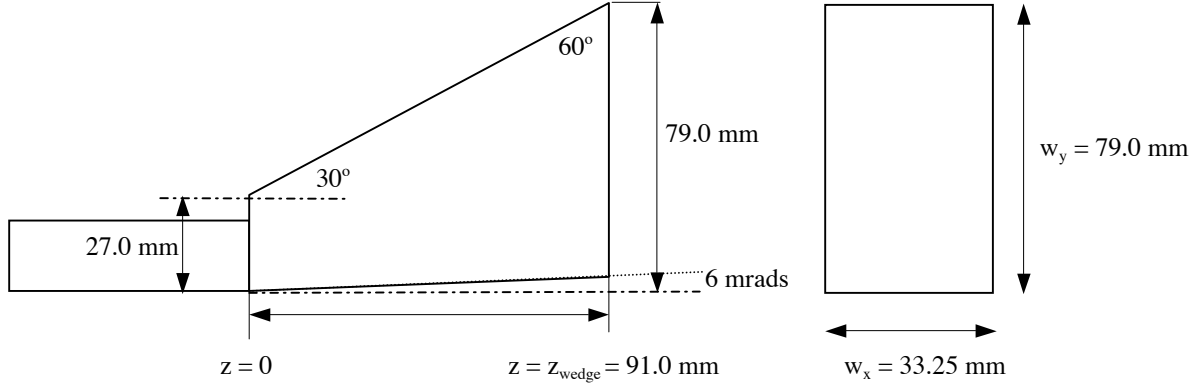
The cylindrical mirror is not much of a help !!

Effect of the wedge

4.2.2008

barw = 3.5; barh = 1.7; barl = 144*2.54;
 wx = 3.325; wy = 7.8; zwedge = 9.1;

The wedge can alter the direction cosines k_x (a sign) and k_y (both a value and a sign).



Assume: Very similar dimensions as in the FDIRC prototype; the same mirror radius.

Ring image at $z = z_{\text{wedge}}$:

4.11.2008

- Add 10 cm of arbitrary y-offset to each separate case:

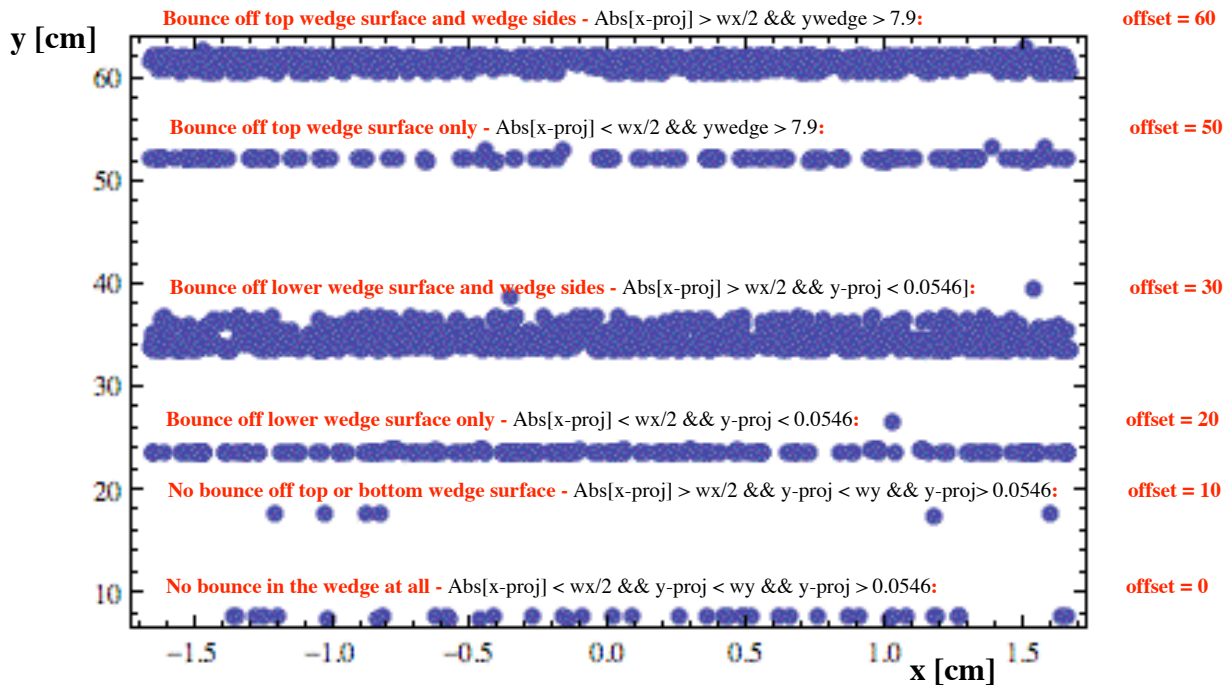
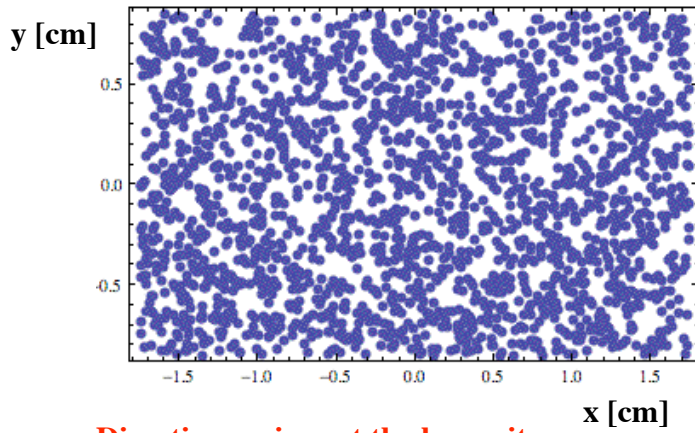
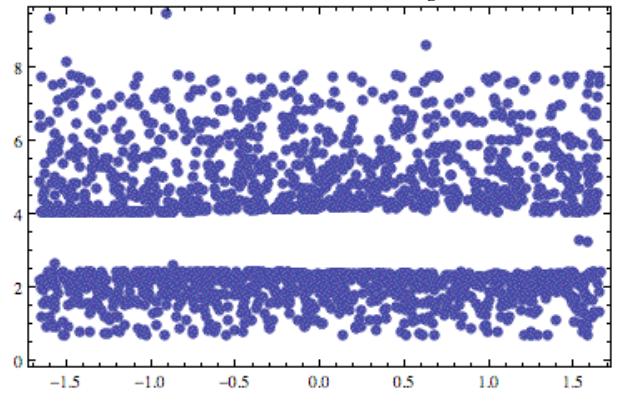


Image at bar and wedge exit:

x-y image at bar exit ($z = 0$):

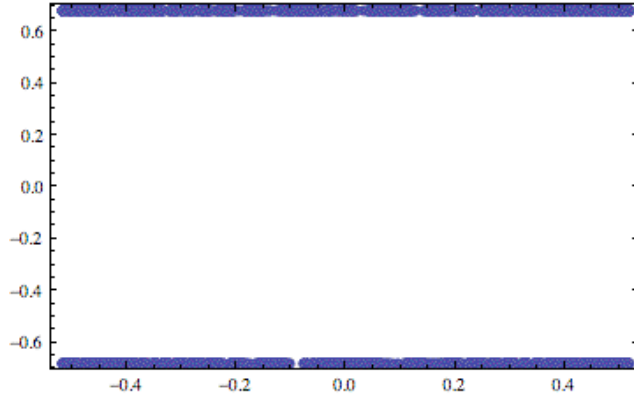


x-y image at bar exit ($z = z_{\text{wedge}}$):

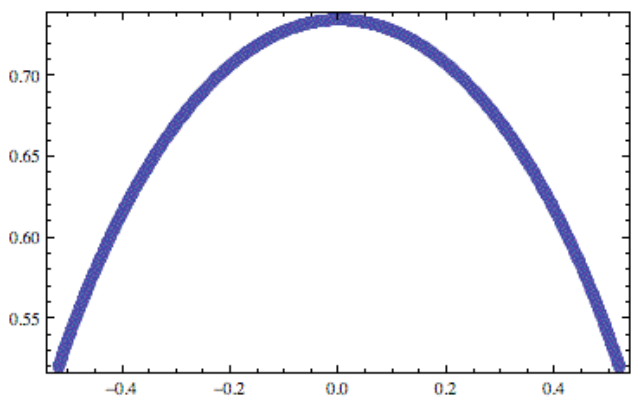


Direction cosines at the bar exit:

k_y vs. k_x :

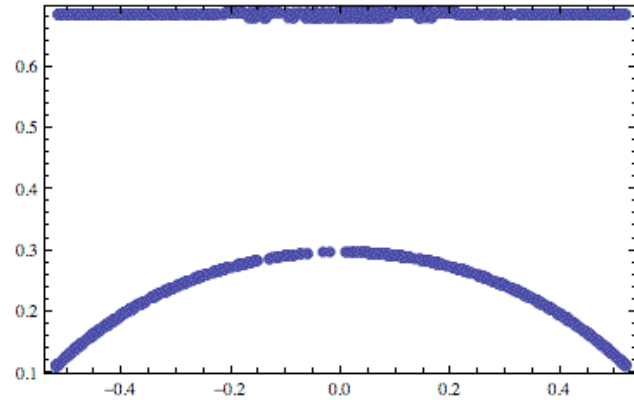


k_z vs. k_x :

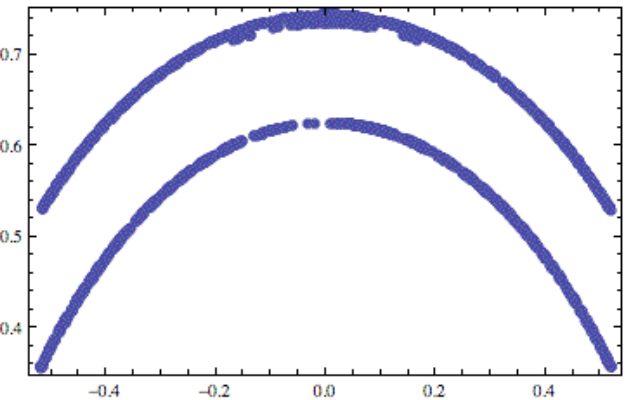


Direction cosines at the wedge exit:

k_y vs. k_x :



k_z vs. k_x :



4.12.2008

Wedge & Circular cylindrical mirror & flat detector plane:

$(y-y_m/a)^2 + (z-z_m/b)^2 - 1 = 0$ (axis along the x-axis)

$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$

$x_m = 0; y_m = 23.0; z_m = -86.6; r = 2*(49.5); a = r; b = r;$

$a_{11} = 0; a_{22} = 1/(a*a); a_{33} = 1/(b*b); a_{12} = 0; a_{13} = 0; a_{23} = 0.0; a_{14} = 0; a_{24} = -y_m/(a*a); a_{34} = -z_m/(b*b); a_{44} = y_m^2/a^2 + z_m^2/b^2 - 1;$

If a new FDIRC would have a **wedge & circular cylindrical mirror** and **similar dimensions** as the FDIRC prototype in ESA, one would get these results (all in the bar coordinate system):

Cylindrical mirror:

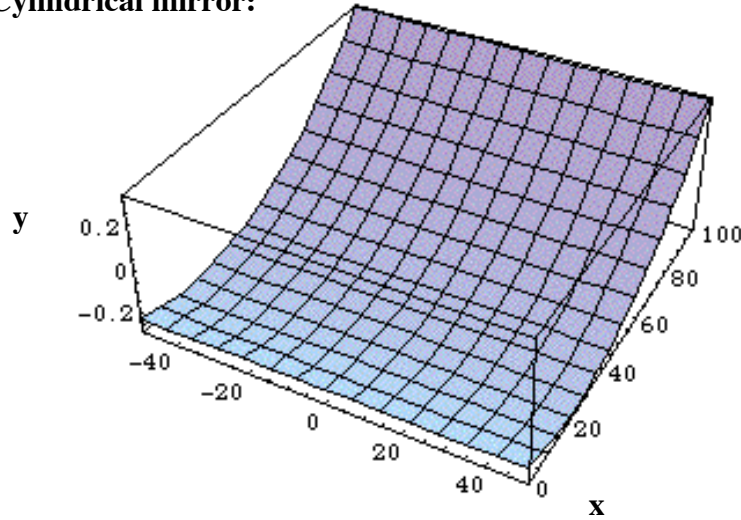
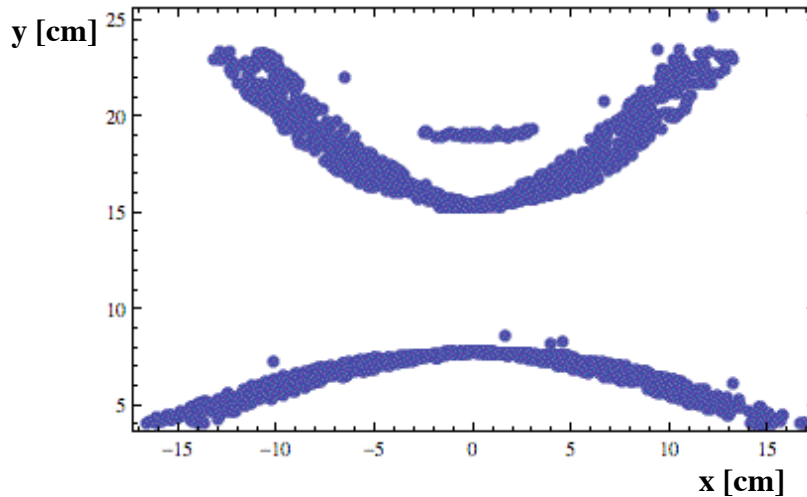
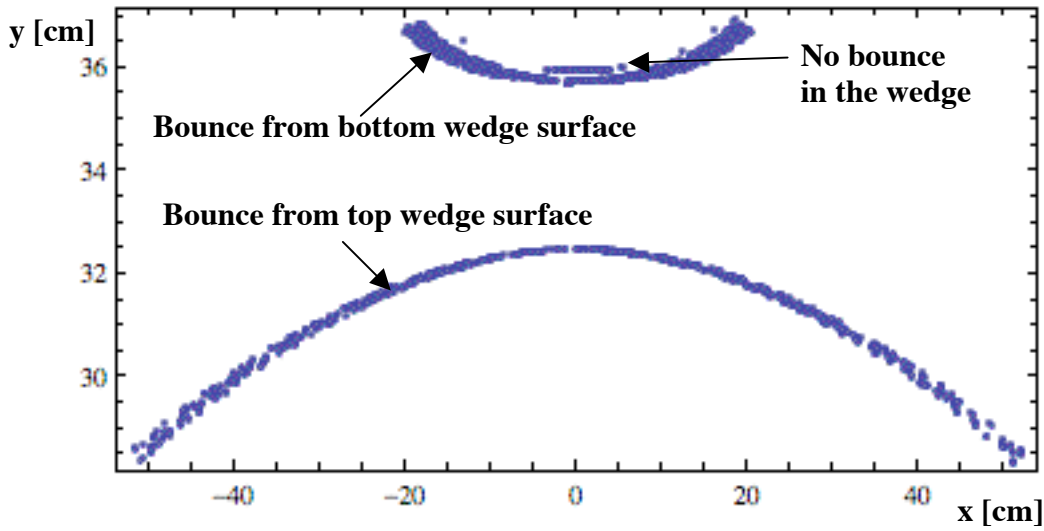


Image at the cylindrical mirror surface:

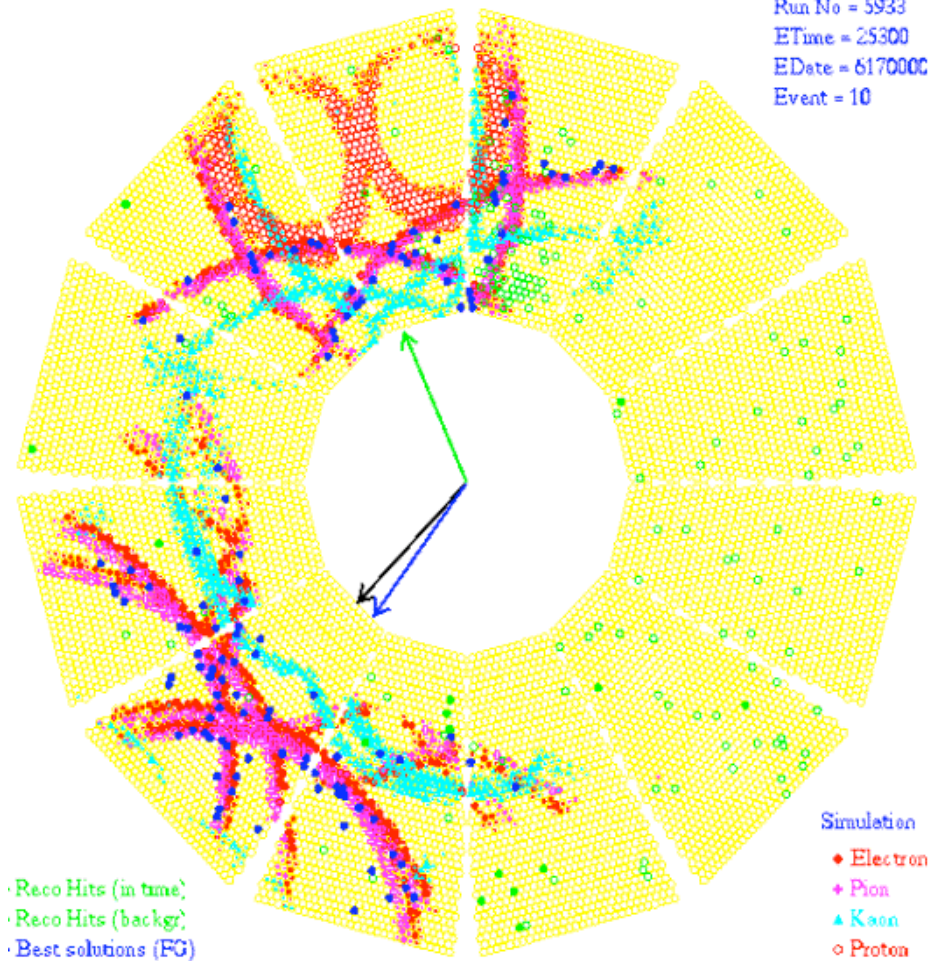


1) Detector plane located in the cylindrical mirror focus:

FDIRC ring images at the detector surface with a cylindrical mirror:

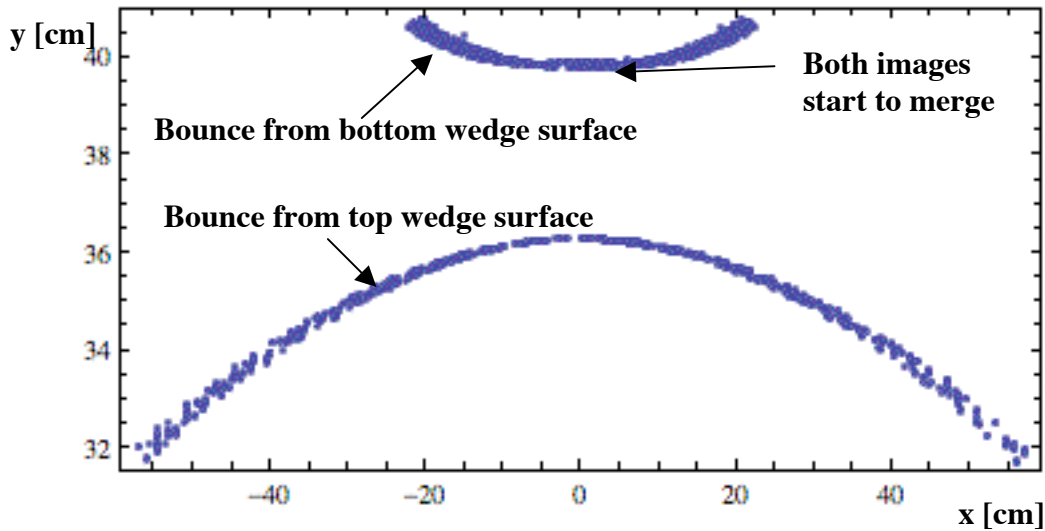


Real DIRC ring images at BaBar:



2) Move the detector plane out of focus ($y = y + 5$ cm):

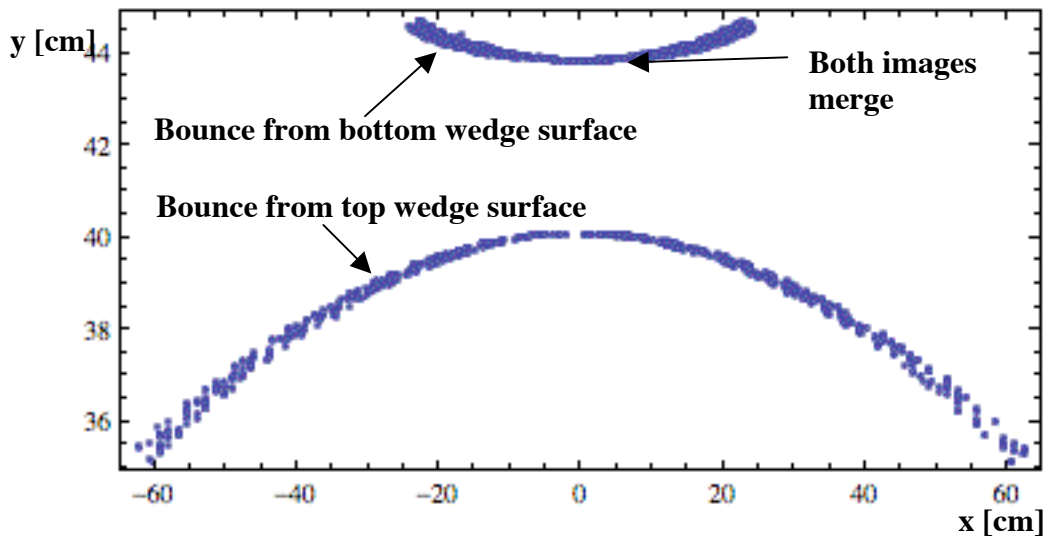
FDIRC ring images at the detector surface with a cylindrical mirror:



- Both images almost merged.

3) Move the detector plane out of focus ($y = y + 10$ cm):

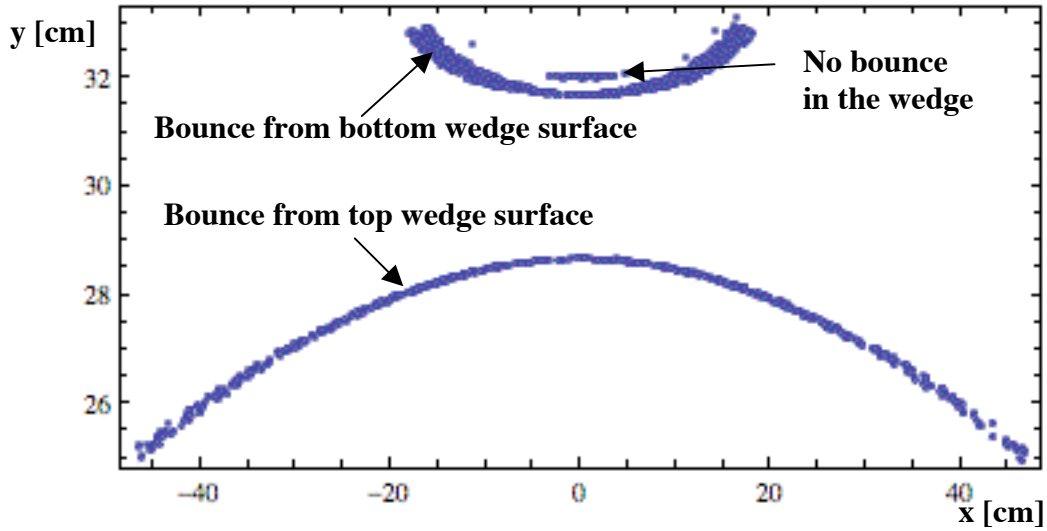
FDIRC ring images at the detector surface with a cylindrical mirror:



- Both images merged.

4) Move the detector plane out of focus other way ($y = y - 5$ cm):

FDIRC ring images at the detector surface with a cylindrical mirror:



- This is another solution: separate images completely and analyze them separately.