Temperature-Dependent Sellmeier Coefficients and Chromatic Dispersions for Some Optical Fiber Glasses

Gorachand Ghosh, Michiyuki Endo, *Member, IEEE*, and Takashi Iwasaki

Abstract— **Temperature-dependent Sellmeier coefficients are necessary to optimize optical design parameters of the optical fiber transmission system. These coefficients are calculated for fused silica (SiO_2), aluminosilicate, and Vycor glasses for the first time to find the temperature dependence of chromatic dispersion at any wavelength from UV to 1.7 μm. The zero dispersion wavelength λ_0 (1.273 μm for SiO_2, 1.393 μm for aluminosilicate, and 1.265 μm for Vycor glasses at 26°C) varies linearly with temperature, and $d\lambda_0/dT$ is 0.03 nm/K for aluminosilicate and Vycor glasses, whereas for SiO_2 it is 0.025 nm/K. This study interprets the recently observed experimental value of $d\lambda_0/dT$ for two dispersion shifted optical fibers; and the dominantly material origin of $d\lambda_0/dT$ is confirmed here as a fundamental property of the optical fiber glasses.**

I. INTRODUCTION

SILICA glass (SiO_2) is the basic optical fiber material, and it has been extensively used to make various kinds of optical fibers [1], amplifiers [2], and fiber lasers [3] with suitable doping materials since the late 1960's. Fiber lasers and amplifiers are the first step to replace the older electronic regenerators and to lead the all-optical complete photonics age in the future. A fiber laser/amplifier with the optical fiber communication system has been identified as ABHISARICA [4] (A—amplification, B—bandwidth, H—high-reliability, I—immunity to electromagnetic interference, S—silica glass/soliton transmission, A—attenuation, R—refractive index, I—impurity, such as rare-earth doping, to make fiber laser/amplifier, C—chromatic dispersion, and A—absorption).

The refractive index, its dispersion, chromatic dispersion, and its variation as a function of temperature are important characteristics of silica-based glasses, which are necessary for the evaluation of optical fiber transmission system designs using such glasses made with optical fibers. In general, they are measured at discrete wavelengths in the transmission range of the glasses, and the suitable interpolation/extrapolation technique is the state of the art. Of the various available techniques [5], the Sellmeier method is the one that is most commonly used because it has a sound physical basis. A knowledge of the refractive index as a function of temperature is necessary to evaluate chromatic dispersion at different temperatures, since it plays a vital role in the optical fiber communication system. Matsuoka *et al.* [6] used a three-pole temperature-dependent Sellmeier equation to represent refractive index variation simultaneously as a function of frequency and temperature only in the UV and visible region from −165.4 to 83.3°C. We, however, use the two-pole Sellmeier formula to represent the refractive index dependence with wavelength from UV to 1.7 μm at room temperature; the temperature effect is then separately introduced as a variation of the Sellmeier coefficients, a method used by Ghosh and Bhar [7]. The two-pole Sellmeier formula is a physically meaningful model to represent refractive indexes with wavelength. One pole is due to electronic resonance absorption, and the other pole is due to the lattice/ionic resonance absorption.

In the following, after first describing computational methods for finding Sellmeier coefficients for three kinds of silica glasses—SiO_2, aluminosilicate, and Vycor glasses—we arrive at relations of Sellmeier coefficients using temperature and smoothed thermooptic coefficients. Two other aspects are also considered from temperature-dependent Sellmeier coefficients, viz., chromatic dispersion and zero dispersion wavelength at different temperatures, and they are compared to the experimental values.

II. DERIVATION OF SELLMEIER COEFFICIENTS

The wavelength-dependent Sellmeier is of the form

$$n^2 = A + B/(1 - C/\lambda^2) + D/(1 - E/\lambda^2) \qquad (1)$$

(λ is the wavelength in μm), where the last term accounts for the decrease in refractive indexes due to lattice absorption, the first and second terms represent, respectively, the contribution to refractive indexes due to higher energy and lower energy gaps of electronic absorption. The method of fitting a set of refractive index data for low refractive index materials to a Sellmeier in order to evaluate constants was described by Maltison [8]. However, his equation was of a slightly different form. The normal approach for any such problem is to first find the initial values of the parameters and then to add corrections by an iterative process so as to minimize the deviation between the measured and computed values. We are able to evaluate the Sellmeier constants by fitting the measured data to an accuracy better than the experimental accuracy. In this way, we evaluate the constants A, B, C, and D, taking beforehand a reasonably estimated value for E, the lattice absorption

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TABLE I

SELLMEIER COEFFICIENTS FOR FUSED SILICA (FS), ALUMINOSILICATE (AS), AND VYCOR GLASSES AT ROOM TEMPERATURE AND AT A HIGHER TEMPERATURE WHICH IS 471°C FOR FS AND 526°C FOR AS AND V GLASSES. $n^2 = A + B/(1 - C/\lambda^2) + D/(1 - E/\lambda^2)$

Glass & E= 100.0	Temp. (©C)	Sellmeier Coefficients				Expt. accuracy & sources	Our fit RMS error
		A	B	C	D		
						<- - - - - 10^{-5} - - - ->	
Fused Silica	26	1.3121622	0.7925205	1.0996732×10^{-2}	0.9116877	±21±9.6	9.5
(SiO_2)	471	1.3148367	0.8034391	1.1248041×10^{-2}	0.9119589	[10], [11]	
Fused Silica	20	1.3107237	0.7935797	1.0959659×10^{-2}	0.9237144	2.8-1.2 [8]	1.6
SiO_2	20.5	1.3156569	0.7901384	1.0993430×10^{-2}	1.0248690	±0.3 [6]	0.5
	45.2	1.3066410	0.7994875	1.0919460×10^{-2}	0.9598566	±0.3 [6]	0.4
alumino-silicate	28	1.4136733	0.9503994	1.3249011×10^{-2}	0.9044591	±21±9.6	3.4
	526	1.5205253	0.8556252	1.5205234×10^{-2}	0.9092824	[10]	4.4
Vycor	28	1.2754213	0.8271916	1.0653107×10^{-2}	0.9384236	±21±9.6	4.1
Glass	526	1.3488048	0.7695233	1.1884981×10^{-2}	0.9460697	[10]	5.1

frequency. The choice of the value for E is not critical since the materials stop transmitting long before the onset of lattice absorption frequency. The E value essentially determines the infrared transmission cutoff of silica glasses [9] and is 1080 cm^{-1}. The significance of fixing E will be further justified in connection with discussion on temperature dependence. Using the refractive index data for fused silica [6], [8], [10], and [11], aluminosilicate [10], and Vycor glasses [10], the Sellmeier coefficients are evaluated and are shown in Table I for room temperature, and at a higher temperature from the measured refractive indexes at specific wavelengths. It is interesting to point out from the analyses that the average energy gaps as defined by $\sqrt{C}$ of (1) are 11.8, 10.8, and 12.0 eV for SiO_2, aluminosilicate, and Vycor glasses, respectively. Due to aluminum doping, the average energy gap is lowered. These gaps are lying in the VUV spectral region, and agree well with experimental VUV absorption peaks [9]. Experimental error and fitted accuracy are also cited in the table. The computed curves of refractive indexes are the solid lines based on (1) and experimental points are shown in Fig. 1 at room temperature. Although the data of Matsuoka *et al.* [6] are analyzed critically in this model with greater accuracy than that of experiment, these data alone are not sufficient to extrapolate refractive indexes up to 1.7 μm to interpret chromatic dispersion and zero-dispersion wavelength of SiO_2 glass. We have used UV to 1.7 μm wavelength data for the interest of optical fibers having three optical transmission windows in this wavelength region.

III. TEMPERATURE DEPENDENCE

Tsay *et al.* [12] have given an excellent account of the variation of refractive index as a function of temperature in transparent crystals. Of the two factors determining the temperature dependence, the electronic effect plays a dominant role over the lattice/ionic effect in diamond-like semiconductor crystals. Recently, Lines [13] has observed the physical origin of the temperature dependence of chromatic dispersion in fused silica. It is shown that all manifestations of chromatic dispersion in the silica optic window possess a temperature modulation which is dominated by a single term, namely, the temperature derivative of the Sellmeier valence to conduction band energy gap. But their analysis is not straightforward. In our formalism, it is the temperature derivative of the average energy gap as defined by $\sqrt{C}$. And since in such glasses the variation of energy gap with temperature (-2.3×10^{-4} eV/K) [6] is much larger than that of the thermal expansion (1.6×10^{-7}/K) [14] factor, a positive dn/dT is observed. We therefore allow variations in the electronic energy gap, keeping the lattice absorption frequency ($\sqrt{E}$) constant (the value is from [9]) while evaluating temperature dependency of Sellmeier coefficients. Again, there is a negligible shift of lattice absorption gap when temperature is changed [15].

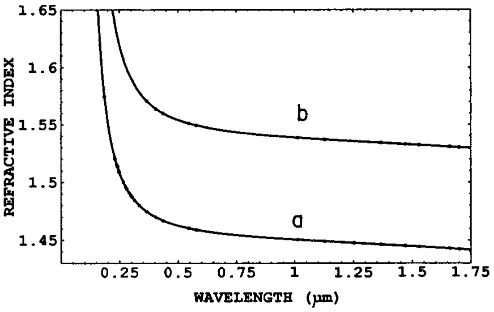

Fig. 1. The schematic plot of refractive index versus wavelength for fused silica (a) and aluminosilicate (b) glasses at 26 and 28°C, respectively: points are experimental and the solid lines are computed from the Sellmeier coefficients.

Most of the semiconductors and chalcopyrite crystals exhibit the constancy of dn/dT for operating temperatures at any given wavelength lying within the transmission range. Such constancy has been observed in fused silica up to 471/828°C, aluminosilicate up to 526°C, and in Vycor glasses up to 526/826°C by Wray and Neu [10]. Williams *et al.* [11] have measured the constancy of dn/dT for fused silica at 0.1849 μm. Malitson [8] has presented the spectral response of thermooptic coefficients of fused silica by measuring the same for the temperature range 20–30°C. Matsuoka *et al.* [6] have measured the refractive indices of SiO_2 glass at seven temperatures from −165.4°C to 83.3°C for the UV and visible wavelength region, and presented the constancy of thermooptic coefficients at 20.5°C. These thermooptic coefficients (dn/dT) and their dispersion have been critically analyzed and smoothed by properly taking into account the band edge dispersion, and will be published elsewhere. The Sellmeier coefficients at any temperature T are computed from the

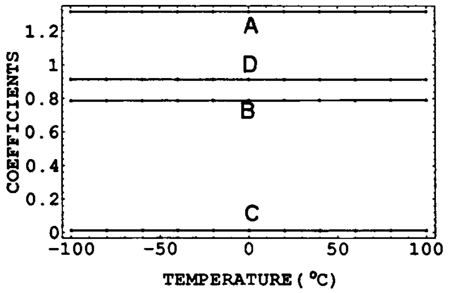

Fig. 2. Typical plot of the temperature-dependent Sellmeier coefficients (A, B, C, D) for SiO_2 glass.

TABLE II
THE LINEAR FITTED CONSTANTS (m = SLOPE, $c1$ = INTERCEPT), OF THE LEAST SQUARES ANALYSES OF THE TEMPERATURE-DEPENDENT SELLMEIER COEFFICIENTS (X) OF FS, AS, AND V GLASSES ($X = mT + c1$, T IS THE TEMPERATURE IN DEGREE CENTIGRADE)

Sellmeier coefficients	Fitted constants	Fused silica SiO_2	Alumino-silicate	Vycor glass
A	m x 10^6	6.90754	24.95380	45.70720
	c1	1.31552	1.41294	1.27409
B	m x 10^5	2.35835	-0.11466	-1.47194
	c1 x 10	7.88404	9.50465	8.27657
C	m x 10^7	5.84758	12.24700	12.35900
	c1 x 100	1.10199	1.32143	1.06179
D	m x 10^7	5.48368	11.60740	12.58560
	c1	0.91316	0.90443	0.93839
E	---	100.0	100.0	100.0

room temperature Sellmeier equation and the smoothed dn/dT values by calculating refractive indexes from the relation

$$n_T = n_R + (T - R)(dn/dT)_{\text{smoothed}} \tag{2}$$

where T is the temperature in degree centigrade, R is the room temperature, n_T and n_R are the refractive indexes at T and room temperature, respectively. We have fitted the calculated refractive indexes of SiO_2, aluminosilicate, and Vycor glasses from −100 to 100°C at 20°C intervals by using (2). All these Sellmeier constants (X) are then separately plotted against temperature. Strangely enough, all are found to fit nicely into straight lines. A typical plot of the coefficients for the fused silica is shown in Fig. 2. Constants of the straight lines are obtained by least square analysis of the data and are shown in Table II. Interestingly, dEg/dT can be calculated from the slope of C coefficients. The variations of bandgap as defined by $\sqrt{C}$ with temperature are −3.2, −5.0, and -7.0×10^{-4} eV/K for SiO_2, aluminosilicate, and Vycor glasses, respectively. These values agree reasonably well with the experiment [6].

IV. CHROMATIC DISPERSION AND ZERO-DISPERSION WAVELENGTH

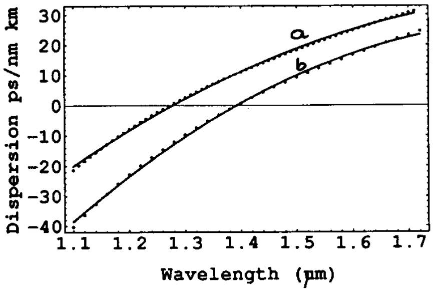

Fig. 3. Dispersion behavior of SiO_2 (a) and aluminosilicate (b) glasses at 26°C from 1.1 to 1.7 μm.

The material chromatic dispersion plays a vital role in the optical fiber communication system, and it manifests through the wavelength dependence of the refractive index $n(\lambda)$ as in (1) by the following relation [16]:

$$M(\lambda) = -\lambda/c \cdot d^2n(\lambda)/d\lambda^2 \tag{3}$$

$$\begin{aligned} M(\lambda) = -1/(cn)[&-4/\lambda^5\{BC^2/(1 - C/\lambda^2)^3 \\ &+ DE^2/(1 - E/\lambda^2)^3\} \\ &+ \lambda(dn/d\lambda)^2 + 3n(dn/d\lambda)] \end{aligned} \tag{4}$$

where

$$\begin{aligned} dn/d\lambda = -1/(n\lambda^3)[&BC/(1 - C/\lambda^2)^2 \\ &+ DE/(1 - E/\lambda^2)^2] \end{aligned} \tag{5}$$

and c is the speed of light.

Chromatic dispersions are computed for these glasses at different temperatures by using temperature-dependent Sellmeier coefficients, and are shown in Fig. 3 for SiO_2 and aluminosilicate glasses at 26°C. The dispersion characteristics are not linear for the whole spectral region. The temperature dependence of chromatic dispersion at 1.53 μm is computed for SiO_2 glass and it is -1.5×10^{-3} ps/nm km K. This value is exactly the same as the experimental one [17]. The dispersion behavior near the zero dispersion wavelength is shown in Fig. 4 for Vycor glass at −100 and 100°C, respectively. The dependency is almost linear as shown. The zero dispersion wavelengths are also computed for these glasses at different temperatures. The zero-dispersion wavelengths are 1.273, 1.393, and 1.265 μms at 26°C for SiO_2, aluminosilicate, and Vycor glasses, respectively. Interestingly, the temperature dependency of zero-dispersion wavelength is linear and $d\lambda_0/dT = 0.025$ nm/K for SiO_2. This study agrees well with the recently experimental [17] values 0.029 ± 0.004 nm/K and 0.031 ± 0.004 nm/K for two dispersion shifted optical fibers within the experimental accuracy. $d\lambda_0/dT$ is 0.03 nm/K for both aluminosilicate and Vycor glasses. Zero-dispersion wavelength as a function of temperature T is shown in Fig. 5 for SiO_2 glass. This analysis implies that $d\lambda_0/dT$ is dominated by the material of the core-glass of the optical fiber instead of the optical fiber design. This is one of the fundamental optical properties of the glass itself.

V. CONCLUSION

We have formulated the temperature-dependent Sellmeier coefficients of three optical fiber glasses for the first time to

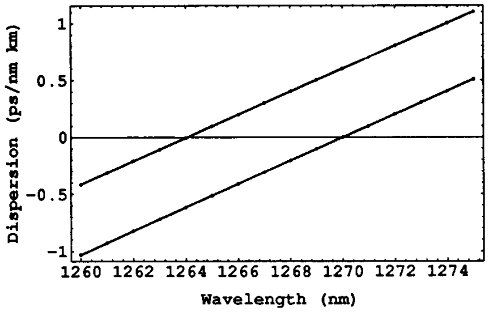

Fig. 4. Dispersion characteristics of Vycor glass at −100°C and +100°C (the upper and lower lines, respectively).

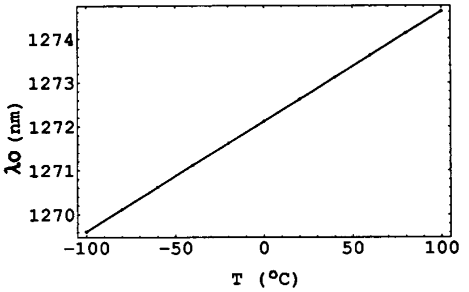

Fig. 5. Zero dispersion wavelength λ_0 as a function of temperature T for SiO_2 glass.

calculate refractive indexes at any wavelength from UV to 1.7 μm, and at any operating temperature. These are useful not only to determine optical design parameters at different temperatures, but also are capable of predicting the operating features of fiber optics as a function of temperature. The variation of average energy gap as defined by $\sqrt{C}$ is the major contributor of refractive indexes, and dEg/dT is -0.5 meV/K for these glasses. Chromatic dispersions as a function of wavelength have been computed at different temperatures, and $dM(\lambda)/dT$ is -1.5×10^{-3} ps/nm km K at 1.53 μm for SiO_2 glass. The zero-dispersion wavelength λ_0 has been analyzed, and it is dominated by the material characteristics of the glass with a value $d\lambda_0/dT$ 0.03 nm/K. This study has confirmed the fundamental dominant material origin of $d\lambda_0/dT$ instead of the optical fiber design.

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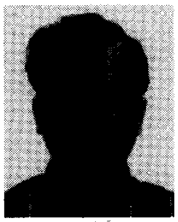

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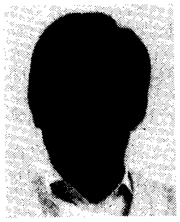

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