MODELING TeV CLASS PLASMA AFTERBURNERS[*]

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Abstract

Plasma wakefield acceleration can sustain acceleration gradients three orders of magnitude larger than conventional RF accelerator. In the recent E164X experiment, substantial energy gain of about $3 - 4$ GeV has been observed. Thus, a plasma afterburner, which has been proposed to double the incoming beam energy for a future linear collider, is now of great interest. In an afterburner, a particle beam drives a plasma wave and generates a strong wakefield which has a phase velocity equal to the velocity of the beam. This wakefield can then be used to accelerate part of the drive beam or a trailing beam. Several issues such as the efficient transfer of energy and the stable propagation of both the drive and trailing beams in the plasma are critical to the afterburner concept. We investigate the nonlinear beam-plasma interaction in such scenario using the 3D computer modeling code QuickPIC. We will report on the preliminary simulation results of both 100 GeV and 1 TeV plasma afterburner stages for electrons including the beam-loading of a trailing beam. Analytic analysis of hosing instability in this regime will be presented.

INTRODUCTION

The state-of-the-art linear collider, i.e., SLC, operates at 100 GeV. SLC colliders 50 GeV electron and positron beams. For the proposed next world-class collider, i.e, the International Linear Collider(ILC), the electron and positron beams will collide at energy of 500 GeV up to 1 TeV. Since conventional and superconducting rf technology can support a maximum acceleration field less than 100 MeV/m, the required length for acceleration is several tens of kilometers for a TeV collider. The plasma wakefield accelerator (PWFA) concept is one of the advanced acceleration schemes which may enable the design of a compact TeV accelerator.

Plasmas can sustain very high acceleration gradients, the plasma wave amplitude scales as $\sqrt{n_0(cm^{-3})}$ eV/cm, where n_0 is the plasma density. In the recent E164X experiment conducted at SLAC, $3 \sim 4$ GeV energy gain over 10 cm long plasma section was reported [1]. This substantial energy gain is obtained by sending a 28.5 GeV electron beam into an underdense Li vapor of $3 \times 10^{17} cm^{-3}$ peak density. This experiment is in the so called "blow-out" regime and the acceleration field is on the order of 10 GeV/m.

An "afterburner" concept based on the PWFA blow-out regime has been proposed as an energy booster for an existing linac such as the SLC to double the energy of electron and positron beams before they collide [2]. In this paper, we will discuss the parameters for possible afterburner stages at both the SLC and the ILC. Several issues are addressed in this paper, including the efficient transfer of energy from the drive beam to a possible trailing beam, the transverse dynamics and stability of ultra-relativistic beams, the X-ray radiation loss and the beam erosion in the under-dense plasma. In order to make clear quantitative predictions, accurate 3D particle-based computer models are needed. Recently, we have developed a fully non-linear quasi-static parallel PIC code which enables us to conduct the first full scale simulation study in TeV class afterburner stages. We describe our simulation model and report on preliminary results in the following sections.

QUASI-STATIC PIC MODEL

To double the energy of a $50 \sim 500$ GeV beam in a PWFA "afterburner" stage requires plasmas of $10^5 \sim 10^6 c/\omega_p$ length. Using a full PIC code, it would take $\sim 10^{13}$ particle pushes to model even a single GeV PWFA stage. On today's fastest computers, such a simulation takes $\sim 5,000$ CPU hours. Therefore, to model 500 GeV stages would take $2,500,000$ CPU hours for a full PIC simulation. Clearly this is impossible. However, for typical plasma "afterburner" parameters, the drive beam might not evolve for over 1000's of time steps. For example, for a PWFA the drive beam evolves on the scale of the betatron wavelength which is $(2\sqrt{\gamma_b})^{1/2}$ times longer than the plasma wavelength, where γ_b is the Lorentz factor of the beam. For a 50 GeV beam this is a factor of ~ 500 times longer. Therefore, it is possible to separate the time scales of the beam and the plasma evolution and use a reduced model for this problem. Here we use the quasi-static parallel PIC code, QuickPIC [3]. QuickPIC solves a reduced set of Maxwell equations under the quasi-static approximation, i.e., the high energy beam is "stiff" and evolves on a time scale much longer than the plasma oscillation period. The quasi-static field equations in Lorenz gauge can be written in the moving window variables ($x, y, s \equiv z, \xi \equiv ct - z$) as:

$$\nabla_\perp^2 \phi = -4\pi\rho \qquad (1)$$

$$\nabla_\perp^2 \mathbf{A} = -\frac{4\pi}{c}\mathbf{J} \qquad (2)$$

$$\nabla_\perp \cdot \mathbf{A}_\perp = -\frac{\partial \psi}{\partial \xi} \qquad (3)$$

where $\psi \equiv \phi - A_z$.

QuickPIC solves for ϕ, ψ and $\mathbf{A}$ using predictor-corrector technique and then $\mathbf{E}$ and $\mathbf{B}$ fields can be obtained to advance particles to the correct positions. We have benchmarked QuickPIC with full PIC code OSIRIS for the parameters in the range of interest, and the agreement is excellent [3].

CHOOSING PARAMETERS

In the non-linear blow-out regime of PWFA, the accelerating electric field can be roughly estimated by the 1D wave-breaking limit $E_{wave-breaking} \approx mc\omega_p/e \approx 96\sqrt{n_0}$ eV/m, where n_0 is in unit of (cm^{-3}). Larger accelerating field can be achieved by using higher density. However, due to technology limitations, the total number of beam particles generated in SLC is about 2×10^{10} and the longitudinal size is on the order of $10 \sim 100$ microns. Therefore, that puts limit on the maximum plasma density in order to operate in the blow-out regime. We use $n_0 \sim 10^{16} cm^{-3}$, which gives $E_{max} \sim 10$ GeV/m.

In general, to efficiently transfer the energy from the drive beam to the wake and then to the trailing beam, the density profile of both beams are of importance. This is because the blow-out process is determined by the ratio Λ_b/Λ_0, where Λ_b and Λ_0 are the charges in unit length of the beam and the plasma. In the large blow-out radius situation, i.e., $r_{bm} >> r_0$, where r_{bm} is the maximum blow-out radius and r_0 is the transverse size of the beam, the dependence on the transverse profile is small, thus the longitudinal profile of the drive beam can determine the rate of energy loss to the wake. The linear theory of beam loading [4] gives the ideal "door-step" profile of the drive beam to achieve constant energy transfer rate to the wake inside the beam. In a future publication [5] we will show that even in the non-linear blow-out regime, a wedge shaped beam is best. In our simulations, the beam density rises linearly from the head to the tail, this profile gives a roughly constant deceleration wake field inside the beam. The trailing beam is shorter and has less charge than the drive beam and it is placed near the end of the blow-out channel where it witnesses an acceleration field larger than the deceleration field acting on the drive beam. A trailing beam with a properly chosen profile can flatten the longitudinal field at the region where the beam resides, reducing the final energy spread of the trailing beam. The ratio between the witnessed acceleration field and the maximum deceleration field is defined as the transformer ratio. It is desirable that this parameter is large so the energy transfer is more efficient.

The above discussion only serves as a general guideline, there is freedom in the parameters for the final design. In

reality, it is difficult to manipulate the total charge, profile and spacing of the beams, it might be relatively easier to fine-tune the plasma density to obtain a good wake field structure inside the ion channel. The parameters for our 100 GeV and 1 TeV "afterburner" stages are summarized in Table 1. These simulations are not intended to be the final design, but rather represent the first attempt to study the relevant physics of beam propagation in an "afterburner" stage.

Table 1: Simulation parameters

Parameter	100 GeV stage	1 TeV stage
Initial energy (GeV)	50/50	500/500
Beam charges (10^{10})	3.0/1.0	3.0/1.0
Emittance ϵ_N $(mm \cdot mrad)$	2230/2230	2230/2230
Spot size σ_r (μm)	15/15	15/15
Driver length (μm)	145	145
Trailer σ_z (μm)	10	10
Beam separation (μm)	100	100
Plasma density (cm^{-3})	5.66×10^{16}	5.66×10^{16}
Plasma length (cm)	300	2188

The 100 GeV stage starts with $\gamma_b = 97,847$ and a matched emittance, while the 1 TeV stage uses a beam of $\gamma_b = 978,473$ and ϵ_N about $1/3$ of the matched emittance. So in the first simulation the beam envelope does not change, but the large emittance causes the beam head to erode rather quickly. In the second simulation, the erosion is minimal for the whole distance which is on the order of $10^6 c/\omega_p$.

SIMULATION RESULTS

Fig. 1 shows the phase space of the drive beam and trailing beam at the end of the 100 GeV simulation. They have the same initial energy, spot size and emittance. The driver has an initial tilt in centroid, but the hosing instability which is triggered by the tilt is stabilized in this matched beam simulation. The final energy of the trailing beam is centered around 100 GeV with 1.12 nC charge. About 0.48 nC charge are lost due to hosing, their energy are around 50 GeV. The head of the drive beam expands and remains at the initial energy while the tail almost stops. The initial (black) and final (grey) longitudinal wakefields are plotted in Fig. 2. The wakefield slips backward due to beam erosion and the transformer ratio drops from 2 to 1.

Shown in Fig. 3 is the phase space plot of the simulation for a 1 TeV stage. Both the drive and trailing beams are on axis, so no hosing growth is triggered in this simulation. Fig. 4 is the longitudinal wakefield at different times of the simulation. The head of the beam erodes away at the beginning and the decelerating field becomes larger while the acceleration field remains relatively constant. The wake structure slips as the beam head diverges and ideal beam-
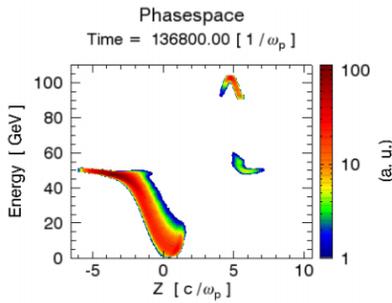
Figure 1: Phase space plot at the end of the 100 GeV stage simulation.
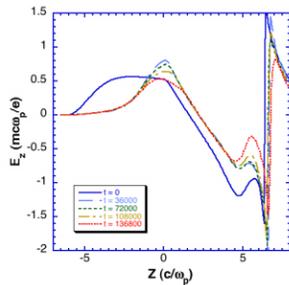
Figure 2: Longitudinal wakefield evolution for the 100 GeV simulation.

loading situation appears later in the simulation. Then the density at the new head position becomes high enough to form the ion channel, which slows down the erosion rate. The wake is very stable for $t > 90000 \times 1/\omega_p$ and the transformer ratio is close to 1.1.
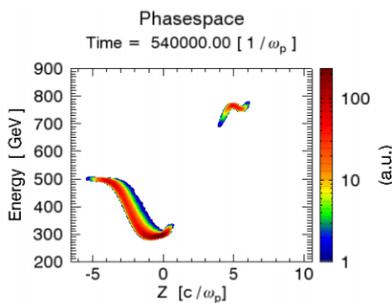
Figure 3: Phase space plot of the 1 TeV stage simulation at time $540,000 \times 1/\omega_p$.

DISCUSSION

In the above sections, we have ignored the radiation loss due to beam betatron oscillations. The energy loss rate can be estimated by $W_{loss} = r_e mc^2 r_b^2 k_p^4 \sigma_r^2/(12e)$ (eV/m). For the 100 GeV simulation parameters, the formula yields $W_{loss} = 1.06$ GeV/m. This is much smaller than $E_{wave-breaking} \approx 24$ GeV/m, therefore one can ignore the radiation loss. However, for a 1 TeV stage with the
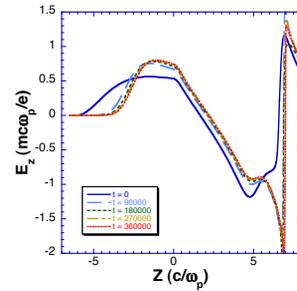
Figure 4: Wakefield evolution in the 1 TeV stage simulation.

beam parameters in Table 1, $W_{loss} = 106.25$ GeV/m. One can reduce the loss to a few percent by compressing the transverse spot size by a factor of 10. A simulation with this parameter is currently not possible because it requires enormous number of grid points. But as long as the spot size is much smaller than the blow-out radius, the physics should depend weakly on the spot size.

Another issue not being addressed in the 1 TeV simulation is the hosing instability. After investgating the mechanism of the hosing instability in the relativistic blow-out regime, we have obtained the coupled equations for beam centroid x_b and channel centroid x_c [6],

$$\partial_\xi^2 x_c + c_1 c_2 c_3 x_c = c_1 c_2 c_3 x_b, \qquad (4)$$

$$\partial_s^2 x_b + k_\beta^2 x_b = k_\beta^2 x_c, \qquad (5)$$

where $c_1(\xi)$, $c_2(\xi)$, $c_3(\xi)$ represent effects on hosing from the charge neutralization radius, the magnetic fields in the wake, and the plasma self force respectively. These three factors can be determined in the simulation and generally reduces the growth rate from the linear prediction.

CONCLUSION

We have developed a quasi-static PIC code to efficiently model TeV class "afterburners". Simulations of a 100 GeV and a 1 TeV stages have been carried out and the energy gain and transfer are studied. Radiation loss and hosing instability are also discussed for the blow-out regime.

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