SCALING LAWS OF A DIELECTRIC OPTICAL ACCELERATOR

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Abstract

Some of the basic scaling laws of future dielectric optical accelerator are re-examined.

1 INTRODUCTION

Indications that solid-state lasers will reach wall-plug to light efficiencies of 30% or more make a laser-driven vacuum-accelerator [1-2] increasingly appealing. Since at the wavelength of relevant lasers, dielectrics may sustain significantly higher electric field and transmit power with reduced loss comparing to metals, the basic assumption is that laser accelerator structures will be dielectrics. For structures that have typical dimensions of a few microns, present manufacturing constraints entail either planar structures or acceleration structures with a higher degree of symmetry, similar to optical fibers. The former require re-evaluation of many of the scaling laws [3] that were developed for azimuthally symmetric structures whereas the latter have some inherent advantages, but thermal gradients as well as heat dissipation may become critical impediments. Efficiency, emittance and heat flow scaling laws that have been developed recently will be presented.

2 EFFICIENCY

One of the major questions that will determine the fate of any future accelerator is the efficiency since it is natural to require acceleration of the largest number of electrons at the minimal input power i.e., maximum efficiency. Denoting the laser power injected in the structure by $P_{\rm L}$ and the resulting gradient at the location of the electrons by $E_{\rm acc}$, it is possible to define the interaction impedance as $Z_{\rm int} \equiv |E_{\rm acc}\lambda|^2 / P_{\rm L}$ which characterizes any acceleration structure; in this expression λ is the wavelength in the vacuum. This laser pulse accelerates a bunch (Q) that as it moves in an arbitrary acceleration structure generates an electromagnetic wake. Associated with this wake there is a decelerating electric field component $(E_{\rm dec})$ which by virtue of the linearity of Maxwell's equations, must be proportional to Q. Therefore, without loss of generality it is possible to assume that there is an effective transverse dimension denoted by $R_{\rm eff}$, determined by the details of the structure and the bunch, such that $E_{\rm dec} = 2Q/4\pi\varepsilon_0 R_{\rm eff}^2$.

Since it is possible to establish the power associated with the wake-field, $P_{\rm wake} = Q\,v\,E_{\rm dec}$, then the impedance definition as previously implies $Z_{\rm wake} \equiv |E_{\rm dec}\lambda|^2 / P_{\rm wake} = Z_0 / 2\pi (R_{\rm eff}/\lambda)^2$ wherein Z_0 is the vacuum impedance. For example, the interaction impedance of the photonic band-gap structure analyzed in Ref. 2, has an interaction impedance of $Z_{\rm int} = 19.5\Omega$ at 1μm and for an accelerating gradient of 1GV/m the total power required is about 50kW. Furthermore, since the radius of the vacuum tunnel is $R = 0.678\lambda$ and it may be demonstrated [3-5] that $R_{\rm eff} \simeq 1.23R$, then $Z_{\rm wake} \simeq 86\,\Omega$.

With these two impedances, it is possible to determine the effective (loaded) gradient as

$$E_{\rm eff} \equiv E_{\rm acc} - E_{\rm dec} = \frac{1}{\lambda}\sqrt{P_{\rm L} Z_{\rm int}} - \frac{QcZ_{\rm wake}}{\lambda^2} \qquad (1)$$

therefore, the gain in the kinetic energy of the bunch in structure of length L is $\Delta U_{\rm kin} \equiv QE_{\rm eff}L$ whereas the total electromagnetic energy stored in the structure is $U_{\rm em} \equiv P_{\rm L}\tau_{\rm f} = P_{\rm L}(L/c)(1-\beta_{\rm gr})/\beta_{\rm gr}$ where $\tau_{\rm f}$ is the fill-up time and $c\beta_{\rm gr}$ is the group velocity. Based on these two energy definitions, the efficiency of the acceleration process is

$$\eta \equiv \frac{\Delta U_{\rm kin}}{U_{\rm em}} = \frac{Q(Q_0 - Q)}{Q_{\rm n}^2} \qquad (2)$$

wherein $Q_{\rm n} \equiv \sqrt{\frac{P_{\rm laser}}{Z_{\rm wake}}\frac{\lambda^2}{c^2}\frac{1-\beta_{\rm gr}}{\beta_{\rm gr}}}, Q_0 \equiv \frac{\lambda\sqrt{P_{\rm laser} Z_{\rm int}}}{cZ_{\rm wake}}$.

Clearly, maximum efficiency occurs for an optimal value of the charge given by $Q_{\rm opt} \equiv Q_0/2$ in which case the value of this efficiency is

$$\eta_{\max} = \left(\frac{1}{2}\frac{Q_0}{Q_{\rm n}}\right)^2 = \frac{1}{4}\frac{\beta_{\rm gr}}{1-\beta_{\rm gr}}\frac{Z_{\rm int}}{Z_{\rm wake}} \qquad (3)$$

implying that the maximum efficiency is determined by the ratio of the impedances and the group velocity. For the PBG structure mentioned above [2], the group velocity is 0.58c therefore the maximum possible efficiency is 7.86%. It is likely that considerations of energy spread will lead to charges and efficiencies below the optimum values.

3 EMITTANCE

In an azimuthally symmetric structure, the ratio of the transverse force to the longitudinal force is virtually negligible since it is proportional to $|F_{\perp}/F_z| \approx \gamma^{-2}$. On the other hand, in a non-symmetric structure of a typical transverse dimension a, the ratio of the two forces is $|F_{\perp}/F_z| \approx \lambda/a$. 3D numerical simulations [3] of a bunch of 30GeV electrons indicate that the relative change in the emittance is drastically affected by the transverse dimension specifically, the relative change in the emittance across 10cm long acceleration structure when the wavelength is $\lambda = 1\mu$m scales like

$$\delta\varepsilon \equiv \frac{\varepsilon^{(\text{out})} - \varepsilon^{(\text{in})}}{\varepsilon^{(\text{in})}} \simeq \left(\frac{b}{a}\right)^{\nu} ; \qquad (4)$$

here E_{acc}=1 [GV/m] and the initial bunch length was 90°. The parameters b and ν depend on the radius of the beam (R_b) and in case $R_b = 3\mu\text{m}$ the coefficients are b=15.4 and $\nu = 4.4$. The bunch length $(\Delta\chi)$ also affects the emittance. A similar analysis shows that if $|\Delta\chi| < 90^o$, the emittance change is quadratic in $\Delta\chi$ namely,

$$\delta\varepsilon \simeq 1.1\times\left(\Delta\chi/180^o\right)^2 ; \qquad (5)$$

here R_b=3μm and a=20μm. In order to maintain low emittance change in a single acceleration module it is necessary to "symmetrise" the structure by constructing each module by sets of four non-symmetric segments each one rotated by 90°. For a typical transverse separation of (a=) 20μm, an initial bunch radius R_b=2μm, an average energy 30GeV with energy spread of 1% and bunch length of 3.6°

$$\delta\varepsilon \simeq 4.14\left[1+\left(N_{st}/15\right)^2\right]^{-1} \qquad (6)$$

where N_{st} is the number segments. For 400 segments the emittance increase in a 1m long acceleration structure is 0.6%, entailing a maximum segment length of 2.5mm.

4 THERMAL CONSIDERATIONS

Although dielectric materials are known to have low ohm loss there still is an important aspect of temperature gradients and heat dissipation. For investigating this phenomenon consider a hollow dielectric (ε) fiber of internal radius $R_{\text{int}}\left(\simeq 1[\mu\text{m}]\right)$ and external radius $R_{\text{ext}}\left(\simeq 30[\mu\text{m}]\right)$ An electromagnetic pulse propagates along this fiber and due to the Ohmic loss, represented here by $\tan\delta$, it is assumed that the thermal energy generated within the fiber volume is extracted from the outer surface where it is in contact with a perfect thermal conductor thus its temperature change vanishes hence $\Delta T(r = R_{\text{ext}}) = 0$. At the inner wall there is no thermal power-flow towards the vacuum thus $\left[\partial\Delta T/\partial r\right]\left(r = R_{\text{int}}\right) = 0$. Both thermal characteristics of the structure, heat conductivity $\sigma_{\text{T}}[\text{J}/\sec\text{m}^o\text{K}]$ and diffusion coefficient $D[\sec/\text{m}^2]$ are assumed to be uniform across the structure. Subject to the boundary conditions mentioned above, the variation in temperature (ΔT) across the structure may be determined using the diffusion equation driven by the electromagnetic power *loss density.* Since on the scale of one period of the wave ($2\pi/\omega_0 \equiv \lambda/c \simeq 3\text{fsec}$) there are no temperature variations, the electromagnetic power loss density is *averaged* over one period of the radiation field. The solution is further simplified by the following assumptions:

(i) $\left(\text{v}_{\text{gr}}\tau_p\right)^{-1} \ll p_s/R_{\text{ext}} \ll D\text{v}_{\text{gr}}$ wherein v_{gr} is the group velocity and p_s/R_{ext} represents the typical transverse variation of the temperature,

(ii) the diffusion time is much longer than the pulse duration (τ_{p}) $\tau_{\text{D}} \equiv DR_{\text{ext}}^2 \gg \tau_{\text{p}}$.

For convenience, the temperature change can be normalized with $T_N \equiv \frac{\tau_{\text{p}}}{\sigma_{\text{T}}D}\varepsilon\tan\delta\left[\frac{P_{\text{L}}}{\pi R_{\text{int}}^2}\frac{\omega_0}{c}\right]$ and the heat flow with $Q_N \equiv \frac{\tau_{\text{p}}}{DR_{\text{ext}}}\varepsilon\tan\delta\left[\frac{P_{\text{L}}}{\pi R_{\text{int}}^2}\frac{\omega_0}{c}\right]$. Figures 1 and 2 illustrate these two normalized quantities for the power distribution shown in Figure 3 representing the energy flux distribution in a Bragg structure acceleration structure [6].

For Zirconia (ZrO_2): $\tan\delta = 10^{-4}$, $\varepsilon = 2$ a total laser power of $P_{\text{L}} = 10[\text{kW}], Z_{\text{int}} = 100[\Omega]$, $R_{\text{ext}} = 30[\mu\text{m}]$, $R_{\text{int}} = 0.5[\mu\text{m}]$, $\tau_{\text{p}} = 1[\text{psec}]$, $\sigma_{\text{T}} = 2[\text{W/m}^o\text{K}]$ and $D = 1.15\times10^6[\sec/\text{m}^2]$. The normalizing heat flux is $Q_N \simeq 46\,[\text{W/cm}^2]$ whereas the normalizing temperature is $T_N \simeq 7^o\,\text{K}$. Zirconium has been chosen since it is a good thermal insulator thus the typical diffusion time $\tau_{\text{D}}\left(\equiv DR_{\text{ext}}^2 \simeq 1[\text{msec}]\right)$ is nine orders of magnitude larger than the pulse duration $\tau_p\left(\sim 1[\text{psec}]\right)$.

Figure 1 reveals a rapid increase in temperature (at $r = R_{\text{int}}$) from zero to a maximum and then exponential decay on a time-scale significantly shorter than the diffusion time (τ_D). Investigating the behavior on the scale of the laser pulse duration τ_p we find a linear increase from zero to the maximum value occurring at the end of the pulse. Using the estimate of T_{N} we conclude that the peak change in temperature is $\Delta T_{\max} \simeq 0.1^o\,\text{K}$.

While the temperature variation at $r = R_{\text{int}}$ occurs virtually instantaneously with the laser pulse, the heat reaches the output radius with a delay of the order of the diffusion time - see Figure 2; accordingly, the maximum heat flow to be dissipated is $Q_{\max} \simeq 0.01[\text{W/cm}^2]$.

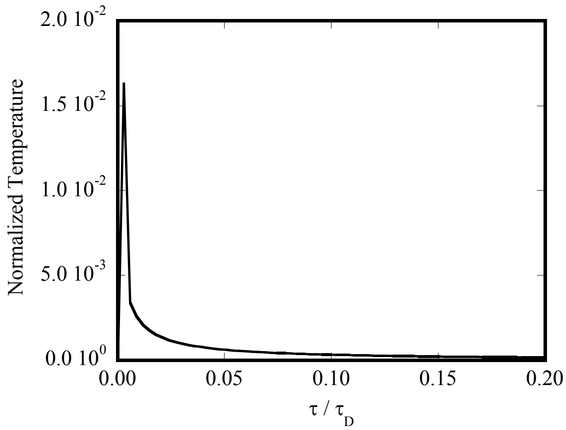

Figure 1**:** Variation in time of the normalized temperature at $r = R_{\rm int}$.

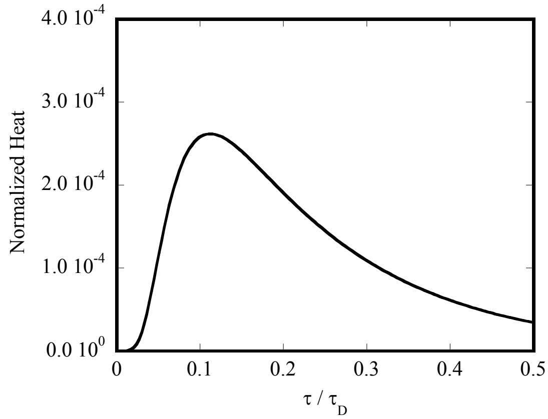

Figure 2**:** The variation in time of the normalized heat flow at $r = R_{\rm ext}$.

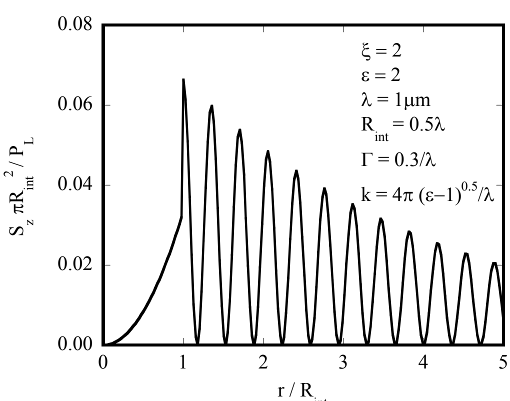

Figure 3**:** Transverse variation of the energy flux.

These two estimates account for a *single* pulse. In practice, a repetition rate of about 50 MHz will be required in order to satisfy the luminosity requirements on the one hand and heat dissipation on the other hand. As already indicated, the heat is deposited on the time scale of the pulse duration ($\tau_p \sim 1$psec) but it is dissipated along a diffusion time ($\tau_D \sim 0.1$msec). Consequently, heat from many laser pulses may accumulate before the dissipation affects the heat balance therefore, rather than considering the instantaneous power-loss ($p_{\rm loss}$) averaged over one radiation period, we consider the density power loss averaged over one period of the repetition rate $\left(T_{rr}\right)$ of the system. With this approach in mind, it is convenient to define the thermal trans-conductance as the ratio of the heat flow to the temperature increase at the two relevant locations thus in normalized terms it reads

$$\bar{G} \equiv \frac{R_{\rm ext} Q_{\rm max}^{\rm (av)}}{\sigma \, \Delta T_{\rm max}^{\rm (av)}} . \qquad (7)$$

Simulations indicate that it is weakly dependent on the internal radius – less than 5% when $0.3 \le R_{\rm int} / \lambda \le 0.8$ - but quite significantly dependent on the confinement parameter (Γ) $\bar{G} \simeq 0.26 + 0.05 / \Gamma\lambda$ for $0.2 \le \Gamma\lambda \le 0.8$. Specifically, for $\Gamma = 0.3/\lambda$ and $R_{\rm int} = 0.5\lambda$ it was found that $\bar{G} \simeq 0.42$.

Maximum average temperature was found to be $\Delta T_{\rm max}^{\rm (av)} = 59^o$[K] whereas the energy flux is $Q_{\rm max}^{\rm (av)} = 160$ [W/cm^2] . This energy flux represents a *passive* dissipation process but by virtue of the linearity of the system, if a fraction of these 160 [W/cm^2] are extracted *actively* with a thermo-coupler, we may actually cause the temperature change at the vacuum-dielectric interface to be significantly reduced.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

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