On the Tolerance to Tail Misalignment in E-157

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In the E-157 Plasma Wakefield Accelerator experiment, the head of the SLC beam displaces plasma electrons creating a plasma wake that is both focusing and accelerating for electrons in the tail of the beam. The goal of the experiment is to measure the energy change along the beam by measuring the beam deflection in a bending magnet. By subtracting the beam deflections measured with the plasma on and off, the energy change caused by the plasma can be probed. The question arises as to errors in the measurement due to transverse beam deflections caused by factors other than the energy change in the plasma. Specifically, this note addresses the following questions: If there is a tail on the beam (i.e, a misalignment of the axis of symmetry of the tail of the beam with respect to the head), what error will result in the diagnosis of beam energy change? Alternatively, what misalignment of the beam can be tolerated without compromising the measurements?

The dispersion of the final bending magnet sets the scale for tolerable displacements. The beam deflection at the diagnostic (i.e., at IP2) is 3 mm for a GeV energy change. Based on the February diagnostic tests, we expect to be able to measure centroid shifts of order 2 pixels or approximately 100 microns in the Cerenkov radiation image of the beam. This corresponds to an energy resolution of 30 MeV.

From the above, one might expect that the tail of the beam must be aligned to better than 100 microns with the axis of the body of the beam. In fact, the answer is not that simple. The tolerance is relaxed by the subtraction techniques used in the diagnostic. On the other hand, the plasma wake becomes very narrow near its peak value. This restricts the tolerance further for the part of the beam in the final measurable slice (i.e, the final ps bin).

To estimate the tolerance on the beam other than the final 1-2 slices, consider the contributions to beam deflection with plasma on and off:

Plasma off: d = d(correlated energy spread) + d(misalignment)

Plasma on: d = d(correlated energy spread) + d(misalignment)*$(1 + k_\beta * s * \sin k_\beta l)$+ d(energy gain)

Here k_β is the wavenumber of betatron oscillations in the plasma ($1/\beta$), l is the plasma length and s is the drift distance after the plasma. Since the plasma is chosen to be an integer number of betatron wavelengths long, $k_\beta l=n\pi$, the misaligned beam exits the plasma at the same orientation at which it entered. That is, the misalignment is the same as in the plasma off case and its effect is subtracted. This is true so long as the misalignment is not larger than the radius of the plasma focusing channel which is of order mm for most of the beam. Thus for most of the beam, misalignments of several hundred microns may be tolerable.

The situation changes for the portion of the beam in the final time resolvable bins. Here the plasma wake is narrowing as it increases (see Fig. 3a of the E-157 proposal). The width of the accelerating structure at the center of the last measurable bin is approximately 50 microns. If the beam is off axis by more than this amount, it will not be accelerated, and worse, it will be deflected by plasma defocusing terms. Thus to measure this last slice, requires that it be aligned to a fraction of this wake width. This sets the scale for the alignment tolerance at the 10 micron range.

Note that the tuning of the plasma density and length (which determines the $\sin k_\beta l$ term) must be fairly precise to assure a negligible effect from betatron phase advance. For small errors ($k_\beta l = n\pi[1+\eta]$ and $\eta \ll 1$), we find from above that η can be expressed in terms of the deflection due to the betatron error (dy_f), the intial misalignment (dy_o) and the other parameters as follows:

$$\eta = \frac{dy_f}{dy_o}\frac{l}{s}\frac{1}{(n\pi)^2}$$

To keep the misalignment from amplifying, we need $dy_f/dy_o<1$. Since l/s = 1m/10m and n is nominally 3, this gives $\eta < .001$. Since k_β is proportional to square root of plasma density, this implies that the plasma density times plasma length squared must be tuned to within 0.2%. This is relaxed somewhat if the misalignment (dy_o) is kept to the 10 micron level. Then to maintain a final energy resolution of 30 MeV, we can tolerate a final deflection error (dy_f) of 100 microns or dy_f/dy_o=10. Thus the plasma density times plasma length squared need only be tuned to within 2%.

In summary, the question of the allowed tolerance on tail misalignment does not have a short answer. While 100 micron displacements are tolerable for the main part of the beam if the plasma length and density are precisely tuned, more realistic tuning errors and the need for measuring the final energy bin indicates that 10 micron alignment is needed.