

# Suppression of Beam-Beam Interaction in a Muon Collider \*

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## ABSTRACT

We examine the suggestion by G. V. Stupakov and P. Chen and independently by A. Skrinsky of using plasma at the interaction point to reduce the effects of beam-beam tune shift in a 4 TeV c-of-m muon collider.

## I INTRODUCTION

In a recent paper G.V. Stupakov and P. Chen[1] have advanced the possibility of beam-beam interaction suppression in circular colliders for heavy particles such as muons and protons, by introducing a plasma at the interaction point.

More recently and independently, A. Skrinsky [2] has suggested to use flowing liquid Li for the same purpose in a muon collider. This is an alternative that is only plausible for muons and indeed is used in the present cooling scenario [3].

The electromagnetic interaction between the colliding beams results in perturbation of the individual particle motion, which leads to unstable motion and loss of the beam. This effect is quantified with the single parameter  $\xi$  beam-beam tune shift [4]. For a round beam  $\xi = \frac{r_c N}{4\pi\epsilon_N}$ , where  $N$  is the number of particles in the bunch,  $r_c$  is the classical radius of the particle in the beam and  $\epsilon_N$  is the normalized emittance.  $\xi \approx 0.044$  for a muon collider.

Recalling the definition of luminosity

$$\mathcal{L} = \frac{N^2 H_D f}{4\pi\sigma_{\perp}^2} = \frac{N H_D f}{\beta_{\perp} r_c} \gamma \left( \frac{N r_c}{4\pi\epsilon_N} \right) \propto \xi, \quad (1)$$

therefore, if we increase the luminosity with a fixed number of particles  $N$  and fixed energy, we have to increase the value of  $\xi$  with the consequence of reduction of the dynamical aperture of the ring.

Physics consideration tends to increase the required luminosity  $\mathcal{L}$ , which means higher values of  $\xi$ ; the natural question is: Is it possible to increase  $\xi$  without a catastrophic beam-beam interaction and reduction of the dynamical aperture?

The possible answer is: it requires to somehow short circuit the electric field of the beams and create counter-propagating currents to cancel the beam magnetic field. P. Chen suggested to include a plasma at the interaction region [1].

When the plasma density  $\rho_p \gg$  than the particle beam density then the plasma electrons responds to the presence of the electric field carried by the beam ( $\mu^+$ ) by moving transversely to neutralize the space charge and also moving longitudinally in order to cancel the magnetic field.

Of course, now we have to confront 2 issues:

- degradation of the beam lifetime due to scattering of the particles (muon) of the beam with plasma particles;
- background at the detector reduced by beam-plasma interaction

In this note we attempt to answer the first concern and show that the change of lifetime is negligible in view that the muon beam itself has a lifetime of  $\simeq 1000$  turns =  $0.036$  s, and on that account alone we may think that an increase of the number of particles in the bunch is a reasonable path to increase the luminosity of the collider.

The second issue is much more involved and requires detailed simulations and is outside the scope of this note.

As a benefit to the reader, we'll repeat in the next section calculations from ref.[1], borrowing heavily from its text and line of reasoning; in the following section we'll present calculations for liquid Li.

## II FORMULATION

The most general charge distribution in 3D of each bunch coming to collision at the IP is assumed in the form of the product of 3 Gaussians. The beam parameters we are considering are approximately  $T = 2$  TeV,  $\epsilon = 25 \cdot 10^{-10}$   $\pi$ m-rad and  $\beta^* = 3$  mm. This yields an spot size at the IP  $\sigma_{x,y} \approx 3\mu$ m and  $\sigma_z = 3$  mm; therefore  $\sigma_z \gg \sigma_x, \sigma_y$ . In this case the most appropriate charge distribution is a Gaussian in 2D with line number density  $n$ , i.e.,

$$\rho(x, y) = \frac{ne}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{x^2}{2\sigma_x} - \frac{y^2}{2\sigma_y}\right] \quad (2)$$

In this case the potential function[5]  $\Phi(x, y; \sigma_x, \sigma_y)$  is

$$\Phi(x, y; \sigma_x, \sigma_y) = \frac{ne}{4\pi\epsilon_0} \int_0^{\infty} dt \frac{\exp\left[-\frac{x^2}{2\sigma_x^2+t} - \frac{y^2}{2\sigma_y^2+t}\right]}{\sqrt{2\sigma_x^2+t}\sqrt{2\sigma_y^2+t}} \quad (3)$$

To simplify the algebra we assume  $\sigma_x \equiv \sigma_y \equiv \sigma_r$  then the electric field  $\vec{E} = (E_r, 0, 0)$  and magnetic field  $\vec{B} = (0, B_{\phi}, 0)$  are respectively,

$$E_r = -\frac{ne}{4\pi\epsilon_0} \frac{\partial}{\partial r} \int_0^{\infty} \frac{\exp\left[-\frac{r^2}{2\sigma_r^2+t}\right]}{2\sigma_r^2+t} dt \quad (4)$$

and

$$B_{\phi} = \frac{ne\beta c\mu_0}{4\pi} \frac{\partial}{\partial r} \int_0^{\infty} \frac{\exp\left[-\frac{r^2}{2\sigma_r^2+t}\right]}{2\sigma_r^2+t} dt \quad (5)$$

From this expression we can evaluate the Lorentz force  $\vec{F} = (F_r, 0, 0)$  between round beams of Gaussian distribution of

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charges in 2D

$$F_r(r) = -\frac{n e^2}{2\pi\epsilon_0 r} (1 + \beta^2) \left[ 1 - e^{-\frac{r^2}{2\sigma_r^2}} \right] \quad (6)$$

$\beta$  is the assumed constant relative normalized velocity of both beams. Clearly for  $r \ll \sigma_r$  the force is attractive and linear in  $x$  and  $y$ , i.e. behaves like a solenoid (or a focusing quadrupole in both transverse planes).

Furthermore G. Stupakov and P. Chen [1] showed using a very general expression for the electromagnetic fields generated by an arbitrary external current density in a cold plasma that

$$E_z \sim \frac{\sigma_r}{\sigma_z} B_\phi, \quad E_r \sim \left(\frac{\sigma_r}{\sigma_z}\right)^2 B_\phi$$

in other words under the conditions of interest the electric fields created by the beam is smaller by  $\left(\frac{\sigma_r}{\sigma_z}\right)$  than the azimuthal magnetic field. This result can be arrived in a less rigorous way by looking at Eqs. 13.58 in Jackson[9] and assuming that  $\sigma_z k_z \sim 1$ ,  $\sigma_r k_\perp \sim 1$  and  $k_p \sigma_r \geq 1$ .

Another result quoted in reference[1] is an expression for the beam beam tune shift

$$\xi = \frac{\xi_o}{2} \int_0^\infty \frac{\eta^3 d\eta}{(\eta^2 + k_p^2 \sigma_r^2)} e^{-0.25\eta^2} \quad (7)$$

where  $\xi_o$  is the beam beam interaction parameter in the absence of the plasma.

Obviously, the introduction of Li at the IP gives place to a deleterious effect on the muon beam due to the collisions of muons with the Li nuclei and electrons; consequently, the plasma parameters must be chosen such that these negative effects do not cancel out the positive consequences of reducing the beam-beam interaction effects. Next we briefly examine, following ref[1], each one of the processes (incoherent collective effects) that affect the dynamics of the muon beam in the collider ring and therefore the lifetime of the beam.

## A Muon beam heating due to small-angle multiple Coulomb scattering on nuclei

The emittance growth by many small-angle scatterings from nuclei is given by[6]

$$\frac{d\epsilon_N}{dz} = \frac{1}{2} \beta \gamma \beta_\perp \frac{d\langle \theta^2 \rangle}{dz} \quad (8)$$

where  $dz$  is the width of the plasma column,  $\beta_\perp$  is the betatron function at the IP and[7]

$$\langle \theta^2 \rangle = \left( \frac{13.6 \text{ MeV}}{\beta c p} \right)^2 \frac{\rho z}{X_o} \left[ 1 + 0.038 \ln \left( \frac{\rho z}{X_o} \right) \right]^2 \quad (9)$$

is the scattering angle Gaussian distribution width. In the last expression  $X_o$  is the radiation length given by[8]

$$X_o = \frac{716.4 A \text{ g cm}^{-2}}{Z(Z+1) \ln \left( \frac{287}{\sqrt{Z}} \right)} \quad (10)$$

In a storage ring denoting with  $f$  the revolution frequency, the rate of change of emittance (growth) is

$$\epsilon_N \approx 0.5 \beta \gamma \beta_\perp \left( \frac{13.6 \text{ MeV}}{\beta c p} \right)^2 \frac{\rho z}{X_o} f \left[ 1 + 0.038 \ln \left( \frac{\rho z}{X_o} \right) \right]^2 \quad (11)$$

and the lifetime of the beam due to multiple scattering is  $\tau = \frac{\epsilon_N}{\dot{\epsilon}_N}$ . Certainly, the probability for single scattering events at large angle as well as plural events becomes negligible at high energy beams. Therefore, although these processes will induce betatron oscillations and if the amplitude of oscillation is larger than the vacuum pipe aperture then the particle is lost, the effects are much smaller than other processes.

## B Bremsstrahlung energy loss

Radiative processes are more important than ionization energy loss at high energy. This is clearly seen in Fig.??

PLOT

In reference[8] the muon energy loss is given by

$$\left| \frac{dE}{dz} \right| = a(E) + b(E)E \quad (12)$$

where  $a(E)$  is the ionization energy loss and  $b(E)$  is the sum of the  $e^+e^-$  pair creation, bremsstrahlung and photo-nuclear contributions

$$a(E) = \frac{K}{A} Z \frac{\rho}{\beta^2} \left[ 0.5 \ln \left( \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} \right) - \beta^2 - 0.5\delta \right] \quad (13)$$

where  $\frac{K}{A} = \frac{0.307075}{A} \text{ MeV g}^{-1} \text{ cm}^2$ ,  $T_{\max}$  is the maximum energy that a free electron may acquire

$$T_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{M_\mu} + \left( \frac{m_e}{M_\mu} \right)^2} \quad (14)$$

$I$  is the mean excitation energy in eV,  $I = 16Z^{0.9} \text{ eV}$  and finally  $\frac{\delta}{2}$  is a shell correction which is negligible at high energy  $0.5\delta \rightarrow \ln(\hbar\omega_p/I) + \ln(\beta\gamma) - 0.5$ .

The lifetime of the beam corresponding to each physical process ionization loss (order  $\alpha^2$ ), bremsstrahlung (order  $\alpha^3$ ), direct pair production (order  $\alpha^4$ ) and muon-nucleus interaction is calculated by  $\tau_i = (n_p \sigma_i l f)^{-1}$  where  $l$  is the length of the Li cell at the IP and  $f$  is the revolution frequency.  $l$  is chosen to be  $l \sim 2\beta^*$ . It can shown that the most important process at this energy is pair creation. In Tb. I we summarize the results. We notice that at 2 TeV the pair creation cross section is  $\sim 90 \text{ mb}$  which is a factor of 9 higher than the pair creation cross section due to bare (no plasma) beam-beam interaction. The lifetime due to pair creation is  $\sim 8.6 \text{ s}$  to be compared with the muon beam lifetime of 0.026 s.

## III CONCLUSION

Flowing liquid Li, or in the form of jet, will significantly reduce the beam beam tune shift at the cost of increasing the number of pair created at the IP. However, these pairs will fly mainly

Table I: Beam parameters and Lifetime

Beam Parameters	$\mu$ Collider
E (TeV)	2
$\gamma$	$2 \times 10^4$
$\sigma_z$ (mm)	3
$\sigma_r$ ( $\mu\text{m}$ )	3
f (Hz)	$4.3 \times 10^4$
N	$2 \times 10^{12}$
$\epsilon_N$ ( $\pi$ mm-mrad)	50
$\xi_o$	0.05
<b>Li Parameters</b>	
$k_p \sigma_r$	126
$\xi/\xi_o$	0.001
$n_p$ ( $\text{cm}^{-3}$ )	$5 \times 10^{22}$
l (cm)	0.6
<b>Beam lifetimes</b>	
$\tau_0$ (s)	0.026
$\tau$ (s)	8.6

in the beam pipe without being deflected by the beam field itself into the detector. Careful simulation of the detector area is urgently needed before to make conclusions regarding the merits of the idea.

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