

Pion Collection from an Intense Proton Beam in a Plasma

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The $\mu\mu$ collider concept requires an intense proton beam smashing into a target to make pions that subsequently decay into muons. As described in the literature, the concept requires a high-field solenoid to enhance the effective final muon yield. Given the difficulty of maintaining the muon beam in the presence of muon decay, it seems interesting to consider concepts that preserve *first* the integrity of the pion beam, and maintain a lower pion beam emittance. In this note we lay out the considerations for application of a plasma chamber together with the target to this end. The concept requires pre-modulation of the proton beam; this we suggest could be accomplished by means of a plasma pre-buncher, with wakefields driven by a Ku-Band or W-Band modulated electron beam, with parameters in the range of those demonstrated at LLNL.

Proton Beam Parameters

The proton beam to be employed for the $\mu\mu$ collider source has not been produced, so the parameters are somewhat in flux. We will use parameters as in Table 1, and the figure of 8 GeV for the proton kinetic energy.¹

Table 1. Proton beam parameters used for examples.

particles in the bunch	N_b	2×10^{14}
bunch length (rms)	σ_z	30 cm
rms normalized emittance	ϵ_n	1×10^{-4} mrad
initial rms beam size	σ_x, σ_y	0.5 cm

Using $m_p = 938.3$ MeV, one sees that the Lorentz factor is $\gamma \approx 9.53$. The beam charge is $Q_b \approx 32 \mu\text{C}$, and the peak current, assuming a Gaussian longitudinal profile is,

$$I_b = \frac{Q_b c}{(2\pi)^{1/2} \sigma_z} \approx 12.8 \text{ kA}.$$

The Aflven constant for a proton beam is

$$I_0 = \frac{m_p c^3}{e} \approx 31.3 \text{ MA},$$

and the Budker parameter is

$$\nu_b = \frac{I_b}{I_0} \approx 4.1 \times 10^{-4}.$$

The basic scalings for plasma focusing can be set down in terms of this constant, the beam current and the normalized emittance. For the plasma, we will refer to the plasma wavenumber

$$k_p = \sqrt{4\pi n_p r_e},$$

¹ "μ+μ- Collider: A Feasibility Study", (July, 1996) BNL-52503. We employ for emittance figures there quoted as "95%" emittance; no doubt revisions will be in order later.

where (in cgs units) $r_e = e^2 / m_e c^2 \approx 2.82 \times 10^{-13}$ cm is the classical electron radius, $-e$ is the electron charge, and m_e is the electron mass. The angular plasma frequency is $\omega_p = k_p c$. When speaking of the injected beam, we will be picturing a long Gaussian bunch with $\sigma_x = \sigma_y = \sigma_r$. "Long" means $k_p \sigma_z \gg 1$.

Finally, for those of us used to working with electrons, it is helpful to note

$$\frac{m_p}{m_e} \approx 1836,$$

and in this connection we note that $r_p = e^2 / m_p c^2 \approx 1.54 \times 10^{-15}$ cm.

Self-Focusing in a Plasma

A relativistic beam injected into a plasma will perturb plasma electrons by means of its electrostatic field. An electron beam expels plasma electrons, while a proton beam will attract them. When the beam current variation is adiabatic on the time-scale of a plasma period, one may picture a quasineutral steady-state, where a surfeit of electrons resides within the beam volume, cancelling the line charge density of the beam. In this limit, the only field component remaining is the magnetic field due to the beam. This we may compute from Ampere's Law,

$$\frac{1}{r} \frac{\partial}{\partial r} r B_{b\theta} = -4\pi e n_{b0} \frac{V}{c} \exp\left(-\frac{r^2}{2\sigma_r^2}\right),$$

with beam drift speed $V \approx c$. The solution is

$$B_{b\theta} = -4\pi e n_{b0} \frac{\sigma_r^2 V}{r c} \left\{ 1 - \exp\left(-\frac{1}{2} \frac{r^2}{\sigma_r^2}\right) \right\},$$

and the net force on a beam proton is given by

$$\begin{aligned} F_r &= -e \left(E_r - \frac{V}{c} B_{b\theta} \right) \approx -4\pi e^2 n_{b0} \frac{\sigma_r^2}{r} \left\{ 1 - \exp\left(-\frac{1}{2} \frac{r^2}{\sigma_r^2}\right) \right\} \\ &\approx -m_p c^2 k_{b0}^2 \frac{\sigma_r^2}{r} \left\{ 1 - \exp\left(-\frac{1}{2} \frac{r^2}{\sigma_r^2}\right) \right\}, \end{aligned}$$

where we introduce the proton beam wavenumber,

$$k_{b0} = \left(4\pi n_{b0} r_p \right)^{1/2} = \left(4\pi \frac{I_{b0}/ec}{2\pi\sigma_r^2} r_p \right)^{1/2} = \left(2 \frac{I_{b0}}{I_0} \right)^{1/2} \frac{1}{\sigma_r} = \frac{(2v_b)^{1/2}}{\sigma_r}.$$

For $\sigma_r = 0.5$ cm, and $\sigma_z = 30$ cm, and 2×10^{14} protons, $n_{b0} \approx 1.7 \times 10^{12}$ cm⁻³ and $k_{b0} \approx 5.7 \times 10^{-2}$ cm⁻¹, scaling in inverse proportion to the beam size. Paraxially,

$$\frac{d\vec{p}_\perp}{dt} = F_r \hat{r} \approx -\frac{1}{2} m c^2 k_{b0}^2 \vec{r}_\perp, \quad \Leftrightarrow \quad \frac{d}{dz} \gamma \vec{r}'_\perp \approx -\frac{1}{2} k_{b0}^2 \vec{r}_\perp.$$

This may be expressed in various conventional forms as

$$K = \frac{1}{2\gamma} k_{b0}^2 \quad \text{or} \quad k_\beta^2 = \frac{1}{2\gamma} k_{b0}^2 = \frac{1}{\sigma_r^2} \left(\frac{v_b}{\gamma} \right). \quad (\text{magnetically self-focused regime}),$$

The ratio v_b/γ will be recognized as the ratio of beam current to Alfvén current. The self-pinched betatron period for $\sigma_r = 0.5$ cm is $\lambda_\beta = 2\pi/k_\beta = 4.8 \times 10^2$ cm.

Let us note the equilibrium self-pinched beam-size, for an initially matched beam, absent scattering and other sources of incoherence. Normalized emittance is constant,

$$\varepsilon_n = \gamma k_\beta \sigma_r^2 = \gamma \frac{1}{\sigma_r} \left(\frac{v_b}{\gamma} \right)^{1/2} \sigma_r^2 = (v_b \gamma)^{1/2} \sigma_r, \quad (\text{MFR equilibrium})$$

so that

$$\sigma_{eq} = \frac{\varepsilon_n}{(v_b \gamma)^{1/2}}, \quad k_\beta = \frac{1}{\sigma_r} \left(\frac{v_b}{\gamma} \right)^{1/2} = \frac{(v_b \gamma)^{1/2}}{\varepsilon_n} \left(\frac{v_b}{\gamma} \right)^{1/2} = \frac{v_b}{\varepsilon_n}. \quad (\text{MFR equilibrium})$$

For our assumed emittance, this corresponds to $\lambda_\beta \approx 1.5 \times 10^2$ cm in equilibrium, with $\sigma_{eq} \approx 0.16$ cm. However, this assumes a plasma density on the order of (or larger than) the corresponding beam density

$$n_b = \frac{I_{b0}/ec}{2\pi\sigma_r^2} = \frac{v_b}{2\pi\sigma_r^2 r_p}.$$

This corresponds to $n_b \approx 1.7 \times 10^{13} \text{ cm}^{-3}$. A 1-m tank with plasma density $n_p \approx 1.7 \times 10^{13} \text{ cm}^{-3}$ does not challenge the state-of-the-art; such a tank with $10\text{-}50 \times$ higher density has been proposed for SLAC E-157, and is currently being tested.² The corresponding plasma wavenumber is $k_p \approx 7.6 \text{ cm}^{-1}$; since $k_p \sigma_{eq} \approx 1.2$, plasma compensation effects afford a small correction to the fields. The plasma frequency is $f_p = ck_p/2\pi \approx 36$ GHz. At the still-modest density of $n_p \approx 1. \times 10^{14} \text{ cm}^{-3}$ one has $f_p \approx 91$ GHz.

Capture in a Proton-Beam Driven Wakefield

The foregoing considerations suggest that the proton beam could be handled by means of self-focusing in a plasma, and focused to a small spot. We can ask that the plasma accomplish still more, however. Pions produced from the target slabs interspersed in the plasma will also be focused. In addition, however, if we pre-modulate the proton beam, an accelerating wakefield can be developed in the plasma, assisting pion capture and beam formation. The frequency of the accelerating wave will be the plasma frequency, in the Ku-Band or W-Band range, and this could be quite appropriate for downstream acceleration, provided the small aperture diameter is acceptable there, and depending on

² T. Katsouleas, *et al.*, "A proposal for a 1 GeV plasma-wakefield acceleration experiment at SLAC", *Proceedings of the 1997 Particle Accelerator Conference* (IEEE, New York, to be published); R. Assmann, *et al.*, "Proposal for a One GeV Plasma Wakefield Acceleration Experiment at SLAC", SLAC-E157.

development of accelerator technology in the CLIC or W-Band frequency ranges.

The maximum accelerating gradient produced by a modulated proton beam resonantly driving the plasma corresponds to wave-breaking in the plasma and is given by

$$E_{wb} \approx \frac{m_e c^2}{e} k_p.$$

This is 3×10^2 MV/m for our example at $n_p \approx 1.7 \times 10^{13} \text{ cm}^{-3}$, with $f_p \approx 36$ GHz (Ku band). (At $n_p \approx 1. \times 10^{14} \text{ cm}^{-3}$, with $f_p \approx 91$ GHz---W-Band---one has 0.92 GV/m.) To appreciate the effect on the pion beam, recall that the pion mass is $m_{\pi^+, \pi^-} = m_\pi = 139.6$ MeV, and the lifetime is $\tau_{\pi^+, \pi^-} = \tau_\pi \approx 26.0$ ns. Thus a 10-m plasma cell would result in typical pion energies of 2 GeV, and $\gamma_\pi \approx 15$, with lifetimes in the lab frame $\tau_l \approx \gamma_\pi \tau_\pi \approx 0.4 \mu\text{s}$ (400 ft). One would expect the effect of focusing and acceleration to be a lower emittance pion beam, with a higher capture fraction. (Things would go somewhat better at W-Band, where the interaction length could be reduced by a factor of 3 and the yield improved due to the higher gradient.)

Self-modulation of the proton beam in the plasma will proceed at a growth rate scaling with the longitudinal proton beam plasma frequency and this is quite low. Thus some means of modulating the proton beam prior to the target chamber is required.

Pre-Buncher

Given the frequency range, a microwave prebuncher would have a rather small aperture (or require large peak powers from sources that don't exist). In addition, the proton beam average power is 8 MW at 30 Hz pulse repetition frequency, and structure damage due to halo would be a major concern. One could employ sub-harmonic bunching. However, let us consider instead bunching at the fundamental frequency (Ku or W-Band), in a second plasma tank. Such a plasma pre-buncher would require a driver.

One thought for a driver is an intense laser employed in the laser wakefield configuration. The gradient produced by a laser with dimensionless vector potential amplitude a_0 is given by

$$E_{lwf} \approx \frac{m_e c^2}{e} k_p \frac{a_0^2}{4}.$$

The laser peak power is then

$$P_l \approx \pi \frac{m_e^2 c^5}{e^2} \left(w \frac{\omega}{c} \right)^2 \frac{a_0^2}{4},$$

where $m_e^2 c^5 / e^2 \approx 8.7$ GW, w is the laser waist, and ω is the laser frequency. Taking $a_0 \approx 2$ (the wave-breaking limit), employing the same plasma frequency as for the target chamber $n_p \approx 1.7 \times 10^{13} \text{ cm}^{-3}$, $w \approx 2\sigma_{eq} \approx 0.32$ cm, and 1 μm laser wavelength we obtain a peak laser power of $P_l \approx 0.3$ PW. The Rayleigh range is quite long $L_R \approx 3 \times 10^3$ cm, so diffraction of the pulse is not an issue for a prebuncher tank less than a few meters in length. This pulse must last for 3 ns, and corresponds then to 0.8 GJ of laser pulse energy. This would be quite a laser, something on the scale of NIF. The simple explanation for these discouraging numbers is that the laser wakefield excitation mechanism is non-resonant. Given that the laser figures are extraordinary, one is interested to consider other

means of driving the plasma to wave-breaking.

A natural alternative driver is an electron-beam employed in the mode of a plasma-wakefield accelerator driver. Such a beam would itself require modulation; however, at Ku-band and W-Band, modulation of the required kA, 10-20 ns beams has been accomplished employing an induction linac with a free-electron laser.³ With 3×10^2 MV/m wavebreaking gradient, the actual plasma tank length could be quite short, perhaps 0.1 m, depending on the drift length to be employed (for bunching) between the prebuncher tank and the target tank. (At W-Band, the tank could be still shorter).

The picture of the pion source one arrives at is that of Fig. 1.

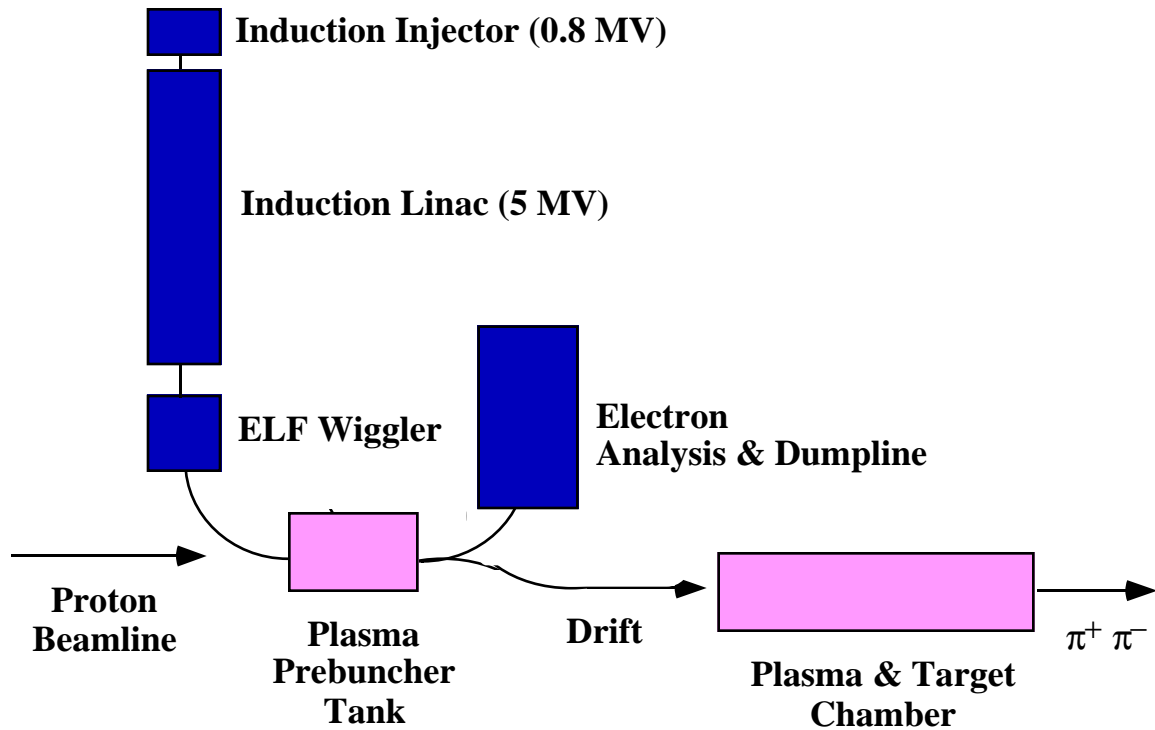


FIGURE 1. Conceptual layout of components for plasma-based pion beam production with an intense proton beam. A kA electron beam with 10 ns pulse length is produced in an induction linac on the scale of the LLNL Experimental Test Accelerator (ETA). The beam is modulated at Ku band, as in the ELF experiment, and drives a plasma. The resulting plasma wakefield modulates the proton beam energy. The proton beam bunches in the drift, and drives an accelerating wakefield in the plasma and target chamber.

Work To Do

There are many interesting issues to examine further here, concerning the different behaviors of the positive and negative pions, beam-loading and stopping of the proton beam by the plasma. This problem is most easily simulated in two pieces. The first simulation would be for the proton driver, arriving at an accurate assessment of the amplitude of the resonantly excited wakefield, with transverse gradients and phase-mixing. The second simulation would sample the pion phase-space, in Monte Carlo fashion, and examine pion capture and yield, with a realistic aperture.

Other features of concern are: H- formation, target slab layout and damage, pion

³ A. L. Thoop, *et. al.*, “Experimental Results of a High-Gain Microwave FEL Operating at 140GHz”, *Nuc. Instr. & Methods in Phys. Res.* **A272**, 15 (1988), S. L. Allen, *et. al.*, “Generation of High Power 140 GHz Microwaves with an FEL for the MTX Experiment”, *Proceedings of the 1993 Particle Accelerator Conference* (Washington, D.C., 1993), 1551.

post-acceleration. Naturally one is interested too in electron beam modulation by means of a relativistic klystron.

In addition, the downstream accelerator complex should be revisited taking account of the improved pion beam characteristics. In particular, it seems likely that a revised collider ring concept could employ a dual-ring design, consisting of a pion storage & decay ring, with fields designed to automatically channel decayed pions into the collider ring proper. For that matter, one could contemplate just a π -storage ring, functioning as a $\pi\pi$ -collider, with $\mu\mu$ and $\mu\pi$ physics functioning as background, or as parasitic or standby experiments.