A CONTINUOUS PLASMA FINAL FOCUS

David H. Whittum

Preface

This report originally was originally presented at the Workshop on the role of Plasmas in Accelerators, Tsukuba, Japan Aug 28 - Sept 1, 1989. It was subsequently published as D. H. Whittum, "Continuous plasma final focus", in *Nonlinear and Relativistic Effects in Plasmas*, V. Stefan, ed., (AIP, New York, 1992) pp. 387-401. I am reissuing it as an ARDB Technical Note as it provides a hard-to-find summary of the zeroth-order phenomena that arise when an intense relativistic electron beam is injected into a plasma.

Some caveats are in order. This report does not include discussion of the electron-hose effect, as it was not known at that time. It also omits discussion of the accelerating wakefield, as the interest at that time was a plasma-based final focus. Results discussed here are relevant to the E157 proposal, and to the collider final focus discussed by Zimmerman. A companion technical note will refer to results noted here, apply them to E157 parameters (these are close to those of column #3 in Table 1), and go on to consider additional phenomena not covered here. Where references used here are "obsolete", the companion note will also make use of more recent references.

Also, Fig. 3 has been lost, so I have edited it out, as it is not too important, in retrospect. It depicted an example of beam-size and plasma density variation for a possible plasma lens experiment with the TRISTAN injector. For those with a die-hard interest in the TRISTAN injector, the original can be found in the reference cited above.

Finally, in the discussion of the TEV collider example, where "100 GeV" is stated, it should have read "100 MeV".

LBL-27965

A CONTINUOUS PLASMA FINAL FOCUS

David H. Whittum
Lawrence Berkeley Laboratory, Berkeley, California 94720

Scaling laws are set down for a plasma cell used for transport, focussing and current neutralization of fine, intense, relativistic electron beams. It is found that there exists a minimum beam spot size, $\sigma_{min} \sim \varepsilon_n (I_A / \gamma I)^{1/2}$, in such a focussing system. Propagation issues, including channel formation, synchrotron radiation, beam ionization and instabilities, are discussed. Numerical examples are given for a proof-of-principle experiment at KEK, an application for luminosity enhancement at the SLC, and a hypothetical TeV electron-positron collider. For a TeV collider, it is found that the effect of ion-motion on focussing, and the effect of the Buneman instability on current neutralization must be considered.

PACS numbers: 41.80Ee, 52.40Mj, 07.77.+p

I. Introduction

A relativistic electron beam (REB) injected into a plasma less dense than the beam expels plasma electrons from the beam volume, producing an "ion-channel." The radial electric field due to the ions then focusses the beam. A plasma more dense than the beam ("overdense") will neutralize the beam charge, so that the REB is focussed by its own magnetic field. A still denser plasma will partially neutralize the current, if the plasma skin depth is short compared to the beam radial size, and if the magnetic diffusion time is long compared to the beam length. Over the last twenty years, these and other features of REB propagation in plasmas have been studied extensively, theoretically and experimentally.[1] Over the last decade, ion-channel focussing has been successfully employed in the transport of high current beams for advanced accelerator work.[2,3,4]

The "adiabatic focusser", proposed by Chen et *al*.[5] extends the underdense-plasma ion-focussing mechanism, for use in a TeV linear electron-positron collider. They propose to increase the plasma density along the direction of beam propagation, so as to focus the beam continuously to a spot size smaller than can be achieved through conventional magnetic optics. They observe that continuous focussing is a means of circumventing the Oide limit on the spot size in a discrete focussing system.[6]

At the same time, a subject of ongoing interest, in TeV linear electron-positron collider design, is the reduction of coherent beam-beam effects: beamstrahlung and disruption.[7,8] One method which has been proposed is current neutralization in an overdense plasma at the interaction point (IP).[9,10] Beamstrahlung and disruption are suppressed due to plasma return currents which reduce the magnetic pinch forces seen by the two colliding beams.

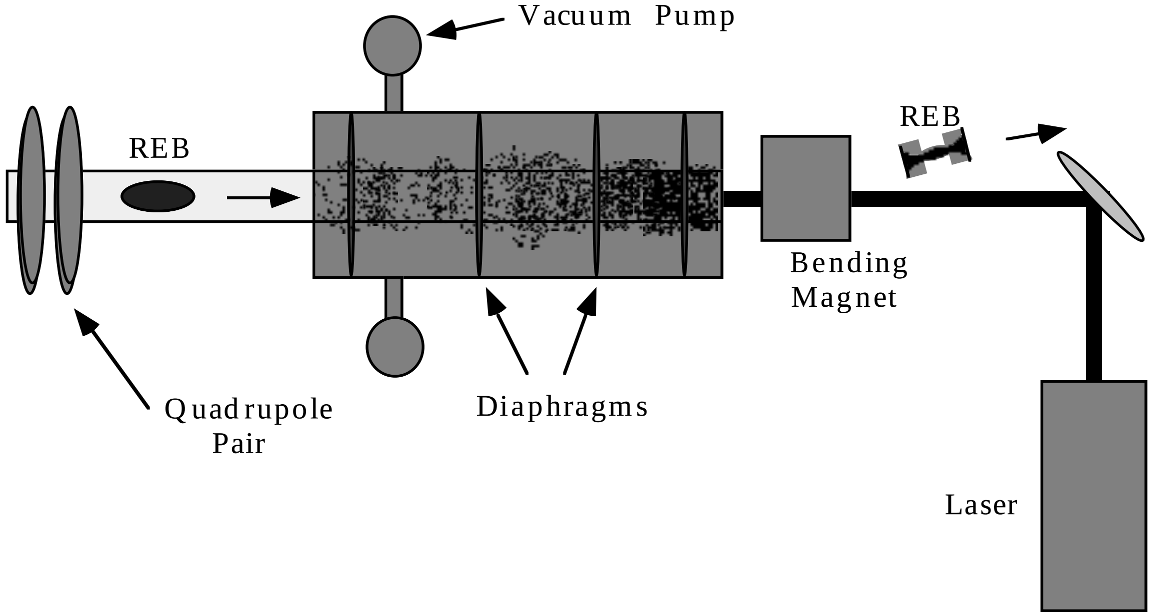

Figure 1. Set-up for a "proof-of-principle" continuous plasma focus experiment.

In this note, a plasma final focussing system consisting of an underdense adiabatic focussing cell, as proposed by Chen *et al.*, followed by an overdense current neutralization cell at the IP, is considered (Fig.1). The parameter range of interest consists of electron bunches which are short (1-100 ps), fine (mm-μm radius), high current (100 A - 1000 A) and highly relativistic (100 MeV - 1 TeV), propagating through a plasma of density 10^{12} cm^{-3} - 10^{22} cm^{-3}. Parameters in this range have attracted growing interest in recent years, in connection with the plasma wakefield accelerator,[11,12,13] the plasma lens,[14] and the plasma beat-wave accelerator.[15]

In the next section, the basic scaling laws for such a "continuous plasma final focus" are set down. In Section III, beam propagation, including scattering and ion channel formation, is discussed. In Section IV, radiation in the ion channel is considered. Beam ionization and instabilities are discussed in Sections V and VI. Finally, numerical examples are given and conclusions are offered.

II. Scaling Laws

In the adiabatic focusser, an axial density gradient in a neutral gas is maintained prior to ionization, through differential pumping.[16] An ionizing laser pulse then produces an axially increasing plasma density. Within less than a recombination time, an REB is injected. The gradient in plasma density results in an axially increasing electrostatic force, due to ion space-charge, on beam electrons as they traverse the cell. Consequently, the beam spot size is continuously reduced; the beam is "adiabatically focussed."

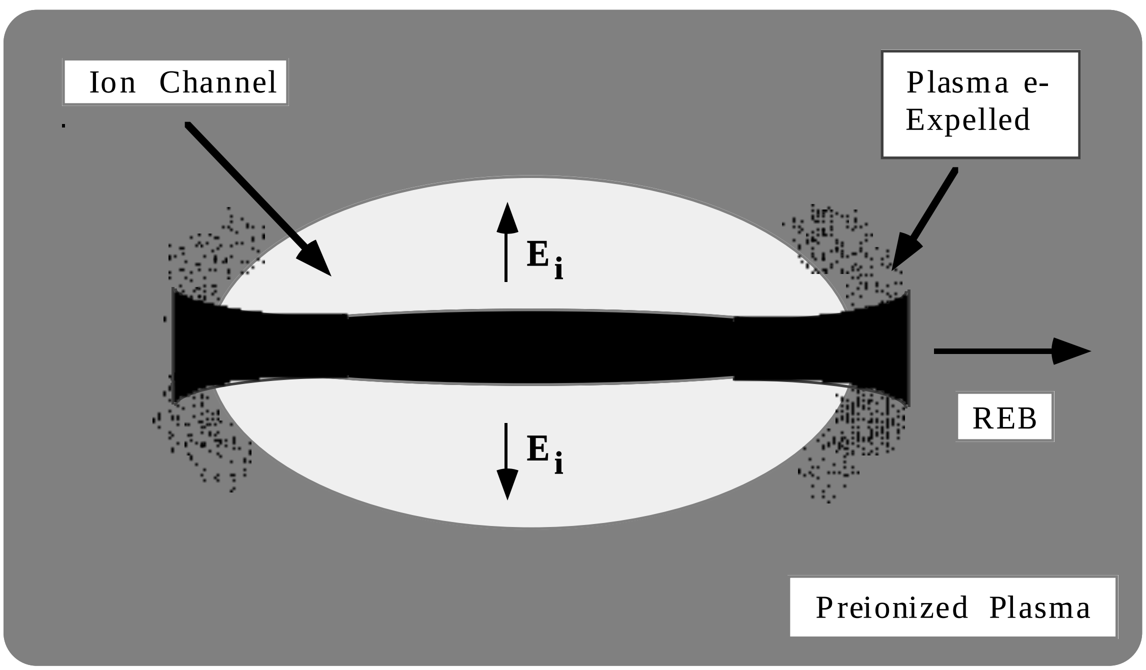

Figure 2. The radial electric field of the beam expels plasma electrons from a large volume, or "channel". Beam electrons are then focussed by the radial electric field of the relatively immobile ions.

The continuous plasma final focus consists of such an adiabatic focussing cell, terminated with an abruptly increased plasma density extending through the IP. As the beam enters this overdense plasma, return currents are induced within the beam volume, reducing the azimuthal magnetic field that would otherwise disrupt the two beams in collision.

In this section, the scaling laws for a continuous focus are set down. Focussing near the axial center of a long cigar-shaped beam, in a perfectly rigid channel, is considered (Fig. 2). Discussion of channel formation is taken up in the next section.

Focussing in the Ion-Channel

As the beam head propagates through the underdense plasma, it continuously expels plasma electrons from the beam volume, forming an "ion-channel", or volume from which plasma electrons have been completely ejected by the beam charge. For a very underdense plasma, the radius of this channel is given by $R \sim \sigma\, (2n_b/n_p)^{1/2}$, where σ is the rms beam radius, n_p is the plasma density prior to channel formation, and n_b is the beam density on axis.[17]

For focussing to be effective, the restoring force due to the ion charge should be much larger than the transverse Lorentz force on the beam due its self-fields. This requires

$$n_b > n_p >> n_b\left(\frac{1}{\gamma^2}+\beta_\perp^2\right), \qquad (1)$$

where $\beta_\perp = v_\perp/c$, with $v_\perp << c$ the transverse velocity. The speed of light is c and γ is the beam energy divided by the electron rest energy, mc^2. The transverse Lorentz force seen by a beam electron in the channel is then

$$\vec{F}(r) = -e\left(\frac{1}{\gamma^2}+\beta_\perp^2\right)\vec{E}_b - e\vec{E}_i \approx -e\vec{E}_i, \qquad (2)$$

where E_b is the radial electric field due to the beam and E_i is the field due to the ion charge, $E_i \sim 2\pi e n_p r$, with r the radial coordinate. The electron charge is -e and its mass is m.

In the potential well of the ion-charge, beam electrons oscillate transversely with wavenumber

$$k_\beta = \frac{k_p}{\beta_z\sqrt{2\gamma}}, \qquad \text{(underdense)} \qquad (3)$$

where $\beta_z = v_z/c$ and $v_z \sim c$ is the beam axial velocity. The quantity $k_p = \omega_p/c$, where ω_p is the plasma frequency, $\omega_p^2=4\pi n_p e^2/m$.

To transport the beam into the plasma without excessive emittance growth, k_β should always vary adiabatically, i.e., the initial beam spot size should be just the equilibrium spot size of the beam in the initial section of plasma. This determines the initial plasma electron density, n_{pi}, in terms of the initial beam spot size, σ_i:

$$n_{pi} = \frac{1}{2\pi}\frac{\varepsilon_n^2}{\gamma r_e \sigma_i^4}, \tag{4}$$

where $\varepsilon_n=\gamma k_\beta \sigma^2$ is the normalized emittance, r_e is the classical electron radius and σ is the rms spot size. Neglecting radiation, scattering, and self-fields, ε_n is an adiabatic invariant. Thus an adiabatic increase in k_p, increases k_β, and decreases σ. This is the principle of the adiabatic focusser.

This adiabaticity requires that the plasma density be tapered over a length of order the initial betatron wavelength. This provides an estimate of the overall length of the plasma cell, $L_p \sim 2\pi\gamma\sigma^2/\varepsilon_n$.

Minimum Spot Size

As the beam is focussed to an ever smaller spot, the plasma density approaches the beam density and the character of the focussing changes. In the overdense regime, the ion space-charge is sufficient to neutralize the beam charge, so that the beam is focussed by its own magnetic field.

This transition, from the underdense regime and ion space-charge focussing, to the overdense regime and beam self-pinching, occurs for a minimum beam radius, σ_{min}, determined by setting $n_p=n_b$,

$$\sigma_{min} = \varepsilon_n \left(\frac{I_A}{\gamma I}\right)^{1/2}. \tag{5}$$

The quantity $I_A=mc^3/e=17.05$ kA, is the Alfven current, and $I=Nec/(2\pi)^{1/2}\sigma_z$ is the peak beam current. The number of electrons per bunch is N.[18] The density at this transition is

$$n_{pt} = \frac{\gamma}{2\pi r_e \varepsilon_n^2} \frac{I^2}{I_A^2}, \tag{6}$$

and the betatron wavenumber at this density is $k_{\beta max} \sim (I/I_A)\varepsilon_n^{-1}$.

In the overdense regime, the effective betatron wavenumber provided by the beam magnetic field is

$$k_\beta \approx \sqrt{\frac{I_{net}}{2\gamma\beta_z I_A} \frac{1}{\sigma}}, \qquad \text{(overdense)} \tag{7}$$

where the net current, I_{net}, is the sum of the beam current and the plasma return current within the beam volume.[19] Since $I_{net} \leq I$, the maximum focussing strength is bounded: $k_\beta \leq k_{\beta max}$.[20]

Therefore, once the beam spot size is focussed to σ_{min}, the adiabatic focussing is complete, and the overdense ion-focussed regime relied on in conventional ion-focussing experments obtains. This establishes a limit on spot size in the adiabatic focusser, neglecting radiation damping. However, for the low emittance, high energy beams of a TeV collider, this limit is far smaller than the beam spot size required. Therefore, the design final spot size, σ_f, will usually be larger than the minimum possible spot size, σ_{min}. In this case, the final density in the focussing section, n_{pf},

$$n_{pf} = \frac{1}{2\pi} \frac{\varepsilon_n^2}{\gamma r_e \sigma_f^4}, \tag{8}$$

will usually be much less than n_{pt}.

Current Neutralization

It has been shown elsewhere that the net current associated with an REB in a collisionless plasma is a function only of $k_p\sigma$, and scales as $I_{net} \sim I/(1+0.5\, k_p^2\, \sigma^2)$.[21] To obtain partial current neutralization, without an increase in beam spot size, the adiabatic focussing cell should be terminated within a distance $\lambda_{\beta f}$ of the IP with a nonadiabatic increase in plasma density to a value, n_{pc}, such that $k_p\,\sigma_f \sim 2^{1/2}$,

$$n_{pc} \approx \frac{1}{2\pi r_e \sigma_f^2}. \tag{9}$$

The length of this cell should be of order a few bunch lengths, and, to avoid defocussing due to plasma return currents, it should be less than the final betatron wavelength at the focusser exit. This implies, $\sigma_z < \lambda_{\beta f}$.

If the adiabatic focusser is terminated with $n_{pf} < n_{pt}$, the beam may pinch as it enters the current neutralization cell. Pinching may be neglected provided the cell length is much less than $\lambda_{\beta min}$; this requires $\sigma_z < \lambda_{\beta min}$.

III. Beam Propagation

The simplified analysis of the last section considered focussing of a long cigar shaped bunch, neglecting the details of channel evolution at the bunch head and tail. No consideration was given to emittance growth due to scattering. These two problems, ion-channel formation and emittance growth due to scattering are known to limit propagation in many beam-plasma applications and, in this section, their effect in the continuous plasma focus is considered.[22]

Scattering

The total cross-section for small angle scattering is[23]

$$\sigma_s = \pi \left(\frac{2 Z r_e}{\gamma} \right)^2 \frac{1}{\theta^2_{min}} . \tag{10}$$

For a fully ionized, quasineutral plasma, $\theta_{min} \sim \hbar/(\lambda_D mc\gamma)$, where λ_D is the Debye wavelength. However, for a partially ionized gas from which plasma electrons have been ejected, $\theta_{min} \sim \hbar/(Rmc\gamma)$, for scattering from ions, and $\theta_{min} \sim \hbar/(amc\gamma)$ for scattering from neutral atoms. The atomic number is Z and $a \sim 1.4\, a_B Z^{-1/3}$, is the screening radius in the Thomas-Fermi model. The constant $\hbar = h/2\pi$, h is Planck's constant, and a_B is the Bohr radius, $a_B = \hbar^2/me^2$. It will be assumed that the ionization fraction, f, is sufficiently low that scattering with neutral atoms dominates.

The mean-square scattered angle per scattering event is

$$\left\langle \theta^2 \right\rangle = 2\theta^2_{min} \ln\left(\frac{\theta_{max}}{\theta_{min}} \right) . \tag{11}$$

The maximum scattering angle is $\theta_{max} = \hbar/(r_n mc\gamma)$ where $r_n \sim 0.5\, r_e A^{1/3}$ is the nuclear radius and A is the mass number. This gives, $\theta_{max}/\theta_{min} \sim 5.26\ 10^4/(AZ)^{1/3}$. $\Theta_{rms}(z)$, the rms scattering angle after traversing a length, z, of gas, varies according to,

$$\frac{d}{dz}\Theta_{rms}^2 = n_0 \sigma_s \langle \theta^2 \rangle$$
$$= 8\pi n_0 \frac{Z^2 r_e^2}{\gamma^2} \ln\left(\frac{\theta_{max}}{\theta_{min}}\right), \qquad (12)$$

where n_0 is the density of neutral atoms.[24] Emittance growth is then given by,[25]

$$\frac{d\varepsilon_n}{dz} = \frac{\gamma}{2k_\beta}\frac{d}{dz}\Theta_{rms}^2, \qquad (13)$$

The change in normalized emittance in passing through the cell is then

$$\Delta\varepsilon_n = 2\frac{r_e Z^2}{f}\int_0^{L_p} dz\, k_\beta \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$
$$\approx \frac{r_e Z^2}{\alpha_0 f}\ln\left(\frac{\theta_{max}}{\theta_{min}}\right)\ln\left(\frac{\lambda_{\beta i}}{\lambda_{\beta f}}\right). \qquad (14)$$

In the overdense regime, envelope expansion is qualitatively different because the quasistatic beam equilibrium is maintained by the beam magnetic field, rather than the (external) field of the ion charge. As the beam expands, the focussing is reduced, with the result that the beam envelope exponentiates, on the scale of the Nordsieck length,[26]

$$L_N = \frac{1}{4\pi n_0 r_e^2}\frac{\gamma I}{I_A}\frac{1}{Z^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)}, \qquad (15)$$

where channel radiation has been neglected. The Nordsieck length is always much longer than the current neutralization cell.

Ion Channel Formation

As the beam moves through the plasma, the beam head must eject electrons from the channel. The ion charge thus exposed provides focussing for electrons to the rear. In the meantime, electrons at the front are not strongly focussed, and expand due to emittance. These and other issues have been discussed in connection with "beam head erosion" of long pulses injected into an unionized gas,[27] and for long pulses in a preionized plasma of radial extent comparable to the beam.[28] The regime of interest here has not been extensively studied, but it is expected that erosion should be negligible, since the plasma is preionized, the electron bunch is short, the emittance is low, the energy is high, and the propagation length is short.[29]

Specifically, regions at the beam head and tail will be less dense than the plasma and will be magnetically focussed by much less than the peak azimuthal magnetic field. Thus a realistic beam profile would appear flared at each end. For best focussing the beam current rise should be adiabatic on the ω_p^{-1} time scale, i.e.,[30]

$$\sigma_z > \frac{1}{\sqrt{4\pi n_{pi} r_e}}, \tag{16}$$

and this condition is typically marginal on injection and well satisfied toward the end of the focusser.

IV. Radiation in the Ion Channel

Radiation in the ion-channel is of interest as a diagnostic, and of possible concern for its effect on beam optics. Two types of radiation are considered: bremstrahlung and synchrotron radiation due to the betatron motion.

Bremstrahlung may be characterized by the radiation length λ_R,[23]

$$\lambda_R^{-1} = \frac{16}{3}\alpha n_0 r_e^2 Z^2 \ln\left(\frac{233}{Z^{1/3}}\right), \tag{17}$$

where $\alpha = e^2/\hbar c$ is the fine structure constant. The fractional energy loss is then

$$\left(\frac{\Delta\gamma}{\gamma}\right)_B \approx \int_0^{L_p} \frac{dz}{\lambda_R} \approx \frac{4\alpha}{3\pi\alpha_0} r_e^2 \lambda_{\beta i}\left(\sqrt{n_{0f} n_{0i}} - n_{0i}\right) Z^2 \ln\left(\frac{233}{Z^{1/3}}\right), \tag{18}$$

where n_{0i} and n_{0f} are the initial and final neutral densities, respectively. This loss is typically very small.

Radiation due to the betatron motion takes on the character of wiggler radiation, for strong focussing ($\gamma\beta_\perp \geq 1$).[31] In principle, the exact single particle trajectory should be used to compute the radiation fields; but for estimates here, it is enough to note the main features.

The spectrum on axis is peaked at frequencies $\omega_W=2\gamma^2 ck_\beta/(1+\gamma^2\beta_\perp{}^2)$. Integrated over all angles, the spectrum is characterized by the critical frequency, $\omega_c=3\gamma^3 c/\rho$, where $\rho = 1/(k_\beta{}^2\,\sigma)$, is the effective bending radius. The angular distribution extends to angles of order $\beta_\perp$. Quantum effects are small provided $\Upsilon < 0.2$, where, $\Upsilon=\gamma^2 \lambda\!\!\!^{-}_c/\rho$, and $\lambda\!\!\!^{-}_c = \hbar/mc$ is the Compton wavelength.[32]

As in a damping ring, synchrotron radiation can decrease the normalized emittance of the beam.[33] However, for the continuous plasma focus it is desirable to limit radiation losses to a small fraction of the beam energy. Fractional energy loss is computed in Ref. 5 and the result, for Υ small, is

$$\left(\frac{\Delta\gamma}{\gamma}\right)_S = -\frac{2\pi^2}{3}\gamma^2\varepsilon_n r_e \frac{\left(1+\alpha_0^2\right)^2}{\alpha_0}\left(\frac{1}{\lambda_{\beta f}^2}-\frac{1}{\lambda_{\beta i}^2}\right). \tag{19}$$

Here, $\Delta\gamma$ is the change in γ and a linear variation in λ_β is assumed: $\lambda_\beta=\lambda_{\beta i} - 4\pi\alpha_0 z$. For the examples, α_0 will be fixed at $\sim 1/4\pi$, corresponding to an overall length, $L_p \sim \lambda_{\beta i}$.

V. Beam Ionization

Ionization by the beam is of concern in determining the actual axial plasma density profile. Beam ionization is also of interest as a means of augmenting laser ionization in the short, dense region at the end of the focusser, and in the neutralization cell. Ionization is produced by the beam through electron impact, gas breakdown, and stripping of atoms and ions in the strong radial electric field at the beam "edge". To accurately compute the net volume rate of ionization requires numerical solution of detailed rate equations, and modelling of the chemistry of the particular gas used. To estimate the effect of impact ionization, a phenomenological estimate must be made for the effective area into which secondary electrons are ejected.[34] In this section, only a few simple estimates are made.

The time scale for ionization in the overdense regime via impact ionization of neutrals by beam electrons is $\tau_b \sim 1/(n_0 \sigma_{bi} c)$, where σ_{bi} is an effective ionization cross-section of order 10^{-18} cm^2.[35] This ionization time is ~ 1 ps at a density of 3 10^{19} cm^{-3}.

The character of breakdown produced by long pulses is determined by the value of E/p, the ratio of radial electric field to pressure.[36] For very fine beams, E/p will be sufficiently large that secondary electrons are ejected far beyond the beam volume before they create additional ionization.

In addition, for short pulses, a key limitation is the formative time required for breakdown. This is roughly the time for one secondary electron accelerated in the beam field, to ionize one neutral, $\tau_e \sim 1/(n_0 \sigma_{ei} v_e)$, where σ_{ei} is the cross-section for ionization by secondaries and v_e is the secondary velocity. The quantity $\sigma_{ei} v_e$ peaks at secondary electron energies of order ~ 100 eV, with $\sigma_{e1} v_e \sim 10^{-7}$ - 10^{-8} cm^3/sec, depending on the gas.[37] The time scale τ_e is then of order ~ 1 ps at a density of 3 10^{19}. Based on this estimate, energetic secondaries, and significant ionization beyond the beam volume may be expected depending on the particular parameters.

The radial electric field at the beam edge will be adequate to strip an atomic electron with ionization potential, $\Delta\varepsilon$, for currents of order

$$I \approx \alpha^4 \frac{\sigma}{r_e}\left(\frac{\Delta\varepsilon}{e^2/a}\right) I_A . \tag{20}$$

For very fine beams, this mechanism may fully ionize a channel larger than the beam, with some multiple ionization.

When field stripping may be neglected, plasma electrons are also lost through recombination on a time scale $\tau_r \sim 1/(\alpha_r n_p)$, and through attachment on a time scale $\tau_r \sim 1/(\alpha_a n_0)$. Here, α_r and α_a are the recombination and attachment coefficients, respectively.[38] Taking recombination in N_2 as an example, $\alpha_r \sim 2\ 10^{-7}$ cm^3/sec, at electron energies ~ 1 eV.[39] At a density of 3 10^{19} cm^{-3}, $\tau_r \sim 0.2$ ps and this is quite short. However, α_r will be lower for more energetic electrons. In addition, despite recombination and attachment, the beam volume will become depleted of plasma electrons, provided the impact ionization time scale is short enough. This occurs because, as electrons go through successive ionizations and recombinations, they diffuse away from the beam center.

Any realistic model of beam ionization, for TeV collider parameters, will have to incorporate all of these effects.

VI. Instabilities

A number of instabilities complicate the equilibrium outlined above, and in this section, the growth rates are noted.

In the focussing cell, the equilibrium discussed so far, consisting of a beam travelling down a static channel, is maintained only to the extent that ions are immobile. In fact, ions at radius R collapse inward, neutralizing the beam charge, on a time scale,

$$\tau_{ion} \approx \frac{1}{4}\frac{R}{c}\left(\frac{m_i}{m}\right)^{1/2}\left(\frac{I_A}{I}\right)^{1/2}, \tag{21}$$

where m_i is the ion mass. For pulses longer than τ_{ion}, focussing is stronger for the beam tail than the head. This should be avoided since it may result in disruption and emittance growth.[40]

In addition, in the underdense regime, it has been suggested that a "transverse two-stream instability" may develop,[41] whereby a displacement of the beam centroid perturbs the channel wall, which then acts back on the beam. However, only preliminary work has been performed on this problem and a growth rate has not yet been derived.

In the current neutralization cell, significant current cancellation requires a low collision rate. However, in the collisionless limit, instabilities may replace collisions in dissipating the energy of the secondaries.[42] In particular, the two-stream (Buneman) instability will couple the electron motion to the ions on a time scale ν_{eiTS}^{-1}, where

$$\nu_{eiTS} = \frac{3^{1/2}}{2}\omega_p\left(\frac{m}{2m_i}\right)^{1/3}, \tag{22}$$

This time scale can be quite short in the current neutralization cell. On the other hand, this instability convects away from the beam, and the carriers of the return current are constantly being replaced with an unperturbed flow of plasma electrons. A thorough analysis of the effect of this instability on current neutralization, including the effects of energy spread, has not been performed.

In addition, in the overdense regime, significant return currents flow within the beam volume and two adjacent plasma electron return current filaments attract. Filaments form and disrupt the intended current neutralization.[43] The growth rate for the Weibel or filamentation instability is

$$\nu_W = \omega_p \left(\frac{n_b}{\gamma n_p} \right)^{1/2} , \tag{23}$$

and typically a few e-folds may develop.[44,45]

VII. Examples

In this section, three applications of a continuous plasma final focus are considered: a proof-of-principle experiment at the TRISTAN injector at the National Laboratory for High Energy Physics (KEK), an application for luminosity enhancement at the Stanford Linear Collider (SLC), and a hypothetical TeV electron-positron collider. A summary of parameters is given in Table I.

TRISTAN Injector

The TRISTAN injector offers the possibility of doing single beam focussing and current neutralization experiments as a "proof-of-principle." A cell of length L_p ~ 3 m, with initial density, n_i ~ 1 10^{11} cm^{-3}, and final density, n_f ~ 2 10^{13} cm^{-3} would focus the spot size from, σ ~ 1 mm, to σ ~ 0.3 mm. An increase in plasma density up to n_c ~ 6 10^{14} cm^{-3} over a length of a few millimeters would produce partial current neutralization.

For these parameters, ion motion, known instabilities, ionization by the beam, breakdown, recombination and attachment are negligible. Scattering produces negligible emittance growth; for example, taking the ionization fraction, f ~ 0.1 % and Z ~ 50, gives $\Delta\varepsilon_n$ ~ 1 10^{-6} m-rad.

As for radiation, quantum effects and fractional energy loss are negligible. Wavelengths extending to 130 nm will be produced.

One complication with these parameters is that the adiabatic current rise condition, Eq. (16), is not satisfied at injection. Thus nonlinear plasma oscillations would be excited and may contribute to emittance growth.

Stanford Linear Collider

At the SLC, a continuous plasma focus could be employed for a proof-of-principle experiment, and to significantly enhance the luminosity in a working collider.[46] The initial plasma electron density would be $n_i \sim 1\ 10^{14}$ cm^{-3} and the overall length of the focusser would be $L_p \sim 1.4$ m. For a final density $n_{pf} \sim 1\ 10^{18}$ cm^{-3}, the spot size would be $\sigma \sim 0.5$ μm. The density required for partial current neutralization would be $\sim 2\ 10^{20}$ cm^{-3}. As a proof of principle, current neutralization experiments would be interesting. However, for the SLC, beamstrahlung and disruption are small and current neutralization is not required.

Assuming an atomic weight, A ~ 100, the time scale for ion motion at the focusser exit is $\tau_{ion} \sim 3$ ps. This is short enough to affect focussing and further work is required to assess its effect. In the current neutralization section, the time scale for filamentation is $\tau_w \sim 2$ ps. The time-scale for the electron-ion two-stream instability is $\tau_{eiTS} \sim 0.1$ ps and this is short. It is unclear what effect this will have on the return current, and further work is required here as well. Scattering will produce small emittance growth; taking the ionization fraction, f ~ 0.1 % and Z ~ 50, gives $\Delta\varepsilon_n \sim 2\ 10^{-6}$ m-rad and this is probably acceptable.

At a density of 10^{18} cm^{-3} the formative time for breakdown will be $\tau_e \sim$ 10 ps, so that breakdown will be marginal. In the neutralization cell, $\tau_e \sim 0.03$ - 0.3 ps depending on the gas, while the impact ionization time scale is $\tau_b \sim 0.1$ ps. Field stripping of atoms will be significant for ionization potentials less than ~ 5 eV. Otherwise, recombination may be significant since a simple estimate gives $\tau_r \sim 10^{-2}$ ps.

Fractional energy loss and quantum effects are negligible. Synchrotron photon energies in excess of 50 MeV will be produced.

TeV Electron-Positron Collider

The normalized emittance used for this example was an order of magnitude larger than in conventional TeV collider designs, and the charge per bunch was taken to be $5\ 10^{10}$, which is a bit higher than is typical. The initial plasma electron density would be $n_i \sim 3\ 10^{15}$ cm^{-3}. The length of the focusser would be $L_p \sim 1.3$ m. For a final density of $3\ 10^{19}$ cm^{-3}, the spot size would be $\sigma \sim 0.1\ \mu$m. At this density the betatron wavelength would be ~ 1.2 cm and the density required for partial current neutralization would be ~ $6\ 10^{21}$ cm^{-3}, and it would be desirable to reach densities of 10^{22} cm^{-3}.

Taking A ~ 100, the time scale for ion motion near the focusser exit is $\tau_{ion} \sim 0.5$ ps and this is shorter than a bunch length. Numerical work is indicated to determine the effect of ion motion near the end of the focussing cell.

In the current neutralization section, the time scale for filamentation is $\tau_w \sim 1$ ps. The time-scale for the electron-ion two-stream instability is $\tau_{eiTS} \sim 4\ 10^{-3}$ ps and further work remains to assess the effect of this on current neutralization.

Scattering may produce some emittance growth depending on the parameters; taking the ionization fraction, f ~ 0.1 % and Z ~ 50, gives $\Delta\varepsilon_n \sim 5\ 10^{-6}$ m-rad, indicating that an ionization fraction f > 0.5% would be desirable.

At a density of $3\ 10^{19}$ cm^{-3} the formative time for breakdown will be $\tau_e \sim 0.3$ ps, while the impact ionization time is $\tau_b \sim 1$ ps. The recombination time will be of order $\tau_r \sim 0.2$ ps. Therefore it is likely that beam ionization will be significant at the focusser exit. In the neutralization cell, $\tau_e \sim 2 - 20$ fs depending on the gas, while $\tau_b \sim 5$ fs. Field stripping of atoms will be significant for ionization potentials less than ~ 30 eV, so that an annulus will be cleared around the beam edge, in which all atoms are at least singly ionized. It is evident from these simple estimates that copious ionization will be produced by the beam in the current neutralization section.

As for radiation, bremstrahlung energy loss is 0.1 % for Z ~ 50. Quantum effects in synchrotron radiation are negligible. Fractional energy loss is ~ 10% and this is roughly the beamstrahlung energy loss in conventional TLC designs. However, energy loss of this size is not an intrinsic feature of the continuous plasma focus and it can be reduced by reducing the emittance. Photon energies will be in excess of 100 GeV.

Table I. Parameters for three example applications of a continuous plasma focus.

	TRISTAN	SLC	TeV LC	(Units)
Beam Parameters				
$mc^2\gamma$	0.25	46	1000	(GeV)
I	0.4	0.25	1.0	(kA)
ε_n	10^{-3}	10^{-5}	10^{-5}	(m-rad)
N	$6\ 10^{10}$	10^{10}	$5\ 10^{10}$	(---)
σ_z	10	2.5	1.0	(ps)
Plasma Parameters				
n_{pi}	$1\ 10^{11}$	$1\ 10^{14}$	$3\ 10^{15}$	(cm^{-3})
n_{pf}	$2\ 10^{13}$	$1\ 10^{18}$	$3\ 10^{19}$	(cm^{-3})
n_{pc}	$6\ 10^{14}$	$2\ 10^{20}$	$6\ 10^{21}$	(cm^{-3})
Focussing Cell Parameters				
σ_i	1000	5.0	1.0	(μm)
σ_f	300	0.5	0.1	(μm)
σ_{min}	300	0.3	0.03	(μm)
$\lambda_{\beta f}$	27	1.4	1.2	(cm)
L_p	3.0	1.4	1.3	(m)
Radiation & Scattering Parameters				
$(\Delta\gamma\ /\gamma)_S$	$8\ 10^{-10}$	$1\ 10^{-4}$	0.1	(---)
Υ	$2\ 10^{-8}$	$3\ 10^{-4}$	$6\ 10^{-2}$	(---)
$\Delta\varepsilon_n$	$2\ 10^{-11}\ Z^2/f$	$8\ 10^{-13}\ Z^2/f$	$2\ 10^{-12}\ Z^2/f$	(m-rad)
Instability Time Scales				
τ_i	$70\ A^{1/2}$	$0.3\ A^{1/2}$	$5\ 10^{-2}\ A^{1/2}$	(ps)
τ_W	100	2	1	(ps)
τ_{eiTS}	$50\ A^{1/3}$	$2\ 10^{-2}\ A^{1/3}$	$1\ 10^{-3}\ A^{1/3}$	(ps)

VIII. Conclusions

The concept of a continuous plasma final focus, consisting of an adiabatic plasma focussing cell, followed by a short, dense current neutralization cell has been outlined. The scaling laws for such a device have been set down, together with three numerical examples.

Beam emittance growth due to gas scattering is small, but non-negligible for the high energy beams and relatively short cell lengths discussed here. Ionization fractions, $f > 1\%$ are favored. The use of lower atomic number gases, in the SLC, or a TeV collider, is inhibited by the need to control ion motion, through the use of a high atomic weight. Ion motion appears unavoidable near the end of the focusser in a TeV collider design, and further work is required to assess the effect on focussing.

Beam ionization occurs through three mechanisms: impact ionization, breakdown, and field "stripping". Beam ionization may helpful in producing plasma in the denser regions in an SLC experiment or in a TeV collider; however, recombination and attachment will also be significant

The well-known beam-plasma instabilities will be insignificant in the adiabatic focussing cell, where the plasma electrons reside beyond the beam volume. However, the possibility exists of a transverse two-stream instability and this remains to be investigated. In the current neutralization cell, the growth rate for the plasma electron-ion two stream instability is high. It remains to perform numerical simulations, including energy spread, to determine the effect on current neutralization.

Much analytical and numerical work remains to be done for a practical experiment. Interesting problems include: (1) studies of the high energy products of beam-plasma collisions, (2) studies of continuous plasma focussing of positron beams, (3) design of the vacuum system, (4) the numerical calculation of the radiation spectrum as a diagnostic for a specified axially-varying focussing strength ($\gamma\beta_\perp$), (5) development of techniques to probe the short-lived ion-channel, (6) studies of realistic beam ionization profiles and their effect on focussing, (7) focussing and channel formation at the beam head, (8) matching of weakly focussed beams (nonlinear wakefield theory in a very underdense plasma), (9) the effects of ion motion at the focusser exit, and (10) numerical simulation of channel formation, including ion-motion, collisions, and dipole perturbations to the beam centroid.

IX. Acknowledgements

Numerous discussions with Andrew M. Sessler were of great help. Conversations with William M. Sharp and Simon S. Yu are greatly appreciated. Comments by John J. Stewart and Yong Ho Chin were quite useful. Thanks go to Atsushi Ogata for encouraging this note.

This work was supported by the Office of Energy Research, U.S. Dept. of Energy, under Contract No. DE-AC03-76SF00098. Work at KEK was supported in part by the XIV International Conference on High Energy Accelerators and The National Laboratory for High Energy Physics (KEK), Tsukuba, Japan.

[1]G. Wallis, K Sauer, D. Sunder, S. E. Rosinskii, A. A. Rukhadze and V. G. Rukhlin, Sov. Phys.-Usp. **17**, 492 (1975); R. Okamura, Y. Nakamura, and N. Kawashima, Plasma Phys. **19**, 997 (1977); P. C. de Jagher, F. W. Sluijter, and H. J. Hopman, Physics Reports **167**, 177 (1988).
[2]W. E. Martin, G. J. Caporaso, W. M. Fawley, D. Prosnitz, and A. G. Cole, Phys. Rev. Lett. **54**, 685 (1985); G. J. Caporaso, F. Rainer, W. E. Martin, D. S. Prono, and A. G. Cole, Phys. Rev. Lett. **57**, 13 (1986).
[3]R. B. Miller, in Physics of Particle Accelerators, edited by Melvin Month and Margaret Dienes, published as AIP Conf. Proc. **184**, Rita G. Lerner, series editor, (AIP, New York, 1989), Vol. 2, p. 1730.
[4]R. F Lucey, Jr., R. M. Gilgenbach, J. D. Miller, J. E. Tucker, and R. A. Bosch, Phys. Fluids B **1**, 430 (1989).
[5]P. Chen, K. Oide, A.M. Sessler, S. S. Yu, in Proceedings of the XIV International Conference on High Energy Accelerators, (Tsukuba, 1989).
[6]K. Oide, Phys. Rev. Lett. **61**, 1713 (1988). The advantage of the ion-channel in this regard is that focussing is continuous, and, for multi-GeV beams, much stronger than is attainable through continuous magnetic focussing. The features of the continuous plasma focus are then rather different from the discrete plasma focus, although the physics is closely related.
[7]R. Hollebeek, Nucl. Instrum. Methods **184**, 333 (1981); P. Chen, in Physics of Particle Accelerators, edited by Melvin Month and Margaret Dienes, AIP Conf. Proc. **184**, (AIP, New York, 1989), Vol. 1, p. 633; G. Bonvicini, E. Gero, R. Frey, W. Koska, C. Field, N. Phinney, A. Minten, Phys. Rev. Lett. **62**, 2381 (1989).
[8]Existing designs for such colliders rely on flat beams, for which beamstrahlung is reduced by a factor of the beam aspect ratio (R. B. Palmer, "The Interdependence of Parameters for a TeV Collider," SLAC-PUB-4295; S. van der Meer, "The CLIC Project and the Design for an e^+e^- Collider, CLIC Note 68, [CERN, 1988]).
[9]D. H. Whittum, A. M. Sessler, S. S. Yu, and J. J. Stewart, "Plasma Suppression of Beamstrahlung," LBL Report No. 25759, (to be published in Part. Acc.). The theory of current neutralization of electron beams in plasmas was first set down in J. L. Cox, Jr., and W. H. Bennett, Phys. Fluids **13**, 182 (1970).
[10]Charge and current neutralization using two-additional beams has also been studied. J. B. Rosenzweig, B. Autin, and P. Chen, in Proceedings of the Lake Arrowhead Workshop on Advanced Accelerator Concepts, (UCLA, 1989), LBL No. 27058.
[11]P. Chen, J. M. Dawson, R. W. Huff, and T. Katsouleas, Phys. Rev. Lett. **54**, 693 (1985); T. Katsouleas, Phys. Rev. A **33**, 2056 (1986); J. M. Dawson, Sci. Am. **260**, 54 (March, 1989).
[12]J. B. Rosenzweig, D. B Cline, B. Cole, H. Figueroa, W. Gai, R. Konecny, J. Norem, P. Schoessow, and J. Simpson, Phys. Rev. Lett. **61**, 98 (1988).
[13]H. Nakanishi, A. Enomoto, K. Nakajima, A. Ogata, T. Urano, Y. Nishida, S. Ohsawa, T. Oogoe, and H. Kobayashi, in Proceedings of the XIV International Conference on High Energy Accelerators, (Tsukuba, Japan, 1989).
[14]P. Chen, Part. Acc. **20**, 171 (1987); P. Chen, J. J. Su, T. Katsouleas, S. Wilks, and J. M. Dawson, IEEE Trans. Nucl. Sci. **PS-15**, 218 (1987).

[15]T. Tajima and J. M Dawson, Phys. Rev. Lett. **43**, 267 (1979); C. Joshi and T. Katsouleas, eds., Laser Acceleration of Particles, AIP Conf. Proc. No. 130 (AIP, New York, 1985).

[16]Design of the vacuum system will not be addressed here. However, the principles of differential pumping are well-understood. See, G. Schmidtke, Rev. Sci. Instrum. **42**, 1431 (1971).

[17]In terms of $s=z-v_z t$, the beam density is assumed to take the form:

$$n_{beam}(r,s) = n_b \exp\left(-\frac{r^2}{2\sigma^2} - \frac{s^2}{2\sigma_z^2} \right),$$

[18]In terms of σ_{min}, the condition that self-fields be negligible, Eq. (1), may be written, $\sigma < \gamma\, \sigma_{min} / (1+\gamma^2 \beta_\perp^2)^{1/2}$.

[19]This is the maximum k_β as a function of radial position. In the overdense regime, focussing is non-linear, so that electrons at the beam edge see a weaker restoring force.

[20]Current enhancement, where $I_{net} > I$, has been observed experimentally in long pulses; but it is not localized to the beam volume, and it is to be avoided here since it is associated with instabilities (B. Hui, R. F. Hubbard, M. Lampe, Y. Y. Lau, R. R. Fernsler, and G. Joyce, Phys. Rev. Lett. **55**, 87 [1985]; S. E. Rosinskii, E. V. Rostomyan, and V. G. Rukhlin, Sov. J. Plasma Phys. **2**, 27 [1976]).

[21]The collisionless approximation requires that $\nu\tau < 1$, where ν is the collision rate of plasma electrons with neutral atoms and ions. If this condition is not satisfied, current neutralization may still be effective, provided only that $\tau_m > \tau$, where $\tau_m = 2(k_p\sigma)^2/\nu$, is the magnetic diffusion time. Numerical simulations show that the large drift induced in the secondary electrons significantly reduces collisions (Ref. 9). This is not surprising since the drift energy is of order ~ 1 keV or larger, while collision cross-sections peak at ~ 10 eV. This makes a simple estimate of ν, based on neutral density and ambient plasma temperature, misleading.

[22]A thorough and concise discussion of propagation issues may be found in A. S Fisher, R. H. Pantell, J. Feinstein, T. L. Deloney and M. B Reid, J. Appl. Phys. **64**, 575 (1988). Estimates are made, and compared to results, in a series of gas-loaded FEL experiments for low current, short bunches (4 ps, 10 A, 42 MeV). The intent in this experiment was to avoid most of the plasma effects discussed here.

[23]J. D. Jackson, Classical Electrodynamics, 2nd ed. (Wiley, New York, 1975).

[24]Evidently, scattering with neutral atoms dominates as long as $f < \ln(a/r_n)/\ln(R/r_n)$. This is the case for f ~ 10%, or less.

[25]B. W. Montague and W. Schnell, in Laser Acceleration of Particles, edited by Chan Joshi and Thomas Katsouleas, AIP Conf. Proc. **130**, (AIP, New York, 1985), p. 146.

[26]T. P. Hughes and B. B Godfrey, Phys. Fluids **27**, 1531 (1984).

[27]W. M. Sharp and M. Lampe, Phys. Fluids **23**, 2383 (1980).

[28]H. L. Buchanan, Phys. Fluids **30**, 221 (1987).

[29]W. M. Sharp and W. M. Fawley, (private communication).

[30]For shorter pulses, nonlinear, radial, plasma oscillations are driven by the rapidly rising beam current, as in a nonlinear plasma wake-field accelerator (J. B. Rosenzweig, Phys. Rev. Lett. **58**, 555 [1987]). However, this regime is relevant only when the initial plasma density, n_{pi}, is low, as for matching of a weakly focussed beam.

If turbulence results in a plasma electron temperature, T_e, the channel edge will have a finite thickness of order a Debye wavelength,

$$\lambda_D = \left(\frac{k_B T_e}{m c^2} \frac{1}{4 \pi n_p r_e} \right)^{1/2},$$

where k_B is Boltzmann's constant. The discussion of Section II assumes $\lambda_D < r_c$, and this requires $k_B T_e < 4mc^2 I / I_A$. For I ~ 1 kA, this requires $k_B T_e < 100$ keV.

[31]A. Hofman, Physics Reports **68**, 253 (1980). Typically $\gamma \beta_\perp \geq 1$ initially and $\gamma \beta_\perp >> 1$ at the focusser exit. For a beam focussed to the minimum spot size, σ_{min}, $\gamma \beta_\perp = (2 \gamma I_{net} / I_A)^{1/2} >> 1$.

[32]A. A. Sokolov and I. M. Ternov, Radiation from Relativistic Electrons, (AIP, New York, 1986).

[33]W. A. Barletta, in Proceedings of the Workshop on New Developments in Particle Acceleration Techniques, (Orsay, 1987) and LLNL No. 96947; E. P. Lee, "Radiation Damping of Betatron Oscillations," UCID-19381 (1982).

[34]D.P. Murphy, M. Raleigh, R.E. Pechacek, and J. R. Grieg, Phys. Fluids **30**, 232 (1987).

[35]A. E. S. Green, Radiation Research **64**, 119 (1975); A. E. S. Green and T. Sawada, J. Atmos. Terr. Phys. **34**, 1719 (1972).

[36]P. Felsenthal, J. M. Proud, Phys. Rev. **139**, 1796 (1965).

[37]M. Mitchner and C. H. Kruger, Jr., Partially Ionized Gases, (Wiley, New York).

[38]Ibid.

[39]F. J. Mehr and M. A. Biondi, Phys. Rev. **181**, 264 (1969).

[40]In addition, the longitudinal two-stream instability will develop due to the relative motion of the beam over the ion background, on a time scale ν_{biTS}^{-1}, where

$$\nu_{biTS} = \frac{3^{1/2}}{2} \omega_i \left(\frac{\omega_b^2}{2 \gamma^3 \omega_i^2} \right)^{1/3},$$

where the limit $\omega_i^2 > \omega_b^2 / \gamma^3$ is assumed, where $\omega_b^2 = 4\pi n_b e^2 / m$. This time scale is long.

[41]W. M. Sharp and S. S. Yu (private communication).

[42]D. Prono, B. Ecker, N. Bergstrom, and J. Benford, Phys. Rev. Lett. **35**, 438 (1975); D. A. McArthur and J. W. Poukey, Phys. Rev. Lett. **27**, 1765 (1971).

[43]R. B. Miller, Intense Charged Particle Beams, (Plenum, New York, 1982).

[44]Resistive instabilities (hose, sausage, hollowing) are neglected in the collisionless limit ($\nu \tau < 1$). For example, the growth time for the resistive hose instability (E. P. Lee, Phys. Fluids **21**, 1327 [1978]) is of order a magnetic diffusion time, which will be longer than a bunch length, in the current neutralization section.

[45]In addition, the beam is subject to the longitudinal two-stream instability, due to the relative motion with respect to the plasma electron drift. The growth rate is

$$\nu_{beTS} = \frac{3^{1/2}}{2}\omega_p\left(\frac{n_b}{2\gamma^3 n_p}\right)^{1/3},$$

and this is typically small.

[46]Plasma focussing of positron beams is not addressed here. To estimate luminosity enhancement, positron beams focussed conventionally or by a discrete plasma lens must be considered.