MATTER EFFECTS ON NEUTRINO PROPAGATION

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ABSTRACT

Neutrino oscillations can be different in matter than in vacuum due to the index of refraction. Examples discussed are the MSW effect for solar neutrinos and effects of passage through the Earth on solar and atmospheric neutrinos.

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Neutrino interactions appear to be well described by the Standard Model. There are the charged current (CC) interactions due to $W^{\pm}$ exchange with quarks

$$\begin{aligned} \nu_i + d &\to \ell_i^- + u, \\ \bar{\nu}_i + u &\to \ell_i^+ + d, \qquad i = e, \mu, \tau, \end{aligned}$$

as well as neutral current (NC) interactions due to Z exchange with quarks

$$\nu_i + q \to \nu_i + q, \qquad i = e, \mu, \tau.$$

The scattering on electrons is slightly more complicated; for ν_μ and ν_τ there is only NC, but for ν_e there is both NC and CC, which can interfere with each other. The Z exchange is in the t-channel, for ν_e-e scattering W exchange is in the u-channel, for $\bar{\nu}_e$-e scattering the W is in the s-channel. Of importance for experiments the cross-sections for elastic scattering on electrons follow approximately

$$\sigma(\nu_e\, e) : \sigma(\bar{\nu}_e\, e) : \sigma(\nu_x\, e) = 6 : 2.5 : 1,$$

where $\nu_x = \nu_\mu$ or ν_τ.

For neutrino energies of 10-20 MeV, the highest energy for solar neutrinos

$$\sigma \sim 10^{-41}\,\mathrm{cm}^2$$

so that the mean free path $L \sim 10^{12}\mathrm{km}/\rho$ where ρ is the density in gm/cc. For ν_μ with energies from 1 to 100 GeV as for accelerator and also atmospheric neutrino experiments

$$\sigma \sim 10^{-38}\,\mathrm{cm}^2\text{-E(GeV)}$$

so that $L \sim 10^9\mathrm{km}/\rho E$. In all these cases the Earth and Sun are essentially transparent. The matter doesn't matter.

Twenty years ago I discovered that in some circumstances the matter could matter.[1] The basic point is that transparent media may have significant indices of refraction. The key is the optical theorem

$$n = 1 + \frac{2\pi N}{p^2} f(0),$$

where N is the number of scatterers per cc, $f(0)$ is the forward scattering amplitude and p is the momentum. The imaginary part of this equation yields $\mathrm{Im}\, n$, which is the absorption coefficient, corresponding to the well-known relation between $\mathrm{Im}\, f(0)$ and the cross-section. However, we are interested in the real part. For a weak interaction, $f(0)$ is real in the first approximation; $\mathrm{Im}\, f(0)$ occurs only in second order and is proportional to G^2, but $\mathrm{Re}\, f(0)$ is proportional to G. Specifically, we find

$$p(n-1) \sim GN,$$

where G is Fermi's constant. Since $p(n-1)x$ determines a phase change, GN defines the inverse of a length over which there is a large phase change. Multiplying G times Avogadro's number determines a length which is approximately the radius of the Earth!

My original idea was to consider massless neutrinos with an off-diagonal neutral current. That is, I considered the possibility that Z exchange changed the flavor so that, for example,

$$\nu_\mu + q \to \nu_\tau + q.$$

My motivation was to find a method to check the Standard Model statement that neutral currents were diagonal. The off-diagonal NC corresponds to an off-diagonal index of refraction, analogous to the case of optical rotation. Thus, as a ν_μ beam passed through the Earth it could "rotate" into ν_τ. As one observed neutrinos coming through the Earth at different angles the transmission probability would oscillate, or, if the angular resolution was bad one would find an average survival probability of about 50%.[2] Exactly this was found 20 years later in the Super Kamiokande experiment. The possibility that the experiment could be explained in this was has been explored by several authors.[3] This is probably not the explanation; in particular, it predicts that the disappearance is energy-independent.

Let me turn now to the case of massive neutrinos with oscillations due to mixing. Considering only two flavors of neutrinos ν_a and ν_b

$$\nu_a = \nu_1 \cos\theta_v + \nu_2 \sin\theta_v, \qquad \nu_b = -\nu_1 \sin\theta_v + \nu_2 \cos\theta_v,$$

where ν_1, ν_2 are mass eigenstates and θ_v is the vacuum mixing angle, one finds the oscillation probability in vacuum

$$\begin{aligned} P(\nu_a \to \nu_b) &= \sin^2 2\theta_v \sin^2(\pi x/\ell_v), \\ \ell_v &= 4\pi p/\Delta m^2 = 2.5\,\mathrm{meters}\,\frac{p(\mathrm{MeV})}{\Delta m^2(\mathrm{eV}^2)}. \end{aligned} \tag{1}$$

In the case of oscillations involving ν_e, the effect of matter can be important. This is because n is different for ν_e than for ν_μ or ν_τ since there is a contribution to ν_e-e scattering from W exchange. (The phase due to Z exchange is the same for all ν_i and so is just an overall phase.) This contribution is

$$p(n-1) = -\sqrt{2}GN_e. \tag{2}$$

The propagation equation then becomes

$$\begin{aligned} i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} &= \frac{1}{2E}\begin{bmatrix} m_{ee}^2 + 2\sqrt{2}GN_eE & m_{e\mu}^2 \\ m_{e\mu}^2 & m_{\mu\mu}^2 \end{bmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \\ m_{ee}^2 &= \frac{1}{2}\left(\mu^2 - \Delta m^2 \cos 2\theta_v\right), \\ m_{\mu\mu}^2 &= \frac{1}{2}\left(\mu^2 + \Delta m^2 \cos 2\theta_v\right), \\ m_{e\mu}^2 &= \frac{1}{2}\Delta m^2 \sin 2\theta_v. \end{aligned} \tag{3}$$

Thus, for values of ℓ_v that are comparable to $(GN_e)^{-1}$ or larger, oscillations in matter can be very different in the mixing angle and oscillation length than in vacuum. For uniform matter the oscillation length ℓ_m and mixing angle θ_m are given by[1]

$$\begin{aligned} \sin 2\theta_m &= \sin 2\theta_v(\ell_m/\ell_v), \\ \ell_m &= \ell_v\left[1 - 2\ell_v/\ell_0 \cos 2\theta_v + \left(\frac{\ell_v}{\ell_0}\right)^2\right]^{-\frac{1}{2}}, \\ \ell_0 &\equiv 2\pi/\sqrt{2}GN_e = 16,000\text{km}/\rho_e, \end{aligned}$$

where ρ_e is N_e in units of Avogadro's number.

An example of particular interest was pointed out by Mikheev and Smirnov.[4] This was the case in which θ_v is relatively small and one considers ν_e emerging from the Sun, thus passing through a region of continuously changing density. One assumes that in the vacuum the lower mass state is mainly ν_e ($m_{ee}^2 \ll m_{\mu\mu}^2$). In the center of the Sun the ee term in Eq. (3) dominates because of large N_e, but this term gradually decreases until it equals $m_{\mu\mu}^2$. This they refer to as a resonance; it can also be called a "level crossing". If we consider the "eigenstates" for propagation we see that for large N_e the top eigenstate is overwhelmingly ν_e but outside the Sun this eigenstate is primarily ν_μ. In the adiabatic approximation the neutrino stays in the top eigenstate and so starting out as ν_e it emerges as mainly ν_μ. This is called the MSW effect. The adiabatic approximation requires that the two "levels" not get too close at crossing; since they are kept apart by the $m_{e\mu}^2$ term ($= \frac{1}{2}\,\Delta m^2 \sin 2\theta_v$), this requires that θ_v be not *too* small. Solar neutrino disappearance rates have been fitted with $\sin^2 2\theta_v \sim 10^{-2}$ and $\Delta m^2 \sim 5\times 10^{-6}\mathrm{eV}^2$. (See the talks by Wilkerson[5] and Murayama.[6]) Even though θ_v is very small the oscillation probabilities end up greater than 50%. It is also possible to exploit this matter effect and fit the data with large $\sin^2 2\theta_v \sim 0.7$ (LMA – MSW).

An important aspect of the MSW effect in the adiabatic approximation is that the neutrinos leave the Sun in a vacuum eigenstate. There are no oscillations between Sun and Earth. However, once these neutrinos enter the Earth they are not in a matter eigenstate; therefore oscillations start again. Thus the flux of solar ν_e coming through the Earth and detected at night can be different than that detected during the day. In particular, it is possible that the ν_e shine more brightly at night than during the day. For certain values of Δm^2 and $\sin^2 2\theta_v$ for the LMA-MSW it is found[7] that a 20% asymmetry ((Night-Day)/Sum) would be possible at Super Kamiokande. Such large asymmetries have been ruled out, but small asymmetries are possible.

Note that this Earth effect is not the MSW effect since the density of the Earth does not vary continuously. It is constant for neutrinos that miss the core and there is an almost discontinuous change for those that go through the core. Note also that the day-night effect could occur even if the ν_e suppression factor is exactly $\frac{1}{2}$, maximal mixing.[8] This is because if the adiabatic condition holds, the neutrinos arrive at the Earth in the eigenstate $(\nu_e + \nu_\mu)/\sqrt{2}$ and in the Earth they oscillate in part to $(\nu_e - \nu_\mu)/\sqrt{2}$ so that they emerge as a coherent mixture

$$\cos\varphi(\nu_e + \nu_\mu)/\sqrt{2} + e^{i\alpha}\sin\varphi(\nu_e - \nu_\mu)/\sqrt{2}$$

Returning now to the atmospheric neutrinos, the question has been raised whether the disappearing ν_μ oscillate to ν_τ or possibly to a new kind of non-interacting "sterile" neutrino ν_s. For the case of $\nu_\mu \to \nu_s$ the oscillations can be affected by matter because ν_μ has an index of refraction

$$p(n-1) = GN_n/\sqrt{2},$$

where N_n is the neutrino number density whereas ν_s does not. As the energy increases the mixing terms proportional to $\Delta m^2/E$ become less important relative to the GN_n term; as a result, the mixing is suppressed. Thus, data on the energy dependence of the ν_μ disappearance has been used as evidence against ν_μ-ν_s possibility. (See the talk by Kearns.[9])

For the value of $\Delta m^2 \sim 3 \times 10^{-3}\text{eV}^2$ which we assume is associated with ν_μ-ν_τ oscillations it is important whether there is some mixing with ν_e described by $\sin\theta_{e3}$. So far we only know that this is small. For atmospheric neutrinos the appearance of upward going ν_e from $\nu_\mu \to \nu_e$ oscillations can be affected by matter. In particular, it is possible that even for small values like $\sin^2\theta_{e3} = 0.03$ the oscillation of $\nu_\mu \to \nu_e$ in the Earth could be almost 50% but only for those of a particularly energy coming through the core. However, even averaging over a range of energies and angles it is possible that a significant enhancement could be present in future atmospheric neutrino data.[10]

One of the reasons for the interest in θ_{e3} is the possibility of CP violation in the neutrino mixing matrix. As in the case of quarks (CKM matrix) it is necessary to consider the full 3×3 matrix. If θ_{e3} is not too small there is the possibility of having a CP-violating difference between $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$. Such an experiment would be conceivable in the future in long baseline experiments with beams from Fermilab or CERN. However, you expect some difference anyway because the matter effect is different for ν_e and $\bar{\nu}_e$. Thus, it is necessary to separate out the matter effect to find CP violation.[11]

In summary, we have considered a number of cases where matter effects could be important:

1. Solar neutrinos:
   - Matter effects inside the Sun could enhance the oscillations (MSW effect).
   - Matter effects inside the Earth could then cause a day-night effect.
2. Atmospheric neutrinos:
   - An off-diagonal neutral current could induce oscillations even for massless neutrinos.
   - Matter effects allow to distinguish $\nu_\mu \to \nu_\tau$ from $\nu_\mu \to \nu_s$.
   - Matter effects could enhance $\nu_\mu \to \nu_e$ transitions.
3. Long baseline experiments:

- Matter effects cause a difference between $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$, thus simulating CP violation.

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