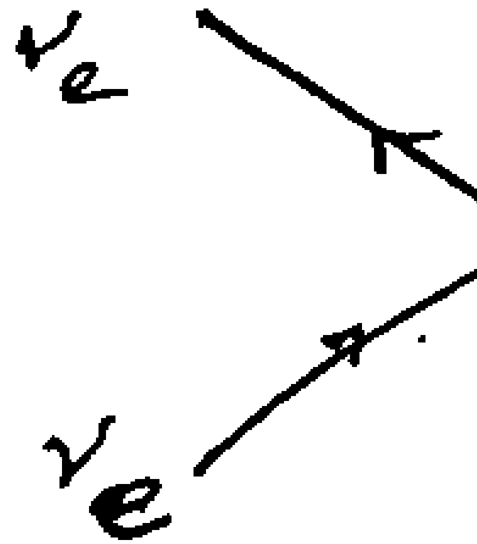
Matter Effects on Neutrino Oscillations

Lincoln Wolfenstein
Carnegie Mellon University

e

$\bar{\nu}_e$ +



$\sigma(\nu_e\, e) : \sigma$

Earth and Sun are Transparent

$E(\nu_e) \sim 10 - 20$ Mev

$\sigma \sim 10^{-41}$ cm^2

M.f.p. $L \sim 10^{12}$ km$/\rho$

or

$E(\nu_\mu) > 1$ Gev

$\sigma \sim 10^{-38}$ cm^2 E (Gev)

$$L \sim \frac{10^9 \text{ km}}{\rho E}$$

Oscillations in Matter

Index of refraction

$$n = 1 + \frac{2\pi N}{p^2} f(0)$$

$$p\,(n-1) \propto G_F N$$

S.M. N.C. all ν $\frac{1}{\sqrt{2}} G_F N_{neut}$

C.C. ν_e $-\sqrt{2}\, G\, N_e$

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{1}{2E} \left\{ m_{ij}^2 - 2E\,(n-1)_{ij} \right\}$$

My original idea

Off-diagonal

$$(n-1)_{\mu\tau} = (n-1)_{\tau\mu} = \frac{1}{\sqrt{2}} G_F N_{neut}$$

(Lipari-Lusignoli PRD 60, 13003 (99))

(Problems with this: PRD 61, 053005 (2000) up-down asymmetry for massless neutrinos

Off diagonal neutral current

$\nu_\mu + N \to \nu_\tau + N$

ν_μ "rotates" into ν_τ in

analogy with optical activity

Oscillation length $\propto \frac{1}{G_F \rho}$

~ 3000 to 6100 km

L.W. PRD 17, 2369 (1978)

M.C. Gonzaly-Garcia + 7 (1998)

Lipari Lusignoli (999)

L. W. in AIP Conference Proceedings No. 52, 1978

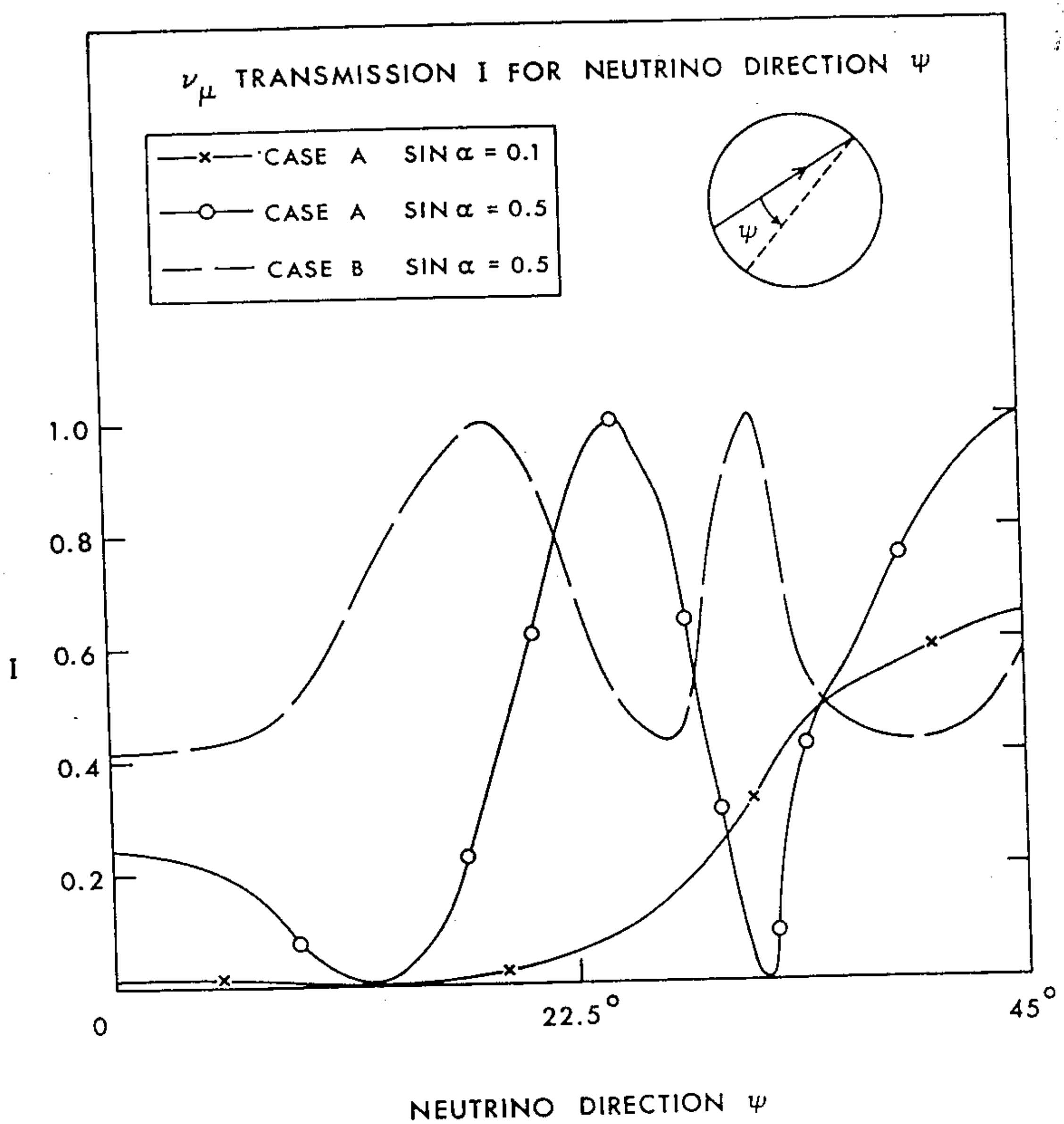


Fig. 1. Angular dependence of the transmission probability I of cosmic-ray muon-type neutrinos coming through the earth.

Neutrino oscillations in vacuum

Two generation example

$$\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta$$

$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta$$

As a function of time

$$\nu_e \to \nu_1 \cos\theta\, e^{-iE_1 t}$$
$$+ \nu_2 \sin\theta\, e^{-iE_2 t}$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta$$
$$\sin^2 (\pi x / l_\nu)$$

$$l_\nu = 4\pi p / \Delta m^2 = 2.5\,m. \frac{E(Me}{\Delta m^2(e}$$

$$\Delta m^2 = m_2^2 - m_1^2$$

where we use $E_2 - E_1 \approx \dfrac{m_2^2 - m_1^2}{2P}$

y includes
ch is pro-

LATIONS

sumptions of
sider how
ios pass
nat the neu-
n as in the
onal and sym-
Considering
given by

eresting
nos is ν_e.
uum oscil-
vacuum

(21)

n_2). Neutrino
e in the phase
e of ν_1 and

in the vacuum

where $\sqrt{2}\, GN_e \equiv k(n_e - 1)$ and n_e is the index of refraction associated with the charged-current scattering. The eigenstates for propagation in matter are

$$|\nu_{1m}\rangle = |\nu_e\rangle \cos\theta_m - |\nu_\mu\rangle \sin\theta_m ,$$

$$|\nu_{2m}\rangle = |\nu_e\rangle \sin\theta_m + |\nu_\mu\rangle \cos\theta_m ,$$

$$\tan 2\theta_m = \tan 2\theta_v \left(1 - \frac{l_v}{l_0} \sec 2\theta_v\right)^{-1} , \tag{23a}$$

$$l_0 \equiv 2\pi/GN_e \approx 2.0 \times 10^9 \text{ cm}/\rho_e \cdot \approx \frac{20{,}000\text{ km}}{\rho_e} \tag{23b}$$

The oscillation length in matter is

$$l_m(k) = l_v(k)\left[1 + \left(\frac{l_v(k)}{l_0}\right)^2 - 2\cos 2\theta_v \left(\frac{l_v(k)}{l_0}\right)\right]^{-1/2} , \tag{24a}$$

and the transformation probability is given by

$$|\langle \nu_e | \nu_\mu(x)\rangle|^2 = \tfrac{1}{2}\sin^2(2\theta_v)(l_m/l_v)^2 \times [1 - \cos(2\pi x/l_m)] . \tag{24b}$$

As long as $l_v \ll l_0$, it is seen from Eqs. (23) and (24) that $l_m \approx l_v$, $\theta_m \approx \theta_v$, and therefore the oscillations in the medium will be very much the same as in the vacuum. For $l_v \gg l_0$ it is seen that $l_m \approx l_0$ independent of θ_v and therefore from Eq. (24b) the amplitude of the oscillation is very small. Some examples of the effect of the medium for the intermediate case $l_v = l_0$ are illustrated in Table II. Independent of the value of l_v/l_0, it follows from Eq. (24b) that as long as $(2\pi x/l_m) < 1$, the oscilla-

ν Propagation in Matter

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \frac{1}{2E}\begin{bmatrix} m^2_{ee} + 2\sqrt{2}\,G N_e E & m^2_{e\mu} \\ m^2_{e\mu} & m^2_{\mu\mu}\end{bmatrix}$$

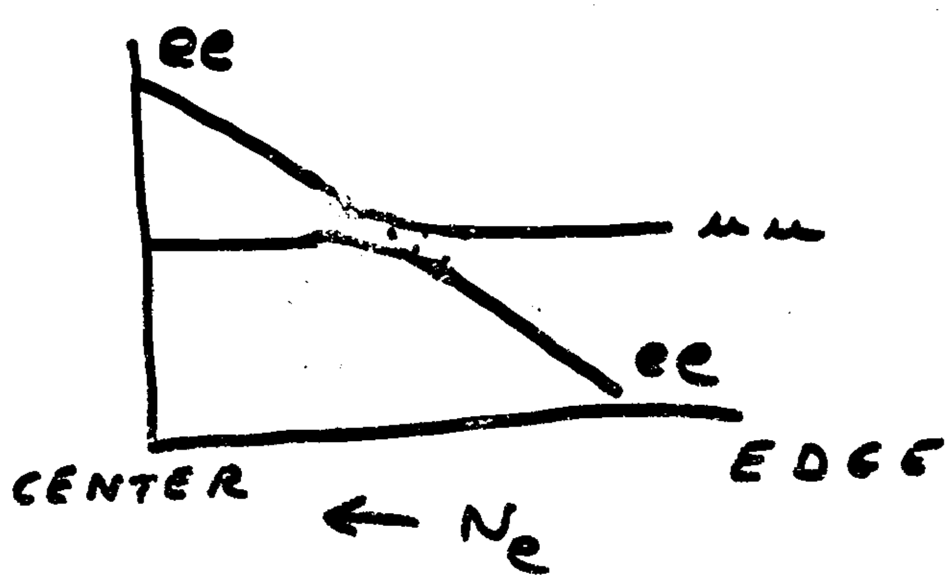
$$m^2_{ee} = \frac{1}{2}\left(\mu^2 - \Delta m^2 \cos 2\theta_v\right)$$

$$m^2_{\mu\mu} = \frac{1}{2}\left(\mu^2 + \Delta m^2 \cos 2\theta_v\right)$$

$$m^2_{e\mu} = \frac{1}{2}\,\Delta m^2 \sin 2\theta_v$$

For ν_e with $\Delta m^2 \approx m^2(\nu_\mu) - m^2(\nu_e)$

θ_v = vacuum mixing angle



E_ν (Mev) ρ_e = 6.5 × 10^6 Δm^2 (eV^2)

MIKHAEYEV - SMIRNOV

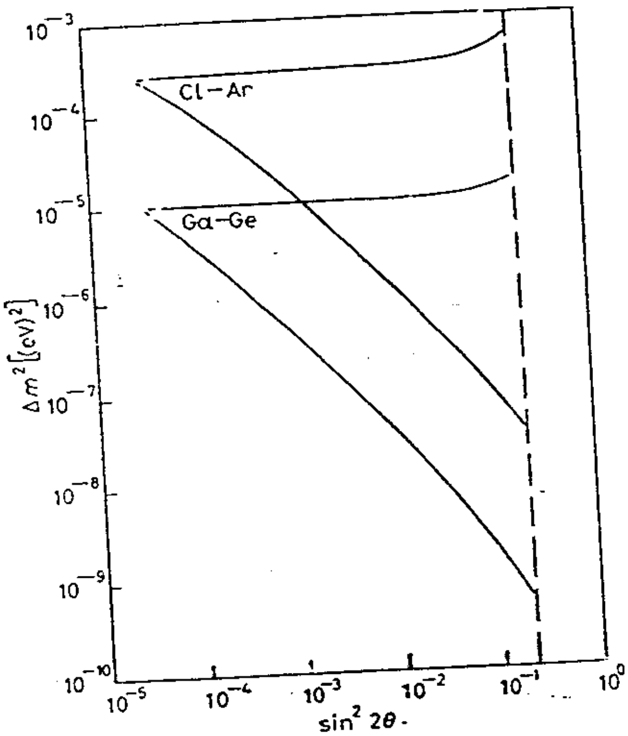


Fig. 2. – The region of neutrino parameters m^2, $\sin^2 2\theta$ for which resonant amplification of oscillations takes place in the Sun. Inside the regions limited by full lines the suppression factor for Cl-Ar and/or Ga-Ge experiments exceed 10%. The dashed line restrict the region of 10% effect due to vacuum oscillations only.

Sun. The system of equations (10) has been solved with the density distribution of standard solar model ([9]). For the point $E/\Delta m^2 = 4\cdot 10^4$ the resonance occurs in the centre of the Sun. In the region $E/\Delta m^2 < 4\cdot 10^4$ there is no resonance at all (case C)). In the wide region $E/\Delta m^2 < (E/\Delta m^2)_{max} = f(\sin^2 2\theta)$ condition (12) is fulfilled. So at $4\cdot 10^{-4} < E/\Delta m^2 < (E/\Delta m^2)_{max}$ case A) takes place. For $E/\Delta m^2 > (E/\Delta m^2)_{max}$ the resonant layer is thin: $R_{res} < L_m/2$ (case B)).

Solid curves in fig. 3 represent the probabilities $P(E/\Delta m^2)$ for neutrino production region with $R_{pr} = 0.2\cdot R_S$. The solid and dashed curves differ appreciably in the region $E/\Delta m^2 < 4\cdot 10^{-4}$. The reason is that the part of neutrinos with $E/\Delta m^2 > 4\cdot 10^{-4}$ produced in the forward hemisphere (with respect to Earth) does not pass through the resonant layer.

Consider now possible changing in solar-neutrino spectrum and in experi- ... resonant oscillation. The solar-neutrino flux near the

Signatures of Oscillation Solutions

A common signature of most neutrino oscillation solutions is distortion of *B*-ıtrino spectrum. The survival probabilities for SMA MSW, LMA MSW and just-VO are shown in Fig.2. The survival probabilities for RSFP are similar to SMA ;W and shown in Fig.3.

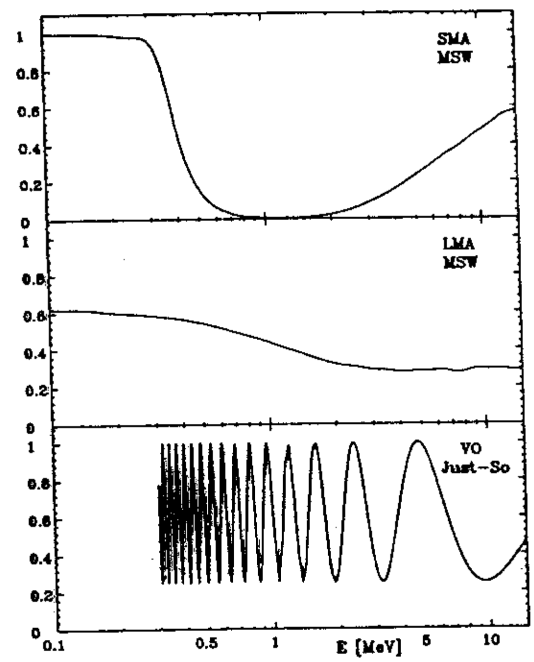


Fig. 2. *Electron neutrino survival probabilities as function of neutrino energy.*

One can see there that LMA MSW (and LOW too) predicts small distortion of -neutrino spectrum spectrum in the region of observation $5-15$ MeV. For EIS VO .e distortion is absent. The strongest spectrum distortion one can expect for SMA SW and just-so VO. However, spectrum of recoil electrons are distorted weaker ıan that of neutrinos, because of cross-section and averaging over energy bins in ›servations (*e.g.* see Fig.8). *The absence of distortion of neutrino or recoil-electron ›ectra is not a general argument against neutrino oscillations.*

Anomalous NC/CC ratio is another common signature of neutrino oscillations hich can be observed in SNO. The NC events will be seen there by detection of ›utrons produced in $\nu + D \to p + n + \nu$ reaction. Oscillation $\nu_e \to \nu_\mu(\nu_\tau)$ does not ıange NC interaction but changes CC interaction and thus the ratio of NC/CC rate. case of oscillation to sterile neutrino the NC/CC ratio is not changed. Therefore, *e normal ratio NC/CC is not a general argument against neutrino oscillations.*

MSW solutions have very specific signatures which are unique. They are day/night

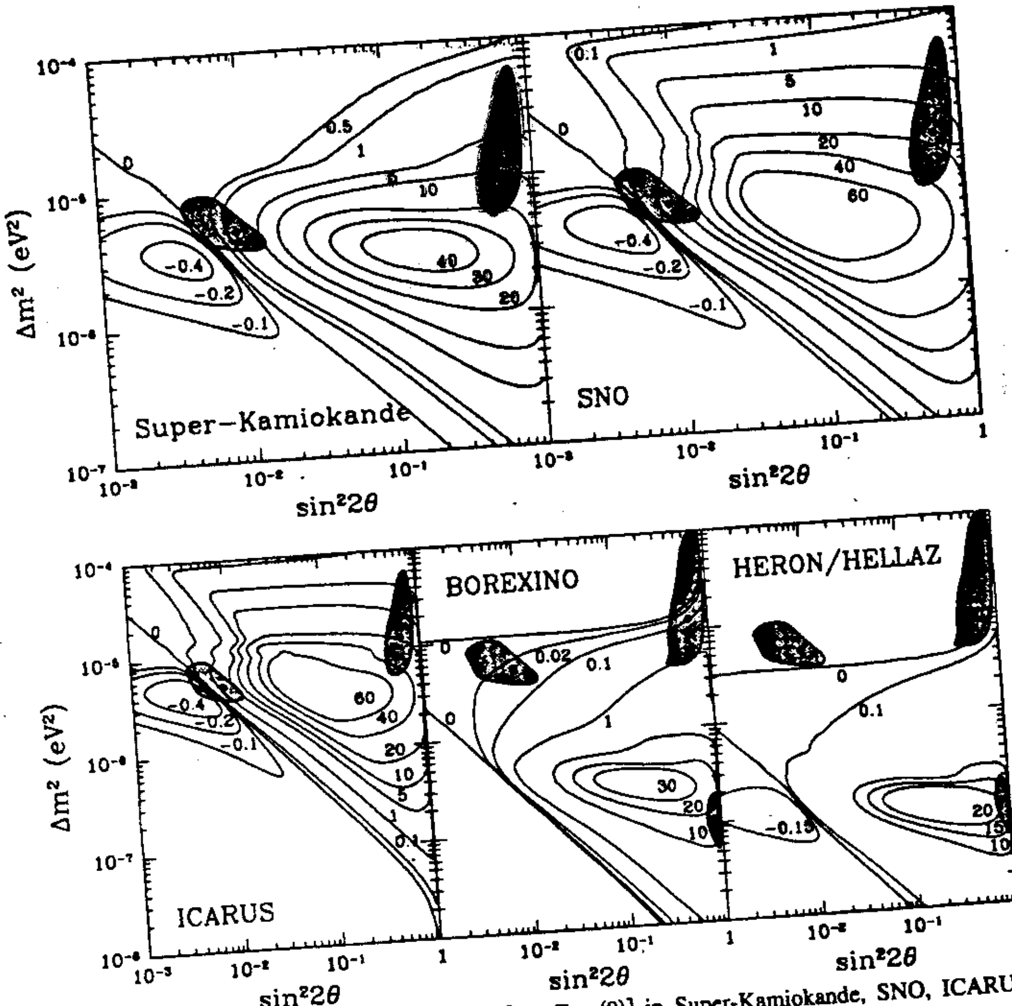


FIG. 10. Contours of constant day-night asymmetry A_{n-d} [see Eq. (9)] in Super-Kamiokande, SNO, ICARUS, BOREXINO, and HERON/HELLAZ. The shaded regions are the same as in Fig. 7.

creases, for both the SMA and the LOW solutions, the average kinetic energy of the recoil electrons in both Super-Kamiokande and SNO. This decrease occurs because in the sun these two solutions preferentially transform low-energy neutrinos from ν_e to ν_μ (or ν_τ) and therefore there is a relatively larger chance at low-energy of regenerating ν_e from ν_μ (or ν_τ) in the Earth. For the LMA solution, regeneration increases the average kinetic energy since in this case the high-energy part of the ^{8}B neutrino energy spectrum is preferentially depleted of ν_e in the Sun.

The shift between day and night of the moments is most significant for the LMA solution. In fact, if Nature has chosen the LMA solution, then the spectral distortion may be highlighted by comparing the day-time and night-time moments.

TABLE IV. Day-night asymmetry in Super-Kamiokande and SNO. The table gives the magnitude of the expected day-night asymmetry [A_{n-d}, see Eq. (9)] (in percent) in Super-Kamiokande and SNO for values of the neutrino oscillation parameters Δm^2 and $\sin^2 2\theta$ corresponding to the best fit SMA and LMA solutions [see Eqs. (1a), (1b) and (2a), (2b)]. The indicated uncertainties describe the expected limits at 95% C.L.

Solution	Super-Kamiokande	SNO
SMA	$1.8^{+4.2}_{-2.0}$	$2^{+6.0}_{-2.4}$

X. SENSITIVITY TO EARTH MODELS AND SOLAR MODELS

We calculate in Sec. X A the sensitivity of the MSW predictions to the assumed density profile and chemical composition.

TABLE V. What statistical test is best? The table compares the statistical power of three methods for analyzing data on the regeneration effect: (a) moments of the zenith-angle distribution, (b) day-night asymmetry (A_{n-d}), and (c) Kolmogorov Smirnov (KS) test of the zenith-angle distribution. The number of σ's listed in the table corresponds to the deviation of the best-fit MSW solutions described in Sec. IV from the undistorted zenith-angle distribution. The numerical results correspond to 30 000 events for the SMA solution and 5000 events for the LMA solution.

Detector	Solution	Moments (σ)	A_{n-d} (σ)	KS test (σ)
Super-Kamiokande	SMA	5	4.4	3.7
	LMA	5.5	9.2	6.3
SNO	SMA	6.5	4.9	4.4
	LMA	10	18.8	12.4

Possible ν_e asymmetry (0.8)
in Super K
~~enhanced~~ by matter effect

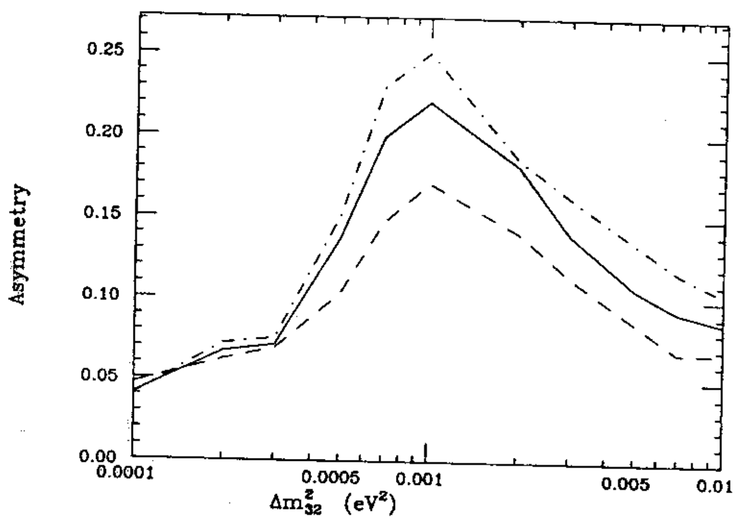


Figure 3c

$- - - \sin^2 2\theta_{e3} = .03$

Akmedov ... Smirnov
hep-ph 9808270