THREE GENERATIONS

solar + atmos

3 generations ⇒ 3 mass eigenvalues

$$m_1 < m_2 < m_3$$

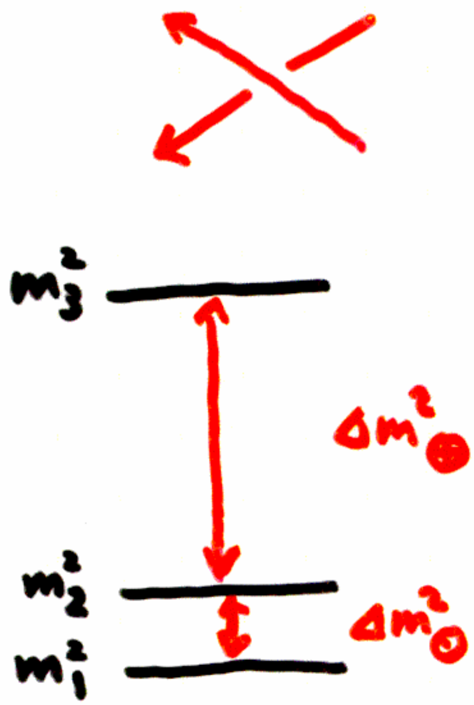
solar ν osc $\longrightarrow \Delta m^2_\odot$

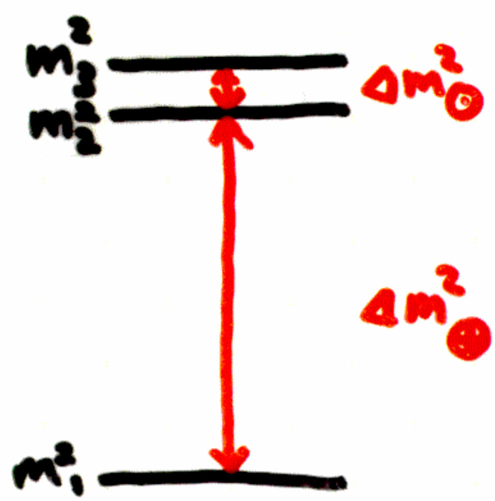
atmospheric ν osc $\longrightarrow \Delta m^2_\oplus$

2 mass^2 differences

$$m_3^2 - m_2^2 \qquad \Delta m^2_\odot$$

$$m_2^2 - m_1^2 \qquad \Delta m^2_\oplus$$


m_3^2
$\Delta m^2_\oplus$
m_2^2
m_1^2
$\Delta m^2_\odot$


m_3^2
m_2^2
m_1^2
$\Delta m^2_\odot$
$\Delta m^2_\oplus$

commonly used parameterization

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s_{12} = \sin\theta_{12}$, $c_{23} = \cos\theta_{23}$ etc

because we fixed our convention

$$m_1 < m_2 < m_3$$

we need to vary

$$0 \leq \theta_{12} \leq \frac{\pi}{2}$$

$$0 \leq \theta_{13} \leq \frac{\pi}{2}$$

$$0 \leq \theta_{23} \leq \frac{\pi}{2}$$

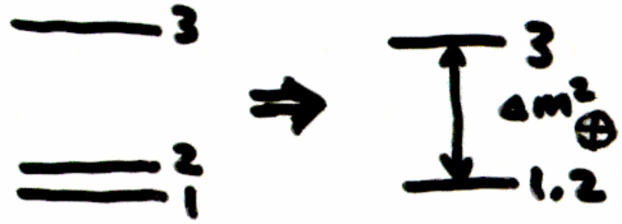
$$0 \leq \delta \leq 2\pi$$

Disappearance channel

$$P(\nu_e \to \nu_e) = 1 - 2 \sum_{\alpha,\beta} |U_{e\alpha}|^2 |U_{e\beta}|^2 \sin^2 \frac{m_\alpha^2 - m_\beta^2}{4E} L$$

simplification if $\frac{\Delta m^2_{\odot}}{4E} L \ll 1$

assume



$$P(\nu_e \to \nu_e) = 1 - 4 \underbrace{(|U_{e1}|^2 + |U_{e2}|^2)}_{\substack{\shortparallel \\ 1 - |U_{e3}|^2 \\ \shortparallel \\ \cos^2\theta_{13}}} \overbrace{|U_{e3}|^2}^{\substack{\sin^2\theta_{13} \\ \shortparallel}} \sin^2 \frac{\Delta m^2_{\oplus}}{4E} L$$

unitarity

$$= 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{\oplus}}{4E} L$$

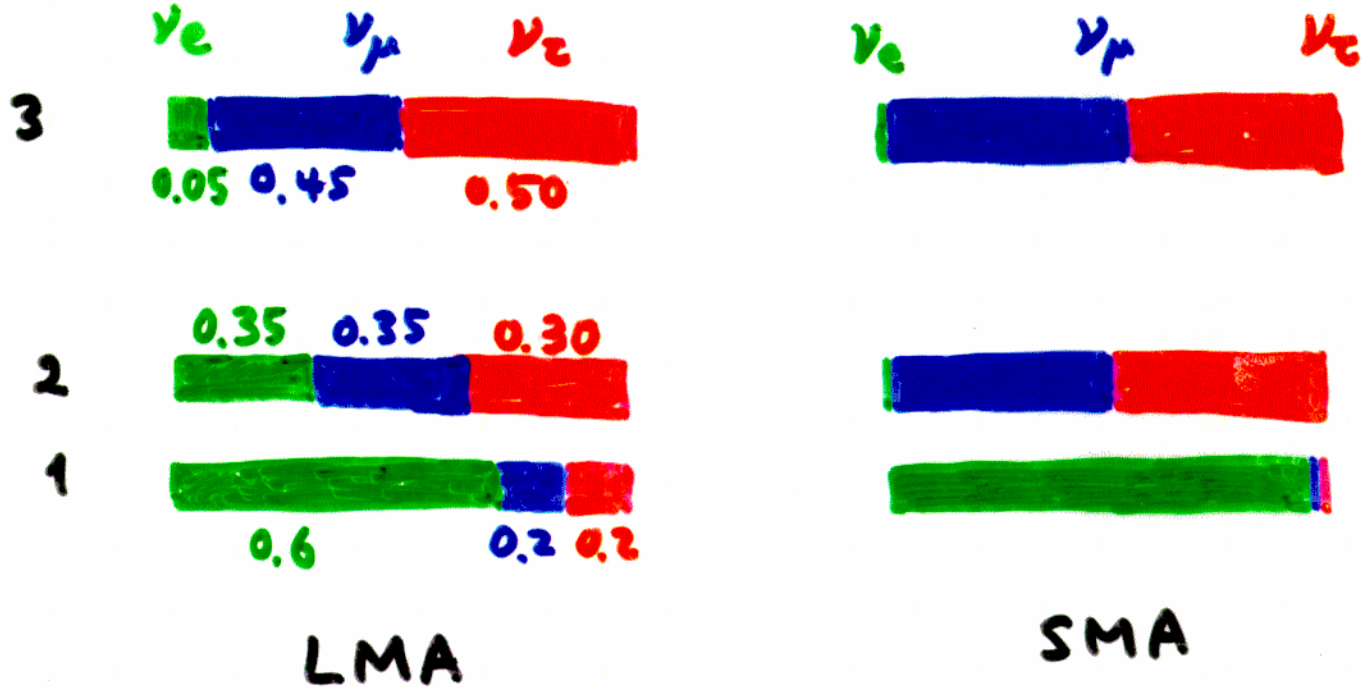
the same as in two-flavor case!

CHOOZ: $\sin^2 2\theta_{13} < 0.05 - 0.2$

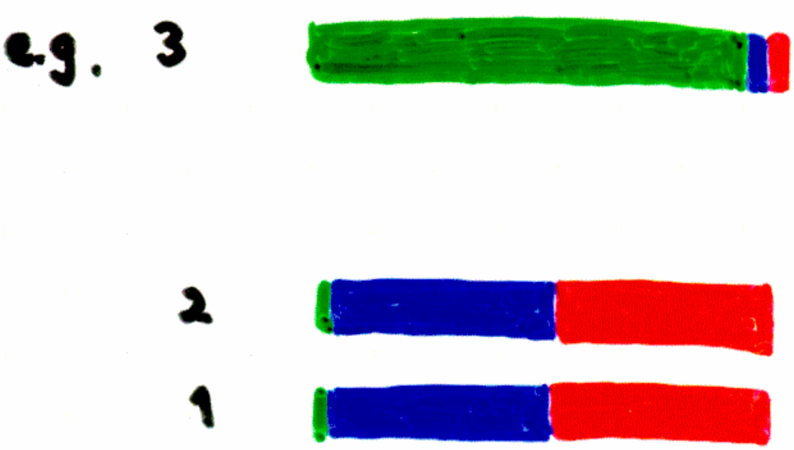
for $\Delta m^2_{\oplus} = 10^{-2} - 10^{-3}\ \mathrm{eV}^2$

$(\sin^2\theta_{13} = |U_{e3}|^2 < 0.05)$

flavor budget

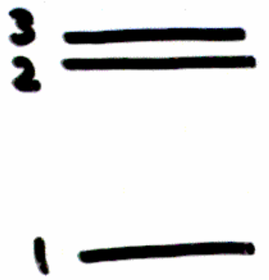


Wait! Doesn't $\sin^2 2\theta_{13} < 0.2$ still allow $|U_{e3}|^2 > 0.95$?



doesn't solve neither solar nor atmos osc

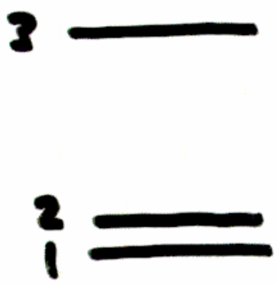
$\Rightarrow \sin^2\theta_{13} < 0.05$

What if 

?

in the absence of data which has matter effects with $\Delta m^2_{\oplus}$.

cannot be distinguished from



i.e. interchange $1 \leftrightarrow 3$

similarly,

$$P(\nu_\mu \to \nu_\mu) = 1 - 4\left(|U_{\mu 1}|^2 + |U_{\mu 2}|^2\right)|U_{\mu 3}|^2 \sin^2 \frac{\Delta m^2_\oplus}{4E} L$$

$$= 1 - 4\left(1 - c_{13}^2 s_{23}^2\right) c_{13}^2 s_{23}^2 \sin^2 \frac{\Delta m^2_\oplus}{4E} L$$

atmospheric data

$$\sin^2 2\theta_\oplus \to 4\left(1 - c_{13}^2 s_{23}^2\right) c_{13}^2 s_{23}^2$$

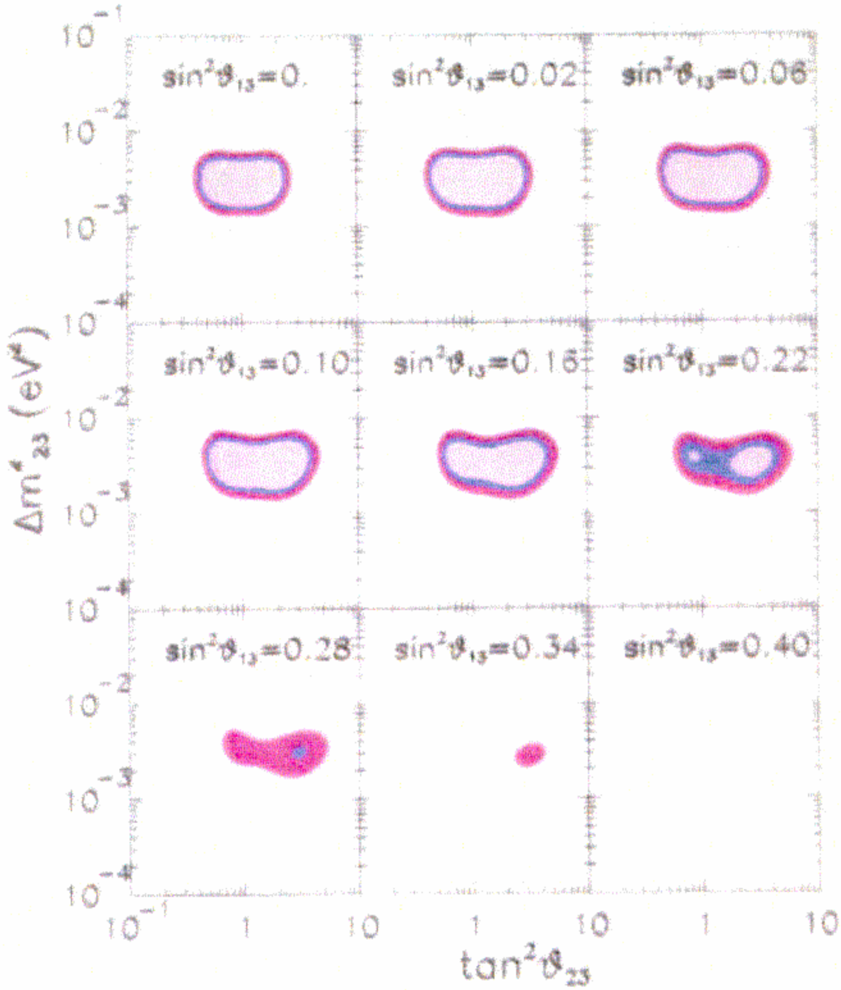
CHOOZ $\quad c_{13}^2 > 0.95$

$$\sin^2 2\theta_\oplus \simeq \sin^2 2\theta_{23}$$

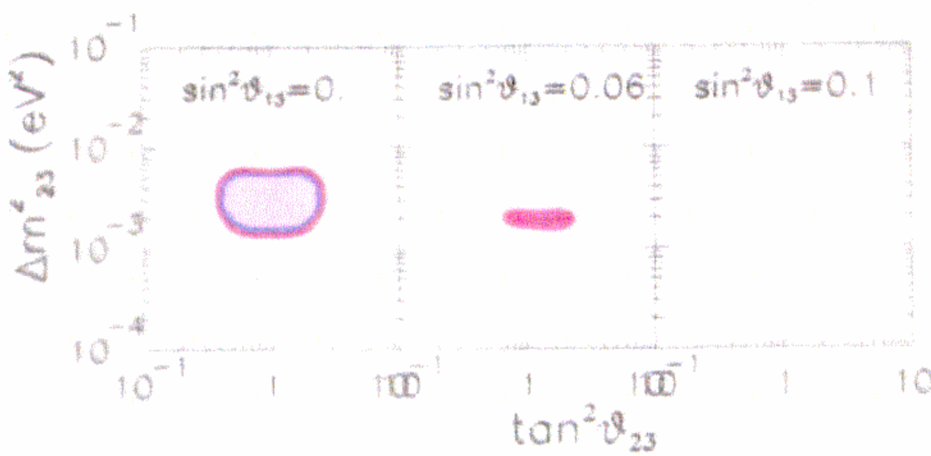
3–ν Atmospheric Neutrino Oscillation Parameters

M.C.G-G, M. Maltoni, C. Peña-Garay, J. Valle, in preparation

From All Atmospheric Neutrino Experiments



Including Also Chooz



similarly,

$$P(\nu_\mu \to \nu_\mu) = 1 - 4\left(|U_{\mu 1}|^2 + |U_{\mu 2}|^2\right)|U_{\mu 3}|^2 \sin^2 \frac{\Delta m^2_\oplus}{4E} L$$

$$= 1 - 4\left(1 - c_{13}^2 s_{23}^2\right) c_{13}^2 s_{23}^2 \sin^2 \frac{\Delta m^2_\oplus}{4E} L$$

atmospheric data

$$\sin^2 2\theta_\oplus \longrightarrow 4\left(1 - c_{13}^2 s_{23}^2\right) c_{13}^2 s_{23}^2$$

CHOOZ $c_{13}^2 > 0.95$

$$\sin^2 2\theta_\oplus \simeq \sin^2 2\theta_{23}$$

solar ν if $\theta_{13} \neq 0$?

$$|\nu_e\rangle = \underbrace{|\nu_1\rangle c_{12} c_{13} + |\nu_2\rangle s_{12} c_{13}} + \underbrace{|\nu_3\rangle s_{13} e^{-i\delta}}$$

interference $\propto \exp\left(-i \frac{\Delta m^2_\odot}{2E} L\right)$

oscillates very quickly $\Rightarrow 0$

two flavor:

$$P(\nu_e \to \nu_e) = f(\Delta m^2_\odot, \theta_\odot, N_e)$$

three flavor:

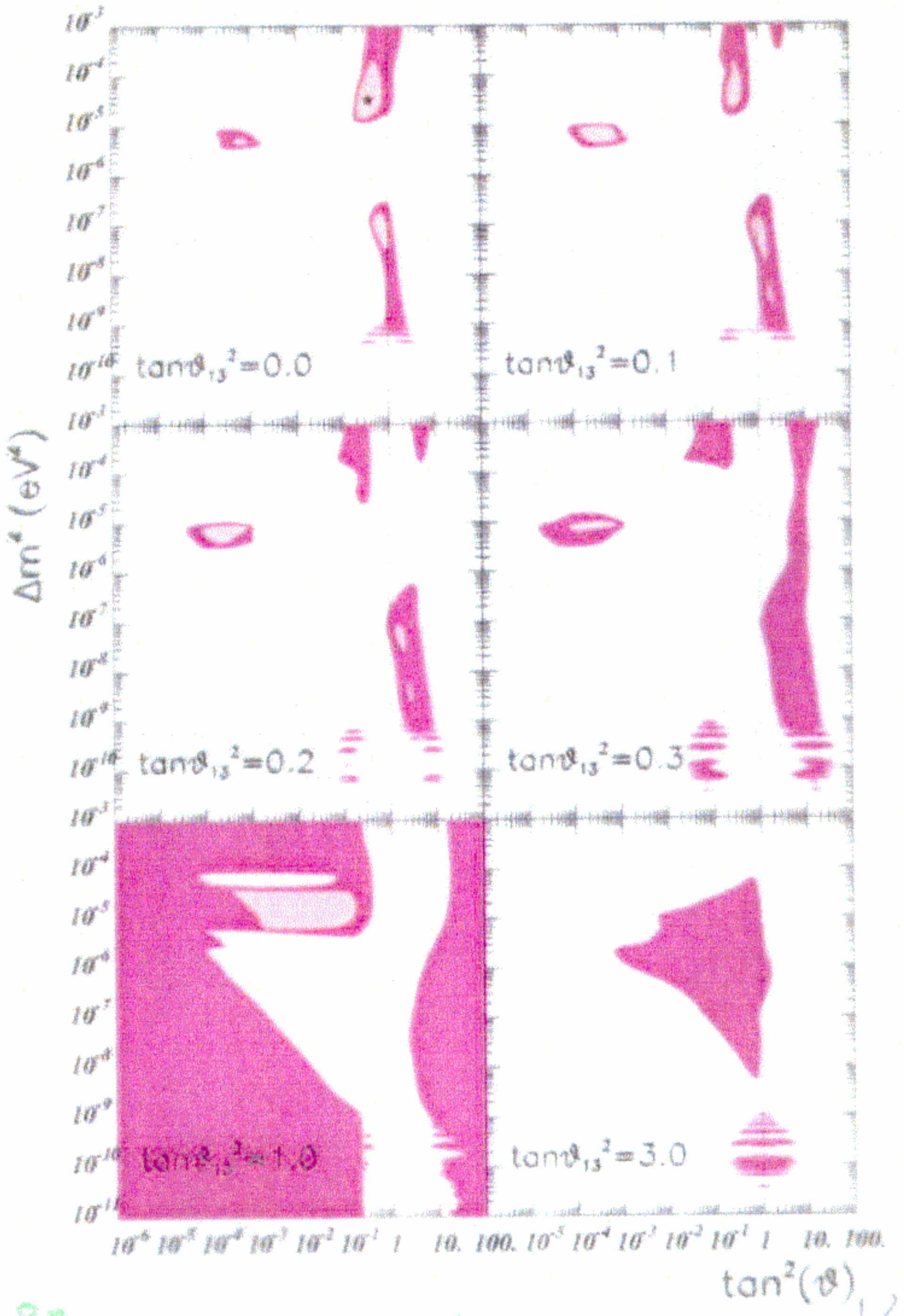
$$P(\nu_e \to \nu_e) = c_{13}^4 f(\Delta m^2_\odot, \theta_{12}, N_e c_{13}^2)$$

$$+ s_{13}^4$$

3–ν Solar Neutrino Oscillation Parameters

M.C.G-G, M. Maltoni, C. Peña-Garay, J. Valle, in preparation

Allowed regions from Global Analysis



$P_{ee}^{3\nu} = C_{13}^4 P_{ee}^{2\nu} + S_{13}^4$

You are about to enter the area whose contents are

prejudiced

driven by aesthetics

sociological

philosophical

subjective

speculative

political

AGREE

DO NOT AGREE

FOUR GENERATIONS

solar + atm + LSND

$$\Delta m^2_{\odot} \lesssim 10^{-4}\,\text{eV}^2$$

$$\Delta m^2_{\oplus} \sim 10^{-3}\text{–}10^{-2}\,\text{eV}^2$$

$$\Delta m^2_{LSND} \sim 0.1-1\,\text{eV}^2$$

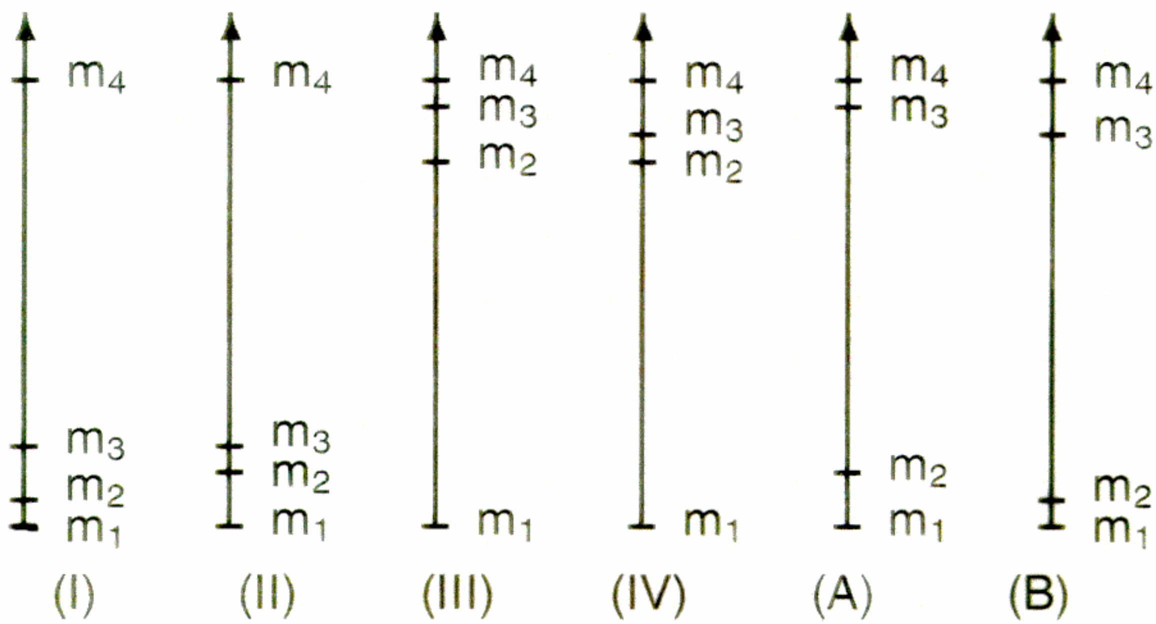
need <u>four</u> states to assign 3 Δm^2's!

Z invisible width $\Rightarrow N_\nu = 3$

one of them needs to be "sterile"

i.e. no coupling to Z

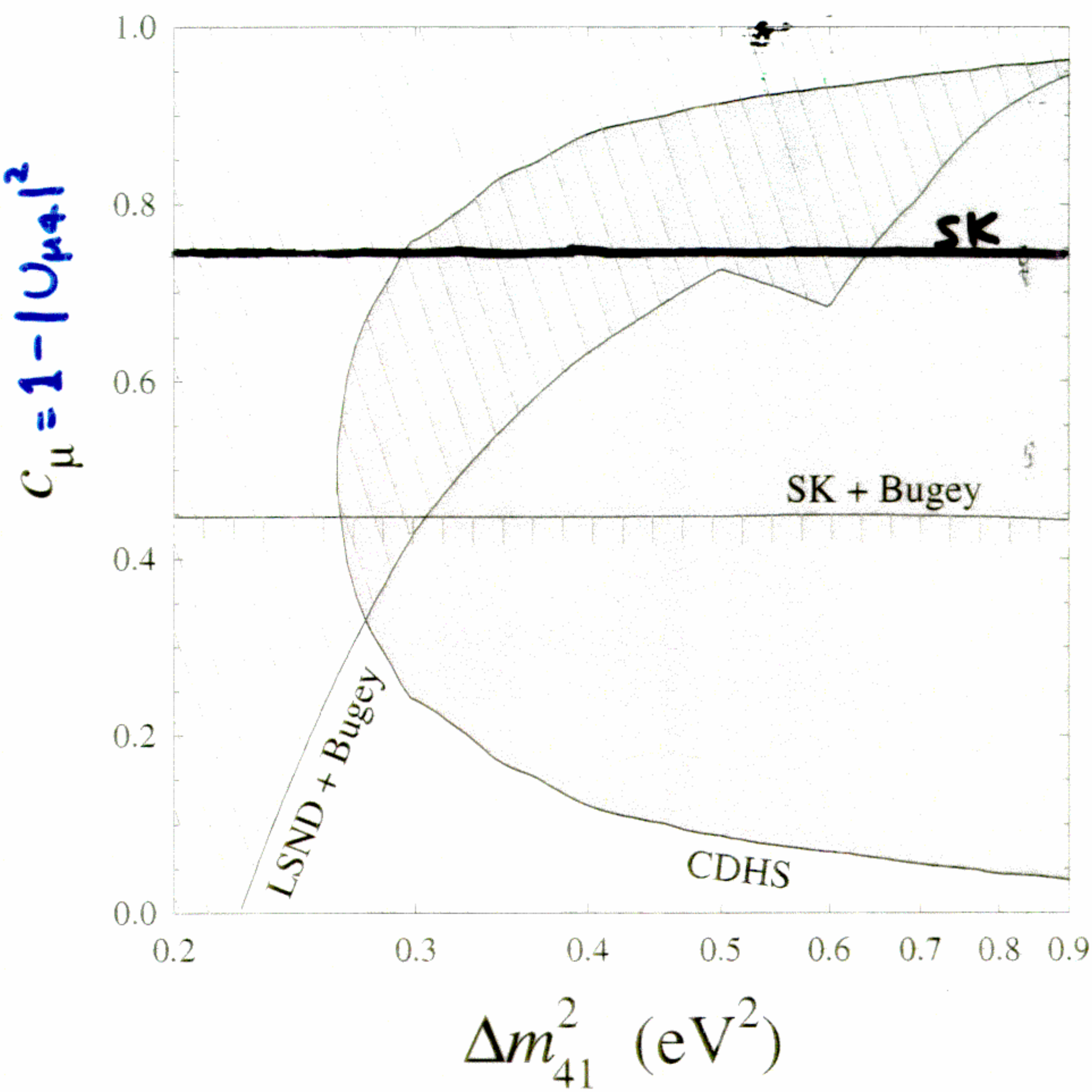
3 Δm^2's $\Rightarrow$ $3! = 6$ ways to order them



Bilenky, Giunti, Grimus, Schwetz

only (A) or (B) possible

Bilenky, Giunti, Grimus, Schwetz, Phys. Rev. D 60, 073007 (1999)



LSND: $\bar{\nu}_\mu \to \bar{\nu}_e$ take case (I)

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = 4\,|U_{e4}|^2\,|U_{\mu 4}|^2 \sin^2 \frac{\Delta m^2_{LSND}}{4E} L$$

neglecting $\Delta m^2_\odot, \Delta m^2_\oplus \ll \Delta m^2_{LSND}$

two-flavor fit

$$\sin^2 2\theta = 2\times10^{-3} - 3\times10^{-2}$$

$$\Rightarrow 4\,|U_{e4}|^2\,|U_{\mu 4}|^2 = 2\times10^{-3} - 3\times10^{-2}$$

Bugey reactor ν expt

no $\bar{\nu}_e$ disappearance @ Δm^2_{LSND}

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 4|U_{e4}|^2(1-|U_{e4}|^2)\sin^2\frac{\Delta m^2_{LSND}}{4E}$$

$$4|U_{e4}|^2(1-|U_{e4}|^2) \lesssim 0.03$$

either
$|U_{e4}|^2 \gtrsim 0.992$
$|U_{e4}|^2 \lesssim 0.008$

$\Delta m^2_\odot = \Delta m^2_{LSND}$

$\Leftarrow$ only choice

$|U_{\mu 4}|^2 \gtrsim 0.06 - 1$

atmospheric up/down

down: only Δm^2_{LSND} because $\frac{\Delta m^2_{\odot}}{4E} L \ll 1$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4|U_{\mu 4}|^2(1-|U_{\mu 4}|^2)\sin^2\frac{\Delta m^2_{LSND}}{4E}L$$

$$\simeq 1 - 2|U_{\mu 4}|^2(1-|U_{\mu 4}|^2)$$

up: $\frac{\Delta m^2_{\odot}}{4E} L \gg 1$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4(|U_{\mu 1}|^2 + |U_{\mu 2}|^2)|U_{\mu 3}|^2 \sin^2\frac{\Delta m^2_{\odot}}{4E}L$$

$$- 4|U_{\mu 4}|^2(1-|U_{\mu 4}|^2)\sin^2\frac{\Delta m^2_{LSND}}{4E}L$$

$$\simeq 1 - 2|U_{\mu 4}|^2(1-|U_{\mu 4}|^2)$$

$$- 2|U_{\mu 3}|^2(1-|U_{\mu 4}|^2-|U_{\mu 3}|^2)$$

take $\frac{\text{up}}{\text{down}} = \frac{1-2y(1-y)-2x(1-x-y)}{1-2y(1-y)}$

$x = |U_{\mu 3}|^2 \quad y = |U_{\mu 4}|^2$

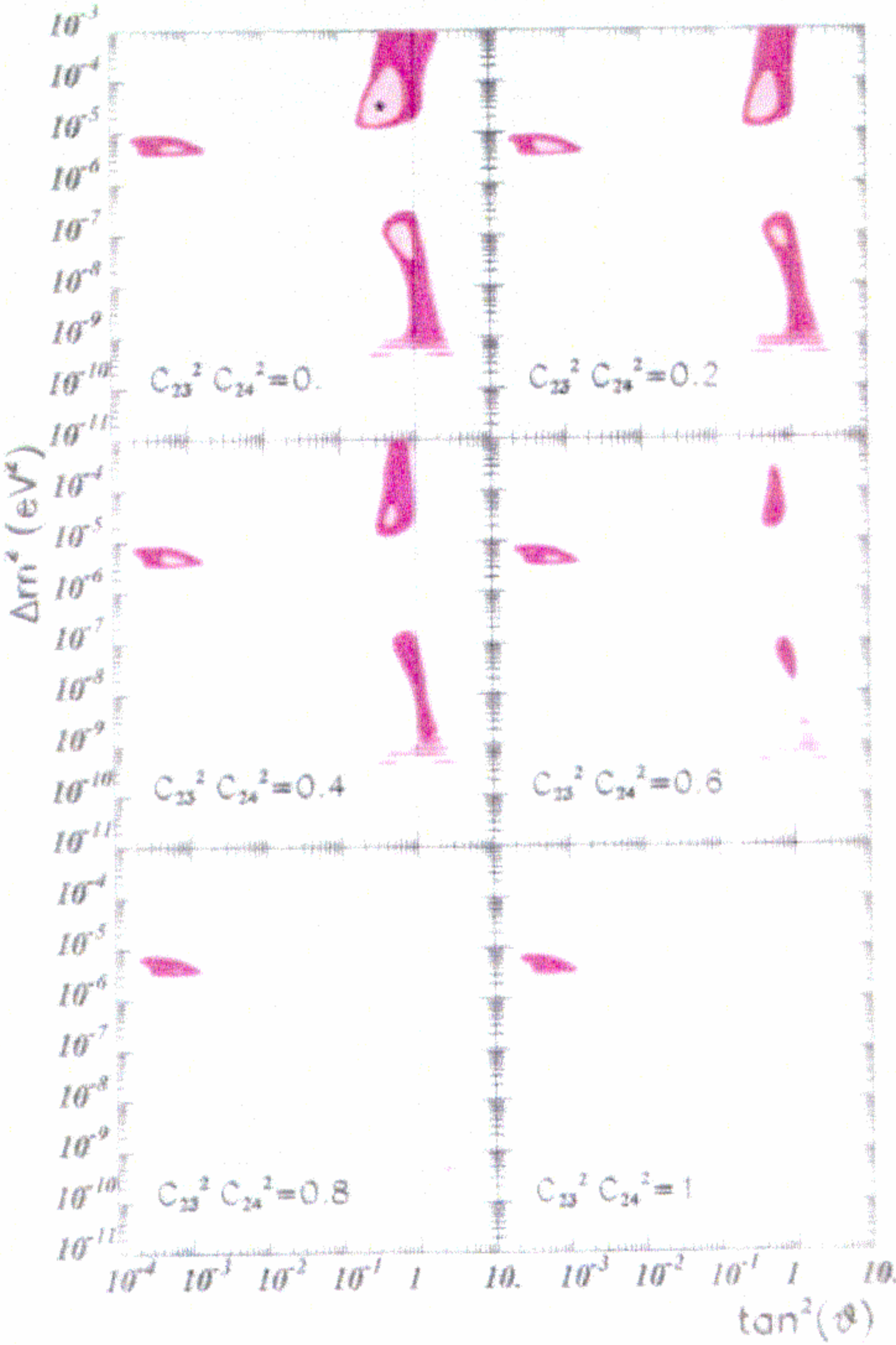
minimize $\frac{\text{up}}{\text{down}}$ w.r.t. $x \Rightarrow x = \frac{1}{2}(1-y)$

$$\left(\frac{\text{up}}{\text{down}}\right)_{min} = \frac{1}{2}\left[1 + \frac{y^2}{(1-y)^2+y^2}\right] \leftrightarrow 1 - \frac{1}{2}\sin^2 2\theta$$

$\sin^2 2\theta < 0.88 \Rightarrow y = |U_{\mu 4}|^2 < 0.27$

Solutions for Four-neutrino Oscillations

Update of C Giunti, MCG-G, C Peña-Garay PRD62 (2000)



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Fogli, E.L., Marrone : PRELIMINARY RESULTS
USING 61 KTy SK data (55 bins)

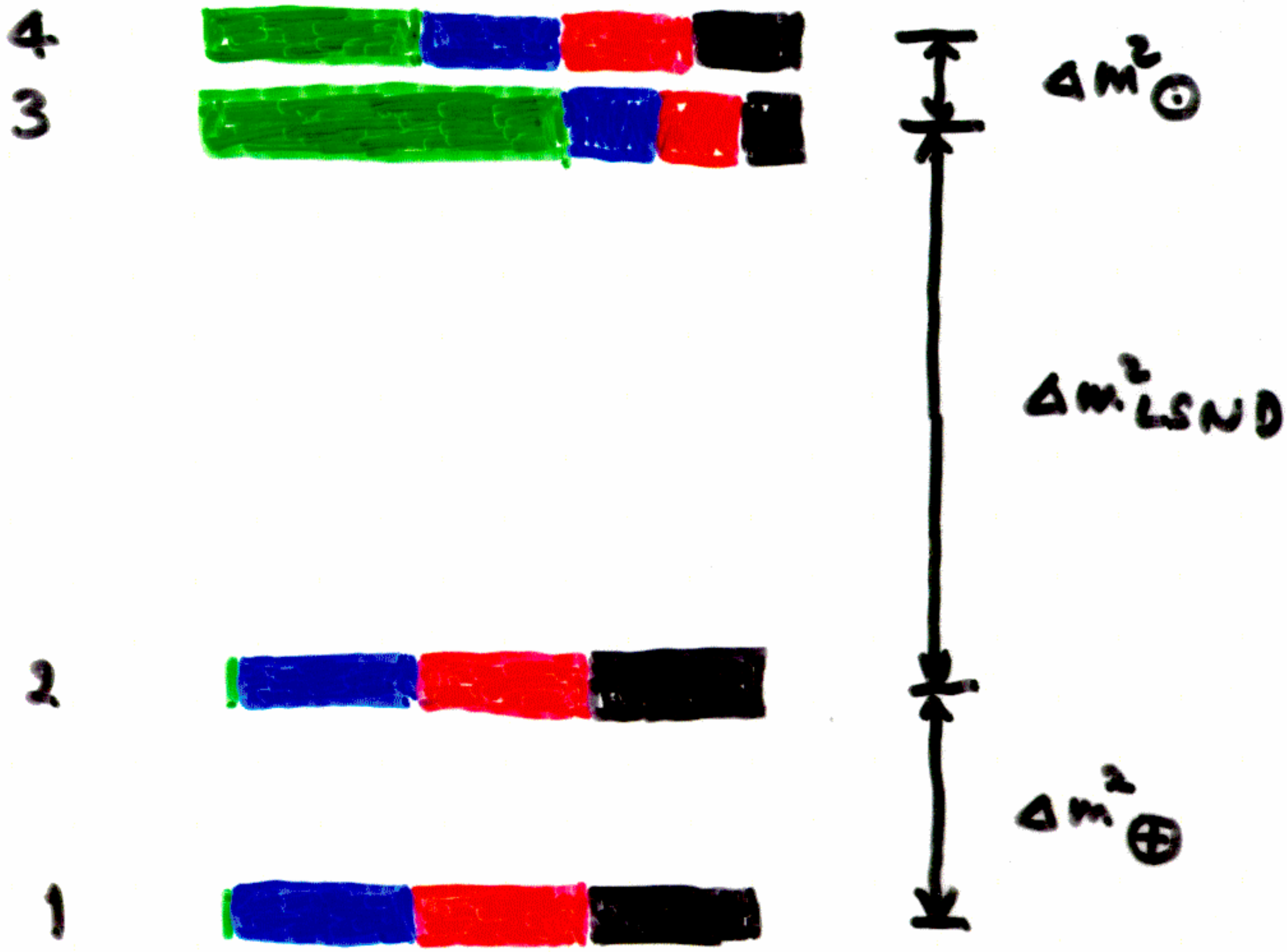
- Best fit at $\sim 3 \times 10^{-3}\,eV^2$, close to left side ($\sim \nu_\mu \to \nu_\tau$)
- Pure $\nu_\mu \to \nu_s$ (right side) disfavored @ 99% C.L.
- However, large $\nu_\mu \to \nu_s$ oscillations, in addition to $\nu_\mu \to \nu_\tau$, cannot be excluded (e.g., $\nu_\mu \to \frac{1}{\sqrt{2}}(\nu_s + \nu_\tau)$)
- At "high" m^2, nonzero $\nu_\mu \to \nu_s$ favored (reduces large μ suppression)
- Constraints increase with energy and s^2_ξ, due to effective mass term in matter

$$A_{eff} = 2\sqrt{2}\, G_F \frac{N_n}{2} \cdot E \cdot s^2_\xi$$

due to effective mass term in matter

types (II), (III), (IV) excluded, too

(A) LMA



If SuperK spectrum still allows SMA

(A) SMA