Theories of Neutrino Masses and Mixings

Standard Model Neutrinos

Vacuum Oscillation

Matter Oscillation

Three Generations solar + atm

Four Generations solar + atm + LSND

Dirac vs Majorana

Models

Future

Standard Model Neutrinos

H. Murayama, SSI 2000

C. D. Ellis
W. A. Wooster
(1927)

$E_e < \Delta m c^2$

energy not conserved in nuclei?? (Niels Bohr)

$(A,Z) \rightarrow (A,Z+1) + e^- (+ \bar{\nu}_e)$

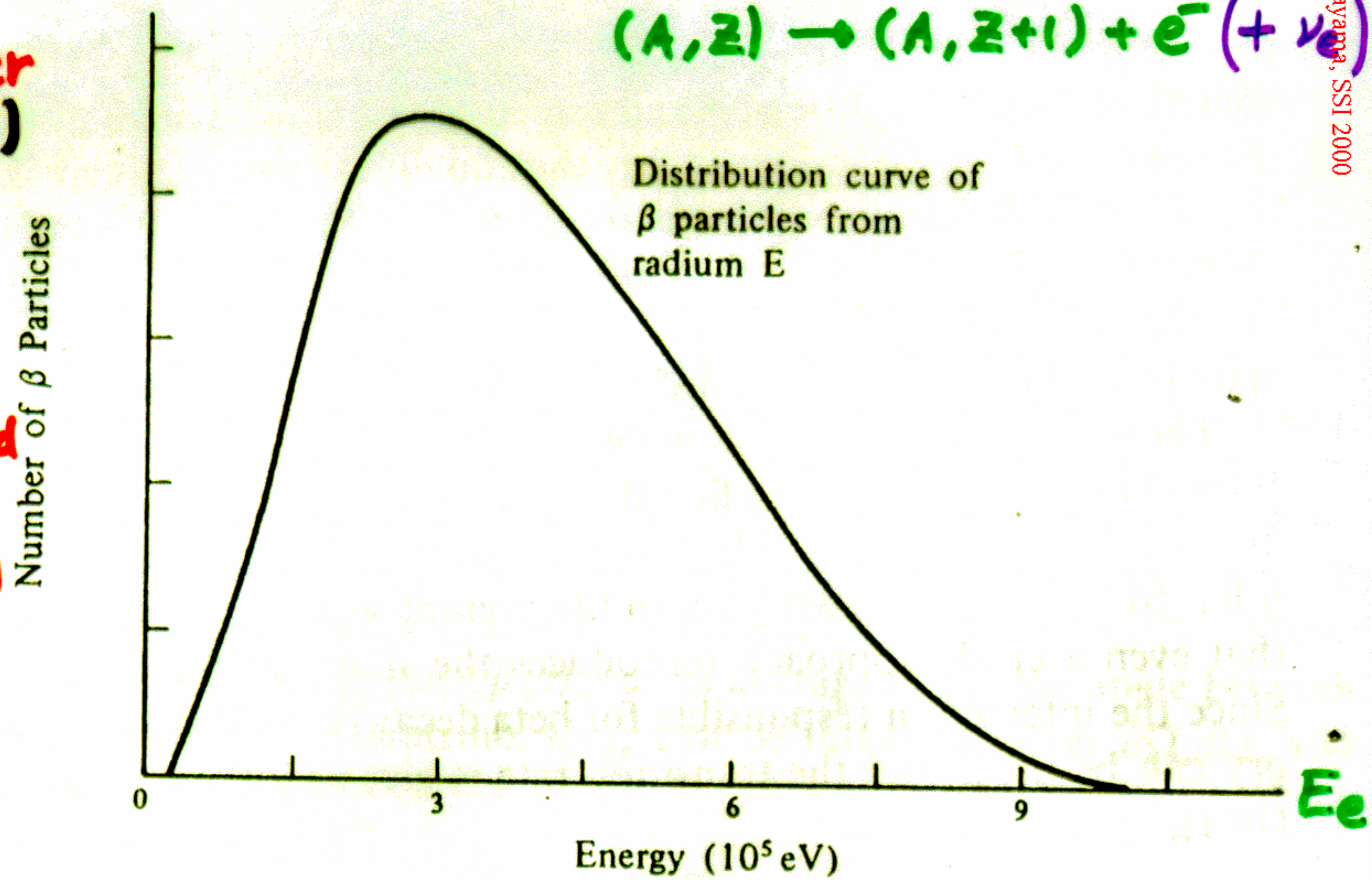


Fig. 11.1. Example of a beta spectrum. [This figure is taken from one of the classic papers: C. D. Ellis and W. A. Wooster, *Proc. Roy. Soc.* (London) A117, 109 (1927).] Present experimental techniques yield more accurate energy spectra, but all essential aspects are already contained in the curve reprinted here.

have found quantized states. The existence of quantized levels was well known in 1920, and the first puzzle posed by the continuous beta spectrum thus was: Why is the spectrum of electrons continuous and not discrete?

At the present stage of atomic theory, however, we may say that we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β-ray disintegrations, and are even led to complications and difficulties in trying to do so. Of course, a radical departure from this principle would imply strange consequences, in case such a process could be reversed. Indeed, if, in a collision process, an electron could attach itself to a nucleus with loss of its mechanical individuality, and subsequently be recreated as a β-ray, we should find that the energy of this β-ray would generally differ from that of the original electron. Still, just as the account of those aspects of atomic constitution essential for the explanation of the ordinary physical and chemical properties of matter implies a renunciation of the classical idea of causality, the features of atomic stability, still deeper-lying, responsible for the existence and the properties of atomic nuclei, may force us to renounce the very idea of energy balance. I shall not enter further into such speculations and their possible bearing on the much debated question of the source of stellar energy. I have touched upon them here mainly to emphasize that in atomic theory, notwithstanding all the recent progress, we must still be prepared for new surprises.

Niels Bohr

ncerning the more general possibility of surprises in those interactions ve today call "weak," Bohr should maintain his point in another respect. ever, his idea that there was only a statistical conservation of energy in interactions seemed unacceptable to both Fermi and me. We had many

Public letter to the group of the Radioactives at the district society meeting in Tübingen:

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dec. 1930
Gloriastr.

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the "exchange theorem"[3] of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin $\frac{1}{2}$ and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. – The continuous β-spectrum would then become understandable by the assumption that in β-decay, a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant. Now the question that has to be dealt with is which forces act on the neutrons? The most likely model for the neutron seems to me, because of wave mechanical reasons (the details are known by the bearer of these lines), that the neutron at rest is a magnetic dipole of a certain moment μ. The experiments seem to require that the effect of the ionization of such a neutron cannot be larger than that of a γ-ray and then μ should not be larger than $e * 10^{-13}$ cm.

For the moment, however, I do not dare to publish anything on this idea and I put to you, dear Radioactives, the question of what the situation would be if one such neutron were detected experimentally, if it

Helicity of Neutrinos*

M. Goldhaber, L. Grodzins, and A. W. Sunyar
Brookhaven National Laboratory, Upton, New York
(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] 0−, we find that the neutrino is "left-handed," i.e., $\boldsymbol{\sigma}_\nu \cdot \hat{\mathbf{p}}_\nu = -1$ (negative helicity).

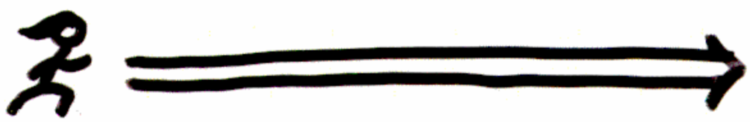
Our method may be illustrated by the following simple example: take a nucleus A (spin $I=0$) which decays by allowed orbital electron capture, to an excited state of a nucleus $B(I=1)$, from which a γ ray is emitted to the ground state of $B(I=0)$. The conditions necessary for resonant scattering are best fulfilled for those γ rays which are emitted opposite to the neutrino, which have an energy comparable to that of the neutrino, and which are emitted before the recoil energy is lost. Since the orbital electrons captured by a nucleus are almost entirely s electrons (K, L_{I}, $\cdots$ electrons of spin $S=\frac{1}{2}$), the substates of the daughter nucleus

All neutrinos left-handed

$\Rightarrow$ massless

ν_L  $\frac{\vec{s}\cdot\vec{p}}{|\vec{p}|} = -\frac{1}{2}$

if $m_\nu \neq 0$, $v < c$

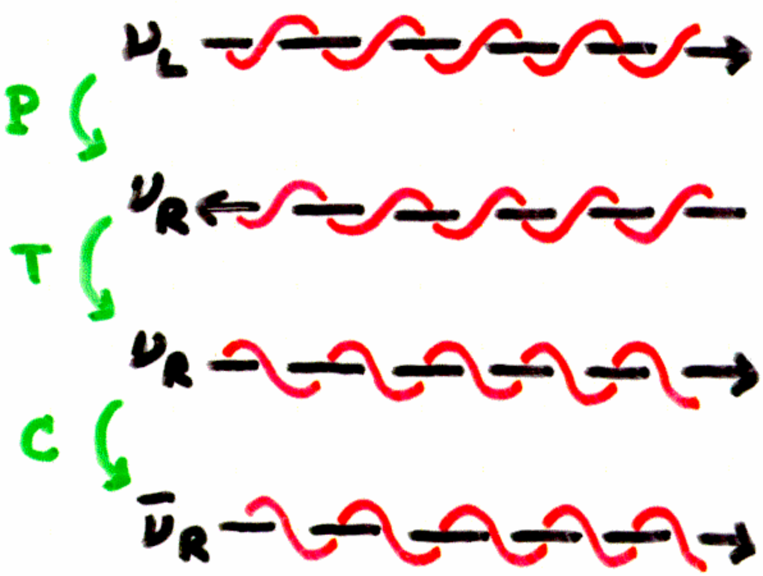
 you can pass it and look back

"ν_R"  $\frac{\vec{s}\cdot\vec{p}}{|\vec{p}|} = +\frac{1}{2}$??

contradiction

All anti-neutrinos right-handed

CPT theorem



ν_L: exists

ν_R: doesn't exist

ν_R: doesn't exist

$\bar{\nu}_R$: exists

three kinds of neutrinos

for three generations of particles

quarks

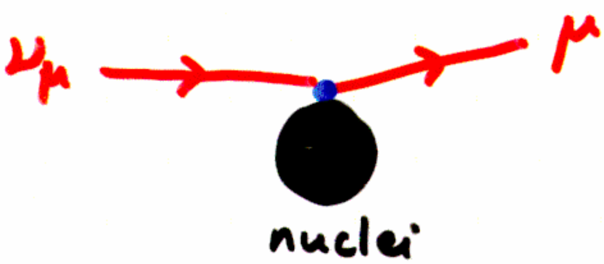
$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{matrix} u_R \\ d_R \end{matrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{matrix} c_R \\ s_R \end{matrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} \begin{matrix} t_R \\ b_R \end{matrix}$$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{matrix} u_R \\ d_R \end{matrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{matrix} c_R \\ s_R \end{matrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} \begin{matrix} t_R \\ b_R \end{matrix}$$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{matrix} u_R \\ d_R \end{matrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{matrix} c_R \\ s_R \end{matrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} \begin{matrix} t_R \\ b_R \end{matrix}$$

leptons

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} e_R \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \mu_R \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \tau_R$$

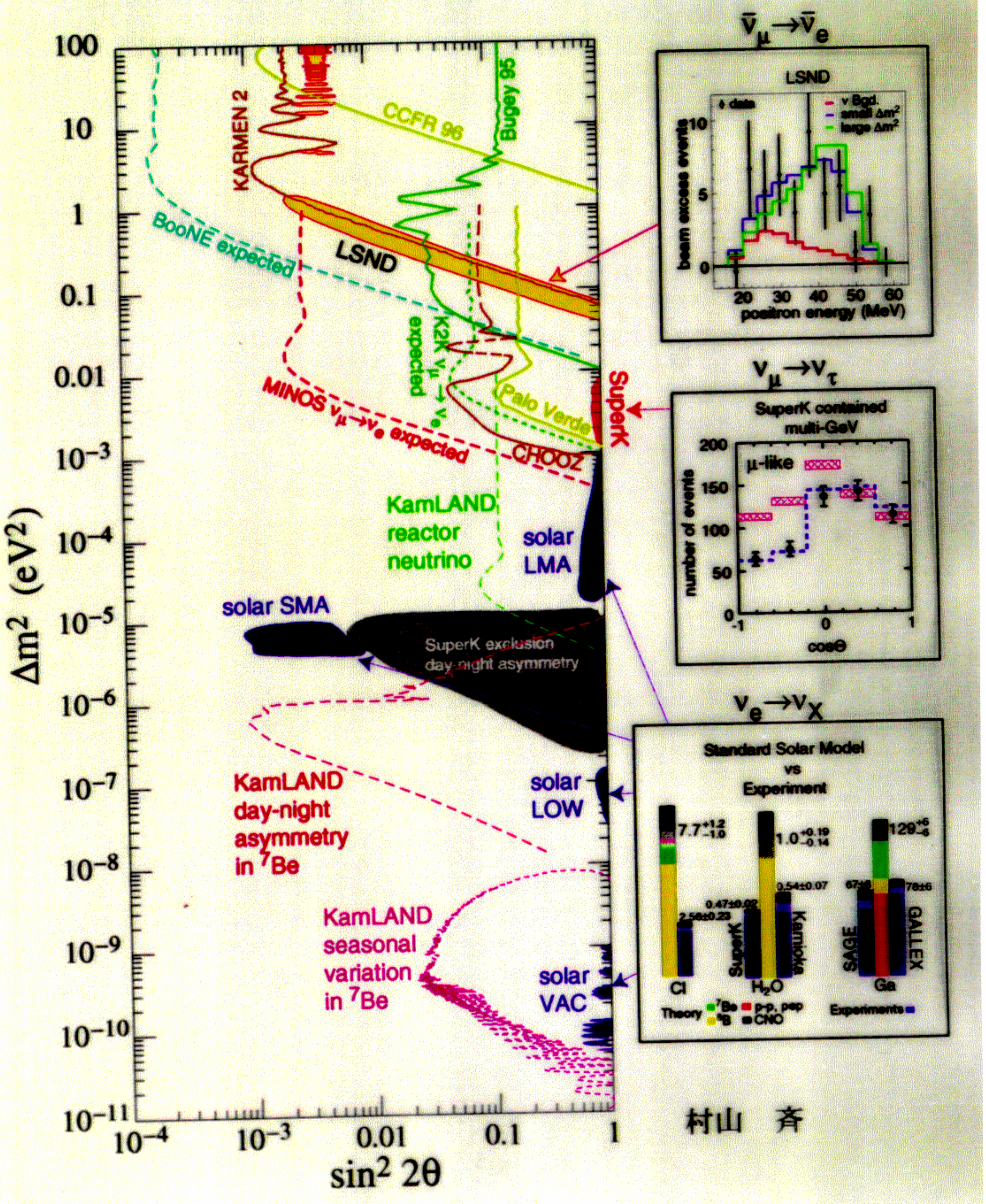


← not e, τ

all three neutrinos left-handed

if $m_\nu \neq 0$

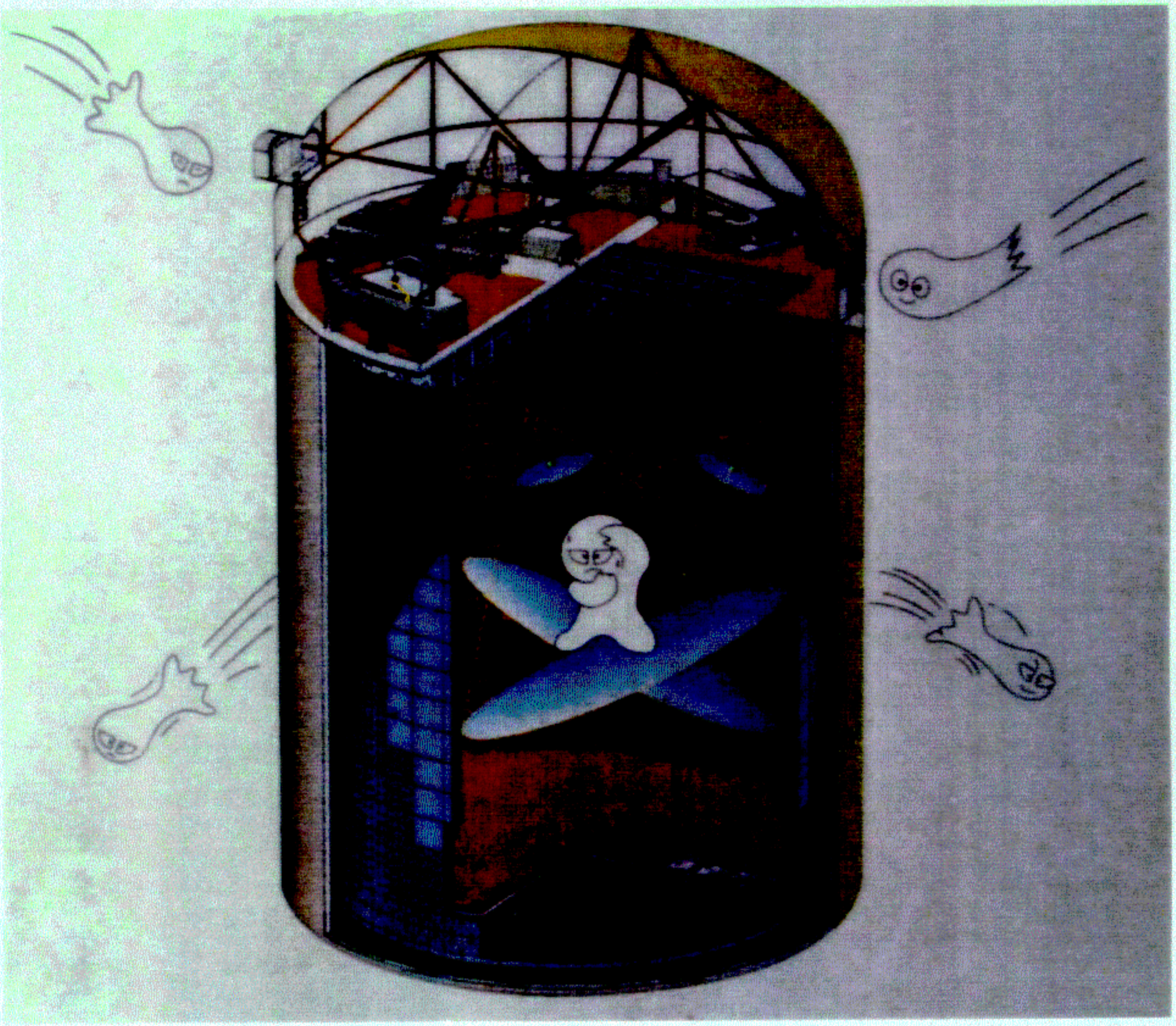
$\Rightarrow$ Standard Model incomplete!



Δm^2 (eV^2)
$\sin^2 2\theta$
KARMEN 2
CCFR 96
Bugey 95
BooNE expected
LSND
K2K $\nu_\mu \to \nu_e$ expected
MINOS $\nu_\mu \to \nu_e$ expected
Palo Verde
CHOOZ
SuperK
KamLAND reactor neutrino
solar LMA
solar SMA
SuperK exclusion day-night asymmetry
KamLAND day-night asymmetry in ^{7}Be
solar LOW
KamLAND seasonal variation in ^{7}Be
solar VAC
$\bar{\nu}_\mu \to \bar{\nu}_e$
LSND
beam excess events
positron energy (MeV)
$\nu_\mu \to \nu_\tau$
SuperK contained multi-GeV
μ-like
number of events
cosΘ
$\nu_e \to \nu_X$
Standard Solar Model vs Experiment
Cl
H_2O
Ga
SuperK
Kamioka
SAGE
GALLEX
Theory
Experiments
村山 斉

VACUUM OSCILLATION

Super-Kamiokande



- Inner Detector(ID): 11,200 Photomult. Tubes, 32,000 metric tons
- Outer Detector(OD): 2,000 Photomult. Tubes, 18,000 metric tons

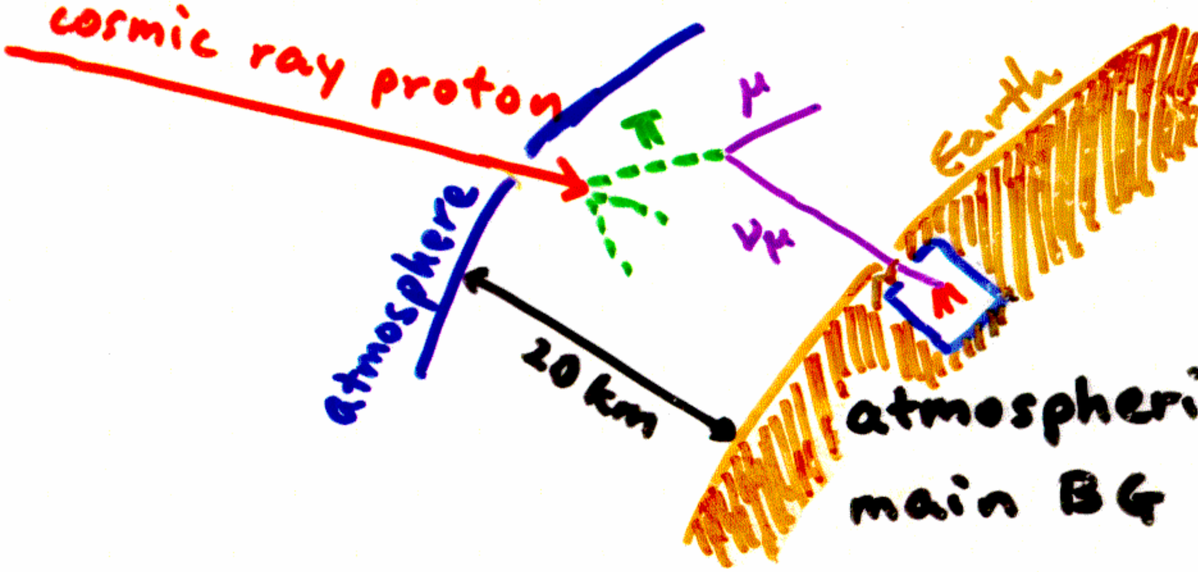
Michael B. Smy, UC Irvine

Super Kamiokande

nucleon decay experiment

$p \to e^+ \pi^0,\ K^+ \bar{\nu}$ etc

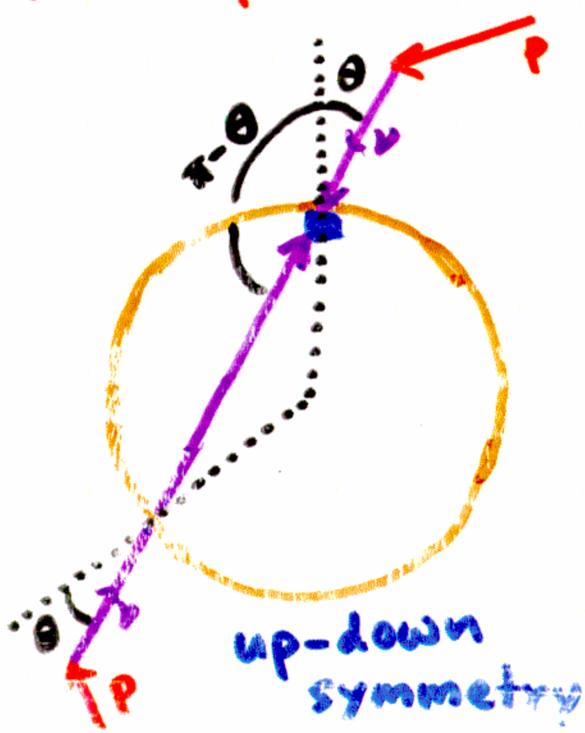
so far not seen



atmospheric ν

main BG

① cosmic rays are isotropic



② ν_μ/ν_e ratio well predicted

esp. $E_\nu \lesssim 2$ GeV

$\pi \to \mu\ \nu_\mu$

$\mu \to e\ \nu_e\ \nu_\mu$

$$\frac{\#\nu_\mu}{\#\nu_e} \approx 2$$

Mann Lepton Photon '99

A Decade-plus of Atm. Flavor Ratios - the Latest:
(Preliminary)

SuperKamiokande:
Water Cherenkov
52 kton - yrs.

$$R\left({}^{\mu}/_{e}\right)_{\text{single-ring}} \begin{cases} = 0.68 \pm 0.02 \pm 0.05 & \text{(Sub-GeV)} \\ = 0.68 \pm 0.04 \pm 0.08 & \left(\text{Multi-GeV, FC + PC}\right) \end{cases}$$

⇕

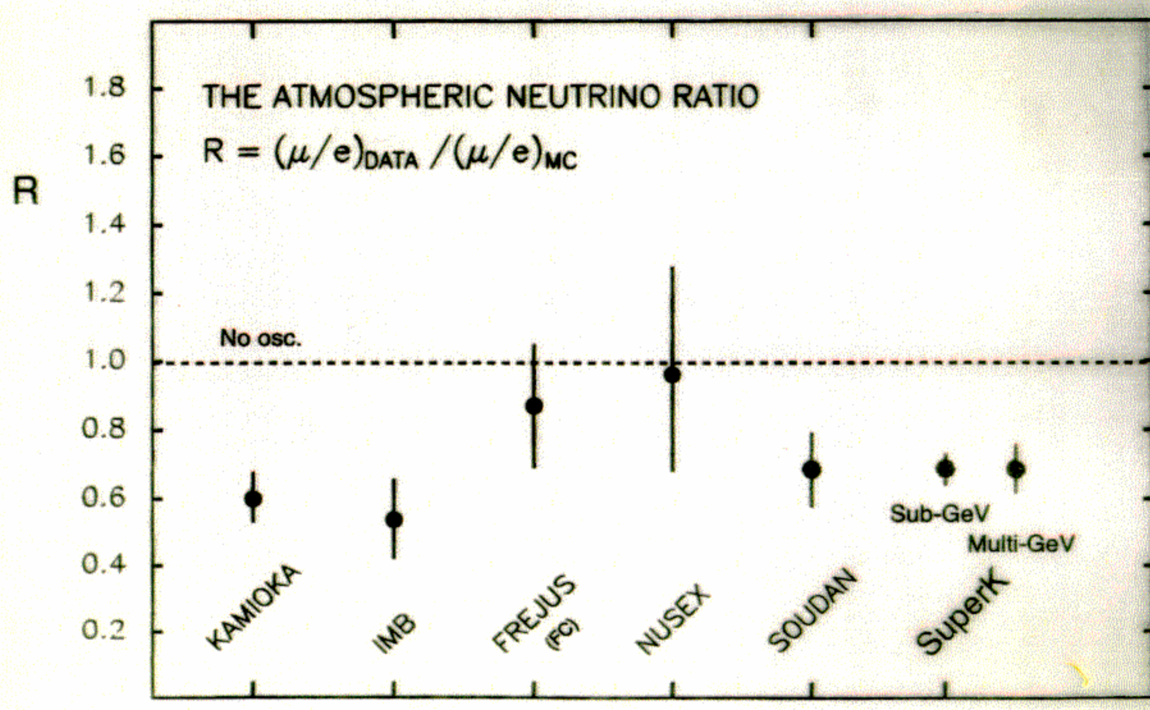
Systematics - limited

Soudan 2:
Iron Tracking Calorimeter
4.6 kton - yrs.

$$R\left({}^{\mu}/_{e}\right)_{\text{single-trk, shwr}} = 0.68 \pm 0.11 \pm 0.06 \quad \left(\text{Mostly Sub-GeV}\right)$$

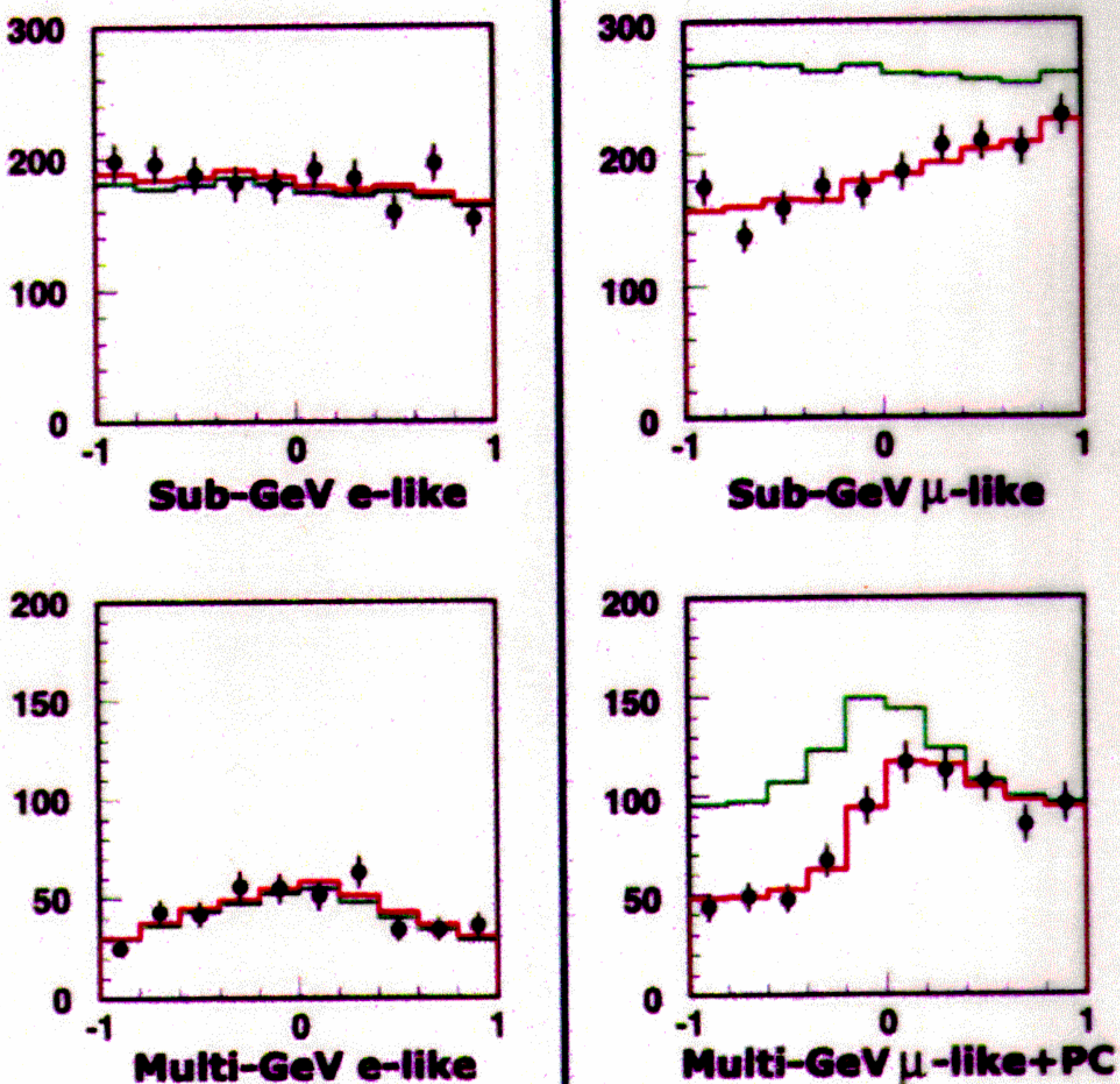
⇕

Statistics - limited



Zenith Angle - 10 bins:

ν_μ depletion at all angles including above-horizon



ν_e - distributions "well-behaved" No overt hints for sub-dominant $\nu_\mu \leftrightarrow \nu_e$.

With increasing E_ν, ν_μ-depletion moves to (mostly) below-horizon.

ν_μ's disappeared (~50%)

<u>neutrino oscillation</u>: best explanation

$$H = \sqrt{\vec{p}^2 c^2 + m^2 c^4} = |\vec{p}|c + \frac{1}{2}\frac{m^2 c^4}{|\vec{p}|c} + \cdots$$

m^2: in general 3×3 matrix

take only 2×2 for simplicity ($c = \hbar = 1$)

$$|\nu_\tau\rangle = -|\nu_1\rangle \sin\theta \quad + |\nu_2\rangle \cos\theta$$

$$|\nu_\mu\rangle = |\nu_1\rangle \cos\theta \quad + |\nu_2\rangle \sin\theta$$

t $\quad e^{-i\frac{m_2^2}{p}t} \quad e^{-i\frac{m_2^2}{p}t}$

t $\quad e^{-i\frac{m_1^2}{p}t} \quad e^{-i\frac{m_2^2}{p}t}$

survival probability

$$P(\nu_\mu \to \nu_\mu) = |\langle \nu_\mu | \nu_\mu \rangle_t|^2$$

$$= 1 - \sin^2 2\theta \sin^2\left(1.27 \frac{\Delta m^2}{p} L\right)$$

$\Delta m^2 = m_2^2 - m_1^2$ in eV^2

p in GeV/c

L in km

rapid oscillation

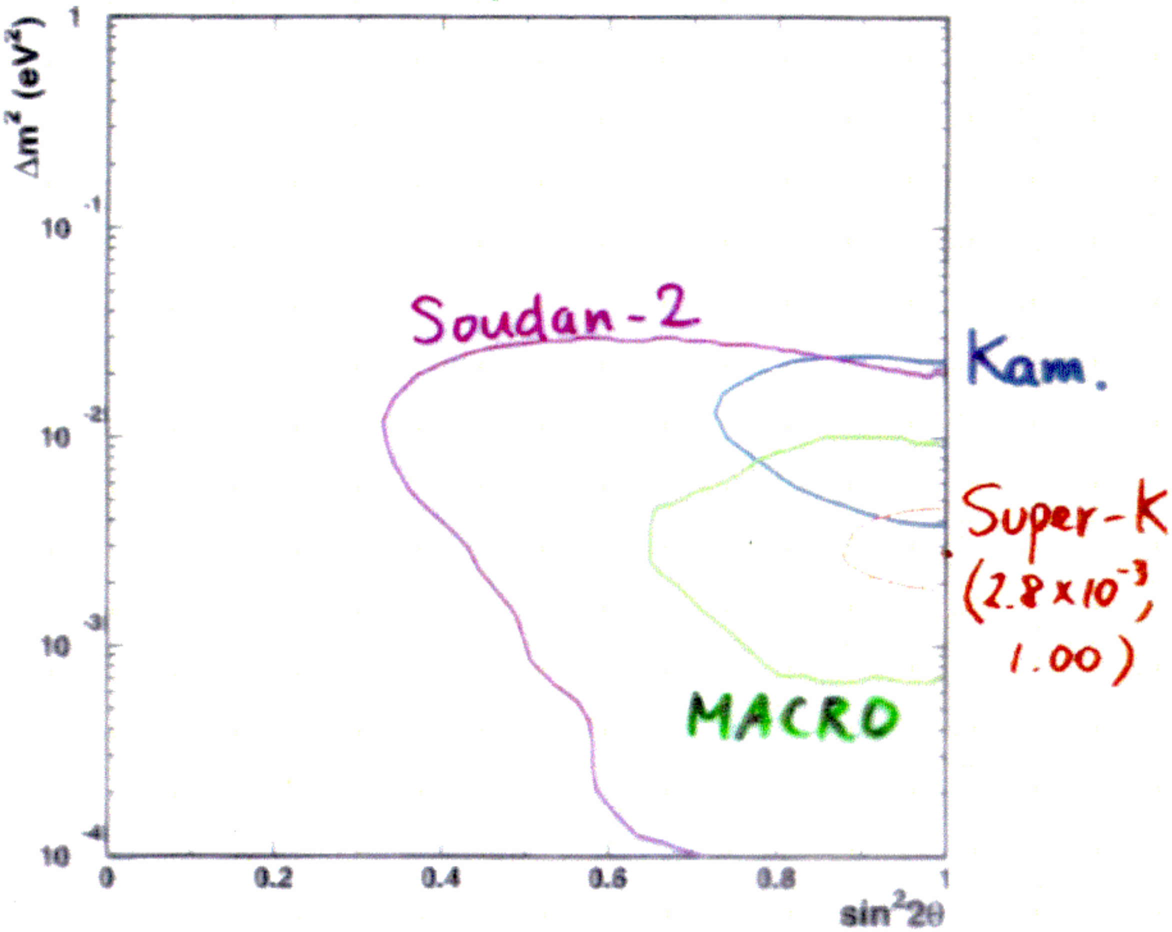
$$\simeq 1 - \sin^2 2\theta \times \frac{1}{2}$$

Kajita PASCOS '99

Allowed region from each exp.

(Contained events + up-going μ's)

$\nu_\mu \to \nu_\tau$ 90% C.L.



$$\begin{cases} \Delta m^2 \sim (2 \sim 5) \times 10^{-3} eV^2 \\ \sin^2 2\theta \geq 0.88 \end{cases}$$

For 3 generations,

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

↑ Maki-Nakagawa-Sakata (MNS) matrix

t=0

$$|\nu_\mu\rangle = |\nu_1\rangle U_{\mu 1} + |\nu_2\rangle U_{\mu 2} + |\nu_3\rangle U_{\mu 3}$$

1

t $\quad e^{-i\frac{m_1^2}{2E}t}$ $\quad e^{-i\frac{m_2^2}{2E}t}$ $\quad e^{-i\frac{m_3^2}{2E}t}$

$$\langle \nu_\tau | = U_{\tau 1}^* \langle \nu_1 | + U_{\tau 2}^* \langle \nu_2 | + U_{\tau 3}^* \langle \nu_3 |$$

$$\langle \nu_\tau | \nu_\mu \rangle_t = U_{\mu 1} U_{\tau 1}^* e^{-i\frac{m_1^2}{2E}t} + U_{\mu 2} U_{\tau 2}^* e^{-i\frac{m_2^2}{2E}t} + U_{\mu 3} U_{\tau 3}^* e^{-i\frac{m_3^2}{2E}t}$$

$$P(\nu_\mu \to \nu_\tau) = |\langle \nu_\tau | \nu_\mu \rangle|^2 = \sum_{\alpha,\beta} U_{\mu\alpha} U_{\tau\alpha}^* U_{\mu\beta}^* U_{\tau\beta}\, e^{-i\frac{m_\alpha^2 - m_\beta^2}{2E}t}$$

In general, in "appearance" channels, $(\ell \neq \ell'$

$$P(\nu_\ell \to \nu_{\ell'}) = \sum_{\alpha,\beta} U_{\ell\alpha} U^*_{\ell'\alpha} U^*_{\ell\beta} U_{\ell'\alpha}\, e^{-i\frac{m_\alpha^2 - m_\beta^2}{2E}t}$$

$$= \sum_{\alpha,\beta} \left\{ -2\,\mathrm{Re}\left(U_{\ell\alpha} U^*_{\ell'\alpha} U^*_{\ell\beta} U_{\ell'\beta}\right) \sin^2 \frac{m_\alpha^2 - m_\beta^2}{4E} t \right\}$$

$$+ \sum_{\alpha,\beta} \mathrm{Im}\left(U_{\ell\alpha} U^*_{\ell'\alpha} U^*_{\ell\beta} U_{\ell'\beta}\right) \sin \frac{m_\alpha^2 - m_\beta^2}{2E} t$$

$-$ $-$

$-$

$U \to U^*$ ↑ CP-violating term

in disappearance channel,

$$P(\nu_\ell \to \nu_\ell) = 1 - 2 \sum_{\alpha,\beta} |U_{\ell\alpha}|^2 |U_{\ell\beta}|^2 \sin^2 \frac{m_\alpha^2 - m_\beta^2}{4E} t$$

$$= P(\bar{\nu}_\ell \to \bar{\nu}_\ell) \qquad \text{CPT}$$

note:

$$P(\nu_\ell \to \nu_{\ell'}) = P(\bar{\nu}_{\ell'} \to \bar{\nu}_\ell) \qquad \text{CPT}$$

$$\sum_{\ell'} P(\nu_\ell \to \nu_{\ell'}) = 1 \qquad \text{unitarity}$$

$0 \leq \sin^2 2\theta \leq 1 \quad \Rightarrow \quad 0 \leq \theta \leq \frac{\pi}{4}$

the light side only

what parameter to use?

if we want to preserve the reflection symmetry for vac. osc.

$\theta \leftrightarrow \frac{\pi}{2} - \theta$

$\sin^2\theta \longleftrightarrow 1 - \sin^2\theta$

good on the linear scale

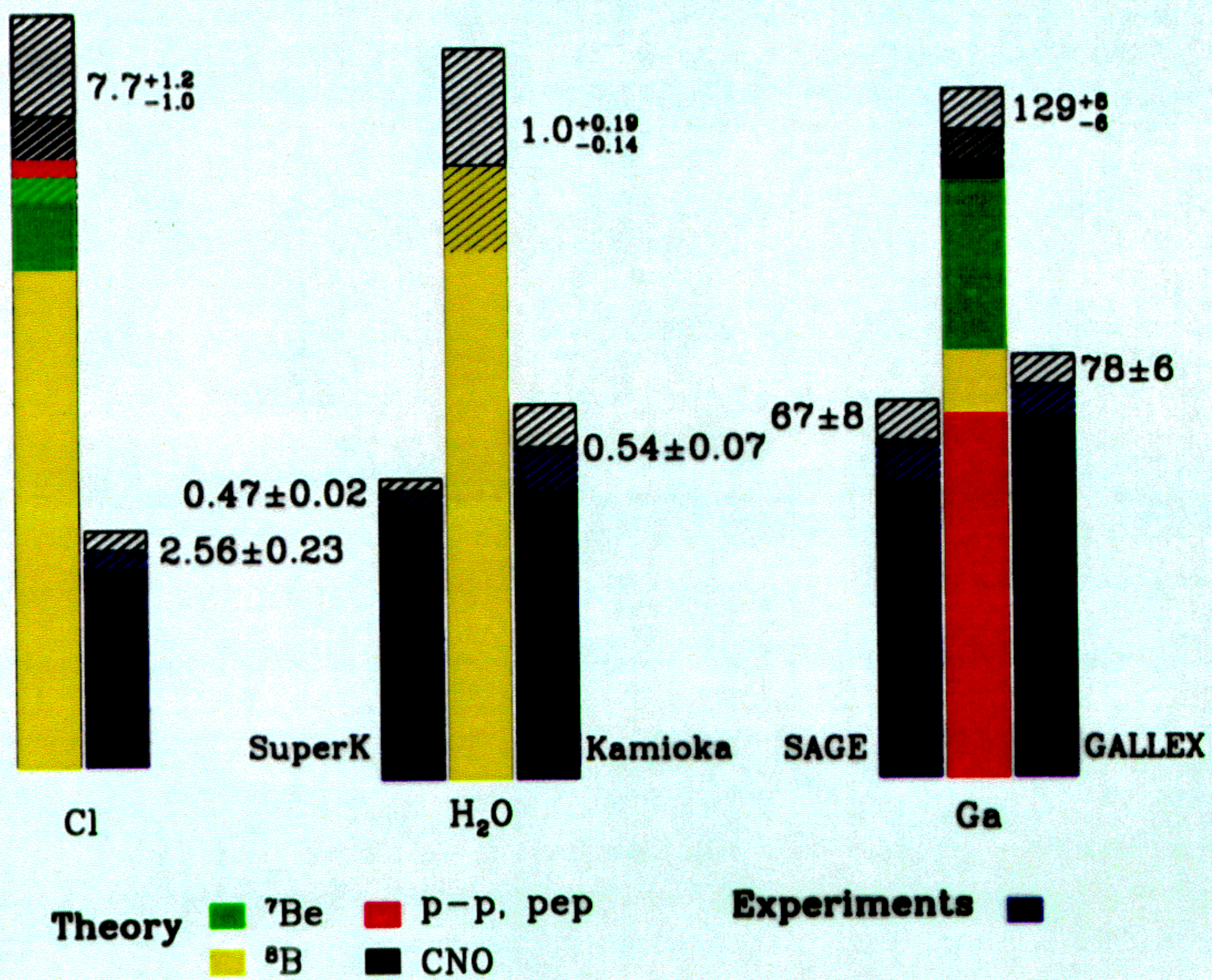
$\tan^2\theta \longleftrightarrow 1/\tan^2\theta$

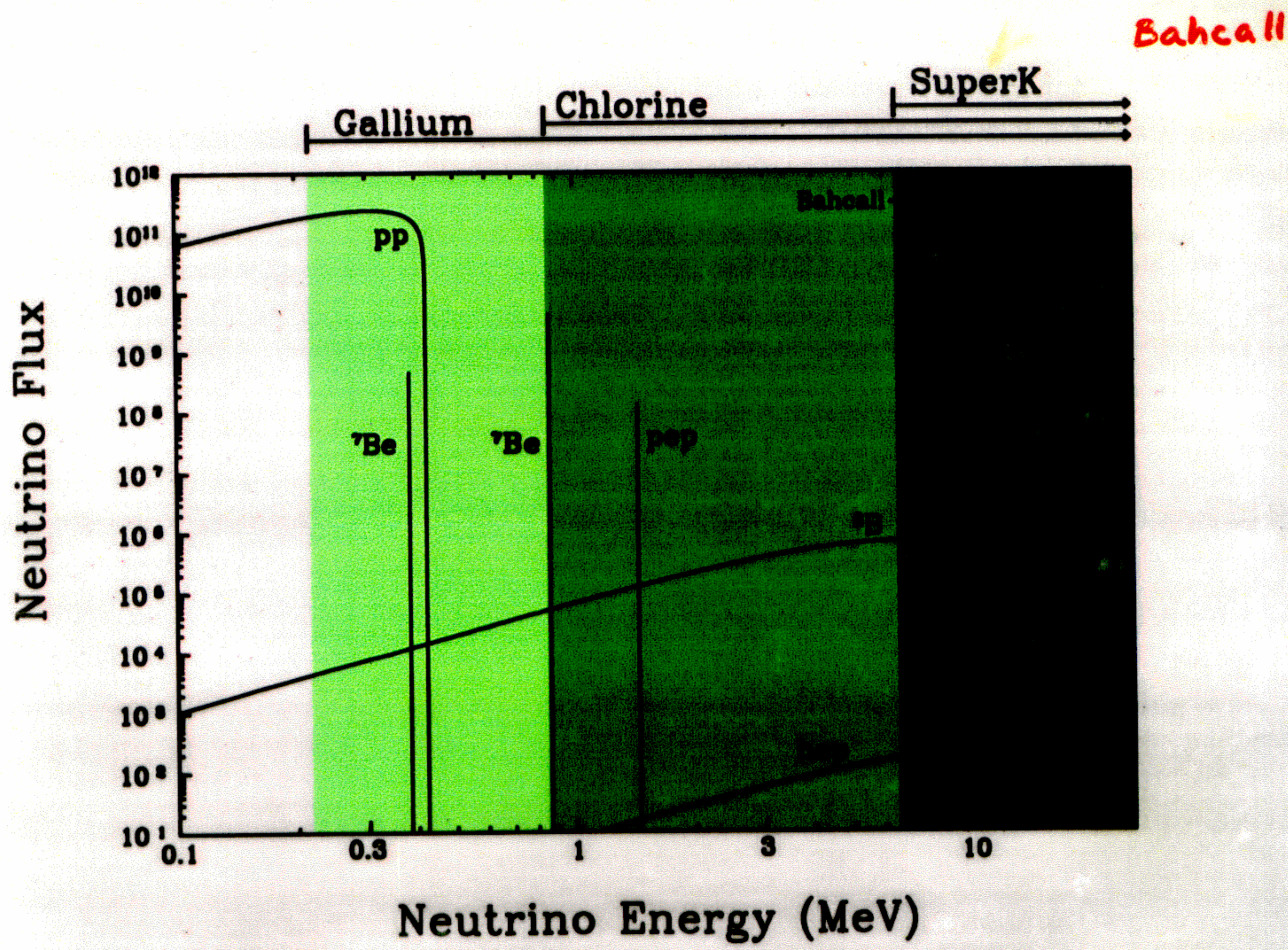
good on the logarithmic scale

MATTER OSCILLATION

Total Rates: Standard Model vs. Experiment

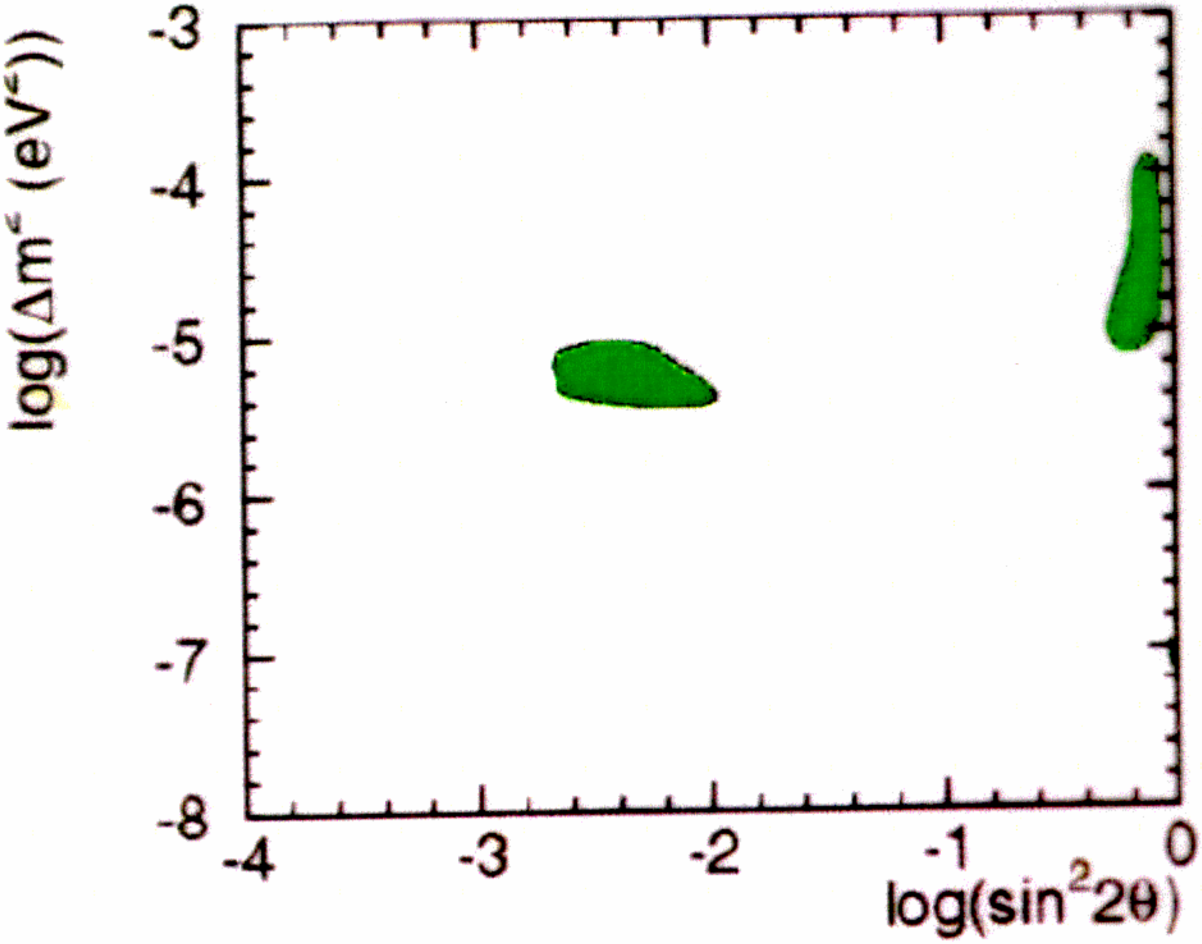
Bahcall–Pinsonneault 98



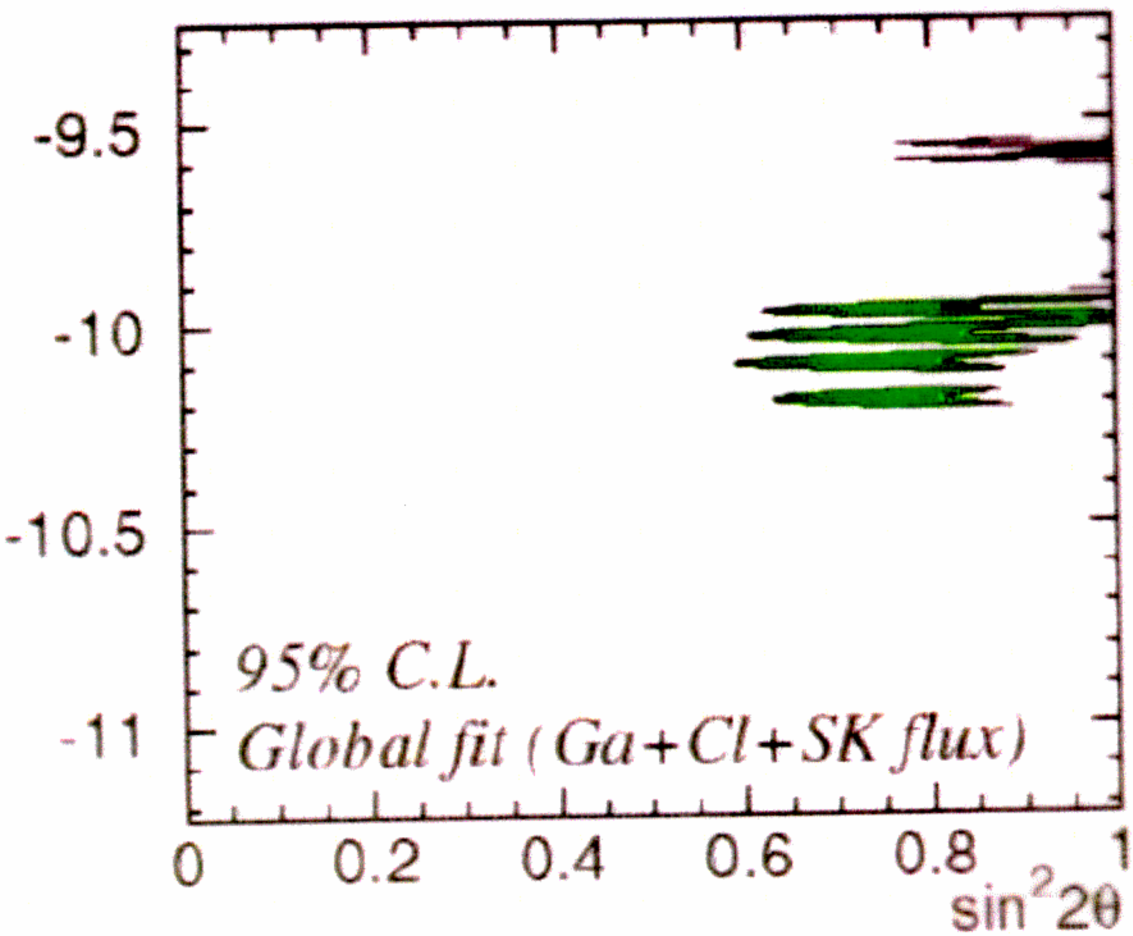


Solar neutrino energy spectrum

Suzuki Neutrino 2000



Allowe
@95%



§ Parameter Space

two-flavor ν oscillation

	mass
$\lvert\nu_1\rangle = \lvert\nu_e\rangle \cos\theta - \lvert\nu_\mu\rangle \sin\theta$	m_1
$\lvert\nu_2\rangle = \lvert\nu_e\rangle \sin\theta + \lvert\nu_\mu\rangle \cos\theta$	m_2

similarity transformations

A. $\theta \to \theta + \pi$ $\quad \lvert\nu_e\rangle \to -\lvert\nu_e\rangle \quad \lvert\nu_\mu\rangle \to -\lvert\nu_\mu\rangle$

B. $\theta \to -\theta$ $\quad \lvert\nu_\mu\rangle \to -\lvert\nu_\mu\rangle \quad \lvert\nu_2\rangle \to -\lvert\nu_2\rangle$

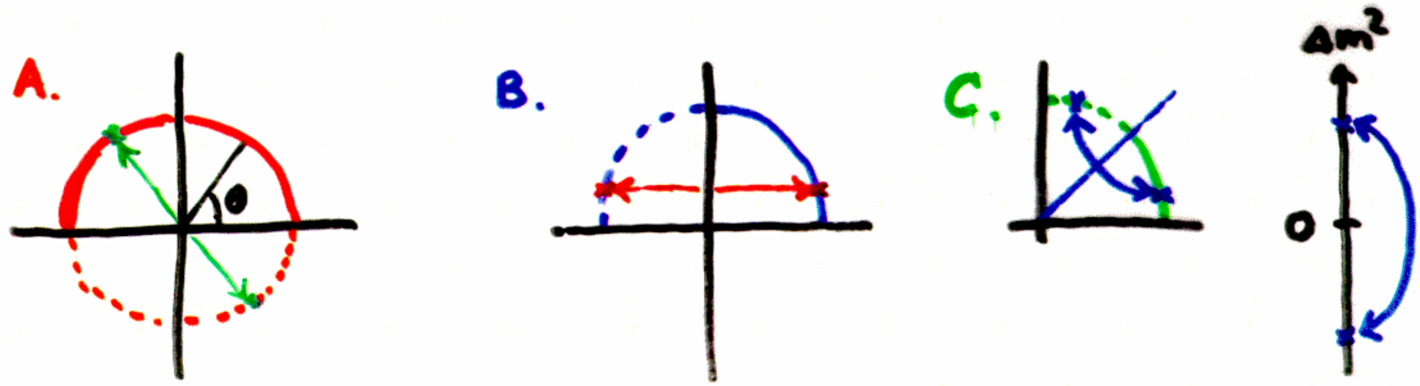
C. $\theta \to \frac{\pi}{2} - \theta$ $\quad \lvert\nu_1\rangle \leftrightarrow \lvert\nu_2\rangle \quad \lvert\nu_\mu\rangle \to -\lvert\nu_\mu\rangle$

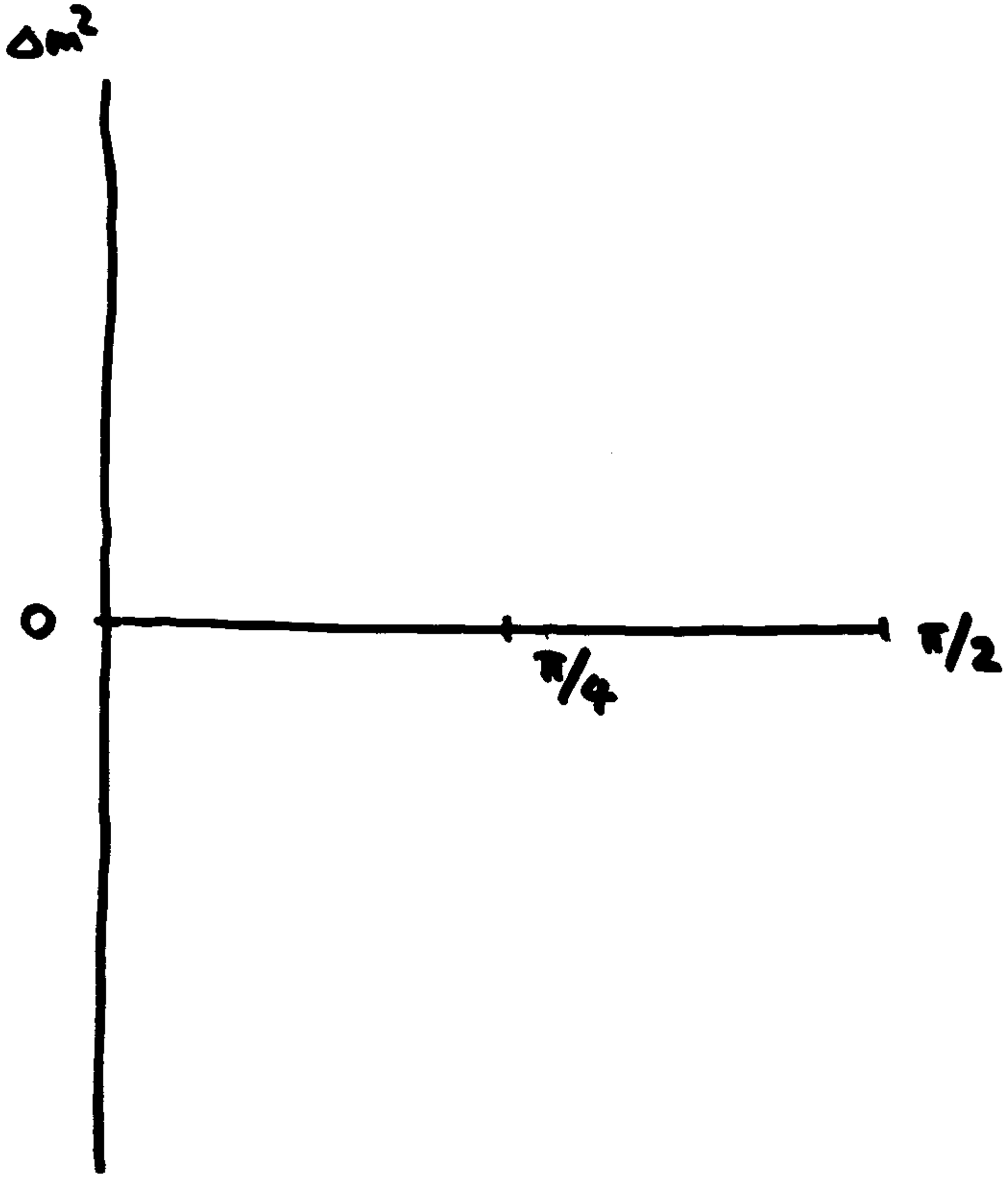
$\Delta m^2 \equiv m_2^2 - m_1^2 \to -\Delta m^2$

two possible choices

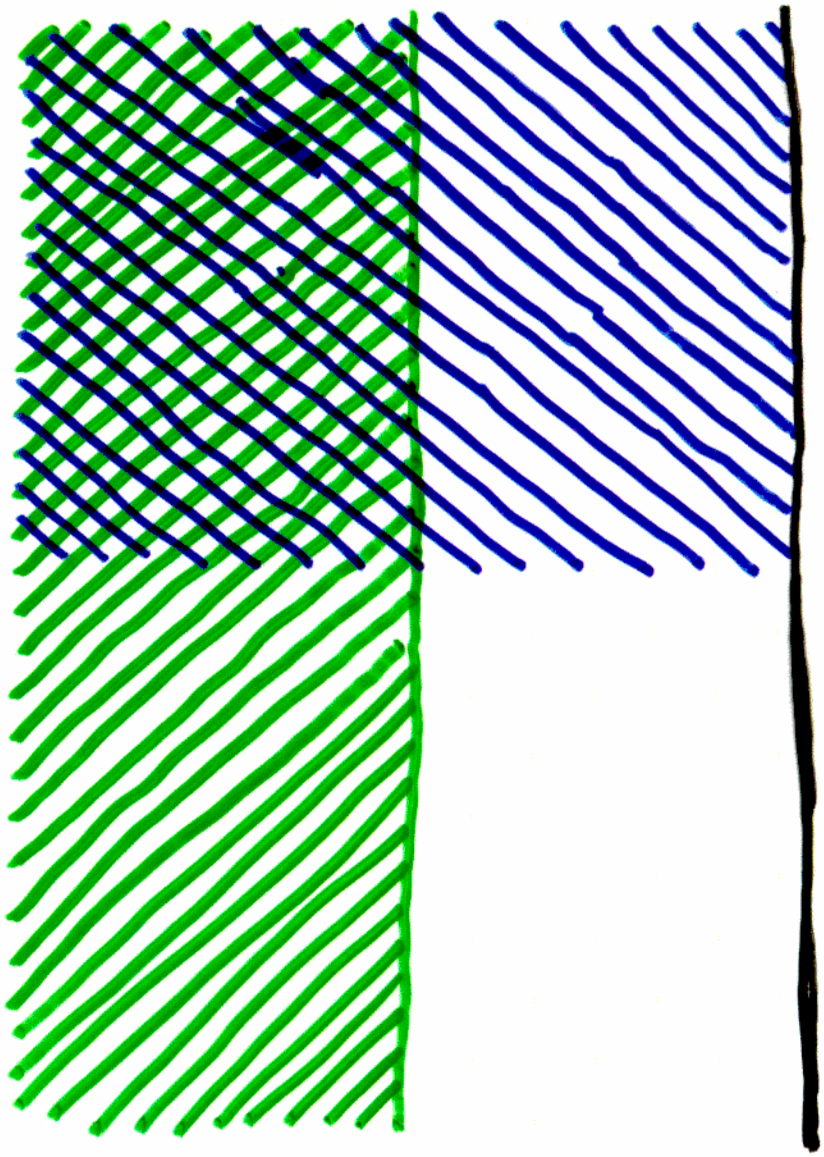
$0 \le \theta \le \frac{\pi}{4}$ $\quad \Delta m^2$ either sign

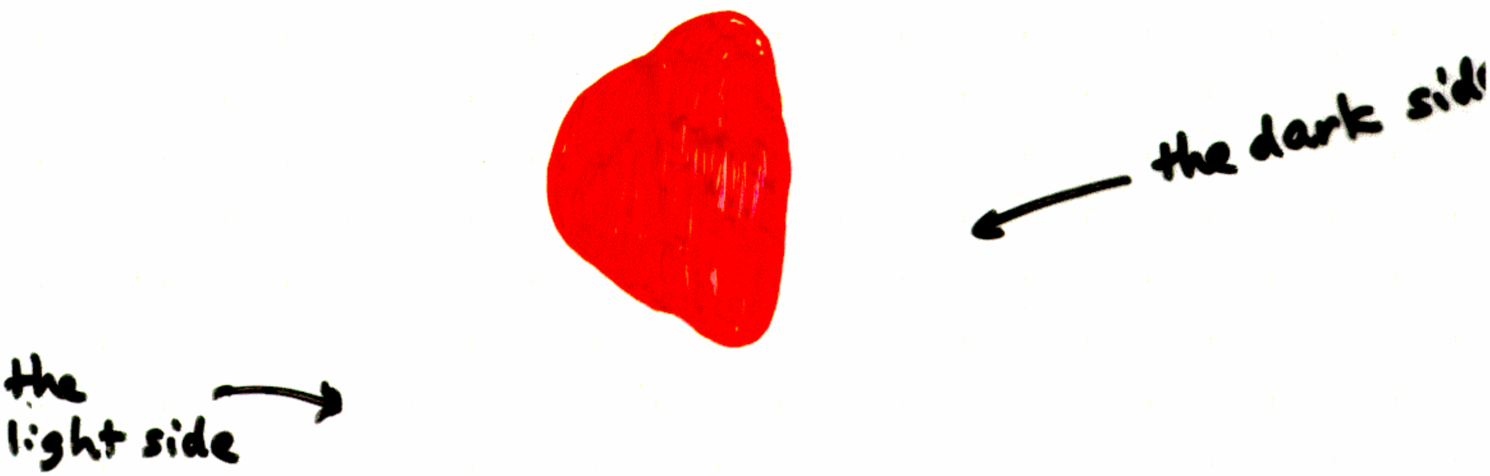
$0 \le \theta \le \frac{\pi}{2}$ $\quad \Delta m^2 \ge 0$





linear scales





NO OSCILLATION

in the presence of matter effects

oscillation in vacuum

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \, \sin^2\left(1.27 \frac{\Delta m^2}{E} L\right)$$

Δm^2 in eV^2

E in GeV

L in km

§ MSW Effect

$$|\nu, t\rangle = a_e |\nu_e\rangle + a_\mu |\nu_\mu\rangle$$

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} a_e \\ a_\mu \end{pmatrix} = H \begin{pmatrix} a_e \\ a_\mu \end{pmatrix}$$

$$H = H_{vac} + H_{matt}$$

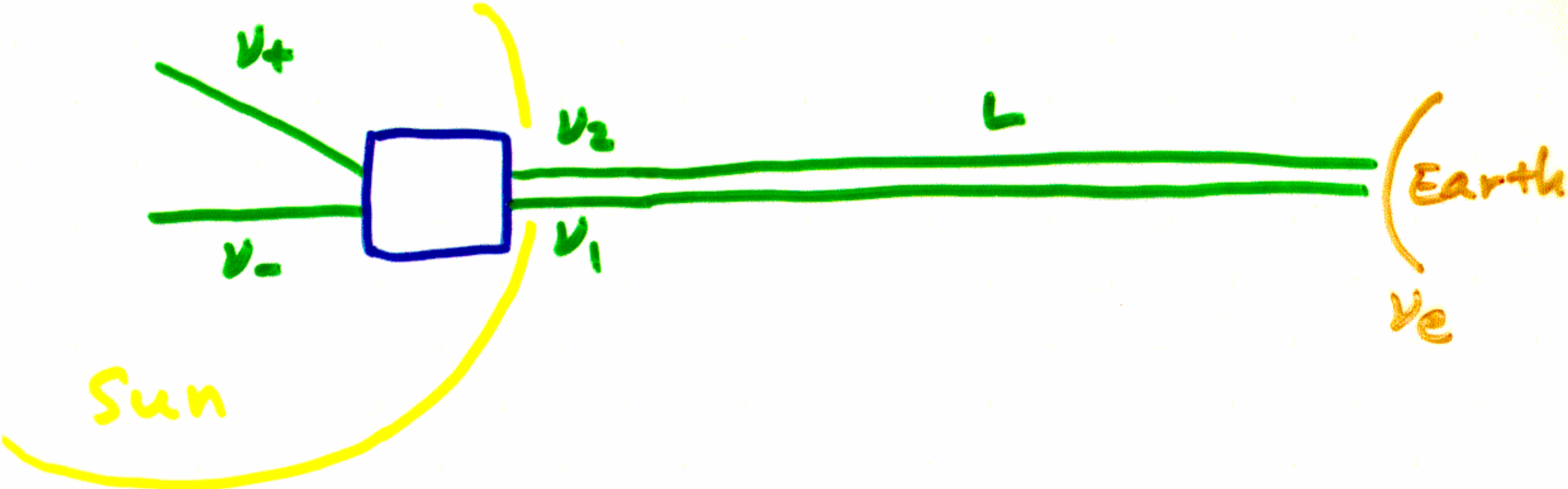
$$H_{vac} = \sqrt{\vec{p}^2 + m^2}$$
$$= p + \frac{m^2}{2p} + O(m^4)$$
$$= p + \frac{m_1^2 + m_2^2}{4p} + \frac{\Delta m^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$H_{matt} = \sqrt{2} G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \frac{1}{\sqrt{2}} G_F N_n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(N_e ↑ CC; N_n ↑ NC)

pieces $\propto 1$ ⇒ only on the overall phase

$$H = \frac{\Delta m^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \sqrt{2} G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$



$$(\cos\theta \quad \sin\theta)\begin{pmatrix} e^{-i\frac{m_1^2}{2p}L} & \\ & e^{-i\frac{m_2^2}{2p}L}\end{pmatrix}\begin{pmatrix} a & b^* \\ -b & a^*\end{pmatrix}\begin{pmatrix} e^{-iE_-t} & 0 \\ 0 & e^{-iE_+t}\end{pmatrix}\begin{pmatrix}\cos\theta_M \\ \sin\theta_M\end{pmatrix}$$

$$|a|^2+|b|^2=1$$

$$(E_+ - E_-)t \sim \sqrt{2} G_F N_e \frac{R_\odot}{10} \sim 1000$$

averaging on production region

$\Rightarrow$ rapid oscillation in "t"

$\Rightarrow$ decoherence between $|\nu_+\rangle$ & $|\nu_-\rangle$

$|b|^2 = P_c$, $|a|^2 = 1 - P_c$

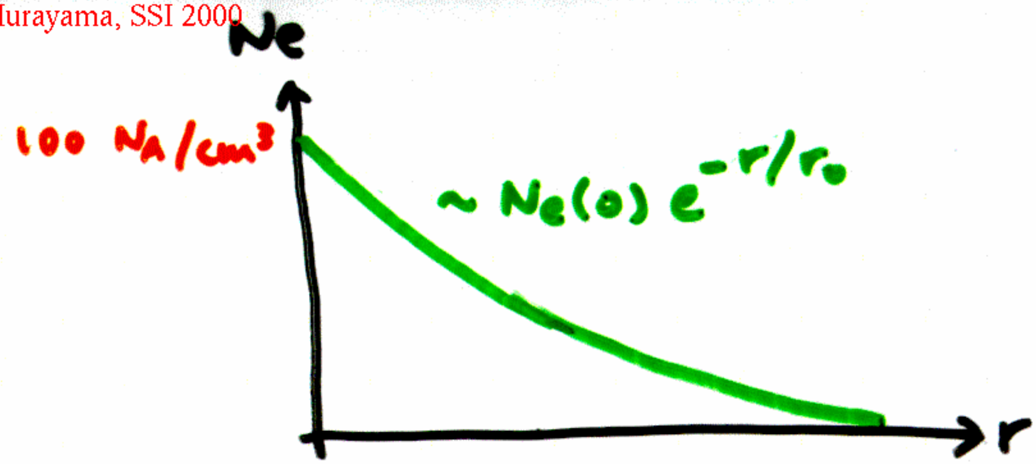
$$P_e = \cos^2\theta_M \left| a \cos\theta \, e^{-i m_1^2 L/2p} - b \sin\theta \, e^{-i m_2^2 L/2p} \right|^2$$

$$+ \sin^2\theta_M \left| b^* \cos\theta \, e^{-i m_1^2 L/2p} + a^* \sin\theta \, e^{-i m_2^2 L/2p} \right|^2$$

$$= \cos^2\theta_M \left((1-P_c)\cos^2\theta + P_c \sin^2\theta \right)$$

$$+ \sin^2\theta_M \left(P_c \cos^2\theta + (1-P_c)\sin^2\theta \right)$$

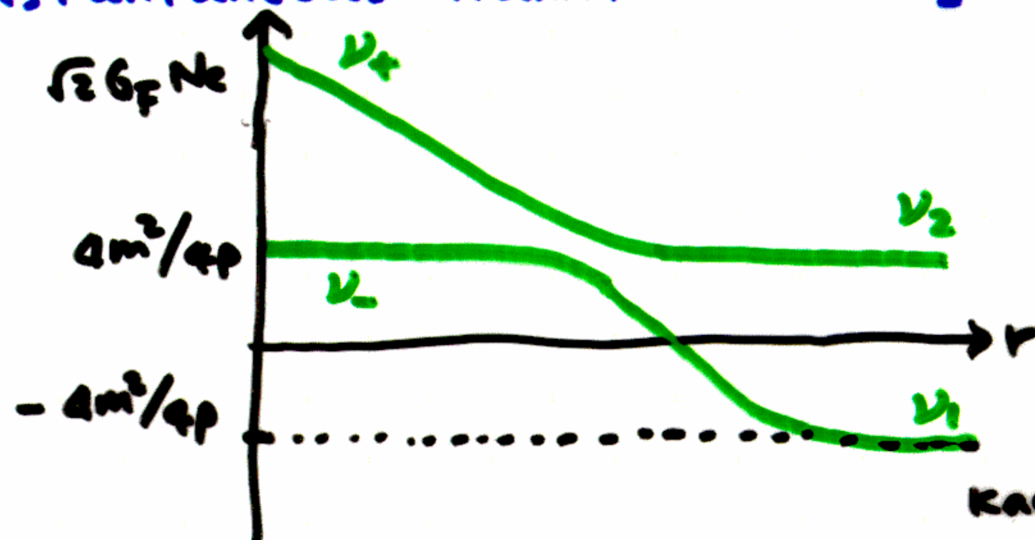
$$- \sqrt{P_c(1-P_c)} \sin 2\theta \cos\left(\frac{\Delta m^2}{2p} L + \delta \right)$$



$R_\odot \simeq 7 \times 10^5$ km

$r_0 \simeq 7 \times 10^4$ km

instantaneous Hamiltonian eigenstates



Kaneko, Tokashev, Petcov

$$P_c = \frac{e^{-\gamma \sin^2\theta} - e^{-\gamma}}{1 - e^{-\gamma}}$$ "hopping probability"

$$\gamma \equiv 2\pi r_0 \frac{\Delta m^2}{2p} = 1.22 \frac{\Delta m^2}{10^{-9}\,\text{eV}^2} \frac{0.862\,\text{MeV}}{E_\nu}$$

$\gamma \gg 1$ $\quad P_c \to 0$ $\quad$ "adiabatic limit"

$\gamma \ll 1$ $\quad P_c \to \cos^2\theta$ $\quad$ "vacuum limit" energy of neutrinos

distributed w/ $\Delta E \sim T \sim$ keV

when $\frac{\Delta m^2}{E} \gtrsim \frac{10^{-8}\,\text{eV}^2}{\text{MeV}}$

ν_1 & ν_2 decohere before reaching $\oplus$

$$\Delta\left(\frac{\Delta m^2}{2p} L\right) \sim 2\times10^3 \frac{\Delta m^2}{10^{-8}\,\text{eV}^2} \frac{E}{\text{MeV}} \frac{\Delta E}{E}$$

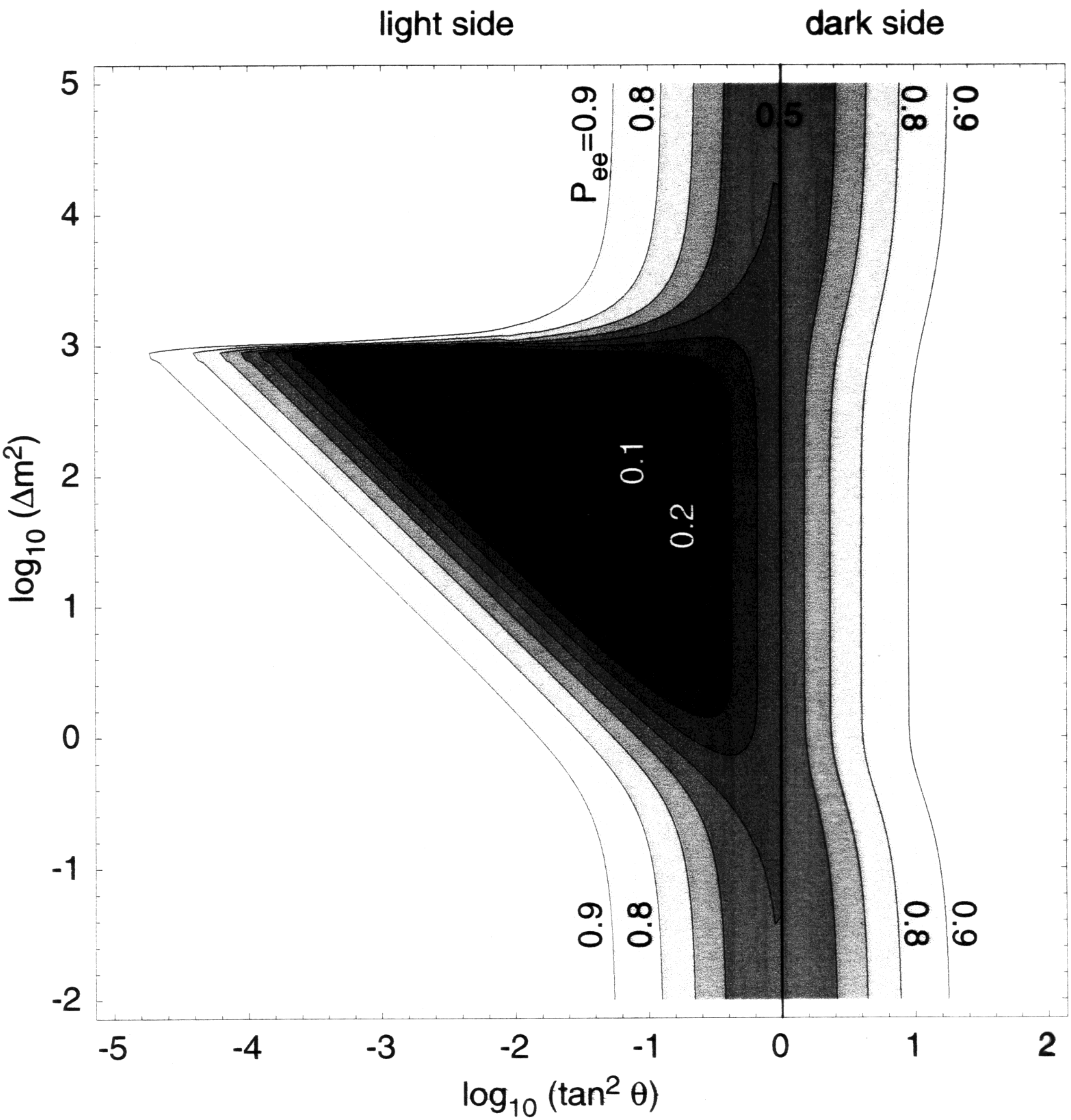
w/ $L = 1.5\times10^8$ km = 1 AU

ν_e exit the Sun as an incoherent mixture o

$|\nu_1\rangle$ prob $P_1 = \cos^2\theta_M (1-P_c) + \sin^2\theta_M P_c$

$|\nu_2\rangle$ prob $P_2 = \cos^2\theta_M P_c + \sin^2\theta_M (1-P_c$

The entire analysis goes through even when you are on the dark side


light side
dark side
P_{ee}=0.9
0.8
0.5
0.8
0.9
0.1
0.2
0.9
0.8
0.8
0.9
$\log_{10}(\Delta m^2)$
$\log_{10}(\tan^2\theta)$

Why was the dark side forgotten?

① theoretical prejudice

"the state closer to ν_e must be lighter"

$\theta < \pi/4$ $\Delta m^2 > 0$

② Landau Zener approx.

$N_e \sim e^{-r/r_0}$ ⇒ linear

$$P_c = \exp\left(-\frac{\gamma}{4}\frac{\sin^2 2\theta}{\cos 2\theta}\right)$$ Parke

cf. $$P_c = \frac{e^{-\gamma \sin^2\theta} - e^{-\gamma}}{1 - e^{-\gamma}}$$

agree when $\gamma \gg 1$, $\sin^2\theta \lesssim 0.3$

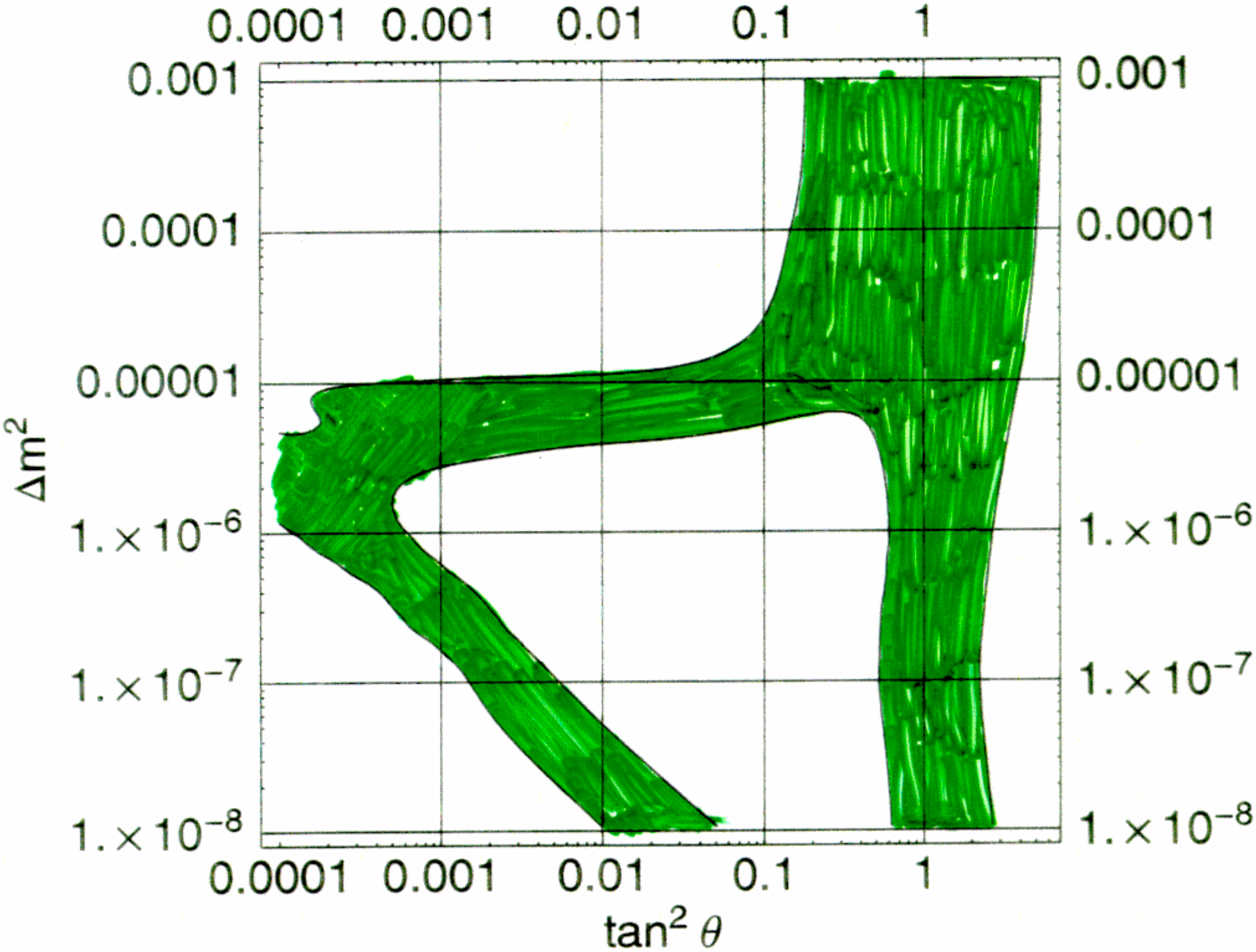
another theoretical prejudice

"mixing angle must be small"

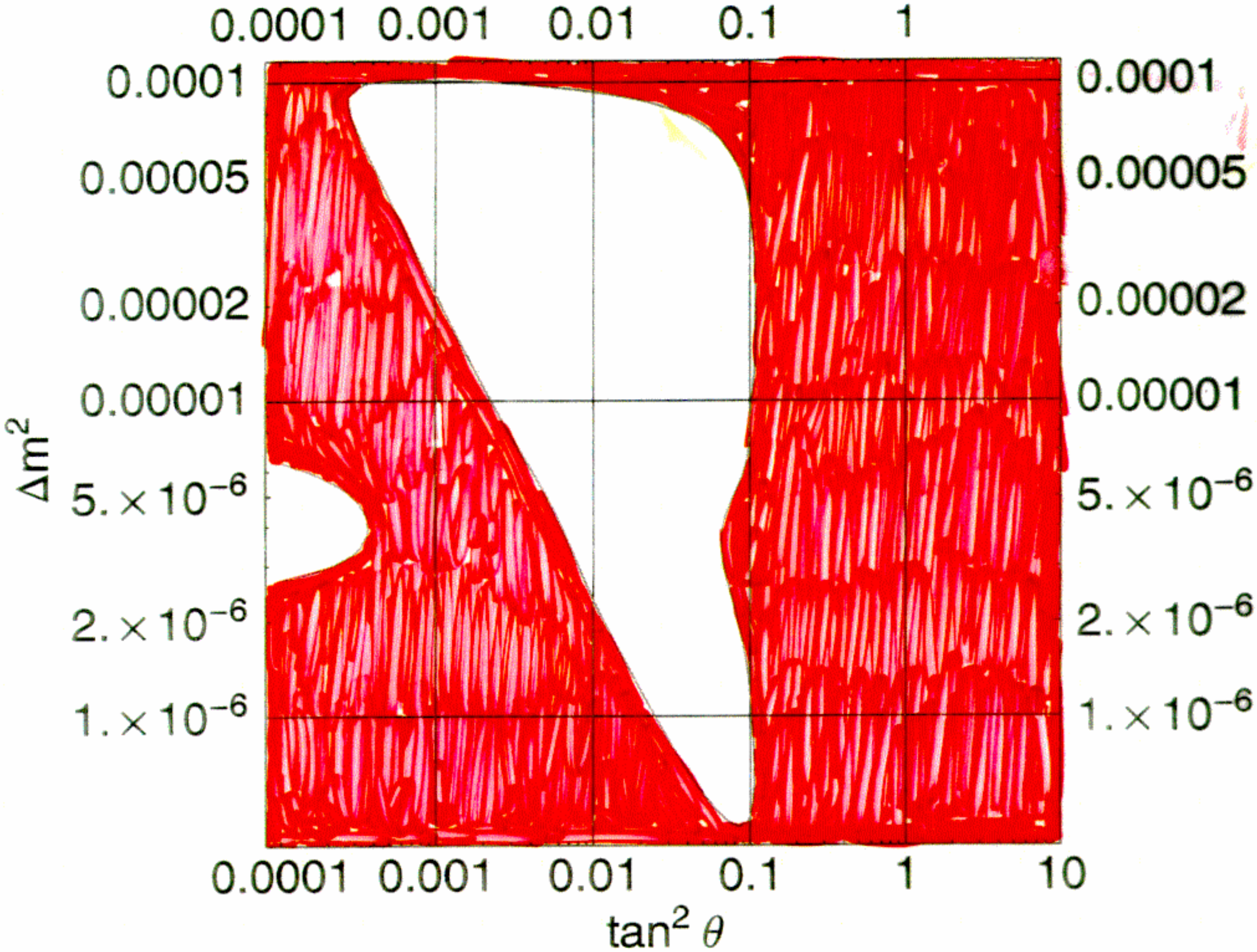
with $\frac{\sin^2 2\theta}{\cos 2\theta} \to \infty$ as $\theta \to \frac{\pi}{4}$

the dark side is infinitely far away

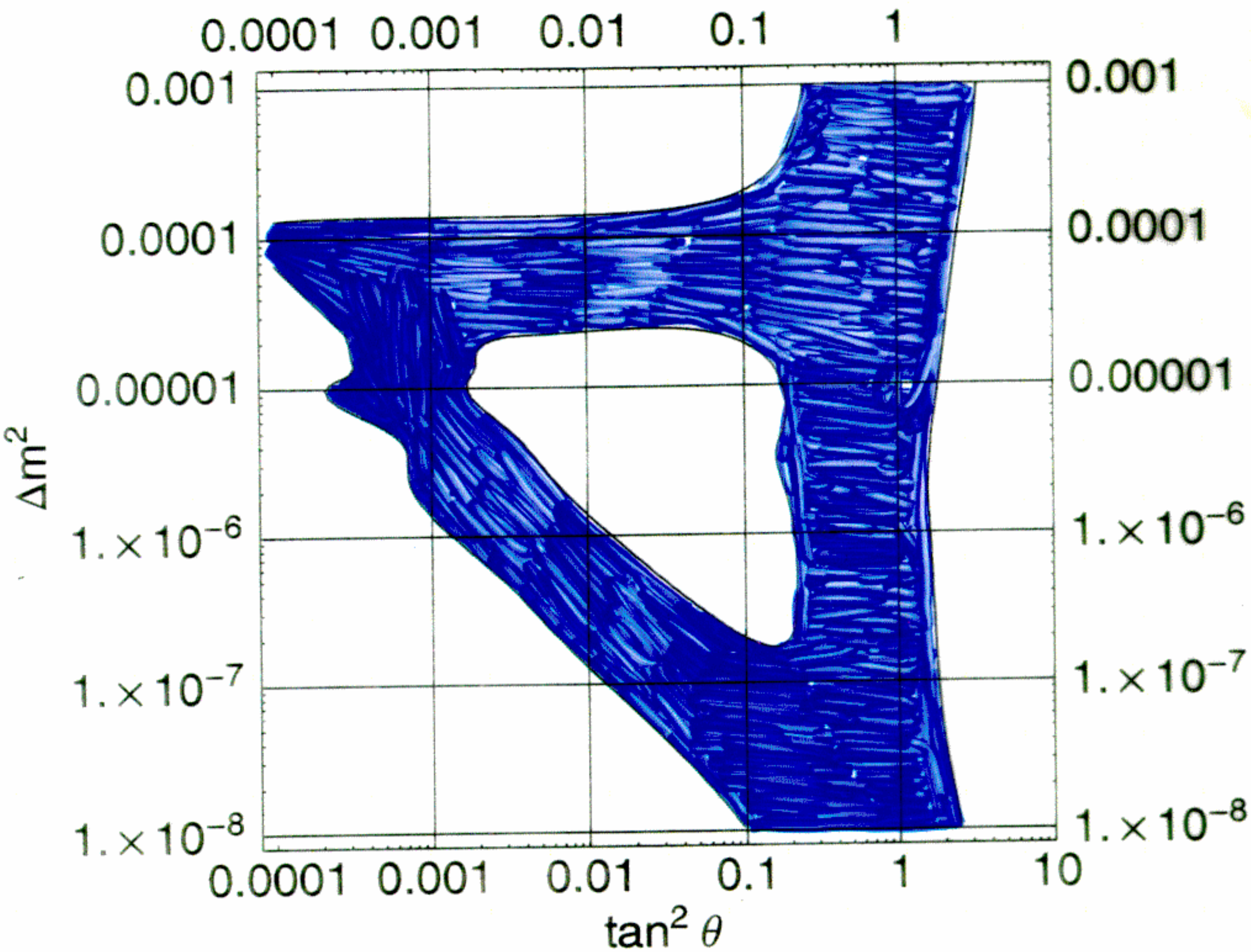
Ga

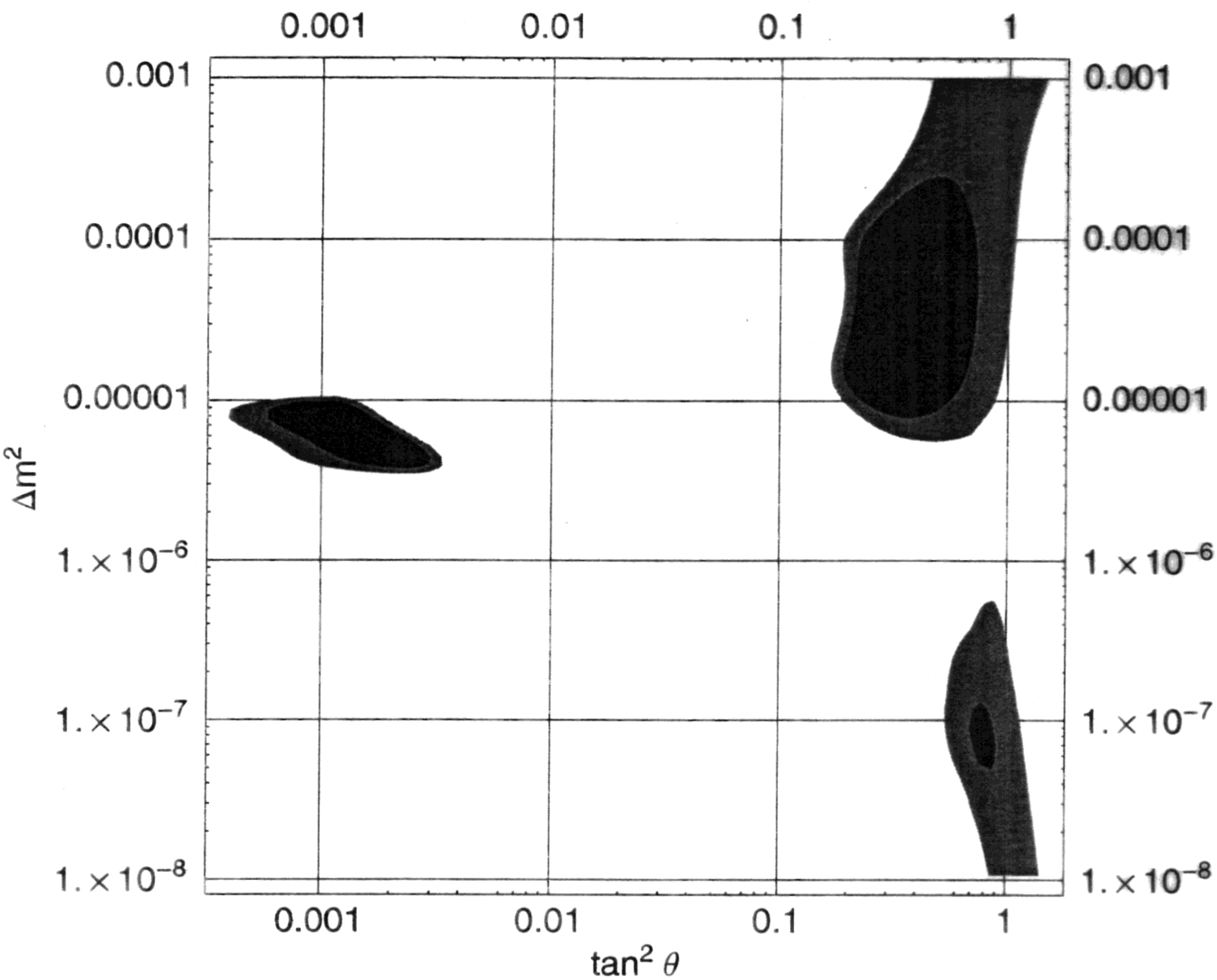

0.0001
0.001
0.01
0.1
1
0.001
0.0001
0.00001
$1.\times10^{-6}$
$1.\times10^{-7}$
$1.\times10^{-8}$
Δm^2
$\tan^2\theta$

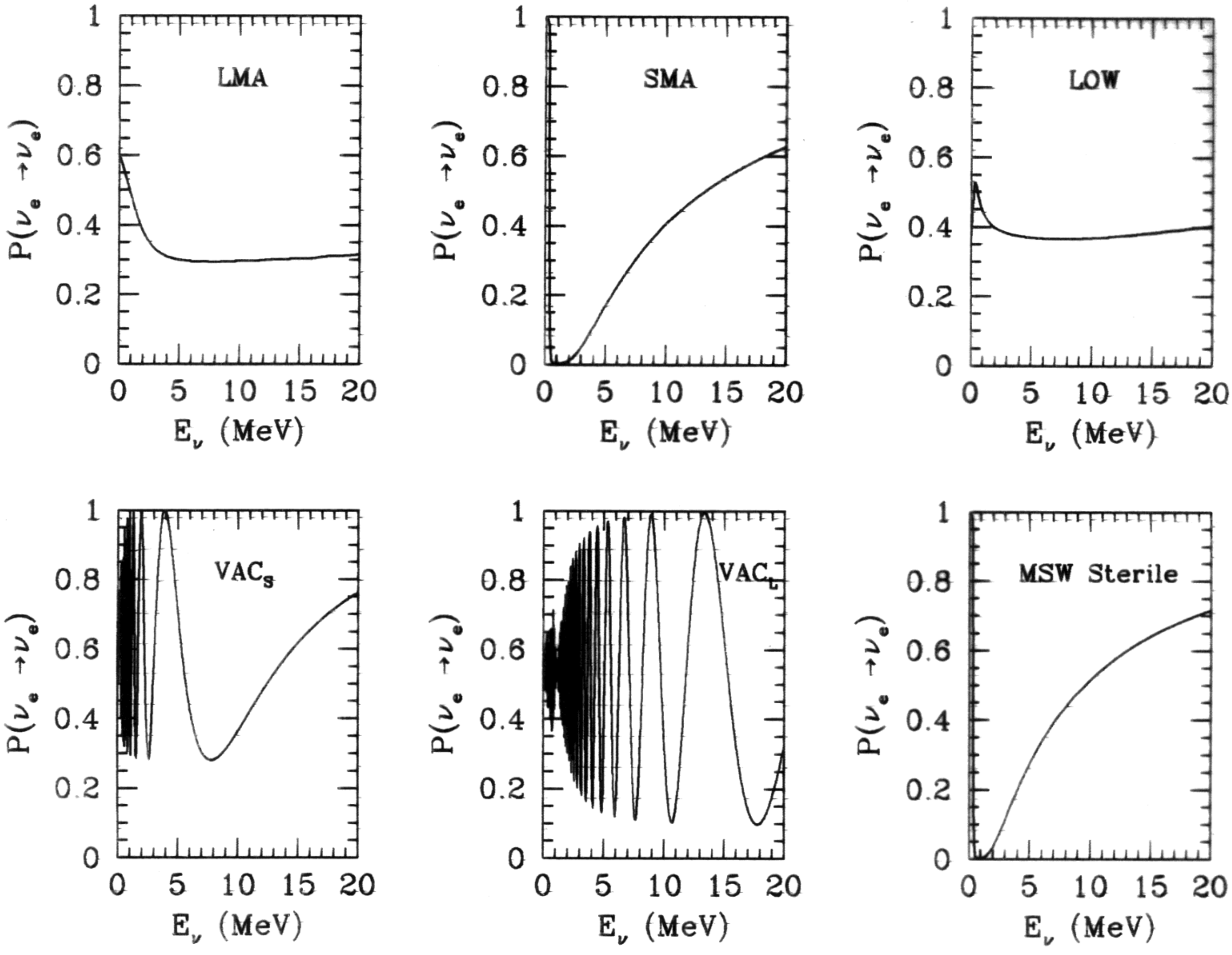
H_2O


0.0001 0.001 0.01 0.1 1
0.0001
0.00005
0.00002
0.00001
5. × 10⁻⁶
2. × 10⁻⁶
1. × 10⁻⁶
Δm²
0.0001 0.001 0.01 0.1 1 10
tan² θ

Cl

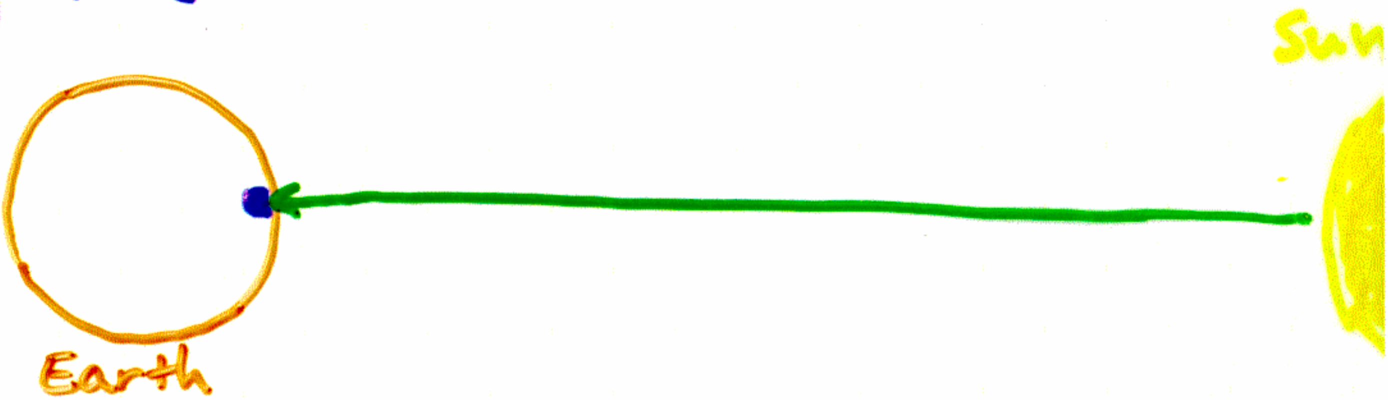

0.0001
0.001
0.01
0.1
1
10
0.001
0.0001
0.00001
1. × 10⁻⁶
1. × 10⁻⁷
1. × 10⁻⁸
Δm²
tan² θ


0.001
0.01
0.1
1
0.001
0.0001
0.00001
1. × 10^-6
1. × 10^-7
1. × 10^-8
Δm^2
tan^2 θ


LMA
SMA
LOW
VAC$_S$
VAC$_L$
MSW Sterile
$P(\nu_e \to \nu_e)$
E_ν (MeV)
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20

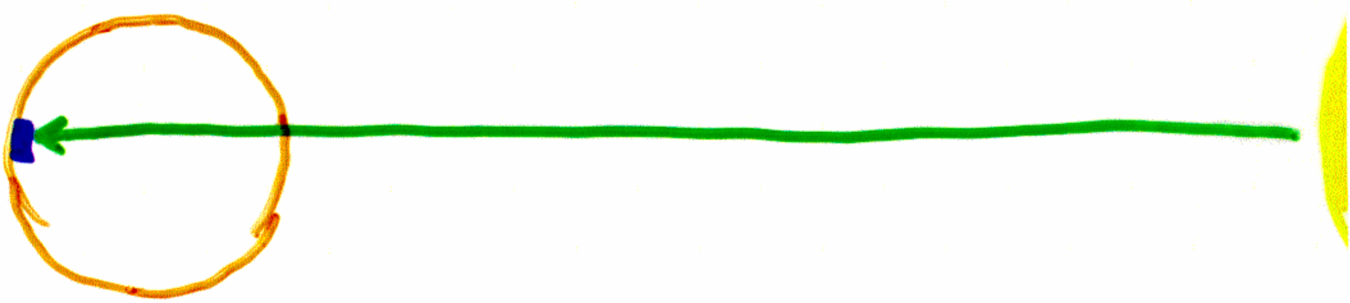
§ Earth Matter Effect

day time

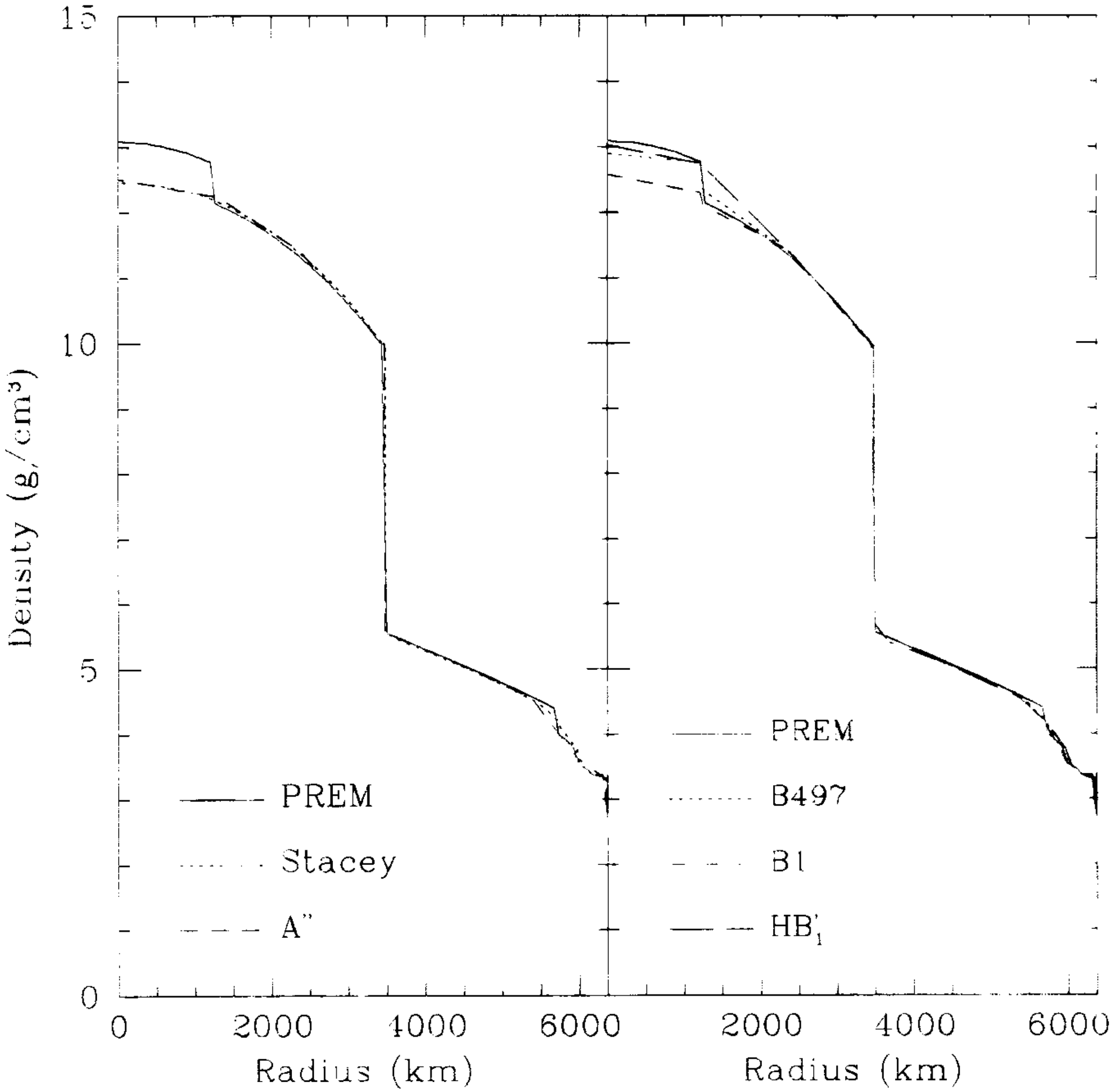


solar neutrinos enter the detector directly

night time

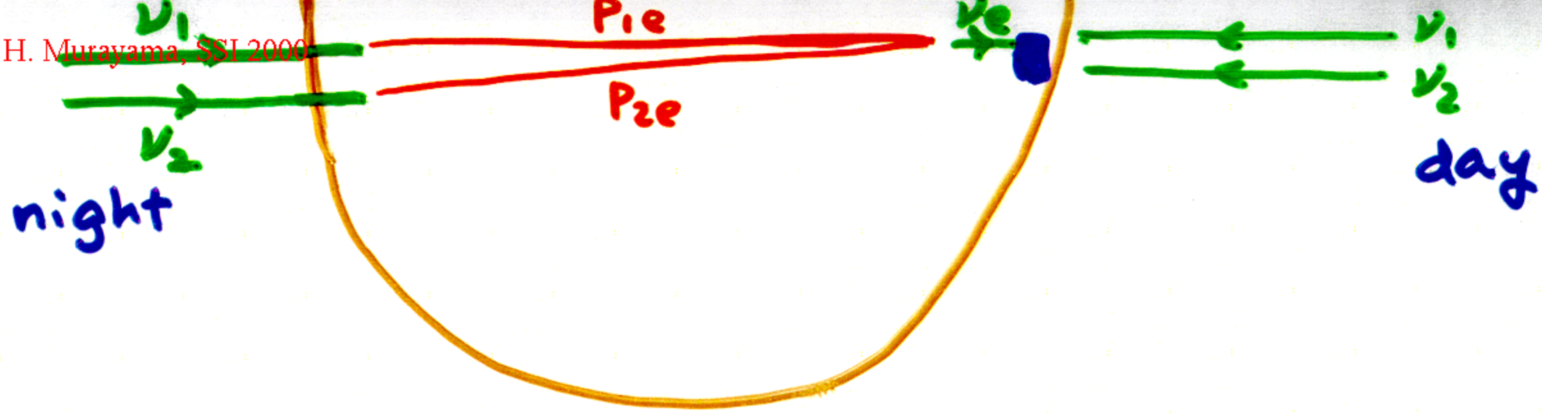


solar neutrinos traverse terrestrial matter before entering the detector



15
10
5
0
Density (g/cm³)
0
2000
4000
6000
Radius (km)
PREM
Stacey
A"
2000
4000
6000
Radius (km)
PREM
B497
B1
HB'1

Figure 2



day: $P_{2e} = \sin^2\theta$

night: $P_{2e}^E \neq P_{2e}$

$$P_e = P_1 P_{1e} + P_2 P_{2e}$$

$$= P_1 + (1-2P_1) P_{2e}$$

$$A_{DN} = \frac{\#N - \#D}{\#N + \#D} = \frac{(1-2P_1)(P_{2e}^E - \sin^2\theta)}{2P_1 + (1-2P_1)(P_{2e}^E + \sin^2\theta)}$$

lore: $A_{DN} \to 0$ as $\sin^2\theta \to \frac{1}{2}$

"50:50 mixture stays 50:50"

fact: $A_{DN} \to 0$ as $P_1 \to \frac{1}{2}$

50:50 mixture of ν_1 + ν_2 → A_{DN} =

$A_{DN} \neq 0$ @ $\theta = \pi/4$ Guth, Randall, Serna hep-ph/9903464

A_{DN} extends smoothly to the dark side

de Gouvêa, Friedland, HM hep-ph/9910286

Why was the dark side forgotten?

③ day-night effect

believed to disappear as $\theta \to \pi/4$

$\Rightarrow$ two sides disconnected (wrong)

even in the correct plots

contours appeared to close at $\theta = \pi/4$

Jacobian $\frac{d \sin^2 2\theta}{d\theta} = 4 \sin 2\theta \cos 2\theta \to 0$

artificial discontinuity

Earth matter effect on ^{7}Be ν

de Gouvêa, Friedland, HM, hep-ph/9910286

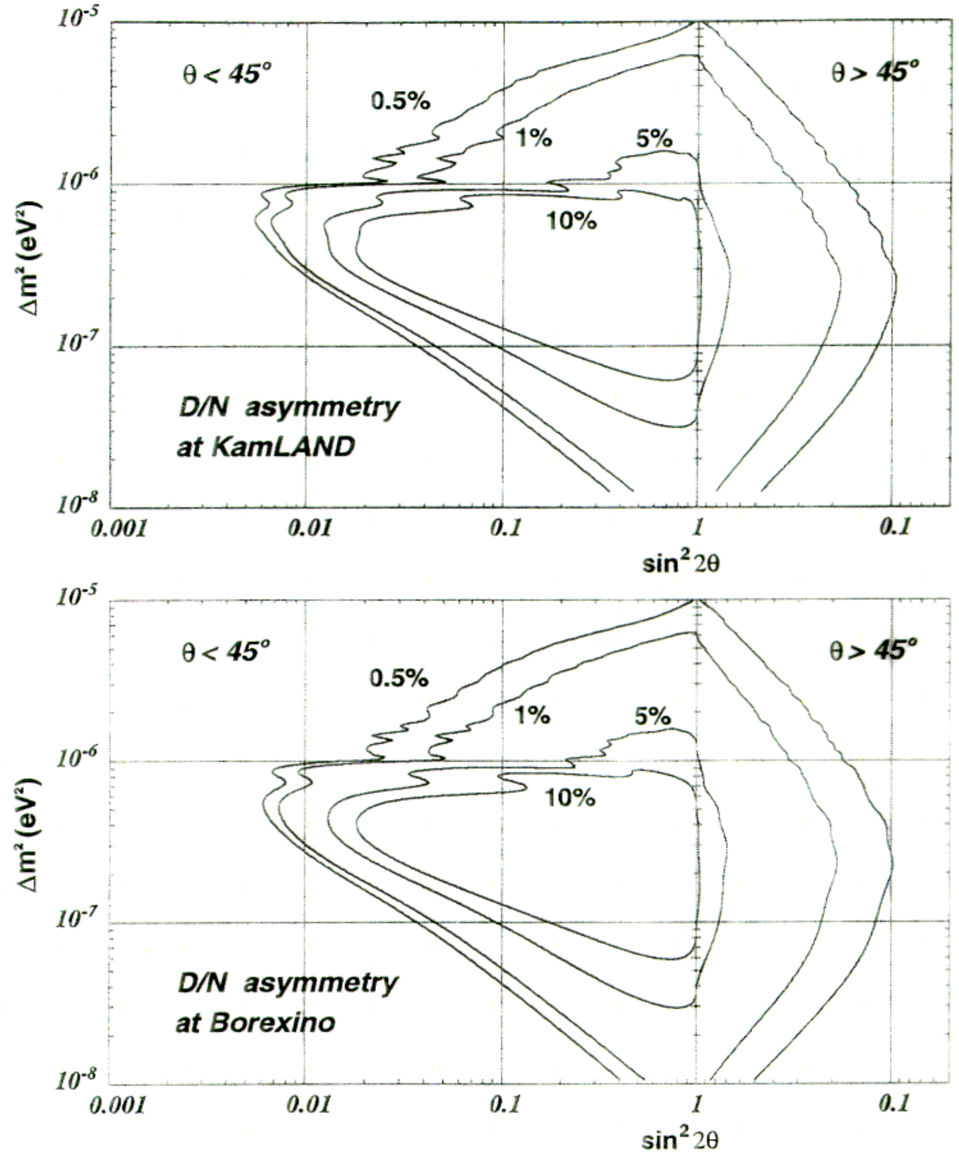
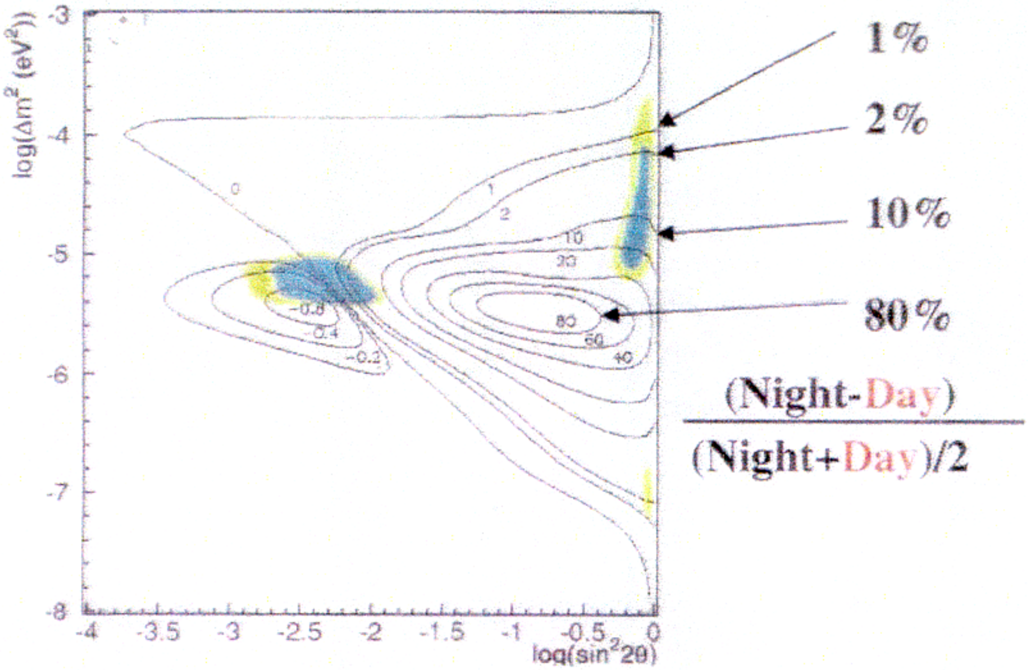
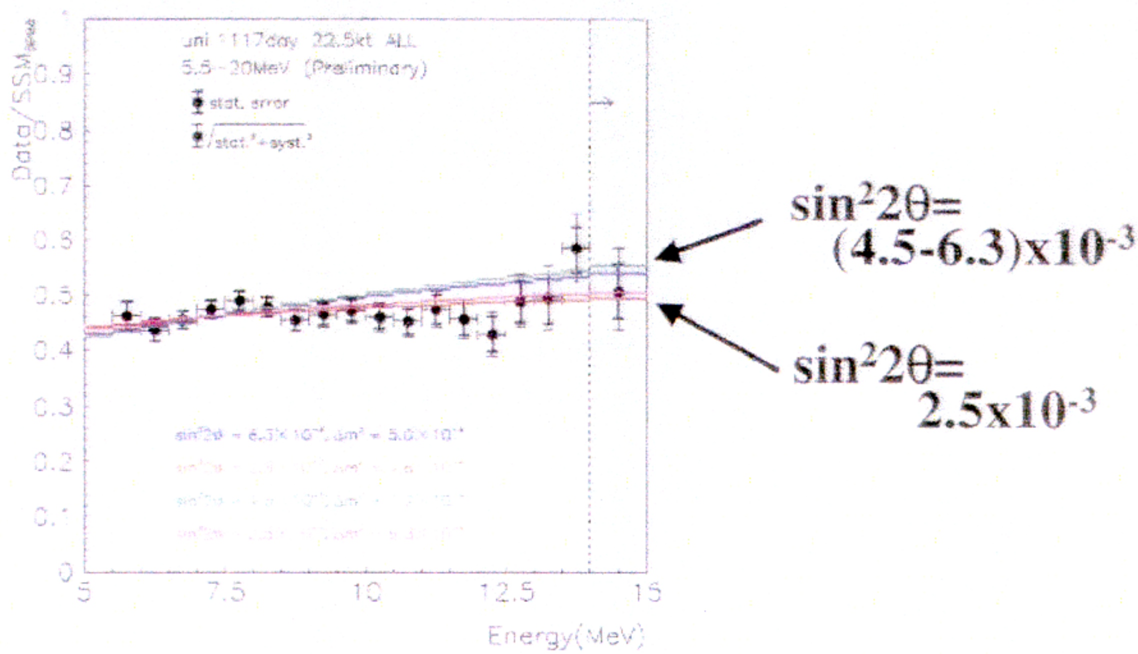


Figure 2: Constant day-night asymmetry contours $(10\%, 5\%, 1\%, 0.5\%)$ in the $(\sin^2 2\theta, \Delta m^2)$-plane for ^{7}Be neutrinos at the KamLAND and Borexino sites. The right side of the plot, with decreasing scale, can also be thought of as $\Delta m^2 < 0$, $\theta < 45^0$.

Expected





Energy(MeV)

Earth matter effect on ^{7}Be ν

de Gouvêa, Friedland, HM hep-ph/9910286

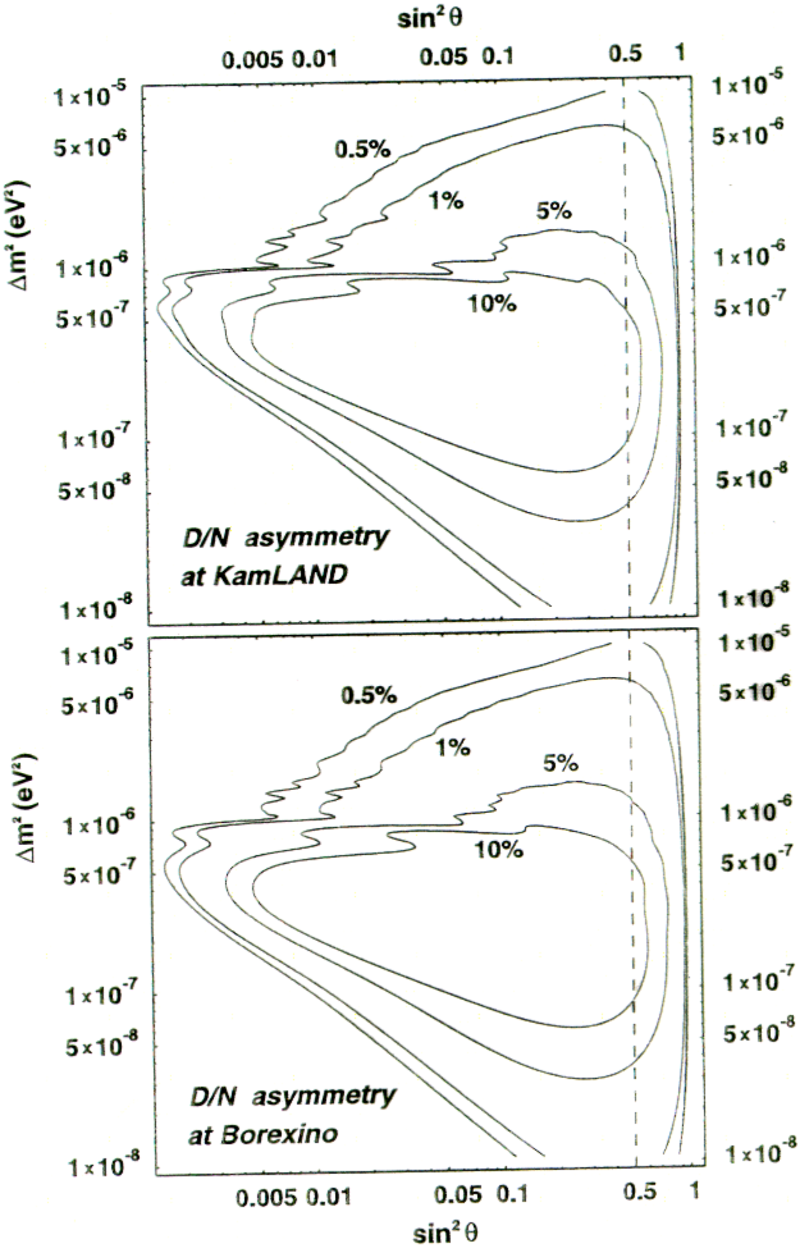
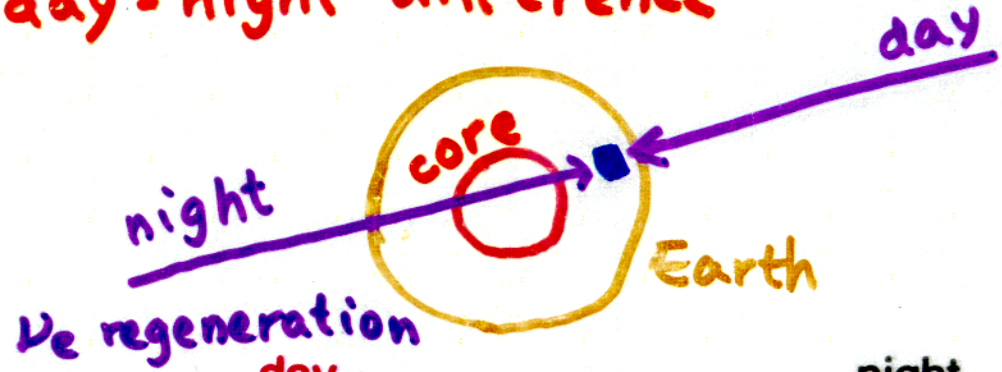
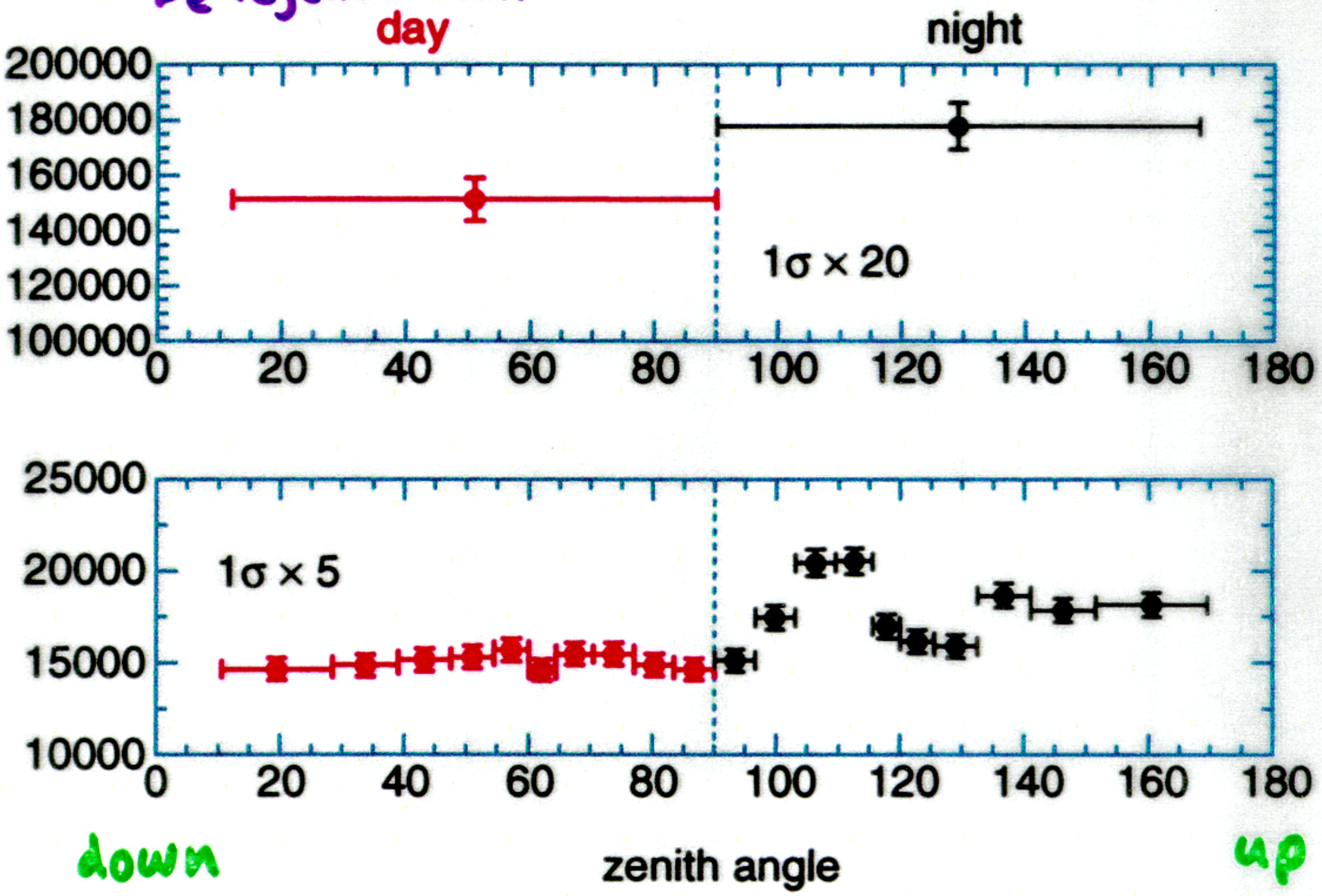


Figure 1: Constant day-night asymmetry contours (10%, 5%, 1%, 0.5%) in the $(\sin^2\theta, \Delta m^2)$-plane for ^{7}Be neutrinos at the KamLAND and Borexino sites. The vertical dashed line indicates $\sin^2\theta = 1/2$, where the neutrino vacuum mixing is maximal.

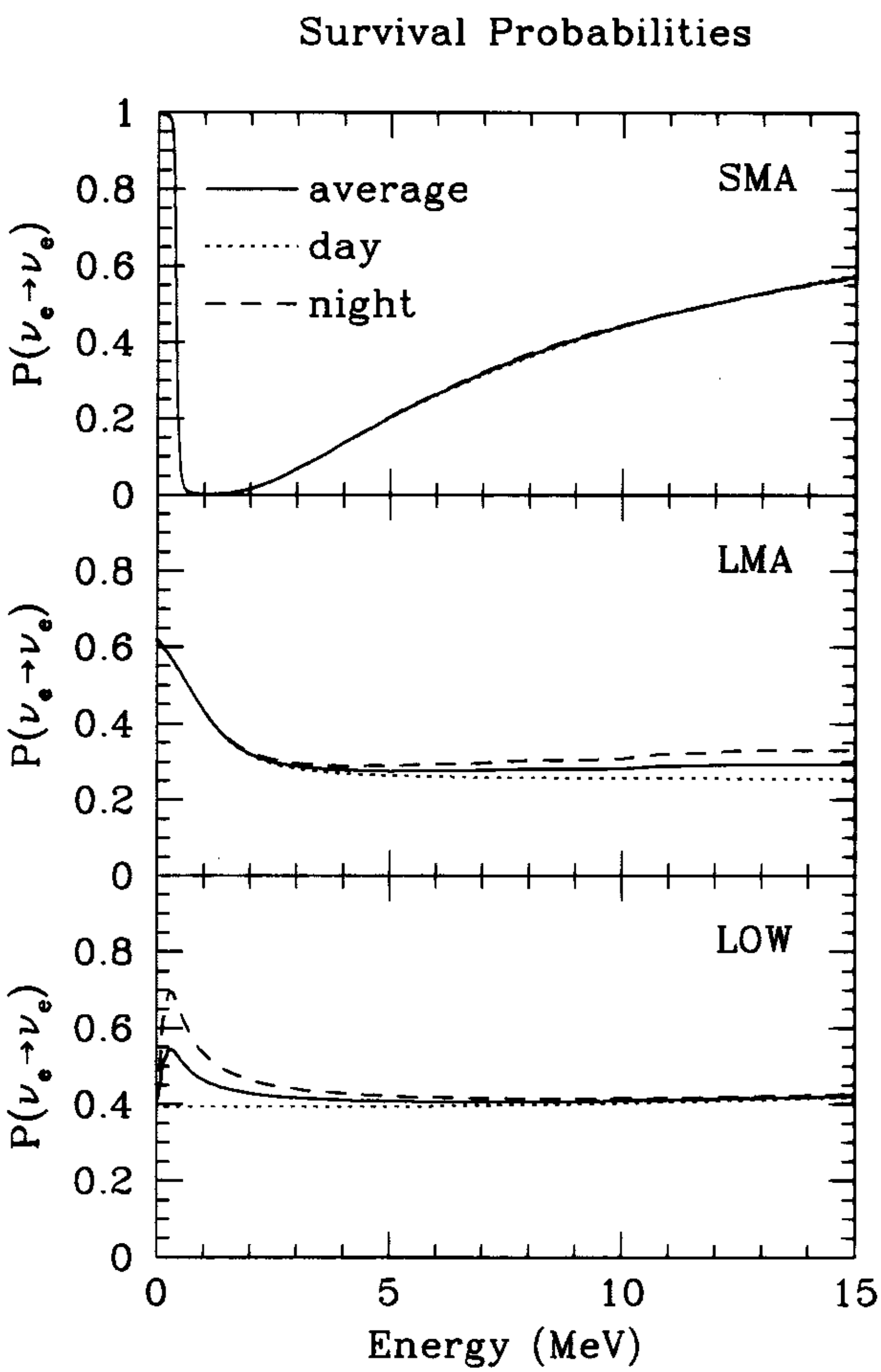
smoking gun signal

day-night difference





Friedland
de Gouvêa
HM
PRC


Survival Probabilities
SMA
LMA
LOW
average
day
night
P($\nu_e \to \nu_e$)
Energy (MeV)
0
5
10
15
0.2
0.4
0.6
0.8
1

§ Matter Effect on "Vacuum Oscillation"

$$P_e = \cos^2\theta_M \left((1-P_c)\cos^2\theta + P_c \sin^2\theta \right)$$
$$+ \sin^2\theta_M \left(P_c \cos^2\theta + (1-P_c)\sin^2\theta \right)$$
$$- \sqrt{P_c(1-P_c)} \sin 2\theta \cos\left(\frac{\Delta m^2}{2p} L + \delta \right)$$

$$P_c = \frac{e^{-\gamma \sin^2\theta} - e^{-\gamma}}{1 - e^{-\gamma}}$$

$$\gamma = 2\pi r_0 \frac{\Delta m^2}{2p}$$
$$= 1.22 \frac{\Delta m^2}{10^{-9}\,\text{eV}^2} \frac{0.862\,\text{MeV}}{E_\nu}$$

"vacuum oscillation solution" (just-so)

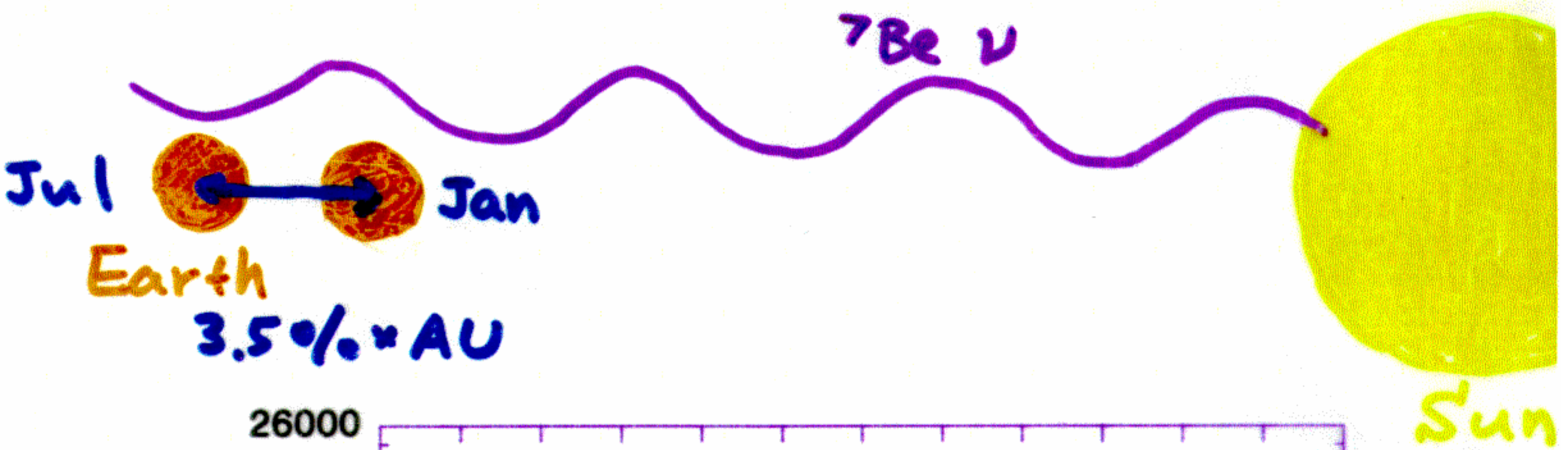
$\gamma \to 0$, $P_c \to \cos^2\theta$, $\cos^2\theta_M \to 0$, $\delta \to 0$

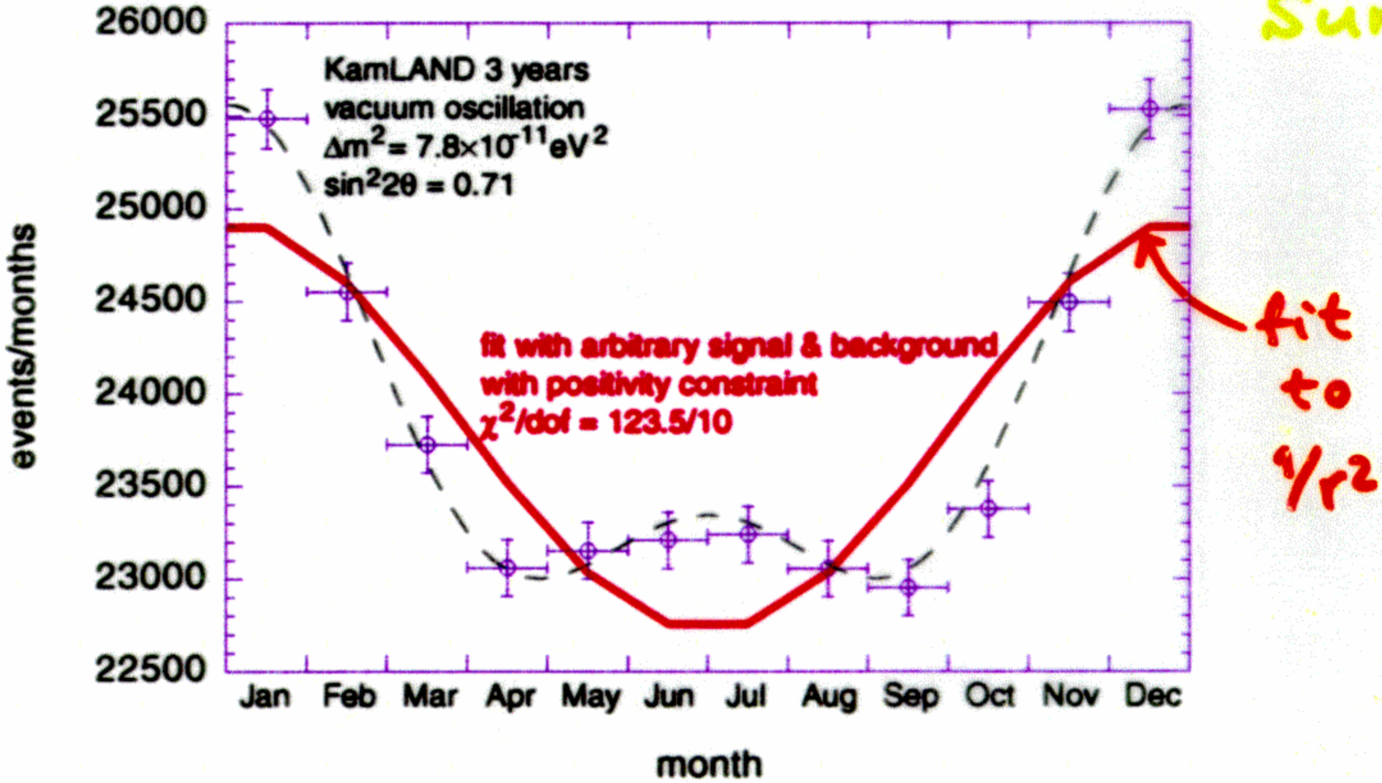
$$P_c \to \cos^4\theta + \sin^4\theta - \cos\theta \sin\theta \sin 2\theta \cos \frac{\Delta m^2}{2p} L$$
$$= 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4p} L$$

$\theta \longleftrightarrow \frac{\pi}{2} - \theta$ reflection symmetric

smoking gun signal

anomalous seasonal variation





Friedland
de Gouvêa
HM

Not yet in this limit even for

$\Delta m^2 \sim 10^{-10} - 10^{-9}\ eV^2$

especially for low-energy neutrinos

$E \gtrsim 0.2\ MeV$

Alex Friedland, hep-ph/0002063

consider $\Delta m^2 = 10^{-11} - 10^{-8}\ eV^2$

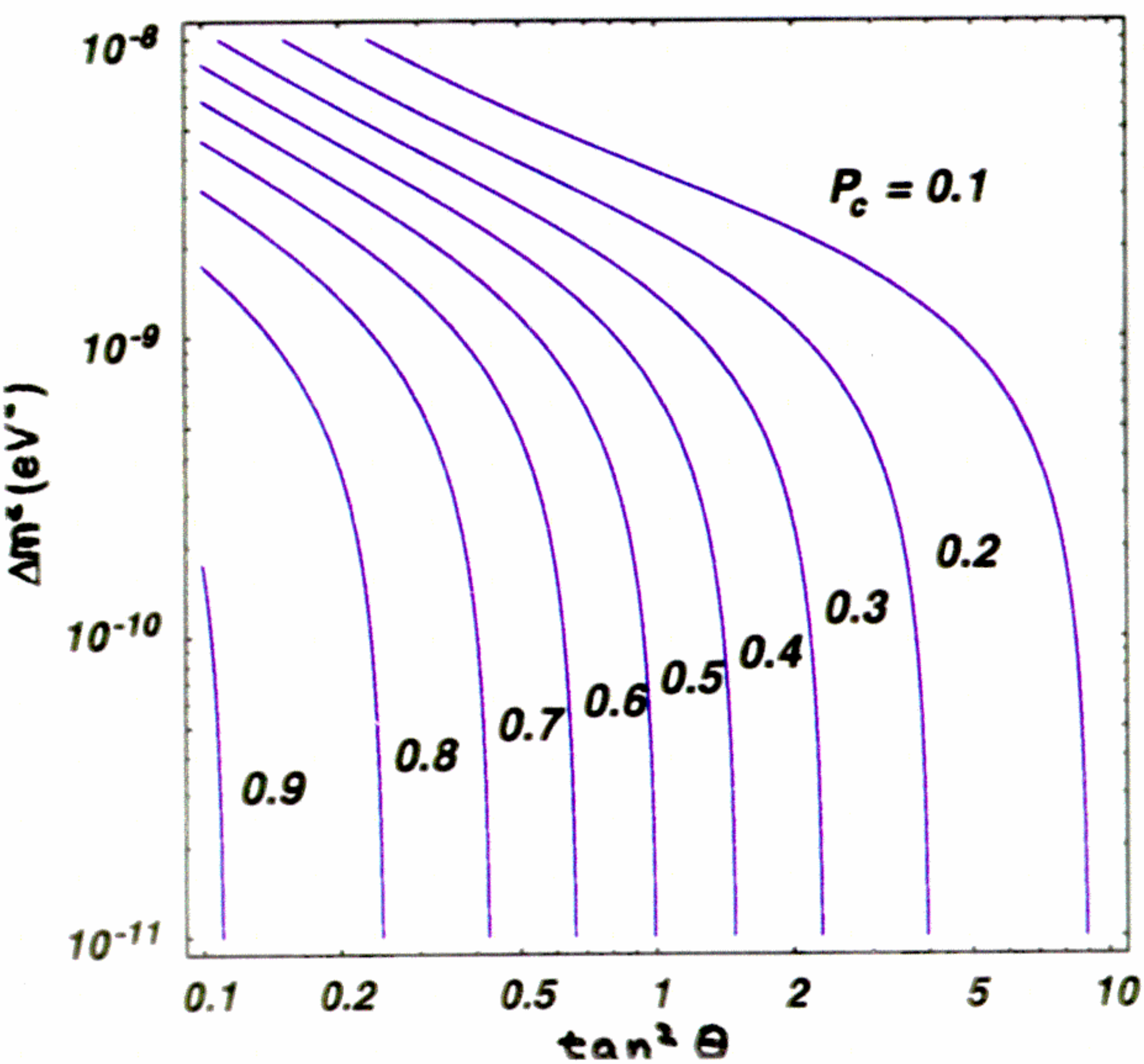
$\sqrt{2}\, G_F N_e r_0 \sim 1000$

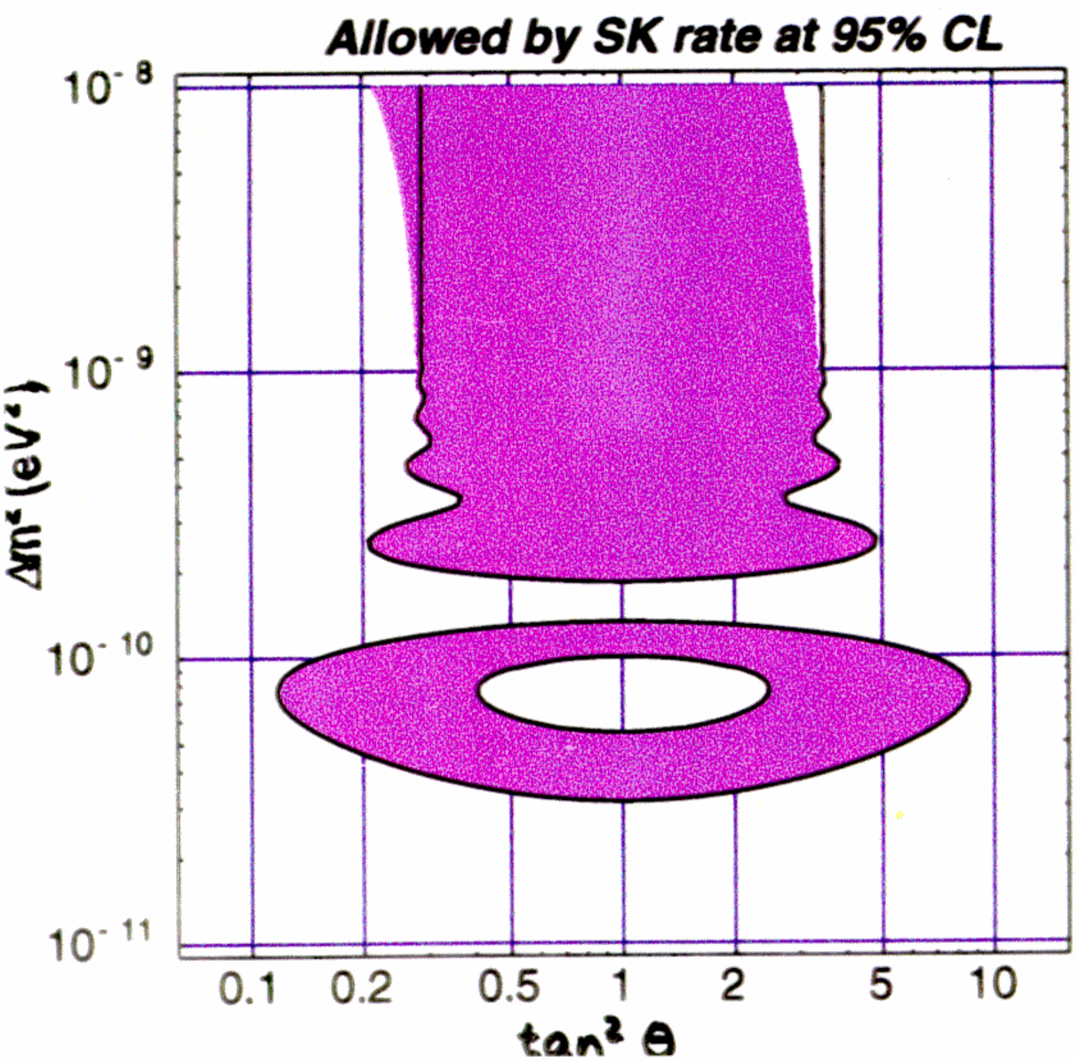
$\frac{\Delta m^2}{2p} r_0 \sim 10^{-1} \frac{\Delta m^2}{10^{-9}\ eV^2} \frac{MeV}{E}$

$\cos^2\theta_M \to 0$

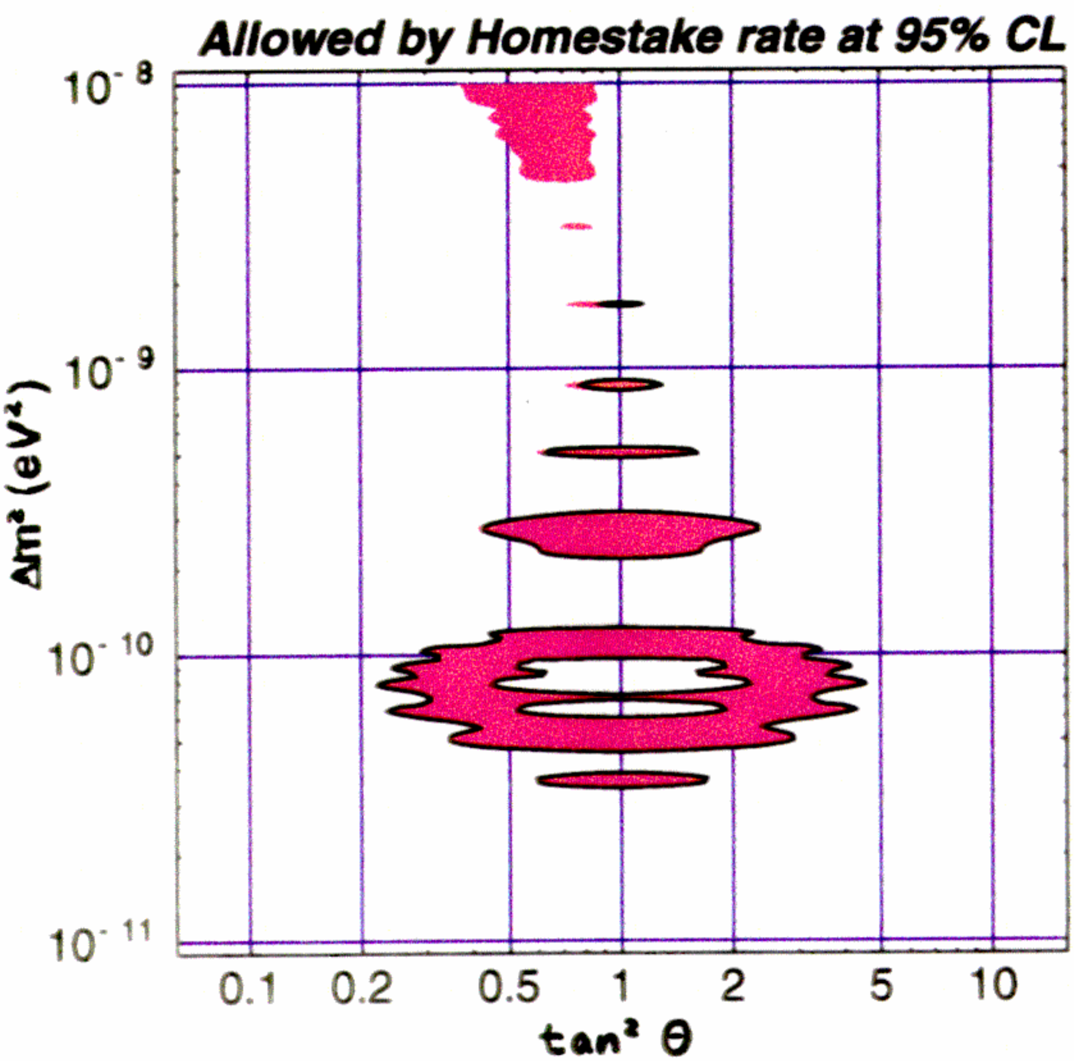
does P_c approach $\cos^2\theta$?

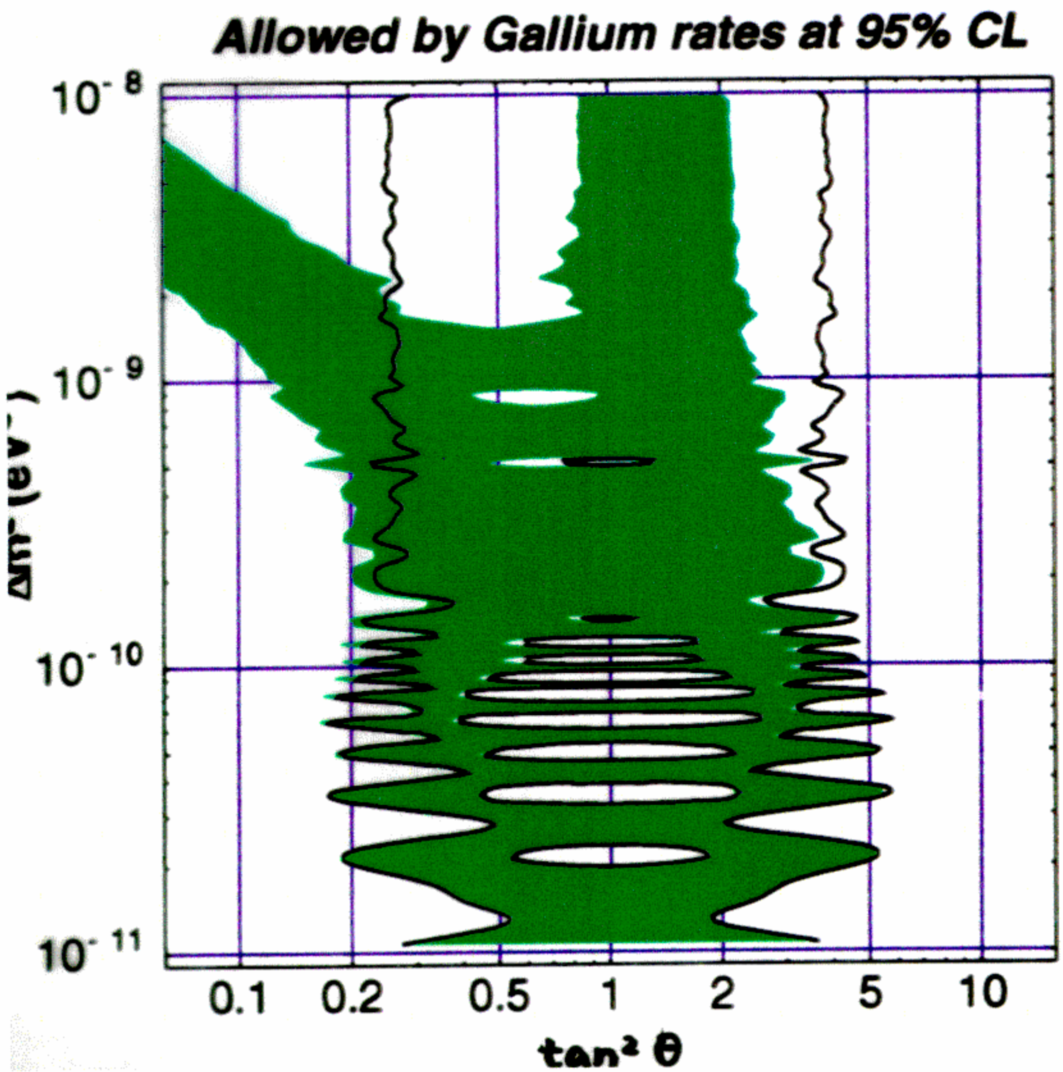
$E_\nu = 0.862$ MeV
(^{7}Be neutrino)

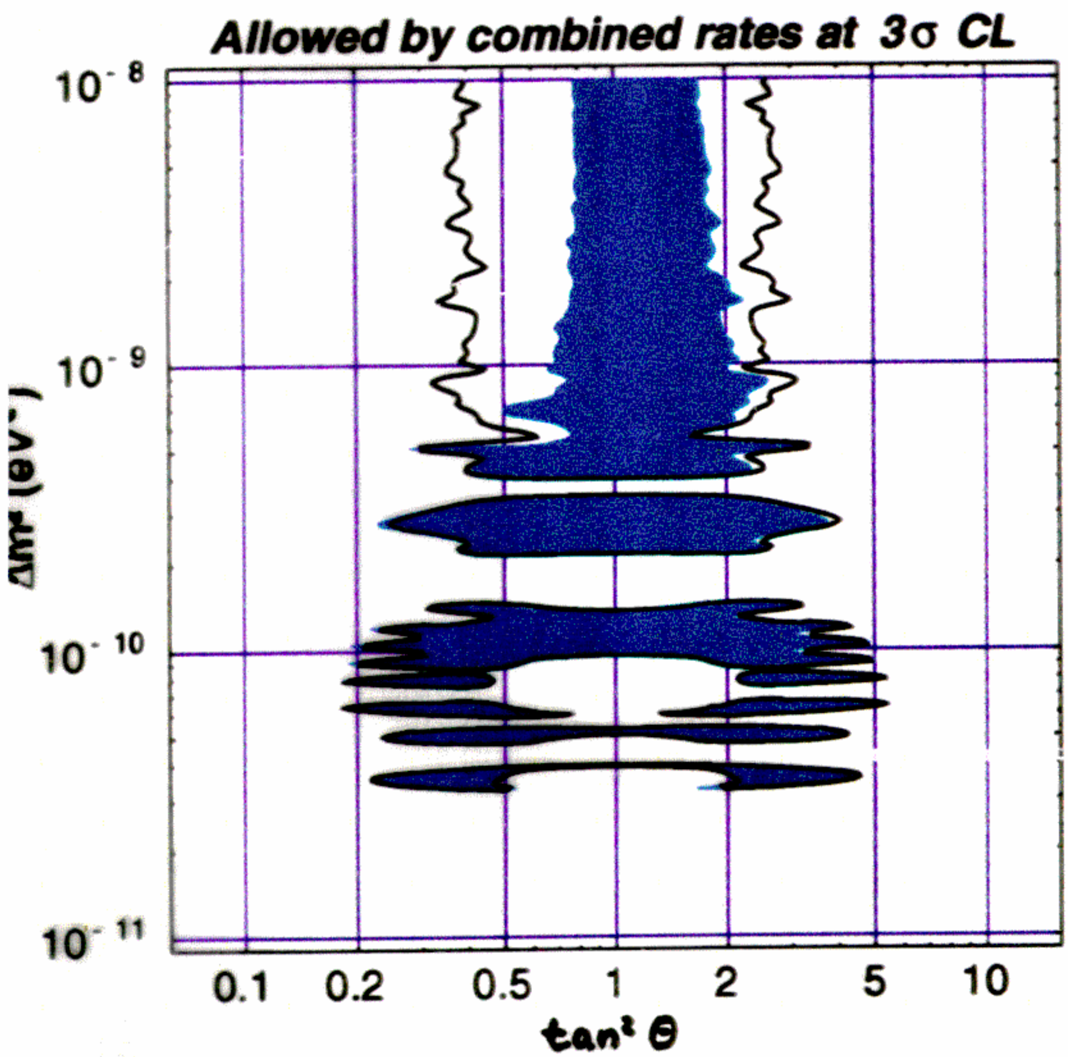




Allowed by SK rate at 95% CL
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
Δm² (eV²)
0.1
0.2
0.5
1
2
5
10
tan² θ


Allowed by Homestake rate at 95% CL
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
Δm² (eV²)
0.1
0.2
0.5
1
2
5
10
tan² θ


Allowed by Gallium rates at 95% CL
10^{-8}
10^{-9}
10^{-10}
10^{-11}
Δm^2 (eV^2)
0.1
0.2
0.5
1
2
5
10
$\tan^2\theta$


Allowed by combined rates at 3σ CL
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
Δm² (eV²)
0.1
0.2
0.5
1
2
5
10
tan² θ

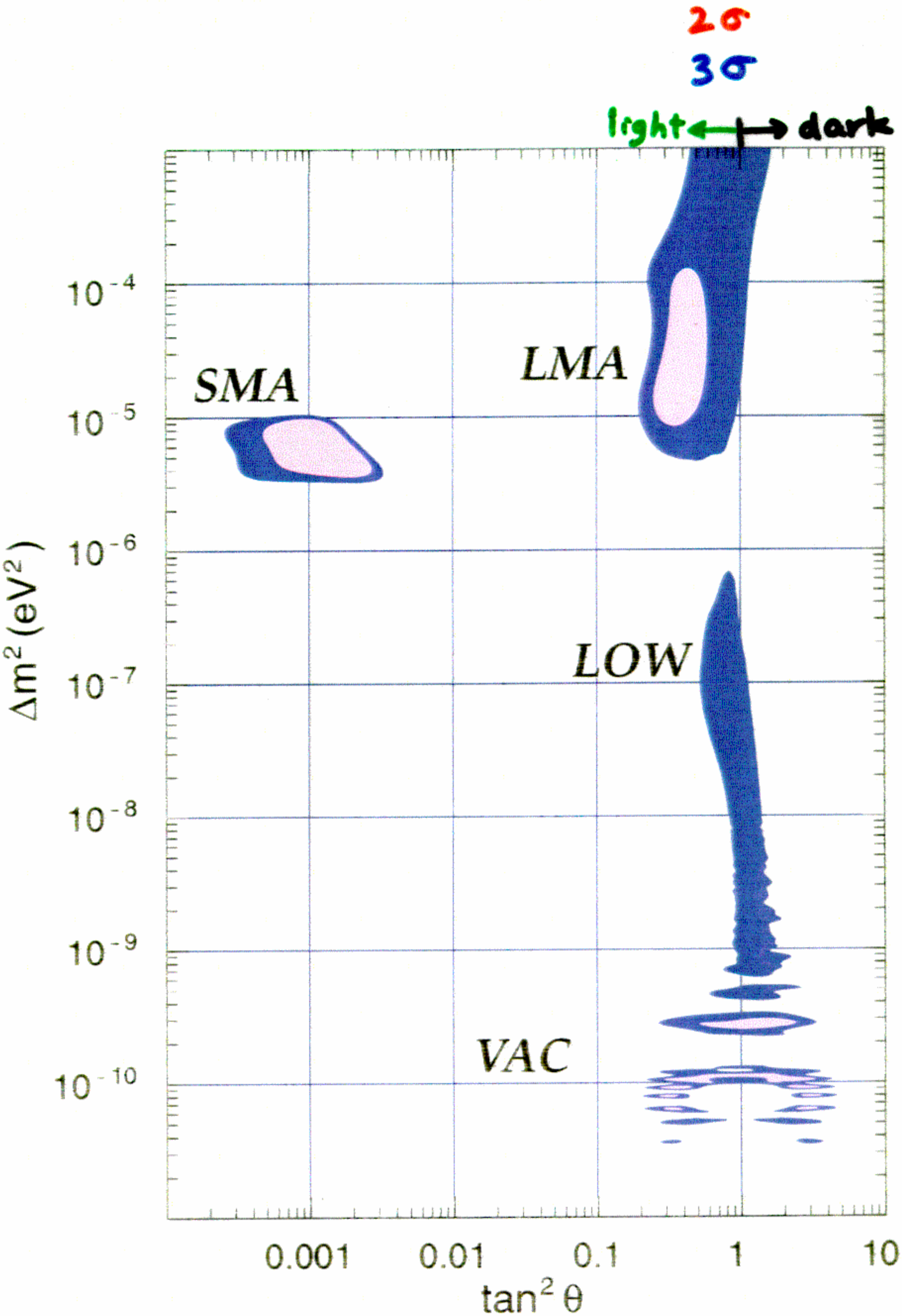
lessons

- one should not discuss "MSW solns" and "vac osc solns" separately
  - MSW effect shuts off very slowly
- the dark side more preferred than the light side for $\Delta m^2 \sim 10^{-9}\ eV^2$

§ Global Fits

performed fits to the rates

- both the light and the dark sides
- continuously from the "MSW region" to the "vacuum oscillation region"


2σ
3σ
light
dark
SMA
LMA
LOW
VAC
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$\Delta m^2 (eV^2)$
0.001
0.01
0.1
1
10
$\tan^2\theta$

Why was the dark side forgotten?

④ data don't prefer the dark side

$P_e \geq \frac{1}{2}$ on the dark side

cf. Homestake data $\sim \frac{1}{4} \times$ SSM

⇓

$\frac{1}{3} \times$ SSM BP '98

Gonzales-Garcia, Peña-Garay
hep-ph/0002186

"On the Size of the Dark Side of the Solar Neutrino Parameter Space"

global fit to all data

the dark side allowed at

CL > 99.3 % w.r.t. global minimum

CL > 82.6 % w.r.t. local minimum (LOW)

We should keep our mind open.

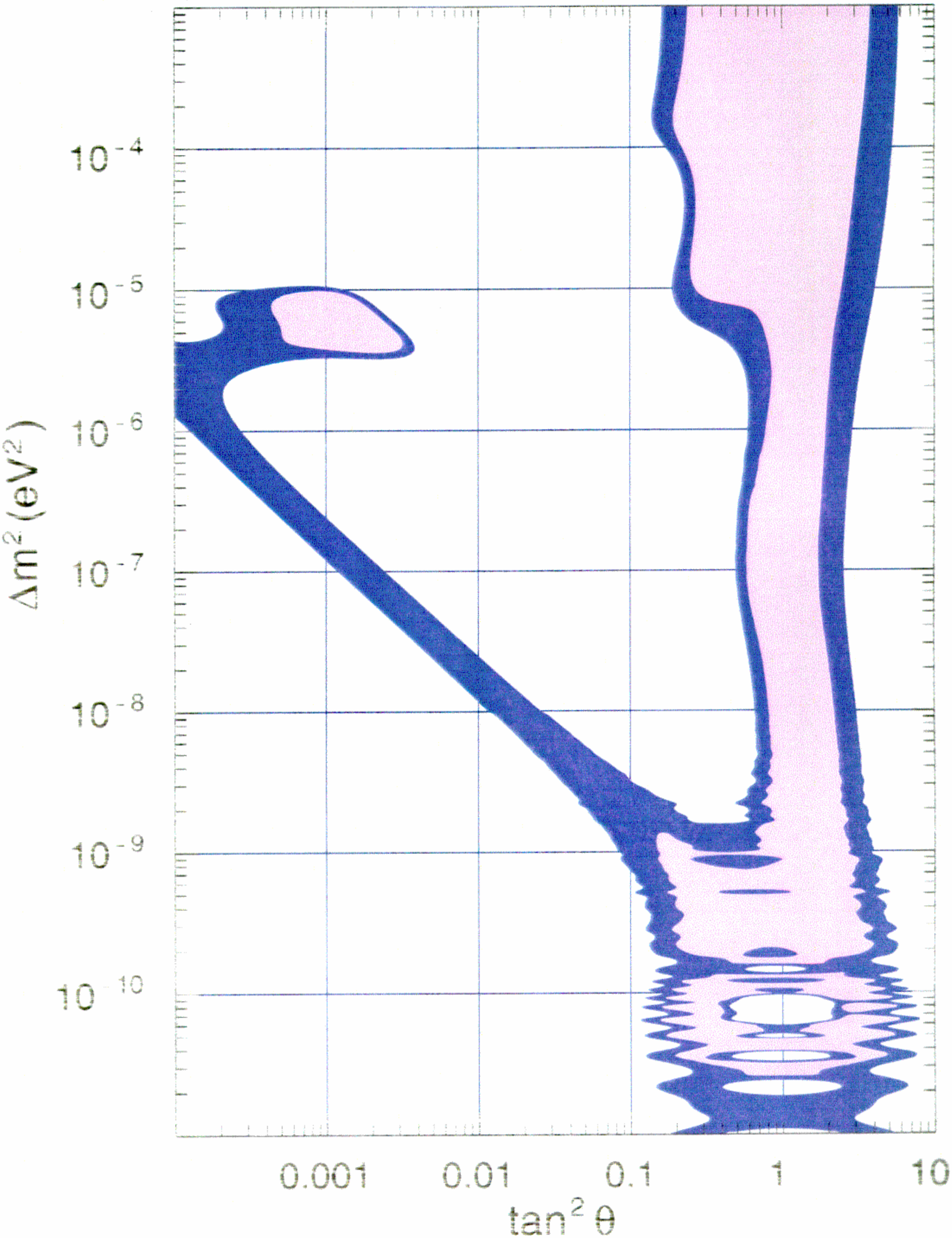
maybe some data weren't quite right

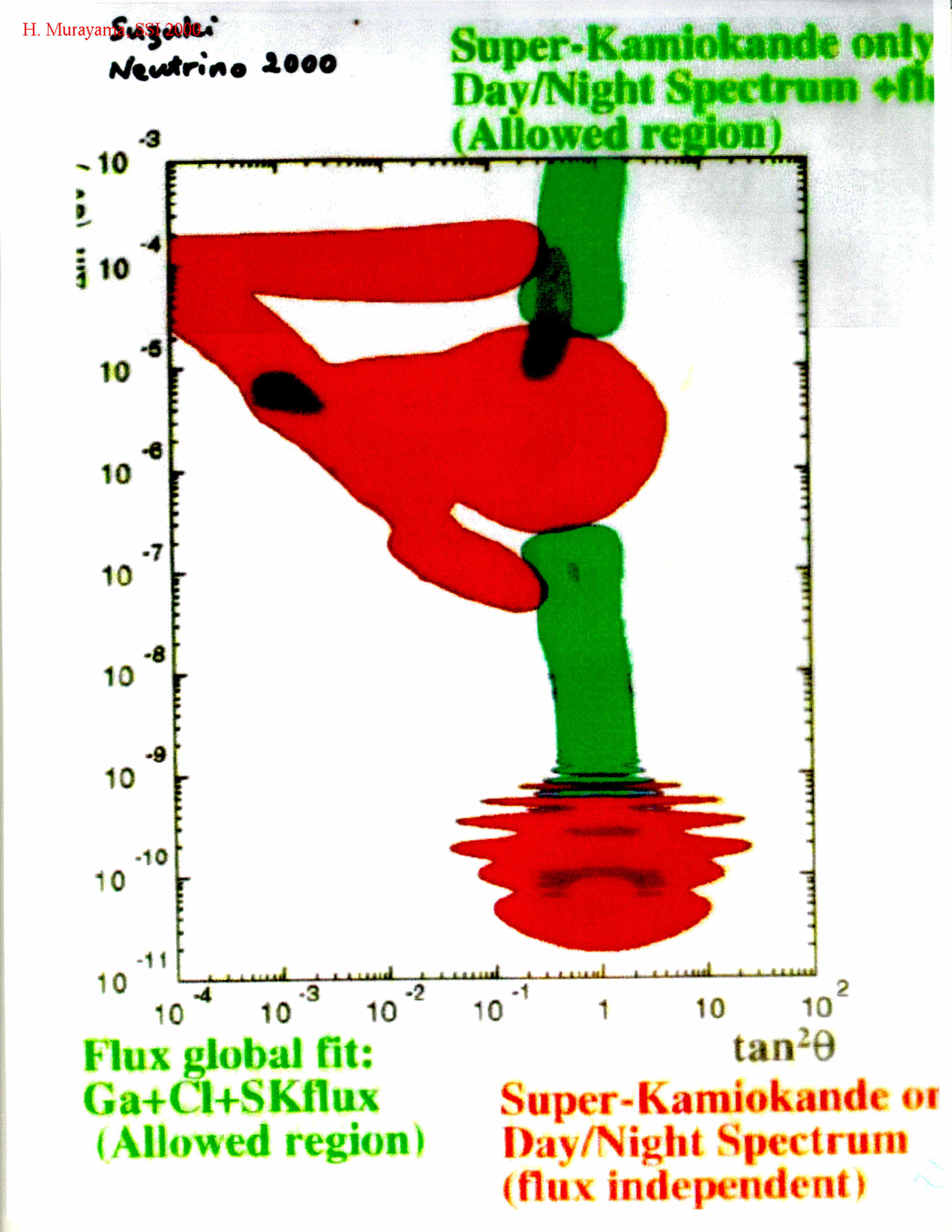
maybe the theory isn't so precise.

⇒ explore the full parameter space
w/o prejudice from the past

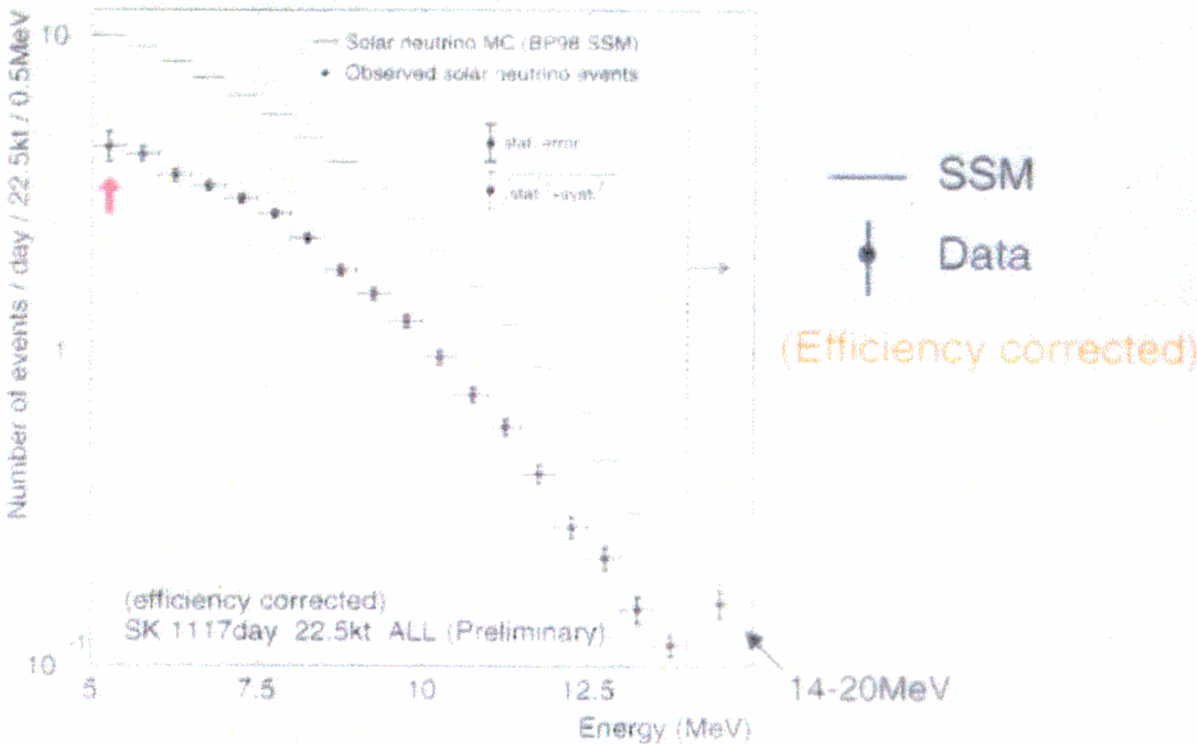
H. Murayama, SSI 2000

dropped Cl data

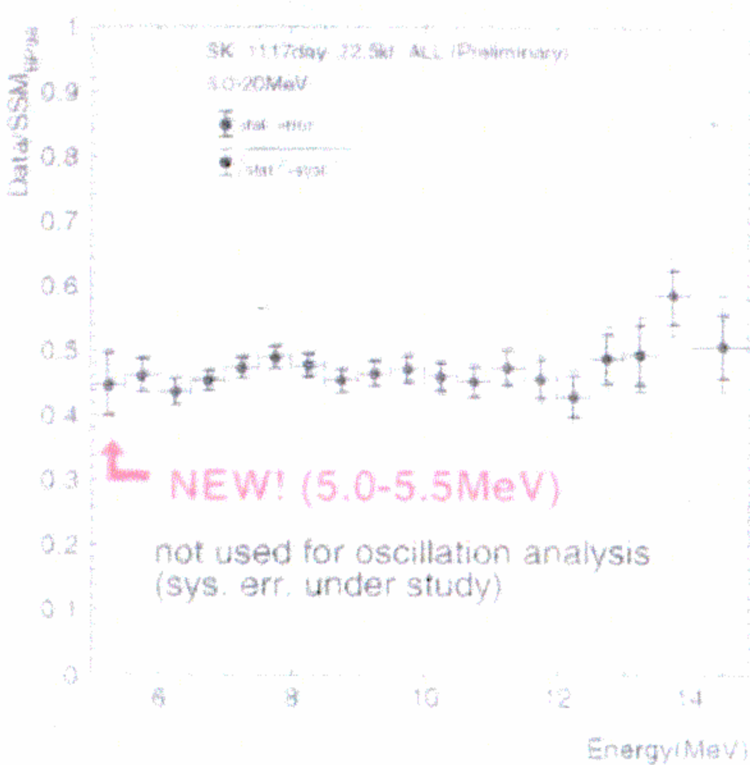



H. Murayama, SSI 2000
Suzuki
Neutrino 2000
Super-Kamiokande only
Day/Night Spectrum +fl
(Allowed region)
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-4
10^-3
10^-2
10^-1
1
10
10^2
tan²θ
Flux global fit:
Ga+Cl+SKflux
(Allowed region)
Super-Kamiokande o
Day/Night Spectrum
(flux independent)

Energy spectrum



Data/SSM(BP98)



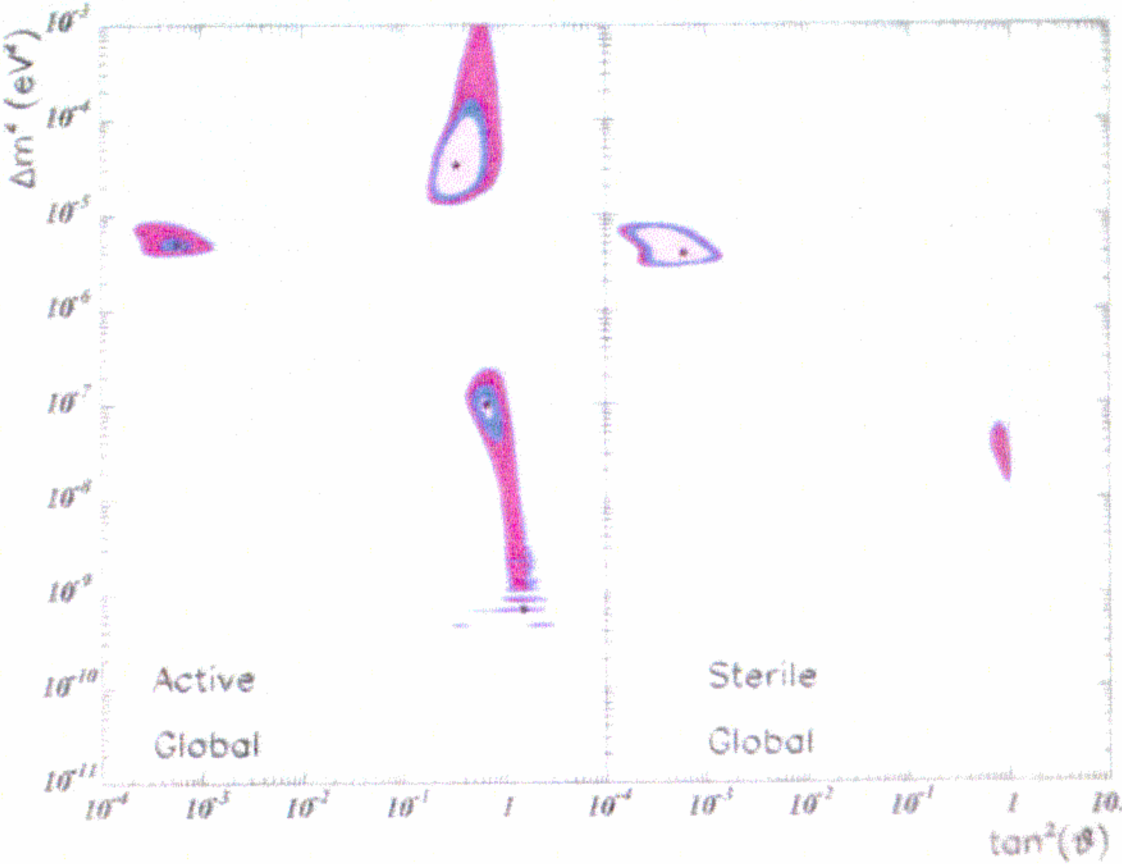
5.5-20MeV
χ^2 for flat = 13.7/17dof
(including sys. err.)
C.L.= 69 %

consistent with flat

Solutions for $\nu_e \to \nu_{active}$ and $\nu_e \to \nu_{sterile}$

Including new GNO, Sage and Super–Kamiokande 1117 Days

M.C.G-G, C. Peña-Garay in preparation



–For Active SMA allowed at 95 % CL but with smaller θ

LOW solution better than SMA after 1117 SK days

–For Sterile SMA: $\Delta m^2 = 3.8 \times 10^{-6}$ $\tan^2\theta = 0.0006$ Prob=31%

– Why differences for oscillations into active or sterile?

Main effect is different contribution to event rates in SuperK

$\nu_{\mu(\tau)} + e \to \nu_{\mu(\tau)} + e \to$ NC events in SuperK

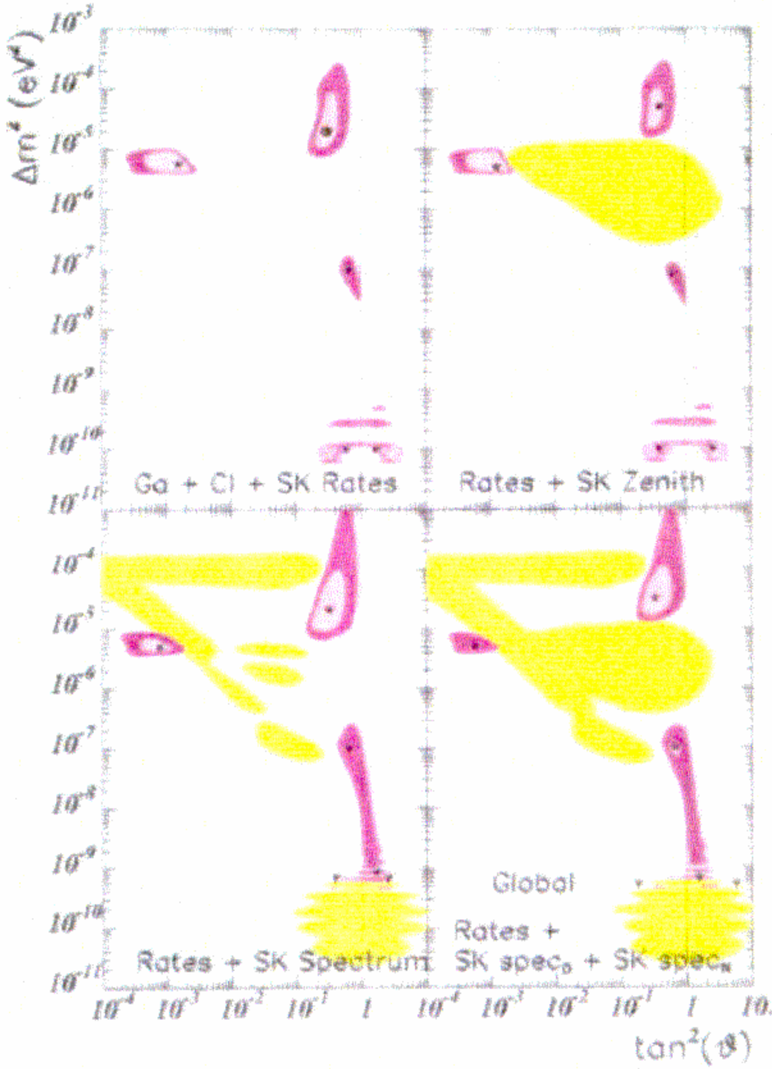
$\nu_s + e \not\to \nu_s + e \to$ no NC events in SuperK

Also slightly different survival probabilities in the sun

Solutions for $\nu_e \rightarrow \nu_{active}$ Including new GNO, Sage and SK 1117 Days

M.C.G-G, C. Peña-Garay in preparation

Effect of Day-Night and Spectrum data:



Observable					
		SMA	LMA	LOW	VAC
Rates	$\Delta m^2/\mathrm{eV}^2$	5.6×10^{-6}	1.9×10^{-5}	$9. \times 10^{-8}$	8×10^{-11}
	$\tan^2\theta$	0.0014	0.2	0.57	0.51 (1.96)
	Prob (%)	38 %	[illegible] %	0.5 %	4 %
Rates+ +Spec$_D$ +Spec$_N$	$\Delta m^2/\mathrm{eV}^2$	5.0×10^{-6}	3.2×10^{-5}	$1. \times 10^{-7}$	6.7×10^{-10}
	$\tan^2\theta$	0.00056	0.33	0.57	1.75 (QVO)
	Prob (%)	34 %	59 %	[illegible] %	32 %