Neutrino Astrophysics and Cosmology

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The "Revolutions"

- Fantastic increase in the capabilities of observational astronomy (*e.g.*, "Keck-Class" telescopes)

- Neutrino Physics has become an experimental science, with very surprising revelations recently. (and more to come!) SNO, Kamland, OMNIS MINOS, K-TOK, etc.

⇓ ⇓ ⇓ ⇓

They May Converge on the r-Process,

Supernovae, cosmology

The Three Families

$$\begin{bmatrix} \mathrm{u} \\ \mathrm{d} \\ e \\ \nu_e \end{bmatrix} \qquad \begin{bmatrix} \mathrm{s} \\ \mathrm{c} \\ \mu \\ \nu_\mu \end{bmatrix} \qquad \begin{bmatrix} \mathrm{t} \\ \mathrm{b} \\ \tau \\ \nu_\tau \end{bmatrix}$$

- Neutrinos Come in Three Flavors

(six species: ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$, ν_τ, $\bar{\nu}_\tau$)

(Majorana or Dirac)

(can also produce "sterile" neutrinos, e.g. SU(2) singlet majorana ν)

- Neutrinos Interact Weakly

$*$ Electromagnetic Cross Section:

$\gamma + e^- \mapsto \gamma + e^-, \quad \sigma \sim 10^{-24}\,\mathrm{cm}^2$

$*$ Weak Cross Section:

$\bar{\nu}_e + \mathrm{p} \mapsto \mathrm{n} + e^+, \quad \sigma \sim 10^{-44}\,\mathrm{cm}^2$

$\sigma \sim \mathrm{G}_\mathrm{F}^2 E_\nu^2$

Experimental Limits on Neutrino Masses

$$m_{\nu_\tau} < 20\,\mathrm{MeV}$$

$$m_{\nu_\mu} < 180\,\mathrm{keV}$$

$$m_{\nu_e} < 4.4\,\mathrm{eV}$$

- Ancillary constraint on Majorana mass of ν_e from absence of observation of neutrinoless double beta decay

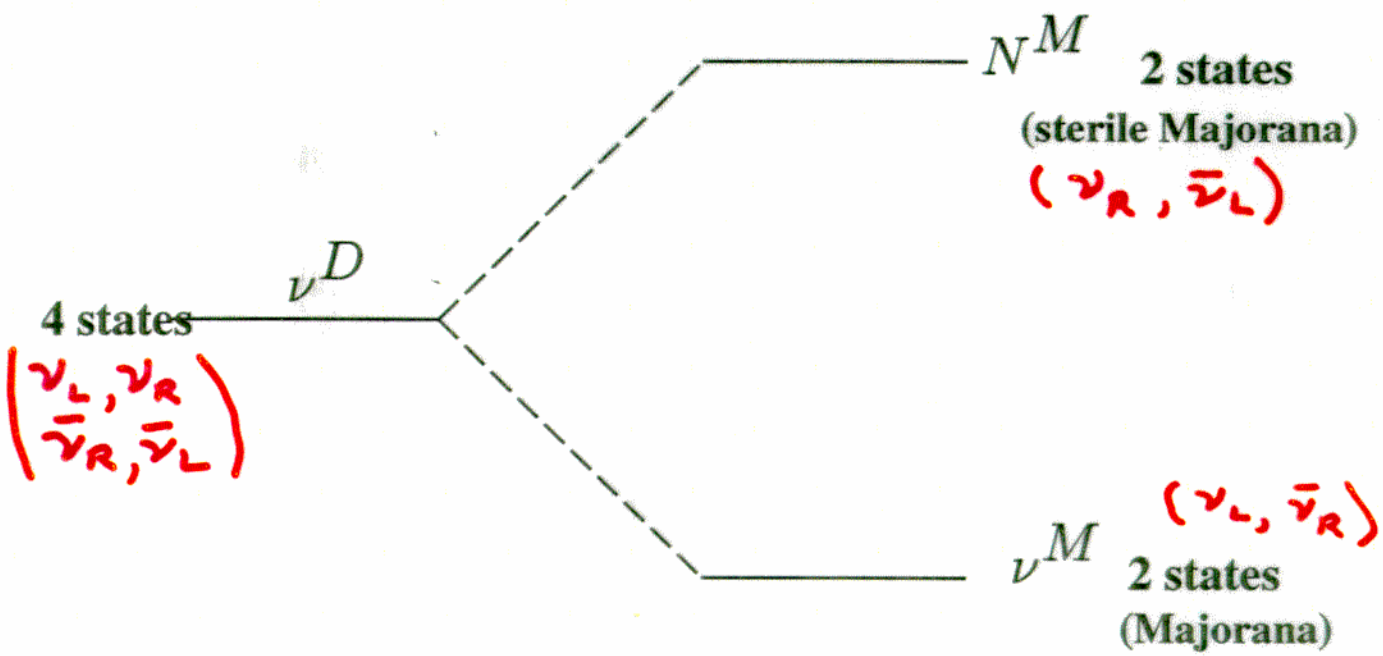
$$m_{\nu_e}^{\mathrm{MAJ}} \lesssim 1\,\mathrm{eV}$$

Why Are Neutrinos So Light?

Dirac Neutrinos: $\nu \neq \bar{\nu}$ $\quad \nu + \bar{\nu} = 4$ **states**

Majorana Neutrinos: $\nu = \bar{\nu}$ $\quad \nu + \bar{\nu} = 2$ **states**

The "See-Saw" Mechanism



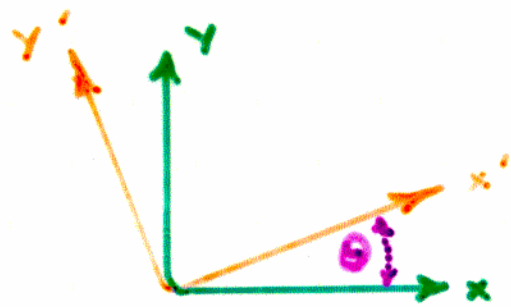
See-Saw Relation: $(m_{\nu^M})(m_{N^M}) \approx m^2_{\nu^D} \sim m_q^2$ **squared-quark mass**

$\sim m_l^2$ **squared-lepton mass**

(See Gell-Mann, Ramond, Slansky; Yanagida; Mohapatra & Senjanovic)

Photons

* 2 polarization states
  photon state $|\psi\rangle$



$$|\psi\rangle = |x\rangle\langle x|\psi\rangle + |y\rangle\langle y|\psi\rangle$$

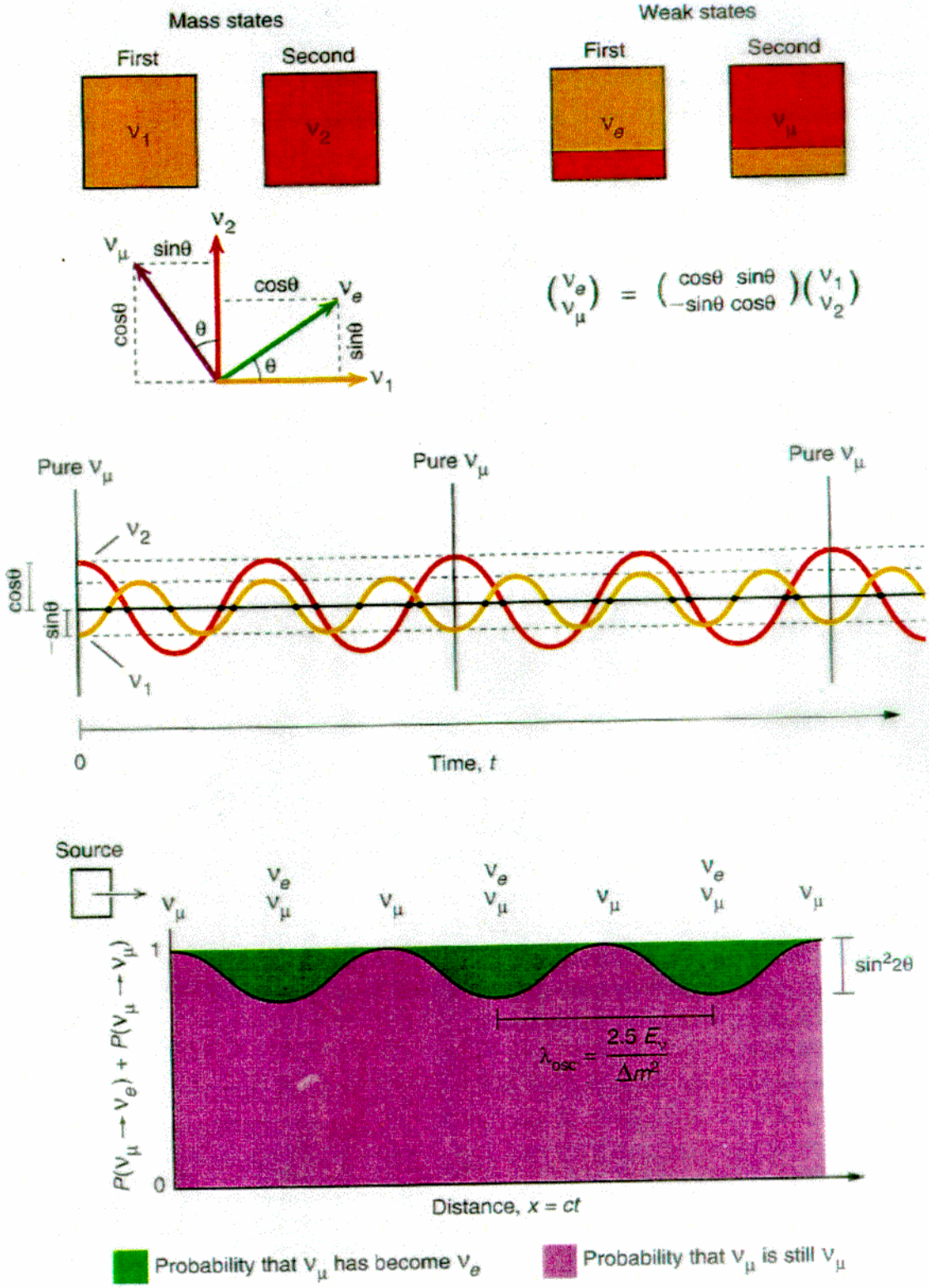
$$|\psi\rangle = |x'\rangle\langle x'|\psi\rangle + |y'\rangle\langle y'|\psi\rangle$$

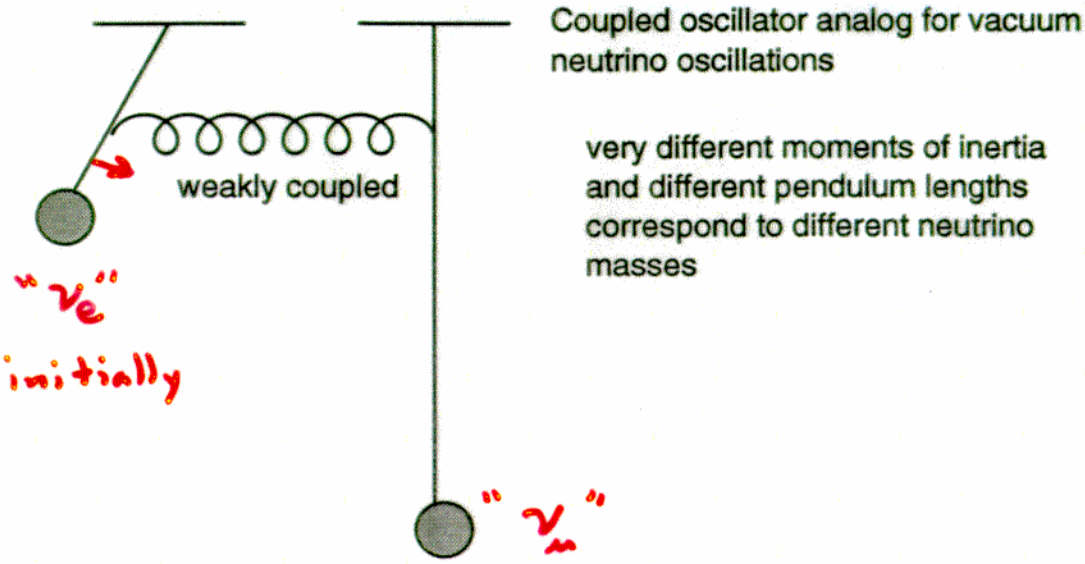
bases related by a
unitary transformation

$$\Downarrow \qquad \Downarrow$$

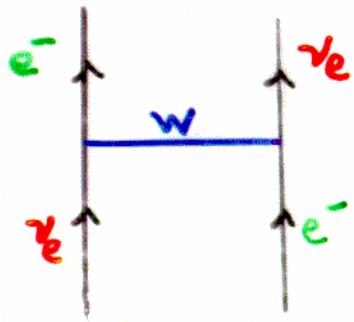
$$|x'\rangle = \cos\theta\,|x\rangle + \sin\theta\,|y\rangle$$

$$|y'\rangle = -\sin\theta\,|x\rangle + \cos\theta\,|y\rangle$$


Mass states
First
Second
ν_1
ν_2
Weak states
First
Second
ν_e
ν_μ
ν_μ
sinθ
ν_2
cosθ
ν_e
cosθ
θ
sinθ
θ
ν_1
$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$
Pure ν_μ
Pure ν_μ
Pure ν_μ
ν_2
cosθ
sinθ
ν_1
0
Time, t
Source
ν_μ
ν_e ν_μ
ν_μ
ν_e ν_μ
ν_μ
ν_e ν_μ
ν_μ
1
0
$P(\nu_\mu \to \nu_e) + P(\nu_\mu \to \nu_\mu)$
$\sin^2 2\theta$
$\lambda_{osc} = \frac{2.5\, E_\nu}{\Delta m^2}$
Distance, $x = ct$
Probability that ν_μ has become ν_e
Probability that ν_μ is still ν_μ



But, as neutrinos move through matter the ν_e acquires an effective mass term which the ν_μ does not acquire. This extra mass term for ν_e stems from charged current exchange



This will effectively make the ν_e "pendulum" longer in this analogy

Photons

* index of refraction n, or "effective mass", from forward scattering

$$\frac{\omega}{k} = \frac{c}{n}$$

* "optically active" materials e.g., calcite crystals

$\Downarrow$

index of refraction depends on photon polarization state

index	polarization
n_e	polarized // to axis
n_o	polarized $\perp$ to axis

$$n_o > n_e$$

Summary of MSW Mechanism

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

- Solve for:

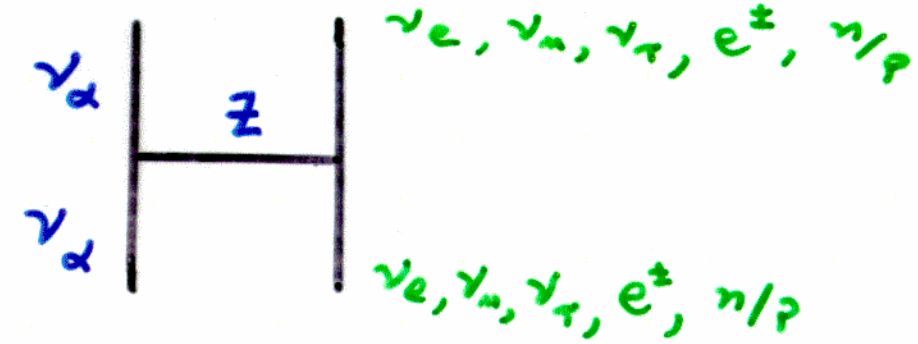
$$|\nu(t)\rangle = a_e(t)|\nu_e\rangle + a_\mu(t)|\nu_\mu\rangle$$

$$i\frac{d}{dt}\begin{bmatrix} a_e(t) \\ a_\mu(t) \end{bmatrix} = H\begin{bmatrix} a_e(t) \\ a_\mu(t) \end{bmatrix}$$

- Define:

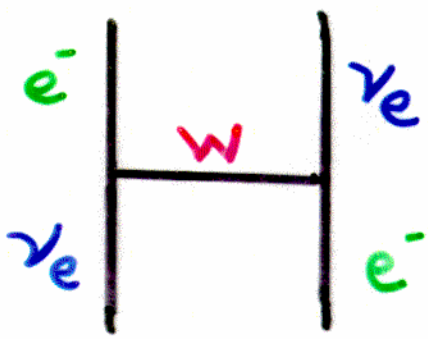
$$\boxed{\delta m^2 \equiv m_2^2 - m_1^2}$$

Forward scattering

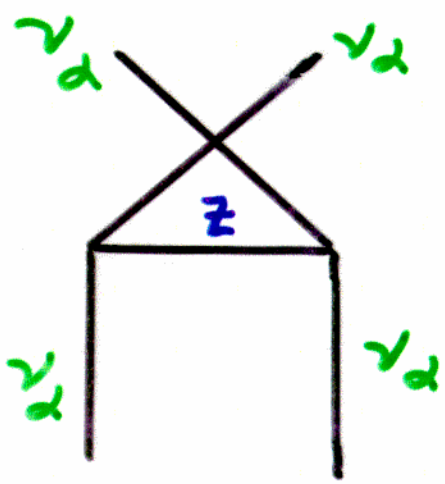


Neutral current

$\alpha = e, \mu, \tau$



charged current exchange



Neutral current exchange

Matter Effects and H

$$\boxed{H = H_e + H_{\nu\nu}}$$

- Neutrino-Electron Charged Current

Forward Exchange Scattering Contribution $\Rightarrow H_e$

In the flavor basis:

$$H_e = \frac{1}{2}\begin{pmatrix} -\Delta\cos 2\theta + A & \Delta\sin 2\theta \\ \Delta\sin 2\theta & \Delta\cos 2\theta - A \end{pmatrix}$$

where $\Delta = \frac{\delta m^2}{2E_\nu}$ and $A = \sqrt{2}G_F(n_{e^-} - n_{e^+})$

- Neutrino-Neutrino Neutral Current

Forward Exchange Scattering Contribution $\Rightarrow H_{\nu\nu}$

$$H_{\nu\nu} = \begin{pmatrix} B & B_{e\mu} \\ B_{\mu e} & -B \end{pmatrix}$$

Neutrino Potentials

for active/sterile transformation ⇒

$$H_{int}(\nu_e) = \sqrt{2}\, G_F \left(2(n_{\nu_e} - n_{\bar{\nu}_e}) + (n_{\nu_\mu} - n_{\bar{\nu}_\mu}) + (n_{\nu_\tau} - n_{\bar{\nu}_\tau}) + (n_{e^-} - n_{e^+}) - \tfrac{1}{2} n_n \right)$$

$$H_{int}(\nu_\mu) = \sqrt{2}\, G_F \left((n_{\nu_e} - n_{\bar{\nu}_e}) + 2(n_{\nu_\mu} - n_{\bar{\nu}_\mu}) + (n_{\nu_\tau} - n_{\bar{\nu}_\tau}) - \tfrac{1}{2} n_n \right)$$

$$H_{int}(\bar{\nu}_e) = -H_{int}(\nu_e)$$

$$H_{int}(\nu_s) = 0$$

note:

$$n_n = n_b - n_p = \rho N_A (1 - Y_e) \qquad Y'_{\nu_x} \equiv \frac{1}{\rho N_A} \left\{ n_{\nu_x}^{eff} - n_{\bar{\nu}_x}^{eff} \right.$$

channel	A	B
$\nu_e \rightleftarrows \nu_s$	$\frac{3\sqrt{2}}{2} G_F \rho N_A (Y_e - \frac{1}{3})$	$\sqrt{2}\, G_F \rho N_A (2Y'_{\nu_e} + Y'_{\nu_\mu} + Y'_{\nu_\tau})$
$\nu_\mu \rightleftarrows \nu_s$	$\frac{\sqrt{2}}{2} G_F \rho N_A (Y_e - 1)$	$\sqrt{2}\, G_F \rho N_A (Y'_{\nu_e} + 2Y'_{\nu_\mu} + Y'_{\nu_\tau})$
$\nu_\tau \rightleftarrows \nu_s$	$\frac{\sqrt{2}}{2} G_F \rho N_A (Y_e - 1)$	$\sqrt{2}\, G_F \rho N_A (Y'_{\nu_e} + Y'_{\nu_\mu} + 2Y'_{\nu_\tau})$
$\nu_\mu \rightleftarrows \nu_e$	$\sqrt{2}\, G_F \rho N_A Y_e$	$\sqrt{2}\, G_F \rho N_A (Y'_{\nu_e} - Y'_{\nu_\mu})$
$\nu_\tau \rightleftarrows \nu_e$	$\sqrt{2}\, G_F \rho N_A Y_e$	$\sqrt{2}\, G_F \rho N_A (Y'_{\nu_e} - Y'_{\nu_\tau})$
$\nu_\mu \rightleftarrows \nu_\tau$	0	$\sqrt{2}\, G_F \rho N_A (Y'_{\nu_\mu} - Y'_{\nu_\tau})$

In high density supernova matter electrons can be relativistically degenerate

$\Rightarrow$ Relativistic Treatment of MSW

$$\mathcal{L}_{tot} = \bar{\Psi}_\nu (i\not{\partial} - m_\nu)\Psi_\nu + \bar{\Psi}_e (i\not{\partial} - m_e)\Psi_e - \frac{G_F}{\sqrt{2}}\left(\bar{\Psi}_\nu \gamma^\mu (1-\gamma_5)\Psi_\nu\right)\left(\bar{\Psi}_e \gamma_\mu (1-\gamma_5)\Psi_e\right)$$

$\Rightarrow$ Define $A^\mu \equiv \frac{G_F}{\sqrt{2}}\left(\bar{\Psi}_e \gamma^\mu (1-\gamma_5)\Psi_e\right)$

$$= (\varphi, \vec{A})$$

then,

$$\underbrace{\mathcal{L}_\nu = \bar{\Psi}_\nu (i\not{\partial} - \not{A}(1-\gamma_5) - m_\nu)\Psi_\nu}_{\left(i\frac{\partial}{\partial t} - \varphi\right)\Psi_\nu = \left[\vec{\alpha}\cdot\left(\frac{1}{i}\vec{\nabla} - \vec{A}\right) + \beta m_\nu\right]\Psi_\nu}$$

This Dirac Equation implies the following kinematic relation

$$(E_\nu - \varphi)^2 = (\vec{P}_\nu - \vec{A})^2 + m_\nu^2$$

$$\Rightarrow E_\nu^2 = P_\nu^2 + \left(m_\nu^2 + \underbrace{\left(2E_\nu\varphi - 2\vec{P}_\nu\cdot\vec{A}\right) + \left(\vec{A}^2 - \varphi^2\right)}_{\text{when averaged over the electron distribution this gives } m^2_{eff}}\right)$$

$$m^2_{eff} = 2E_\nu\langle\varphi\rangle - \langle 2\vec{P}_\nu\cdot\vec{A}\rangle + \left(\langle\vec{A}^2\rangle - \langle\varphi^2\rangle\right)$$

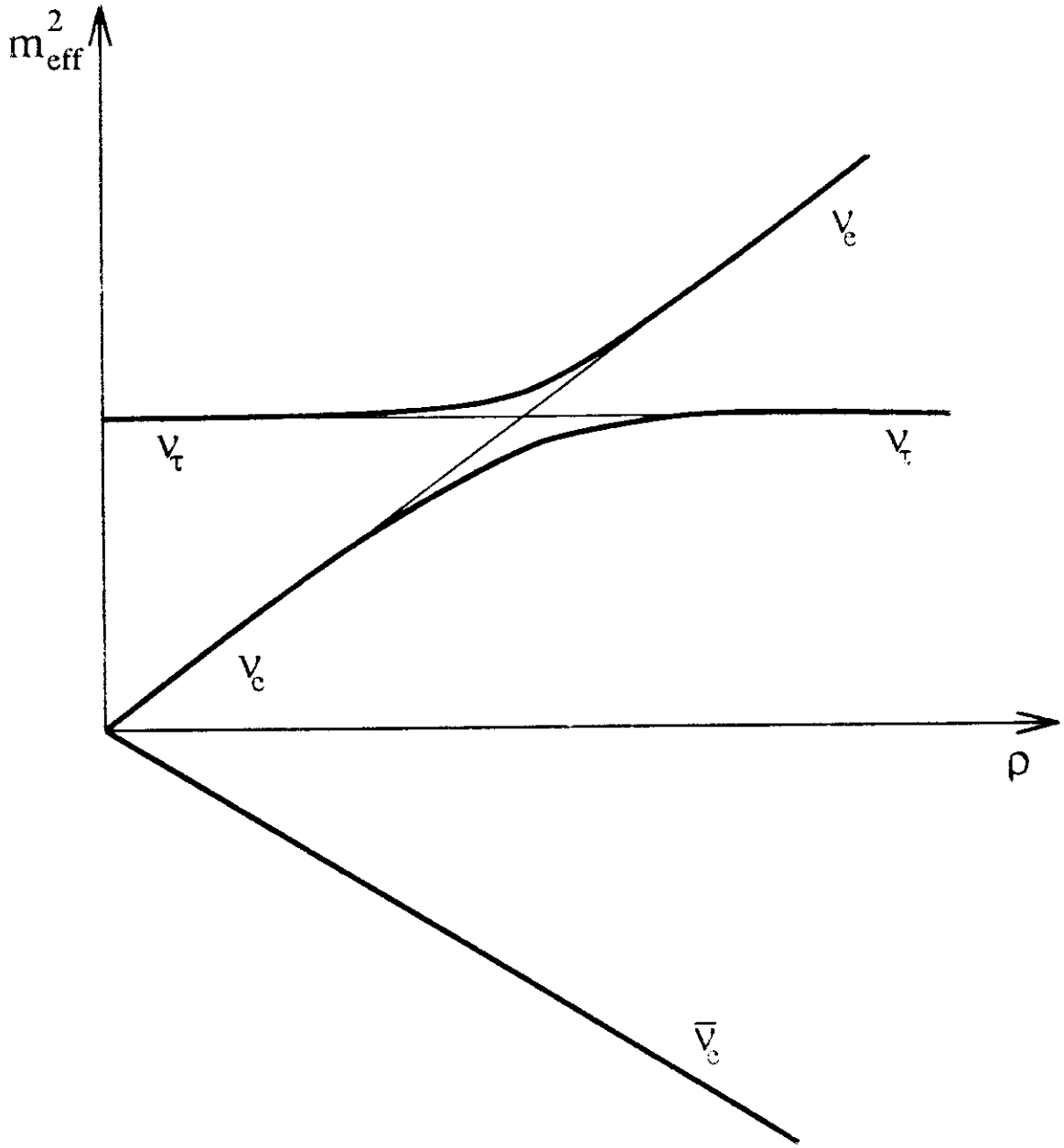
MSW result

$$2E_\nu \frac{2G_F}{\sqrt{2}}\left(\bar{\psi}_e\gamma^0(1-\gamma_5)\psi_e\right)$$

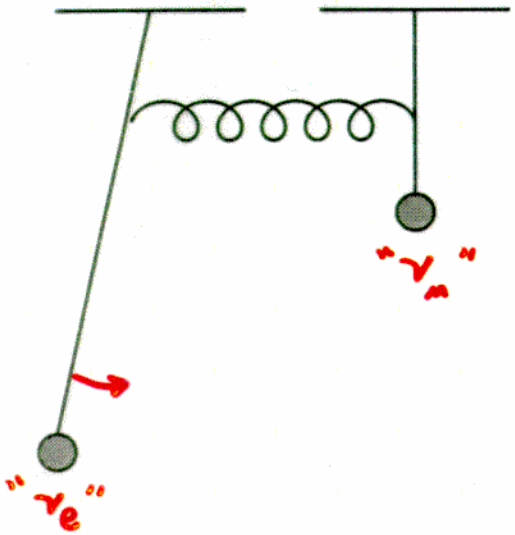
$$\simeq 1.518\times10^{-7}\,\mathrm{eV}^2\left(\rho N_a Y_e\right)\left(\frac{E_\nu}{\mathrm{MeV}}\right)$$

$\Downarrow$ isotropic electron distrib $\Rightarrow = 0$

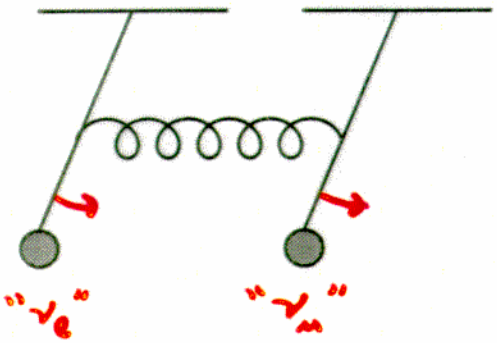
$\downarrow$ $\sim G_F^2$ negligibl


m^2_{eff}
ν_e
ν_τ
ν_τ
ν_e
ρ
$\bar{\nu}_e$

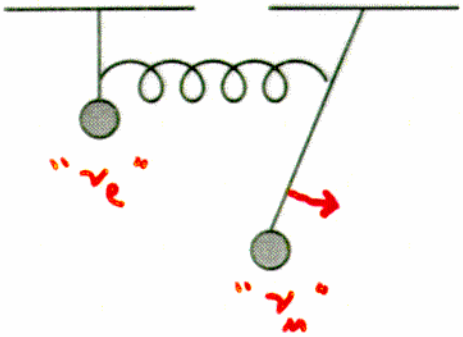
Coupled Oscillator Analog for Matter-Enhanced Neutrino Transformation



Electron neutrino created at very high density, above resonance



At resonance, intermediate density



At very low density, well below the resonance density

Here, all the "oscillation" and energy has been transferred to the muon neutrino

Example: active-active matter enhancement

- Vacuum Propagation Hamiltonian in extreme relativistic limit $E = (p^2 + m^2)^{1/2} \approx p + \frac{m^2}{2p} \Rightarrow$

$$i\frac{d\psi_m}{dt} = \left\{ pI + \frac{1}{2p}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \right\} \psi_m$$

Here the vacuum energy (or "mass") eigenstates are $|\nu_1\rangle$ and $|\nu_2\rangle$ and the amplitudes to be in these states are:

$$\psi_m = \begin{pmatrix} \langle \nu_1 | \Psi \rangle \\ \langle \nu_2 | \Psi \rangle \end{pmatrix}$$

We can re-write the above using

$$\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} = \frac{(m_1^2 + m_2^2)}{2} I + \frac{\delta m^2}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

to obtain

$$i\frac{d\psi_m}{dt} = \left\{ \left(p + \frac{(m_1^2 + m_2^2)}{4p} \right) I + \frac{\delta m^2}{4p}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \right\} \psi_m$$

Now we can transform to the flavor basis:

$$\psi_f = \begin{pmatrix} \langle \nu_e | \Psi \rangle \\ \langle \nu_\mu | \Psi \rangle \end{pmatrix} = U_m \psi_m$$

where

$$U_m = \begin{pmatrix} \langle \nu_e | \nu_1 \rangle & \langle \nu_e | \nu_2 \rangle \\ \langle \nu_\mu | \nu_1 \rangle & \langle \nu_\mu | \nu_2 \rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

So we get $i\frac{d\psi_f}{dt} = (U_m H_m U_m^\dagger)\,\psi_f$, or

$$i\frac{d\psi_f}{dt} = \left\{ \left(p + \frac{m_1^2 + m_2^2}{4p}\right) I + \frac{\delta m^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \right\} \psi_f$$

where we note that

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- Now add the flavor basis matter-interaction term with $A \equiv \sqrt{2} G_F n_e$

$$\begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} = \frac{A}{2} I + \frac{A}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Therefore, in the flavor basis we now have:

$$i\frac{d\psi_f}{dt} = \left\{ \alpha I + \frac{1}{2} \begin{pmatrix} A - \Delta\cos 2\theta & \Delta \sin 2\theta \\ \Delta \sin 2\theta & \Delta\cos 2\theta - A \end{pmatrix} \right\} \psi_f$$

where $\alpha = \left(p + \frac{m_1^2 + m_2^2}{4p} + \frac{A}{2}\right)$ simply gives an overall common phase to the amplitudes and can be ignored in computations of flavor transformation rates (but note that it will remain important for calculating effective masses); and where we define

$$\Delta \equiv \frac{\delta m^2}{2p} \approx \frac{\delta m^2}{2E}.$$

- The flavor basis evolution equation in matter can now be re-written in terms of an "effective matter mixing angle" $\theta_M(t)$, where we anticipate that this angle will depend on density and, hence, on the evolution parameter for the neutrino, t. In what follows we will not append the explicit time dependence in this expression but simply write θ_M. However it must be remembered that, unlike the vacuum case, the mixing angle between the flavor basis and the *instantaneous* mass basis will be time dependent in general.

Note that we can simply re-write the above flavor basis matter evolution equation with an appropriate definition:

$$\imath \frac{d\psi_f}{dt} = \left\{ \alpha I + \frac{\Delta_{\text{eff}}}{2} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix} \right\} \psi_f ;$$

where we have *defined*:

$$\cos 2\theta_M \equiv \frac{\Delta \cos 2\theta - A}{\Delta_{\text{eff}}};$$

$$\sin 2\theta_M \equiv \frac{\Delta \sin 2\theta}{\Delta_{\text{eff}}};$$

and

$$\Delta_{\text{eff}} \equiv \sqrt{(\Delta \cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta}$$

- Now we can note that θ_M really is an instantaneous matter mixing angle and define the time dependent unitary transformation between the intantaneous mass basis ($|\nu_1(t)\rangle$ and $|\nu_2(t)\rangle$) and the flavor basis amplitudes as $U_M(t)$:

$$U_M(t) \equiv \begin{pmatrix} \langle \nu_e | \nu_1(t) \rangle & \langle \nu_e | \nu_2(t) \rangle \\ \langle \nu_\mu | \nu_1(t) \rangle & \langle \nu_\mu | \nu_2(t) \rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{pmatrix},$$

so that $\psi_f = U_M(t)\psi_M$. Note that we can write:

$$\begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix} = U_M(t) \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} U_M^\dagger(t).$$

We also note that

$$\imath \frac{d(U_M(t)\psi_M)}{dt} = U_M(t)\left(\imath \frac{d\psi_M}{dt}\right) + \imath \left(\frac{dU_M(t)}{dt}\right)\psi_M$$

and

$$U_M^\dagger(t)\dot{U}_M(t) = \begin{pmatrix} 0 & \dot{\theta}_M \\ -\dot{\theta}_M & 0 \end{pmatrix}$$

with for example, $\dot{\theta}_M \equiv d\theta_M/dt$.

• So that by mathematical analogy with the above vacuum case we can immediately write down the full matter evolution equation in the instantaneous mass basis:

$$i\frac{d\psi_M}{dt} = \left\{\alpha I + \frac{\Delta_{\rm eff}}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -i\dot{\theta}_M \\ i\dot{\theta}_M & 0 \end{pmatrix}\right\}\psi_M$$

Of course, the degree to which the instantaneous mass states evolve independently depends on the magnitude of the off-diagonal terms proportional to $\dot{\theta}_M$. If the effective matter density is changing slowly enough, then we could make the **ADIABATIC APPROXIMATION** and set $\dot{\theta}_M = 0$. In this limit it is clear that there will be no jump between the evolutionary tracks for the different mass states. In other words, a neutrino born in a flavor state which is approximately the heavy mass eigenstate at high density will **remain** as the heavy mass eigenstate as the neutrino propagates to lower density and eventually into vacuum. In this case the neutrino would be smoothly "rotated" from the flavor state in which it was born at high density to a different flavor state in vacuum.

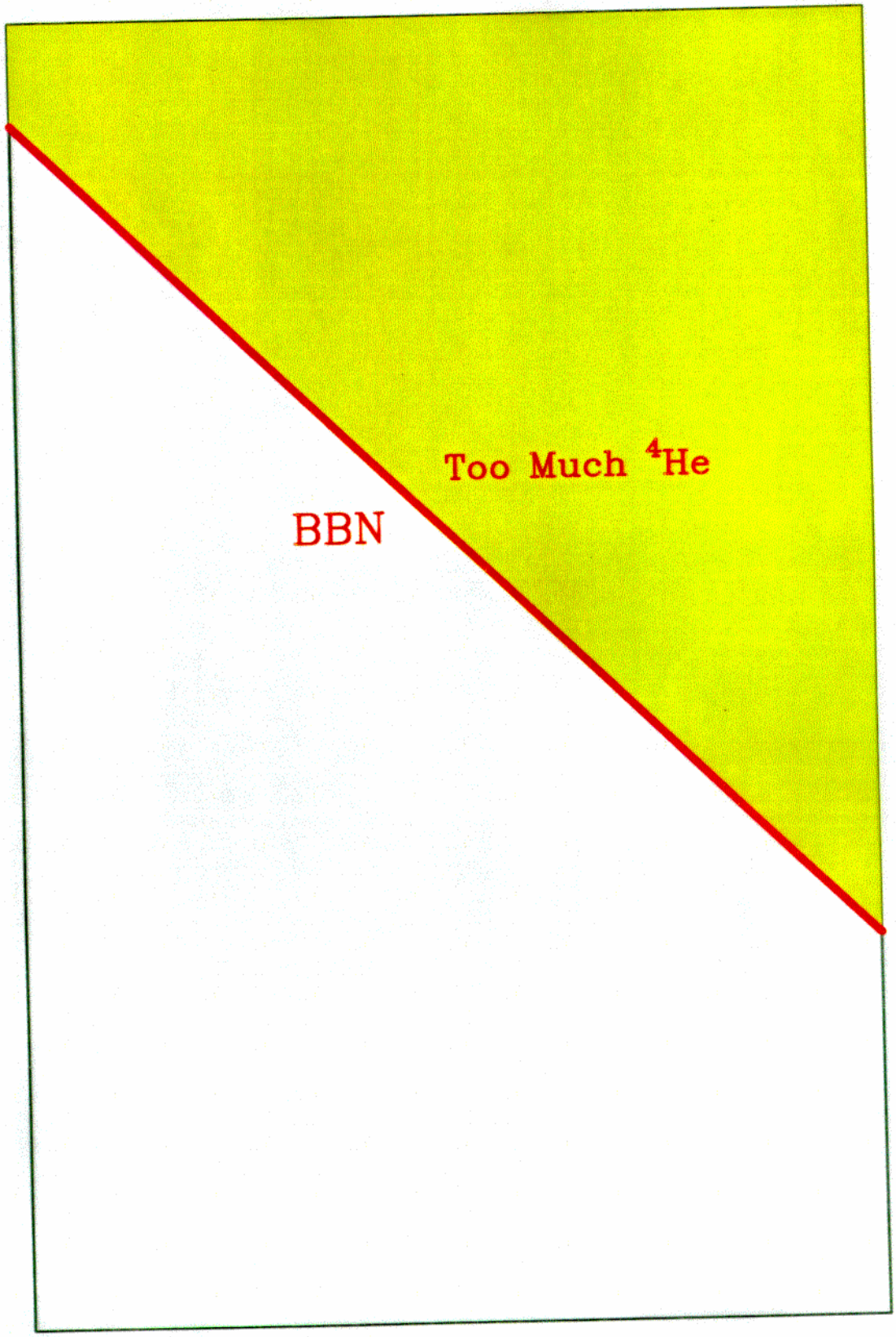
In this example, a resonance (level crossing) occurs where $\Delta \cos 2\theta = A$, and at this point the effective matter mixing angle is maximal, $\pi/4$. The adiabatic approximation will be valid when the oscillation length at resonance is very small compared to the resonance width. The oscillation length at resonance is:

$$L_{\rm osc}^{\rm res} = \frac{4\pi E}{\delta m^2 \sin 2\theta};$$

while the resonance width is

$$\delta r = H \tan 2\theta,$$

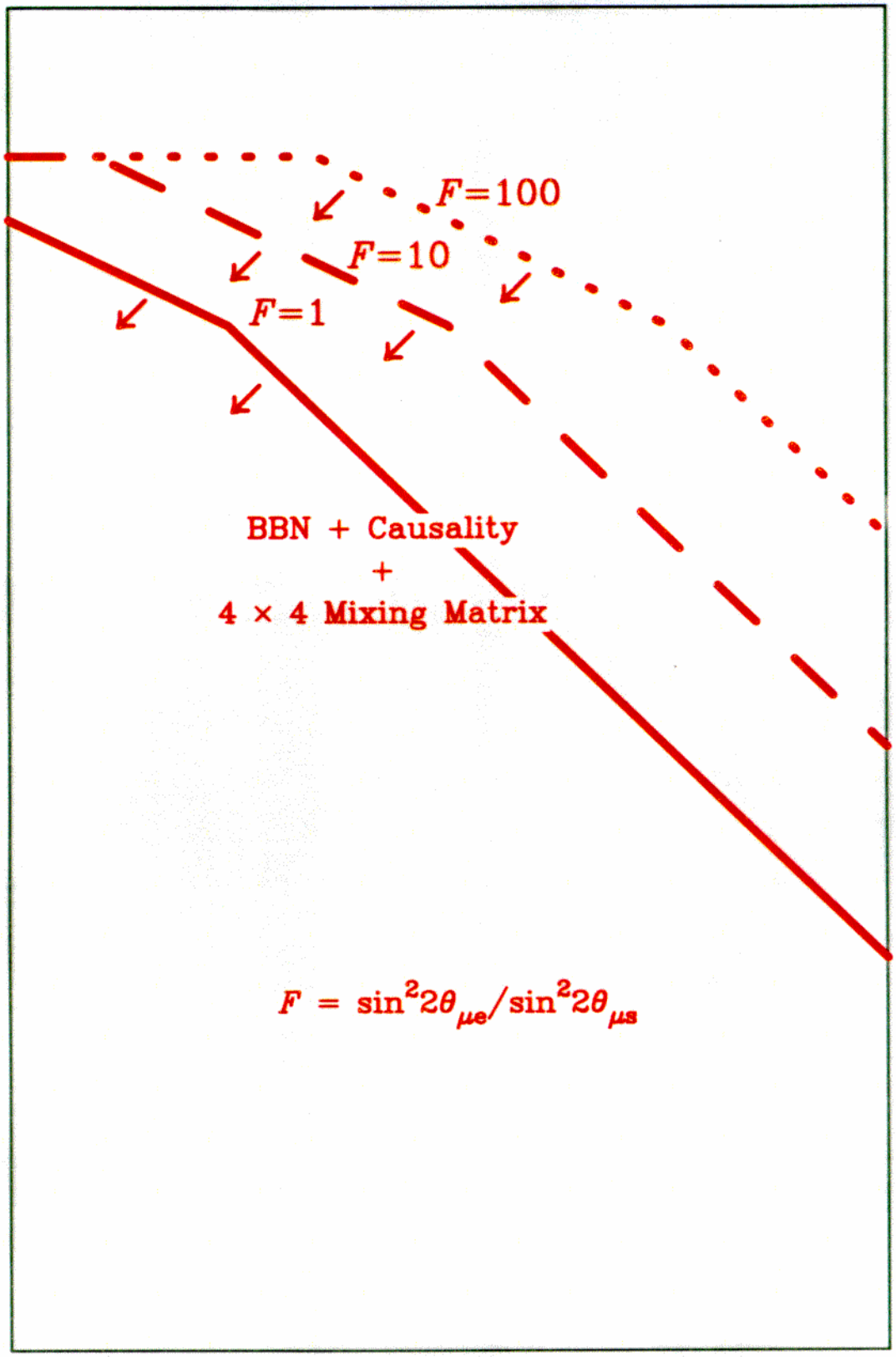
where H is the effective density scale height of weak charges (in this example, the net number of electrons).



(Enqvist *et al.*, 1992)
(Shi, Schramm and Fields, 1993)

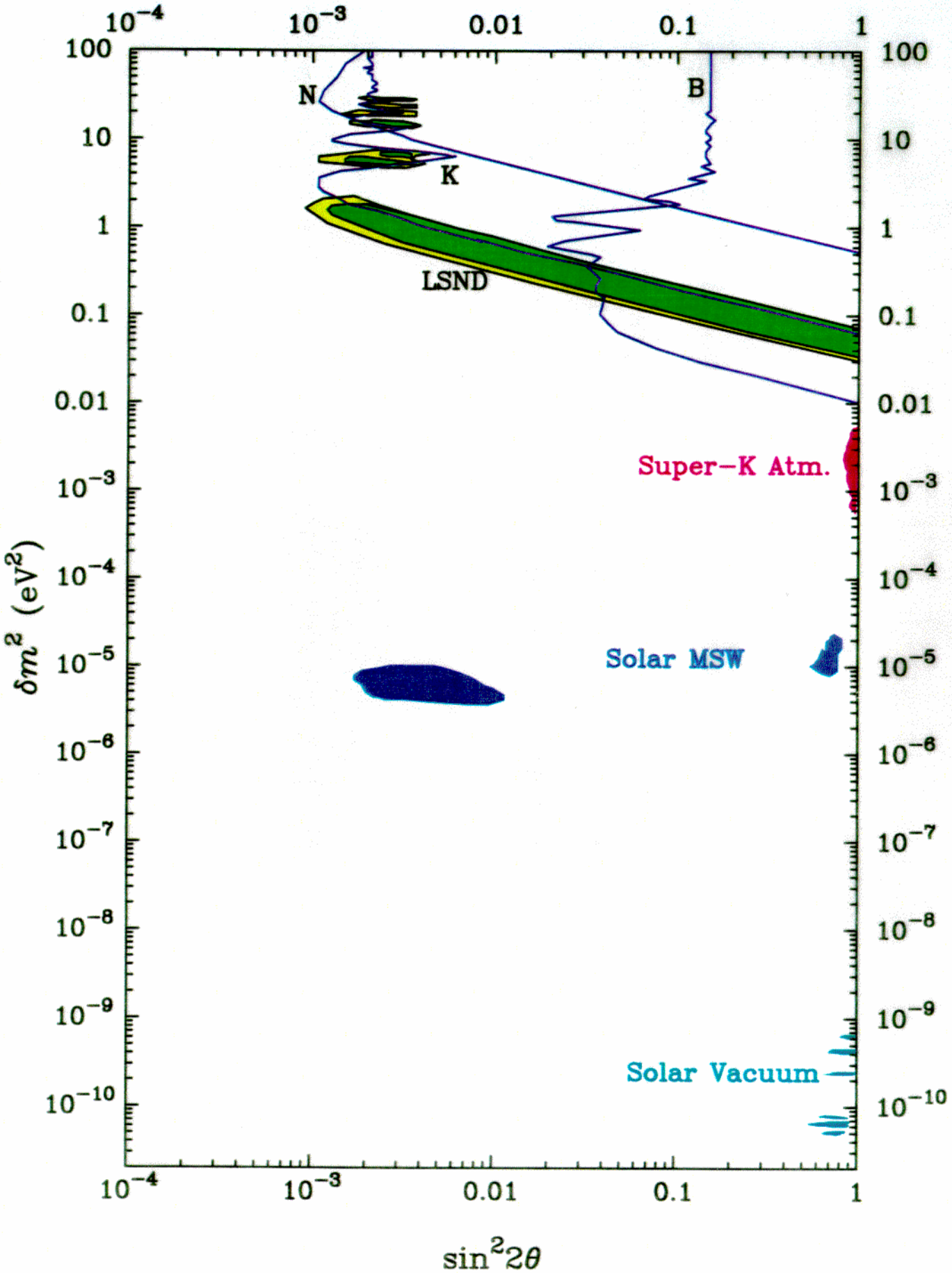
Supernova Nucleosynthesis

(Fuller *et al.*, 1992)
(Qian and Fuller, 1995)



(Shi, Fuller and Abazajian, 1999)

G. Fuller, SSI 2000



Summary of Neutrino Data

• **All** of the current data/problems (LSND, Atmospheric Muon Neutrino Deficit and associated zenith angle dependence in SuperK, and the Solar Neutrino Deficit) **cannot** be fit with just three active neutrino species: **Either**:

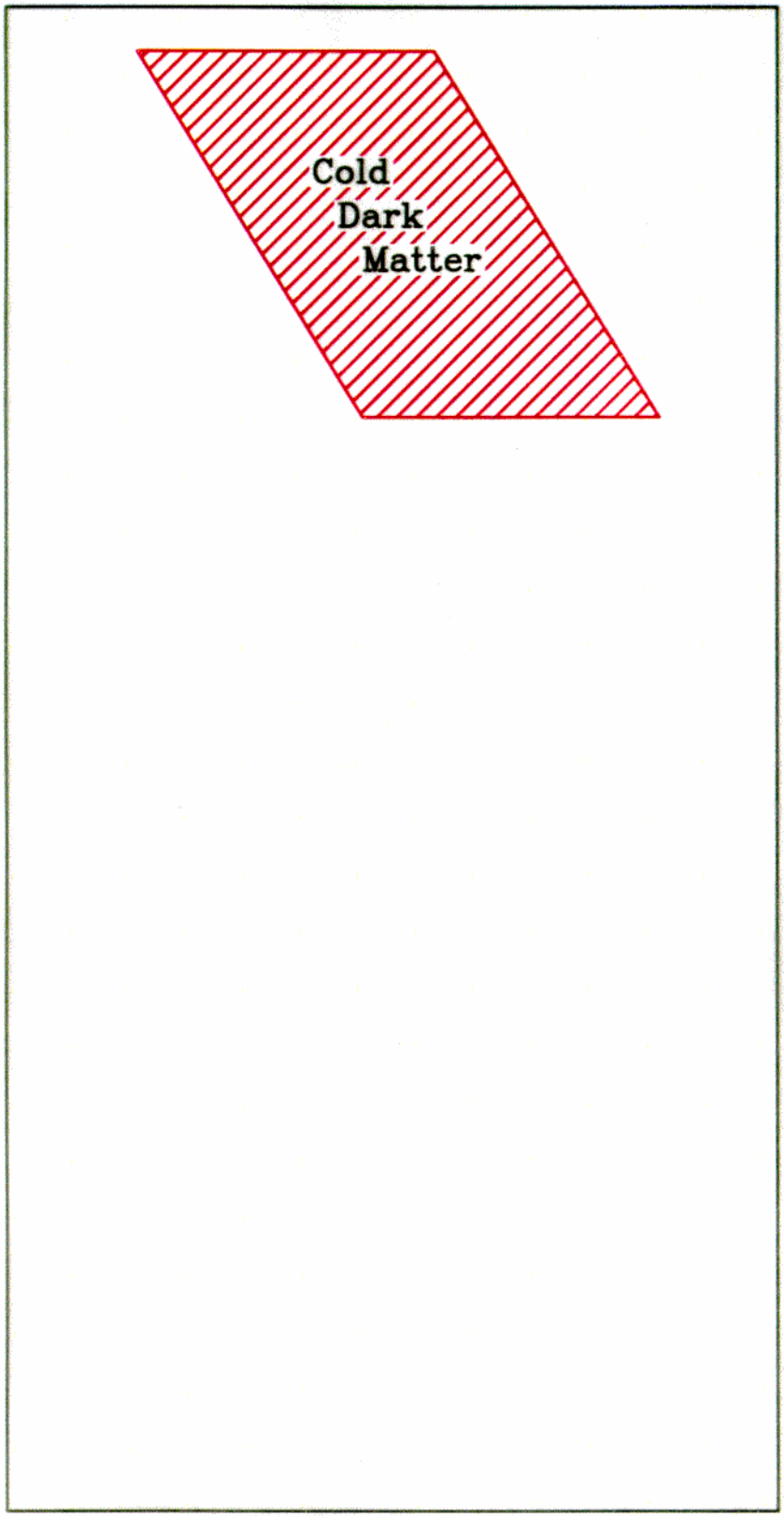
⇒ (1) One or more of the interpretations of these experiments/problems in terms of neutrino flavor mixing is **wrong**:

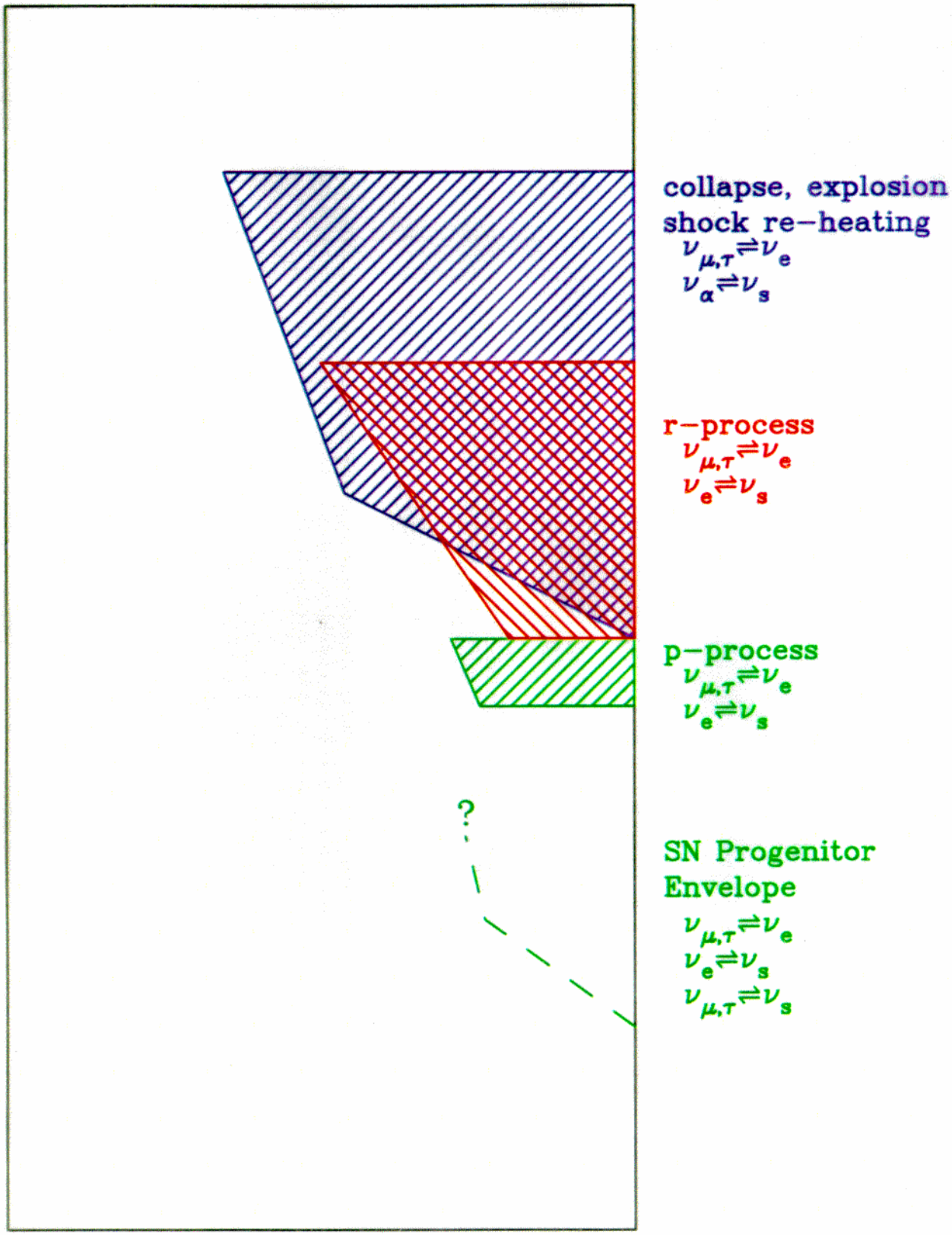
OR

⇒ (2) We must introduce a fourth neutrino species which, because of the LEP/SLC Z^0-width limits, must be a **"sterile" neutrino**, e.g. an SU(2) singlet Majorana neutrino.

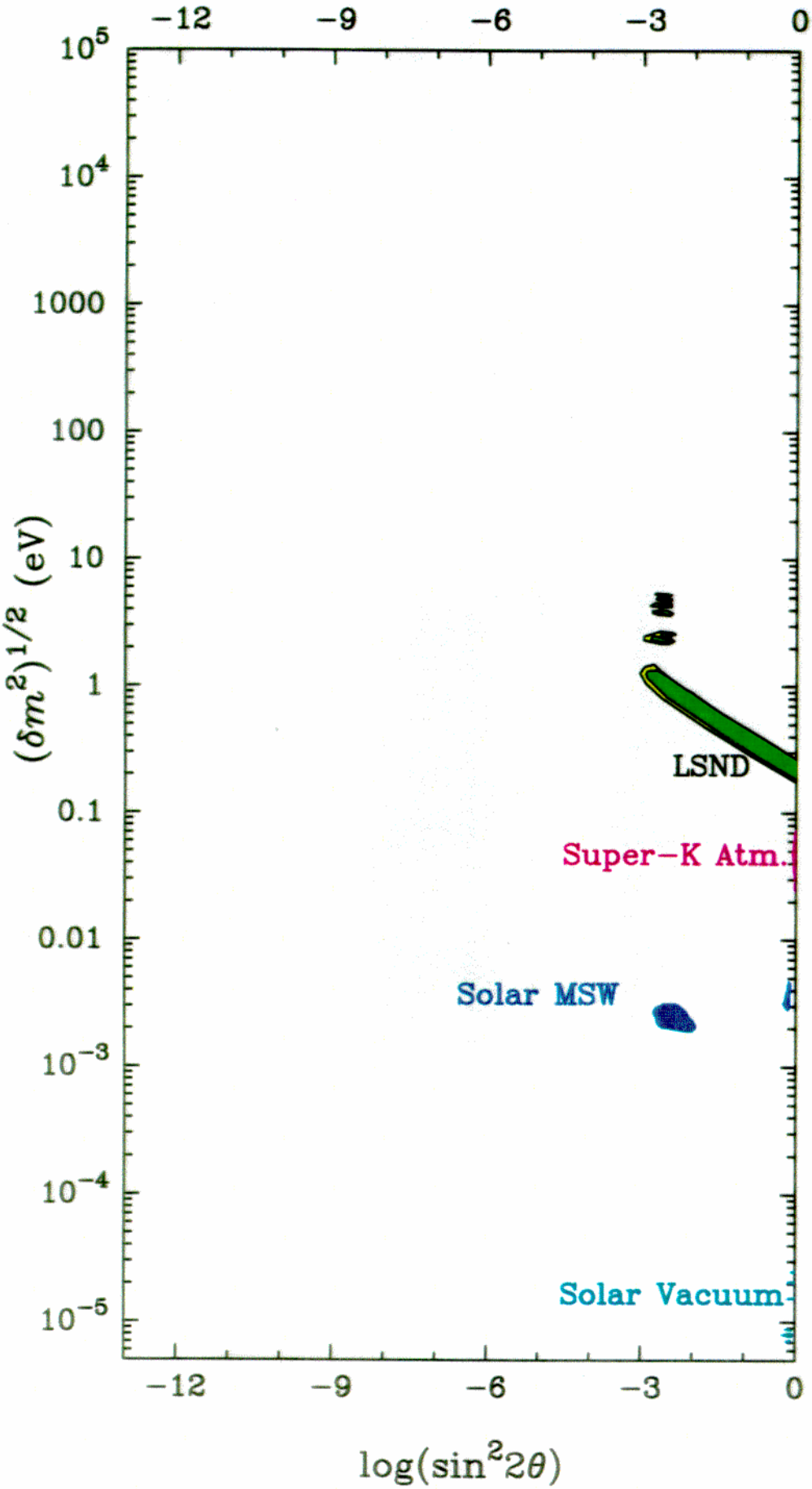
• Adopting the latter view (2) and accepting the existence of a light sterile neutrino which mixes with active species presents particle physics and astrophysics with huge challenges. We would have to abandon the "See-Saw Model" for light neutrino masses, and we may have to give up cherished notions and constraints on **Neutrino Dark Matter** and re-think our models for **Big Bang Nucleosynthesis**, and **Supernova Explosions and Heavy Element Nucleosynthesis**.

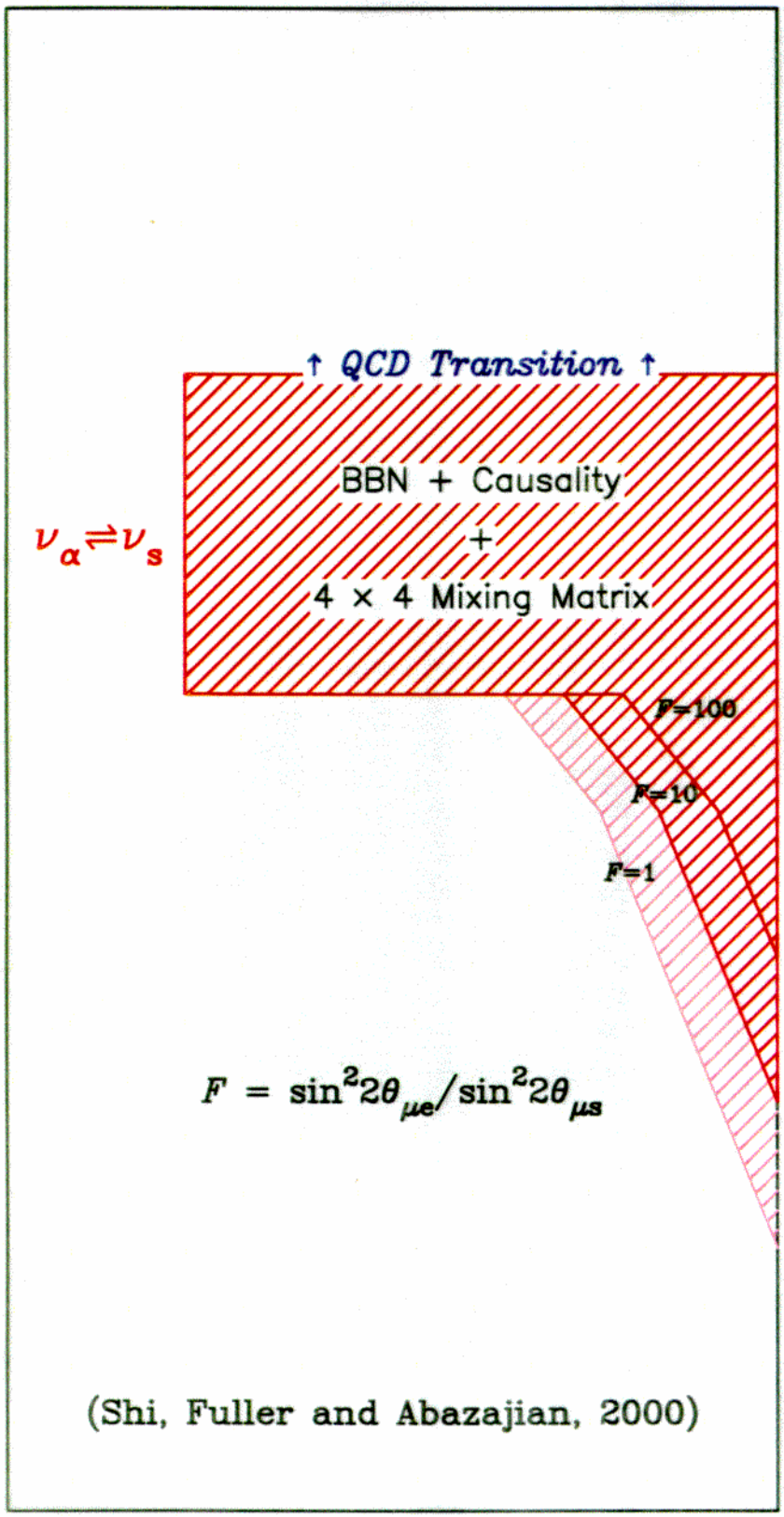
CDM $\nu_\alpha \rightleftharpoons \nu_s$



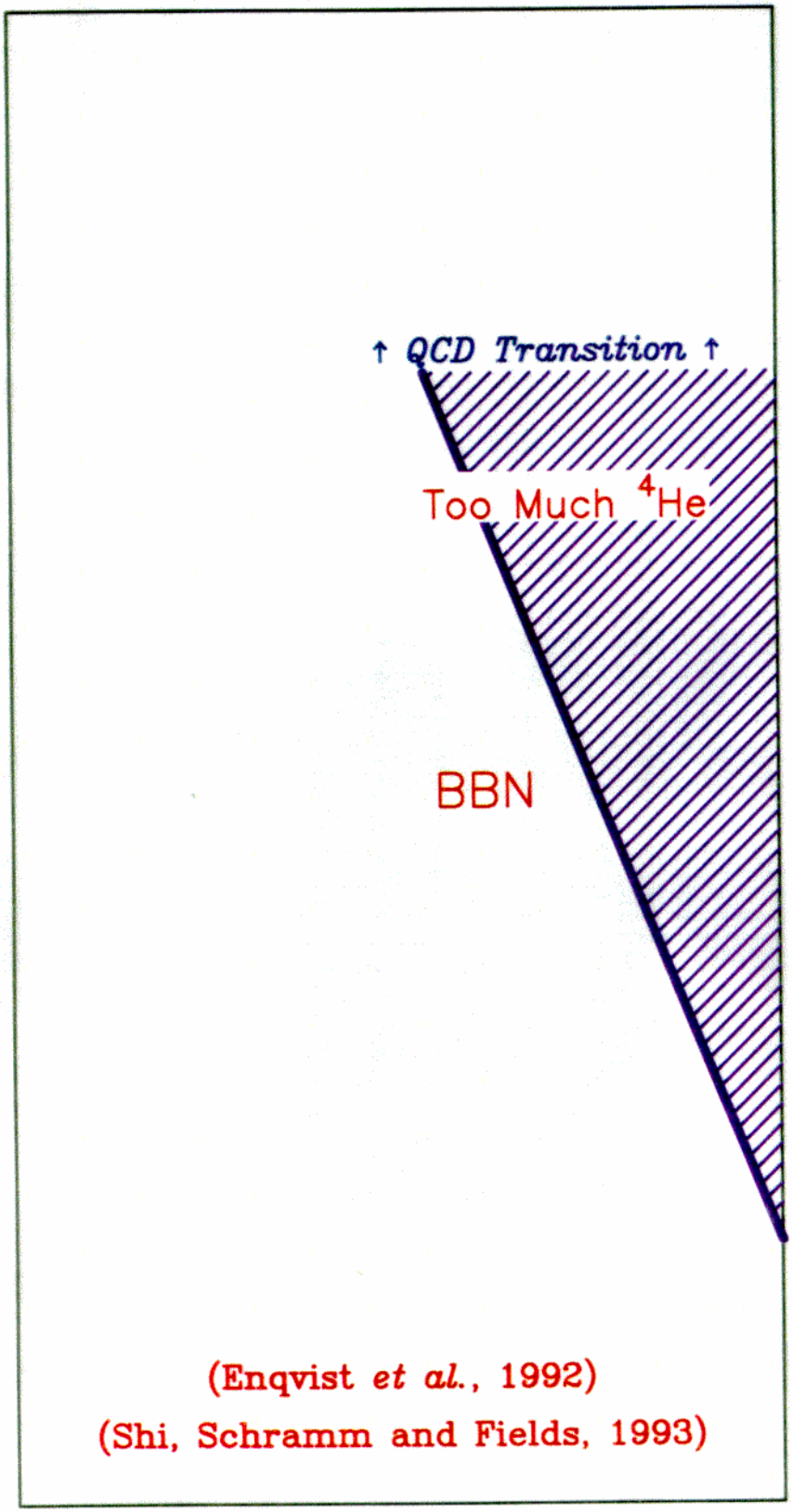

collapse, explosion
shock re−heating
$\nu_{\mu,\tau} \rightleftharpoons \nu_e$
$\nu_\alpha \rightleftharpoons \nu_s$
r−process
$\nu_{\mu,\tau} \rightleftharpoons \nu_e$
$\nu_e \rightleftharpoons \nu_s$
p−process
$\nu_{\mu,\tau} \rightleftharpoons \nu_e$
$\nu_e \rightleftharpoons \nu_s$
?
SN Progenitor
Envelope
$\nu_{\mu,\tau} \rightleftharpoons \nu_e$
$\nu_e \rightleftharpoons \nu_s$
$\nu_{\mu,\tau} \rightleftharpoons \nu_s$

G. Fuller, SSI 2000





(Shi, Fuller and Abazajian, 2000)



(Enqvist *et al.*, 1992)

(Shi, Schramm and Fields, 1993)

Big Bang Cosmology

• A. Friedmann in 1922 and independently G. Lemaitre in 1927 (and later generalized by H. P. Robertson and A. G. Walker in 1935) found the solution to Einstein's Field Equations for a homogeneous and isotropic distribution of mass-energy: the FLRW coordinates (t, r, θ, ϕ) with line element:

$$ds^2 = -dt^2 + a(t)^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right\}$$

Here the curvature parameter determines the fate of the universe and is:

$$k = \begin{cases} -1 & \text{“open”} \quad \Omega < 1 \\ 0 & \text{“flat”} \quad \Omega = 1 \\ +1 & \text{“closed”} \quad \Omega > 1 \end{cases}$$

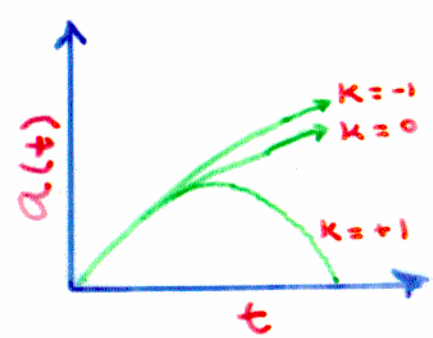
Here the closure parameter is the ratio of the total mass-energy density at time t to the density required to just critically close the universe, ρ_{crit},

$$\Omega = \frac{\rho(t)}{\rho_{\text{crit}}}$$

• The Scale Factor $a(t)$ can be found by solving the mass-energy conservation and Field Equations:

$$a^3 \frac{dP}{dt} = \frac{d}{dt} \left\{ a^3 \left(\rho + P \right) \right\}$$

$$\left(\frac{da}{dt} \right)^2 + k = \frac{8}{3} \pi G \rho a^2$$



G. Lemaitre and G. Gamow were the first to extrapolate this picture backward in time to the earliest epochs of the universe. There material is so compressed and hot that photons and, at high enough temperatures, even neutrinos are in **thermal equilibrium** with matter.

• The expansion rate of the universe is given by the Hubble Parameter:

$$H = \frac{1}{a}\frac{da}{dt}$$

For radiation dominated conditions $a(t) \propto t^{1/2}$ and

$$H \approx \left(\frac{8\pi^3}{90}\right)^{1/2} g^{1/2}\frac{T^2}{m_{\rm pl}} \approx 0.7\,{\rm s}^{-1}\left(\frac{g}{10.75}\right)^{1/2}\left(\frac{T}{1\,{\rm MeV}}\right)^2$$

where the Planck Mass is $m_{\rm pl} = 1/G^{1/2} \approx 1.221 \times 10^{22}\,{\rm MeV}$, T is the temperature of the universe, and g is the statistical weight of the **relativistic particles** in the plasma.

• Every neutrino species in equilibrium contributes 7/8 to g. The more kinds of neutrinos there are, the faster the expansion rate of the universe!

$$g = \sum_i (g_b)_i + \tfrac{7}{8}\sum_j (g_f)_j$$

Cosmology

* homogeneity & isotropy ⇒
FRW spacetime (t, r, θ, φ)

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = -dt^2 + a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right\}$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$

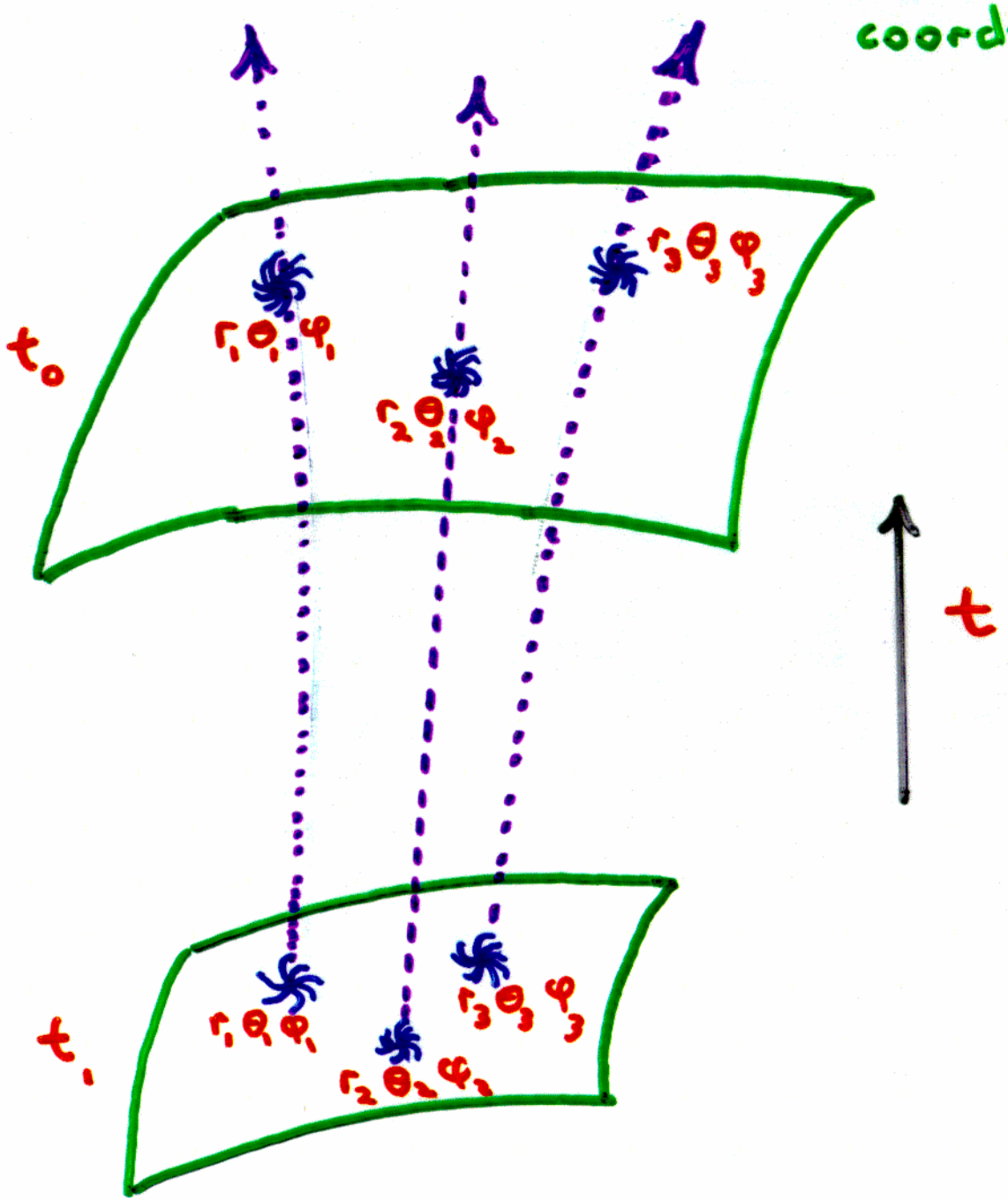
$a(t)$ = "scale factor"

$$k = \begin{cases} -1 & \text{"open"} \quad \Omega < 1 \\ 0 & \text{"flat"} \quad \Omega = 1 \\ +1 & \text{"closed"} \quad \underbrace{\Omega > 1}_{\text{no cosm. const } \lambda} \end{cases}$$

Hubble parameter $\Big\}$ $H \equiv \dfrac{\dot{a}(t)}{a(t)}$

closure parameter $\Big\}$ $\Omega \equiv \dfrac{\rho(t)}{\rho_{crit}}$

LFRW Coordinates $(t, \underbrace{r, \theta, \varphi}_{\text{co-moving coordinates}})$



where $\rho(t)$ is the total mass-energy density @ time t

* stress-energy (perfect fluid)

$$T^{\mu\nu} = P g^{\mu\nu} + (\rho + P) u^\mu u^\nu$$

$$T^{\mu\nu}{}_{;\nu} = 0$$ energy/momentum cons.

* Einstein Field Equations (MTW convention on $R^\alpha{}_{\beta\mu\nu}$)

$$R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R = 8\pi G T^{\mu\nu}$$

$$G \equiv \frac{1}{m_{pl}^2}$$ Newton's Constant

$$m_{pl} \approx 1.2211 \times 10^{22} \text{ MeV}$$

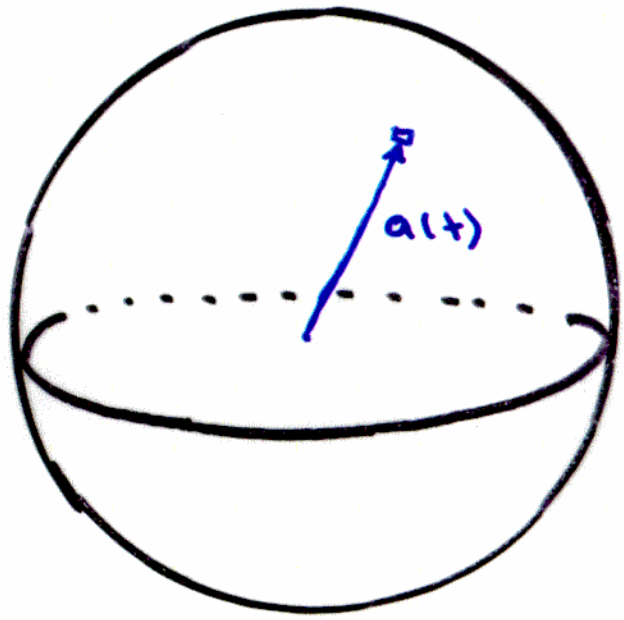
$$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$$

1. $$a^3 \frac{dP}{dt} = \frac{d}{dt}\left[a^3(\rho + P)\right]$$

2. $$\dot{a}^2 + k = \tfrac{8}{3}\pi G \rho a^2$$

an imaginary sphere,
riding along with expansion
(co-moving)

radius to a
"test particle"
is $a(t)$



Birkhoff's Theorem

$\Downarrow$

Kinetic Energy + Grav. Pot. Energy
= constant

$$\frac{1}{2} m \dot{a}^2 + \left[- \frac{G m \left(\frac{4}{3} \pi \rho(t) a^3 \right)}{a} \right] = \text{const}$$

$$\Rightarrow \quad \dot{a}^2 + k = \frac{8}{3} \pi G \rho(t) a^2$$

Kinds of Energy

relativistic particles

nonrelativistic particles

vacuum energy*

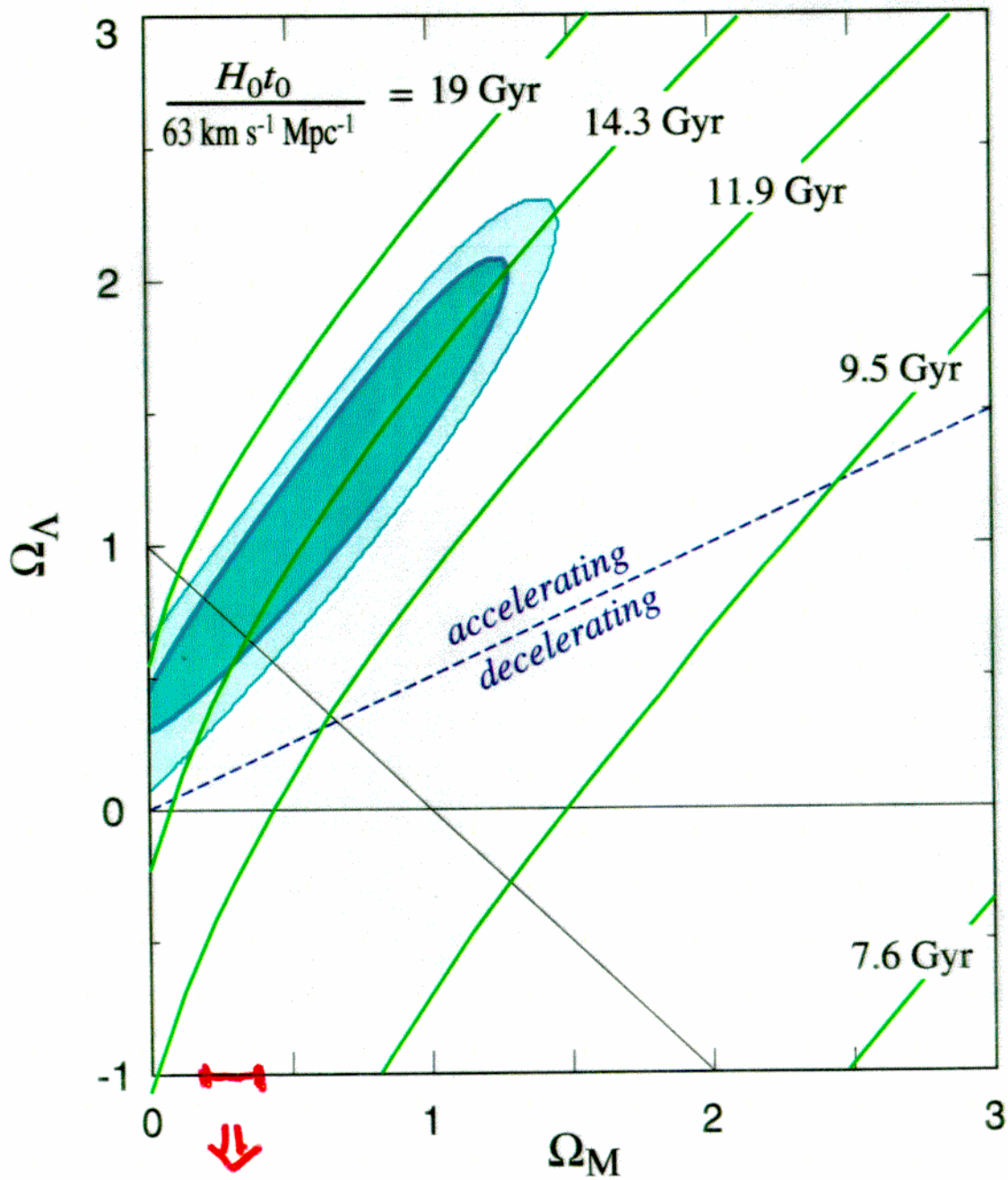
* an energy per unit physical volume

$$p = -\left.\frac{\partial E}{\partial \mathrm{Vol}}\right|_s < 0$$

CMBR (Boomerang & MAXIMA) ⇒ $k=0$, Flat st

Deuterium ⇒ $\Omega_b \approx 0.05 \left(\frac{65\ \mathrm{km\ s^{-1}\ Mpc^{-1}}}{H_0}\right)^2$

Supernova Cosmology Project
Perlmutter *et al.* (1998)



$\Omega_M = 0.2 - 0.3$

Best fit age of universe: $t_o = 14.5 \pm 1\ \ (0.63/h)$ Gyr

Best fit in flat universe: $t_o = 14.9 \pm 1\ \ (0.63/h)$ Gyr

www–supernova.LBL.gov

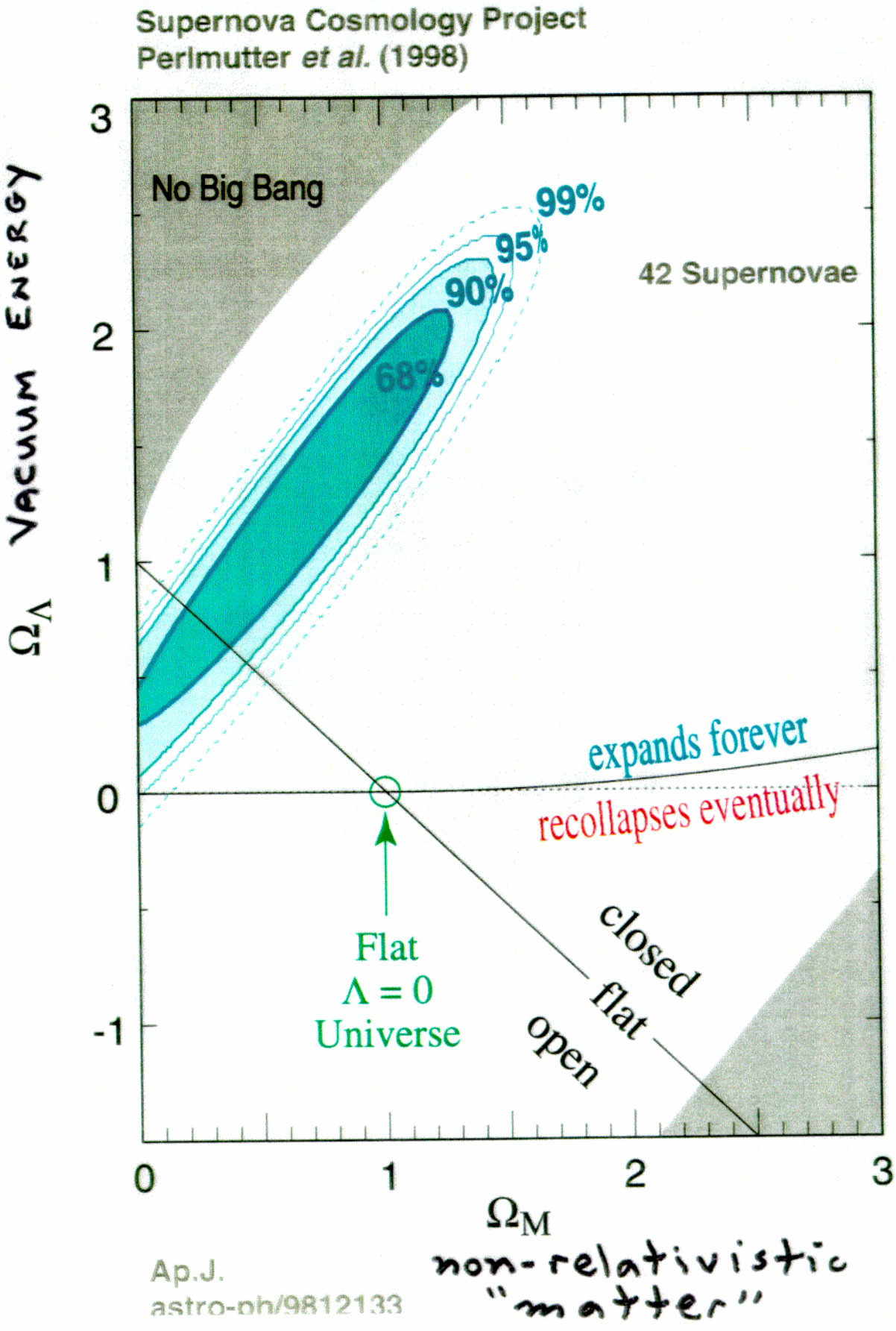
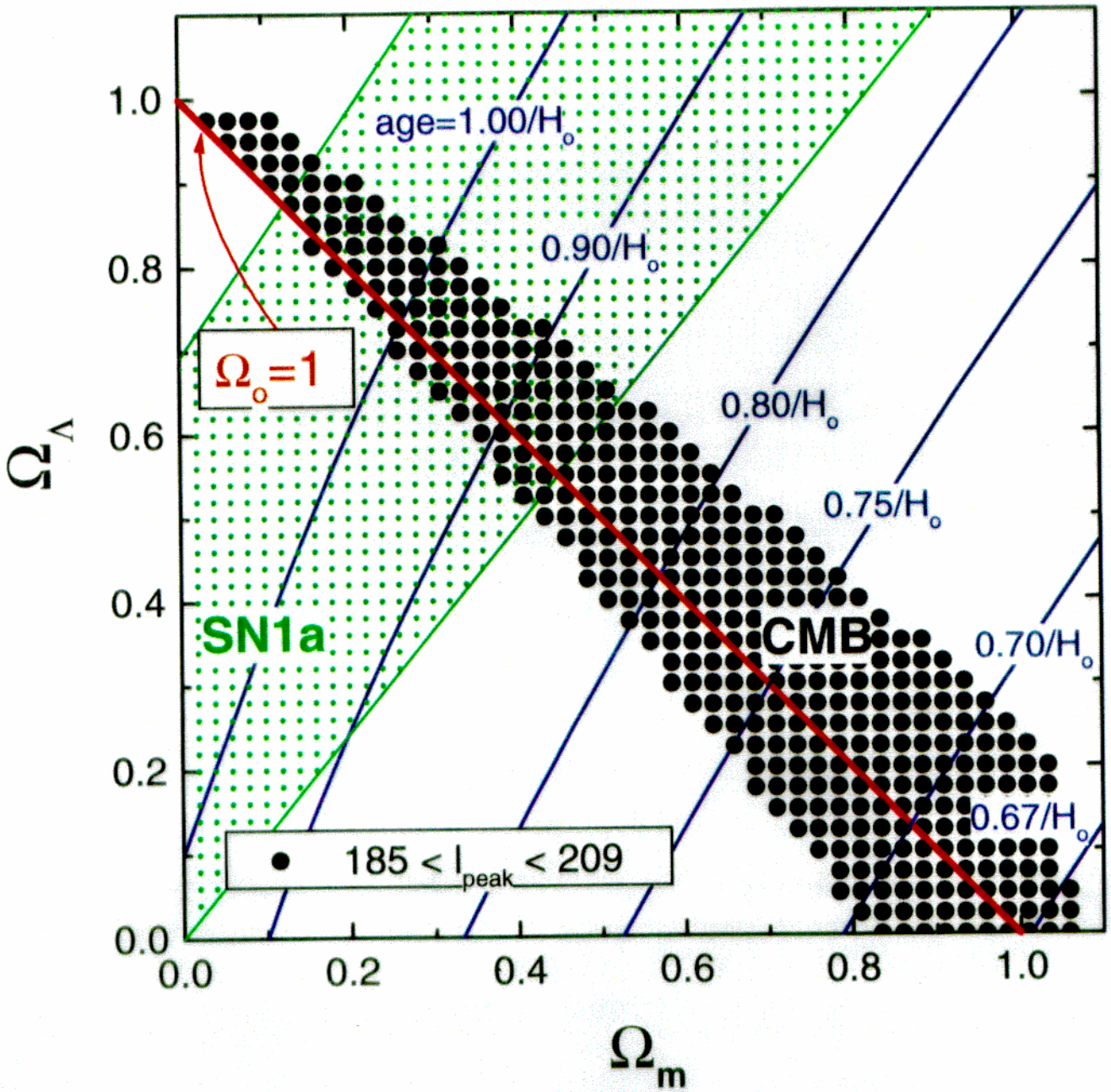

Supernova Cosmology Project
Perlmutter et al. (1998)
No Big Bang
99%
95%
90%
68%
42 Supernovae
expands forever
recollapses eventually
Flat
Λ = 0
Universe
closed
flat
open
Ω_Λ VACUUM ENERGY
Ω_M
non-relativistic
"matter"
Ap.J.
astro-ph/9812133

Figure 3


age=1.00/H$_o$
0.90/H$_o$
0.80/H$_o$
0.75/H$_o$
0.70/H$_o$
0.67/H$_o$
Ω_o=1
SN1a
CMB
185 < l_{peak} < 209
Ω_Λ
Ω_m
0.0
0.2
0.4
0.6
0.8
1.0

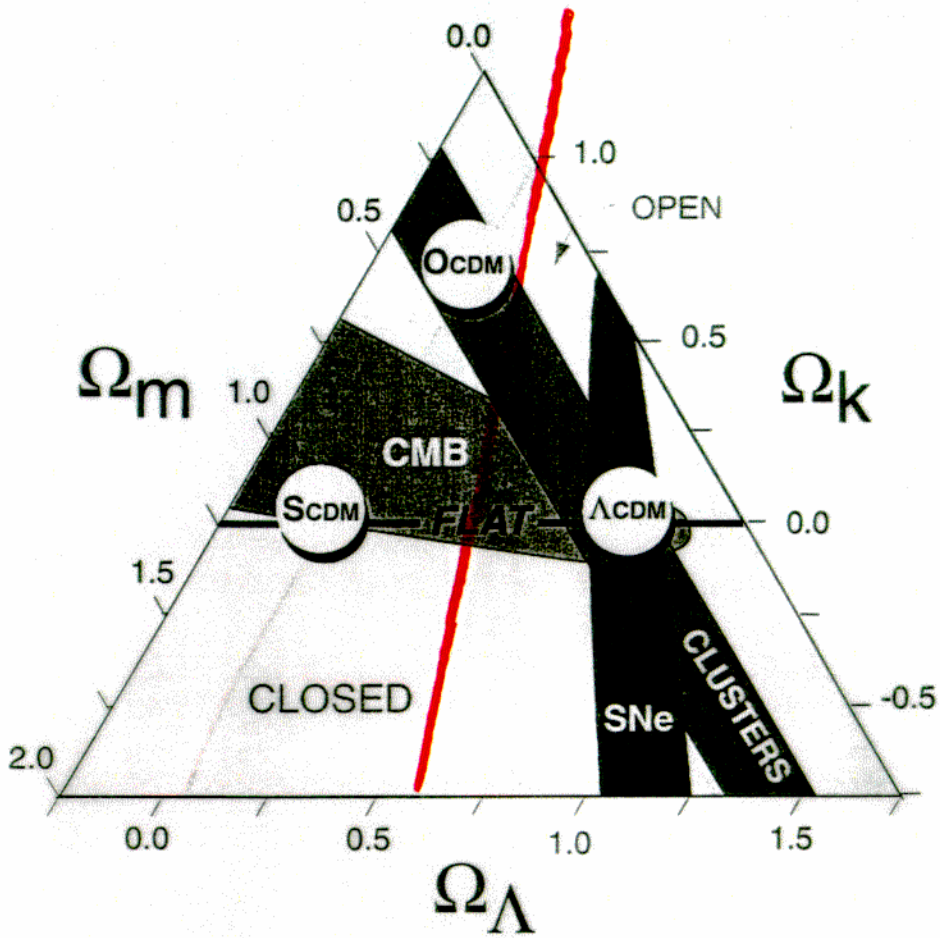


Figure 2: **The Cosmic Triangle Observed** represents current observational constraints. The tightest constraints from measurements at low red shift (clusters, including the mass-to-light method, baryon fraction, and cluster abundance evolution), intermediate red shift (supernovae), and high red shift (CMB) are shown by the three color bands (each representing 1-σ uncertainties). Other tests discussed in the paper are consistent with but less constraining than the constraints illustrated here. The cluster constraints indicate a low-density universe; the supernovae constraints indicate an accelerating universe; and the CMB measurements indicate a flat universe. The three independent bands intersect at a flat model with $\Omega_m \sim 1/3$ and $\Omega_\Lambda = 2/3$; the model contains a cosmological constant or other dark energy.

Freeze-Out From Equilibrium

• The history of the early universe is a succession of "Freeze-Out" processes. When the rates of production and destruction and scattering of a particle are large compared to the expansion rate, then that particle is in equilibrium. However, as the universe expands and the temperature drops, reaction rates slow down and eventually they fall below the expansion rate of the universe. They are then said to be frozen out.

$\Rightarrow$ Photons Freeze Out at $T \approx 0.2\,\mathrm{eV}$, where the rate of photon electron scattering $\gamma + e^- \to \gamma + e^-$ becomes small compared to H. Today, the left over, frozen out photons have a Planck spectrum with a temperature $T \approx 2.714 \pm 0.022 K \sim 10^{-4}\,\mathrm{eV}$, the Cosmic Microwave Background Radiation.

$\Rightarrow$ Nuclear Reactions Freeze Out at $T \sim 100\,\mathrm{keV}$.

$\Rightarrow$ The neutron-to-proton ratio freezes out at $T \approx 700\,\mathrm{keV}$.

$\Rightarrow$ Active Neutrinos Freeze Out of Thermal Equilibrium at $T \approx 1\,\mathrm{MeV}$. $\Rightarrow$ should then be a cosmic neutrino background, with each neutrino species having a "Black Body" Fermi-Dirac spectrum with $T_\nu \sim 1.9\,K \sim 10^{-4}\,\mathrm{eV}$

Particle Horizon

consider a photon world line $\Rightarrow$ null geodesic $ds^2 = 0$ (assume radial)

$$\Rightarrow \boxed{dt = a(t)\frac{dr}{\sqrt{1-kr^2}}} \quad (*)$$

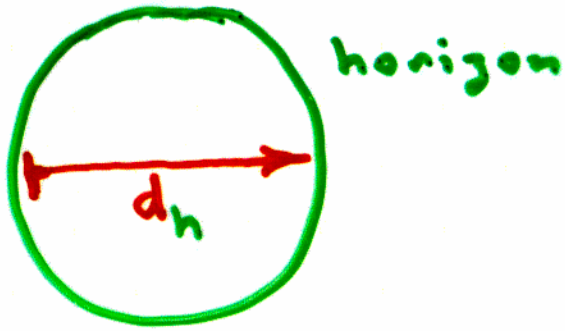
so a photon starts @ $t=0$, propagates to time t where it has reached FRW radial coordinate r_h, so $(*) \Rightarrow$

$$\int_0^t \frac{dt'}{a(t')} = \int_0^{r_h} \frac{dr}{\sqrt{1-kr^2}} = \begin{cases} \sin^{-1} r_h & k=+1 \\ r_h & 0 \\ \sinh^{-1} r_h & -1 \end{cases}$$

Particle Horizon is the proper distance traveled by a photon in the age of univ. t

$$d_h(t) = \int_0^{r_h} \sqrt{g_{rr}}\, dr = a(t)\int_0^{r_h} \frac{dr}{\sqrt{1-kr^2}}$$

$$= a(t)\int_0^t \frac{dt'}{a(t')}$$



assume that . . . .

* radiation dominated

$$\rho \propto a^{-4}$$

$$a(t) \propto t^{1/2}$$

$$d_h = 2t$$

* matter dominated

$$\rho \propto a^{-3}$$

$$a(t) \propto t^{2/3}$$

$$d_h = 3t$$

	QCD Epoch	Weak Decoupling	BBN before $e^{\pm}$ annih.	BBN after $e^{\pm}$ annih.
T [MeV]	100	1	0.1 100 keV	0.01 10 keV
g	~50	10.75	10.75	3.36
t [sec.]	$\sim 3.4\times10^{-5}$	0.74	74	1.3×10^{4}
horizon d_h [km]	~10	2×10^{5} $\sim 1/3\ R_\odot$	2.2×10^{7} $\sim 1/7$ A.U.	4×10^{9} 27 A.U.
M_h^{tot} [$M_\odot$]	~1	2×10^{4}	2×10^{6}	3×10^{8}
M_h^{b} [$M_\odot$]	$\sim 10^{-9}$	2×10^{-3}	2	3×10^{3}

Radiation Dominated Epoch

energy density $\rho = g\, a\, T^4 \qquad a \equiv \pi^2/30$

relativistic statistical weight $g \equiv \sum_i g_b^i \left(\frac{T_i}{T}\right)^4 + \sum_j \frac{7}{8} g_f^j \left(\frac{T_j}{T}\right)^4$

Pressure $p = \frac{1}{3}\rho = \frac{1}{3} g\, a\, T^4$

Proper Entropy Density $S \equiv \frac{\rho + p}{T} = \frac{4}{3} g_s\, a\, T^3 = \frac{2\pi^2}{45} g_s T^3$

$$g_s \equiv \sum_i g_b^i \left(\frac{T_i}{T}\right)^3 + \sum_j \frac{7}{8} g_f^j \left(\frac{T_j}{T}\right)^3$$

proper number densities $n_b = g_b \frac{\zeta(3)}{\pi^2} T^3 \qquad n_f = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_f T^3$

$\zeta(3) \simeq 1.202$

For Example

photons: $g_\gamma = 2 \qquad n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3 \simeq (3.171 \times 10^{31}\ \mathrm{cm}^{-3}) \cdot (T/\mathrm{MeV})^3$

Neutrinos: $\underbrace{\nu_e\ \bar{\nu}_e\ \ \nu_\mu\ \bar{\nu}_\mu\ \ \nu_\tau\ \bar{\nu}_\tau}_{\text{6 species, each has } g=1}$ + "steriles"?

$g^\nu_{tot} = 6$

Statistical weight

QCD epoch $T \gtrsim 100$ MeV

$$g_{\gamma e \nu_\mu} = g_\gamma + \tfrac{7}{8}(g_e + g_\mu + g_\nu) = \frac{57}{4} = 14.25$$

$$g_{qg} = g_{gluon} + \tfrac{7}{8} g_q = 16 + \tfrac{7}{8}(12\, n_q) = 16 + 10.5\, n_q$$

$$g_Q \equiv g_{\gamma e \nu_\mu} + g_{qg} = 30.25 + 10.5\, n_q \quad \begin{cases} \to 51.25 & n_q = 2 \\ \to 61.75 & n_q = 3 \end{cases}$$

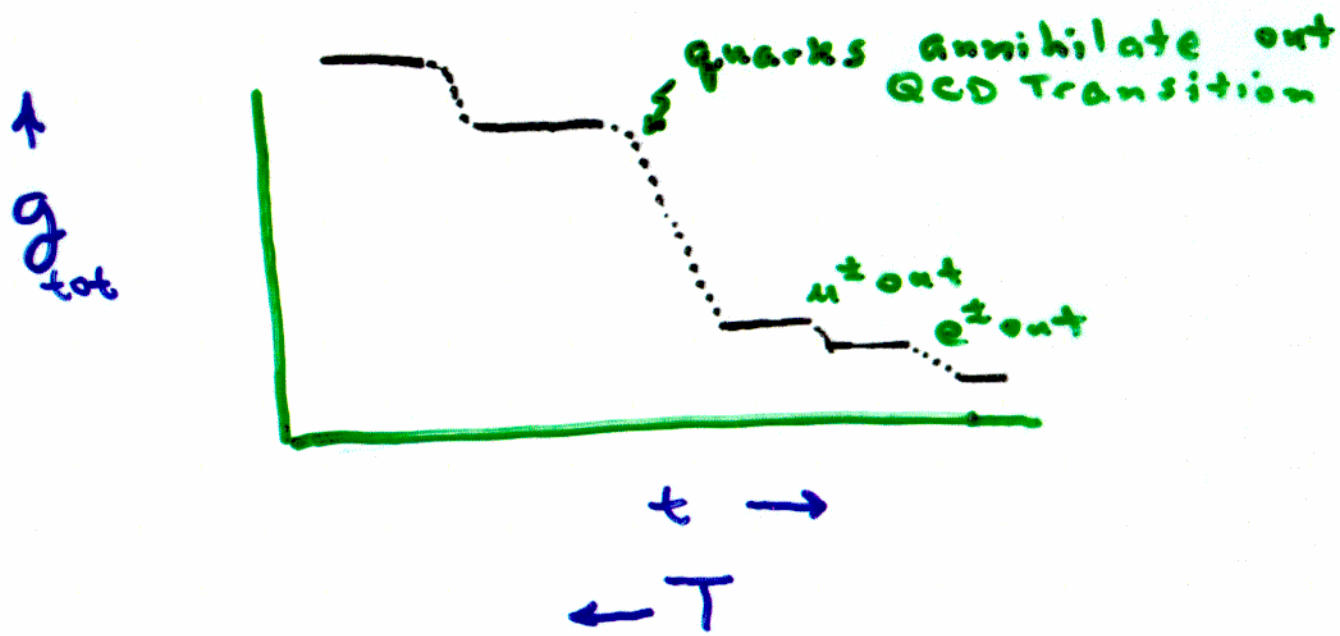
6 if top quark relativistic

Electro-weak Epoch $T \gtrsim 100$ GeV

$$g_{EW} = g_{\gamma e \mu \nu} + g_{qg}(n_q) + g_{WZ}$$

$$= 93.25 + g_{WZ} = 102.25$$

$g_Z = 3$, $g_{W^\pm} = 6$



* Baryon-to-photon ratio

$$\eta = \frac{n_b}{n_\gamma} \simeq 2.68 \times 10^{-8} \left(\frac{2.75\,K}{T_\gamma^0}\right)^3 \Omega_b h^2$$

where $\Omega_b \equiv n_b m_b / \rho_{crit}$

and where latest COBE data gives
$T_\gamma^0 \approx 2.714 \pm 0.022$ @ 95% C.L.

so we should estimate,

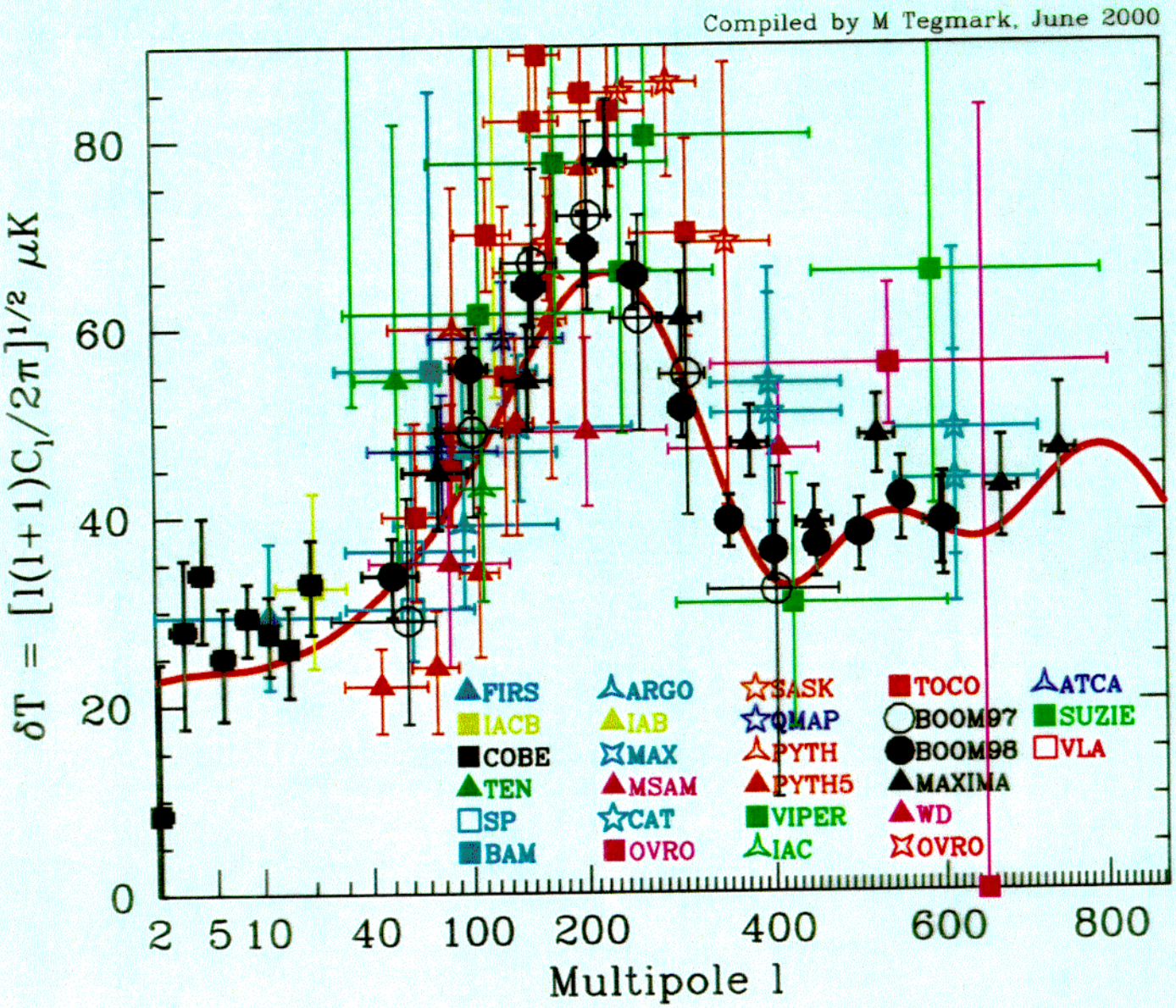
$$\boxed{\eta \approx 2.79 \times 10^{-8}\, \Omega_b h^2}$$

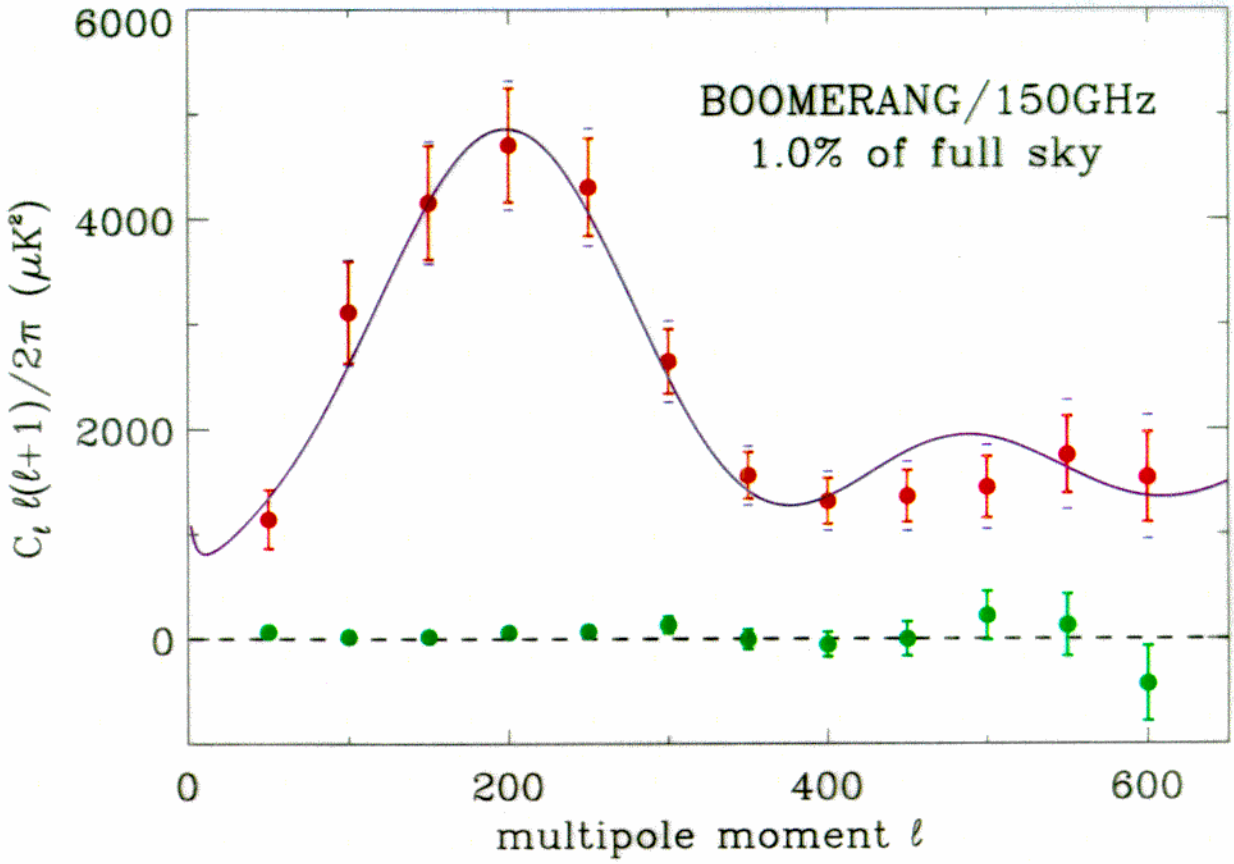
note: $n_b = n_b^0 \left(a^0/a\right)^3$ so η nearly const

$$\eta \simeq \frac{2\pi^4}{45} \zeta^{-1}(3) \frac{g_s}{g_\gamma} s^{-1}$$

entropy per baryon constant

g_s can change


Compiled by M Tegmark, June 2000
$\delta T = [l(l+1)C_l/2\pi]^{1/2}$ μK
0
20
40
60
80
2
5
10
40
100
200
400
600
800
Multipole l
FIRS
IACB
COBE
TEN
SP
BAM
ARGO
IAB
MAX
MSAM
CAT
OVRO
SASK
QMAP
PYTH
PYTH5
VIPER
IAC
TOCO
BOOM97
BOOM98
MAXIMA
WD
OVRO
ATCA
SUZIE
VLA



? $\Omega_b h^2 \simeq 0.03$?

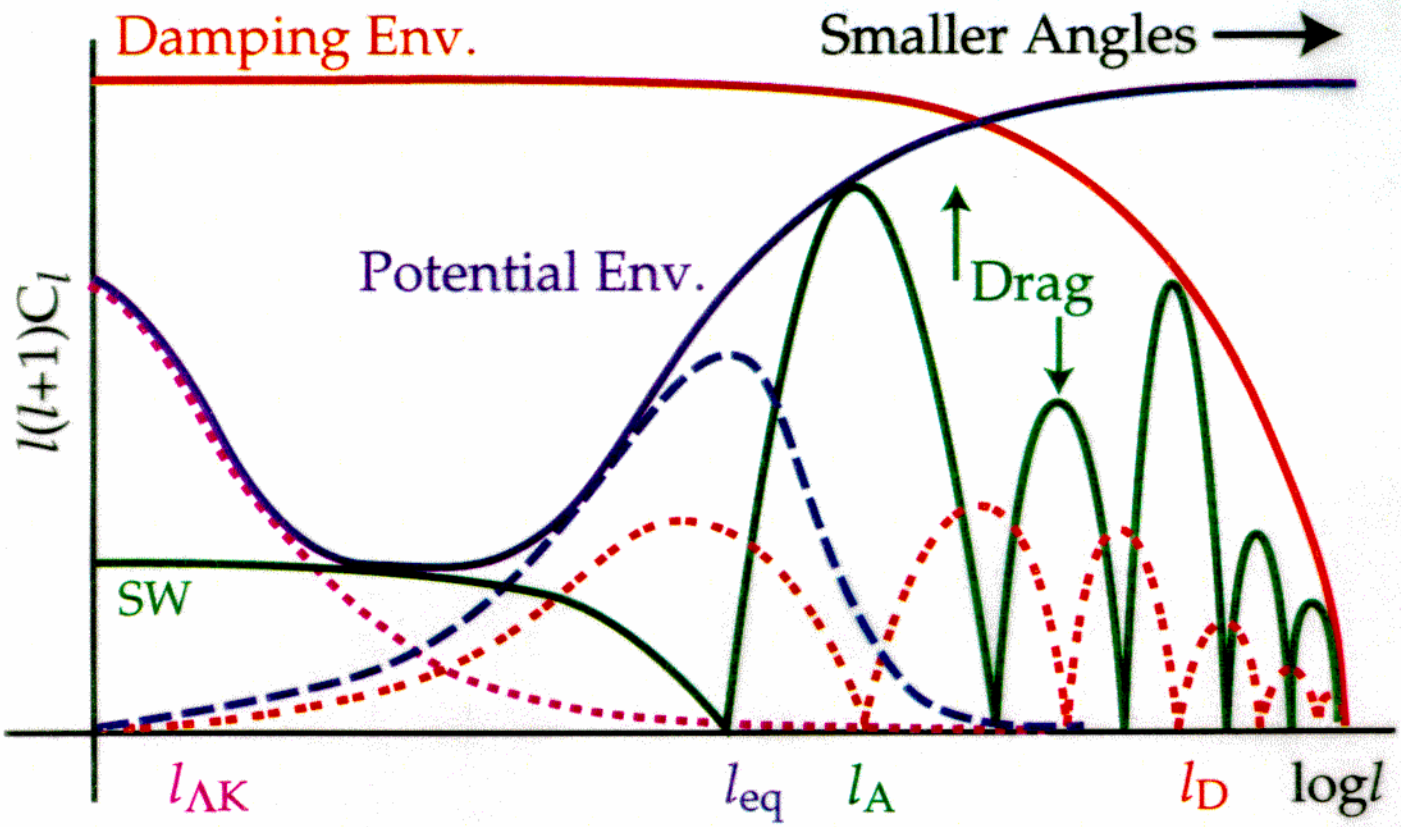
Figure 2

That is 50% higher than $^2H/H$ - Determined $\Omega_b h^2$

more likely: "red tilted" primordial power spectrum

$n=1 \Rightarrow n=0.9$

G. Fuller, SSI 2000



	Ω_K	Ω_Λ	$\Omega_0 h^2$	$\Omega_B h^2$
$l_{\Lambda K}$	↑	↑	•	•
l_{eq}	↑	↓	↑	•
l_A	↑	↓	↓	↑
l_D	↑	↓	↓	↑

Late ISW

Early ISW

Eff. Temp.

Doppler

photon fluid only

(1)

$\Rightarrow$ sound wave

(2.) $\lambda_\gamma \ll \frac{2\pi}{k}$ photon/(proton) baryon fluid

$\Rightarrow$ very tightly coupled

greater compression due to baryons

(3.) $\lambda_\gamma \sim \frac{2\pi}{k}$ photon/baryon fluid

$\Rightarrow$ drag + damping

not as much contrast $\Rightarrow$ reduced rarefaction

Entropy-per-Baryon

$$s = \frac{S}{n_b} \approx \frac{2\pi^4}{45}\,\zeta(3)^{-1}\,\frac{g_s}{g_\gamma}\,\eta^{-1}$$

note that proper baryon number density $n_b = \frac{a_0^3}{a^3}\,n_b^0$

and we therefore expect s to be conserved (since $a^3 S = \text{const}$)

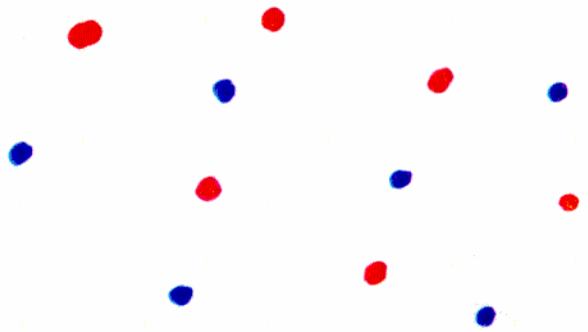
units: Boltzmann constant k per baryon

$$\boxed{\begin{aligned} s &\approx 2.63\times 10^8 \left(\frac{T_\gamma^0}{2.75}\right)^3 \Omega_b^{-1} h^{-2} \\ &\approx 2.53\times 10^8\, \Omega_b^{-1} h^{-2} \sim 10^{10} \end{aligned}}$$

absent black holes, SN, shocks, nuclear burning, etc., s has been constant

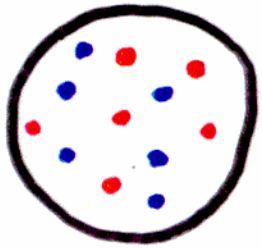
ENTROPY, a better way of thinking

* Disordered / high entropy



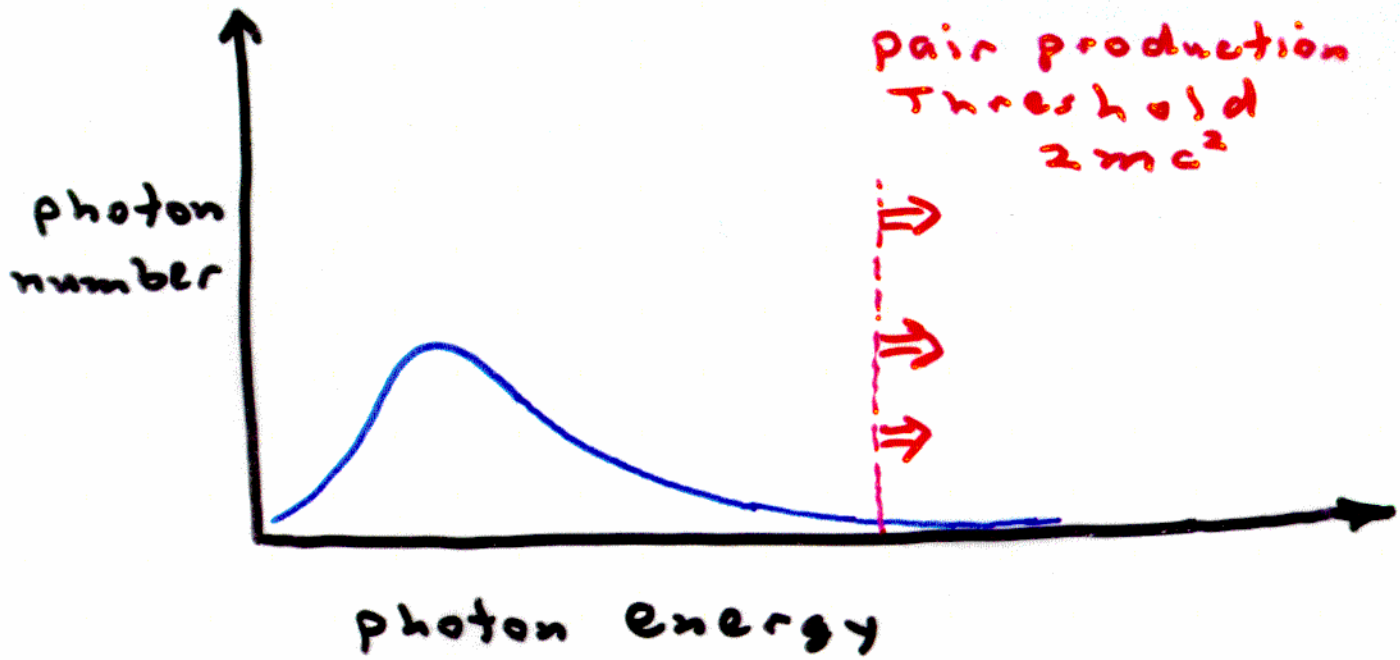
12 "lumps"

* ordered / low entropy



one big "lump"

HIGH ENTROPY SLIGHT OF HAND



⇒ γ's can produce pairs only when they have energies $> 2mc^2$

⇒ way out on exponential tail of Planck distribution

⇒ But, there are $\sim 10^{10}$ γ's per baryon

Nucleosynthesis in a Freeze-Out from Nuclear Statistical Equilibrium

We need to specify **FIVE QUANTITIES:** (not all independent !)

(1) The mean **Entropy per Baryon**

(2) The **Spatial** distribution of entropy (inhomogeneity)

(3) The **Material Expansion Rate**

(4) The **Lepton Numbers / Lepton Energy Spectra** ν-spectra

(5) The **Neutron-to-Proton Ratio,** or $Y_e = (1 + n/p)^{-1}$

* **BBN:** assume homogeneous distribution of entropy; $\underbrace{\text{lepton numbers small}}$

* **SNN:** assume homogeneous outflow

Woosley, et al. '94 Hoffman & Woosley '92
Mayer et al. '92

$$L_{\nu_\alpha} \equiv \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma}$$

by "small" we mean $L_{\nu_e, \mu, \tau} \sim \eta \sim 10^{-10}$

LEP/SLC 3 Neutrinos

expansion rate $H \approx \left(\frac{8\pi^3}{90}\right)^{1/2} g^{1/2} \frac{T^2}{m_{planck}}$

relativistic degrees of freedom $g = \sum g_b + \frac{7}{8} \sum g_f$

(γ's) ($e^\pm$ + "N_ν")

Ultra-Cold Neutron Measurements

Neutron Lifetime $\tau \approx 887.0\,\mathrm{s}$

Montanet *et. al.* 1994

see recent work by LBL group

(e.g., B. Fujikawa et al.)

Freeze-Out From Nuclear Statistical Equilibrium

$\Rightarrow$ Nucleosynthesis Determined By Three Quantities (Plus 2 Assumptions)

- **Mean Entropy-per-Baryon**

baryon-to-photon ratio $\eta \approx 2.79 \times 10^{-8} \Omega_b h^2$

$\eta_{10} \equiv 10^{10} \eta \approx 2.79 \left(\Omega_b h^2 / 10^{-2}\right)$

- **Expansion Rate**

Parameterized by "N_ν," equivalent number of light neutrino flavors.

- **Neutron-to-Proton Ratio**

Nuclear Statistical Equilibrium

$$Z\,p + N\,n \rightleftharpoons A(Z,N) + \gamma$$

$$Z\,\mu_p + N\,\mu_n = \mu_A + Q$$

$$\Downarrow \qquad \Downarrow$$

Saha Equation

$$\boxed{X_{A(Z,N)} \approx \left[S^{1-A}\right] G\, \pi^{\frac{3}{2}(A-1)}\, 2^{\frac{(A-3)}{2}}\, A^{5/2} \cdot \left(\frac{T}{m_b}\right)^{\frac{3}{2}(A-1)} X_p^{Z}\, X_n^{N}\, e^{Q/T}}$$

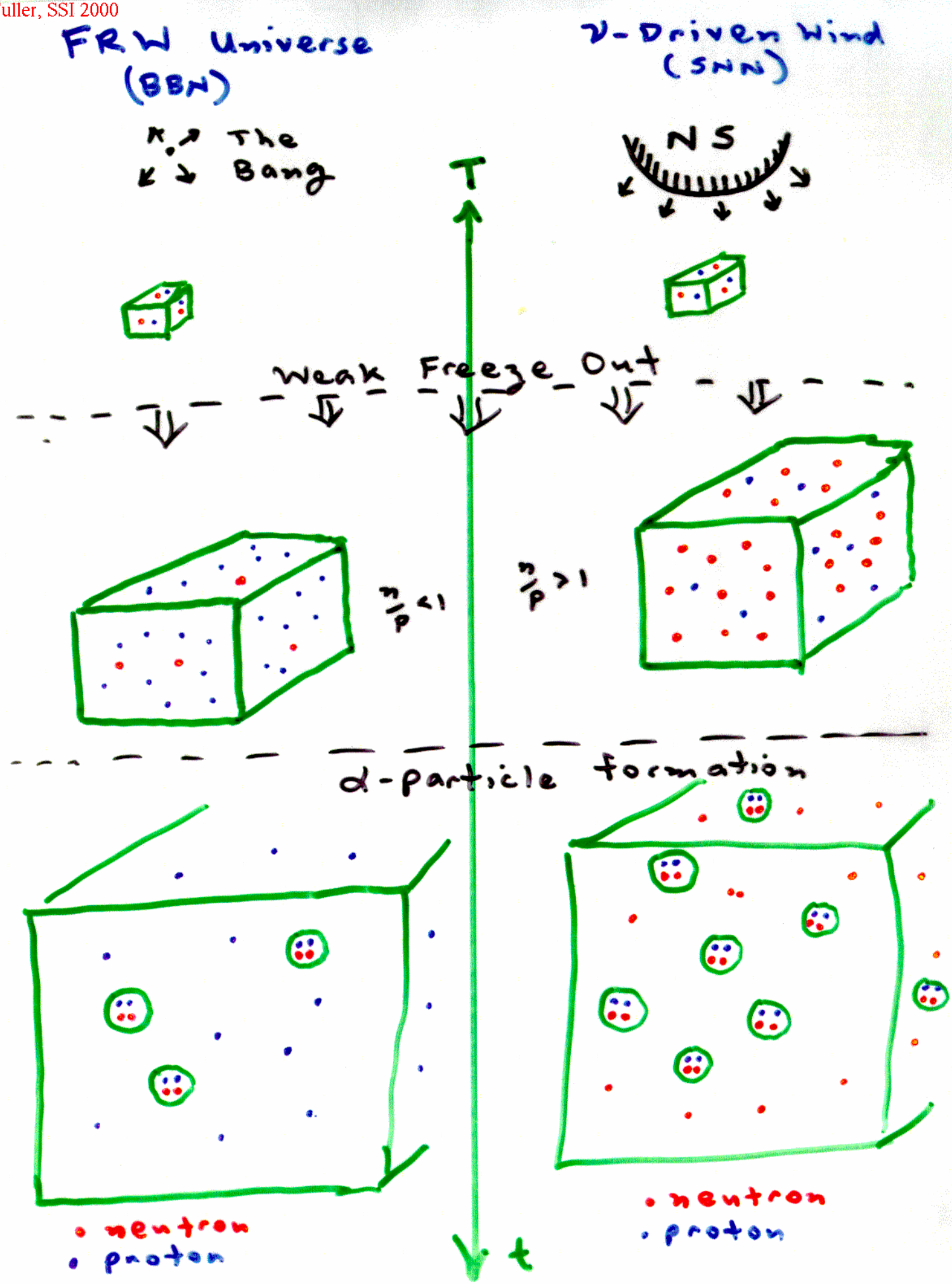
$$\left[S \approx 2.53\times 10^{8}\, \Omega_b^{-1} h^{-2} \sim 10^{10}\right.$$

Boltzmann's const. k per baryon

$$\left.\right]$$

[$S \sim 100$ neutrino-heated supernova ejecta]

G. Fuller, SSI 2000



Freeze-Out Nucleosynthesis

- Nucleosynthesis in a Freeze-Out from Nuclear Statistical Equilibrium is characterized largely by **Three** parameters:

  $\Rightarrow$ electron fraction Y_e see Meyer '9

  $\Rightarrow$ entropy S

  $\Rightarrow$ freeze-out timescale τ

- Freeze-Out from High Entropy

  $Y_e > 0.5 \Rightarrow$ **proton-rich**$\Rightarrow$ Big Bang

  $Y_e < 0.5 \Rightarrow$ **neutron-rich**$\Rightarrow$ r-Process

The Two Assumptions

- Distribution of baryon-to-photon ratio η
  - $\Rightarrow$ Homogeneous (HBBN)
  - $\Rightarrow$ Inhomogeneous (IBBN)

- Net electron, muon, & tau lepton numbers
  - $\Rightarrow$ usually all assumed very small ($\sim \eta$)
  - $\Rightarrow$ Degeneracy?
  - $\Rightarrow$ High Neutrino Masses?

limits from BBN & structure considerations

$$-0.04 \lesssim L_{\nu_e} \lesssim 0.84$$

$$|L_{\nu_\mu,\nu_\tau}| \lesssim 27.5$$

Kang & Steigman '92

Weak Interactions

- **Weak Decoupling**

$T \approx 1 - 3\,\mathrm{MeV}$

neutrinos drop out of thermal equilibrium

- **Weak Freeze-Out**

$T \approx 0.7\,\mathrm{MeV}$

chemical equilibrium (n/p ratio) freeze out

$$\nu_e + n \rightleftharpoons p + e^-$$

$$\bar{\nu}_e + p \rightleftharpoons n + e^+$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

in equilibrium $n/p \approx e^{-\delta m/T} \approx e^{-\frac{1.29\ \mathrm{MeV}}{T}}$

Smith, Kawano, & Malaney '93

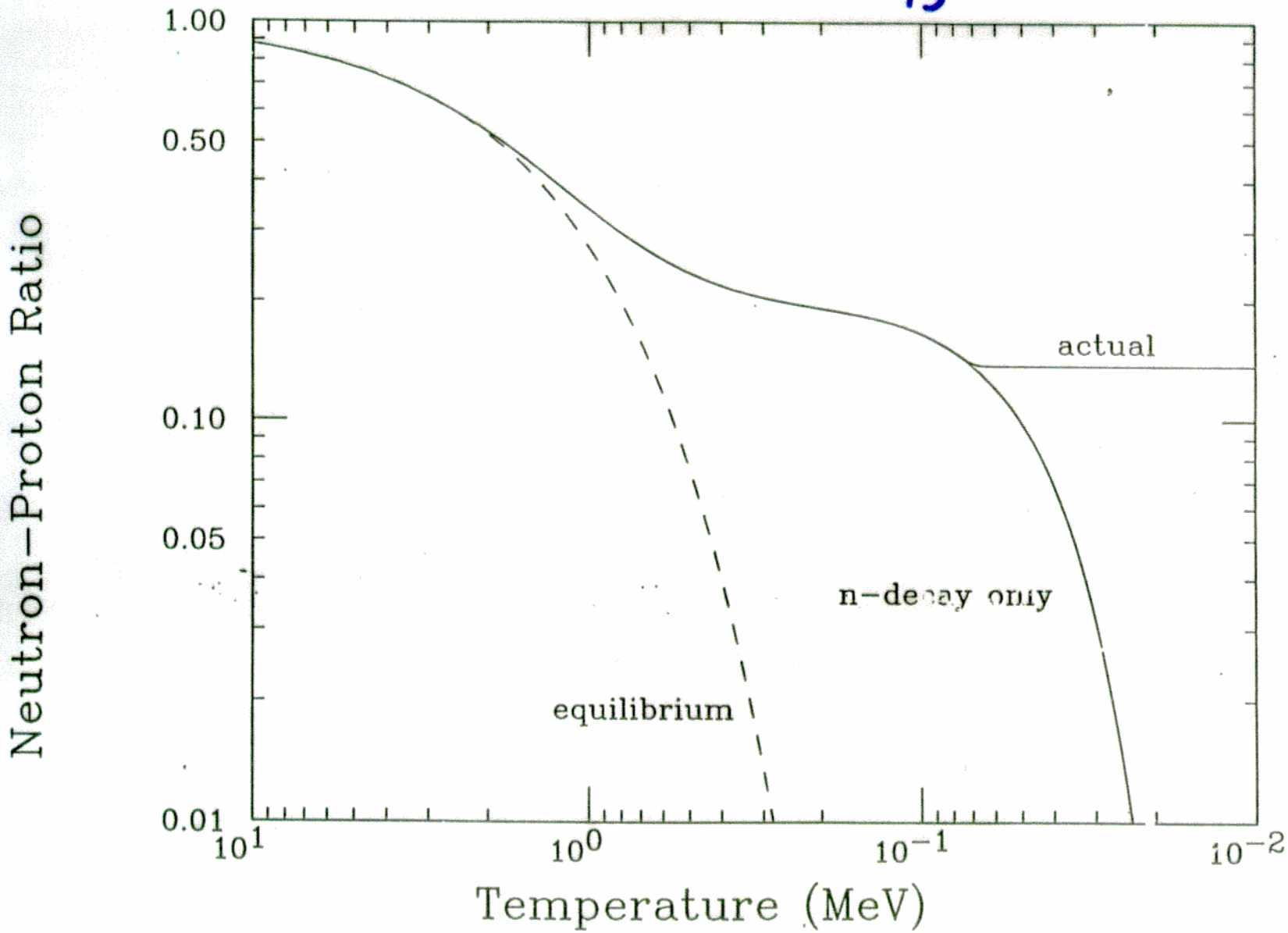


Figure 1.

G. Fuller, SSI 2000

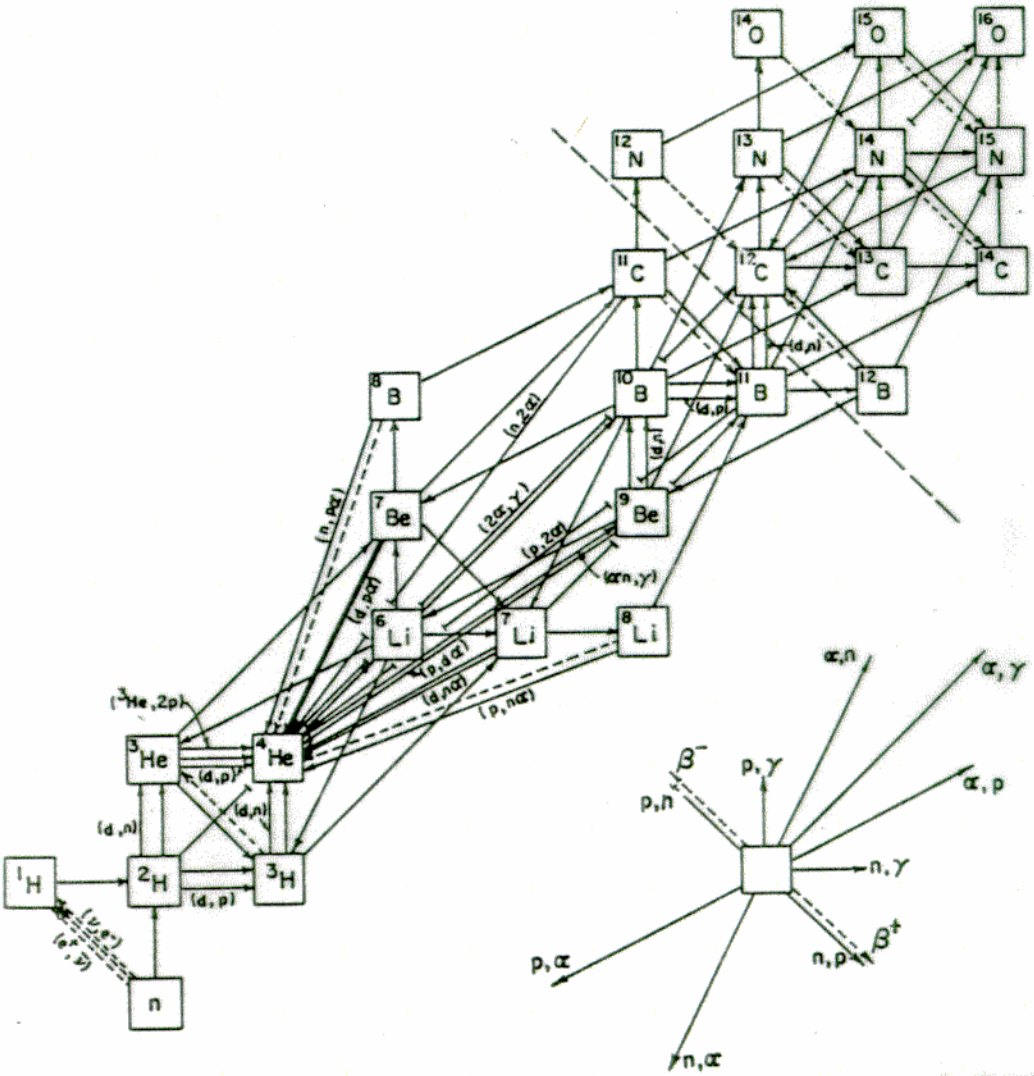


FIG. 1.—Diagram of all nuclear reactions included in the calculation. The exoergic directions are indicated by the arrows. The dashed line indicates that individual nuclear abundances were computed only for $A < 12$.

Freeze-out From Nuclear Statistical Equilibrium (NSE)

$$Z\,p + N\,n \rightleftharpoons A(Z,N) + \gamma$$

NSE Freeze Out

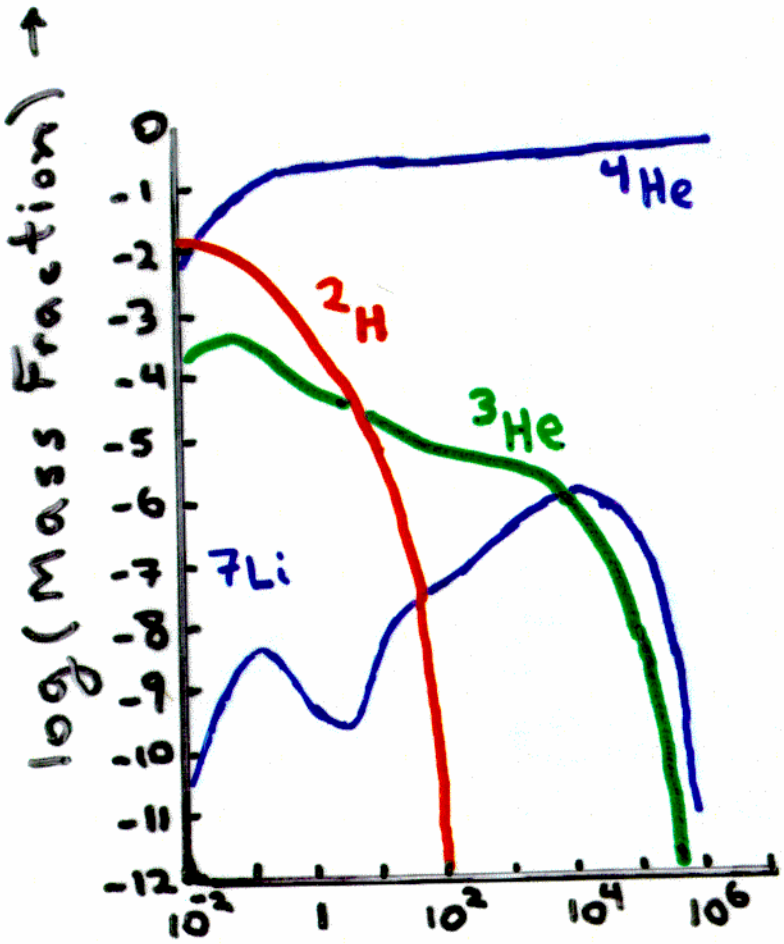
	T [MeV]
n/p	~ 0.7
^4He	~ 0.6
^3He, ^3H QSE with ^2H	~ 0.2
$n + p \rightleftharpoons {}^2H + \gamma$	~ 0.07

BBN Abundance Yields

(Wagoner, Fowler, Hoyle ApJ 148, 3, 1967)

$N_\nu = 2$ $\tau_n \simeq$ 1016 seconds

modern value is $\tau_n \simeq 887.0$ s Montanet et. al. (1994)



$\eta_{10} \equiv 10^{10}\eta$ →

← S

Schramm & Turner
RMP '97

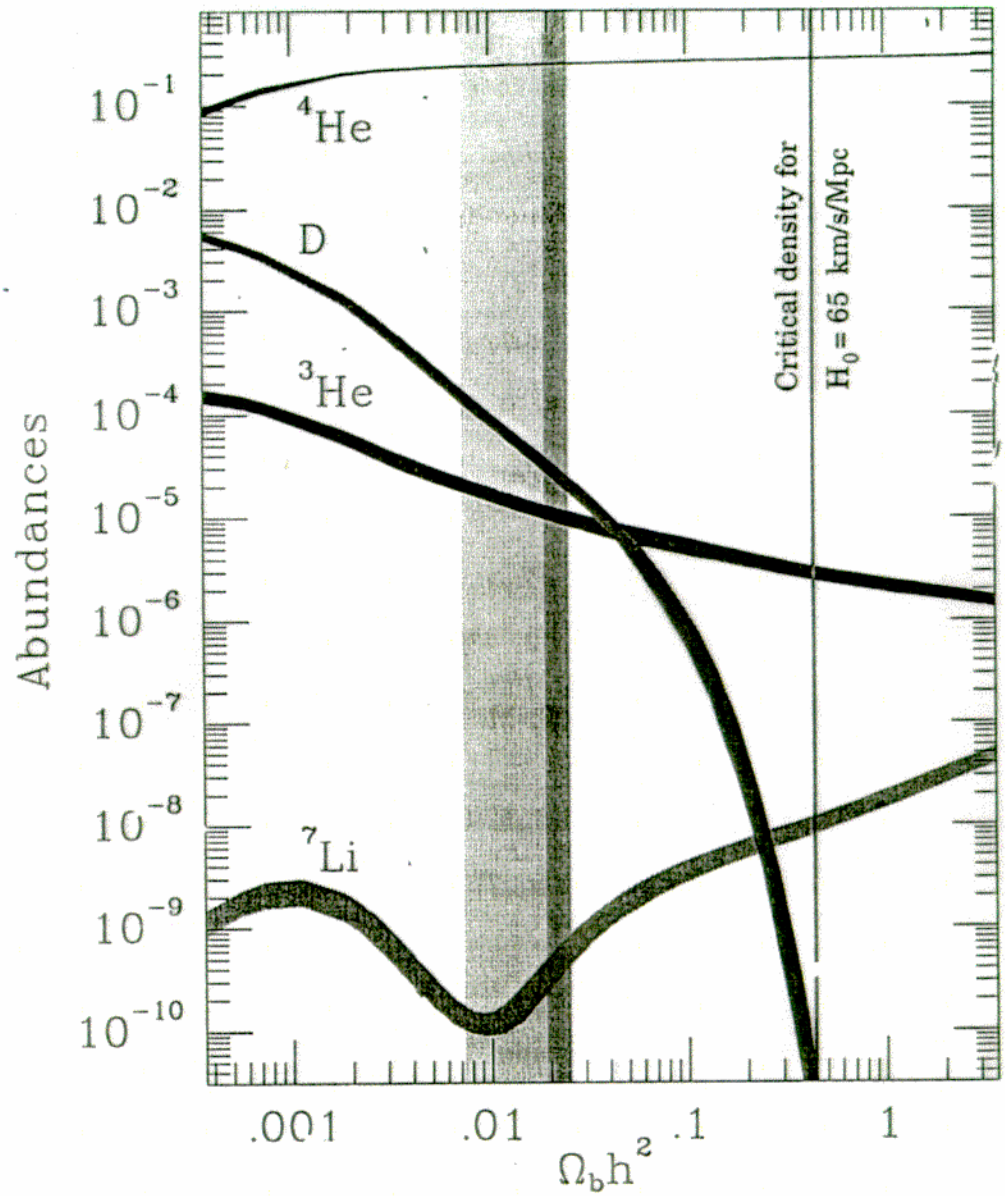


Figure 1: Summary of big-bang production of the light elements. The widths of the curves indicate the 2σ theoretical uncertainties, and the vertical band is the Copi et al [16] consistency interval where the predicted abundances of all four light elements agree with their measured primeval abundances. The darker band in the consistency interval corresponds to Tytler et al's determination of the primeval deuterium abundance (Figure courtesy of K. Nollett).

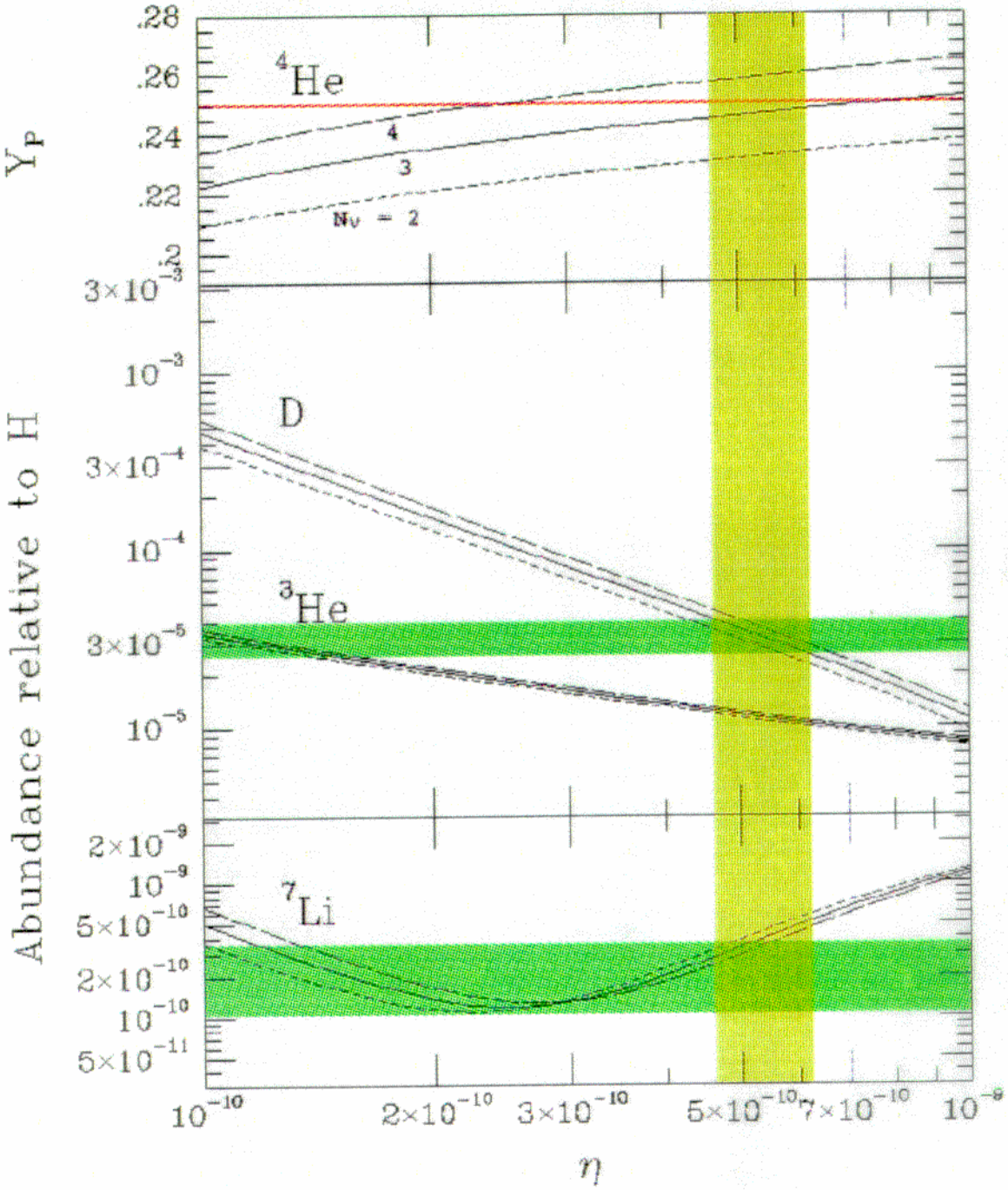
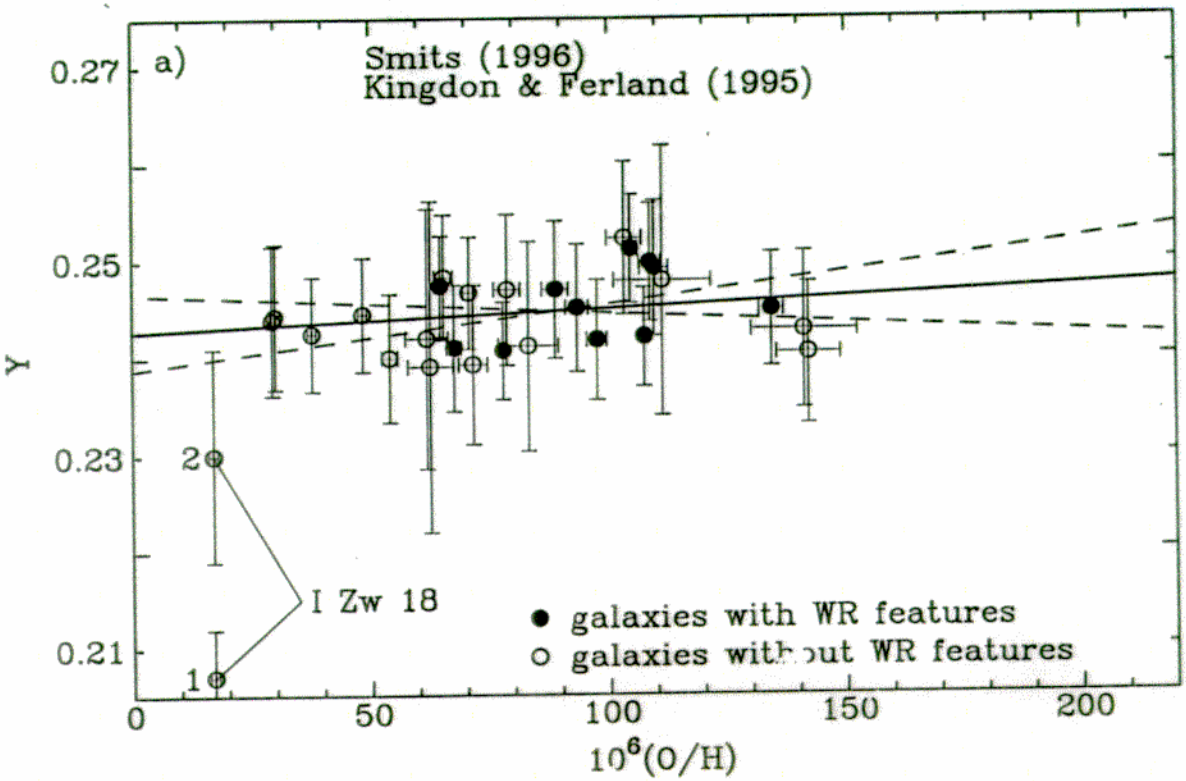
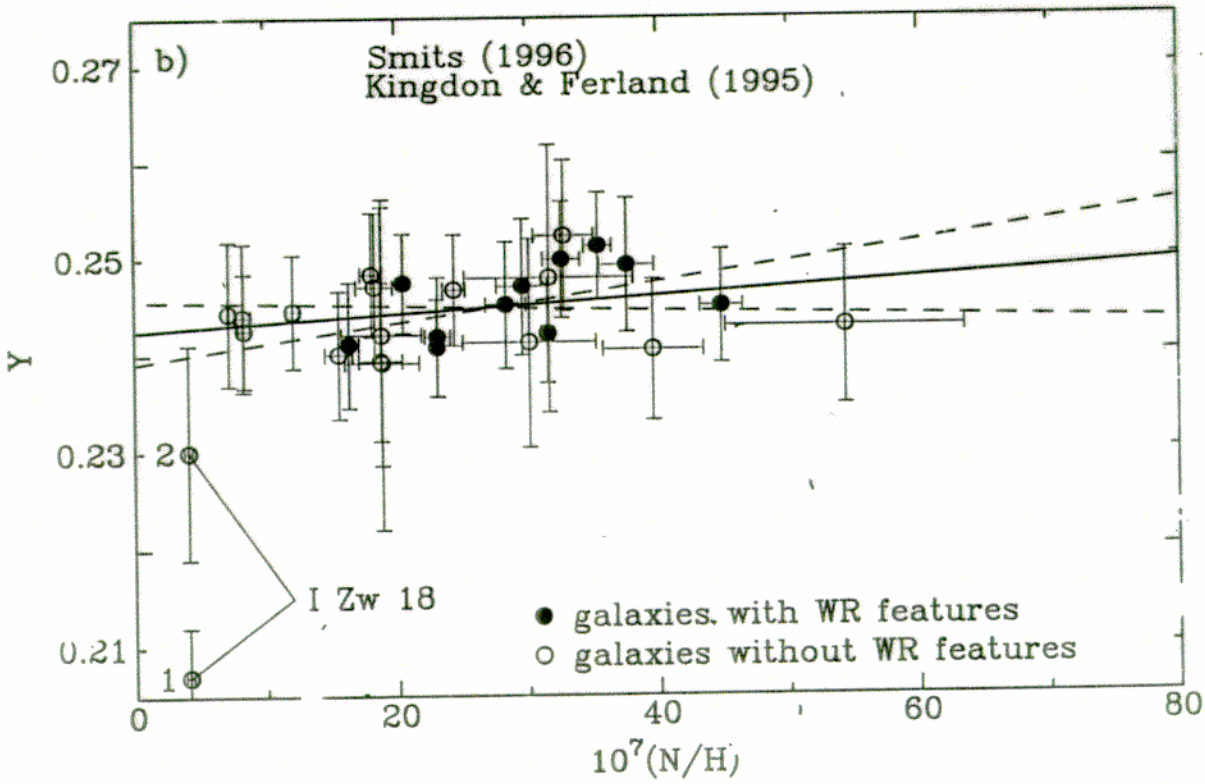


Figure adopted from Copi, Schramm & Turner (1996)

Izotov, Thuan, & Lipovetsky (1996)

$Y_p \approx 0.243 \pm 0.003$





Zero Order BBN

- Abundance of free neutrons $\Rightarrow Y_{\rm n} = \frac{\rm n}{\rm n+p} = \frac{\rm n/p}{1+{\rm n/p}}$

- Abundance of α's $\Rightarrow Y_\alpha = X_\alpha/A_\alpha = X_\alpha/4$, where X_α is the α particle mass fraction.
(In general for species i with mass number A_i, the abundance is $Y_i = X_i/A_i$.)

$\Rightarrow$ Roughly, **every** neutron at $T \approx 0.1$ MeV is incorporated into an α particle!!

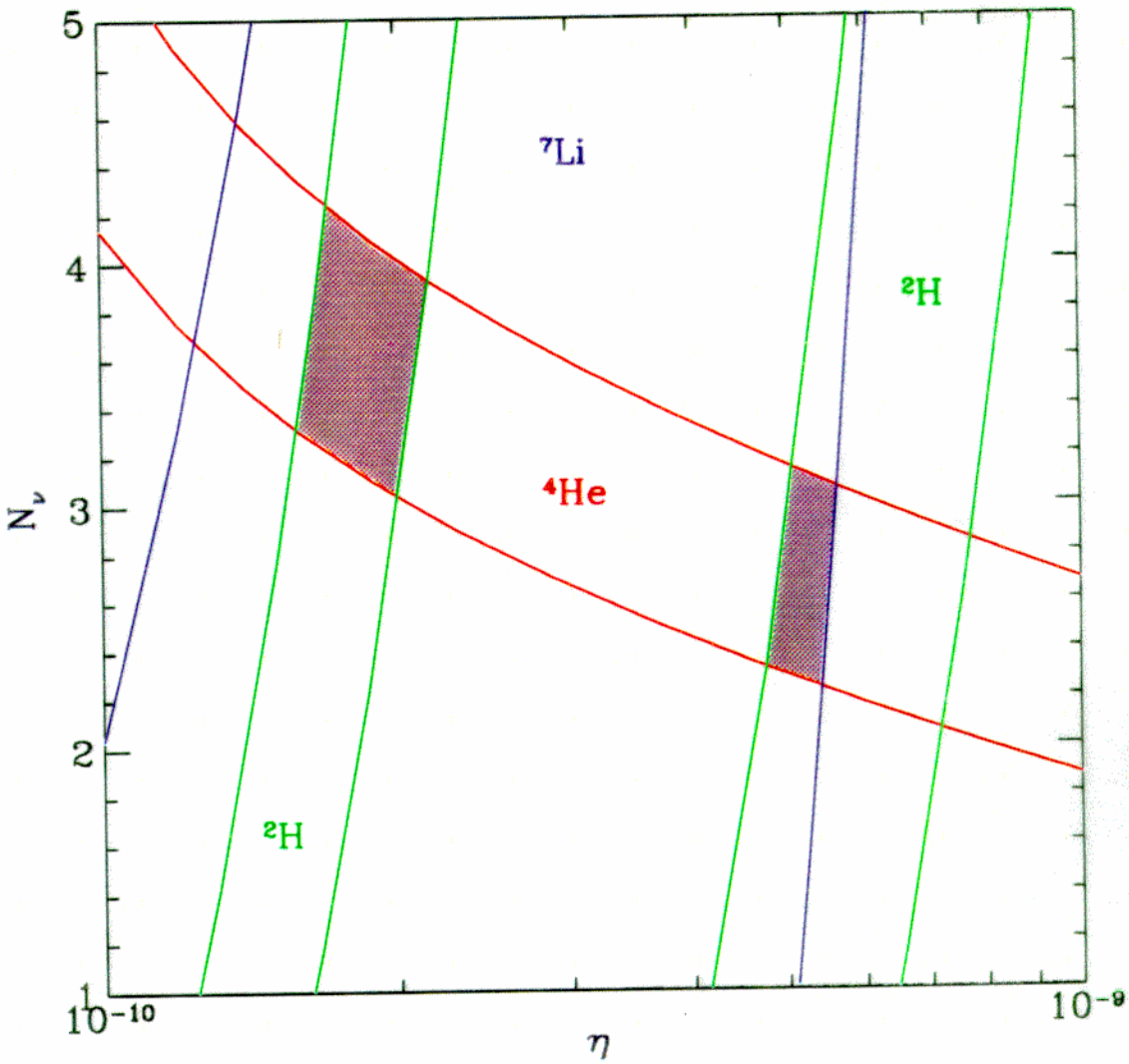
$\Rightarrow$ Therefore, $Y_{\rm n} \approx 2Y_\alpha$
or $X_\alpha \approx (A_\alpha/2)Y_{\rm n} = 2Y_{\rm n}$

- Weak Freeze-Out n/p $\approx 1/6$

- At ^{4}He Synthesis n/p $\approx 1/7$

$\Rightarrow$ Therefore, at ^{4}He synthesis ($T \approx 100$ keV),
$X_\alpha \approx \frac{2{\rm n/p}}{1+{\rm n/p}} \approx \frac{2(1/7)}{1+1/7} = 0.25$

Cardall & Fuller 1996



Hogan 1995 and Burles et al. 1996 ^{2}H/H

$0.237 \leq {}^4\mathrm{He} \leq 0.249$ (Izotov et al. 1996)

$0.7 \leq 10^{10}\ {}^7\mathrm{Li/H} \leq 3.8$ (Olive 1995, depletion + CR production)

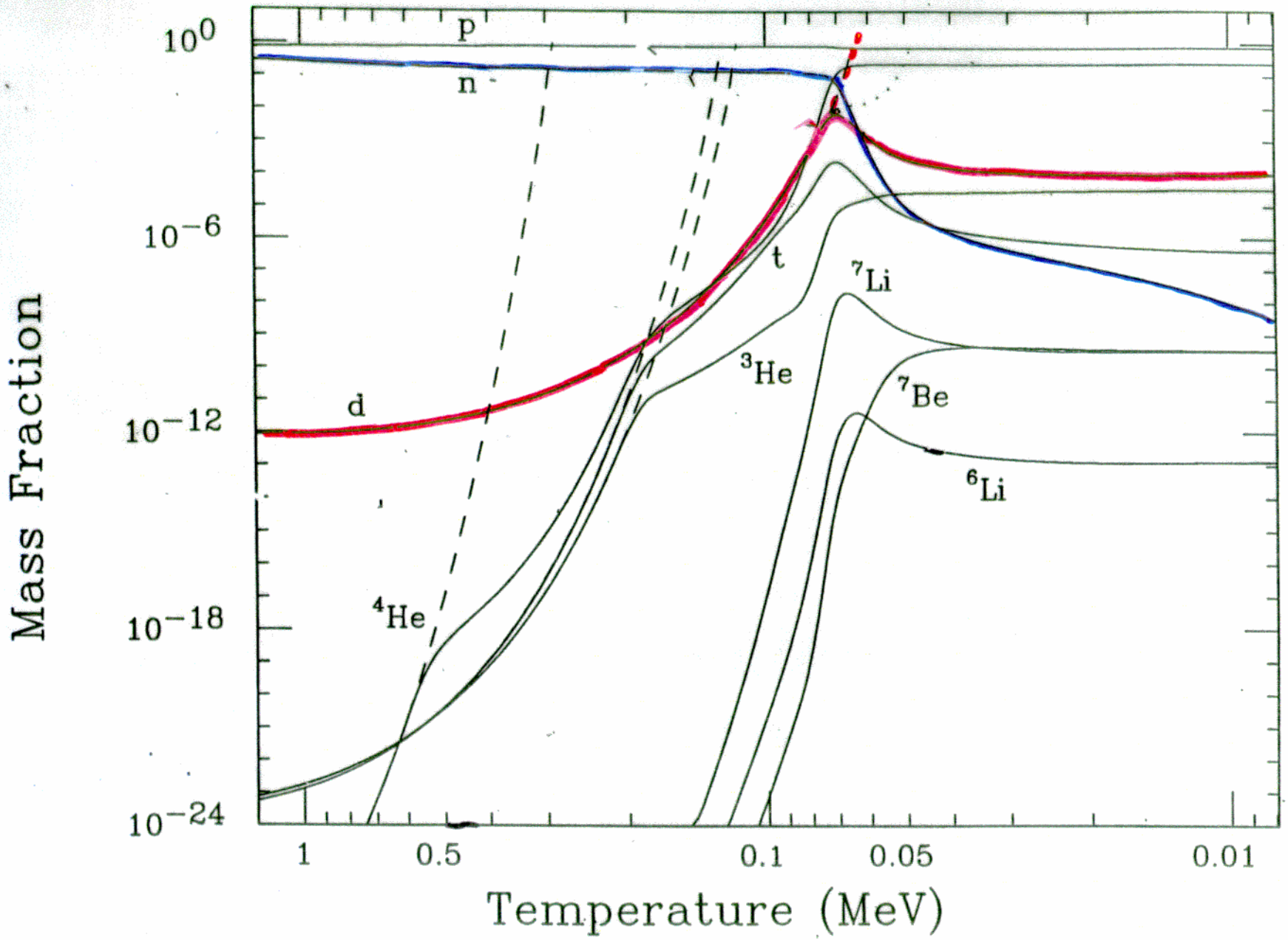
$$n + p \rightleftharpoons d + \gamma$$

- in NSE abundance of ^2H (deuterium) grows with falling temperature

$$X_d \propto e^{B.E./T} \approx e^{2.2\,MeV/T}$$

- NSE ends when free neutrons disappear

Figure 1.



Smith, Kawano, & Malaney (1993)

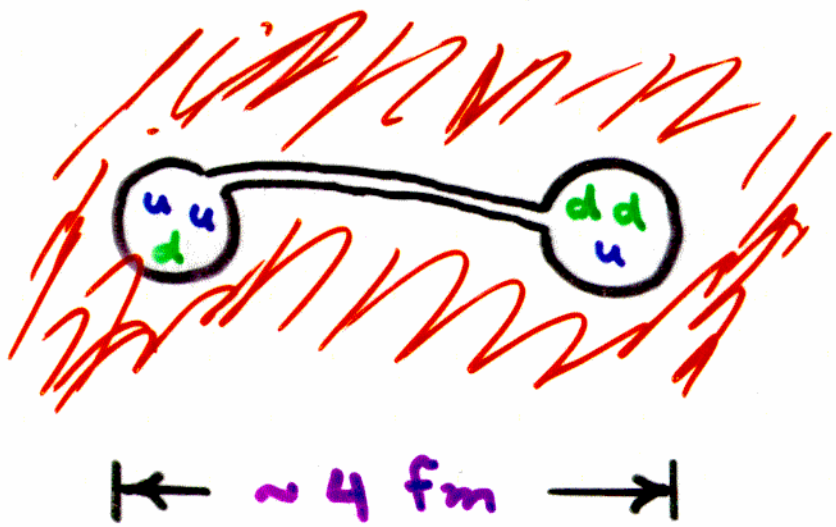
Figure 2.

G. Fuller, SSI 2000

The Deuteron
- a Bizarre Nucleus!

- Binding Energy $E_B \approx 2.225\,\mathrm{MeV}$
$J^\pi = 1^+$, ($^3S_1$ plus $2\% - 6\%$ $^3D_1$)

- The Deuteron is Big and Fragile:



Why Is It So Hard To Make Deuterium?

$\Rightarrow$ Because Neutrons Are Unstable!!

$\Rightarrow$ **Can Get Neutrons By:**

- Weak Interaction $\bar{\nu}_e + \mathrm{p} \mapsto \mathrm{n} + e^+$

But G_F is small!!

- "Spall" Neutrons from Nuclei

$\gamma + \mathrm{A(Z,N)} \mapsto \mathrm{A(Z,N-1)} + \mathrm{n}$

But typical neutron binding $\sim 8\,\mathrm{MeV}$!

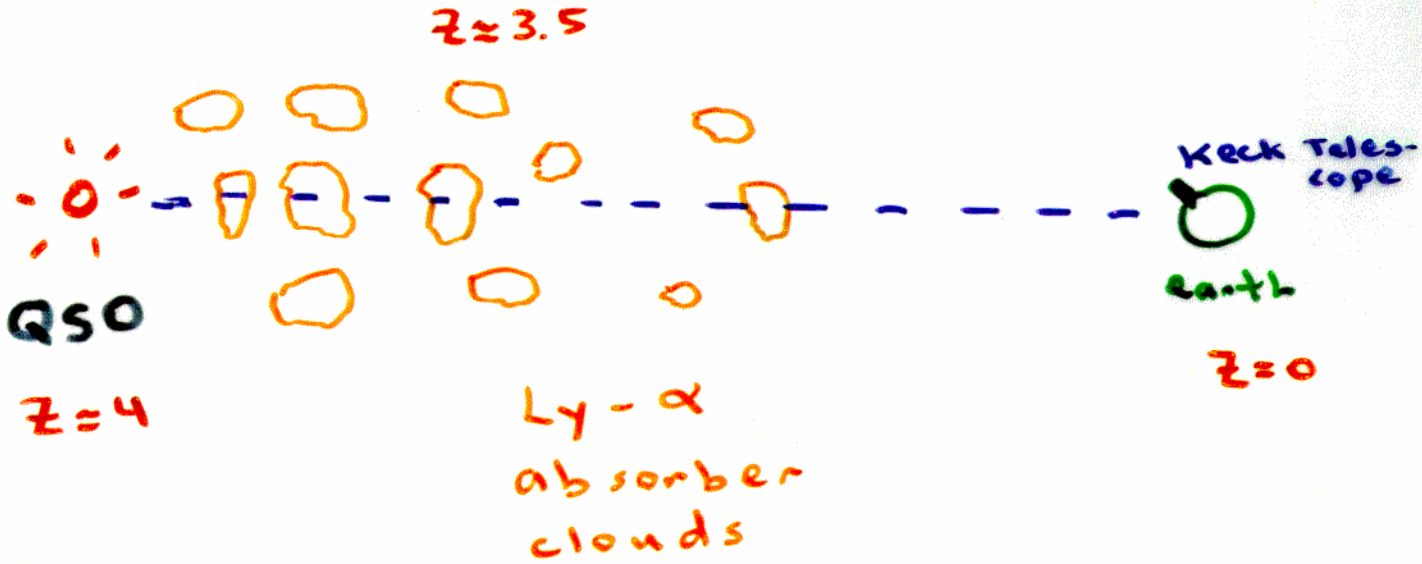
- "Spall" Neutrons from Neutron Stars

(*e.g.*, Binary Neutron Star Coalescence)

But typical neutron binding $\gtrsim 100\,\mathrm{MeV}$!!

$\Rightarrow$ **PLUS:**

- Must have $\mathrm{n(p,\gamma)^2H}$ **FAST**;
- **but** $^2\mathrm{H(p,\gamma)^3He}$ **SLOW**.

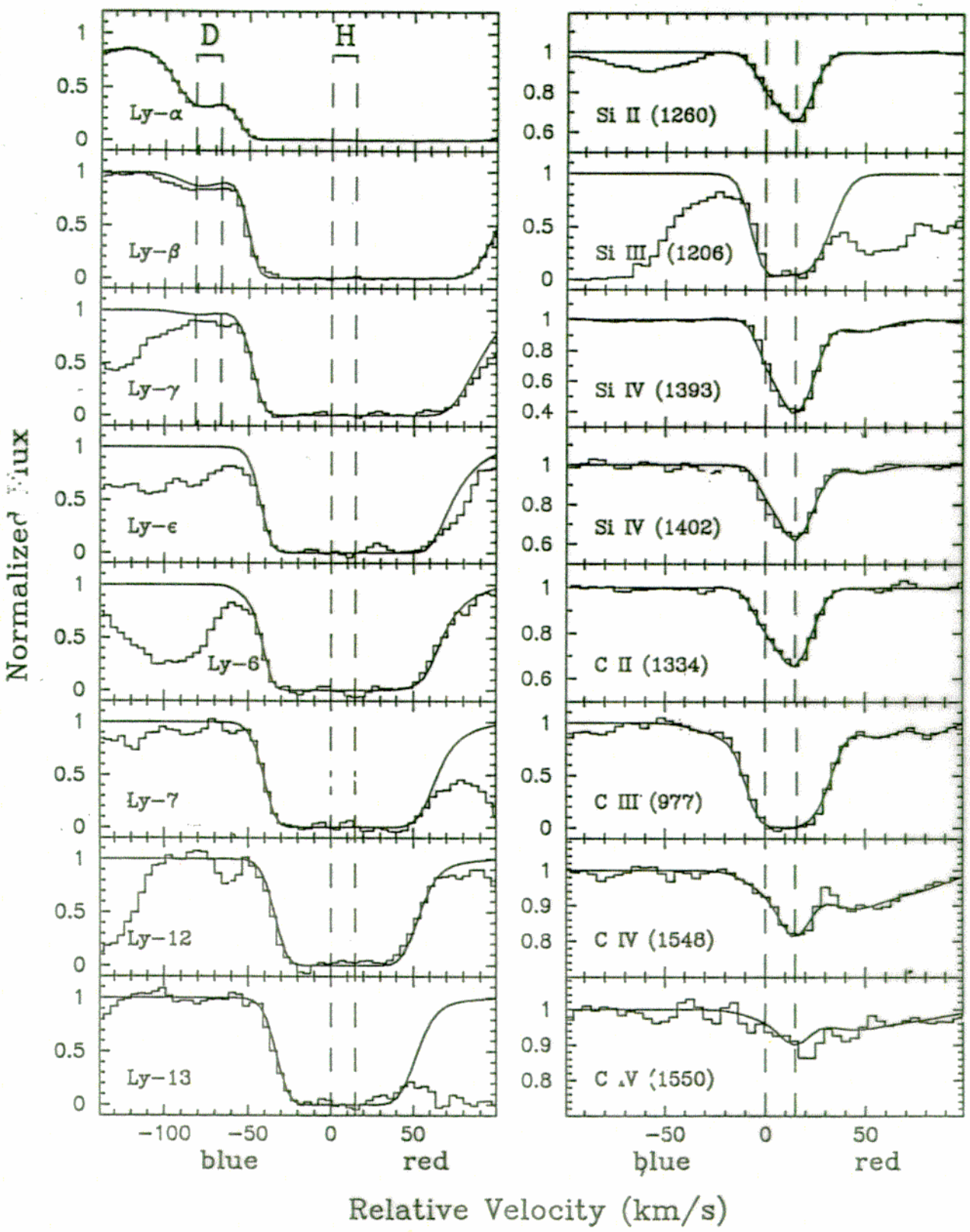


look for isotope-shifted Deuterium abs. lines

* need narrow lines
* need reliable HI column density

$z \approx 3.57$

Tytler, Fan, & Burles (1996)



Neutrino Transformation in BBN

(1.) Active-Sterile Only

$\nu_{\mu,\tau} \rightleftharpoons \nu_s$ **and** $\bar{\nu}_{\mu,\tau} \rightleftharpoons \bar{\nu}_s$

c.f., Shi, Schramm, & Fields 1995,93
Shi 1996

general constraints { Barbieri & Dolgov '91; Enqvist, Kainulainen, Thomson '92; J.M. Cline '92; Foot, Thomson, Volkas 96

(2.) Active-Active Only $\nu_\alpha \rightleftharpoons \nu_\beta$

$$\nu_\alpha, \nu_\beta = \nu_e, \nu_\mu, \nu_\tau$$

c.f., Savage, Malaney, & Fuller 1992

$\Rightarrow$ **can "hide" an initially large lepton number in neutrinos**

$\Rightarrow$ **can relax Helium abundance limits on neutrino degeneracy**

(3.) Active-Sterile Plus Active-Active

c.f., Foot & Volkas 1997; 1998 Lipari et al. '99

Shi & Fuller 1998 Abazajian, Shi, Fuller '00

(a.) $\overset{(-)}{\nu}_\mu \rightleftharpoons \overset{(-)}{\nu}_s$ **creates "large" net mu lepton number**

$$|L_{\nu_\mu}| > 0$$

(b.) $\overset{(-)}{\nu}_\mu \rightleftharpoons \overset{(-)}{\nu}_e$ **re-distributes lepton number to create a net electron lepton number** $L_{\nu_e} > 0$

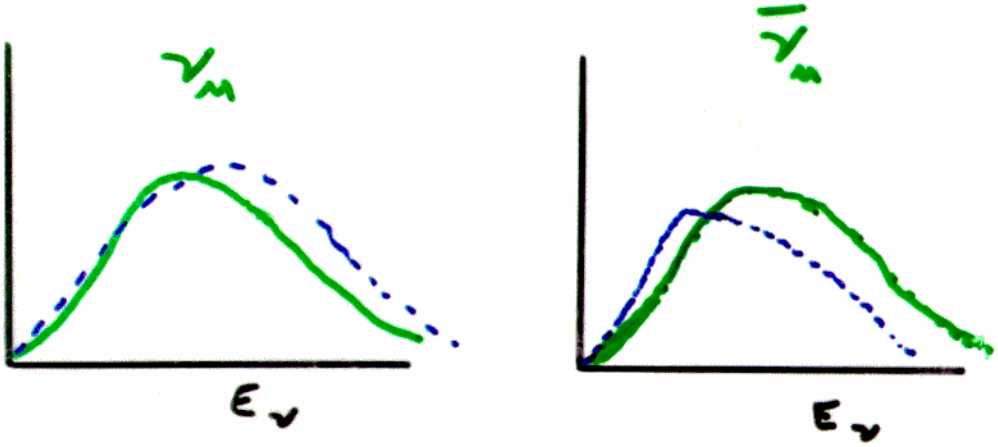
Note that in both steps (a.) and (b.) we could employ tau rather than mu neutrinos (or both)

(4.) Any lepton no. generated at all?!

S. Hansen, S. Dolgov, Semikoz '00

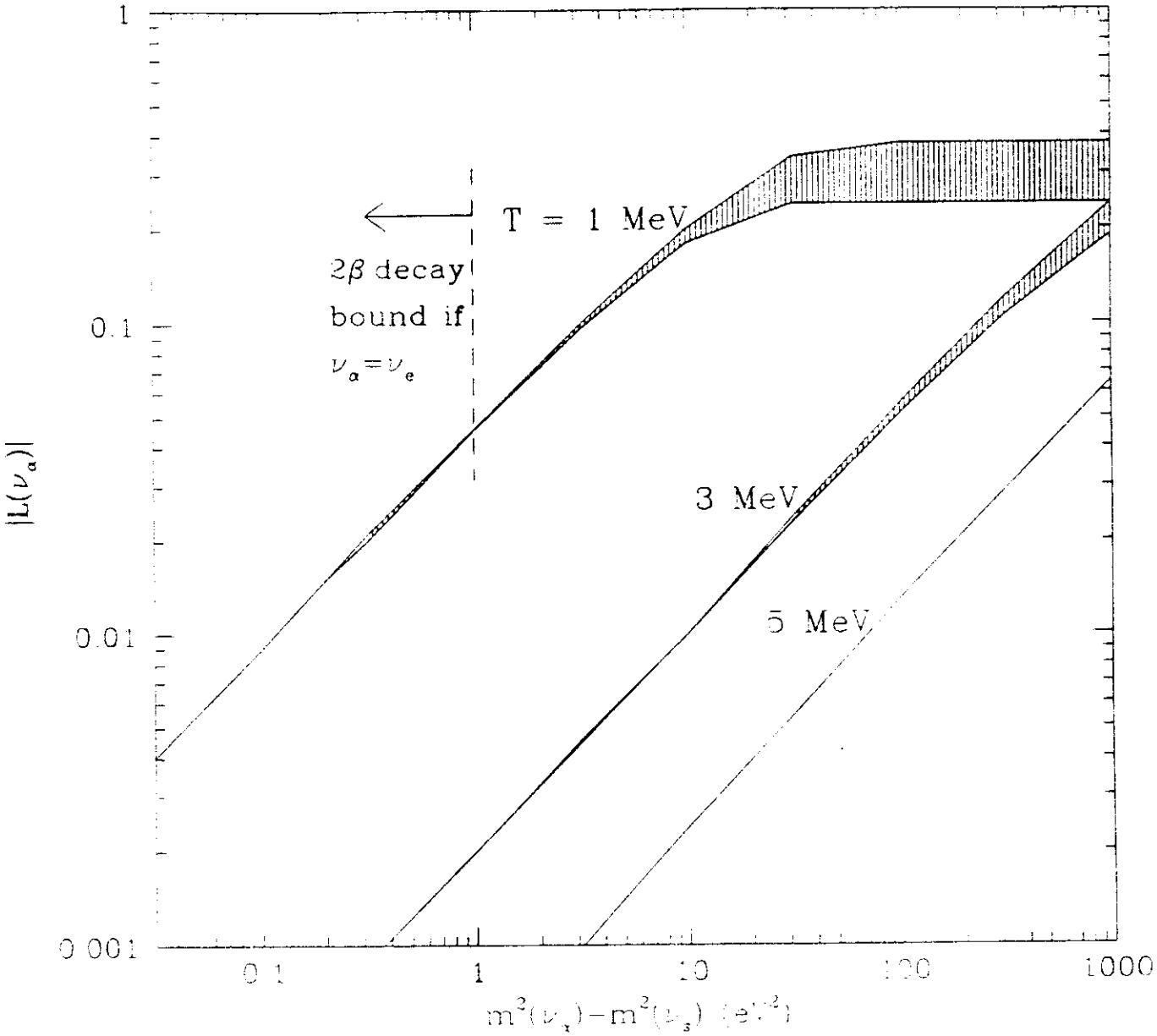
$\bar{\nu}_m \rightleftharpoons \bar{\nu}_s$

MODIFICATION BY neutrino SCATTERING, PAIR PRODUCTION, ANNIHILATION:
e.g., $\nu + \bar{\nu} \rightleftharpoons e^- + e^+ \rightleftharpoons 2\gamma$



more ν_m !!! Fewer $\bar{\nu}_m$!!!

------ Final Distributions


1
0.1
0.01
0.001
0.1
1
10
100
1000
|L(ν_α)|
m²(ν_τ)−m²(ν_s) (eV²)
T = 1 MeV
2β decay
bound if
ν_α=ν_e
3 MeV
5 MeV

Active-Sterile Neutrino Transformation & BBN

• X. Shi, K. Abazajian, & G.M.F. : First semi-self consistant calculation of the evolution of all neutrino energy distribution functions and nuclear reaction rates through the BBN epoch. Two cases were considered:

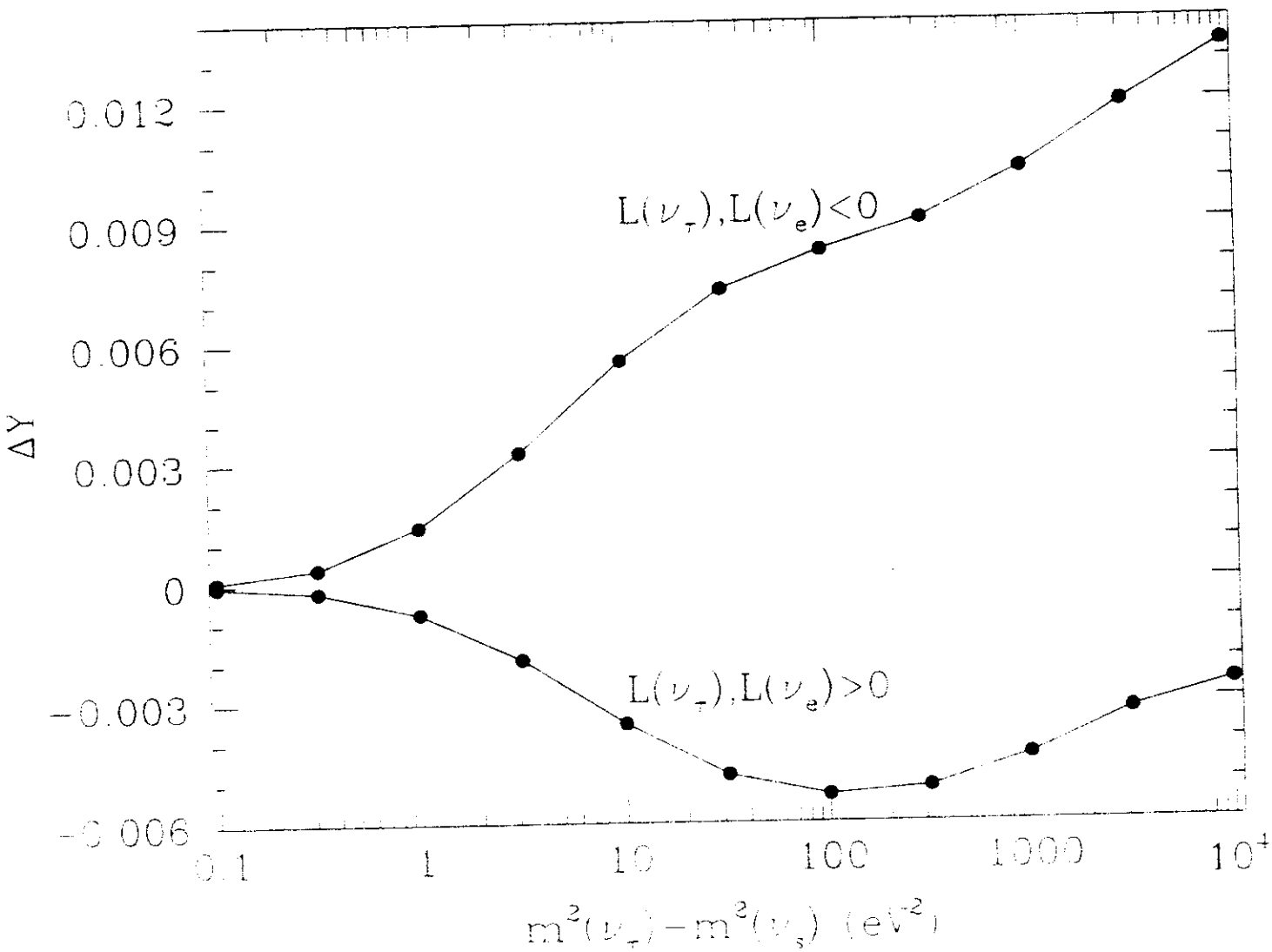
• (1) Lepton Number generation via direct $\nu_e \rightleftharpoons \nu_s$.

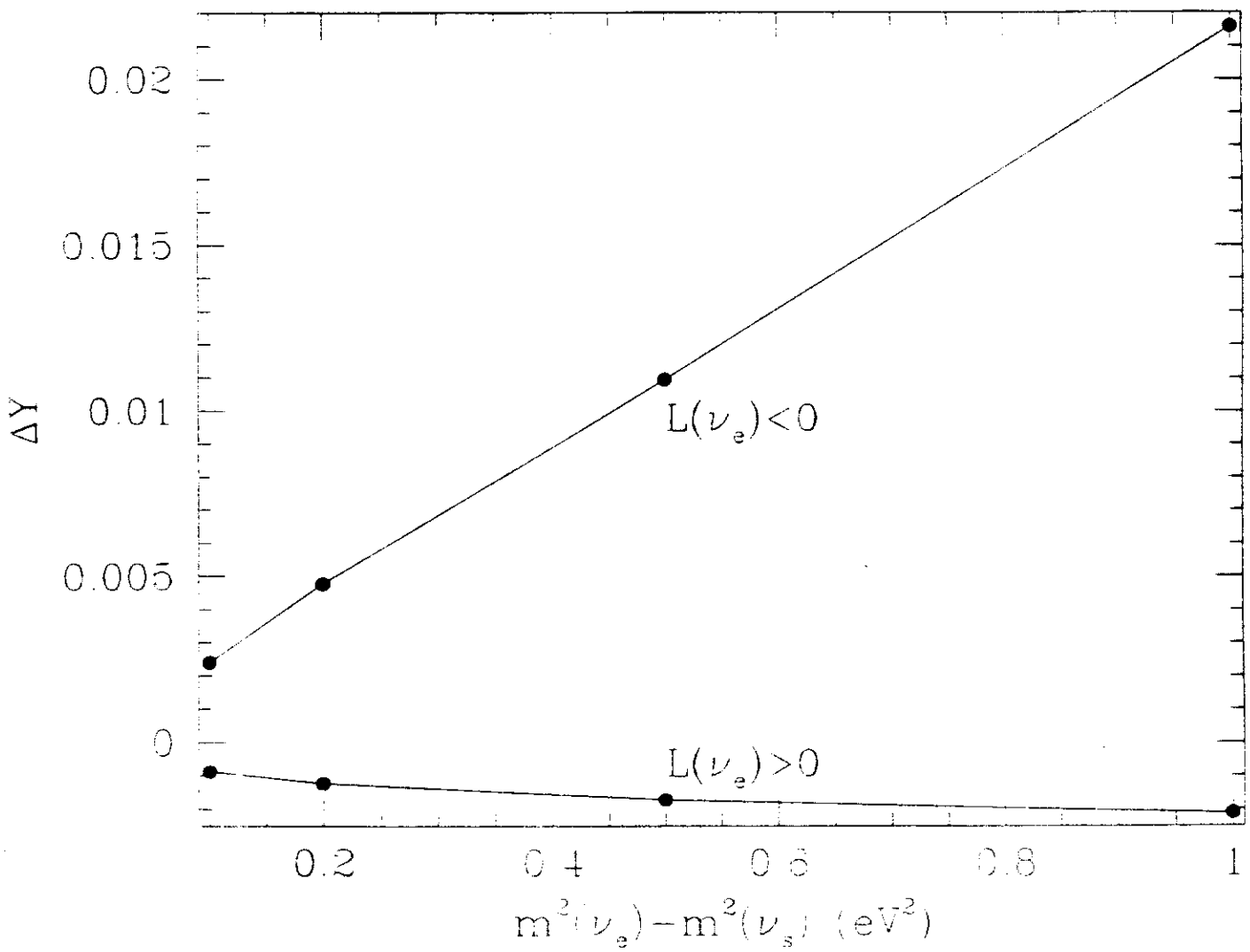
$\Rightarrow$ Find that change in primordial ^{4}He relative to SBBN is $\Delta Y \approx -1\%$ to $+9\%$, with the sign depending on sign of Lepton Number.

• (2) Examine Lepton Number Asymmetry in the mu and/or tau sector generated via $\nu_{\mu,\tau} \rightleftharpoons \nu_s$ followed by partial transfer of this Lepton Number to the $\nu_e\bar{\nu}_e$-sector via matter-enhanced active-active transformation.

$\Rightarrow$ Find that maximal reduction of ^{4}He relative to SBBN is $\sim 2\%$ if the Lepton Number in neutrinos is positive. Otherwise the increase in Y is $\lesssim 5\%$ for $m^2_{\nu_\mu,\nu_\tau} - m^2_{\nu_s} \lesssim 10^4\,\mathrm{eV}^2$.

$*$ **Overall Conclusion**: Change in ^{4}He yield induced by neutrino-mixing-generated lepton number asymmetry can be large in the upward direction, but is quite limited in the downward direction (the direction for ameliorating the "Crisis").


0.012
0.009
0.006
0.003
0
−0.003
−0.006
ΔY
L(ν_τ),L(ν_e)<0
L(ν_τ),L(ν_e)>0
0.1
1
10
100
1000
10^4
m²(ν_τ)−m²(ν_s) (eV²)

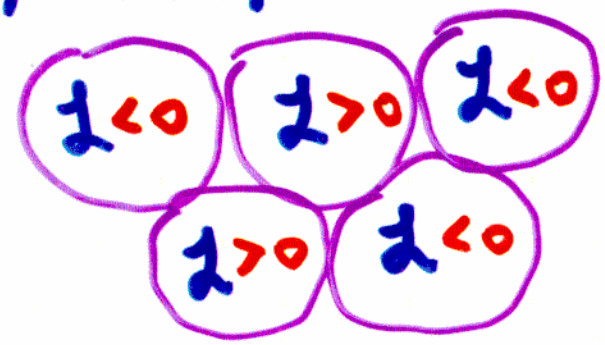

0.02
0.015
0.01
0.005
0
ΔY
L(ν_e)<0
L(ν_e)>0
0.2
0.4
0.6
0.8
1
m²(ν_e)−m²(ν_s) (eV²)

Lepton Number Generation and The Causal Structure of Spacetime

X. Shi & G.M. F
PRL (in press '99)

X. Shi (1996) has shown that the lepton number generation process is **chaotic**

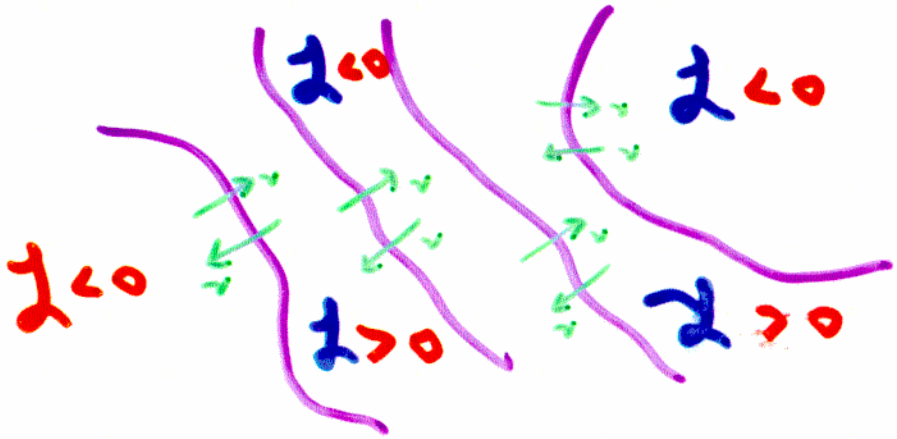
⇒ will create domains of lepton number which are bounded by the particle horizon size



lepton no. generation epoch

⇓ (some complicated percolation process)

(just prior to...) BBN epoch



$\mathcal{L}$ gradients drive ν_s production

BBN, CFQ, LSND and all PDQ

X. Shi, G. M. F., K. Abazajian, PRD (submitted)

$$\delta m^2_{\mu-\tau} \sim 10^{-2}\,\mathrm{eV}^2$$

ν_μ ——— ν_τ

$$\delta m^2_{\mathrm{doublets}}$$

ν_s - - - - - - - - - - ν_e ———

$$\delta m^2_{\mathrm{se}} \leq 10^{-5}\,\mathrm{eV}^2$$

MSW in ⊙

BBN ⇒

$\sin^2 2\theta_{\mu s} \lesssim 10^{-10}$

[all active-sterile level crossings at domain boundaries must be nonadiabatic]

ν_s -·-·-·-·-·-·- ν_e

$$\delta m^2_{e-s} \sim 10^{-10}\,\mathrm{eV}^2$$

"just so" for ⊙

BBN ⇒

$$F \equiv \frac{\sin^2 2\,\text{"}\theta_{\mu e}\text{"}}{\sin^2 2\,\text{"}\theta_{\mu s}\text{"}}$$

Neutrino Mixing & MSW Resonance

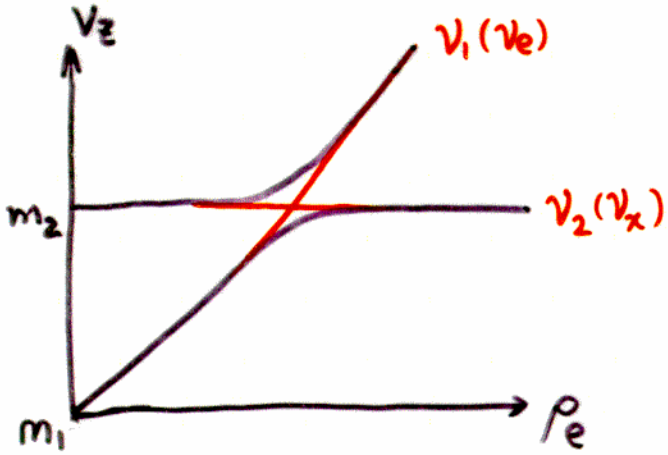
$$i\frac{\partial}{\partial t}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} V_z & \frac{V_x+iV_y}{2} \\ \frac{V_x-iV_y}{2} & 0 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

in vacuum: $V_x = -\frac{m_2^2-m_1^2}{2E}\sin 2\theta$, $V_y = 0$

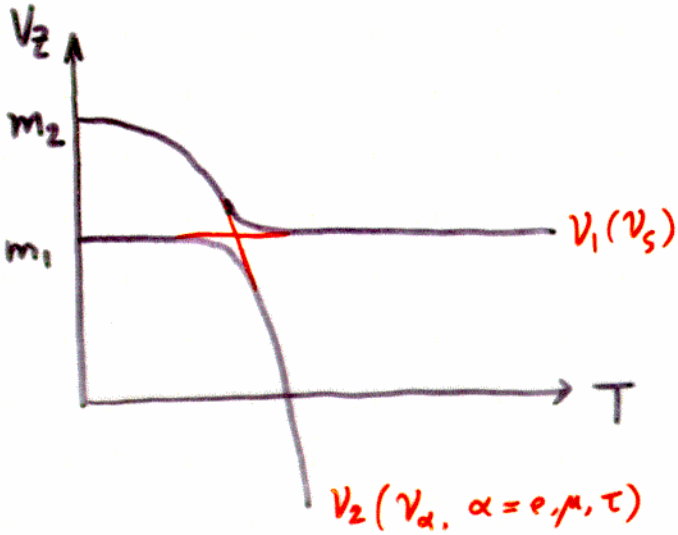
$$V_z = -\frac{m_2^2-m_1^2}{2E}\cos 2\theta$$

in matter medium: $V_z = -\frac{m_2^2-m_1^2}{2E}\cos 2\theta + V_m$

↑ MSW effect



← solar interior ($V_m \propto \rho_e$)



← the early universe ($V_m^{(T)} \propto T^5$, $L \lesssim 10^{-9}$)

Active-Sterile Neutrino Oscillations in BBN Epoch

- Density matrix formalism:

$$\rho_\nu = \begin{pmatrix} \rho_{\alpha\alpha} & \rho_{\alpha s} \\ \rho_{s\alpha} & \rho_{ss} \end{pmatrix} = \frac{P_0\, I + \vec{P}\cdot\vec{\sigma}}{2}$$

$$\Rightarrow \quad \rho_{\alpha\alpha} = \frac{P_0 + P_z}{2}, \quad \rho_{ss} = \frac{P_0 - P_z}{2}.$$

- Boltzmann equations:

$$\dot{P}_0 = \sum_{i=e,\nu_\beta;\beta\neq\alpha} \langle\Gamma(\nu_\alpha\bar{\nu}_\alpha \to i\bar{i})\rangle(n_i n_{\bar{i}} - n_{\nu\alpha} n_{\bar{\nu}\alpha})$$

$$\dot{\vec{P}} = \vec{V}\times\vec{P} + \dot{P}_0\hat{z} - D\vec{P}_\perp$$

where n's are normalized to their equilibrium number density, and:

$$\langle\Gamma\rangle = 2D \sim 4G_F^2 T^5$$

$$V_x = \frac{\delta m^2}{2E}\sin 2\theta, \quad V_y = 0, \quad V_z = -\frac{\delta m^2}{2E}\cos 2\theta + V_\alpha^T \pm V_\alpha^L$$

$$V_\alpha^T = \begin{cases} -\sqrt{2}G_F N_\gamma\left[12.61ET(n_{\nu\alpha} + n_{\bar{\nu}\alpha})/4M_Z^2 + 12.61ET/M_W^2\right], & \alpha = e; \\ -\sqrt{2}G_F N_\gamma\left[12.61ET(n_{\nu\alpha} + n_{\bar{\nu}\alpha})/4M_Z^2\right], & \alpha = \mu,\tau. \end{cases}$$

$$V_\alpha^L = \sqrt{2}G_F N_\gamma\left\{L_0 + 0.375\left[2L_{\nu\alpha} + \sum_{\beta\neq\alpha} L_{\nu\beta}\right]\right\} \quad \text{with} \quad L_\nu \equiv \frac{3}{8}(n_\nu - n_{\bar{\nu}})$$

Cosmologically Significant Neutrinos

- $\Omega_\nu = \frac{m_\nu n_\nu^0}{\rho_{\rm crit}} \Rightarrow \; m_\nu \approx (92\,{\rm eV})\,\Omega_\nu h^2$

$$\Downarrow \qquad \Downarrow$$

Important Mass Range for
Light Neutrino Dark Matter:

$$0.3\,{\rm eV} \lesssim m_\nu < 100\,{\rm eV}$$

From the San Diego Deuterium Abundance (Burles & Tytler 1997) we know that the baryon density in the universe contributes

$$\Omega_b h^2 \approx 0.02$$

$$\Downarrow \qquad \Downarrow$$

Therefore to be as **dynamically** important as baryons, neutrino need only contribute $\Omega_\nu \sim \Omega_b$. In this case the sum of the **light (stable) neutrino masses** need only be:

$$\sum m_\nu \gtrsim 1.5\,{\rm eV}$$

Are Neutrinos the Dark Matter?

- Large scale structure formation simulations, plus observational constraints from CMBR observations, large scale structure, and Type I supernovae all suggest that light neutrinos with a "thermal" energy spectrum cannot contribute more than $\Omega_\nu \lesssim 0.2$. In turn this would imply that the sum of the **light active (stable) neutrino masses** is

$$\sum m_\nu \lesssim 8\,\mathrm{eV}$$

$$\Downarrow \quad \Downarrow$$

Basic Problem: Light neutrinos will **damp** density fluctuations by freely streaming. This will continue until they become nonrelativistic, i.e., when the temperature drops below their rest masses. Therefore, the existence of structure at high redshift puts a limit on m_ν and Ω_ν.

- These constraints might not apply to sterile neutrinos with a non-thermal energy spectrum created by MSW matter-enhancement processes (Shi & Fuller 1998).

Is There Trouble with standard CDM ??

(see talx by P. Gondolo)

Maybe ⇒ simulations suggest too many small halos and small scale cores relative to observations

ways out of this "problem"

⇓ ⇓ ⇓ ⇓ ⇓

1.) fault the observations
2.) tune the inflation potential
3.) posit new interactions for CDM particles (annihilate when density high)

Another Way .....

Shi & Fuller '99, Fuller 00,
Fuller & Patel 00,
Fuller, Hogan, Wadsley 00

A New Kind of Neutrino Dark Matter: Matter-Enhanced Production of Sterile Neutrinos with Rest Masses $\sim 100\,\mathrm{eV}$ to $\sim 10\,\mathrm{keV}$

X. Shi & G. M. Fuller, Phys. Rev. Lett. **82**, 2832 (1999)

- The basic idea: Produce these sterile neutrinos in the early universe via matter-enhanced (MSW) conversion of an initial small excess (lepton number) in one or more active neutrino species. MSW is not a thermal production process. Therefore, the resulting sterile neutrino energy spectrum can be grossly nonthermal and, in particular, "cold."

- These sterile neutrinos will be **Cold Dark Matter** on larger scales but may behave in a "soft" manner on scales smaller than, *e.g.*, Dwarf Galaxy scales.

⇒ This could be an "elegant" solution to the CDM core/"clumpiness" problem

cf. Moore et al. ApJL (1999) ⇒ trouble
Klypin et al. astro-ph/9901240 ⇒ no trouble
(Talks of Silk, Kamionkowski, Primack)

- Define net lepton number in neutrino species ν_α relative to photons:

$$L_{\nu_\alpha} \equiv \left(n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}\right)/n_\gamma$$

Here $\alpha = e, \mu, \tau$.

- Define $\mathcal{L} \equiv 2L_{\nu_\alpha} + \sum_{\beta \neq \alpha} L_{\nu_\beta}$

$\Downarrow \qquad \Downarrow \qquad \Downarrow$

Require $\mathcal{L} \sim 10^{-3}$ to ~ 0.1
for sterile neutrino rest masses
$m_{\nu_s} \approx 10\,\mathrm{keV}$ to $\approx 100\,\mathrm{eV}$, respectively.

$\Downarrow \qquad \Downarrow \qquad \Downarrow$

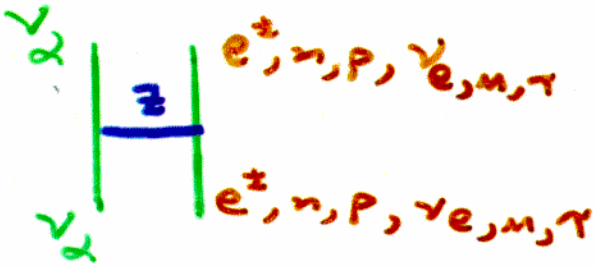
Produces fractional contribution to closure density:

$$\Omega_{\nu_s} \approx \left(\frac{m_{\nu_s}}{343\,\mathrm{eV}}\right)\left(\frac{0.5}{h}\right)^2\left(\frac{\mathcal{L}}{0.1}\right)$$

(h is the Hubble parameter in units of $100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$)

$$\Omega_{\nu_s} \approx 0.17\left(\frac{m_{\nu_s}}{1\,\mathrm{keV}}\right)\left(\frac{0.65}{h}\right)^2\left(\frac{\mathcal{L}}{10^{-2}}\right)$$

Active Neutrinos Acquire Effective Masses via Forward Scatterings



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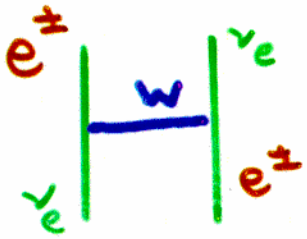
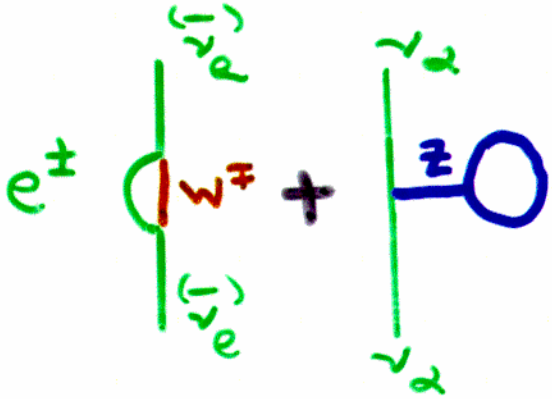


G_F



thermal terms (negative)

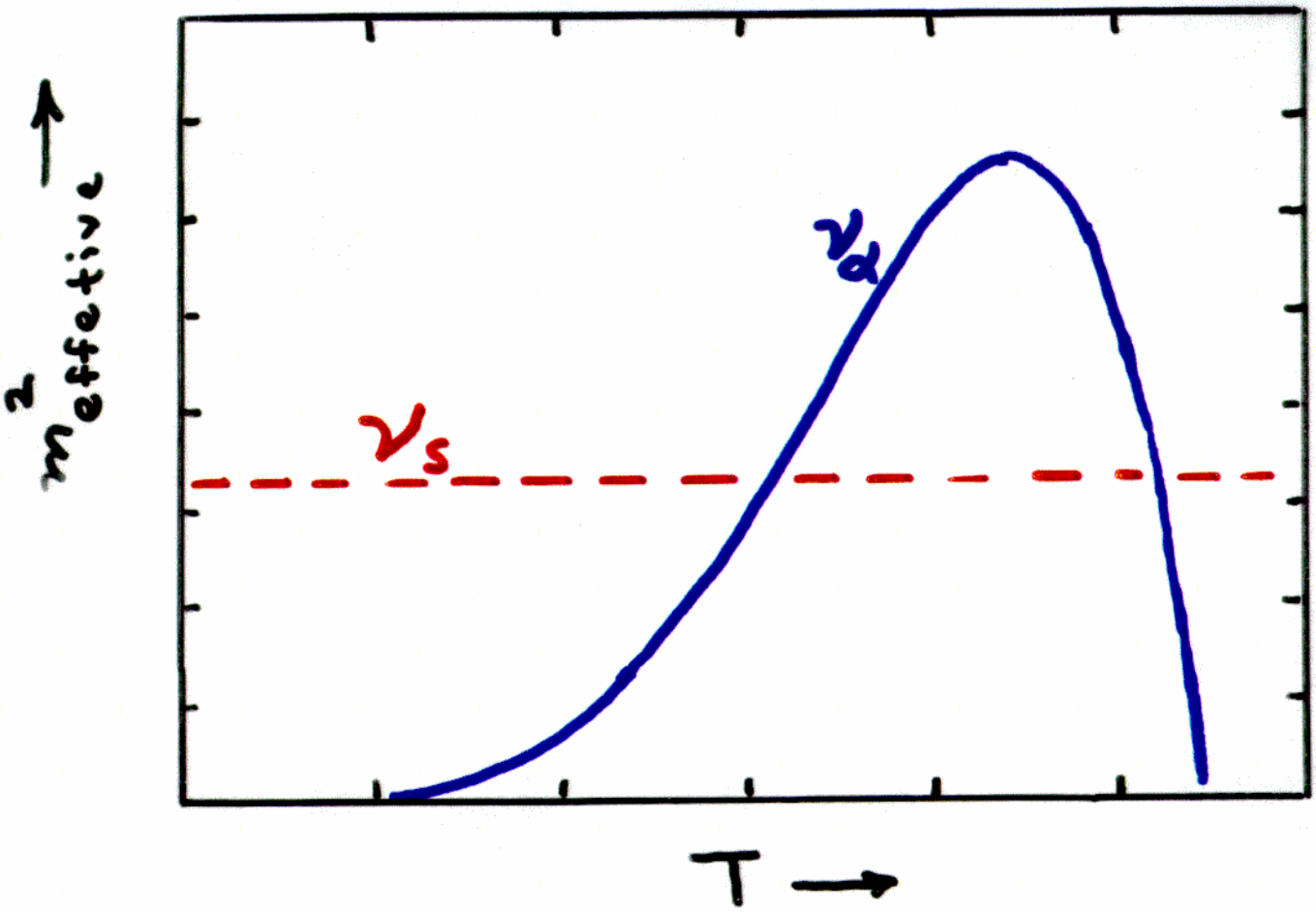
G_F^2

total Weak Potential $V = A + B + C$

square of "effective mass"

$$m_{eff}^2 = 2ETV \approx (8.03\ \mathrm{keV}^2)\,\varepsilon\left(\frac{\alpha \pm \eta/4}{10^{-2}}\right)\left(\frac{T}{100\ \mathrm{MeV}}\right)^4 - \begin{Bmatrix} 0.022\ \mathrm{keV}^2 \\ 0.006\ \mathrm{keV}^2 \end{Bmatrix} \varepsilon^2 \left(\frac{T}{100\ \mathrm{MeV}}\right)^6$$

$$\varepsilon \equiv E_\nu / T$$

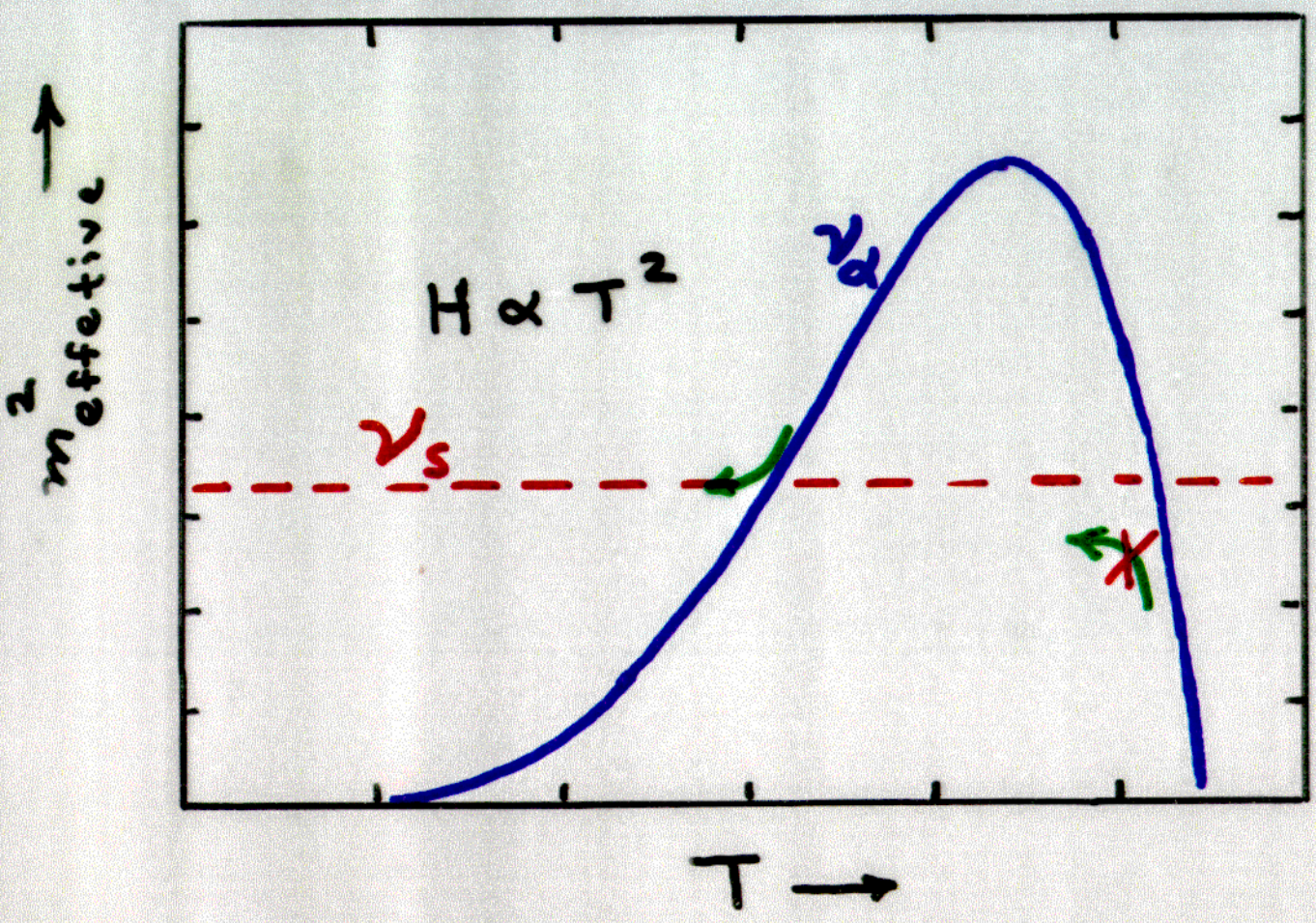


1. small excess of ν_α over $\bar{\nu}_\alpha$ ($\sim 10^{-2} - 10^{-3}$)

2. MSW (matter-enhanced) transfer $\nu_\alpha \rightarrow \nu_s$

$\Downarrow$

{ only efficient for Low Energy $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$



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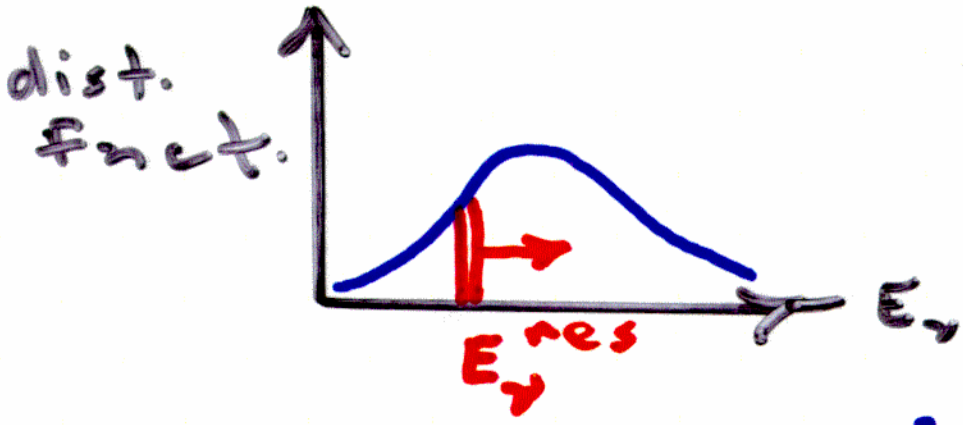
adiabatic condition

resonance "sweep rate" through the active ν energy distribution must be slow compared to local ν-oscillation frequency at resonance



$$\varepsilon_{res} = \frac{E_\nu^{res}}{T} \simeq 0.125 \left(\frac{m_{\nu_s}}{1\,\text{keV}}\right)^2 \left(\frac{\mathcal{L}}{10^{-2}}\right)^{-1} \left(\frac{100\,\text{MeV}}{T}\right)^4$$

adiabaticity parameter

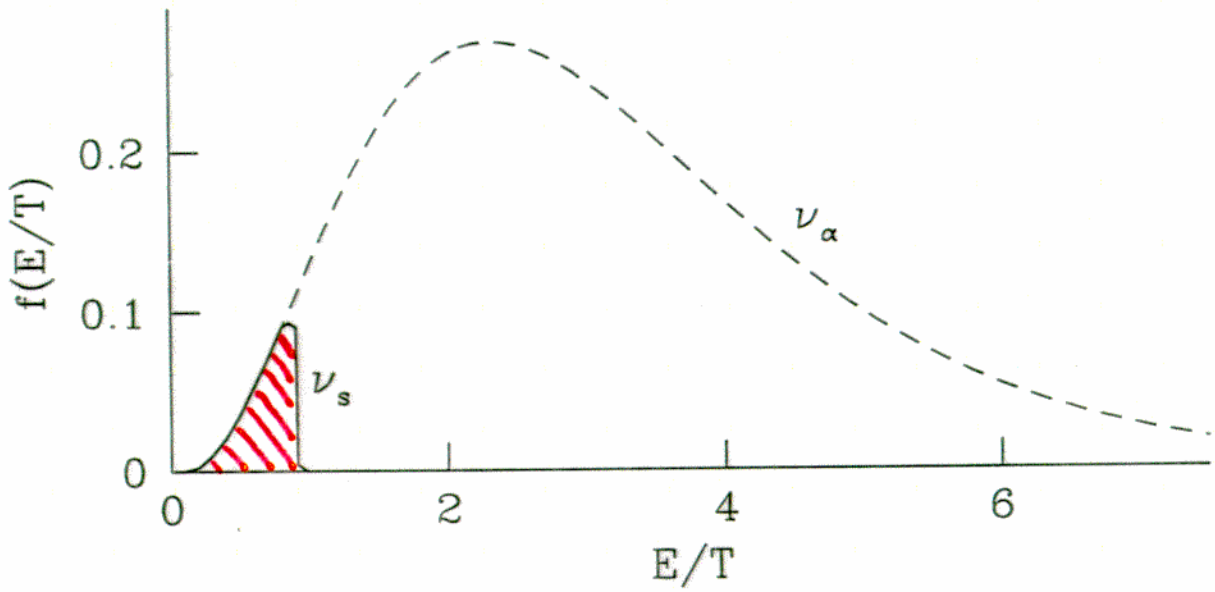
$$\gamma \equiv 2\pi \frac{\delta t}{L_\nu^{osc}} \simeq \left(\frac{m_{\nu_s}}{\text{keV}}\right)^{1/2} \varepsilon^{-1/2} \left(\frac{\mathcal{L}}{10^{-2}}\right)^{3/4} \left[\frac{\sin^2 2\theta}{7\times 10^{-10}}\right]$$

$\gamma \gtrsim 2 \Rightarrow$ complete conversion $\nu_\alpha \to \nu_s$

Shi & Fuller '98

Matter-enhanced processes $\nu_\alpha \rightleftharpoons \nu_s$
Preferentially produce lower energy ν_s

Cool Dark Matter



⇒ ν_s distribution significantly nonthermal

(different from Warm Dark Matter of S. Dodelson & L. Widrow PRL 72, 1 (1994))

⇒ Gives sub-Mpc free streaming scale structure cut-off $\sim 10^{10} M_\odot - 10^5 M_\odot$

⇒ Observable, testable effects at high z ???

Contribution to closure

$$\Omega_{\nu_s} \approx \left(\frac{m_{\nu_s}}{92h^2\,\mathrm{eV}}\right)\left(\frac{2n_{\nu_\alpha}}{n_{\nu_s}}\right)^{-1} \approx \left(\frac{m_{\nu_s}}{350\,\mathrm{eV}}\right)\left(\frac{0.5}{h}\right)^2\left(\frac{L}{0.1}\right)$$

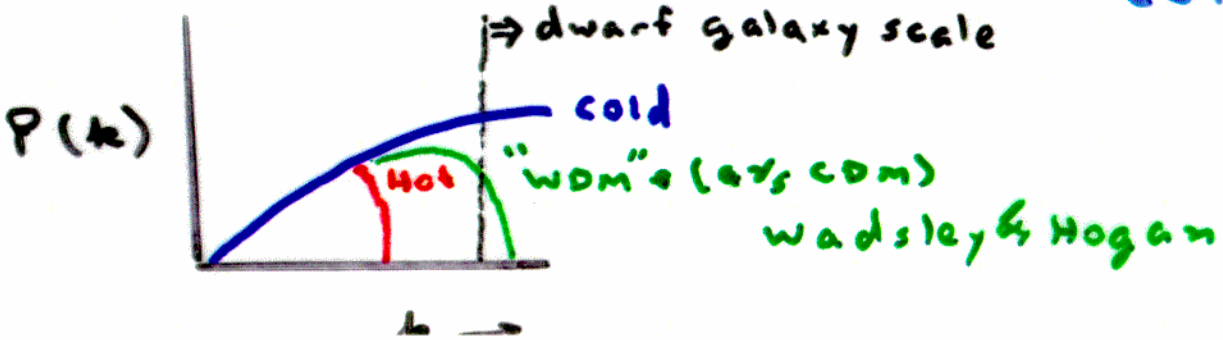
Free streaming length

$$\lambda_{\mathrm{f.s.}} \sim 40\,\mathrm{Mpc}\left(\frac{30\,\mathrm{eV}}{m_{\nu_s}}\right)\left(\frac{E_{\mathrm{c.o.}}/T}{3.15}\right)$$

If we require $\Omega_{\nu_s} = 0.4 \qquad h = 0.5$

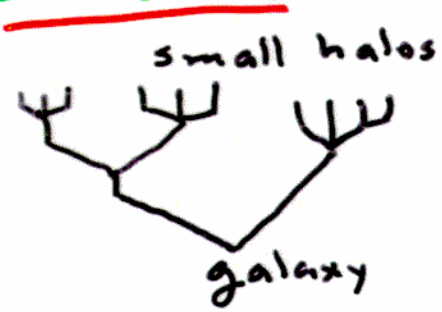
then we can achieve this with the following:

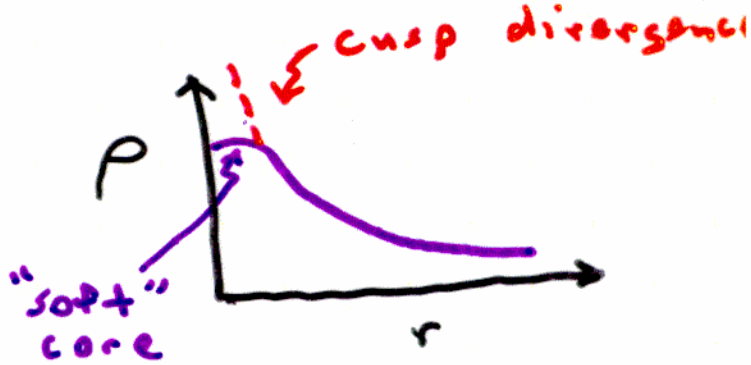
m_{ν_s}	L	$E_{\mathrm{c.o.}}/T$	$\lambda_{\mathrm{f.s.}}$	**Mass Cut-Off**
$200\,\mathrm{eV}$	0.12	~ 2	$\sim 8\,\mathrm{Mpc}$	$\sim 10^{12}\,\mathrm{M}_\odot$ warm
$1\,\mathrm{keV}$	0.023	~ 0.7	$\sim 0.3\,\mathrm{Mpc}$	$\sim 10^{9}\,\mathrm{M}_\odot$ cool
$10\,\mathrm{keV}$	0.002	~ 0.4	$\sim 15\,\mathrm{kpc}$	$\sim 10^{5}\,\mathrm{M}_\odot$ cold



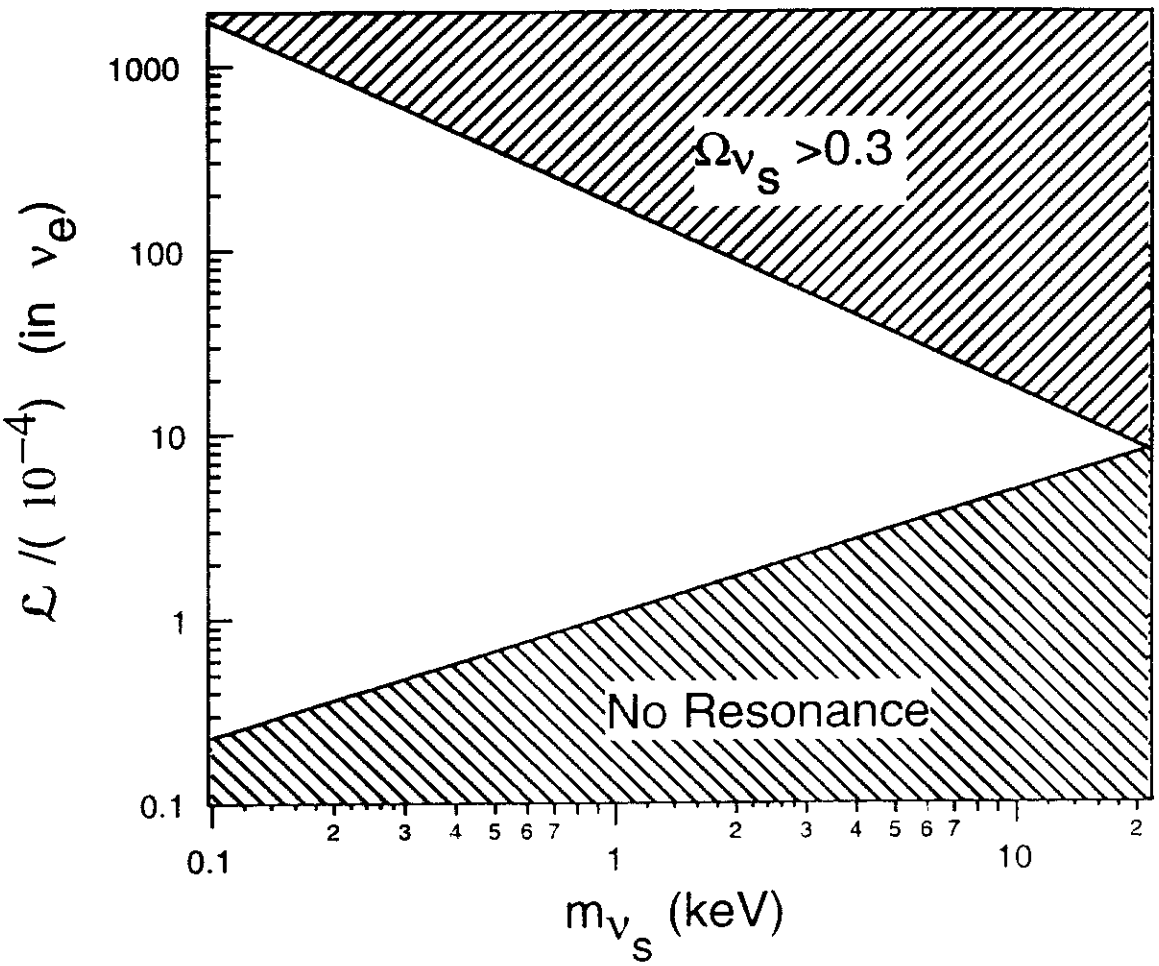
This STUFF* really is CDM on large scales. But the twist is that there will be a cut off at small scales

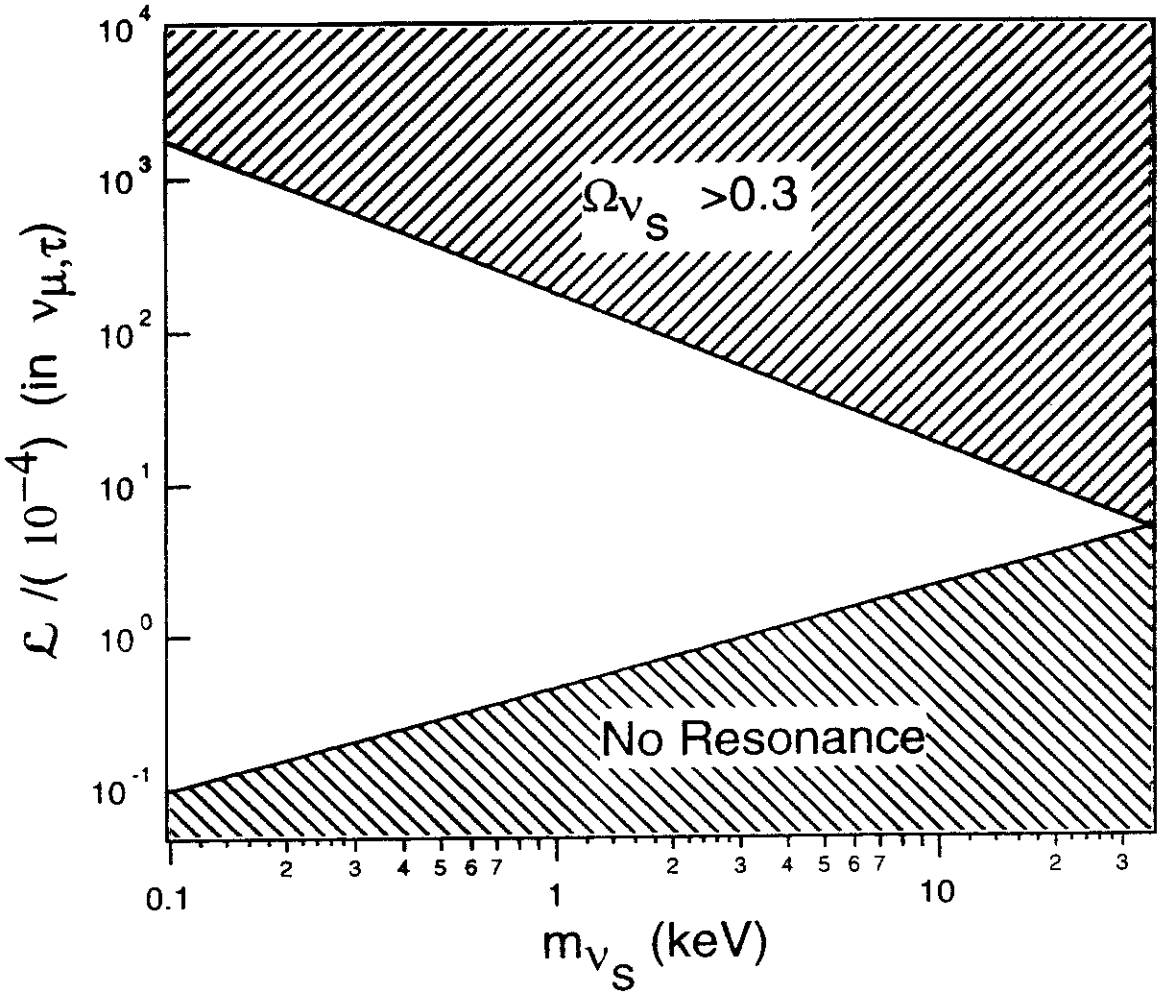
signature: $\lambda_{f.s.}$ small but there!! will give a different galaxy formation history and different sub-galactic structure





* CoolDM ?
MSWDM ?
Cut Off Sterile CDM ?
Utter Nonsense CDM ?


1000
100
10
1
0.1
$\mathcal{L}$ /(10^{-4}) (in ν_e)
$\Omega\nu_s$ >0.3
No Resonance
0.1
2
3
4
5
6
7
1
2
3
4
5
6
7
10
2
m_{ν_s} (keV)


10^4
10^3
10^2
10^1
10^0
10^{-1}
$\mathcal{L}$ /(10^{-4}) (in $\nu_{\mu,\tau}$)
Ω_{ν_s} >0.3
No Resonance
0.1
2 3 4 5 6 7
1
2 3 4 5 6 7
10
2 3
m_{ν_s} (keV)

need

1. sterile Neutrinos
2. lepton number

1. suggested by experimental neutrino data (also r-process?)
   &
   string models
   (extra dimensions, bulk fermions)

2. all we know for sure is BBN limit
   (e.g. Kang & Steigman, N.P. B372, 494, 1993)

$$-4.1\times10^{-2} \lesssim L_{\nu_e} \lesssim 0.75$$

$$-4.7 \lesssim L_{\nu_{\mu,\tau}} \lesssim 4.7$$