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CP Violation in the  
Heaviest Leptons, Quarks, and Bosons

Keith Riles  
University of Michigan

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# Two Caveats

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1. CP Violation will be addressed in more general framework of anomalous gauge Boson and Fermion couplings
2. Lectures will not form comprehensive review:
  - Subject too vast for detailed coverage
  - Instead, will focus pedagogically on particular approaches & signatures
  - Will touch on other topics you can pursue on your own

# Outline (1)

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## Gauge Boson Couplings

- Standard Model
  - Lagrangian terms
    - $\implies$  Expected triple- and quadruple-Boson couplings
- Beyond the Standard Model
  - More general  $WW\gamma$  and  $WWZ$  Lagrangian terms
    - $\implies$  Many possible couplings, including  $\mathcal{CP}$
  - Possible treatments of anomalous couplings
    - \* Effective Lagrangian with light Higgs (linear)
    - \* Chiral Lagrangian with strong coupling (non-linear)
- Experimental signatures for  $WW\gamma$ ,  $WWZ$  couplings
  - Low-energy experiments
  - $e^+e^-$  colliders
  - Hadron colliders
- Other gauge Boson couplings ( $\mathcal{CP}$ )
  - Sampling of couplings & signatures

# Outline (2)

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## Top Quark Couplings

- Standard Model
- Possible sources of non-SM couplings
- Experimental signatures

## Tau Lepton Couplings

- Standard Model
- Possible sources of non-SM couplings
- Experimental signatures

# Standard Model

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S.M. Electroweak Lagrangian – Bosonic interactions:  
 (following notation/convention of Renton text)

Unbroken  $SU(2)_L \times U(1)_Y$ :

$$\begin{aligned} \mathcal{L}_{\text{Bosons}} = & -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu(NA)} \cdot \vec{W}_{(NA)}^{\mu\nu} \\ & + [D_\mu \phi]^\dagger [D^\mu \phi] - \mu^2 \phi^\dagger \phi - \lambda [\phi^\dagger \phi]^2 \end{aligned}$$

where

$$\begin{aligned} D^\mu & \equiv \partial^\mu + i\frac{g}{2} \vec{\tau} \cdot \vec{W}^\mu + i\frac{g'}{2} y B^\mu \\ B^{\mu\nu} & \equiv \partial^\mu B^\nu - \partial^\nu B^\mu \\ \vec{W}_{(NA)}^{\mu\nu} & \equiv \partial^\mu \vec{W}_{(NA)}^\nu - \partial^\nu \vec{W}_{(NA)}^\mu - g \vec{W}_{(NA)}^\mu \times \vec{W}_{(NA)}^\nu \end{aligned}$$

where

$$\begin{aligned} \phi & \equiv \text{Complex scalar doublet} \\ \vec{W} & \equiv \text{Unbroken } SU(2)_L \text{ field} \\ B & \equiv \text{Unbroken } U(1)_Y \text{ field} \end{aligned}$$

Physical fields:

$$\begin{aligned} A^\mu & = c_W B^\mu + s_W W_3^\mu && \text{(Electromagnetic)} \\ Z^\mu & = -s_W B^\mu + c_W W_3^\mu && \text{(Weak neutral)} \\ W^{\pm\mu} & = \frac{1}{\sqrt{2}} (W_1^\mu \pm i W_2^\mu) && \text{(Weak charged)} \end{aligned}$$

where

$$s_W \equiv \sin \theta_W \quad c_W \equiv \cos \theta_W$$

# Standard Model

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Consider Gauge Boson **self interactions** (ignore Higgs terms):

- Invert relations to obtain  $B^\mu$ ,  $W_1^\mu$ ,  $W_2^\mu$  and  $W_3^\mu$  in terms of physical fields  $A^\mu$ ,  $Z^\mu$  and  $W^{\pm\mu}$
- Substitute into first two terms of unbroken Lagrangian

$$\mathcal{L}_{\text{Gauge}} = \mathcal{L}_{\text{Abelian}} + \mathcal{L}_{\text{TGC}} + \mathcal{L}_{\text{QGC}}$$

where

$$\begin{aligned} \mathcal{L}_{\text{Abelian}} &= -\frac{1}{4} {}^{(\gamma)}F_{\mu\nu} {}^{(\gamma)}F^{\mu\nu} - \frac{1}{4} {}^{(z)}F_{\mu\nu} {}^{(z)}F^{\mu\nu} - \frac{1}{2} {}^{(w)}F_{\mu\nu}^\dagger {}^{(w)}F^{\mu\nu} \\ &\implies \text{Abelian kinetic energy} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{TGC}} &= i g (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) W^{-\nu} (c_W Z^\mu + s_W A^\mu) \\ &\quad + i g (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) W^{+\mu} (c_W Z^\nu + s_W A^\nu) \\ &\quad + i g (W^{-\mu} W^{+\nu} - W^{+\mu} W^{-\nu}) \partial_\mu (c_W Z_\nu + s_W A_\nu) \\ &\implies \text{Non-Abelian Triple-Gauge-Couplings:} \\ &\quad \quad \quad \text{WW}\gamma, \text{WW}Z \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{QGC}} &= -g^2 W_\mu^+ W^{-\mu} (c_W Z_\nu + s_W A_\nu) (c_W Z^\nu + s_W A^\nu) \\ &\quad + g^2 W^{+\nu} W^{-\mu} (c_W Z_\mu + s_W A_\mu) (c_W Z_\nu + s_W A_\nu) \\ &\quad + \frac{1}{2} g^2 W_\nu^- W_\mu^+ (W^{-\nu} W^{+\mu} - W^{-\mu} W^{+\nu}) \\ &\implies \text{Non-Abelian Quadruple-Gauge-Couplings:} \\ &\quad \quad \quad \text{WW}\gamma\gamma, \text{WW}ZZ, \text{WW}\gamma Z, \text{WWWW} \end{aligned}$$

where

$${}^{(\gamma)}F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu, \quad {}^{(z)}F^{\mu\nu} \equiv \partial^\mu Z^\nu - \partial^\nu Z^\mu, \quad {}^{(w)}F^{\mu\nu} \equiv \partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}$$

# Standard Model

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Focus on Triple Gauge Couplings (TGC):

$$WW\gamma, WWZ$$

Rewrite  $\mathcal{L}_{\text{TGC}}$ :

$$\mathcal{L}_{\text{TGC}} = \sum_{V=\gamma,Z} i g_{WWV} [ {}^{(w)}F_{\mu\nu}^\dagger W^{-\mu} V^\nu - W_\mu^{-\dagger} V_\nu {}^{(w)}F^{\mu\nu} + W_\mu^{-\dagger} W_\nu^- {}^{(v)}F^{\mu\nu} ]$$

where

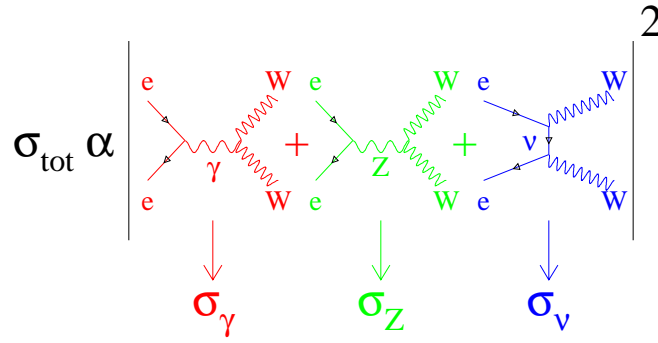
$$\begin{aligned} g_{WW\gamma} &= -g \sin \theta_W &= -e \\ g_{WWZ} &= -g \cos \theta_W &= -e \cot \theta_W \end{aligned}$$

Remarks:

- Gauge invariance explicitly retained
- TGC (and QGC) strengths predicted by Standard Model from measured  $e$  and  $\sin^2 \theta_W$
- All S.M. terms contain one derivative (momentum)

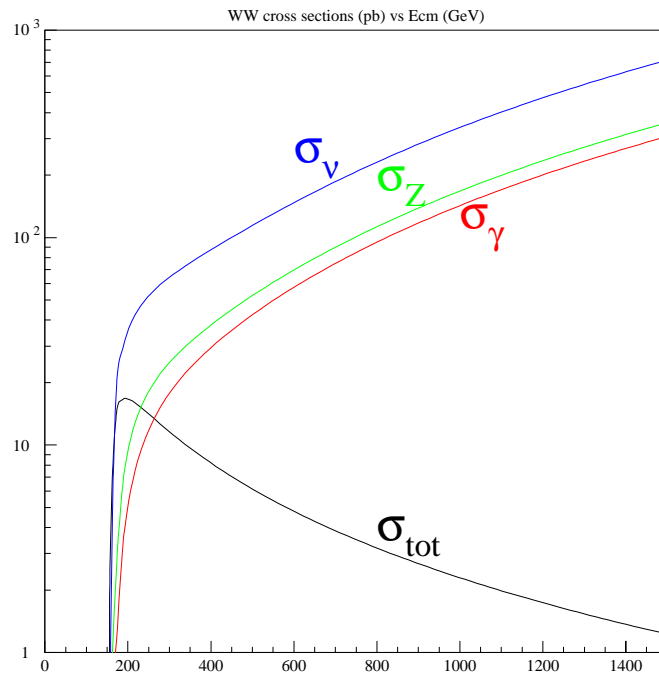
# Application: $e^+e^- \rightarrow W^+W^-$

Three diagrams contribute to  $e^+e^- \rightarrow W^+W^-$ :



Each **diverges** with increasing  $\sqrt{s}$

But the sum is finite (in Standard Model):



$\Rightarrow \sigma_{\text{tot}}$  sensitive to tiny deviations in  $WWV$  couplings



# Beyond the Standard Model

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Generic Lagrangian form for triple Boson vertex

(Hagiwara, Peccei, Zeppenfeld, Hikasa, NPB 282: 253 (1987))

$$\begin{aligned}
 \mathcal{L}_{WWV}/g_{WWW} &= i g_1^V \left( {}^{(w)}F_{\mu\nu}^\dagger W^{-\mu} V^\nu - W_\mu^{-\dagger} V_\nu {}^{(w)}F^{\mu\nu} \right) \\
 &+ i \kappa_V W_\mu^{-\dagger} W_\nu^- {}^{(v)}F^{\mu\nu} \\
 &+ \frac{i \lambda_V}{M_W^2} {}^{(w)}F_{\lambda\mu}^\dagger W_\nu^- {}^{(v)}F^{\nu\lambda} \\
 &- g_4^V W_\mu^{-\dagger} W_\nu^- (\partial^\mu V^\nu + \partial^\nu V^\mu) \\
 &+ g_5^V \epsilon^{\mu\nu\rho\sigma} (W_\mu^{-\dagger} \overleftrightarrow{\partial}_\rho W_\nu^-) V_\sigma \\
 &+ \tilde{\kappa}_V W_\mu^{-\dagger} W_\nu^- {}^{(v)}\tilde{F}^{\mu\nu} \\
 &+ \frac{i \tilde{\lambda}_V}{M_W^2} {}^{(w)}F_{\lambda\mu}^\dagger W_\nu^- {}^{(v)}\tilde{F}^{\nu\lambda} \tilde{V}^{\nu\lambda}
 \end{aligned}$$

where

$$\begin{aligned}
 (A \overleftrightarrow{\partial}_\mu B) &\equiv A(\partial_\mu B) - (\partial_\mu A)B \\
 {}^{(v)}\tilde{F}_{\mu\nu} &\equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} {}^{(v)}F^{\rho\sigma}
 \end{aligned}$$

# Beyond the Standard Model

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Seven  $\times$  two independent couplings

with

$g_4^V, \tilde{\kappa}_V, \tilde{\lambda}_V$  terms CP-violating

$g_5^V$  term C, P violating

(these terms often ignored in studies)

Standard Model:

$$g_1^V = \kappa_V = 1 \quad \lambda_V = g_4^V = g_5^V = \tilde{\kappa}_V = \tilde{\lambda}_V = 0$$

Higher order operators with additional derivatives equivalent to momentum-dependent couplings:

$$\kappa_V = \kappa_V(q^2/\Lambda^2) \quad \text{Form factor couplings}$$

$\Rightarrow$  Important at hadron colliders

# Beyond the Standard Model

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In more familiar terminology ...

W magnetic dipole moment:

$$\mu_W \equiv \frac{e}{2 M_W} (1 + \kappa_\gamma + \lambda_\gamma)$$

W electric quadrupole moment:

$$Q_W \equiv \frac{e}{M_W^2} (\kappa_\gamma \leftrightarrow \lambda_\gamma)$$

W electric dipole moment:

$$d_W \equiv \frac{e}{2 M_W} (\tilde{\kappa}_\gamma + \tilde{\lambda}_\gamma)$$

W magnetic quadrupole moment:

$$\tilde{Q}_W \equiv \frac{e}{M_W^2} (\tilde{\kappa}_\gamma \leftrightarrow \tilde{\lambda}_\gamma)$$

Deviations from SM:

$$\Delta g_1^Z \equiv g_1^Z \leftrightarrow 1 \qquad \Delta \kappa_V \equiv \kappa_V \leftrightarrow 1$$

$$g_1^\gamma(q^2 \rightarrow 0) \equiv 1 \quad - \quad \text{W electric charge}$$

# Beyond the Standard Model

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Many possibilities!

But what is reasonable?

Two common model types:

- Effective Lagrangian with light Higgs (linear model)
- Chiral Lagrangian with strong coupling (non-linear model)

Model parameters can be mapped to generic set:

$$\Delta\kappa_\gamma, \lambda_\gamma, \text{ etc.}$$

# Beyond the Standard Model

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Effective Lagrangian:

$$L_{eff} = L_{SM} + L_{NR}$$

(NR  $\equiv$  Non-Renormalizable in finite order)

where (Einhorn / Wudka notation)

$$L_{NR} \equiv \frac{1}{\Lambda} \sum_i \alpha_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2} \sum_i \alpha_i^{(6)} O_i^{(6)} + \dots$$

and

$$\begin{aligned} O_i^{(N)} &\equiv \text{Local operator of dimension } N \\ \Lambda &\equiv \text{Scale of new physics} \end{aligned}$$

$O_i^{(5)}$  not physical

$\implies$  Dimension 6 operators next in line

$\implies$  Keep terms to this order ( $\Lambda$  large)

# Beyond the Standard Model

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## Example

(Hagiwara, Ishihara, Szalapski, Zeppenfeld)

Use only known light fields (gauge Bosons plus Higgs) and covariant derivatives:

(only C, P conserving operators listed)

$$\begin{aligned} L_{NR} &= \sum_{i=1}^7 \frac{f_i}{\Lambda^2} O_i = \frac{1}{\Lambda^2} \times \\ & ( f_{\Phi,1} (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi) \\ & + f_{BW} \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi \\ & + f_{DW} \text{Tr}([D_\mu, \hat{W}_{\nu\rho}][D^\mu, \hat{W}^{\nu\rho}]) \\ & \Leftrightarrow f_{DB} \frac{g'^2}{2} (\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho}) \\ & + f_B (D_\mu \Phi)^\dagger \hat{B}_{\mu\nu} (D_\nu \Phi) \\ & + f_W (D_\mu \Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu \Phi) \\ & + f_{WWW} \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\nu\rho} \hat{W}_\rho^\mu] \end{aligned}$$

where

$$[D_\mu, D_\nu] = \hat{B}_{\mu\nu} + \hat{W}_{\mu\nu} \equiv i \frac{g'}{2} B_{\mu\nu} + i g \frac{\sigma^a}{2} W_{\mu\nu}^a$$

# Beyond the Standard Model

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First four operators ( $O_{\Phi,1}$ ,  $O_{BW}$ ,  $O_{DW}$ ,  $O_{DB}$ ) affect 2-point boson functions at tree level

⇒ Severely constrained by LEP and other data

(*c.f.* S, T, U parameters of Peskin / Takeuchi)

Remaining operators ( $O_B$ ,  $O_W$ ,  $O_{WWW}$ ) contribute to anomalous triple boson couplings

“Relaxed” HISZ Scenario:

$$\begin{aligned}\Delta\kappa_\gamma &= (f_B + f_W) \frac{M_W^2}{2\Lambda^2} \\ \Delta\kappa_Z &= (f_W \Leftrightarrow s_W^2(f_B + f_W)) \frac{M_Z^2}{2\Lambda^2} \\ \Delta g_1^Z &= f_W \frac{M_Z^2}{2\Lambda^2} = \Delta\kappa_Z + \frac{s_W^2}{c_W^2} \Delta\kappa_\gamma \\ \lambda_\gamma &= f_{WWW} \frac{3M_W^2 g^2}{2\Lambda^2} = \lambda_Z\end{aligned}$$

“Full” H.I.S.Z. Scenario adds the constraint:

$$f_B = f_W$$

# Beyond the Standard Model

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## Remarks on Full HISZ scenario:

- Only two free parameters:

$$\kappa_\gamma, \quad \lambda_\gamma$$

- Anomalous couplings are of order  $f_i \frac{M_W^2}{\Lambda^2}$

⇒ If  $f_i \approx O(1)$ , then

$$\begin{aligned} \Delta\kappa_V &\approx O\left(\frac{M_W^2}{\Lambda^2}\right) \\ &\approx 10^{-1} \quad \text{for } \Lambda \approx 250 \text{ GeV} \\ &\approx 10^{-2} \quad \text{for } \Lambda \approx 1 \text{ TeV} \end{aligned}$$

- It gets worse...

No renormalizable underlying theory for this effective Lagrangian can generate non-vanishing  $O_W$ ,  $O_B$ ,  $O_{WWW}$  at tree level (Artz / Einhorn / Wudka)

⇒ Loop diagrams needed

⇒ Further large suppression [ $O(\frac{1}{16\pi^2})$ ]



# Beyond the Standard Model

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Alternative model to go beyond Standard Model:

Chiral Lagrangian with strong coupling

What exactly is strong coupling?

Useful to consider earlier  $e^+e^- \rightarrow W^+W^-$  example:

- I claimed that  $\sigma_{\text{tot}}$  is well behaved at large  $s$  because diagrams cancel
- But that was a lie...

# Beyond the Standard Model

Previous calculation neglected electron mass ( $m_e$ )!

Residual axial vector term gives:

$$\left| \begin{array}{c} e \\ \gamma \\ e \\ + \\ W \\ e \\ + \\ Z \\ e \\ + \\ \nu \\ W \\ e \\ W \\ e \end{array} \right|^2 \propto m_e^2 s$$

⇒ Divergence at very high S

⇒ Not a practical problem in our lifetimes

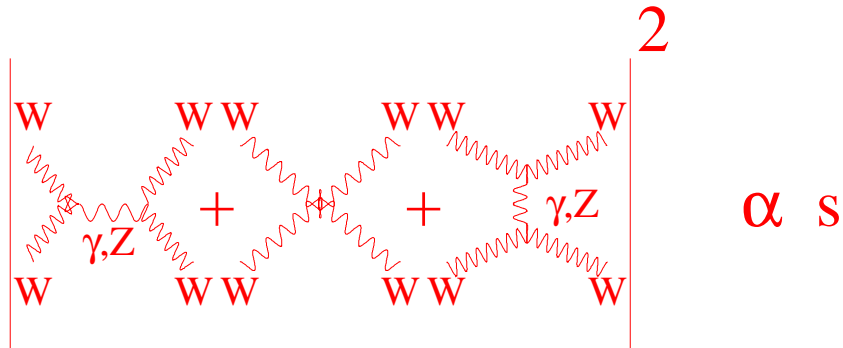
⇒ But suggests eventual need for Higgs cancellation:

$$\left| \begin{array}{c} e \\ H \\ e \\ W \\ W \end{array} \right|^2 \propto m_e^2 s$$

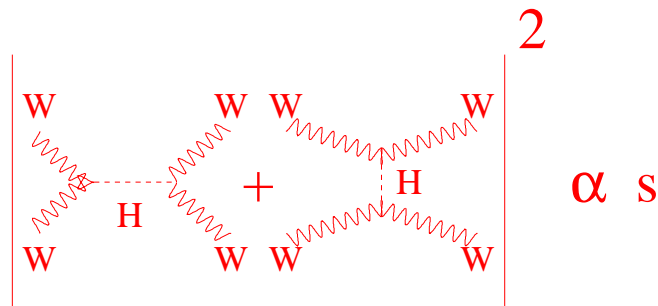
# Beyond the Standard Model

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More pressing divergence:



Again, need something Higgs-like for cancellation:



This works in perturbation theory if

$$m_H < 800 \text{ GeV}$$

# Beyond the Standard Model

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What if  $m_H > 1 \text{ TeV}$ ?

Or if no fundamental Higgs exists?

Use Equivalence Theorem to relate  $W_L W_L$  scattering to Goldstone Boson scattering:

$$W_L W_L \text{ scattering} \quad \Leftrightarrow \quad \phi\phi \text{ scattering}$$

One can go further...

Exploit similarity between Goldstone Boson scattering and low-energy pion scattering:

$$W_L W_L \text{ scattering} \quad \Leftrightarrow \quad \pi\pi \text{ scattering}$$

$$v \text{ (250 GeV)} \quad \Leftrightarrow \quad f_\pi \text{ (90 MeV)}$$

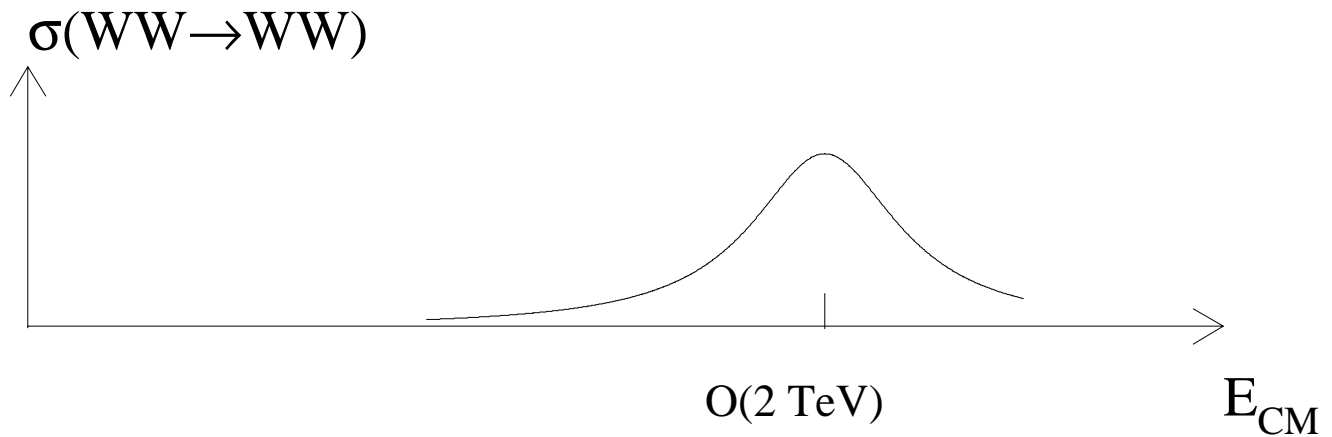
$\Rightarrow$  Just scale everything up by

$$\frac{250}{0.09} \quad \approx \quad 2800 !$$

# Beyond the Standard Model

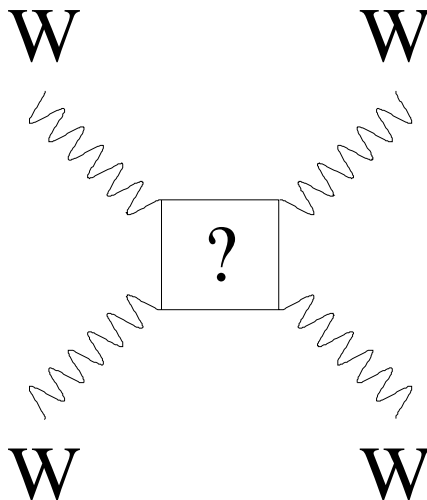
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Easy to imagine a “ $\rho$ ” resonance:



This is not guaranteed

But a resonance would probably indicate higher-mass states occurring in loops:



# Beyond the Standard Model

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TECHNICOLOR

(QCD all over again...)

Take the  $\pi\pi$  analogy to logical extreme:

Longitudinal boson = techni-fermion condensate

$W_L \Leftrightarrow \text{“}\pi\text{”} \Leftrightarrow \text{“}q\bar{q}\text{” (techni-pion)}$

$W_L W_L \text{ resonance} \Leftrightarrow \text{“}(q\bar{q})_V\text{” (techni-rho)}$

In general, technicolor models have many difficulties  
(theoretical and experimental)

But variants still cling to life

# Beyond the Standard Model

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## Chiral Lagrangian Approach

(only C, P conserving terms shown here)

- No Standard Model Higgs

But retain would-be-Goldstone-Boson fields  $w_i$

- Define non-linear  $2 \times 2$  matrix:

$$\Sigma \equiv e^{i\vec{w}\cdot\vec{\sigma}/v}$$

with covariant derivative:

$$D_\mu \Sigma = \partial_\mu \Sigma + \frac{i}{2} g W_\mu^a \sigma^a \Sigma - \frac{i}{2} g' B_\mu \Sigma \sigma_3$$

- Construct effective Lagrangian from the fields and covariant derivatives

Dimension 6 terms giving anomalous  $WWV$  couplings:

$$\begin{aligned} & -ig \frac{v^2}{\Lambda^2} L_{9L} \text{Tr}[W^{\mu\nu} D_\mu \Sigma D_\nu \Sigma^\dagger] \\ & -ig' \frac{v^2}{\Lambda^2} L_{9R} \text{Tr}[B^{\mu\nu} D_\mu \Sigma^\dagger D_\nu \Sigma] \end{aligned}$$

# Beyond the Standard Model

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Coupling parameters  $L_{9L}, L_{9R}$  can be mapped onto generic set:

$$\Delta g_1^Z = \frac{e^2}{2 c_W^2 s_W^2} \frac{v^2}{\Lambda^2} L_{9L}$$

$$\Delta \kappa_\gamma = \frac{e^2}{2 s_W^2} \frac{v^2}{\Lambda^2} (L_{9L} + L_{9R})$$

$$\Delta \kappa_Z = \frac{e^2}{2 c_W^2 s_W^2} \frac{v^2}{\Lambda^2} (L_{9L} c_W^2 + L_{9R} s_W^2)$$

No  $\lambda_V$  terms in dimension 6



# Beyond the Standard Model

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## Parametrizing a $W_L W_L$ Vector Resonance

Alternative parametrization of techni-rho resonance via complex form factor (Peskin)

Multiply Standard Model  $e^+e^- \rightarrow W^+W^-$  amplitude by

$$F_T = \exp\left[\frac{1}{\pi} \int_0^\infty ds' \delta(s', M_{\rho, \rho}) \left\{ \frac{1}{s' \leftrightarrow s \leftrightarrow i\epsilon} \leftrightarrow \frac{1}{s'} \right\}\right]$$

where

$$\delta(s) = \frac{1}{96 \pi v^2} \frac{s}{s} + \frac{3 \pi}{8} \left[ \tanh\left(\frac{s \leftrightarrow M_{\rho, \rho}^2}{M_{\rho, \rho}}\right) + 1 \right]$$

and

$$M_{\rho, \rho} = \text{techni-rho mass, width}$$

[Note:

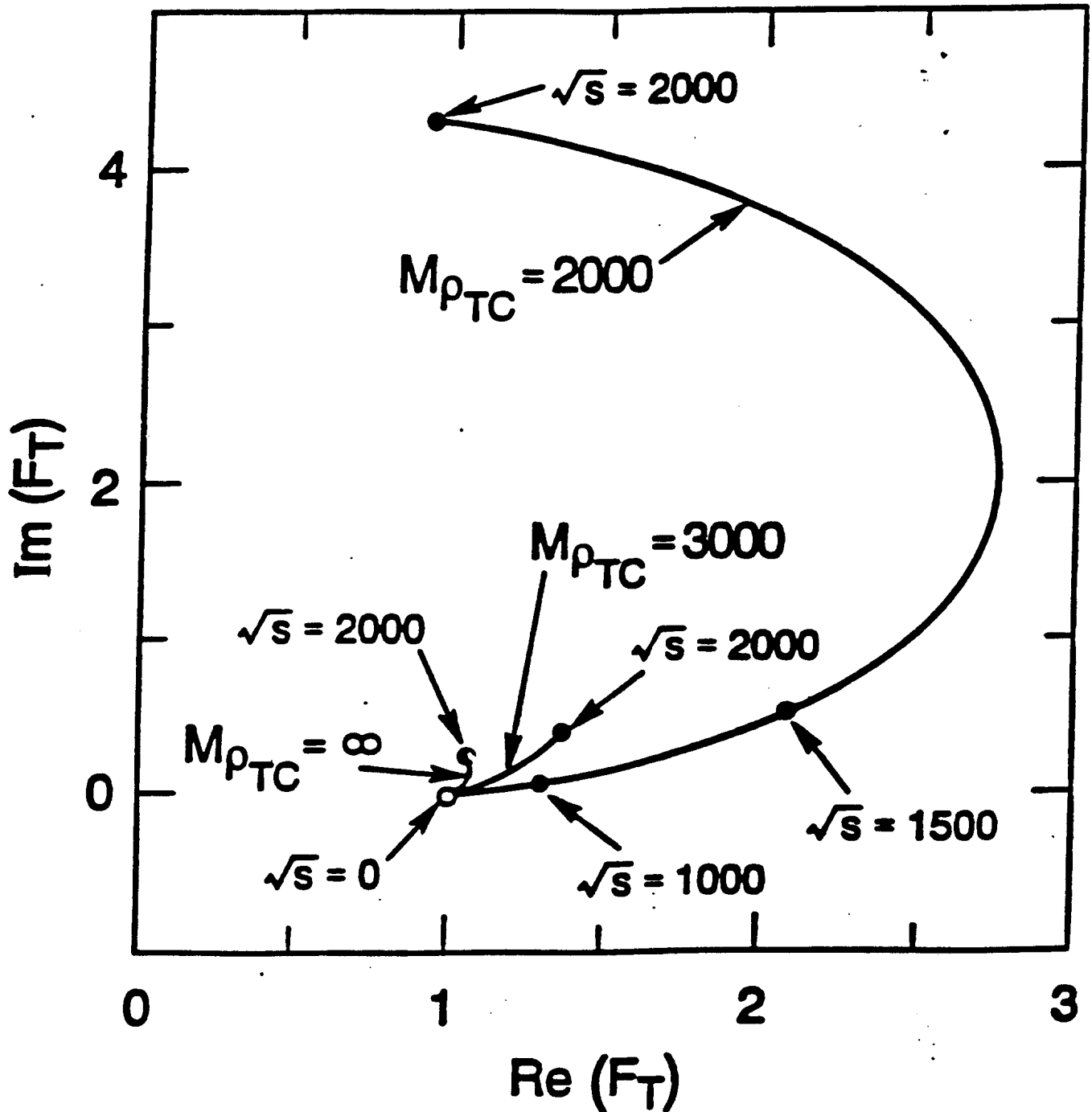
$$\text{As } M_{\rho} \rightarrow \infty, \quad \delta(s) \rightarrow \frac{1}{96 \pi v^2} \frac{s}{s} \quad (\text{L.E.T.})]$$

Extract  $\text{Re}(F_T)$ ,  $\text{Im}(F_T)$  from data

$\Rightarrow$  Limits / evidence for techni-rho

# Beyond the Standard Model

What do we expect for  $F_T$  in this model?



# Experimental Signatures

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## Low-Energy Experimentss

(Indirect – based on loop corrections)

CP-Conserving Couplings:

- Limits from  $b \rightarrow s\gamma \Rightarrow O(1)$
- Limits from  $(g \Leftrightarrow 2)_\mu \Rightarrow O(10^{-1})-O(1)$   
(will improve soon to  $O(10^{-2})-O(10^{-1})$ )
- Limits from “oblique” corrections to  $W/\gamma/Z$  propagators  $\Rightarrow O(10^{-2})$

CP-Violating Couplings:

- Limits from neutron EDM  $\Rightarrow O(10^{-4})$

Remarks:

- Indirect constraints from loop processes can be
  - Powerful within a given model
  - Persuasive under “naturalness”
  - Irrelevant in general case

$\implies$  Above limits are model dependent

# Experimental Signatures

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Return to  $e^+e^- \rightarrow W^+W^-$

- Total cross section sensitive to anomalous couplings
- Additional information in angular distributions of four-Fermion final state
- Seven possible  $W^-W^+$  helicity states  $(\lambda_-, \lambda_+)$  for  $s$  channel production via  $\gamma, Z$ :

$(+, 0)$   $(+, +)$   $(0, +)$   $(0, 0)$   $(0, \Leftrightarrow)$   $(\Leftrightarrow, 0)$   $(\Leftrightarrow, \Leftrightarrow)$

- Note: five states involve longitudinal  $W$ 's
- Two additional helicity combinations allowed by  $t$ -channel  $\nu_e$  exchange diagram:

$(+, \Leftrightarrow)$   $(\Leftrightarrow, +)$

(forbidden by  $\vec{J}$  conservation in  $s$  channel)

# Experimental Signatures

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- Total of nine helicity combinations ( $3 \times 3$ ) in matrix element amplitude
- But we don't detect  $W$ 's; we detect Fermion daughters  
 $\implies$  Interference possible  
 $\implies 9 \times 9 = 81$  components in production tensor:

$$d\sigma \propto P_{\lambda'_- \lambda'_+}^{\lambda_- \lambda_+} D_{\lambda'_-}^{\lambda_-} D_{\lambda'_+}^{\lambda_+}$$

where

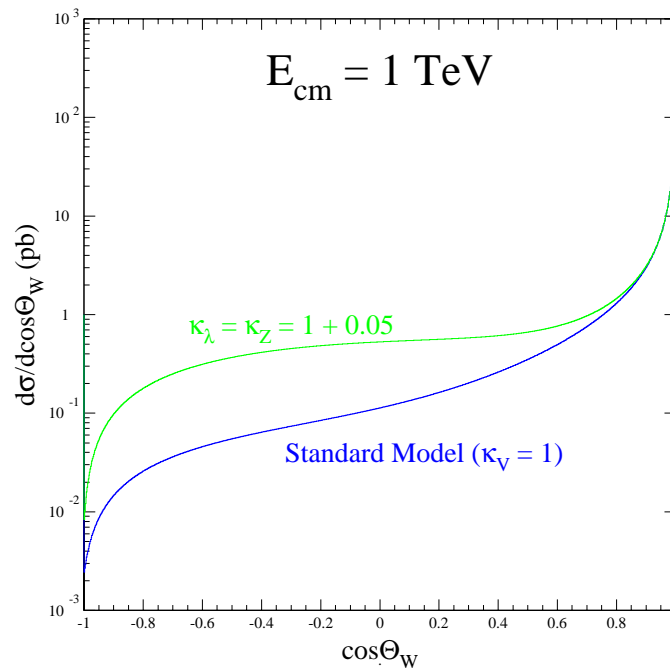
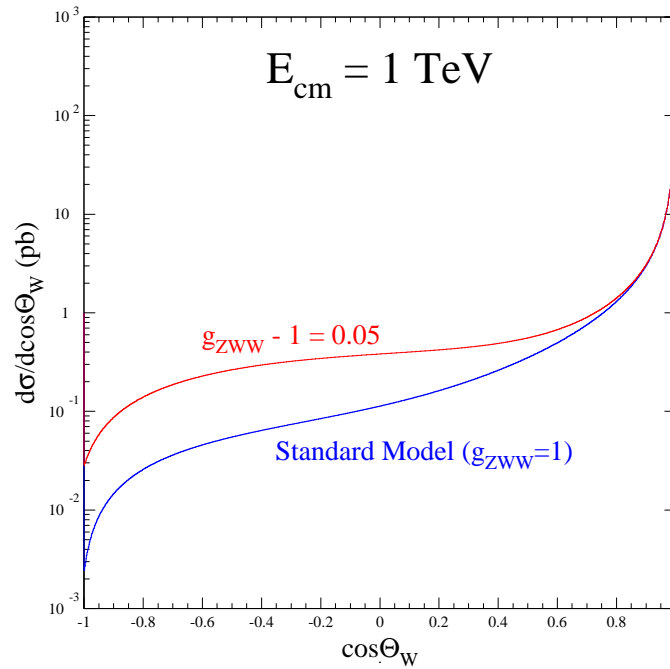
$$\begin{aligned} P_{\lambda'_- \lambda'_+}^{\lambda_- \lambda_+} &= \text{Production tensor} \\ D_{\lambda'_-}^{\lambda_-} &= \text{Decay tensor} \end{aligned}$$

(see Hagiwara *et al.* for explicit expressions)

- In principle, can measure all components from 81 different angular distributions defined by projection operators
- In practice, one chooses smaller set of parameters and angular distributions

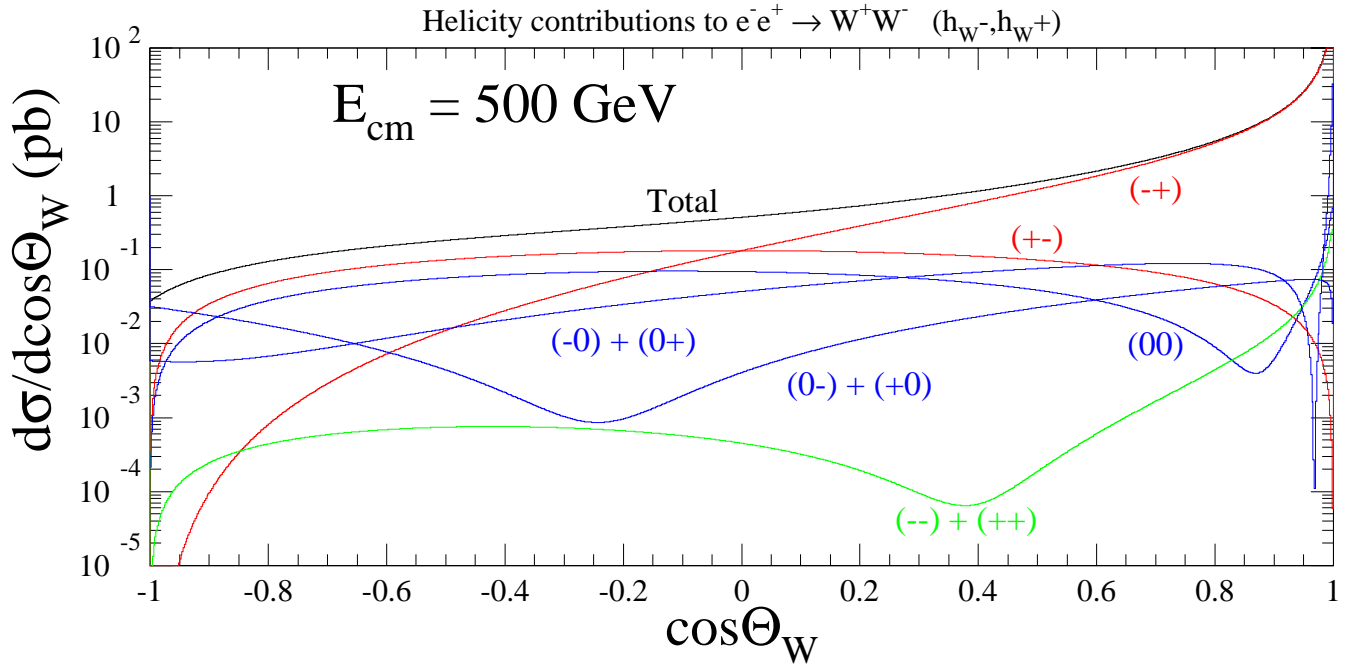
# Experimental Signatures

Example: Polar production angle of  $W^-$ :

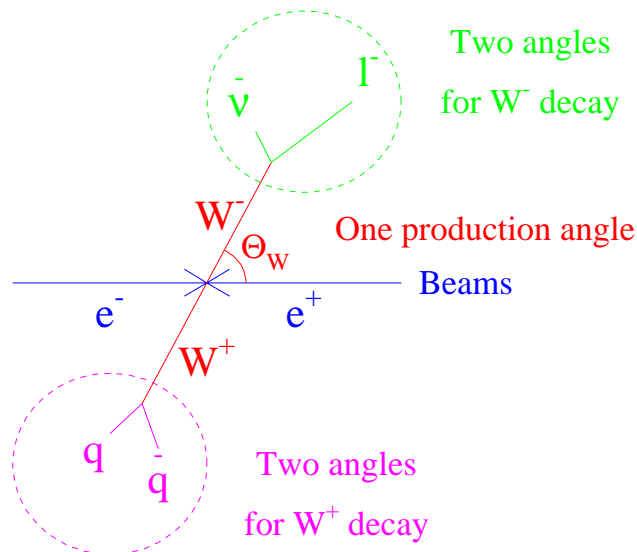


# Experimental Signatures

Single  $d\sigma/d\cos\Theta_W$  distribution hides complex structure:



Use **five** production / decay angles to extract helicity amplitudes:



# Experimental Signatures

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Which  $WW$  decay channel is most useful?

- $WW \rightarrow q_1 \bar{q}_2 q_3 \bar{q}_4$  (4-jet final state)
  - Branching ratio product  $\approx (\frac{2}{3})^2$  (SM = 46%)
  - 3-fold ambiguity in jet assignment
  - 2-fold ambiguity in  $\theta_W$
  - 2-fold ambiguity in  $\theta_1, \phi_1$
  - 2-fold ambiguity in  $\theta_2, \phi_2$   
(jet charge tagging helps resolve angle ambiguities)
  - 6-C kinematic fit improves experimental resolution  
(E,  $\vec{p}$  conservation + two  $M_W$  constraints)
- $WW \rightarrow q_1 \bar{q}_2 \ell \bar{\nu}$  (2-jet + lepton +  $\cancel{E}$  final state)
  - Branching ratio product  $\approx 2 \times (\frac{2}{3}) \times (\frac{1}{3})$  (SM = 44%)
  - 2-fold ambiguity in  $\theta_1, \phi_1$
  - 3-C (2-C) kinematic fit improves resolution for  $e/\mu$  ( $\tau$ )
- $WW \rightarrow \ell \bar{\nu} \bar{\ell} \nu$  (2-lepton +  $\cancel{E}$  final state)
  - Branching ratio product  $\approx (\frac{2}{3})^2$  (SM = 10%)
  - $WW$  reconstruction possible only for  $\ell = e, \mu$
  - 0-C “fit” leaves 2-fold global angular ambiguity,  
no improvement in experimental resolution



# Experimental Signatures

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Examples of analysis methods for extracting couplings or testing Standard Model

- Fits to 1-D angular distributions ( $\Theta_W, \theta_1, \phi_1, \theta_2, \phi_2$ )
- Fits to 2-D (and higher) angular distributions (statistics limited – requires great care)
- Optimal observables:

$$d\sigma(\Omega, \vec{\alpha}) = s^{(0)}(\Omega) + \sum_i \alpha_i \cdot s_i^{(1)}(\Omega) + \sum_{i,j} \alpha_i \alpha_j \cdot s_{ij}^{(2)}(\Omega)$$

where  $\vec{\alpha} \equiv$  set of coupling parameters and  $s^{(0)}$ ,  $s^{(1)}$  and  $s^{(2)}$  are known functions.

All available information contained in observables:

$$o_i^{(1)}(\Omega) = s_i^{(1)}(\Omega)/s^{(0)}(\Omega) \quad o_{ij}^{(2)}(\Omega) = s_{ij}^{(2)}(\Omega)/s^{(0)}(\Omega)$$

Can look at distributions or moments of observables

# Experimental Signatures

---

- Decay spin density matrix: Reduce 81-component  $WW$  production tensor to 9-component  $W^- \rightarrow \ell^- \bar{\nu}$   
Single-W spin density matrix:

$$\rho_{\lambda\lambda'}(\Omega) \equiv \frac{\Sigma_{\lambda_+, \lambda'_+} P_{\lambda'_- \lambda'_+}^{\lambda_- \lambda_+} D_{\lambda'_-}^{\lambda_-} D_{\lambda'_+}^{\lambda_+}}{\Sigma_{\lambda_-, \lambda_+, \lambda'_- \lambda'_+} P_{\lambda'_- \lambda'_+}^{\lambda_- \lambda_+} D_{\lambda'_-}^{\lambda_-} D_{\lambda'_+}^{\lambda_+}}$$

Measured experimentally from projection operators:

$$\rho_{\lambda\lambda'}(\Omega) = \frac{1}{N} \sum_{i=1}^N \Lambda_{\lambda\lambda'}(\cos \theta_1, \phi_1)$$

(see Gounaris *et al.* for explicit  $\Lambda_{\lambda\lambda'}$  expressions)

## Remarks

- First three methods used to extract coupling parameters
- Spin density matrix is model independent  
 $\implies$  Test of Standard Model
- Off-diagonal spin matrix elements complex in general  
 $\implies$  CP violation gives non-zero imaginary components

# Experimental Signatures

---

So what anomalous couplings do we try to fit?

Many free parameters to determine...

⇒ Tempting / customary to allow only  
one / two parameter(s) to vary at a time

Example:

Fit for  $\Delta\kappa_\gamma$  with all other couplings fixed by SM

Complications:

- Convenient but not well motivated theoretically
- $\kappa_V, \lambda_V, g_1^V$  strongly correlated in observables
- Unnatural to vary only  $\gamma$  or only  $Z$  couplings
- Hard to separate  $\gamma, Z$  couplings in  $e^+e^- \rightarrow W^+W^-$  without polarized beams

(bad for LEP II, okay for Linear Collider)

Common choices – Full (relaxed) HISZ scenario

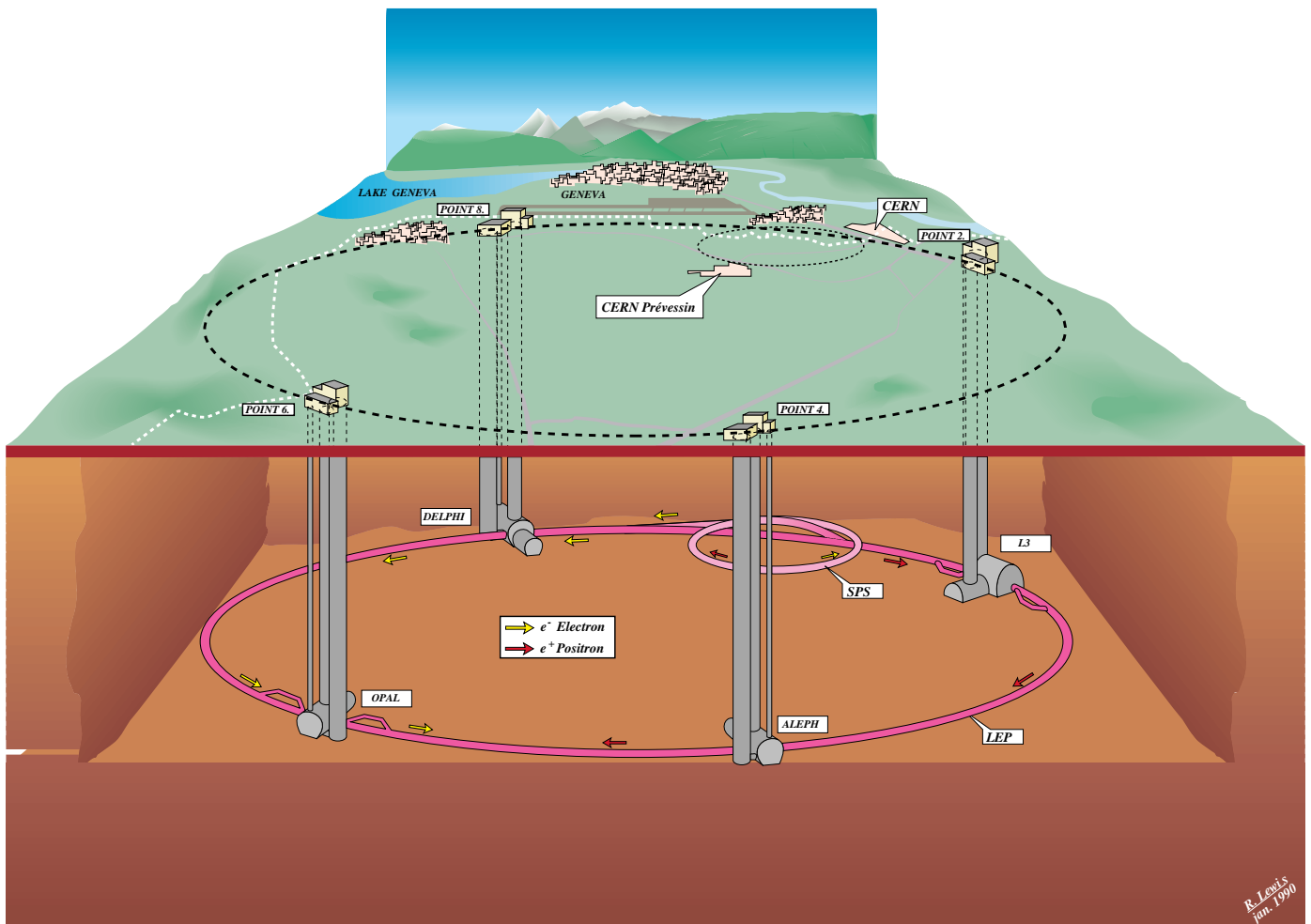
⇒ Two (three) free parameters to fit:

$$\kappa_\gamma, \quad \lambda_\gamma \quad (g_1^Z)$$

# Experimental Signatures

## LEP Ring:

- Electron-positron synchrotron
- 27-km circumference (see figure)
- Four major detectors: ALEPH, DELPHI, L3, OPAL
- Ring magnets permit  $\sqrt{s} \approx 240$  GeV
- RF cavities / power (\$) = real limitations



# Experimental Signatures

---

## LEP Running History:

- Turned on in 1989
- Ran at  $91 \pm 3$  GeV 1989-1995
- Provided millions of  $Z$ 's / experiment  
Standard Model confirmed with depressingly high precision
- Short run at 130-140 GeV in November 1995 (“LEP 1.5”)
- LEP II began in summer 1996 with  $25 \text{ pb}^{-1}$  at 161 GeV
- Subsequent runs at 172, 183, 189 GeV (1996-1998)
- Started 1999 at 192 GeV, now running at 196 GeV
- LEP II integrated luminosity / experiment  $> 350 \text{ pb}^{-1}$

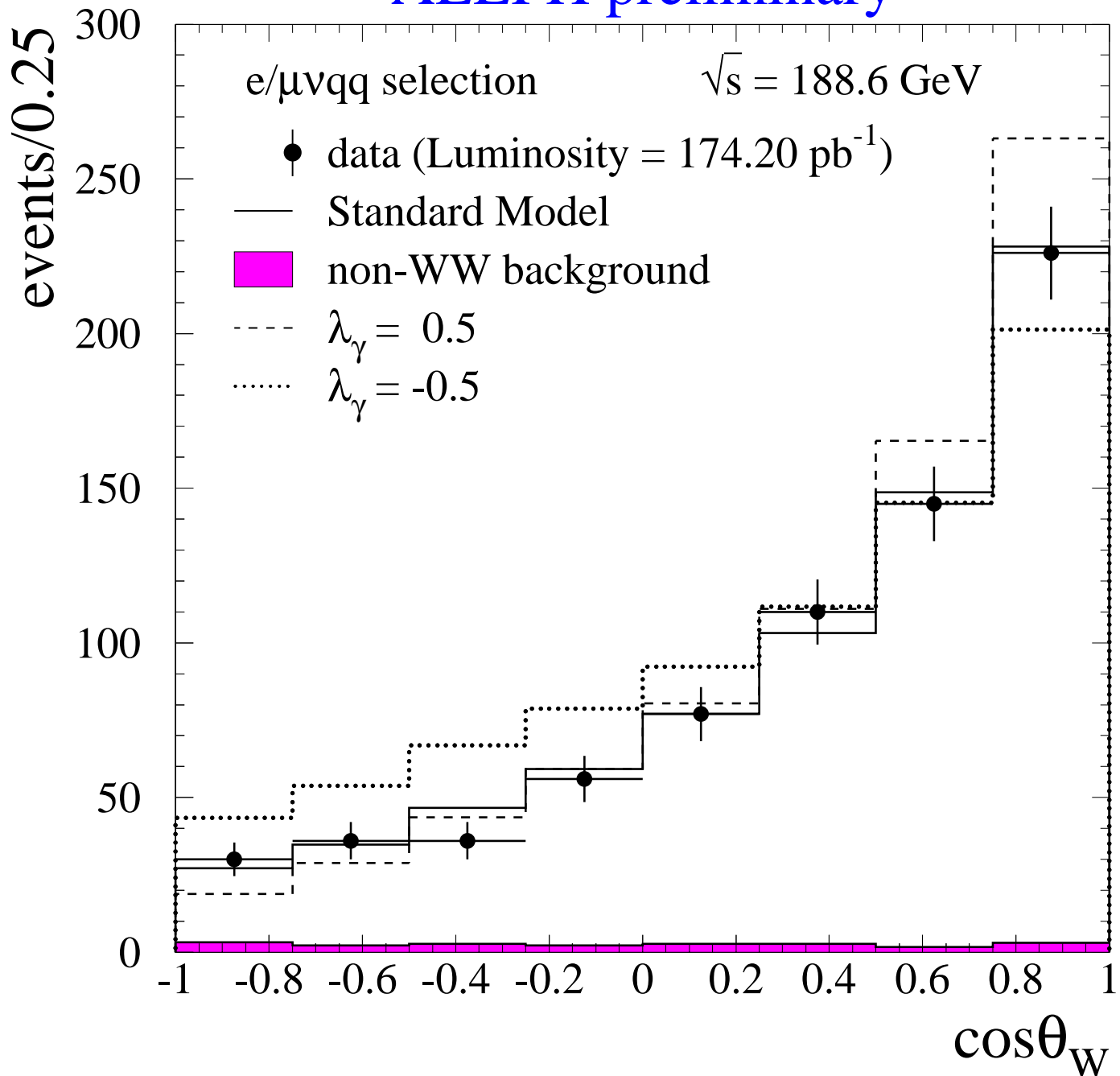
## LEP Running Plans:

- Collect data through 2000 with
  - $\sqrt{s} > 200$  GeV
  - Total LEP II luminosity  $> 500 \text{ pb}^{-1}$
- Shut down for 2001 LHC tunnel construction
- If dramatic new physics seen by end of 2000, running in 2002 possible

# Experimental Signatures

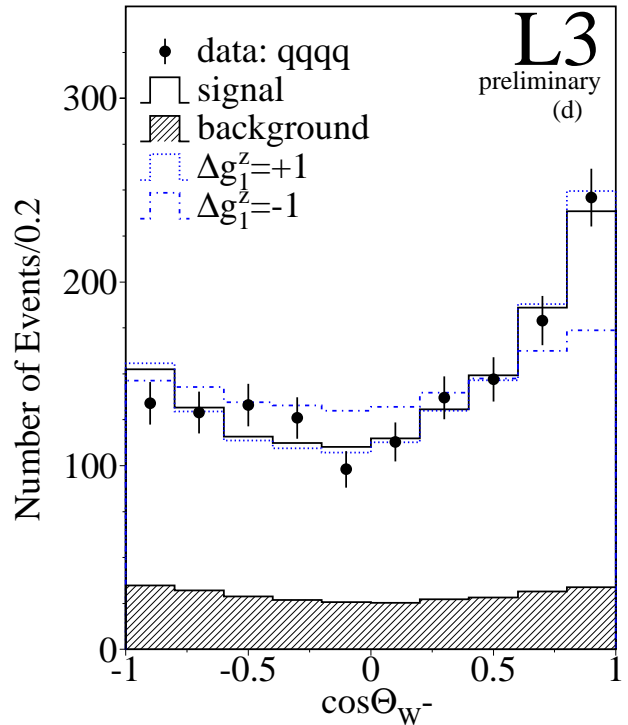
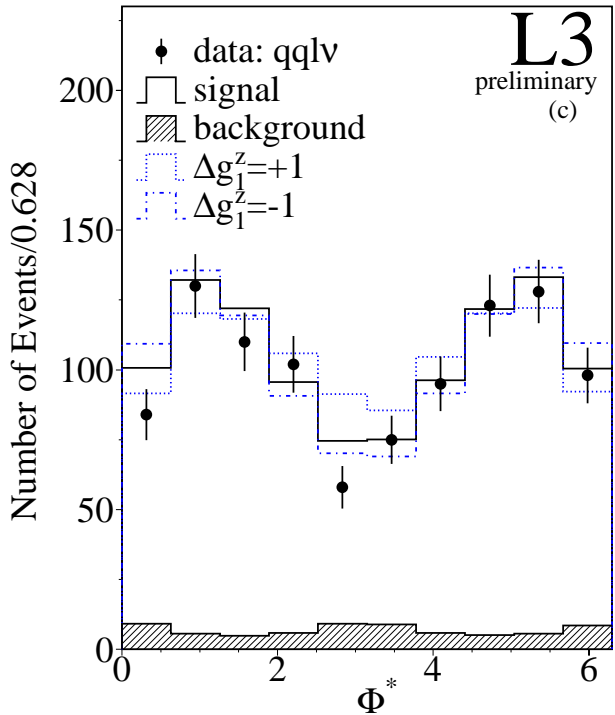
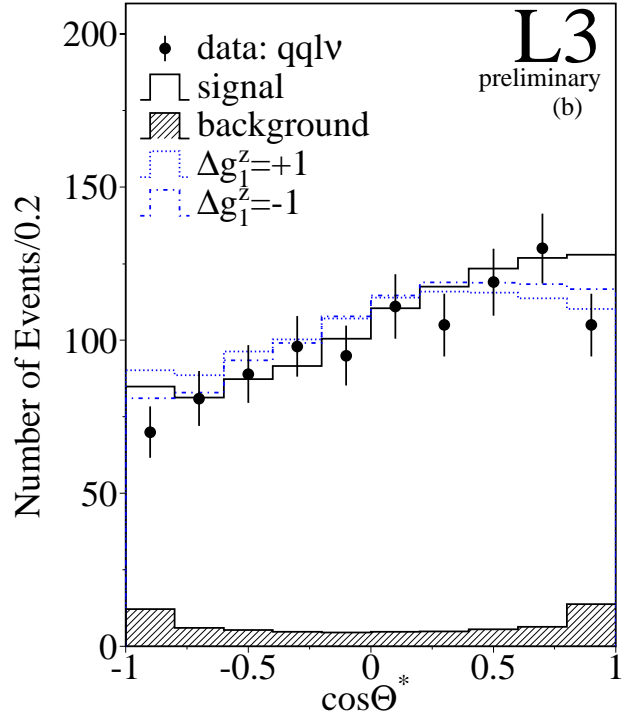
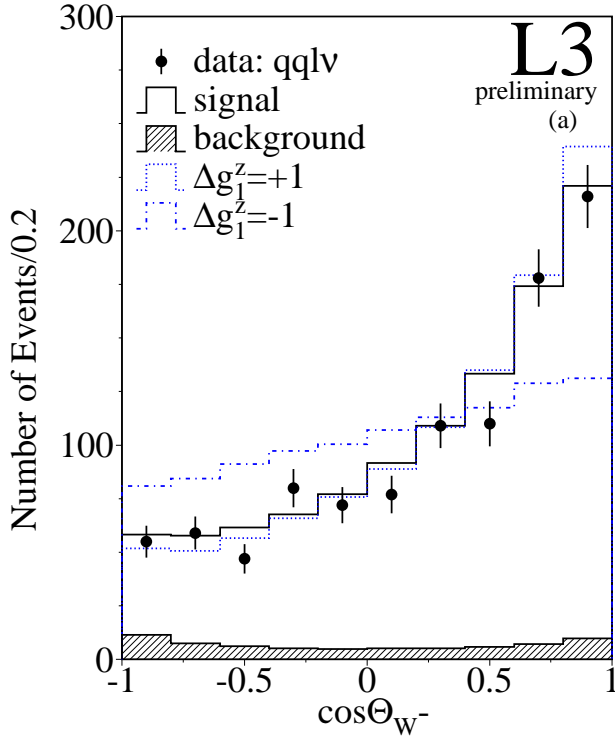
Measurement from ALEPH of  $\cos \Theta_W$  distribution:

**ALEPH preliminary**



# Experimental Signatures

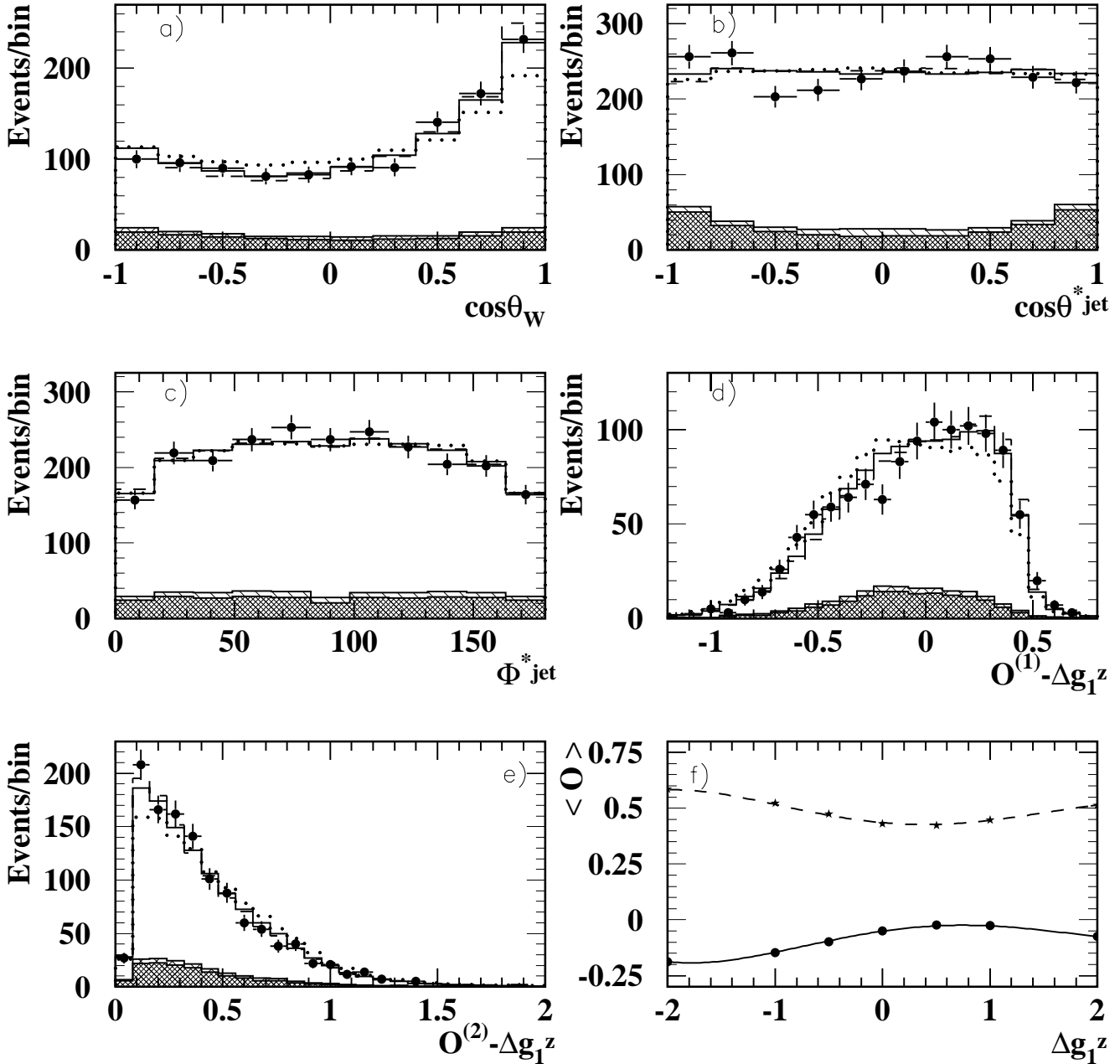
Measurement from L3 of several 1-D distributions:



# Experimental Signatures

OPAL angular distributions & optimal observables (prelim)

(curves for  $\Delta g_1^Z = \Leftrightarrow 0.5$ (dotted), 0(solid),  $+0.5$ (dashed))

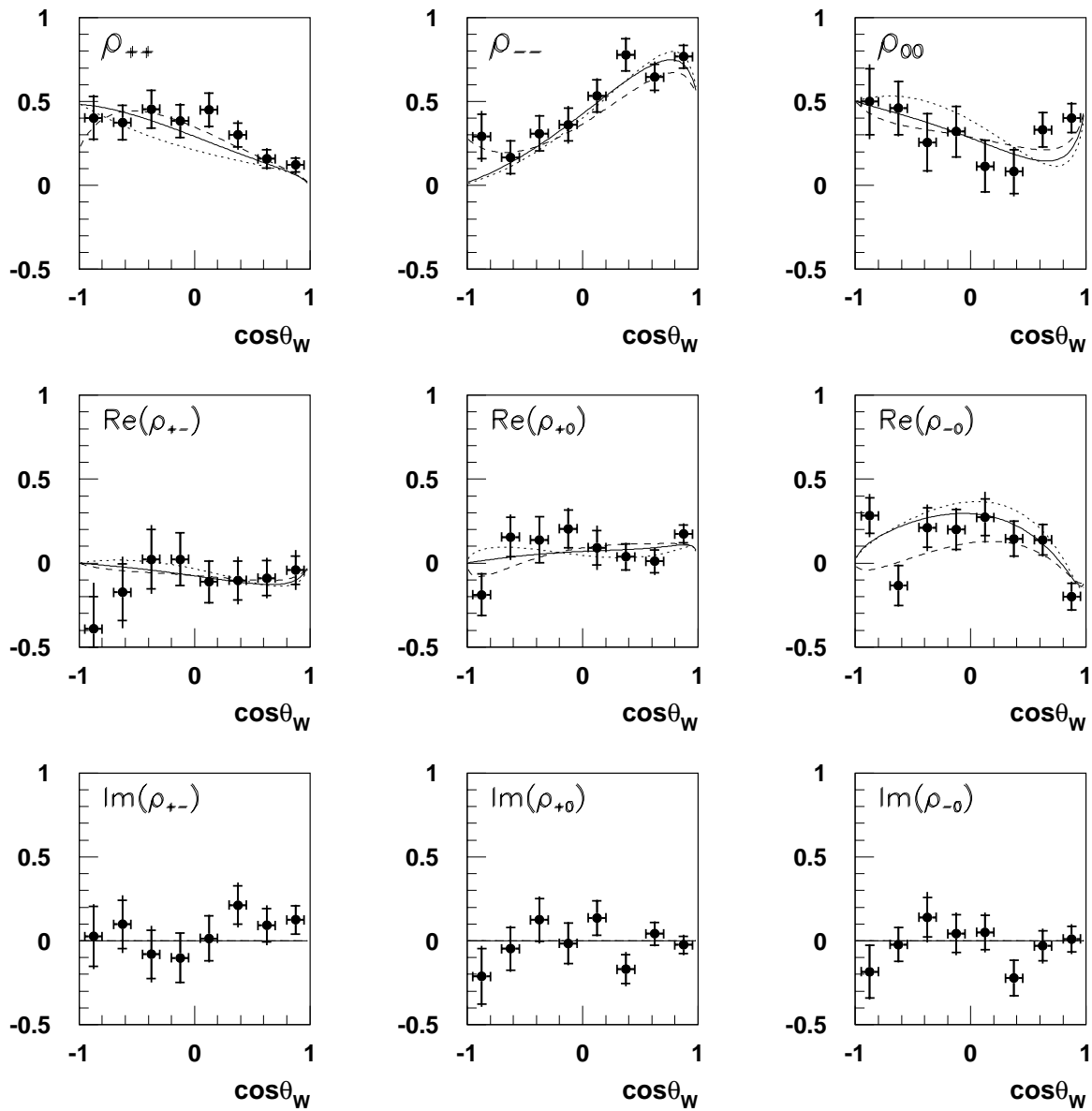




# Experimental Signatures

OPAL spin density matrix measurements:

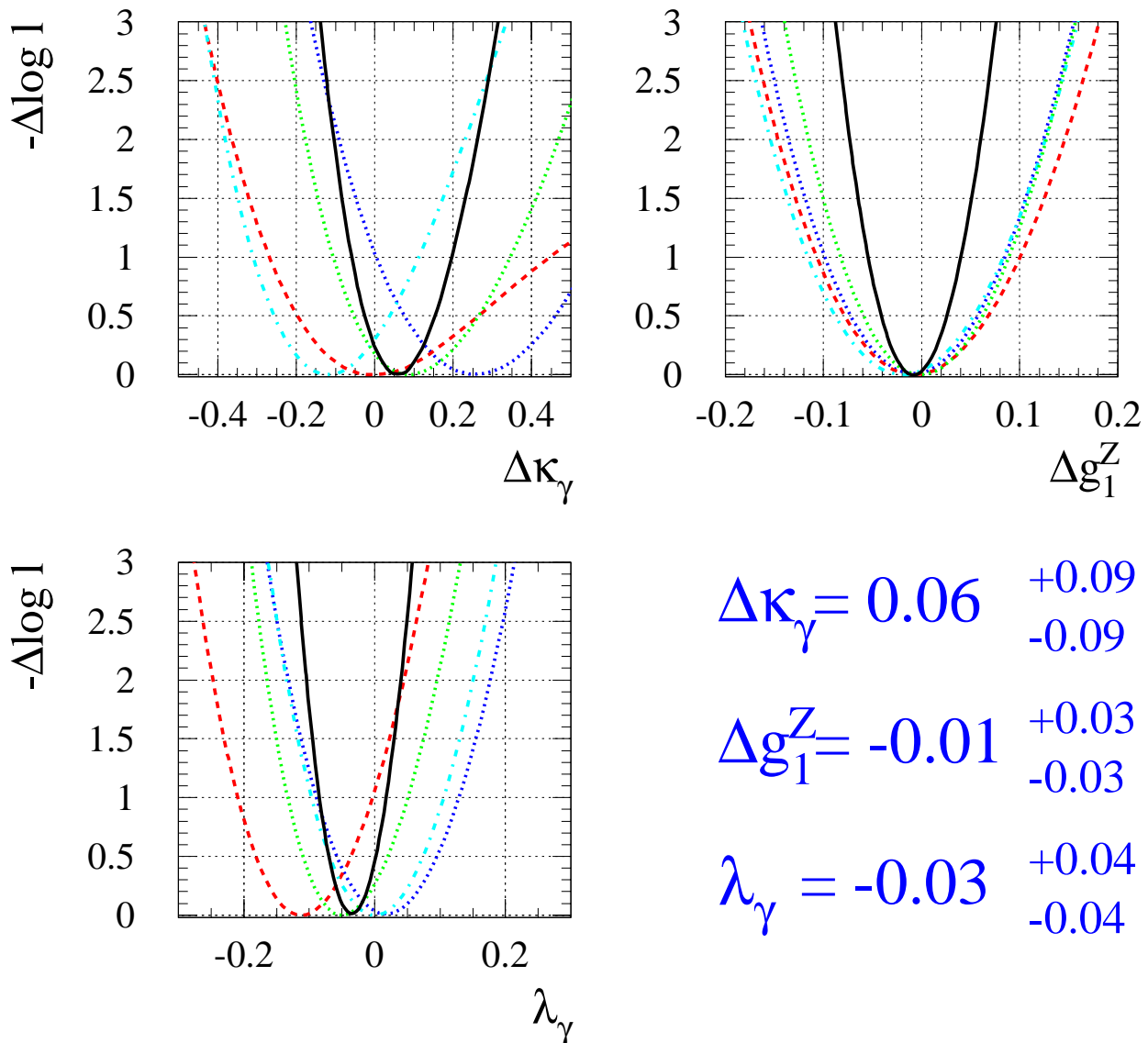
OPAL Preliminary



# Experimental Signatures

Results for HISZ coupling parameters:

**ALEPH + DELPHI + L3 + OPAL**

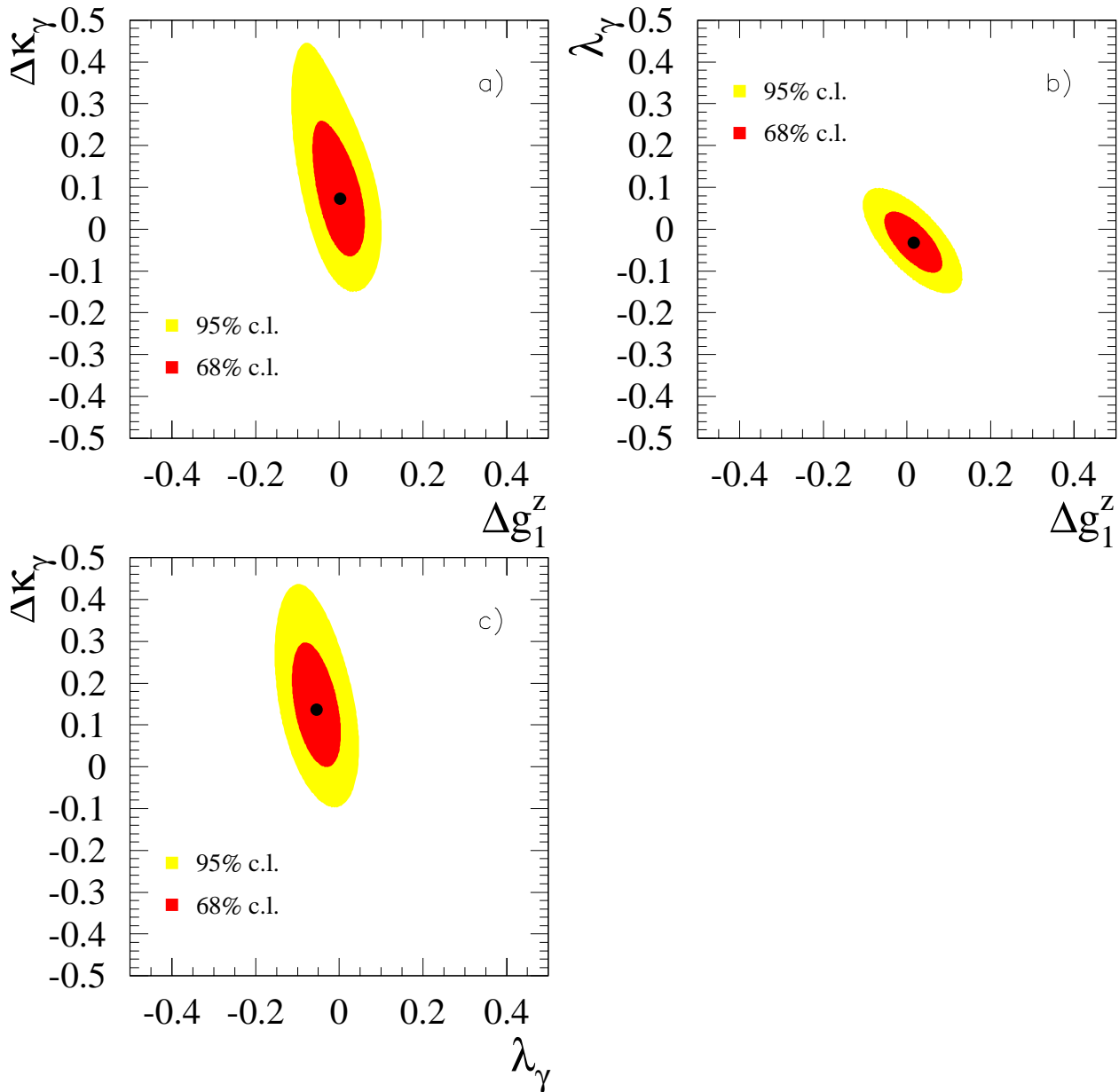


(Fits courtesy of LEP Electroweak Working Group)

# Experimental Signatures

Correlations cannot be neglected:

LEP combined 189 GeV (and 183 GeV) TGC fit



(Fits courtesy of LEP Electroweak Working Group)

# Experimental Signatures

---

## A Next Linear Collider

Who will build it?

- Germany? – TESLA or SBLC
- Japan? – JLC (S, C, or X)
- Russia? – VLEPP
- Europe? – CLIC
- U.S.A.? – NLC

⇒ World-wide, collaborative R & D effort

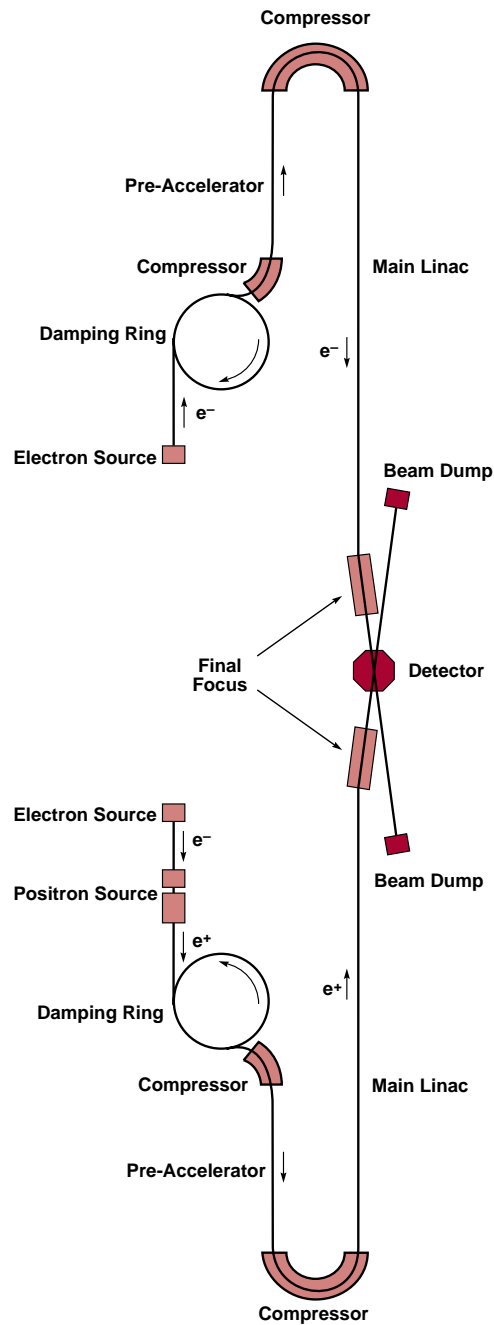
U.S. R & D effort centered at SLAC

- Three-stage concept:
  - Turn on with  $\sqrt{s} = 500$  GeV
  - Increase  $\sqrt{s}$  “adiabatically” to 1 TeV (more/better klystrons)
  - Lengthen machine to achieve 1.5 TeV

# Experimental Signatures

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## Layout of NLC



# Experimental Signatures

---

Some NLC machine parameters:

$\sqrt{s}$	500 GeV	1 TeV
Length (km)	16	18
RF Frequency (GHz)	11	11
Klystron Power (MW)	50	72
# Klystrons	3900	9200
Gradient (MV/m)	50	85
Wall Plug Power (MW)	105	200
Beam spot $\sigma_x$ (nm)	320	250
Beam spot $\sigma_y$ (nm)	5.5	4.3
$\mathcal{L}$ ( $\text{cm}^{-2}\text{s}^{-1}$ )	$5 \times 10^{33}$	$1.4 \times 10^{34}$

$\Rightarrow \approx 6$  years construction

- “Zero-order” Design Report (ZDR) completed 1996
- Detailed engineering work has begun at SLAC

# Experimental Signatures

---

Other Accelerator Options:

- $e^-e^-$  Collider (Get for “free”)
- $e^-\gamma$  Collider
- $\gamma\gamma$  Collider

$\gamma$  colliders based on backscattered laser photons  
(Ginzburg *et al.*, Akerlof 1981)

Some advantages:

- Opens up new Physics channels  
(Spin, Isospin, Charge)
- Isolation of  $\gamma$  from  $Z$  contributions
- Look for  $\gamma\gamma \rightarrow H$
- Look for Majorana neutrinos ( $e^-e^- \rightarrow \nu\nu W^-W^-$ )

Potential pitfall:

- Making  $\gamma$  beams with competitive luminosity

# Experimental Signatures

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Early studies by Barklow of  $q\bar{q}\ell\bar{\nu}$  channel

Use:

Measured energy of lepton

Angles of lepton, jets (require  $|\cos \Theta_W| < 0.8$ )

Velocities of jets ( $\beta_i = p_i/E_i$ )

Kinematic constraint to  $\ell\bar{\nu}j_1j_2$

Require:

$$\chi^2 \equiv \frac{(M_{\ell\bar{\nu}}^{fit} \Leftrightarrow M_W)^2}{2} + \frac{(M_{j_1j_2}^{fit} \Leftrightarrow M_W)^2}{2} < 2$$

Perform unbinned maximum likelihood fit in five reconstructed angles to extract coupling parameters

- No detector smearing\*
- But realistic efficiencies used
- Following exclusion contours defined by covariance matrix elements

\*Studies for Snowmass 96 (KR) confirmed that planned detector resolutions cause no serious degradation



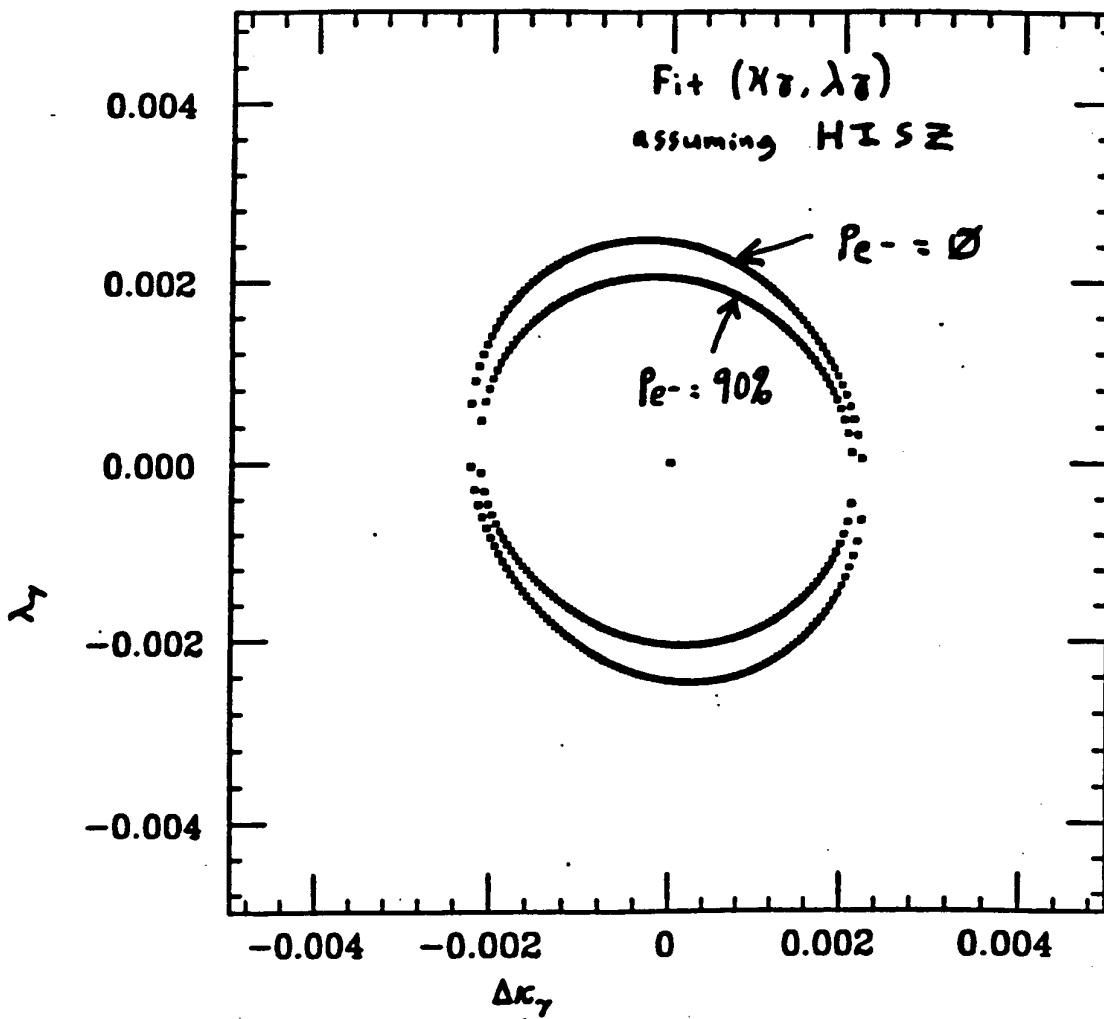
# Experimental Signatures

NLC studies (Barklow)

With and without  $e^-$  beam polarization

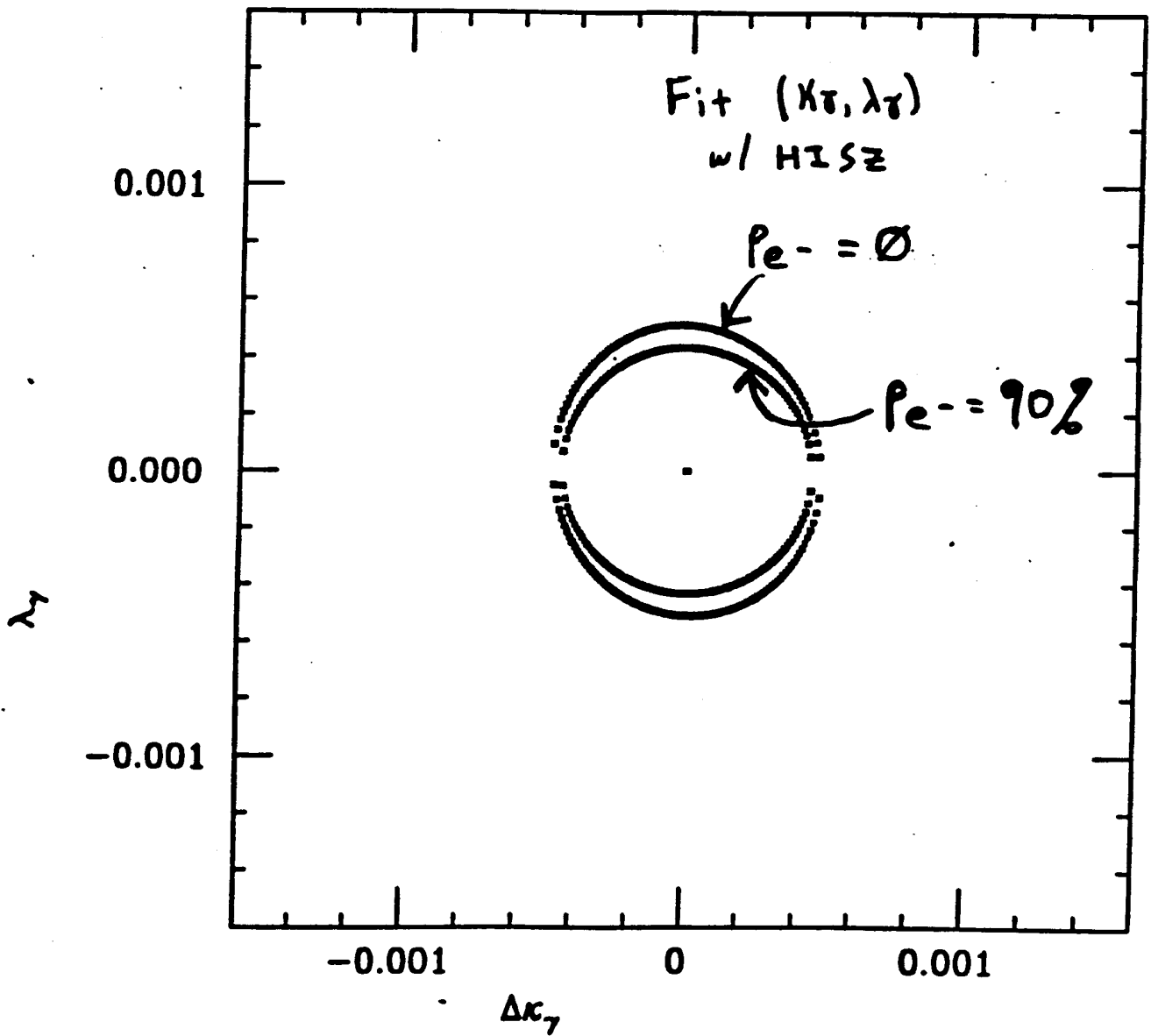
$80 \text{ fb}^{-1}$  at  $\sqrt{s} = 500 \text{ GeV}$

Assumes HISZ scenario



# Experimental Signatures

Same plots with  $190 \text{ fb}^{-1}$  at  $\sqrt{s} = 1.5 \text{ TeV}$



Conclusion:

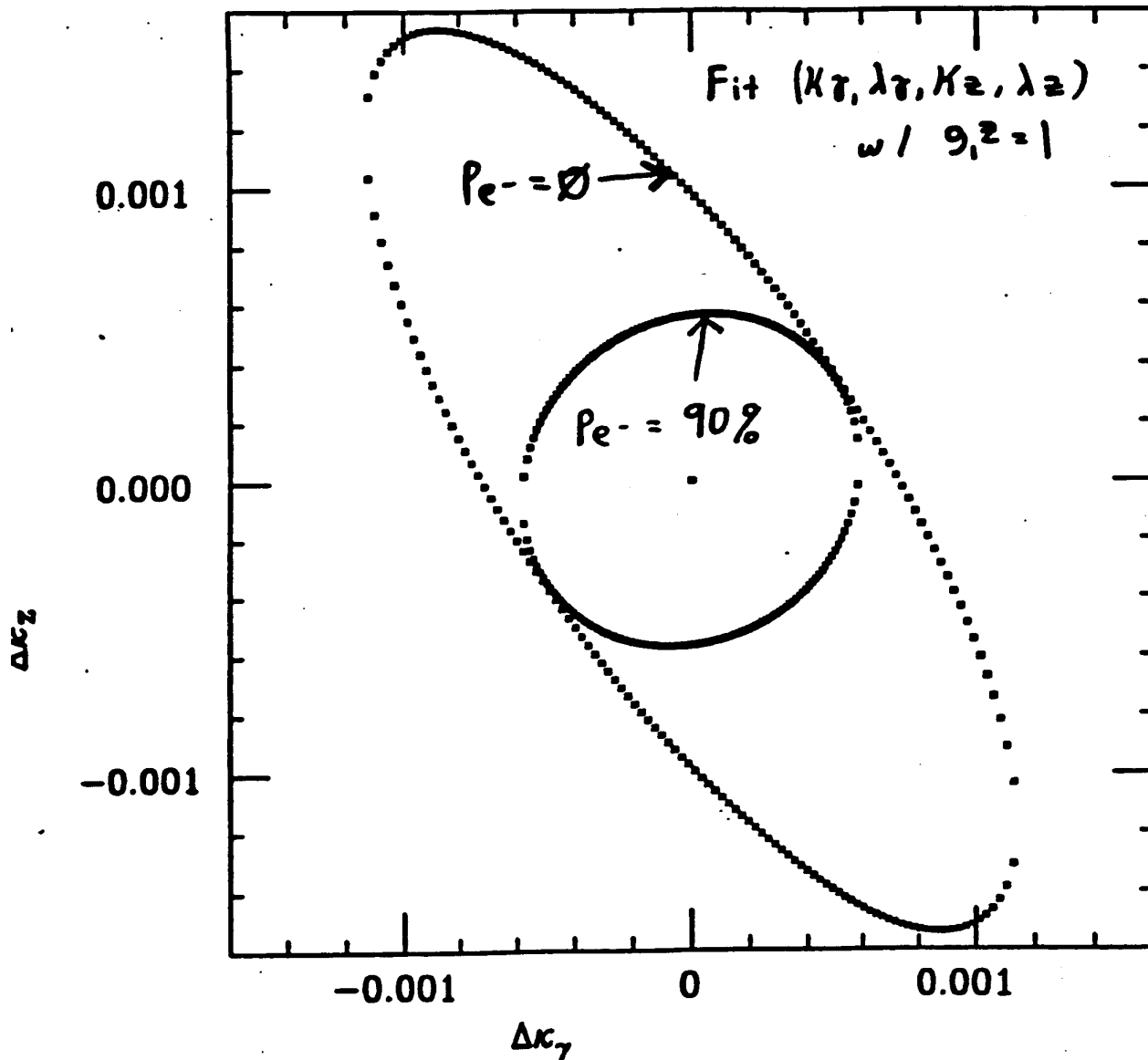
Beam polarization useful even in HISZ Scenario

# Experimental Signatures

In more general model, however,

beam polarization critical

in disentangling  $\gamma$ ,  $Z$  couplings

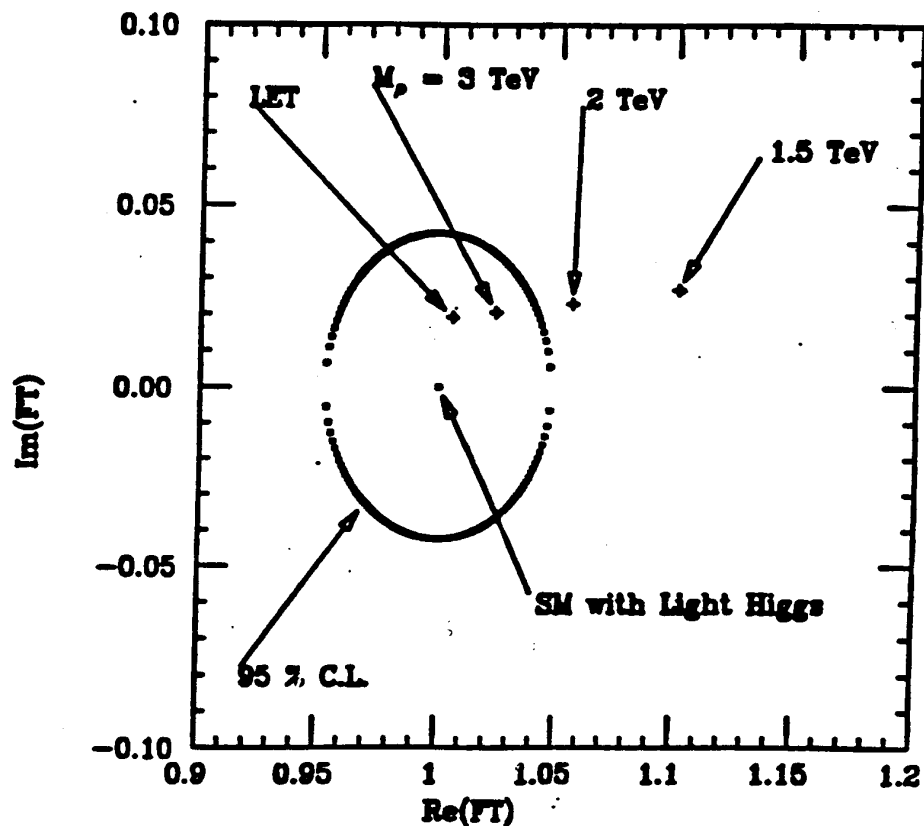


# Experimental Signatures

Can one start to see a techni-rho at NLC?

Preceding helicity analysis (Barklow) of  $e^+e^- \rightarrow W^+W^-$  extended to fit for real/imaginary components of complex form factor:

(S.M.  $W_LW_L$  amplitude multiplied by  $F_T$ )

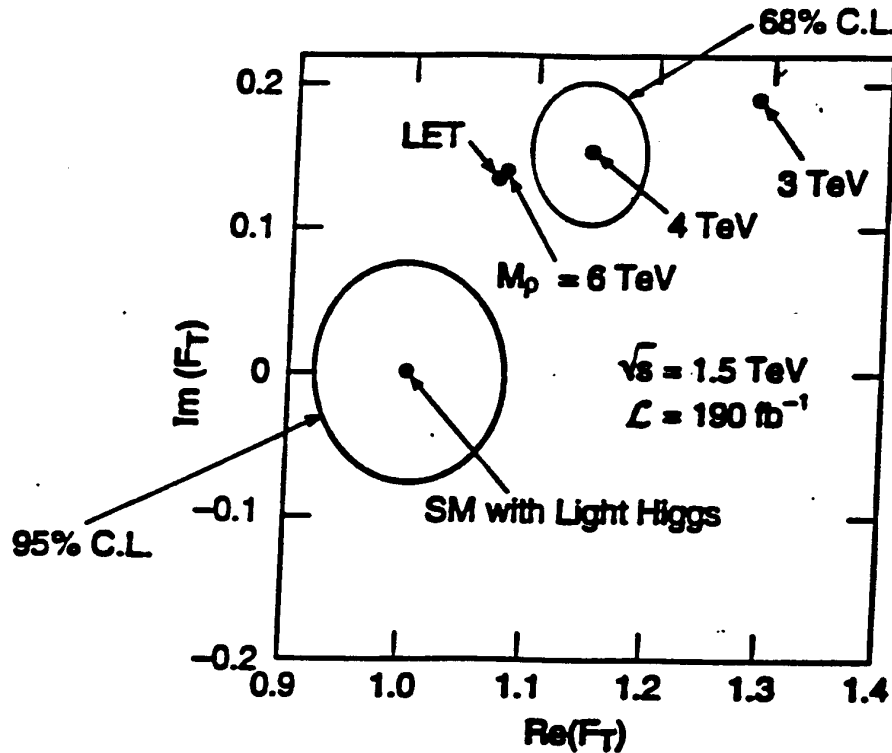


⇒ At 500 GeV:

- Exclude techni-rho with  $M_\rho < 2.5$  TeV (95% CL)
- Discover techni-rho with  $M_\rho < 1.5$  TeV ( $5\sigma$ )

# Experimental Signatures

At higher  $\sqrt{s}$ , expected deviations of  $F_T$  from (1,0) become larger for given  $M_\rho, \rho$ :



⇒ At 1.5 TeV:

- Exclude any techni-rho in this model
- Discovery potential:
  - 4.5  $\sigma$  for L.E.T.
  - 4.8  $\sigma$  for  $M_\rho = 6$  TeV
  - 6.5  $\sigma$  for  $M_\rho = 4$  TeV
- Distinguish L.E.T. from  $M_\rho < 4$  TeV

# Experimental Signatures

---

What about  $W_L W_L$  fusion?

Analysis by Barger / Cheung / Han / Phillips uses  $e^+e^- \rightarrow \nu\bar{\nu}W^+W^-$ ,  $e^+e^- \rightarrow \nu\bar{\nu}ZZ$  channels at 1.5 TeV to measure ratio

$$R_{W/Z} \equiv \frac{\sigma(W_L^+W_L^- \rightarrow W^+W^-)}{\sigma(W_L^+W_L^- \rightarrow ZZ)}$$

Ratio varies with strong coupling model

- S.M. with  $M_H = 1$  TeV: expect  $R_{W/Z} \approx 2$
- L.E.T. ( $M_H \rightarrow \infty$ ): expect  $R_{W/Z} \approx 2/3$
- Technicolor: expect  $R_{W/Z}$  very large  
(no resonance for  $Z_L Z_L$ )

$e^+e^-$  cleanliness allows selection of hadronic decays with modest backgrounds

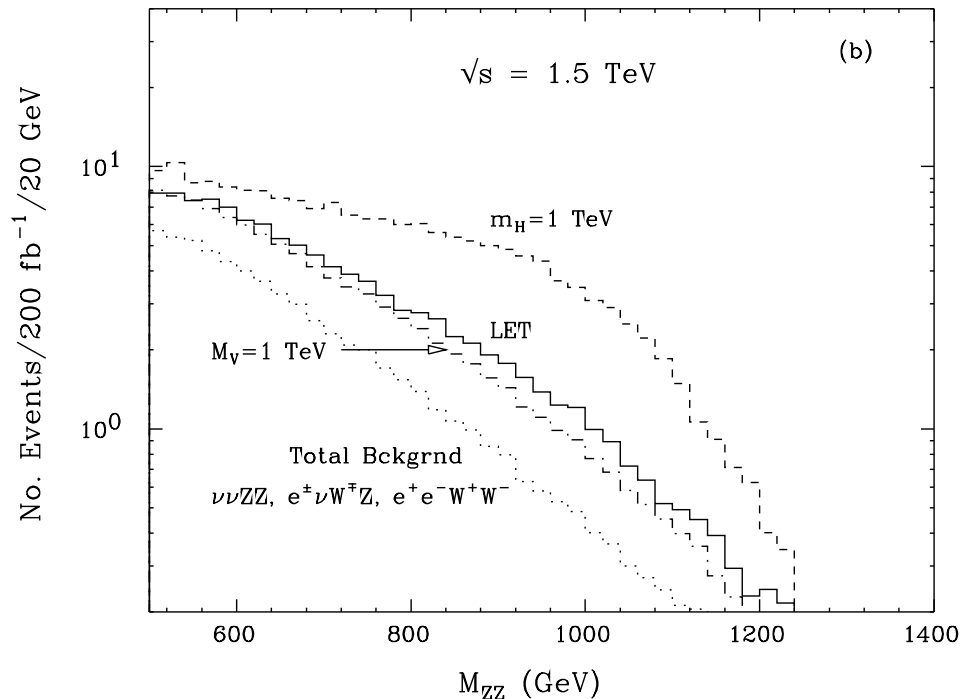
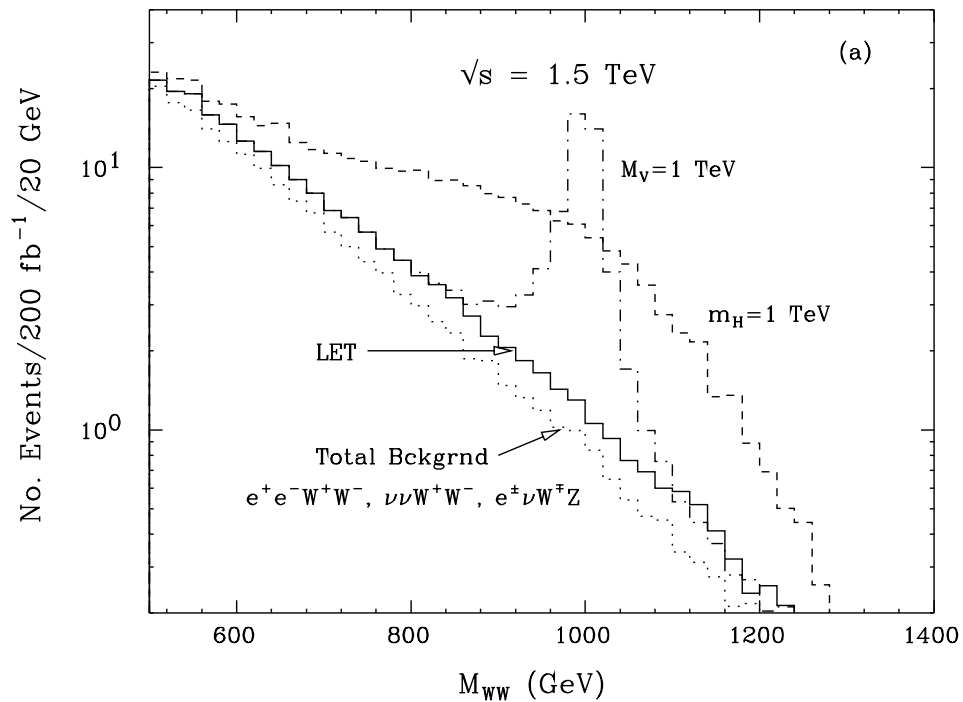
$W^+W^-$  and  $ZZ$  separated statistically by dijet mass:

$$68 \text{ GeV} < M_W < 86 \text{ GeV} \quad 86 < M_Z < 105 \text{ GeV}$$

# Experimental Signatures

How many events?

Following figure assumes  $200 \text{ fb}^{-1}$  at  $1.5 \text{ TeV}$



# Experimental Signatures

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Other Coupling Measurements at  $e^+e^-$  Collider:

Process	Couplings probed
$e^+e^- \rightarrow Z\gamma$	$ZZ\gamma, Z\gamma\gamma$
$e^+e^- \rightarrow WW\gamma$	$WW\gamma, WWZ\gamma, WW\gamma\gamma$
$e^+e^- \rightarrow WWZ$	$WWZ, WWZZ, WWZ\gamma$
$e^+e^- \rightarrow e\bar{\nu}W$	$WWZ, WW\gamma$
$e^+e^- \rightarrow \nu\bar{\nu}\gamma$	$WW\gamma$
$e^+e^- \rightarrow \nu\bar{\nu}Z$	$WWZ$

Coupling Measurements at  $e^-e^-, \gamma\gamma, e^-\gamma$  Colliders:

Process	Couplings probed
$e^-e^- \rightarrow e^-\nu W^-$	$WW\gamma, WWZ$
$e^-e^- \rightarrow e^-e^-Z$	$ZZ\gamma, Z\gamma\gamma$
$e^-e^- \rightarrow e^-\nu W^-\gamma$	$WW\gamma, WWZ$
$e^-e^- \rightarrow \nu\nu W^-W^-$	$WWWW$ (Isospin 2 poss.)
$\gamma\gamma \rightarrow W^+W^-$	$WW\gamma$
$\gamma\gamma \rightarrow W^+W^-Z$	$WWZ, WW\gamma$
$\gamma\gamma \rightarrow ZZ$	$ZZ\gamma, Z\gamma\gamma$
$\gamma\gamma \rightarrow W^+W^-W^+W^-$	$WWWW$ (Isospin 2 poss.)
$\gamma\gamma \rightarrow W^+W^-ZZ$	$WWZZ$
$e^-\gamma \rightarrow W^-\nu$	$WW\gamma$
$e^-\gamma \rightarrow e^-Z$	$ZZ\gamma, Z\gamma\gamma$
$e^-\gamma \rightarrow W^+W^-e^-$	$WWZ, WW\gamma, WWZ\gamma$

Note: can polarize both beams in  $e^-e^-$



# Experimental Signatures

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## WHAT CAN HADRON COLLIDERS TELL US?

Sampling of accessible couplings / processes:

Coupling	Processes
$WW\gamma$	$q\bar{q}' \rightarrow W^* \rightarrow W\gamma$
$WW\gamma/WWZ$	$q\bar{q} \rightarrow \gamma^*/Z^* \rightarrow WW$
$WWZ$	$q\bar{q}' \rightarrow W^* \rightarrow WZ$

Complications:

- Parton initial state energies / longitudinal momenta unknown *a priori*
- Parton collisions have poorly defined maximum  $\sqrt{s'}$  (unlike at  $e^+e^-$ ,  $e^-e^-$ ,  $\gamma\gamma$  colliders)
- Form-factor dependence critical in setting sensible limits

Example:

$$\Delta\kappa_V(s') = \frac{\Delta\kappa_\gamma^0}{\left(1 + \frac{s'}{\Lambda_{FF}^2}\right)^2}$$

where  $\Lambda_{FF} \approx$  scale of new physics

Quoted limits must specify assumed  $\Lambda_{FF}$

Typical choices: 1.0, 1.5, 2.0 TeV

# Experimental Signatures

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## The Tevatron Collider

Two Detectors: CDF D0

Run 1 (1992-95)

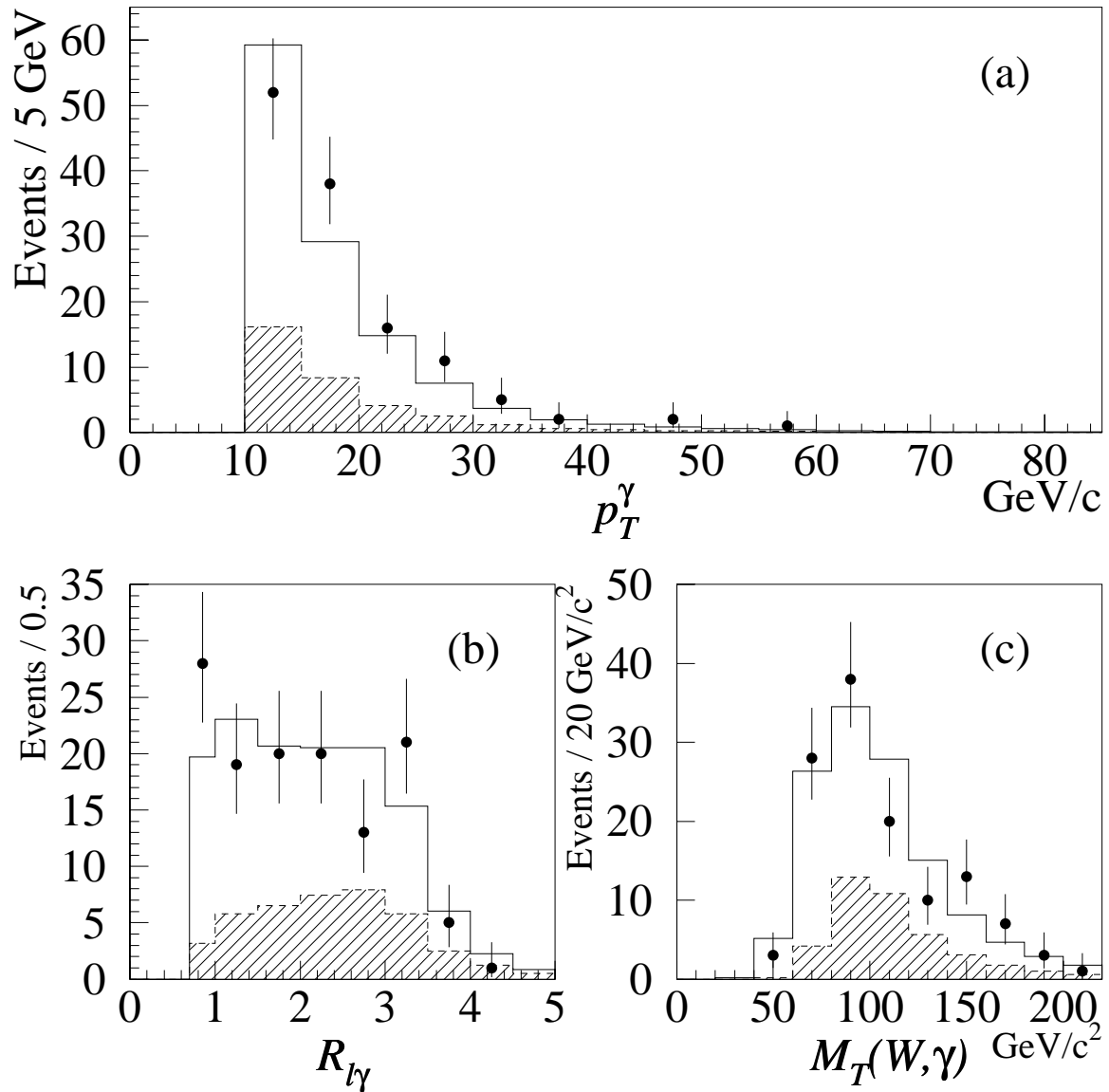
- $>100 \text{ pb}^{-1}$  at  $\sqrt{s_{p\bar{p}}} = 1.8 \text{ TeV}$
- Many TGC results now final & published

Run 2 (2000-200?)

- $>2 \text{ fb}^{-1}$  at  $\sqrt{s_{p\bar{p}}} = 2.0 \text{ TeV}$
- Extension to  $\geq 10\text{-}30 \text{ fb}^{-1}$  (TEV33)

# Experimental Signatures

Measurement of  $W\gamma$  production from D0:

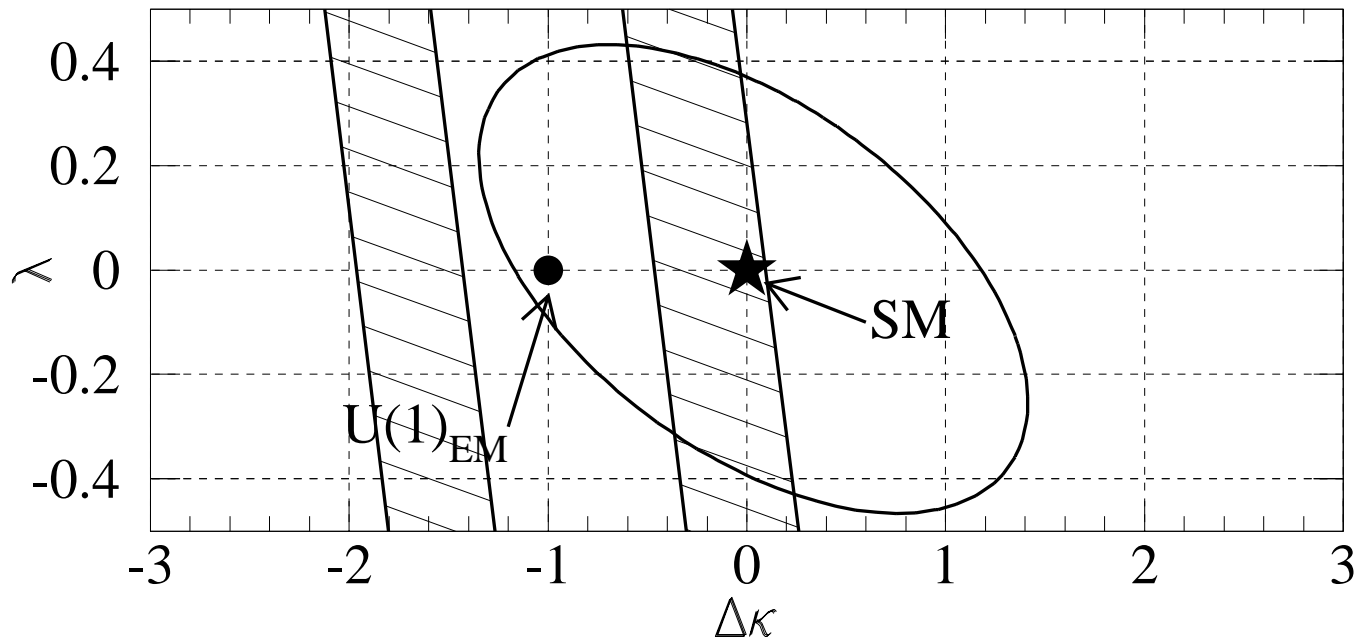


$$R_{l\gamma} \equiv \sqrt{(\delta\eta)^2 + (\delta\phi)^2}$$

$$M_T(W, \gamma) = W\text{-}\gamma \text{ transverse mass}$$

# Experimental Signatures

Resulting D0 limits on  $\Delta\kappa_\gamma, \lambda_\gamma$ : ( $b \rightarrow s\gamma$  limits shown too)



Remarks:

- Better sensitivity to  $\lambda_V$  than to  $\Delta\kappa_V$  (like LEP)
- $WW\gamma$  cleanly isolated from  $WWZ$  (unlike LEP)
- Assumes  $\Lambda_{FF} = 1.5$  TeV

# Experimental Signatures

CDF limits from  $WW$ ,  $WZ$  production:

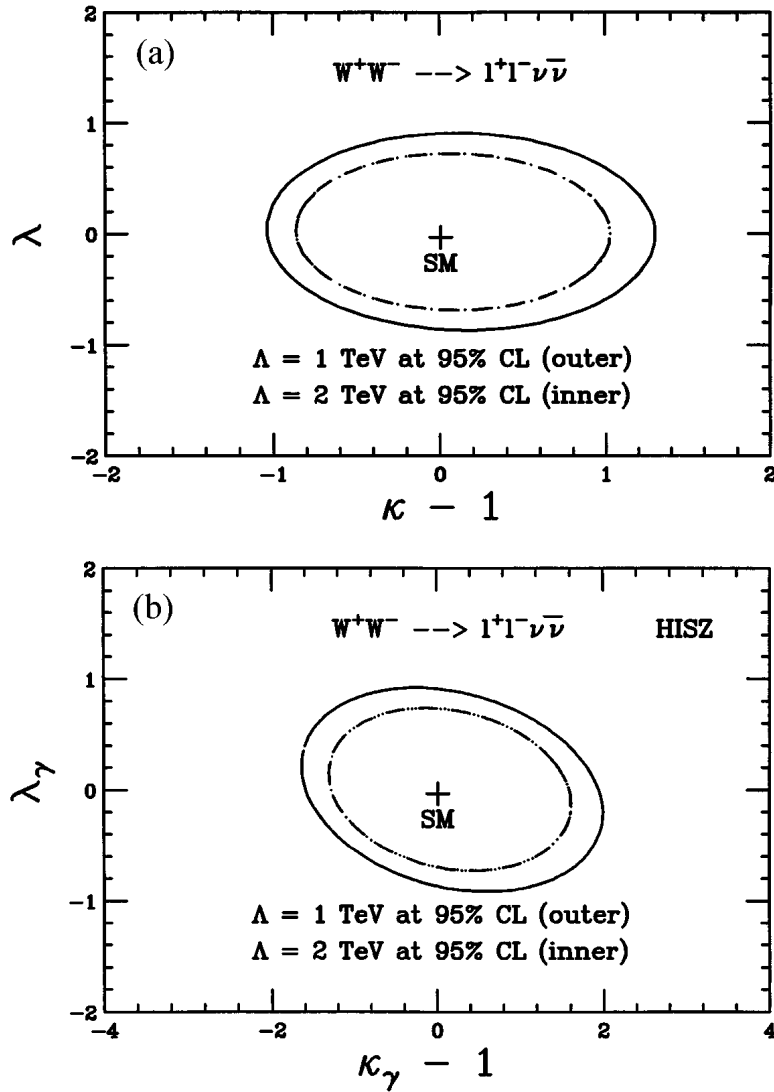


FIG. 2. Limits on anomalous couplings: (a) Assuming  $\kappa_\gamma = \kappa_Z = \kappa$  and  $\lambda_\gamma = \lambda_Z = \lambda$ . (b) The HISZ scenario where  $\kappa_\gamma$  and  $\lambda_\gamma$  are used as independent parameters. The standard model value is located at the center. The outer (inner) contour is the 95% CL limits with the energy scale  $\Lambda = 1 \text{ TeV}$  ( $2 \text{ TeV}$ ).

# Experimental Signatures

Tevatron – Run 2

Expect 1-10 fb<sup>-1</sup> at  $\sqrt{s} = 1.8/2.0$  TeV

Analysis by Errede

Assumes full HISZ scenario

For 1 fb<sup>-1</sup>:

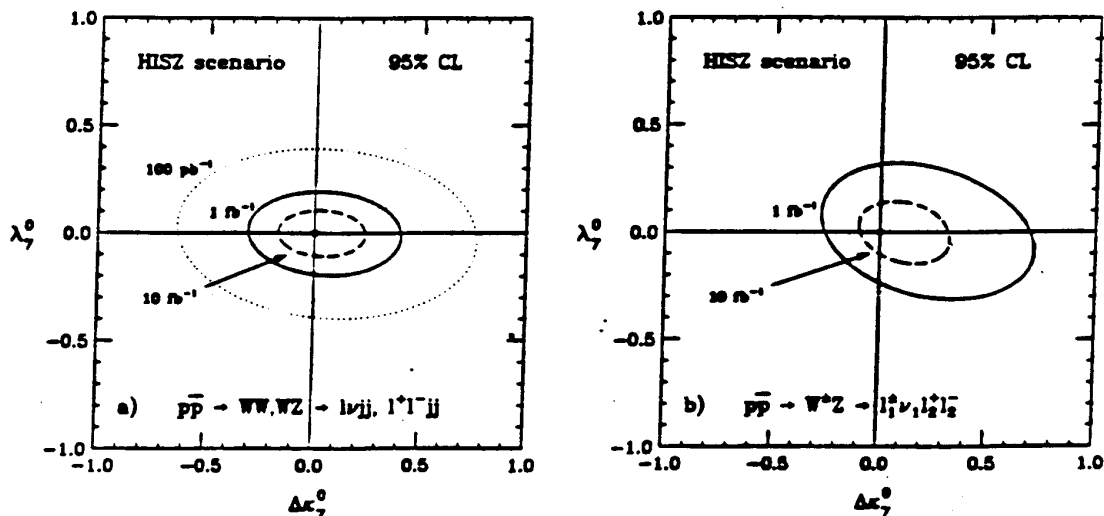
$$\Leftrightarrow 0.31 < \Delta\kappa_\gamma < 0.41 \quad (\lambda_\gamma = 0)$$

$$\Leftrightarrow 0.19 < \lambda_\gamma < 0.19 \quad (\Delta\kappa_\gamma = 0)$$

For 10 fb<sup>-1</sup>:

$$\Leftrightarrow 0.17 < \Delta\kappa_\gamma < 0.24 \quad (\lambda_\gamma = 0)$$

$$\Leftrightarrow 0.10 < \lambda_\gamma < 0.11 \quad (\Delta\kappa_\gamma = 0)$$



# Experimental Signatures

What about the LHC?

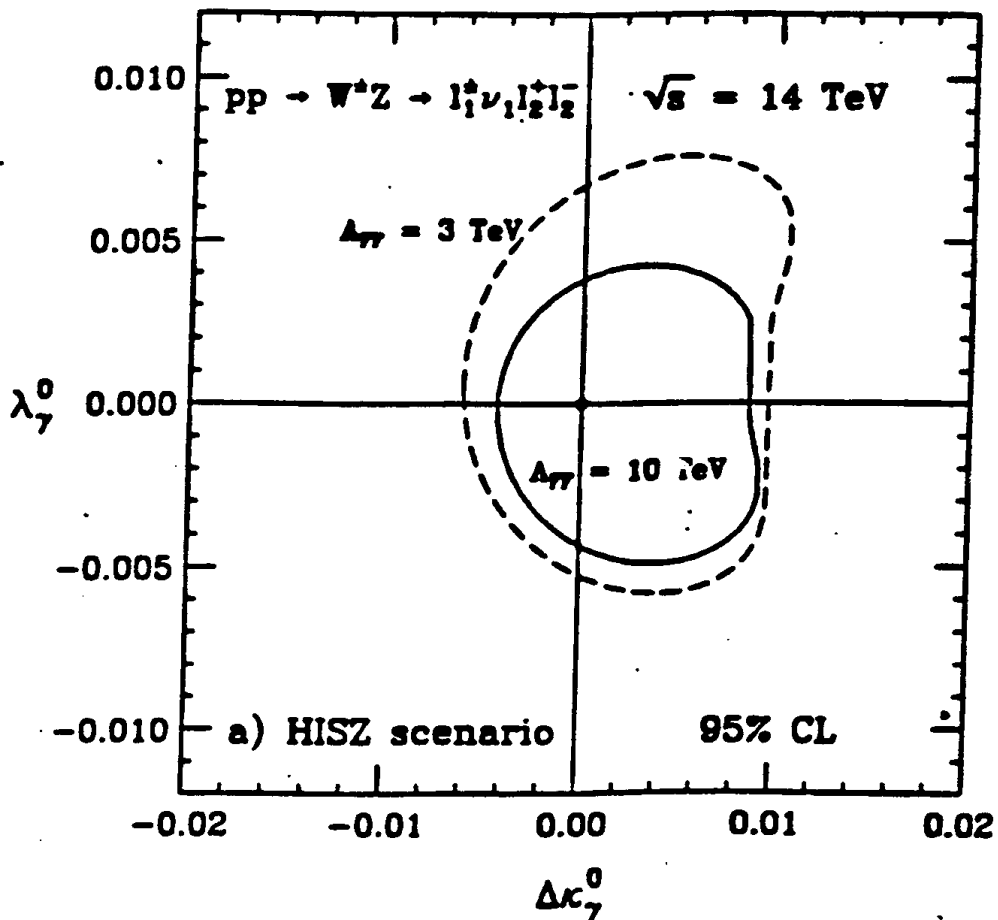
Assume  $100 \text{ fb}^{-1}$  at  $\sqrt{s} = 14 \text{ TeV}$

Analysis for ATLAS TDR

Assumes HISZ scenario

For  $100 \text{ fb}^{-1}$ :

$$\Leftrightarrow 0.006 < \Delta\kappa_\gamma < 0.0097 \quad (\lambda_\gamma = 0)$$
$$\Leftrightarrow 0.0053 < \lambda_\gamma < 0.0067 \quad (\Delta\kappa_\gamma = 0)$$

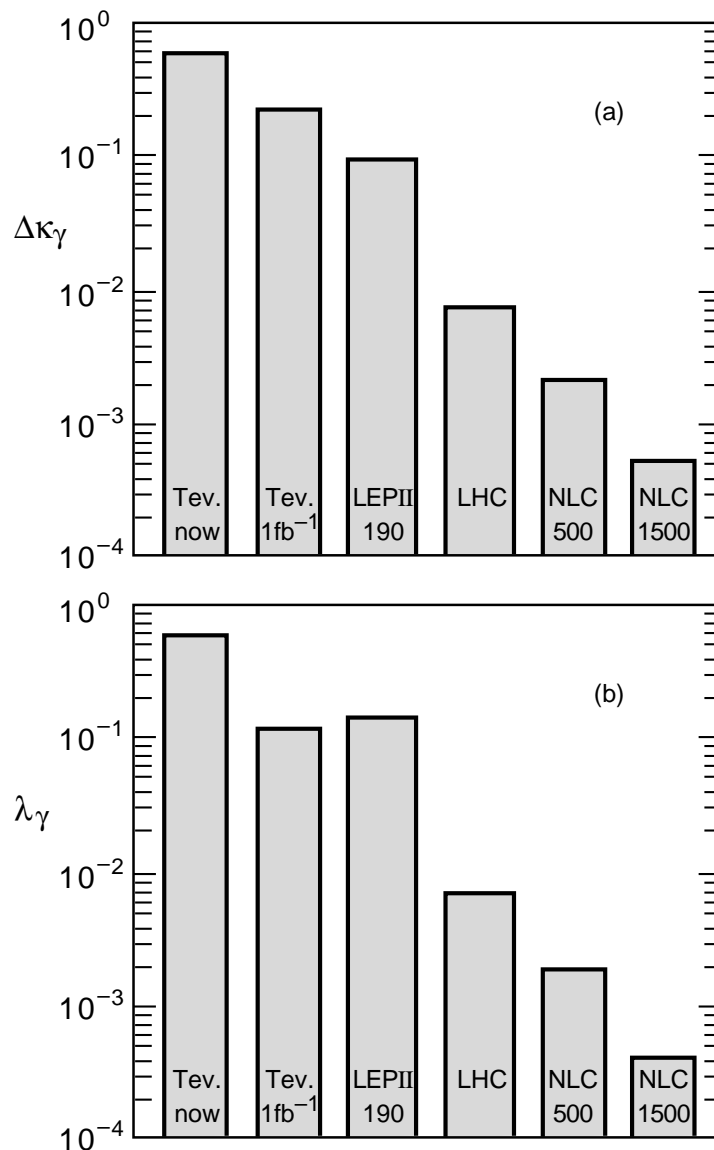


# Experimental Signatures

Summary of CP-conserving  $WWV$  measurement prospects

NLC should improve dramatically upon LEP II / Tevatron and substantially upon LHC

Figure from Barklow/Dawson/Haber/Siegrist:



3-95

7903A10



# Experimental Signatures

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What about CP violation? (!)

Remarks

- Total cross section less sensitive to small  $\mathcal{CP}$  couplings
- Why?

$$\sigma \propto |M_{\text{tot}}|^2$$

where

$$M_{\text{tot}} = M_{\text{S.M.}} + M_{\mathcal{CP}}$$

At tree level,

$$\Im\{M_{\text{S.M.}}\} = 0 \quad \Re\{M_{\mathcal{CP}}\} = 0$$

$$\implies \sigma_{\text{tot}} = \sigma_{\text{S.M.}} + \sigma_{\mathcal{CP}}$$

$\implies$  No interference

- Contrast with CP-even anomalous couplings which disturb large destructive interference in S.M.
- Similar problem occurs in simple angular distributions
- Better sensitivity from
  - Correlations between  $W^+$  and  $W^-$  decay distributions
  - Manifestly CP-odd observables

# Experimental Signatures

---

Examples in  $e^+e^- \rightarrow W^+W^-$ :

- Multi-dimensional maximum likelihood fitting
- Look at following 1-D distributions (Hagiwara *et al.*)
  - $\sin \theta_1 \sin \theta_2$
  - $\sin(\phi_1 \Leftrightarrow 2\phi_2) \Leftrightarrow \sin(2\phi_1 \Leftrightarrow \phi_2)$
  - $\sin(\phi_1 \Leftrightarrow \phi_2)$
- Look at imaginary components of off-diagonal single-W spin density matrix elements (Gounaris *et al.*)
  - $\Im\{\rho_{+-}^{W^+}\} + \Im\{\rho_{+-}^{W^-}\}$
  - $\Im\{\rho_{+0}^{W^+}\} \Leftrightarrow \Im\{\rho_{-0}^{W^-}\}$
  - $\Im\{\rho_{-0}^{W^+}\} \Leftrightarrow \Im\{\rho_{+0}^{W^-}\}$

# Experimental Signatures

---

Present *direct* limits on  $\mathcal{CP}$   $WW\gamma$  couplings:

- D0 used  $p\bar{p} \rightarrow W\gamma + X$  to derive

$$\Leftrightarrow 0.92 < \tilde{\kappa}_\gamma < 0.92 \quad \Leftrightarrow 0.31 < \tilde{\lambda}_\gamma < 0.30$$

from the  $p_t^\gamma$  spectrum

- DELPHI used  $e^+e^- \rightarrow W^+W^-$  and  $e^+e^- \rightarrow e\bar{\nu}W$  events to derive

$$\tilde{\kappa}_\gamma = 0.11_{-0.88}^{+0.71} \pm 0.09 \quad \tilde{\lambda}_\gamma = 0.19_{-0.41}^{+0.28} \pm 0.11$$

(based on small 161-172 GeV data sample)

- OPAL verified imaginary components of off-diagonal single-W spin density matrix consistent with zero, but no explicit limit on  $\mathcal{CP}$  couplings derived

Remarks:

- More stringent limits possible from present LEP II data
- Many experimentalists regard such limits are artificial, since they require setting other anomalous couplings to zero
- Indirect neutron EDM limits on  $\tilde{\kappa}_\gamma$  and  $\tilde{\lambda}_\gamma$  encourage confidence (complacency?) in considering only CP conserving couplings

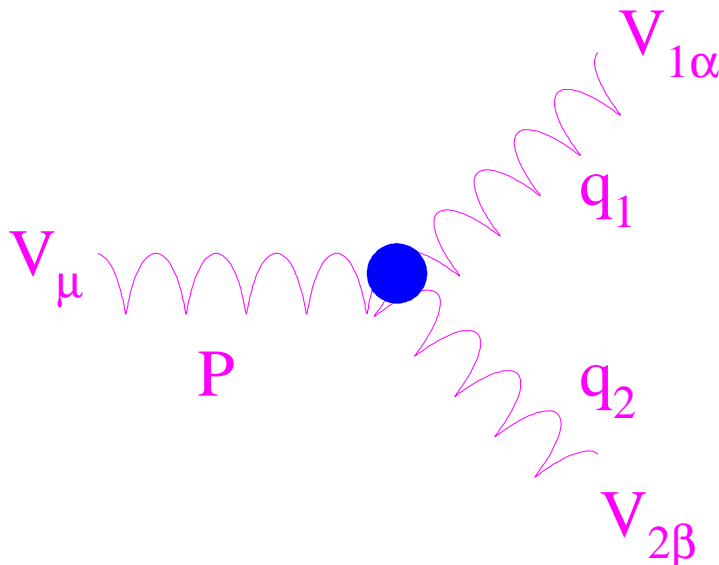
# Other Gauge Boson Couplings ( $\mathcal{CP}$ )

---

## Anomalous $ZZ\gamma$ , $Z\gamma\gamma$ Couplings

- Couplings vanish at tree-level in SM
- Bose symmetry / gauge invariance forbid non-zero values when all bosons on mass shell
- Parametrization of  $Z\gamma V$  vertex function: ( $V \equiv Z\gamma$ )

$$\begin{aligned}
 \Gamma_{Z\gamma V}^{\alpha\beta\mu}(q_1, q_2, P) &\equiv \frac{P^2 \Leftrightarrow m_V^2}{m_Z^2} \times \\
 & \left[ h_1^V (q_2^\mu g^{\alpha\beta} \Leftrightarrow q_2^\alpha g^{\mu\beta}) + \frac{h_2}{m_Z^2} P^\alpha (P \cdot q_2 g^{\mu\beta} \Leftrightarrow q_2^\mu P^\beta) \right. \\
 & \left. + h_3^V \epsilon^{\mu\alpha\beta\rho} q_{2\rho} + \frac{h_4^V}{m_Z^2} P^\alpha \epsilon^{\mu\beta\rho\sigma} P_\rho q_{2\sigma} \right]
 \end{aligned}$$



# Other Gauge Boson Couplings (CP)

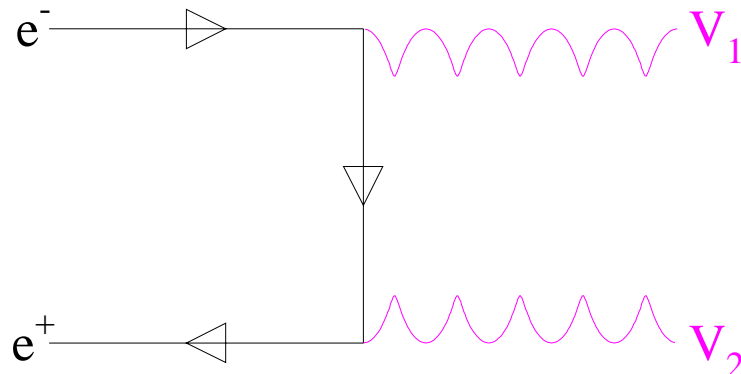
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Anomalous  $Z\gamma V$  couplings:

- CP-odd couplings:  $h_1^V, h_2^V$
- CP-even couplings:  $h_3^V, h_4^V$
- The  $h_i^V$  couplings are really form factors:

$$h_i^v(P^2) \equiv \frac{h_{i0}^V}{\left(1 + \frac{P^2}{\Lambda_V^2}\right)^{n_i^V}}$$

- Unitarity requires  $n_1^V, n_3^V, \geq 3, \quad n_2^V, n_4^V \geq 4$
- Signatures at  $e^+e^-$  collider:  $\gamma\gamma, Z\gamma, ZZ$  production
- Standard Model background:



# Other Gauge Boson Couplings (CP)

---

Example from L3:

Probe all eight  $h_i^V$  couplings via  $Z\gamma$  production

- Look at final state  $q\bar{q}\gamma$  (2 jets + hard photon)
- Look at final state  $\nu\bar{\nu}\gamma$  (single hard photon)
- Couplings determined by matrix element reweighting of Monte Carlo events to match data
- Five kinematic variables used in  $q\bar{q}\gamma$  analysis  
( $E_\gamma, \theta_\gamma, \phi_\gamma, \theta_q^*, \phi_q^*$ )
- The three photon variables used in  $\nu\bar{\nu}\gamma$  analysis

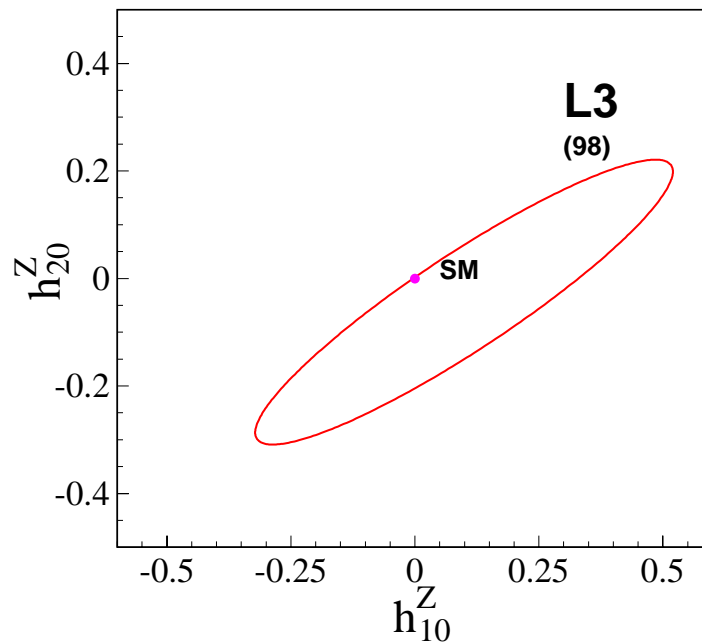
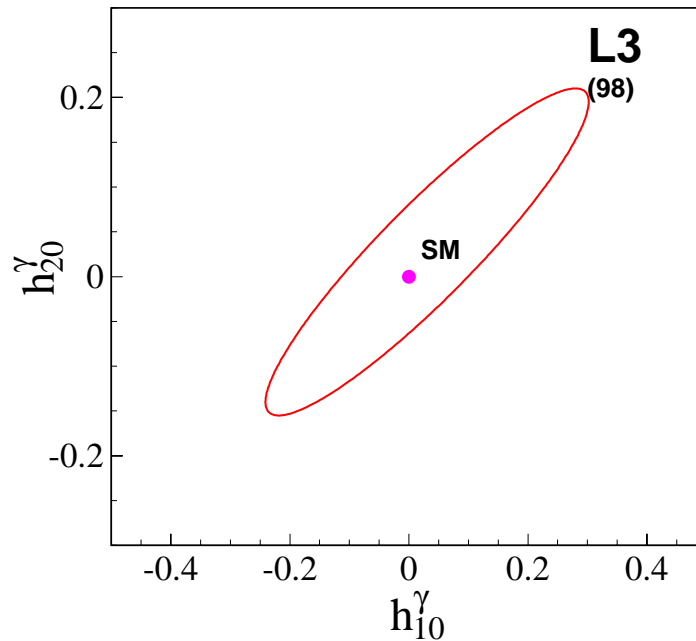
Preliminary Results:

95% CL Limits
$\Leftrightarrow 0.09 < h_1^Z < 0.20$
$\Leftrightarrow 0.12 < h_2^Z < 0.06$
$\Leftrightarrow 0.16 < h_3^Z < 0.15$
$\Leftrightarrow 0.09 < h_4^Z < 0.10$
$\Leftrightarrow 0.09 < h_1^\gamma < 0.08$
$\Leftrightarrow 0.05 < h_2^\gamma < 0.07$
$\Leftrightarrow 0.09 < h_3^\gamma < 0.07$
$\Leftrightarrow 0.05 < h_4^\gamma < 0.06$

# Other Gauge Boson Couplings ( $\mathcal{CP}$ )

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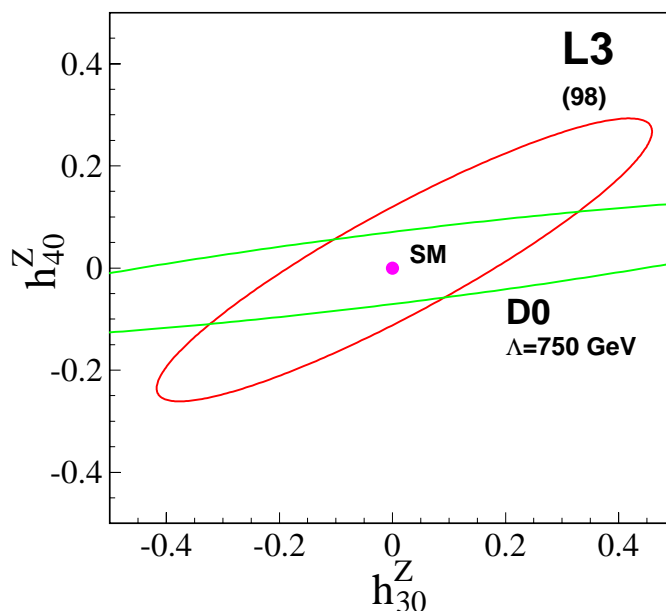
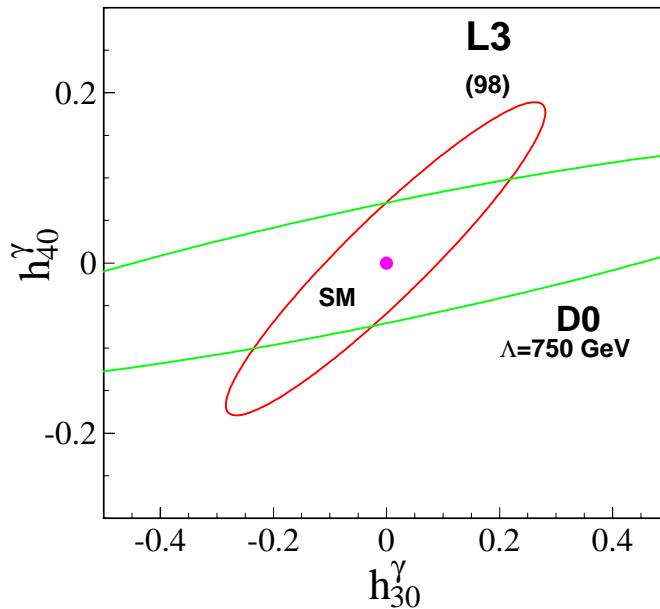
2-Dimensional contour limits on  $\mathcal{CP}$  couplings: (preliminary)



# Other Gauge Boson Couplings ( $\mathcal{CP}$ )

---

2-Dimensional contour limits on  $\mathcal{CP}$  conserving couplings from D0 and L3 (L3 limits preliminary)





# Other Gauge Boson Couplings ( $\mathcal{CP}$ )

---

## Anomalous $ZZ\gamma$ , $ZZZ$ Couplings ( $ZZ$ production)

- Bose symmetry permits two couplings
- Parametrization of  $ZZV$  vertex function:

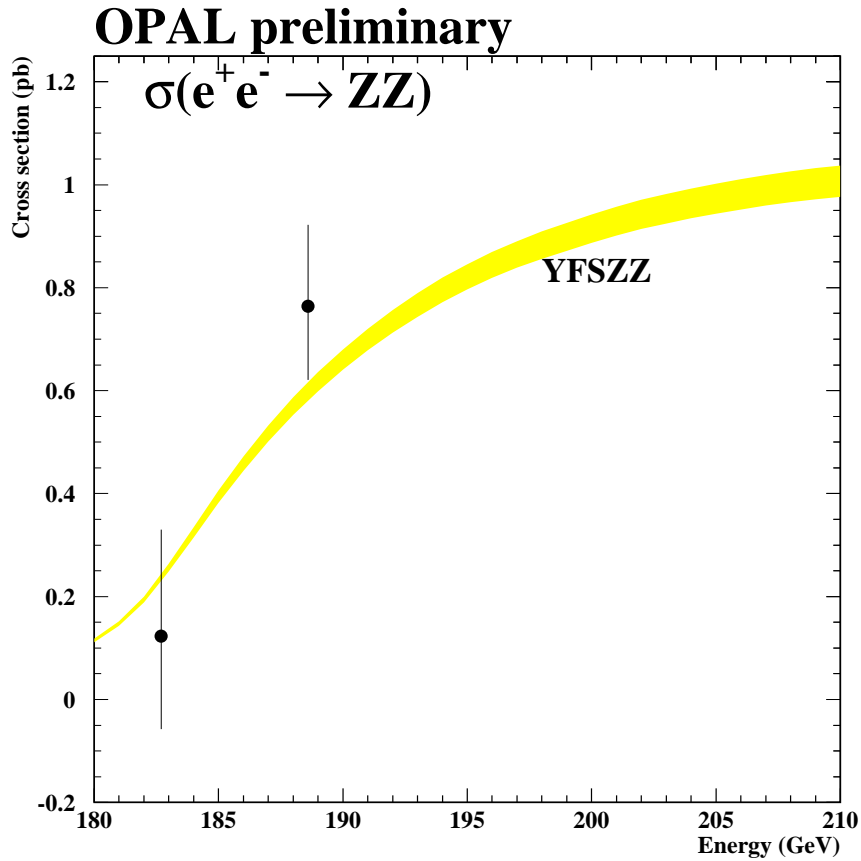
$$i \Gamma_{ZZV}^{\alpha\beta\mu}(q_1, q_2, P) \equiv \frac{P^2 - m_V^2}{m_Z^2} \times [i f_4^{ZZV} (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) + i f_5^{ZZV} \epsilon^{\mu\alpha\beta\rho} (q_1 - q_2)_\rho]$$

- CP-Violating:  $f_4^{ZZV}$
- CP-Conserving:  $f_5^{ZZV}$

Preliminary analysis from OPAL:

- Examines total cross section and  $d\sigma/d\cos\theta_Z$  distribution
- Probes both real & imaginary  $f_i^{ZZV}$  components
- Severely limited by  $ZZ$  statistics ( $\sqrt{s} \leq 189$  GeV)

# Other Gauge Boson Couplings ( $\mathcal{CP}$ )



Resulting limits on  $f_i^{ZZV}$ :

95% CL Limits	
$\Leftrightarrow 2.0 < \Re\{f_4^{ZZZ}\} < 2.0$	
$\Leftrightarrow 2.0 < \Im\{f_4^{ZZZ}\} < 1.9$	
$\Leftrightarrow 5.1 < \Re\{f_5^{ZZZ}\} < 3.6$	
$\Leftrightarrow 5.2 < \Im\{f_5^{ZZZ}\} < 5.4$	
$\Leftrightarrow 1.2 < \Re\{f_4^{ZZ\gamma}\} < 1.2$	
$\Leftrightarrow 1.2 < \Im\{f_4^{ZZ\gamma}\} < 1.2$	
$\Leftrightarrow 3.2 < \Re\{f_5^{ZZ\gamma}\} < 3.0$	
$\Leftrightarrow 3.2 < \Im\{f_5^{ZZ\gamma}\} < 3.2$	

# Gauge Boson Couplings

---

## Summary

- Anomalous TGC and QGC probed most directly at  $e^+e^-$  and hadron colliders
- Best sensitivity: High energy  $e^+e^-$ ,  $e^-e^-$ ,  $\gamma\gamma$ ,  $e^-\gamma$  colliders
- LEP I and low-energy measurements suggest anomalous couplings will not be observed soon

Especially  $\mathcal{O}P$   $WW\gamma$  couplings

- LEP II measurements confirm analysis techniques, but do not challenge Standard Model
- Optimistic perspective:

If non-zero anomalous couplings measured,  
Then dramatic New Physics imminent

# Top Quark $\mathcal{CP}$ Couplings

---

Standard Model: (see Nir lectures):

Expect  $\mathcal{CP}$  in only charged-current quark interactions

Cabibbo-Kobayashi-Maskawa Matrix:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Standard parametrization:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{KM}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{KM}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{KM}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{KM}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{KM}}} & c_{23}c_{13} \end{pmatrix}$$

$\Rightarrow$  Explicit  $\mathcal{CP}$  phase in  $V_{td}, V_{ts}$  elements

$\Rightarrow$  Easy to see CP violation in top decay?

Nope...

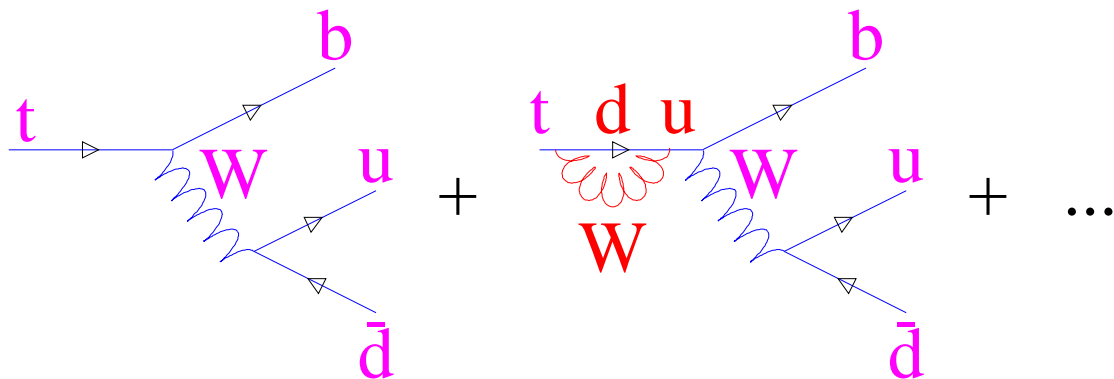
# Top Quark $\mathcal{CP}$ Couplings

---

Obstacles:

- $\text{Max}(|V_{td}|, |V_{ts}|) < \approx 0.01$
- $t$  decays ( $\tau \approx 1.8 \text{ GeV}$ ) before neutral  $t\bar{c}$ ,  $t\bar{u}$  states can form  
 $\implies$  No time for mixing effects
- Any identifiable exclusive decay is “rare”
- Interference possible between tree + loop diagrams

Example:



(exploit imaginary term from on-shell  $W$  in tree diagram)

# Top Quark $\mathcal{CP}$ Couplings

---

- Can look for Partial Rate Asymmetry (PRA):

$$\text{PRA} \equiv \frac{\Gamma(t \rightarrow X_i) - \Gamma(\bar{t} \rightarrow \bar{X}_i)}{\Gamma(t \rightarrow X_i) + \Gamma(\bar{t} \rightarrow \bar{X}_i)}$$

for some  $X_i$  final state

- But loop contribution suppressed by off-diagonal CKM elements and by GIM mechanism

- Result:  $(\text{PRA})^2 \times B(t \rightarrow X_i) < \approx 10^{-15}$

$\implies \mathcal{CP}$  not visible in S.M. top decay / production

$\implies$  If  $\mathcal{CP}$  seen in top sector, new physics at work

# Top Quark $\mathcal{CP}$ Couplings

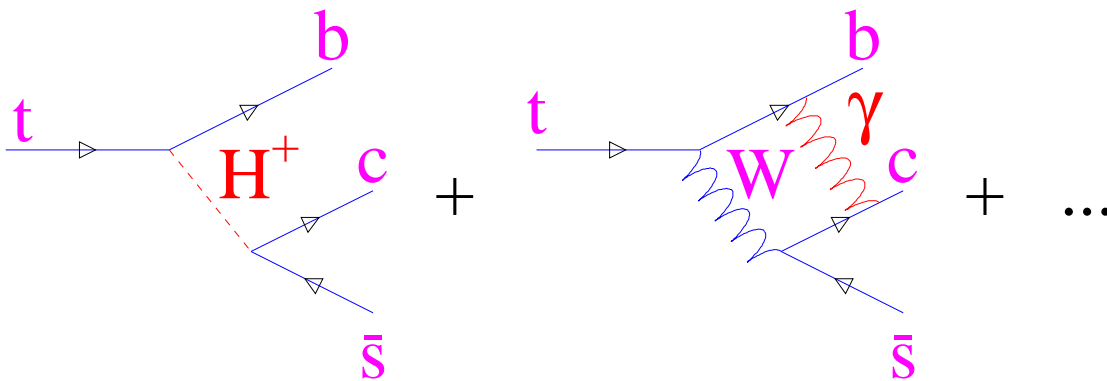
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## Beyond the Standard Model

$\mathcal{CP}$  in the top sector:

- Charged current processes:
  - Add charged Higgs to decay route

Example:



Remarks:

- Non-SM Higgs gives large effects because  $m_t$  large
- Fast  $t$  decay has important advantage:
  - Spin of  $t$  undiluted by hadronization
  - $\implies$  Polarimetry feasible via  $V \Leftrightarrow A$  coupling in decay

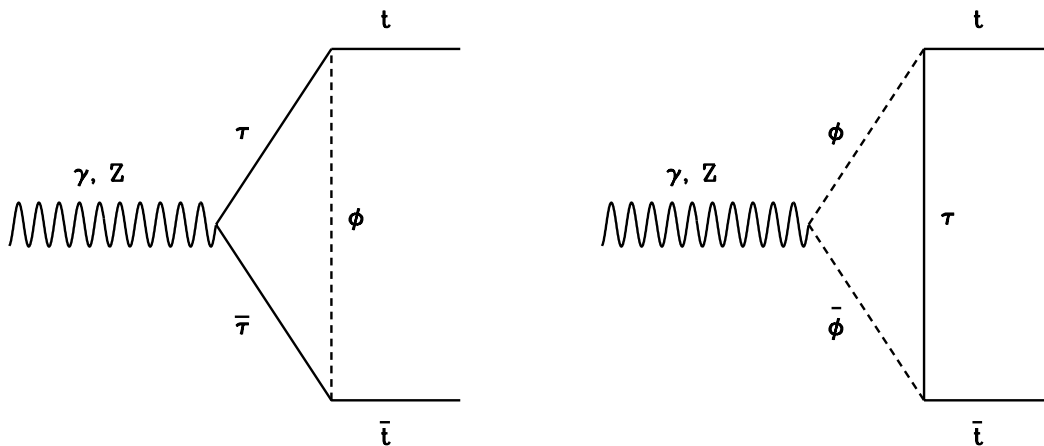
# Top Quark $\mathcal{CP}$ Couplings

---

$\mathcal{CP}$  in the top sector:

- Neutral current processes:
  - Electric dipole moment (SM prediction  $\approx 10^{-30}$  e-cm)
  - Weak and Chromo analogs
  - Much larger moments can arise from
    - \*  $\mathcal{CP}$  in neutral Higgs sector (multi-Higgs models)
    - \*  $\mathcal{CP}$  in charged Higgs sector ( “ “ “ )
    - \*  $\mathcal{CP}$  in MSSM:  $\tilde{t}_L, \tilde{t}_R$  mixing
    - \* Scalar leptoquarks

Example:



(figure from Poulose & Rindani)



# Top Quark $\mathcal{CP}$ Couplings

---

Comparing single  $t$  decay to single  $\bar{t}$  decay...  
(correlations in  $t\bar{t}$  production discussed below)

PRA enhanced in many non-SM scenarios, but still tiny

One can go beyond partial decay widths:

- Energy asymmetry in  $t \rightarrow b\bar{\ell}\nu$  vs  $\bar{t} \rightarrow \bar{b}\ell\bar{\nu}$

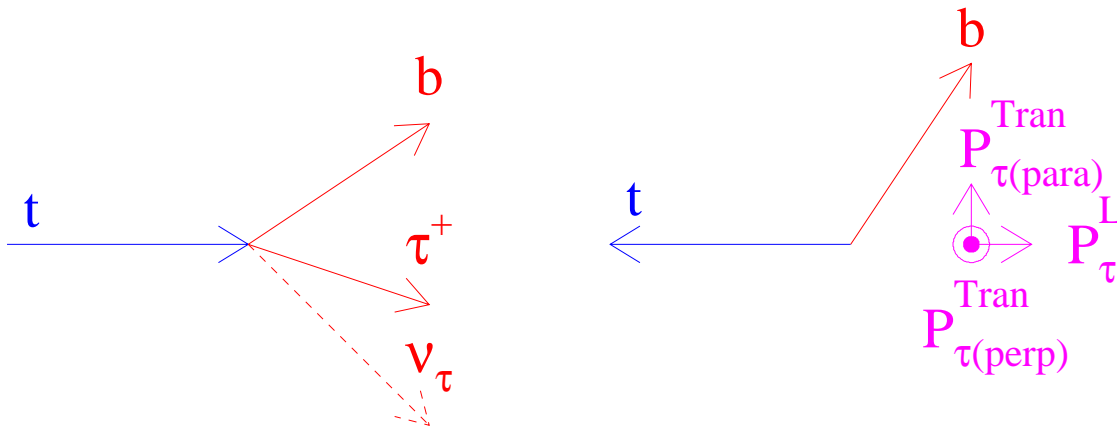
$$A_E \equiv \frac{\langle E_\ell \rangle \Leftrightarrow \langle E_{\bar{\ell}} \rangle}{\langle E_\ell \rangle + \langle E_{\bar{\ell}} \rangle}$$

- $\ell = e$  (Schmidt & Peskin, 1992)
- $\ell = \tau$  (Atwood *et al.*, 1993)  
(enhanced over  $\ell = e$  if Higgs-induced)

- T-odd  $W$  polarization asymmetry in  $t \rightarrow bW$  ( $t$  vs  $\bar{t}$ )  
(Ma and Brandenburg, 1992)
- Partially Integrated Rate Asymmetry (PIRA)  
(partial phase space integration, Atwood *et al.*, 1993)
- Tau transverse polarization asymmetry in  $t \rightarrow b\bar{\tau}\nu$   
(Atwood, Eilam & Soni, 1993)

# Top Quark $\mathcal{CP}$ Couplings

Example – Tau transverse polarization in  $t \rightarrow b\bar{\tau}\nu_\tau$   
 (Atwood, Eilam & Soni, 1993)



Lab frame

$\tau$  frame

Look for difference between  $\langle P_{\tau(\text{para})}^{\text{Tran}} \rangle$ ,  $\langle P_{\tau(\text{perp})}^{\text{Tran}} \rangle$   
 for  $t$  vs  $\bar{t}$  decays

Quoted precision on asymmetry for 500 GeV NLC:

$$3\sigma \text{ for } A_{pol} \approx 6\%$$

For charged Higgs of  $m_{H^+} = 400$  GeV,  
 “might” expect  $A_{pol} \approx 5\text{--}20\%$

Can reach  $\approx 50\%$  for  $m_{H^+} = 200$  GeV

# Top Quark $\mathcal{CP}$ Couplings

---

$\mathcal{CP}$  easier to see in  $t\bar{t}$  production

Especially at lepton, photon colliders

As with gauge bosons,  $\mathcal{CP}$  couplings fall under more general category of anomalous couplings

$\implies$  Parametrized as multipole moments

Electroweak neutral current: (notation of Frey *et al.*, 1996)

$$\begin{aligned} \mu_{t\bar{t}(\gamma/Z)} &= e\bar{t} \left\{ \gamma^\mu [Q_V^{\gamma,Z} F_{1V}^{\gamma,Z} + Q_A^{\gamma,Z} F_{1A}^{\gamma,Z} \gamma_5] \right. \\ &\quad \left. + \frac{ie}{2m_t} \sigma^{\mu\nu} k_\nu [Q_V^{\gamma,Z} F_{2V}^{\gamma,Z} + Q_A^{\gamma,Z} F_{2A}^{\gamma,Z} \gamma_5] \right\} t \end{aligned}$$

where in the S.M.  $F_{1V}^\gamma = F_{2V}^Z = F_{2A}^Z = 1$   
and all other form factors are zero

Normalization:

$$\begin{aligned} Q_V^\gamma &= Q_A^\gamma = \frac{2}{3} \\ Q_V^Z &= (1 \Leftrightarrow \frac{8}{3} \sin^2 \theta_W) / (4 \sin \theta_W \cos \theta_W) \\ Q_A^Z &= \Leftrightarrow 1 / (4 \sin \theta_W \cos \theta_W) \end{aligned}$$

$F_{2V}^{\gamma,Z}, F_{2A}^{\gamma,Z}$  = E.W. magnetic, electric dipole form factors

# Top Quark $\mathcal{CP}$ Couplings

---

Weak charged current: (neglecting flavor violation)

$$\begin{aligned}
 \mathcal{L}_{tbW}^{\mu} &= \frac{g}{\sqrt{2}} \bar{b} \{ \gamma^{\mu} [P_L F_{1L}^W + P_R F_{1R}^W] \\
 &\quad + \frac{i}{2m_t} \sigma^{\mu\nu} k_{\nu} [P_L F_{2L}^W + P_R F_{2R}^W] \} t
 \end{aligned}$$

where  $P_L, P_R$  = left, right projection operators  
 and where in the Standard Model  $F_{1L}^W = 1$  and all other form factors are zero. ( $F_{1R}$  describes  $V+A$  coupling)

SU(3) current: (notation of Rizzo, 1996)

$$\mathcal{L}_{ttg} = g_s \bar{t} T_{\alpha} (\gamma^{\mu} + \frac{i}{2m_t} \sigma^{\mu\nu} (\kappa - i \tilde{\kappa} \gamma_5) q_{\nu}) t G_{\alpha}^{\mu}$$

where  $g_s$  = strong coupling constant,

$T_{\alpha}$  = color generators,

$G_{\alpha}^{\mu}$  = gluon field

$\kappa, \tilde{\kappa}$  = chromo-magnetic, chromo-electric dipole moments  
 (form factors in general)

# Top Quark $\mathcal{CP}$ Couplings

---

Looking for  $\mathcal{CP}$  in top production (a sampling)  
(some techniques automatically probe  $\mathcal{CP}$  in top decay too)

- Top polarization asymmetry in  $e^+e^- \rightarrow t\bar{t}$  and  $gg \rightarrow t\bar{t}$   
(from  $W$  kinematics, Kane, Ladinsky, & Yuan, 1992)
- Azimuthal  $W$  correlations in  $e^+e^- \rightarrow t\bar{t}$   
(Kane, Ladinsky, & Yuan, 1992)
- Energy asymmetry  $A_E$  (Schmidt & Peskin, 1992)
- Manifestly CP-odd observables in  $e^+e^- \rightarrow t\bar{t}$

Correlation Tensor:

$$\hat{T}_{ij} \equiv (\hat{q}_- \Leftrightarrow \hat{q}_+)_i \frac{(\hat{q}_- \times \hat{q}_+)_j}{|\hat{q}_- \times \hat{q}_+|} + (i \Leftrightarrow j)$$

where  $\hat{q}_-, \hat{q}_+ =$  unit vectors along, e.g.,  $b, \bar{b}$  directions  
(Bernreuther, Schröder & Pham, 1992)

# Top Quark $\mathcal{CP}$ Couplings

---

- Optimal observables in  $e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}\ell\bar{\ell}'\nu_\ell\bar{\nu}_{\ell'}$   
Analogous to TGC observables discussed earlier, but with terms projecting out  $\Re\{d_t^{\gamma,Z}\}$ ,  $\Im\{d_t^{\gamma,Z}\}$  where

$$d_t^{\gamma,Z} = \text{Electric, weak electric dipole moment}$$

$$\equiv \frac{e}{2m_t} F_{2A}^{\gamma,Z}$$

(Atwood & Soni, 1992)

- Decay lepton up/down asymmetry in  $gg \rightarrow t\bar{t} \rightarrow b\bar{\ell}\nu X$   
(Grzadkowski & Gunion, 1992)  
(Up/down refers to  $\vec{p}_e$  in  $W$  rest frame w.r.t.  $t \leftrightarrow b$  plane)
- Manifestly CP-odd observables in  $e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}X\bar{X}$   
with polarized beams (Cuypers & Rindani, 1994)

$$O_1 \equiv (\vec{p}_b \times \vec{p}_{\bar{b}}) \cdot \hat{p}_{e^+}$$

$$O_2 \equiv (\vec{p}_b + \vec{p}_{\bar{b}}) \cdot \hat{p}_{e^+}$$

- Multi-distribution fitting with matrix element reweighting  
(reweighting by one or more form factor parameters)  
(Frey *et al.*, 1996)

# Top Quark $\mathcal{CP}$ Couplings

---

More exotic possibilities: (sampling)

- CP-odd observables in  $e^+e^- \rightarrow t\bar{t}H$ ,  $e^+e^- \rightarrow t\bar{t}Z$   
– need hefty  $\sqrt{s}$  for this! (Bar-Shalom *et al.*, 1996;  
Bar-Shalom, Atwood & Soni, 1998)

$$O \equiv \vec{p}_{e^-} \cdot (\vec{p}_t \times \vec{p}_{\bar{t}})$$

- Charge and forward/backward asymmetries of  $\ell^+\ell'^-$  in  
 $\gamma\gamma \rightarrow t\bar{t} \rightarrow b\bar{b} \ell^+\ell'^- \nu_\ell \bar{\nu}_{\ell'}$  (Poulose & Rindani, 1998)

# Top Quark $\mathcal{CP}$ Couplings

---

What about chromo-magnetic/electric dipole moments?

- Gluon energy spectrum in  $e^+e^- \rightarrow t\bar{t}g$  (NLC)  
(Rizzo, 1996)
- Top quark polarization & polarization asymmetry in  $e^+e^- \rightarrow t\bar{t}$  (NLC)  
(Rindani & Tung, 1999)
- $M_{t\bar{t}}, p_t^{t,\bar{t}}$  distributions in  $gg \rightarrow t\bar{t}$   
(Sensitive to anomalous couplings at LHC,  
sensitive to low-scale gravity theory at Tevatron Run II)  
(Review by Rizzo, 1999)



# Top Quark $\mathcal{CP}$ Couplings

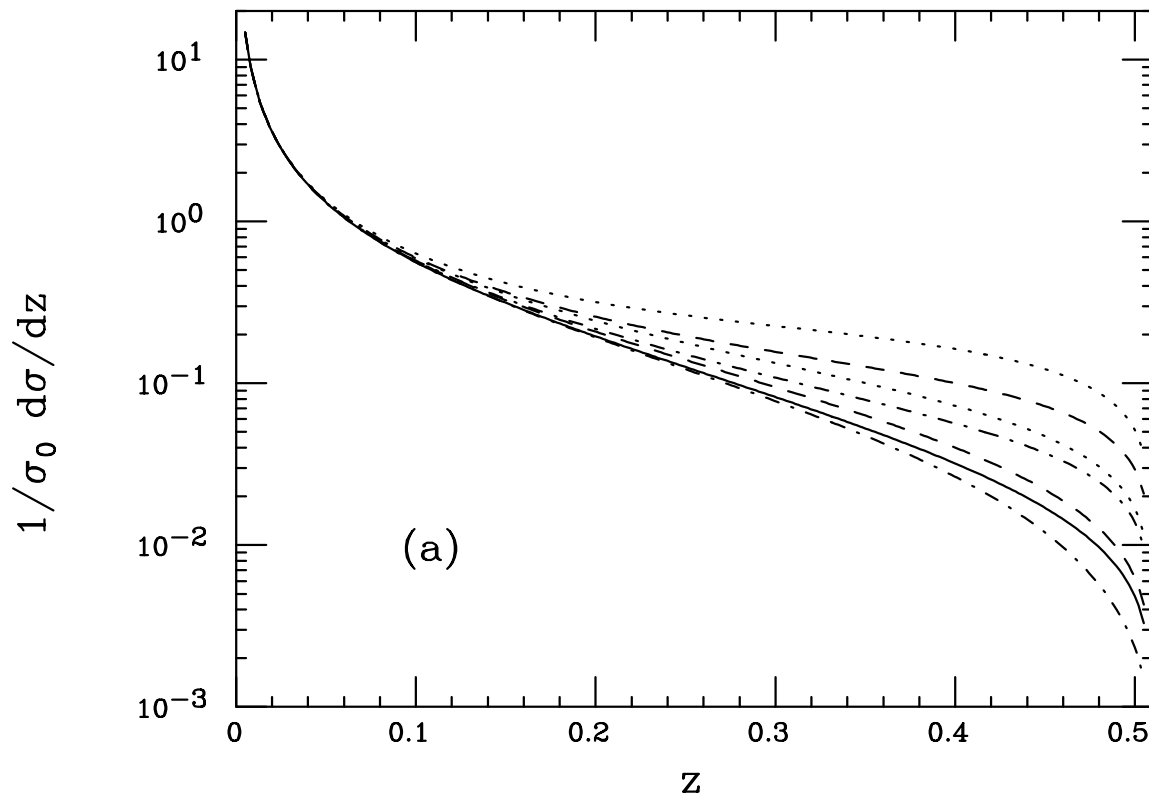
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Example (Rizzo, 1996)

Chromo-magnetic( $\kappa$ ), Chromo-electric( $\tilde{\kappa}$ ) dipole moments

Measure gluon energy distributions at a  $e^+e^-$  linear collider

$E_g$  shapes at 500 GeV NLC for  $\kappa = 0, \pm 1, \pm 2, \pm 3$

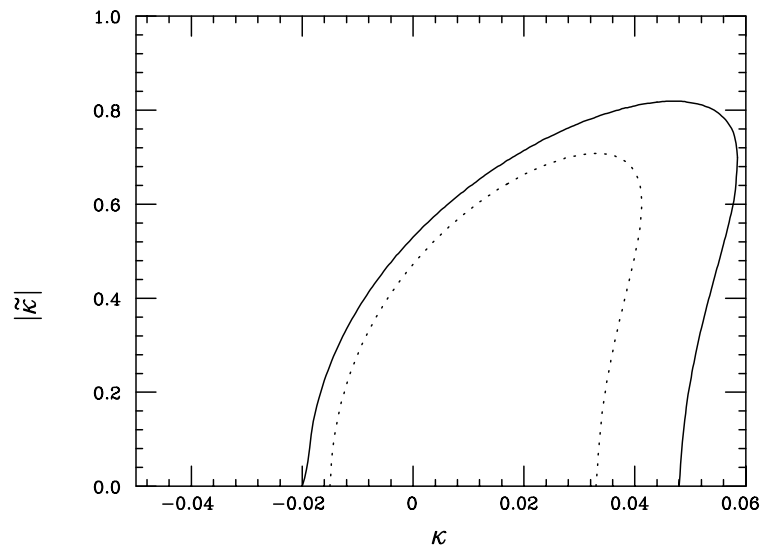


where  $z \equiv E_g/E_{\text{beam}}$

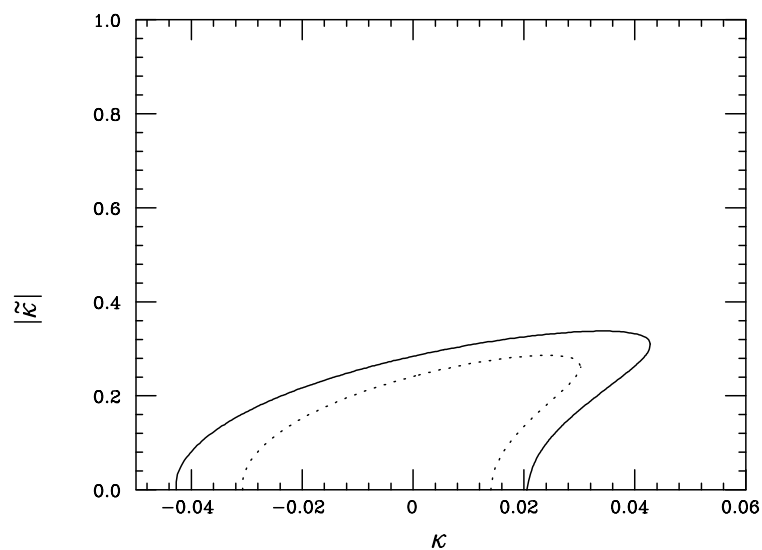
# Top Quark $\mathcal{CP}$ Couplings

---

Resulting limits at a 500 GeV NLC ( $\int \mathcal{L} dt = 50, 100 \text{ fb}^{-1}$ )



and at a 1 TeV NLC ( $\int \mathcal{L} dt = 100, 200 \text{ fb}^{-1}$ )



# Top Quark $\mathcal{CP}$ Couplings

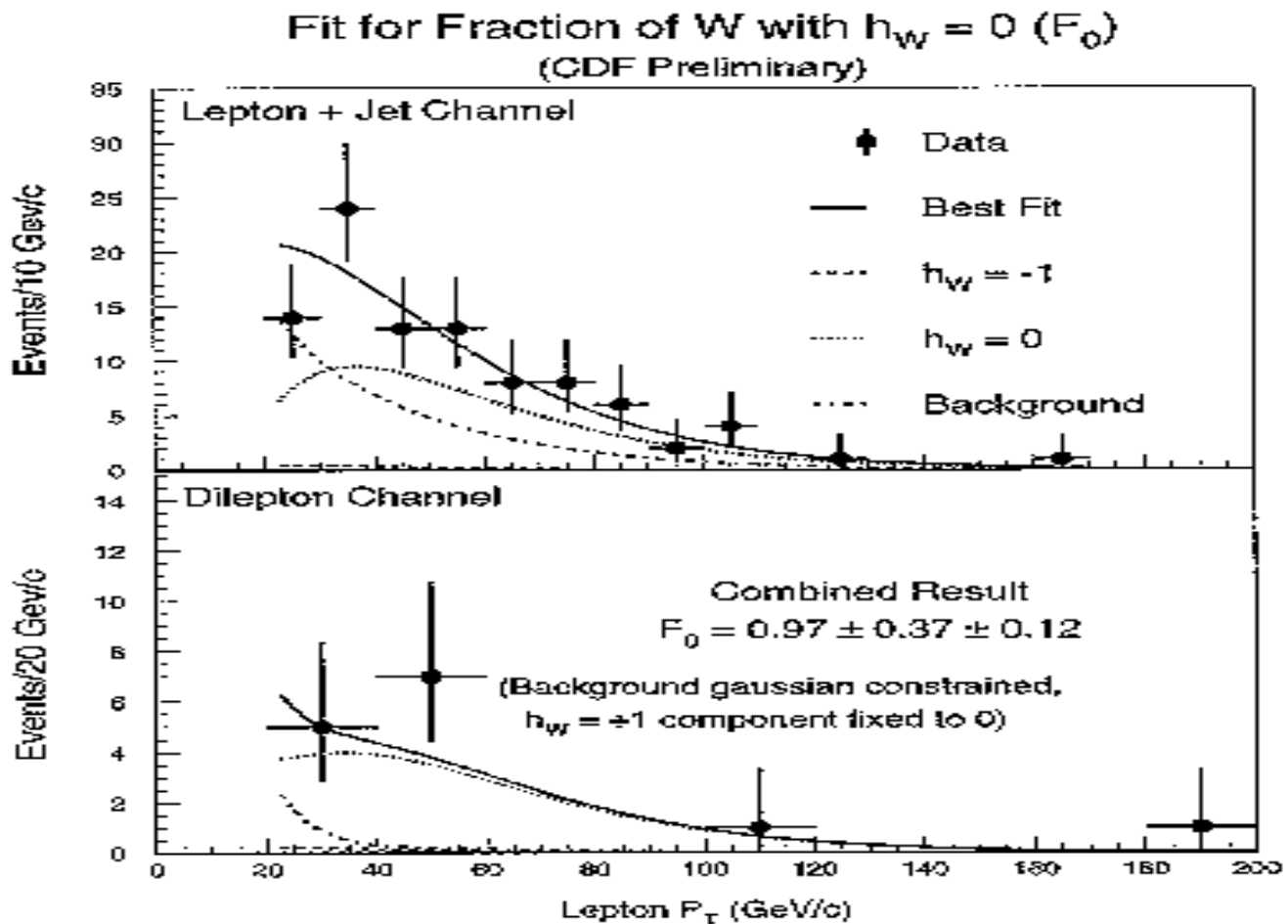
Can any Tevatron Run I data be used?

Not really...too few events

Interesting proof-of-principle analyses from CDF & D0:

- $W$  polarization from top decay
- Spin-spin correlations from  $t$  and  $\bar{t}$  decay products

Example – CDF result on  $W$  long. polarization (SM = 70%)



# Tau Lepton $\mathcal{CP}$ Couplings

---

Standard Model: (see Nir lectures):

If neutrinos massless, then no  $\mathcal{CP}$

But much evidence for  $\nu$  oscillation

$\implies m_{\nu_i} \neq 0$

$\implies$  Possible (likely)  $\mathcal{CP}$  phase in leptonic CKM matrix

Can we therefore detect “S.M.”  $\mathcal{CP}$  in  $\tau$  decay?

Probably not

(cannot use the  $\Im\{W\leftrightarrow\text{tree}\} \times \Re\{\text{loop}\}$  trick because intermediate  $W$  far off resonance)

In principle, one can measure  $\delta_{\text{KM}}^l$  from high-statistics  $\nu$  oscillation asymmetries, but not anytime soon...

$\implies$  As for top decay, any measured  $\mathcal{CP}$  means New Physics

# Tau Lepton $\mathcal{CP}$ Couplings

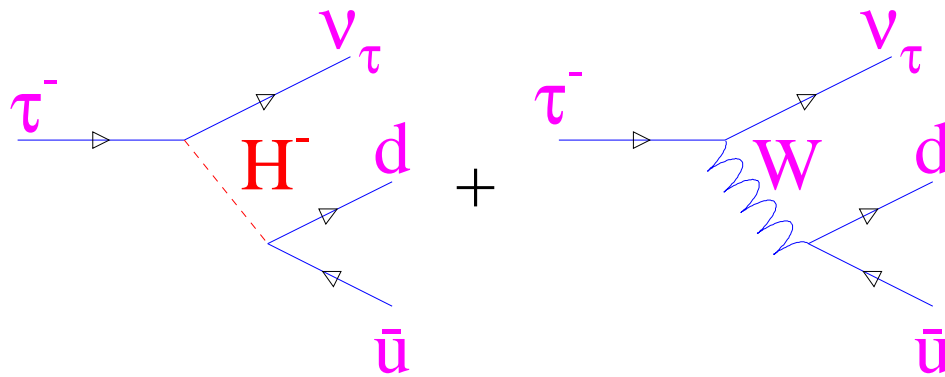
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Beyond the Standard Model

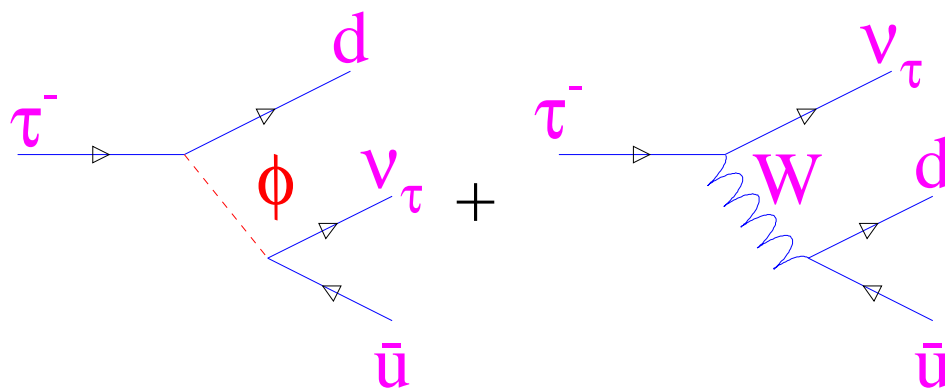
$\mathcal{CP}$  in the tau sector:

- Charged current processes:

- As for top quark, add charged Higgs to decay route:  
(Tsai, 1989)



- Or add a scalar leptoquark:  
(Choi, Hagiwara & Tanabashi, 1994)



# Tau Lepton $\mathcal{CP}$ Couplings

---

- Can enhance the  $\mathcal{CP}$  interference with final state containing possible non-zero CP-conserving phase
  - “Stage Two Spin Correlation” in  $\tau^- \rightarrow \rho^- \rightarrow \pi^- \pi^0 \nu_\tau$   
(interference of two allowed  $\rho$  helicity states)\*  
(Nelson, 1994)
  - Double resonance in  $\tau^- \rightarrow (3\pi)^- \nu_\tau$   
(interference of  $a_1$  ( $J^P=1^+$ ) and  $\pi'$  ( $J^P=0^-$ ) states)  
(Choi, Hagiwara & Tanabashi, 1994)
  - Double resonance in  $\tau^- \rightarrow (K\pi)^- \nu_\tau$   
(interference of  $K^*(892)$  ( $J^P=1^-$ ) and  $K_0^*(1430)$  ( $J^P=0^+$ ) states)  
(Kühn & Mirkes, 1996)

Remark:

Cabibbo suppression of strange channels offset by mass-dependent coupling for multi-Higgs models

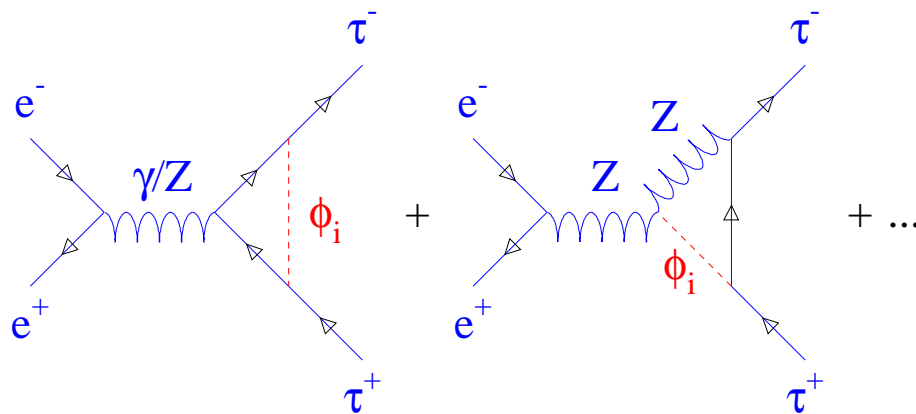
\*Requires CPT violation – shown by Tsai, 1996B

# Tau Lepton $\mathcal{CP}$ Couplings

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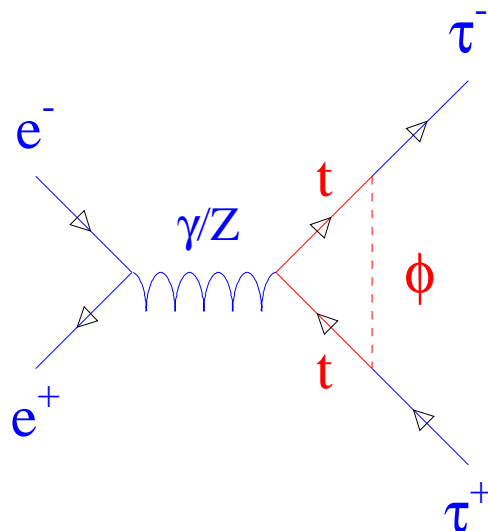
$\mathcal{CP}$  in the tau sector:

- Neutral current processes:
  - Two-Higgs-Doublet models with explicit  $\mathcal{CP}$  phase (Bernreuther, Schröder & Pham)



- Tau-Stau coupling or scalar leptoquarks (Bernreuther, Brandenburg & Overmann, 1997)

Example – Leptoquark loop:



# Tau Lepton $\mathcal{CP}$ Couplings

---

Comparing single  $\tau^-$  decay to single  $\tau^+$  decay...  
(correlations in  $\tau^-\tau^+$  production discussed below)

- Forward-backward asymmetries and optimal observable in  $\tau^- \rightarrow \pi^-\pi^+\pi^-$   
(Choi, Hagiwara & Tanabashi, 1994)
- Partial rate asymmetry of  $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ ,  $\tau^- \rightarrow K \Leftrightarrow \pi^0\nu_\tau$   
(Tsai, 1995)
- $\mu$  polarization in  $\tau$  decay with polarized  $e^+e^-$  beams  
(Tsai, 1995)

Look at

$$(\vec{w}_{\text{beam}} \times \vec{p}_\mu) \cdot \vec{w}_\mu$$

where  $\vec{w}_{\text{beam}/\mu}$  = polarization of beam / muon

Not easy to measure!

- Similar T-odd terms in  $\tau \rightarrow \nu_\tau + (\geq 2\text{hadrons})$   
with polarized  $e^+e^-$  beams (requires  $\geq 2$  final spin states)  
(Tsai, 1996A)



# Tau Lepton $\mathcal{CP}$ Couplings

---

- CP-odd kinematic asymmetries in  $\tau^- \rightarrow (K\pi)^- \nu_\tau$   
(with and without full  $\tau$  kinematic reconstruction)  
(Kühn & Mirkes, 1996)
- Enhance  $\mathcal{CP}$  signal in  $\tau^- \rightarrow (3\pi)^- \nu_\tau$   
with  $\tau$  polarization (Tsai, 1996A/1998)

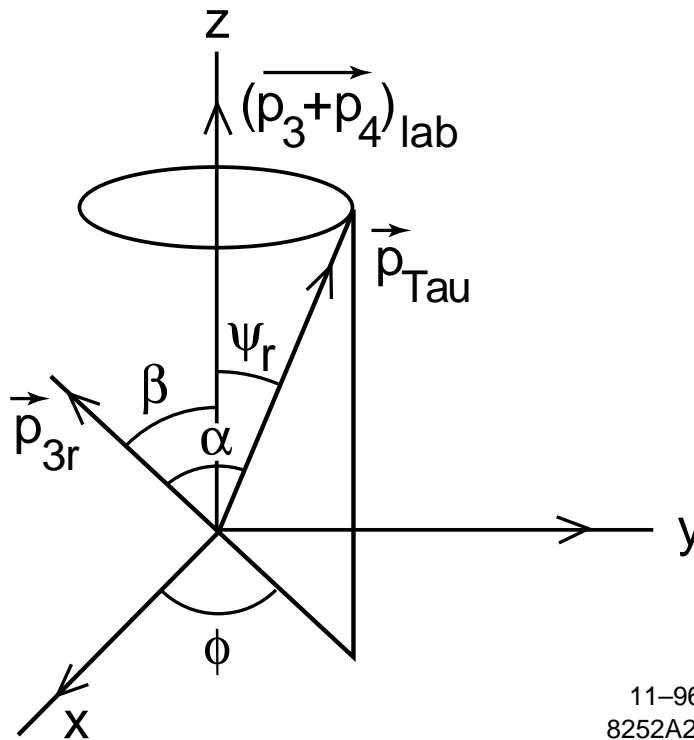
# Tau Lepton $\mathcal{CP}$ Couplings

---

Example –  $\tau^- \rightarrow (K\pi)^- \nu_\tau$

(Tsai, 1996B; based on Kühn & Mirkes, 1996)

Kinematics in  $K\text{--}\pi$  rest frame:



where  $\vec{p}_3, \vec{p}_4 = K, \pi$  momenta and  $\psi_r$  is known even if  $\vec{p}_{\text{Tau}}$  not reconstructed

Define following observable:

$$O_{\tau^-} \equiv \cos \beta \cos \psi_r$$

Non-KM  $\mathcal{CP}$  indicated by differing  $O_{\tau^-}, O_{\tau^+}$  distributions

# Tau Lepton $\mathcal{CP}$ Couplings

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Observable  $O_\tau$  used in search for  $\mathcal{CP}$  by [CLEO] (1998)

( $e^+e^-$  collisions at  $\sqrt{s} = 10.6$  GeV,  $4.4 \times 10^6$   $\tau^-\tau^+$  events)

- Examined  $\tau^\pm \rightarrow K_s^0 \pi^\pm \nu_\tau$  events with  $K_s^0 \rightarrow \pi^+\pi^-$
- Defined following asymmetry in bins of  $O_\tau$ :

$$A \equiv \frac{N^+(\cos \beta \cos \psi_r) \Leftrightarrow N^-(\cos \beta \cos \psi_r)}{N^+(\cos \beta \cos \psi_r) + N^-(\cos \beta \cos \psi_r)}$$

with  $N^\pm =$  Number of  $\tau^\pm$  decays in  $\cos \beta \cos \psi_r$  bin

- Ideally,

$$A \propto [\cos \beta \cos \psi_r] (g \sin \theta_{CP})$$

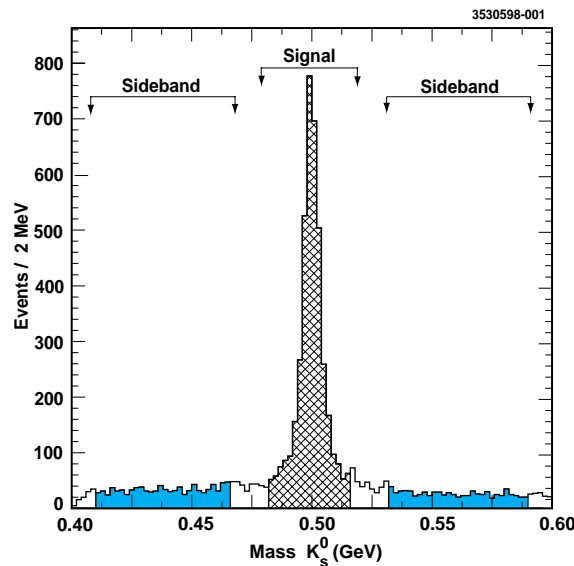
where  $g e^{i\theta_{CP}} =$  scalar/vector coupling strength ratio

(*e.g.*, from  $\tau-\nu_\tau-H^+$  vertex)

# Tau Lepton $\mathcal{CP}$ Couplings

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- In practice, detector displays “C asymmetry” in  $\pi^\pm$  reconstruction efficiencies
- To remove fake  $\mathcal{CP}$  due to detector imperfection, carry out sideband subtraction from  $K_s^0$  mass spectrum:



- Raw asymmetries for two bins of  $\cos \beta \cos \psi_r$ :

	$A_{observed}(\cos \beta \cos \psi < 0)$	$A_{observed}(\cos \beta \cos \psi > 0)$
Signal	$0.058 \pm 0.023$	$0.024 \pm 0.021$
Sideband	$0.049 \pm 0.030$	$0.034 \pm 0.033$

Result:

$g \sin \theta_{CP} < 1.7 \quad \text{at } 90\% \text{ C.L.}$

# Tau Lepton $\mathcal{CP}$ Couplings

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$\mathcal{CP}$  easier to see in  $\tau^+\tau^-$  production

Again, parametrize as multipole moment form factors

Focus on electromagnetic and neutral weak dipole moments:  
(parametrization & notation used by LEP experiments)

- Anomalous magnetic moment  $a_\tau^{\gamma,Z}$   
(dimensionless, CP-even; SM( $\gamma, Z$ ):  $O(10^{-3}), O(10^{-6})$ )
- Electric dipole moment  $d_\tau^{\gamma,Z}$   
(dimensional,  $\mathcal{CP}$ ; SM:  $O(10^{-37}$  e-cm))

Effective Lagrangian terms:

$$\mathcal{L}_{\tau\tau V}^{eff} = \sum_V \left[ -\frac{i}{2} d_\tau^V \bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau^{(v)} F_{\mu\nu} + \frac{1}{2} \frac{e a_\tau^V}{2 m_\tau} \bar{\tau} \sigma^{\mu\nu} \tau^{(v)} F_{\mu\nu} \right]$$

# Tau Lepton $\mathcal{CP}$ Couplings

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Looking for  $\mathcal{CP}$  in  $\tau^-\tau^+$  production (a sampling)  
 (some techniques automatically probe  $\mathcal{CP}$  in  $\tau$  decay too)

- Deviation from S.M. in  $Z \rightarrow \tau^+\tau^-$   
 (Bernreuther & Nachtmann, 1989)

- Manifestly CP-odd observables

Correlation Tensor:

$$\hat{T}_{ij} \equiv (\hat{q}_+ \Leftrightarrow \hat{q}_-)_i \frac{(\hat{q}_+ \times \hat{q}_-)_j}{|\hat{q}_+ \times \hat{q}_-|} + (i \Leftrightarrow j)$$

where  $\hat{q}_+, \hat{q}_- =$  unit vectors along  $\tau$  daughter momenta  
 (e.g.,  $\pi^+, e^-$  in  $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}_\tau e^- \bar{\nu}_e \nu_\tau$ )  
 (Bernreuther & Nachtmann, 1989)

- Optimal observables

(Atwood & Soni, 1992, see also review by Vermes, 1996)

$$O^{\Re} \equiv \frac{M_{CP}^{\Re}}{M_{SM}}; \quad O^{\Im} \equiv \frac{M_{CP}^{\Im}}{M_{SM}};$$

- Photon kinematics in  $e^+e^- \rightarrow \tau^+\tau^-\gamma$   
 (probes  $d_\tau^\gamma, a_\tau^\gamma$  at  $q^2 = 0$ )

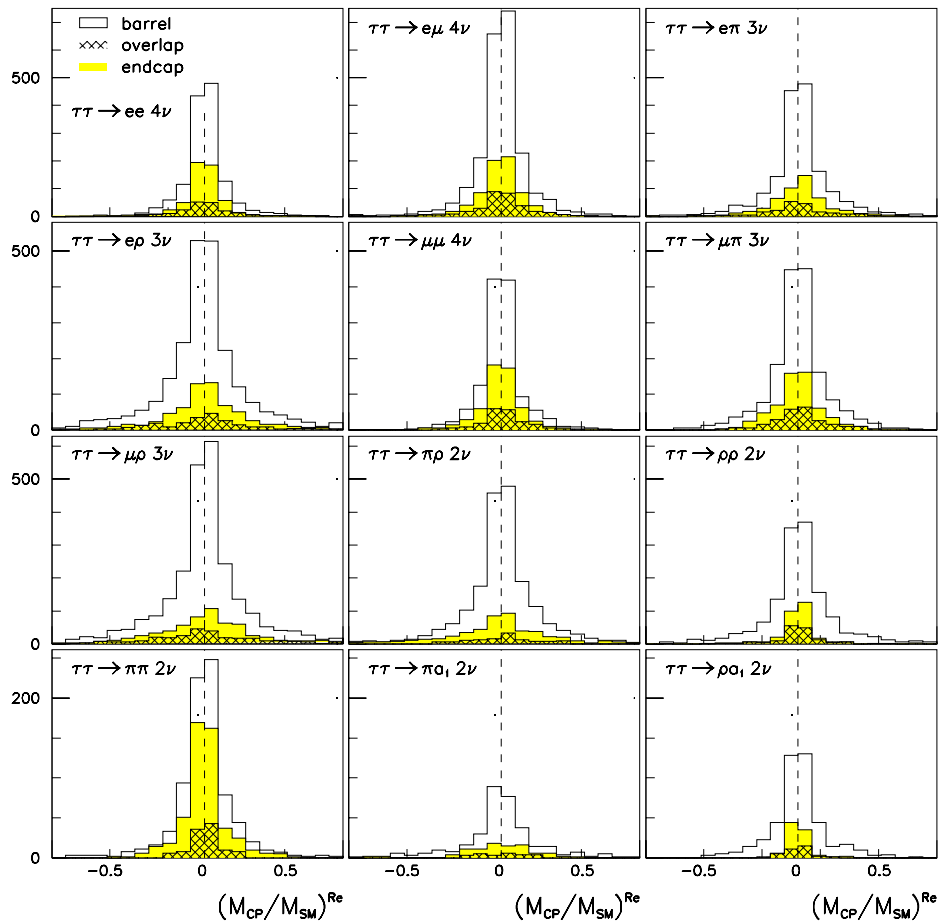
# Tau Lepton CP Couplings

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Example from OPAL (1994):

- Looked at optimal observables derived from  $\tau^+\tau^-$  daughter momenta from variety of decay topologies
- Must worry about “CP symmetry” of detector
  - Biggest worry: twisted tracking chamber
  - Nailed down with  $e^+e^- \rightarrow \mu^+\mu^-$  events
  - Other detector asymmetries checked with event mixing

Sample of CP-odd observable distributions:



# Tau Lepton $\mathcal{CP}$ Couplings

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$\mathcal{CP}$  results from full LEP data samples

Deviation in  $\sigma_{Z \rightarrow \tau^+ \tau^-}$ :

- $|d_\tau^Z| < 1.8 \times 10^{-17}$  e-cm    Wermes review, 1996
- $|d_\tau^\gamma| < 1.1 \times 10^{-17}$  e-cm    Escribano & Massó, 1996

Analyses of optimal observables in  $e^+e^- \rightarrow \tau^+\tau^-$  events:

- $\Re\{d_\tau^Z\} = (\Leftrightarrow 0.29 \pm 2.59 \pm 0.88) \times 10^{-18}$  e-cm    ALEPH
- $\Re\{d_\tau^Z\} = (\Leftrightarrow 1.48 \pm 2.64 \pm 0.27) \times 10^{-18}$  e-cm  
   $\Im\{d_\tau^Z\} = (\Leftrightarrow 0.44 \pm 0.77 \pm 0.13) \times 10^{-17}$  e-cm    DELPHI
- $\Re\{d_\tau^Z\} = (0.72 \pm 2.46 \pm 0.24) \times 10^{-18}$  e-cm  
   $\Im\{d_\tau^Z\} = (0.35 \pm 0.57 \pm 0.08) \times 10^{-17}$  e-cm    OPAL
- $|\Re\{d_\tau^Z\}| < 3.6 \times 10^{-18}$  e-cm  
   $|\Im\{d_\tau^Z\}| < 1.1 \times 10^{-17}$  e-cm    Wermes review, 1996

Analysis of  $e^+e^- \rightarrow \tau^+\tau^-\gamma$  events:

- $|d_\tau^\gamma| < 3.1 \times 10^{-16}$  e-cm    L3 1998



# Tau Lepton $\mathcal{CP}$ Couplings

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Other determinations of anomalous  $\tau$  couplings

- Analysis of azimuthal angular asymmetries  
in  $e^+e^- \rightarrow \tau^+\tau^- \rightarrow h^-h^+\nu_\tau\bar{\nu}_\tau$  L3, 1998

$$|\Re\{a_\tau^Z\}| < 4.5 \times 10^{-3} \quad |\Im\{a_\tau^Z\}| < 9.9 \times 10^{-3}$$

(same analysis gives weak limits on  $|\Re\{d_\tau^Z\}|$ )

- Measurements of Michel parameters and  $\nu_\tau$  helicity from LEP, SLD and CLEO
- Limits on charged-current magnetic / electric dipole moment form factors from
  - $\tau$  lifetime and lepton energy spectrum  $E_\ell$  in  $\tau \rightarrow \ell\nu\bar{\nu}$  (Rizzo, 1997)
  - Apparent  $\tau$  polarization in  $\tau \rightarrow \pi\pi^0\nu_\tau$  and  $B(\tau \rightarrow \pi\pi^0\nu_\tau)$  (Dova *et al.*, 1999)

# Tau Lepton $\mathcal{CP}$ Couplings

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Summary on top and tau  $\mathcal{CP}$  couplings

- Third generation fermions especially interesting because of possible anomalous couplings from Higgs
- Detecting direct  $\mathcal{CP}$  in top, tau decays difficult
- Detecting  $\mathcal{CP}$  in neutral couplings easier, especially at  $e^+e^-$  colliders
- Much work already carried out in  $\tau$  physics with no hint of signal (LEP, SLD & CLEO)
- Verifying “Standard Model” leptonic  $\mathcal{CP}$  not likely in foreseeable future

My prejudice:

$\mathcal{CP}$  will be seen in top couplings at LHC or NLC before gauge boson or tau effects are seen