Lecture 2

3 Examples

$B \to \rho\pi$ and α

Isospin, Interfering resonances

$B \to K\pi$ and γ

All the tools

$B \to X\ell\nu$ and V_{ub}

Very few tools to help

Isospin

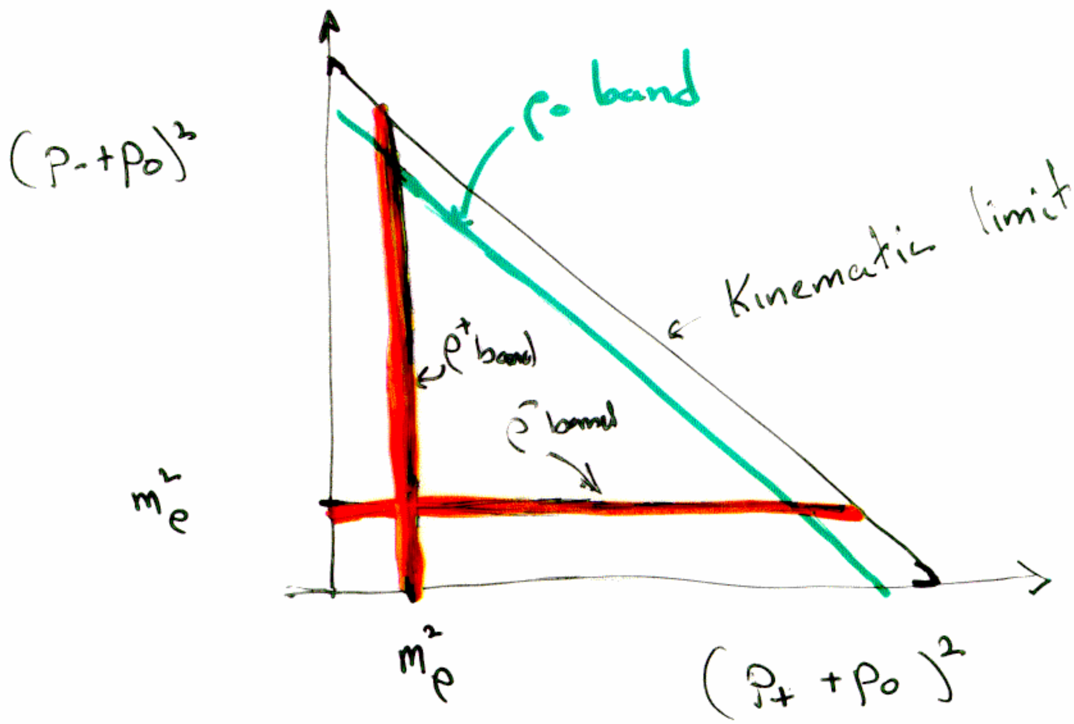
- Extracting α from $B \to \rho\pi$

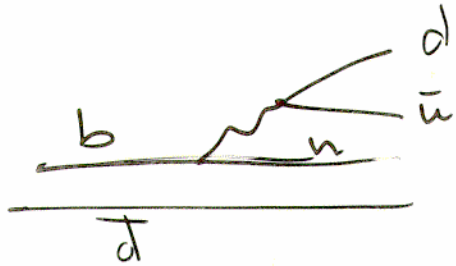
Quinn Snyder PRD 48 2139
BABAR Book Ch 6

$$B^0 \to \rho^+ \pi^-$$
$$\to \rho^- \pi^+ \qquad \to \qquad \pi^+ \pi^- \pi^0$$
$$\to \rho^0 \pi^0$$

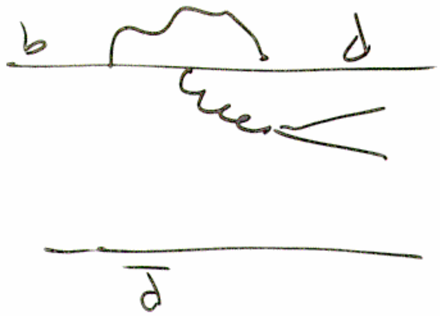
Breit Wigner form for $\rho \to \pi\pi$ introduces KNOWN strong phase variation

Dalitz plot ⇒ resonance bands overlap ⟺ interference





$V_{ub}V_{ud}^*$



$$\frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

$$V_{tb}V_{td}^* f(m_t) + V_{cb}V_{cd}^* f(m_c) + V_{ub}V_{ud}^* f(m_u)$$

$\Longrightarrow$ (unitarity)

$$V_{tb}V_{td}^* \left[f(m_t) - f(m_c) \right] + V_{ub}V_{ud}^* \left[f(m_u) - f(m_c) \right]$$

same weak phase as mixing

same weak phase as tree

$$\frac{q}{p}\frac{\bar{A}}{A} = \frac{T e^{-i\alpha} + P}{T e^{i\alpha} + P}$$

$\left.\begin{array}{l}\Delta I = \frac{1}{2}\ I_f = 0,1 \\ \Delta I = \frac{3}{2}\ I_f = 1,2\end{array}\right\}$ 3 tree amplitudes (4 if charged B's are also included)

$\Delta I = \frac{1}{2}\ I_f = 0,1$ but only 2 penguin amplitudes

$\Rightarrow$ Combinations of A_{+-} A_{-+} A_{00} with no penguins (pure $I_f = 2$)

$\Rightarrow$ should be able to isolate $(\sin 2\alpha)$

In fact can do better -- also get $(\cos 2\alpha)$

Provided $\rho^0\pi^0$ rate not too small

– need to be sensitive to the interference of this amplitude with $\rho^+\pi^-$ and $\rho^-\pi^+$

i.e. linear not quadratic in A_{00}

↳ an advantage compared to $B \to \pi\pi$

$$A(B^0 \to \rho^+ \pi^-) = T_{+-} + P_1 + P_0$$

$$A(B^0 \to \rho^- \pi^+) = T_{-+} - P_1 + P_0$$

$$A(B^0 \to \rho^0 \pi^0) = T_{00} - 2P_0$$

Each T & P amplitude has independent
<u>CONSTANT</u> strong phase

$$A(B^0 \to \pi^+ \pi^- \pi^0)$$

$$= f^{BW}(\rho^+)(T_{+-} + P_1 + P_0)$$
$$+ f^{BW}(\rho^-)(T_{-+} - P_1 + P_0)$$
$$+ f^{BW}(\rho^0)(T_{00} - 2P_0)$$

and similarly for $\overline{B^0}$

$f^{BW} \Leftrightarrow$ kinematically varying strong phase from ρ decay helicity 0 ρ · decay angular distribution

$$f_{BW} = \frac{\cos\theta_H}{s - m_\rho^2 + i\,\Pi(s)}$$

$$\Pi(s) = \frac{m_\rho^2}{\sqrt{s}} \left(\frac{p}{p_0}\right)^3 \Gamma_\rho(m_\rho^2)$$

threshold kinematics of $\rho \to \pi\pi$

$$s = (p_{\pi_1} + p_{\pi_2})^2$$

An approximate representation of $\pi\pi$ phase shift due to ρ resonance

Fit to ρ-bands of Dalitz plot

5 real magnitudes

4 constant strong phases

weak phase α (tree phase - mixing phase)

+ some parameters for background (mostly fixed by region far from ρ bands)

$\Rightarrow$ Multiparameter search for maximum likelihood α

Experimental complications

- Continuum background — CUTS
- Non-resonant $B \to \pi\pi\pi$ background
- Other contributing resonances — FITS
- $B \to \rho K$, $K^* \pi$ with K/π confusion

$\Rightarrow$ multiparameter fit to ? limited data

$\rightarrow$ long term effort.

CLEO

$$B_R(B^\pm \to \rho^0 \pi^\pm) = (1.5 \pm 0.5 \pm .4)\, 10^{-5}$$

$$B_R(\underset{\substack{\uparrow \\ B^0 \text{ or } \bar{B}^0}}{B} \to \rho^\pm \pi^\mp) = (3.5^{+1.1}_{-1.0} \pm 0.5)\, 10^{-5}$$

γ from $B^{\pm} \to K\pi$ and $B^{\pm} \to \pi\pi$

Gronau, London, Rosner | Neubert + Rosner | Neubert hep-ph/9812396

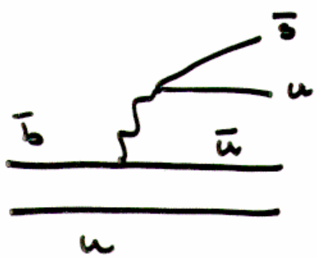
Uses OPE

Isospin

SU(3)

+ Corrections to SU(3) assuming factorization

TREE

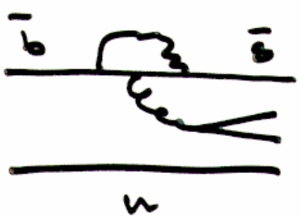


$b \to u\bar{u}s$ $\Delta I = 0, 1$ $I_f = \frac{1}{2}, \frac{3}{2}$

$\propto V_{ub}V_{us}^* = e^{i\gamma}|\lambda_u|$

BUT $\mathcal{O}(\lambda^4)$

GLUONIC Penguin



$u\bar{u} + d\bar{d}$ $\Leftarrow$ gluon has $I = 0$

$\therefore$ $I_f = \frac{1}{2}$ ONLY

u, t, and c quark parts

$$V_{cb}V_{cs}^*(P_c - P_t) + V_{ub}V_{us}^*(P_u - P_t)$$

EWP
- t quark term enhanced by m_t^2/m_W^2 $V_{tb}V_{ts}^*$
- Z has $\Delta I = 1$ part

Isospin $\Rightarrow$ Isolate $I_f = 3/2$ part

$$\boxed{3 \lambda_{3/2} = A(B^+ \to \pi^+ K^0) + \sqrt{2}\, A(B^+ \to \pi^0 K^+)}$$

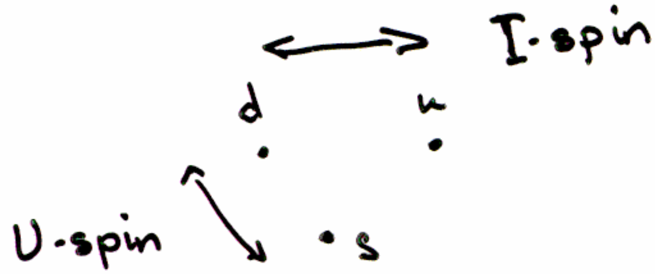
No gluonic penguin contribution

But EWP $\propto V_{tb} V_{ts}^* = V_{cb} V_{cs}^* + \mathcal{O}(\lambda^4)$

comparable to tree term

$\therefore$ must be included in $A_{3/2}$

SU(3)



In SU(3) Limit treat K, π as identical particles related by U-spin

- Bose Statistics $\Leftrightarrow$ no U-odd part to state

c.f. no $I = 1$ in $\pi\pi$ from B-decay.

$\rightarrow$ Only one independent operator matrix element contributes for both tree + EWP.

$\Delta_* \propto I_{f=3/2}$ part of amplitudes

$$R_* = \frac{Br(B^+ \to \pi^+ K^0) + Br(B^- \to \pi^- \bar{K}^0)}{2\left[Br(B^+ \to \pi^0 K^+) + Br(B^- \to \pi^0 K^-)\right]}$$

$$R_* = (1 - \Delta_*)^2$$

CLEO $\Rightarrow 0.47 \pm 0.24$

$$\Delta^* = \varepsilon(\cos\gamma - \delta_{EW})$$

$$\varepsilon = \left|\frac{3A^{Tree}_{3/2}}{P_{ct}}\right| \cos(\delta_T^{3/2} - \delta_P)$$

$$\delta_{EW} = \frac{A^{EWP}_{3/2}}{A^{Tree}_{3/2}}$$

To extract γ must calculate

ε and δ_{EW}

one operator → same strong phase

→ ratio given by calculable e_i – operator coefficients

$\boxed{\text{for Tree and EWP}}$

- Extract value of operator from $B^+ \to \pi^+\pi^0$ rate

+ U-spin

In factorization approximation

$$\langle \pi | j_A | 0 \rangle \langle \pi | j | B \rangle$$

vs $\langle K | j_A | 0 \rangle \langle \pi | j | B \rangle$

SU(3) correction f_K vs. f_π

$$\boxed{f_K / f_\pi = 1.2}$$

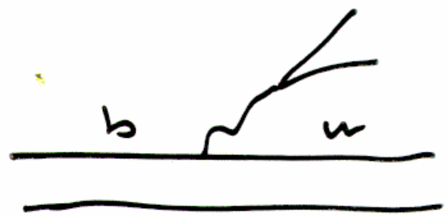
By using all the tools
reduce theoretical uncertainty ✓

But problem of quantifying
residual uncertainty remains

In general • introduce parameters to
quantify residual effects
• constrain these parameters
from models or other measurements

this "end game" can seldom be avoided
but will become better constrained
as knowledge of many modes
is accumulated.

Extracting V_{ub} — hadronic effects



$W \to \ell\nu$ $\qquad \propto V_{ub} \sum_f \langle f | \bar{b}\gamma_\mu(1-\gamma_5)u | B \rangle$

Exclusive $\Leftrightarrow$ matrix element to specific f

Inclusive $\Leftrightarrow$ background from $b \to c$

Cuts to exclude $b \to c$

On lepton momentum

On hadronic invariant mass

How well does hadron spectrum reflect quark spectrum?

Recent suggestion (Aleksan et al)

$W \to \bar{c}s \to D_s$

Use $B \to D_s X$ – assume factorization

Cut on p_{D_s} to exclude $b \to c$

Same issue for quark $\Leftrightarrow$ hadron spectrum

"quark-hadron duality"

– clearly wrong at end point

– less model-dependent as more of spectrum is included

None of the tools discussed above give much help in constraining theoretical uncertainty here.