Hadronic Effects in B decays

Lecture 1

What is the problem?

- 2 body decays

in

- extracting CKM amplitudes

What tools can be used

To control theoretical uncertainties

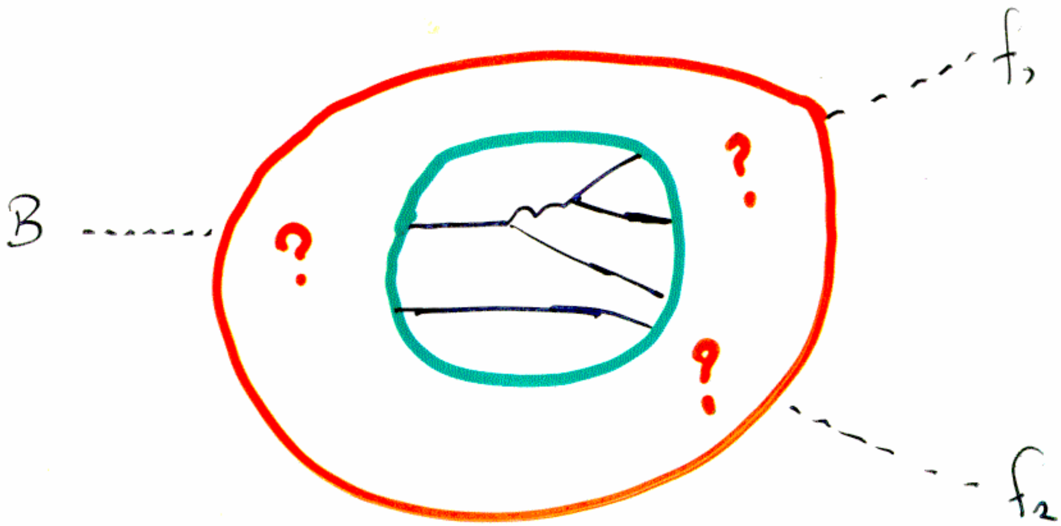
Lecture 2.

Some examples:

- Isospin + Dalitz plot analysis
  e.g. $B \to \rho\pi$
- + SU(3) $B \to K\pi$
- Extracting V_{ub}

What is the problem?

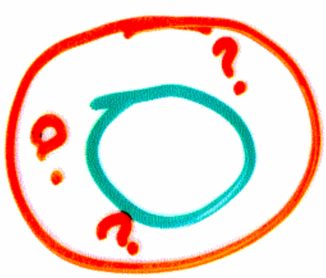
e.g. 2 body decays



Perturbative quark diagrams

Short distance ✓

- weak decay
- hard QCD effects

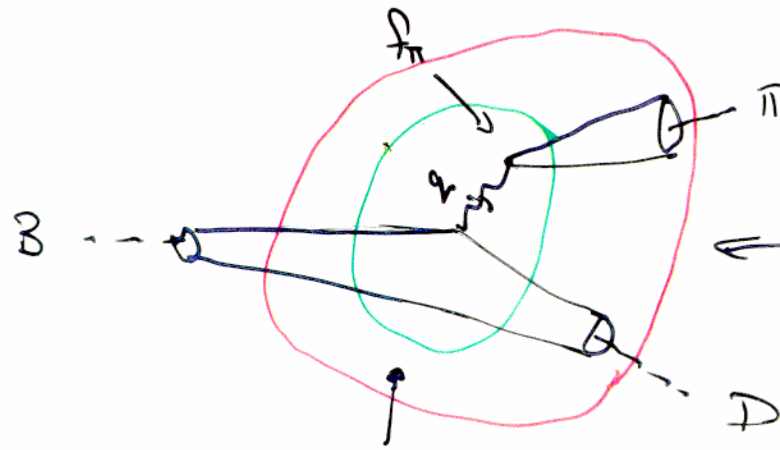


Long distance ✗

- Hadronization
- Form factors
- B structure

- Final state interaction "rescattering"

Usual approach e.g. $B \rightarrow D\pi$



• Ignore final state interactions

$B \rightarrow D$ transition amplitude

$\rightarrow 1$ in $m_b, m_c \rightarrow \infty$ limit for q such that D is at rest in B rest frame

• Mesons treated as simple $q\bar{q}$ states

Theoretical tools

→ See Adam Falk's BaBar Book Chapter 2

- Operator Product expansion OPE
  - formalizes hard | soft division
  - introduces unphysical separation

scale μ

operator matrix element

$$\langle f|\mathcal{H}|i\rangle = \sum_j c_j(\mu)\, \langle f|\, O_j(\mu)\,|i\rangle$$

calculable order by order in $\alpha_s(\mu)$

$$\boxed{\ln(\mu/m)\ ? \quad \rightarrow \mu \sim m_b \ \checkmark \text{ OK}}$$

$\langle f|\, O_j(\mu)\,|i\rangle$ operator matrix elements

calculable on lattice ?

contain all long distance effects

Theoretical tools

Exact symmetry limits

→ systematic expansion in small parameter

OR as a constraint on models

- Heavy quark limit

$m_b, m_c \to \infty$ $\left(m_c/m_b \text{ fixed} \right)$

expansion $\frac{\Lambda_{QCD}}{m_b}$ ✓ , Λ_{QCD}/m_c ?

Useful in extracting
- CKM elements
- B→D transitions

- Isospin $m_u = m_d$

expansion $\frac{m_u - m_d}{\Lambda_{QCD}}$ ✓

$\frac{m_u - m_d}{m_u + m_d}$?

Useful because gluons have $I=0$

∴ Can help separate trees from QCD penguins

Isospin

Iso $\Rightarrow$ equal $\nearrow$ Isobars Equal A
Isotopes Equal Z

spin $\Rightarrow$ SU(2) $\begin{pmatrix} u \\ d \end{pmatrix}$ algebra same as that of spin

Note not quite the same as

$$SU(2)_{Weak} \begin{pmatrix} u \\ d\cos\theta_{12}\cos\theta_{13} + s\sin\theta_{12}\cos\theta_{13} \\ + b\sin\theta_{13}\, e^{-i\delta_{13}} \end{pmatrix}$$

Approximate symmetry because $m_d - m_u \ll \Lambda_{QCD}$

Λ_{QCD} sets scale of MOST baryon masses

But pion masses are $\sqrt{m_q \Lambda_{QCD}}$

$\Rightarrow$ some large isospin breaking effects $\sim \dfrac{m_d - m_u}{m_d + m_u}$

Isospin multiplets

$\begin{pmatrix} u \\ d \end{pmatrix}$ $I = 1/2$ doublet   all other quarks $I = 0$

$\Rightarrow I$ for any hadrons   e.g. $\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$ $I = 1$ triplet

But Isospin breaking also modifies states

$\eta - \pi$ mixing $\Rightarrow \pi^0$ has small $I=0$ part

$I=1 \quad I_z = 0 \quad \frac{u\bar{u} - d\bar{d}}{\sqrt{2}} \qquad I=0 \quad \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$

Also γ and Z couplings do not respect isospin

$$g_1^V \bar{u} \gamma_\mu u + g_1^A \bar{u} \gamma_\mu \gamma_5 u$$
$$+ g_2^V \bar{d} \gamma_\mu d + g_2^A \bar{u} \gamma_\mu \gamma_5 u$$

$\gamma \qquad g_i^V = q_i \qquad g_i^A = 0$

$Z \qquad g_i^V = t_{3L}(i) - 2 q_i \sin^2\theta_W$

$\qquad g_i^A = t_{3L}(i)$

$t_3 =$ Weak Isospin

$\pm \frac{1}{2}$

Isospin Amplitudes

eg $B \to \pi\pi, \rho\pi, \rho\rho, \ldots\ldots$

$b \to u\bar{u}d$ $\qquad \Delta I = 1/2 \quad$ or $\quad 3/2$

\+ spectator $\qquad I_f = 0 \qquad 1 \qquad 2$

excluded by Bose statistics for $\pi\pi$, $\rho\rho$

In general 4 tree amplitudes $A^T_{\Delta I, I_f}$

BUT gluons have $I = 0$

$\dfrac{(u\bar{u} + d\bar{d})}{\sqrt{2}} + \ldots \Rightarrow \Delta I = 1/2$ only

Only 2 penguin contributions (gluonic)

$$\boxed{\text{No penguin } I_f = 2 \text{ parts (except electroweak penguins)}}$$

Note: Multibody states can rescatter to 2 body

Tree and penguin diagrams have different overlaps with each state

$\therefore$ Different strong phases even for same $\Delta I, I_f$

Exact symmetry limits (cont.)

- SU(3) $m_u = m_d = m_s$

expansion $\dfrac{m_s - m_d}{\Lambda_{QCD}}$? not so small

SU(3) corrections typically of order 20%

∴ Trick is to restrict use of SU(3) to small contributions not dominant terms

e.g. Neubert $K\pi$ hep-ph/9812396

- Chiral Limit $m_u = m_d = 0$ Goldstone boson π

Chiral expansion for soft pions p_π / Λ_{QCD}

Not very useful in B-decays

Theoretical Tools (cont)

- QCD sum rules

  ⇒ provide some constraints on models for matrix elements

  not discussed further here – see BABAR book

- **Lattice Calculations for matrix elements**

  In principle can calculate accurately

  e.g. • Transition matrix elements

  In practice introduces new approximations

  e.g. "quenching" light quarks

  For 2 body decays current methods

  do not include final state interactions

  because they use reduction formalism (LSZ)

Theoretical tools (cont.)

Models and Approximations

- Factorization
- Explicit form factor models

problem of μ-dependence

problem of quantifying uncertainty

- provide a good first approximation

"Parameterization"

Relations between amplitude contributions to different decays from similar diagrams

This is quite a large bag of tricks
— how far does it take us ?

→ Depends on the process

Best case $B \to J/\psi\ K_s$

don't need any of it !

hadronic matrix elements almost cancel out
in asymmetry ratios

Why ?

Because all but a tiny contribution
have same CKM phase.

Problems are worst when two
terms with different CKM phases
make comparable contributions

There are many cases of interest where both tree and penguin

quark diagrams contribute significantly. and with different CKM factors

$B \to DD$ $\quad$ $B \to \pi\pi, \rho\pi \ldots$ $\quad$ $B \to K\pi$ $\quad$ etc

Different tools are applicable in different cases.

Game is to use them all, try to find those approaches that minimize theoretical uncertainties

- Hope is that eventually data will constrain residual model dependence & thereby control theoretical uncertainties

Remember

$$\lambda_f = \frac{q}{p}\,\frac{\bar{A}_f}{A_f} \qquad a = \mathrm{Im}\,\lambda_f \sin \Delta m t$$

Consider $\quad A_f = T e^{i(\phi_T + \delta_T)} + P e^{i(\phi_P + \delta_P)}$

δ = strong phase

ϕ = weak phase

$A_f \to \bar{A}_f$

$\phi_i \to -\phi_i$

$\delta_i \to \delta_i$

$$\bar{A}_f = T\, e^{-i\phi_T + i\delta_T} + P e^{-i\phi_P + i\delta_P}$$

$$\lambda_f = e^{2i(\phi_B - \phi_T)} \left\{ 1 + \frac{\frac{2P}{T} e^{i(\delta_P - \delta_T)} \sin(\phi_T - \phi_P)}{1 + \frac{P}{T} e^{i(\phi_P - \phi_T)} e^{i(\delta_P - \delta_T)}} \right\}$$

vanishes if $\frac{P}{T} = 0$ or $\phi_T = \phi_P$

$$\mathrm{Im}\,\lambda_f = \sin 2(\phi_B - \phi_T) + \frac{2P}{T} \sin(\phi_T - \phi_P) \sin\left[2(\phi_B - \phi_T) + (\delta_P - \delta_T)\right] + O\left(\tfrac{P}{T}\right)^2$$

Factorization $\Leftrightarrow$ No Final State Interactions

$$\langle f_1 f_2 | \text{ 4quark operator } | B \rangle$$

? $\overset{\sim}{=}$ sum over Fierz combinations

$$\langle f_2 | \bar{q}()q' | 0 \rangle \langle f_1 | \bar{q}''()q''' | B \rangle$$

must be color singlets

or at least have some color singlet part.

$$\sum_{\alpha\beta} \langle \bar{q}_\alpha q'_\beta \rangle \langle \bar{q}''_\beta q'''_\alpha \rangle \Rightarrow 1/N_c \quad (\alpha = \beta)$$

"color supressed contribution"

$$\sum_\alpha \langle \bar{q}_\alpha q'_\alpha \rangle \quad \sum_\beta \langle \bar{q}_\beta q_\beta \rangle$$

color allowed

Problems with factorization

- Approximation gives μ-independent matrix element

  $\otimes$ μ-dependent coefficient

  $\Rightarrow$ unphysical μ-dependent result.

  OR further assumptions about scale at which factorization is correct.

- Approximation neglects any long range interaction between f_1 and f_2

- Implicitly assumes mesons contain only $\bar{q}q$, no higher Fock states

Can be shown to be valid for $m_b \to \infty$, $m_c \to \infty$

limit case

All methods start from quark diagrams

⇒ Operator Product Expansion

= QCD "dressed" quark diagrams

explicitly including hard gluons

then

⇙ OR ⇓

Parameterized amplitudes

- T Color allowed tree
- C Color supressed tree
- P Color allowed penguin
- P_c Color supressed penguin
- P_{EW} Electroweak penguin

↑

- Convenient for relating similar effects in different decays

Isospin amplitudes

$A^T_{\Delta I, I_f}$

$A^P_{\Delta I, I_f}$

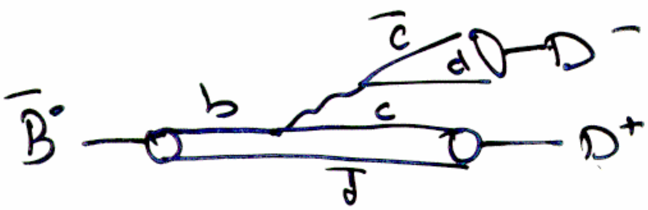
↑

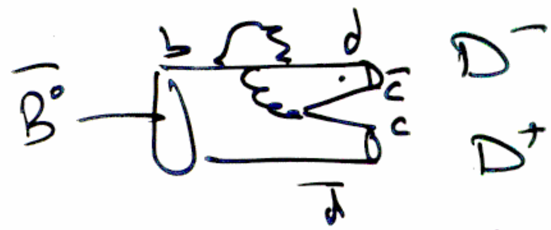
- Convenient for including rescattering
- Possibly for isolating penguin-free terms

Quark diagrams may relate
magnitudes of different I-spin amplitudes

e.g. $B \to D D$

$b \to c \bar{c} d$ $\Delta I = 1/2$ only ; $I_f = 0, 1$





Both tree and penguin give $B \to D^+ D^-$
But not $B \to D^0 \bar{D}$ without
rescattering $d\bar{d} \to u\bar{u}$

Isospin

$$A(B \to D^+ D^-) \propto A_0 - A_1$$

$$A(B \to D^0 \bar{D}^0) \propto A_0 + A_1$$

$\Rightarrow$ $\boxed{\text{Without rescattering} \quad A_0 = -A_1}$

$\boxed{T_0 = -T_1 \qquad P_0 = -P_1}$

Rescattering simply adds phases

$$A_j^x \to A_j^x e^{i\delta_j^x}$$

→ measurement of $B \to D^0 \bar{D}^0$

would demonstrate rescattering

$$A(B \to D^0 \bar{D}^0) = A^T\left(e^{i\delta_0^T} - e^{i\delta_1^T}\right) + A^P\left(e^{i\delta_0^P} - e^{i\delta_1^P}\right)$$

$$\propto \sin\left(\frac{\delta_0 - \delta_1}{2}\right)$$

Also direct CP violation in charged B decays where T & P have different CKM factors

$$\propto \sin(\phi_T - \phi_P)\,\sin(\delta_T - \delta_P)$$

difference of weak phases

difference of strong phases

Theoretical estimates of strong phases are model dependent ⟷ not very reliable.

Estimation of uncertainty in model-dependent results is a guessing game

Usual approach

range of errors = range of model estimates

but often all models contain similar assumptions

→ Very difficult to TEST Standard Model using model-dependent theory calculations