Inelastic Scattering Sum Rules

- in honour of Sid Drell

C. H. Llewellyn Smith

1) Historical overview

(+ comparison with data)

Crucial role of sum rules → scaling, partons, quarks → QCD

(NB bona fide sum rules built into parton fits: consistency → sum rules OK)

2) Two outstanding issues

The singlet nucleon spin sum rule, the Drell-Hearn Gerasimov Iddings sum rule, and their connection

Climate in the mid 60s

- Electron & photon interactions a minority interest
- Very little interest in inelastic lepton scattering – only one submitted paper at 1967 SLAC conference
- Foms on nuclear democracy, bootstraps....
- Quarks generally considered (at best) a heuristic tool
- "Field theory will not die but, like an old soldier, simply fade away" Chew

1965 Adler Sum Rule

(Derivation →)

$$\int_0^1 \left(F_2^{\nu n}(x,q^2) - F_2^{\nu p}(x,q^2) \right) \frac{dx}{x} = 2 \; *$$

in modern notation

→ $$\lim_{E\to\infty} \frac{d\sigma^{\nu n}}{dq^2} - \frac{d\sigma^{\nu p}}{dq^2} = \frac{G^2}{\pi}$$

[Parton interpretation: $n_u + n_{\bar{d}} - n_d - n_{\bar{u}} = 1$
but sum rule is <u>exact</u>]

→ highly suggestive of point-like constituents and scaling, as - uniquely - recognised by Bj

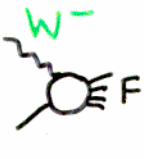
* Note: using Adler's "forward neutrino theorem" $\xrightarrow[\text{PCAC}]{q^2 \to 0}$ Adler-Weissberger relation (1964) between g_A and $\sigma_{\pi N}$

Derivations :

Adler (all Q^2)

$$W_{\mu\nu} = \sum \frac{d^4x}{4\pi} e^{iq\cdot x} \langle P | [J_\mu^+(x), J_\nu^-(0)] | P \rangle$$

$$= \sum_F \Big[\Big| \Big|^2_{\nu>0} - \Big| \Big|^2_{\nu<0} \Big]$$



$$= -g_{\mu\nu} W_1 + \frac{P_\mu P_\nu}{M^2} W_2 + i \frac{\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{2M^2} W_3 + \cdots$$

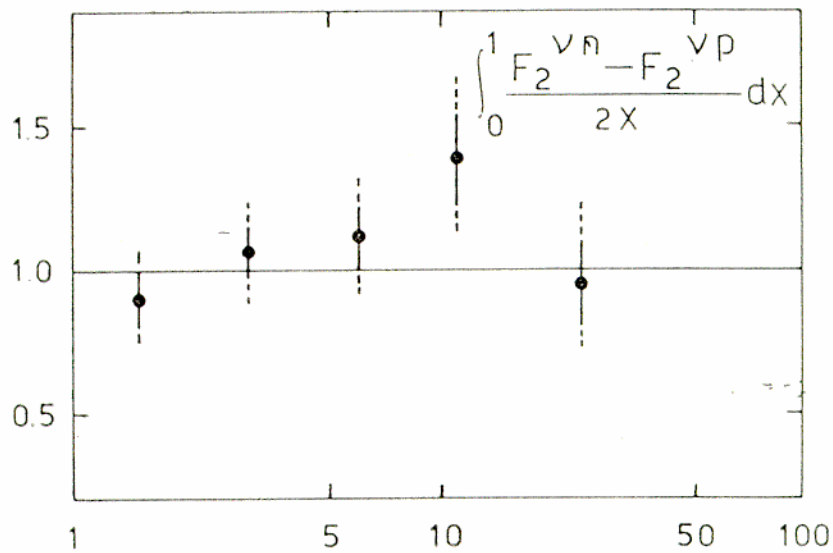
$$\lim_{|\vec{P}| \to \infty} \int dq_0 \, W_{00} \quad \text{in frame } \vec{q}\cdot\vec{P} = 0$$

$$\text{so } dq_0 = \frac{d\nu}{\sqrt{\vec{P}^2 + M^2}}, \quad q^2 = \frac{\nu^2}{\vec{P}^2 + M^2} - \vec{q}^{\,2} \longrightarrow -\vec{q}^{\,2}$$

$$\stackrel{?}{=} \int_0^\infty \left(W_2^{\bar{\nu}}(\nu, q^2) - W_2^{\nu}(\nu, q^2) \right) d\nu = 2M^2$$

Test of Adler sum rule,

WA 25 (BEBC), 1985:



$\Rightarrow$ Average $Q^2 = 1 - 40\ \mathrm{GeV}^2$

$= 1.01 \pm 0.08 \pm 0.18$

$\xrightarrow[\text{latest value of } \sigma_\nu]{\text{corrected with}}$ $1.08 \pm 0.08 \pm 0.18$

1966 Bj's Polarisation Sum Rule etc

$$\int \left(g_1^p(x,q^2) - g_1^n(x,q^2) \right) dx = \frac{1}{6} \left| \frac{g_A}{g_V} \right| \left(1 - \frac{\alpha_s}{\pi} \ldots \right.$$

QCD correction known to $O(\alpha_s^3)$

(discussion / interpretation later: derivation →)

- described as "worthless" (but now tested →) but used ⇒ inequality for unpolarised scattering "inelastic scattering large .... comparable to scattering off point-like charges"
- Same techniques

→ $\sigma_{\bar{e}e} \sim 1/E^2$ "The idea that the total hadronic yield from colliding $[e^+e^-]$ beams should be approximately the same as the $\mu^+\mu^-$ yield is folklore[38]" ← B. Richter private communication

→ $\sigma^{\nu N} \sim E$

Drell-Hearn Gerasimov Iddings (1965) (1965)

→ later

Derivations (cont):

Bjorken Johnson Low Cornwall Norton

(large Q^2)

Spinless currents:

$$T = i\int \theta(x_0)\, e^{iq\cdot x} \langle p|[J^+(x), J(0)]|p\rangle\, d^4x$$

+ polynomial in q_0

$$\xrightarrow{q_0 \to i\infty} \sum_{n=0}^{\infty} \left(\frac{i}{q_0}\right)^{n+1} \int e^{-i\vec{q}\cdot\vec{x}} \langle p|\left[\frac{\partial^n J^+(\vec{x},0)}{\partial t^n}, J(0)\right]|p\rangle$$

+ poly. in q_0

$$= T(0, q^2) + 2\nu \int_{-\infty}^{+\infty} \frac{W(\nu', q^2)}{\nu'(\nu'-\nu)}\, d\nu'$$

- limit $q_0 \to i\infty$ here also, frame $\vec{q} = 0$
- equate coeffs. of $(1/q_0)^n$
- divide by $(p_0)^n$ + let $p_0 \to \infty$

eg Neutrino case:

With $F_1^{\pm}(\omega) = \lim_{q^2\to\infty} \left(W_1^{\nu}(\omega, q^2) \pm W_1^{\bar{\nu}}(\omega, q^2)\right)$

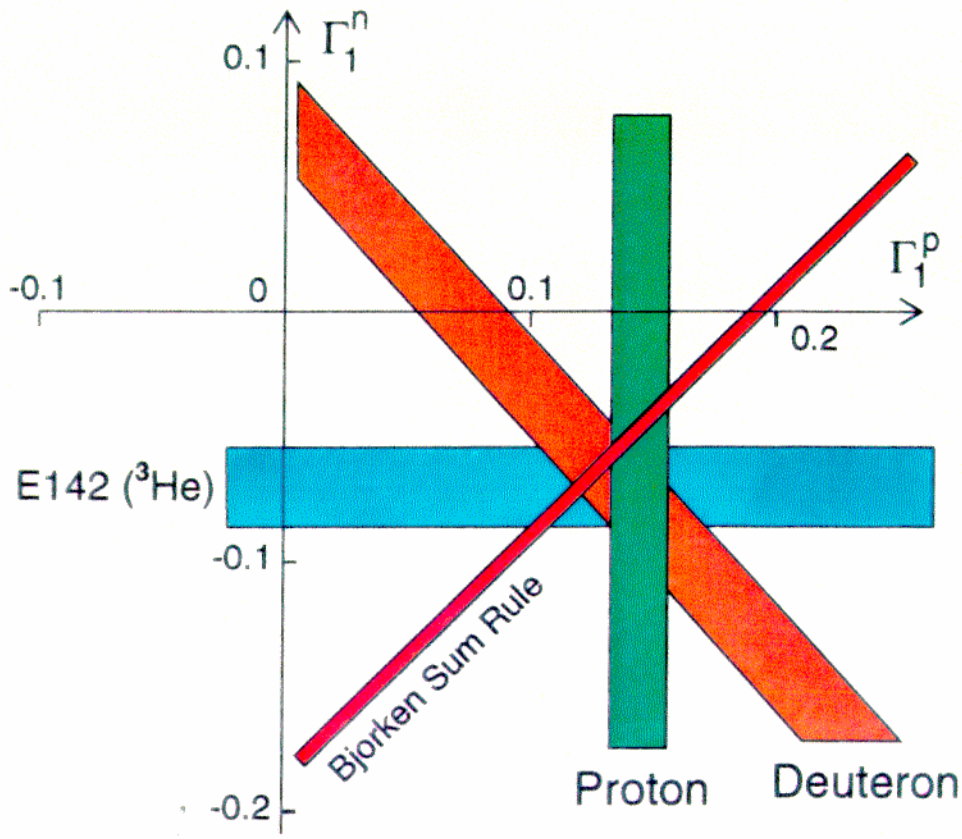
assumed to exist

$$2\int F_1^{\pm} \frac{d\omega}{\omega^{n+2}} = \lim_{p_0\to\infty} i\left(\frac{i}{2p_0}\right)^{n+1} \int \langle p|\left[\frac{\partial^n J^+(\vec{x},0)}{\partial t^n}, J^-(0)\right]|p\rangle\, d^3x$$

+ if n even

− if n odd

The Bjorken Sum Rule



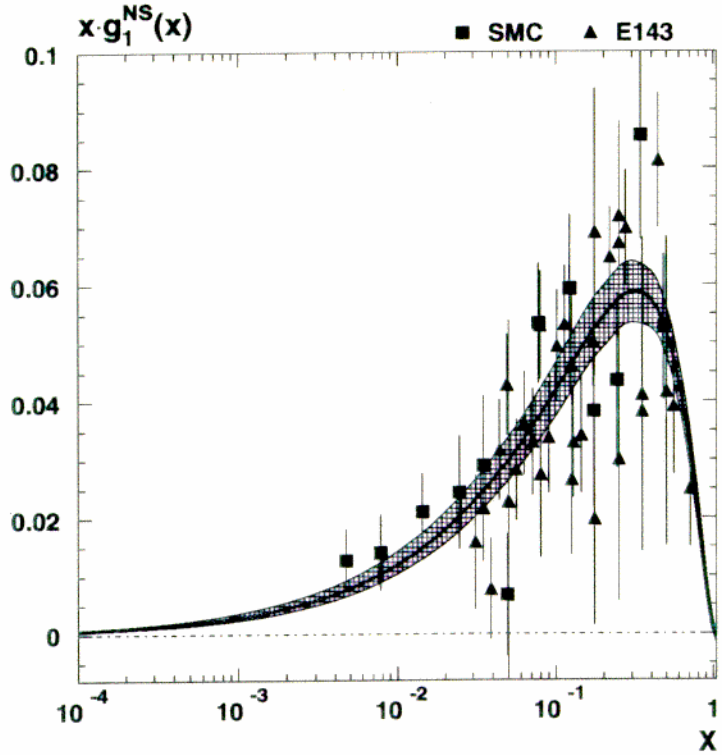
- Combine data at g_1 level, use SMC p & n data for small-x extrapolations, at $Q^2 = 5$ GeV^2:

$$\Gamma_1^p - \Gamma_1^n = 0.202 \pm 0.022$$

- Prediction ($Q^2 = 5$ GeV^2):

$$\Gamma_1^p - \Gamma_1^n = 0.181 \pm 0.003$$

Non-Singlet Fit Result: Bjorken sum rule



$$\left|\frac{g_A}{g_V}\right| = 1.20^{+0.08}_{-0.07}(\text{stat})\pm 0.12(\text{exp.syst})^{+0.10}_{-0.04}(\text{theory})$$

$$[\Gamma_1^p - \Gamma_1^n]\,(Q_0^2 = 5\text{GeV}^2) = 0.181^{+0.012}_{-0.011}(\text{stat})\pm 0.018(\text{exp.syst})^{+0.015}_{-0.006}(\text{theory})$$

$$= 0.181^{+0.026}_{-0.021} \Longleftarrow \text{SMC} - \text{NS Result}$$

Excellent agreement with the theoretical calculation

$\Gamma_1^p - \Gamma_1^n = 0.181 \pm 0.003$ at $Q^2 = 5$ GeV^2

Statistical error larger than the global fit result as expected

Systematic uncertainties comparable or smaller!

1967 Bj - SLAC Conference + Varenna Lectures

Adler sum rule ⇒ description in terms of incoherent scattering off point-like constituents →

→ partons (pre Feynman), but no explicit statement of scaling

+ Gottfried →

1968

- Bj's "derivation" of scaling (published 1969)

but "a more physical approach into what is going on is, without question, needed" !

- First deep inelastic data from SLAC →

Bj, SLAC Conference 1967:

. . . . . the result is that as the incident energy goes to infinity the difference of neutrino and antineutrino cross sections is a constant independent of q^2:

$$\lim_{E \to \infty} \frac{d\sigma(\bar{\nu} p)}{dq^2} - \frac{d\sigma(\nu p)}{dq^2} = \frac{G^2}{\pi}\left(\cos^2\theta_c + 2\sin^2\theta_c\right) \tag{8}$$

where θ_c is the Cabibbo angle. This result would also be true were the nucleon a point-like object, because the derivation is a general derivation. Therefore, the difference of these two cross sections is a point-like cross section, and it is big. Therein lies its interest to us.

How can we interpret this sum rule ? I would like to suggest an interpretation based essentially on history and not much more. We assume that the nucleon is built out of some kind of point-like constituents which could be seen if you could really look at it instantaneously in time, rather than in processes where there is a time averaging and in which the charge distribution or matter distribution of these constituents is smeared out. Then if we go to very large energy and large q^2, which is the interesting region because of the constant on the right-hand side of the sum rule, we can expect that the scattering will be incoherent from these point-like constituents. Suppose, just for example, these point-like constituents had isospin one-half. The antineutrino, because it goes to a μ^+ and must pick up a charge, scatters only from the isospin up constituents. Therefore, its cross section is proportional to the expectation value of the number of isospin up objects in the nucleon times the point cross section

$$\frac{d\sigma^{(\bar{\nu} p)}}{dq^2} = \frac{G^2}{\pi} \langle N\uparrow \rangle . \tag{9}$$

Likewise, the same thing applies for the neutrinos with isospin up replaced by isospin down. Therefore, what the sum rule says is simply $\langle N\uparrow\rangle - \langle N\downarrow\rangle = 1$ for any configuration of constituents in the proton. This gives a very simple minded picture of this process which may look a little bit better if you really look at it, say, in the center-of-mass of the lepton and the incoming proton. In this frame the proton is Lorentz contracted into a very thin pancake and the lepton scatters essentially instantaneously in time from it in the high energy limit. Furthermore the proper motion of any of the constituents inside the hadron is slowed down by time dilatation. Provided one doesn't observe too carefully the final energy of the lepton to avoid trouble with the uncertainty principle, this process looks qualitatively like a good measurement of the instantaneous distribution of matter or charge inside the nucleon. That, of course, is folklore. If we take this idea over to the electromagnetic processes, what we say by analogy is that the limit as the energy goes to infinity of $d\sigma/dq^2$ for lepton-proton scattering (with either the lepton an electron or a muon) should be the Rutherford point cross section times the sums of the squares of the charges of those point-like constituents inside the nucleon.

$$\lim_{E \to \infty} \frac{d\sigma^{ep}}{dq^2} = \underbrace{\left(\frac{4\pi\alpha^2}{q^4}\right)}_{\text{Rutherford}} \sum_i Q_i^2 \tag{10}$$

It is big; it is point-like.

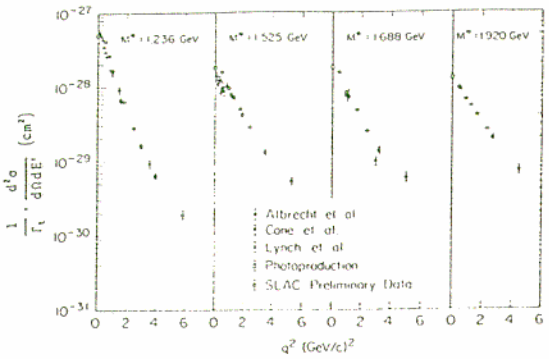


Fig. 21 Plots of $(d^2\sigma/d\Omega dE')/\Gamma_t$ for the first four resonances, compiling data reports from various laboratories.

The inelastic muon data reported by Zipf et al.[27] cover inelasticities up to an equivalent photon energy of 7 GeV and a range of $q^2 < 0.9$ $(GeV/c)^2$. An optical spark chamber technique is used; the statistical accuracy is, of course, well below that of the electron data. Within this limited accuracy, there is fair agreement in the region of overlap between the electrons and muons.

The qualitatively striking fact is that these cross-sections for inelastic electron and muon scattering leading to the continuum are very large and decrease much more slowly with momentum transfer than the elastic scattering cross-sections and the cross-sections of the specific resonant states; in fact, indications are that they probably decrease even more slowly than would be predicted from a simple ρ-vector dominance propagator. Therefore theoretical speculations are focused on the possibility that these data might give evidence on the behaviour of point-like, charged structures within the nucleon.

Treating the proton by a non-relativistic point quark model, Godfrey has derived a sum rule for the integral $\int W_2(q^2,\nu)d\nu$. Evaluation of the integral over the SLAC data gives about 60% of the required amount. There is no visible quasi-elastic peak at a defined inelasticity $\nu = q^2/2m$, where m is some characteristic mass, but the apparent success of the parametrization of the cross-sections in the variable ν/q^2 in addition to the large cross-section itself is at least indicative that point-like interactions are becoming involved. Numerical evaluation of the sum rules is difficult since the integrals will converge only if the curves shown in Fig. 23 eventually decrease more rapidly with (ν/q^2) than over the region covered by present data.

I have only attempted to point out the qualitative features of the data; specific comparisons with models and sum-rules are discussed in the theoretical sessions. However, a great deal more fundamental experimental material must be developed in this field before a clear picture can emerge.

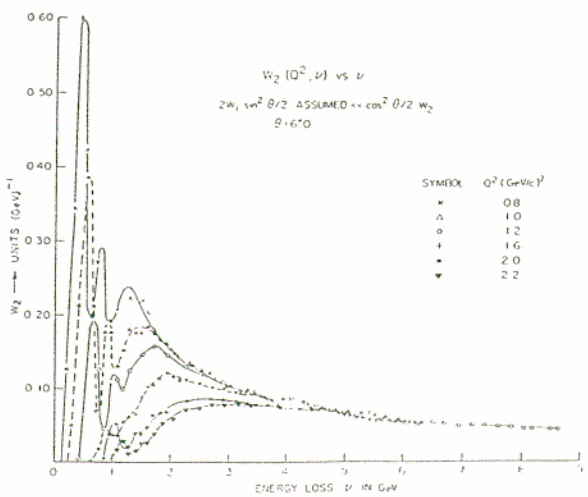


Fig. 22 Plot of $W_2(q^2,\nu)$ versus q^2 for various values of $\nu = E - E'$.

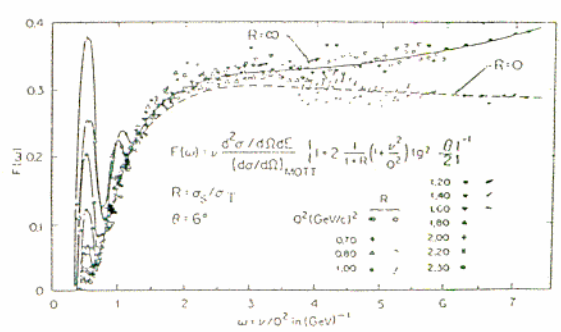


Fig. 23 Plot of $F(\omega) = \nu W_2(q^2,\nu)$, as a function of $\omega = \nu/q^2$.

back to **Gottfried** (1967)

Noted that in "breathtakingly crude" naive 3 quark model:

$$\sum_{i,j} Q_i Q_j \equiv \sum_i Q_i^2 + \sum_{i \neq j} Q_i Q_j$$

$\llcorner$ 0 for p and n

→ correlations vanish, so using "closure approx." (cf atomic & nuclear sum rules):

$$\int_{\nu \text{ inel. thres}} W_2^{ep}(\nu, q^2)\, d\nu = 1 - \frac{G_E^2 - q^2 G_M^2 / 4m^2}{1 - q^2/4m^2}$$

+ observed: oversaturated (?) for γ prod.

γ asked: idiotic? or some truth?

⇒ want "derivation that a well educated person could believe"

Bj (SLAC Conference 1967) ⇒ diffractive contributions should be excluded ⇒

$$\int \left(F_2^{ep}(x, q^2) - F_2^{en}(x, q^2) \right) \frac{dx}{x} = \frac{1}{3}$$ in modern notation

$$= \frac{1}{3}\left(n_u + n_{\bar{u}} - n_d - n_{\bar{d}}\right) = \frac{1}{3} + \frac{2}{3}\left(n_{\bar{u}} - n_{\bar{d}}\right) \xrightarrow[n_{\bar{d}} > n_{\bar{u}}]{\text{data}}$$

$(u\bar{u}) < (d\bar{d})$ ⊕ $p \leftrightarrow \pi^+ n + \pi^0 p$ ....

Pauli → $\bar{d} > \bar{u}$

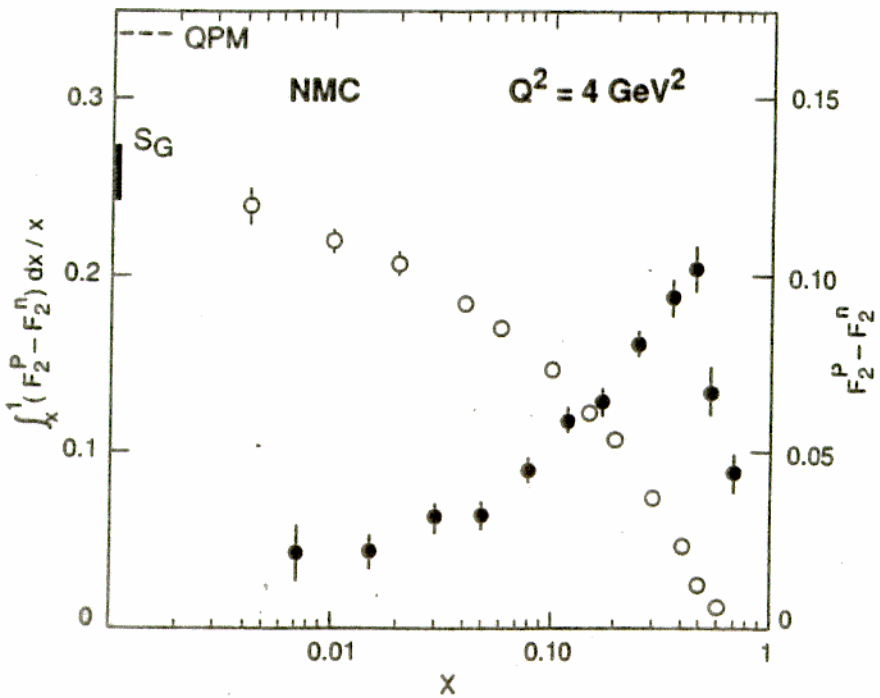


Figure 7: *NMC test of the Gottfried sum rule. The closed symbols show the experimental data and the open symbols the extrapolation to $x \to 0$. Also shown are the extrapolated value S_G and the naive QPM prediction.*

1969

• Partons

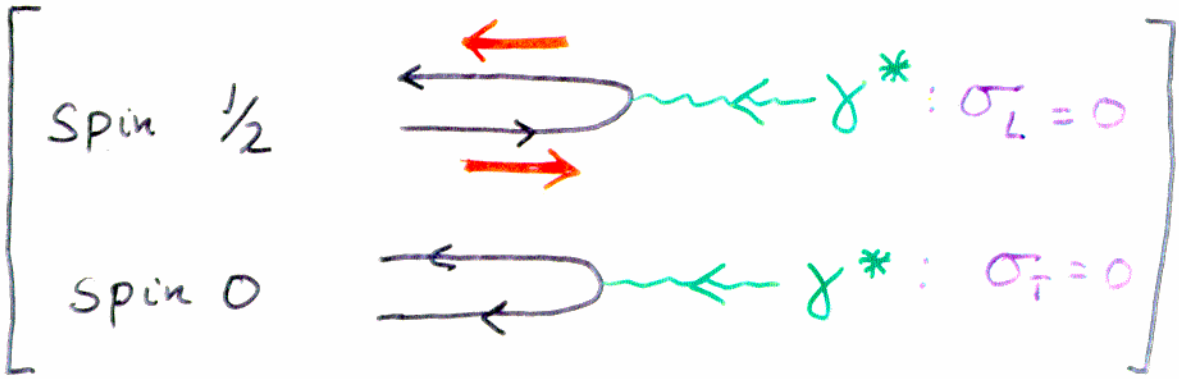
- Feynman - unpublished
- Bjorken & Paschos - explicit model (no gluons)
- Drell, Levy, Yan - TTN field theory with $|\vec{p}_T|$ cut off $\Rightarrow$ applications to other processes

• Callan - Gross

Related $\int F_2 \, d\omega$ and $\int \omega F_1 \, d\omega$ to

$[\dot{J}, J]$ → test model (spin of constituents)

$$\frac{\sigma_L}{\sigma_T} = \begin{cases} 0 & \text{quark model, spin } 1/2 \\ \infty & \text{algebra of fields, spin } 0 \end{cases}$$



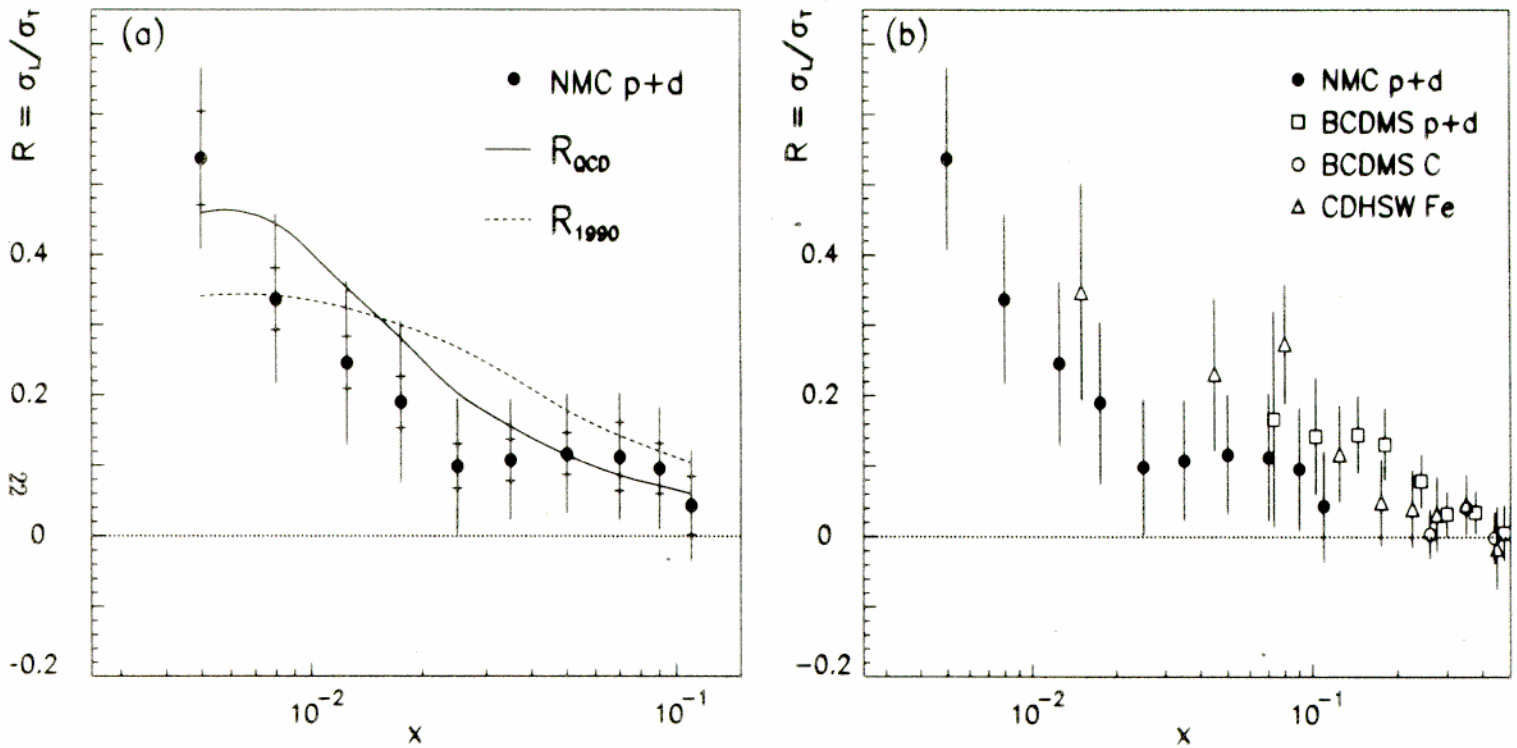


Figure 10: (a) The ratio R measured in this experiment, averaged for the proton and the deuteron. The outer error bars represent the quadratic sum of the statistical and systematic uncertainties; the inner bars indicate the statistical errors. The dashed line represents the R parametrisation from ref. [2] at the average Q^2 of each x bin. The solid line indicates a QCD calculation described in the text. (b) The ratio R measured in this experiment (filled circles), compared to previous results at comparable x and Q^2 obtained by BCDMS [10, 20] (open squares and circles) and by CDHSW [21] (open triangles). The error bars represent the total errors.

$\boxed{1969 \text{ (cont)}}$

- **Gross - LLS**

- Algebra of fields $F_3 = 0$
- Quark model : $\int F_3^{\nu N} \frac{dx}{2x} = 3 \left(1 - \frac{\alpha_s}{\pi} \cdots\right.$ known to $O(\alpha_s^3)$

$= N_u + N_d - N_{\bar{u}} - N_{\bar{d}}$

- critical test of quark model →

- **Failure of Bj limit in pert. th.**

Adler & Tung ; Jackiw & Preparata

Typical reaction (G-LLS):

"No reason to believe relevant: field theory ⇒ $F_2 \sim \ln q^2 \to \infty$ - contradicted by experiment" !

- $\frac{\sigma_L}{\sigma_T} = 0.2 \pm 0.2$ (September)

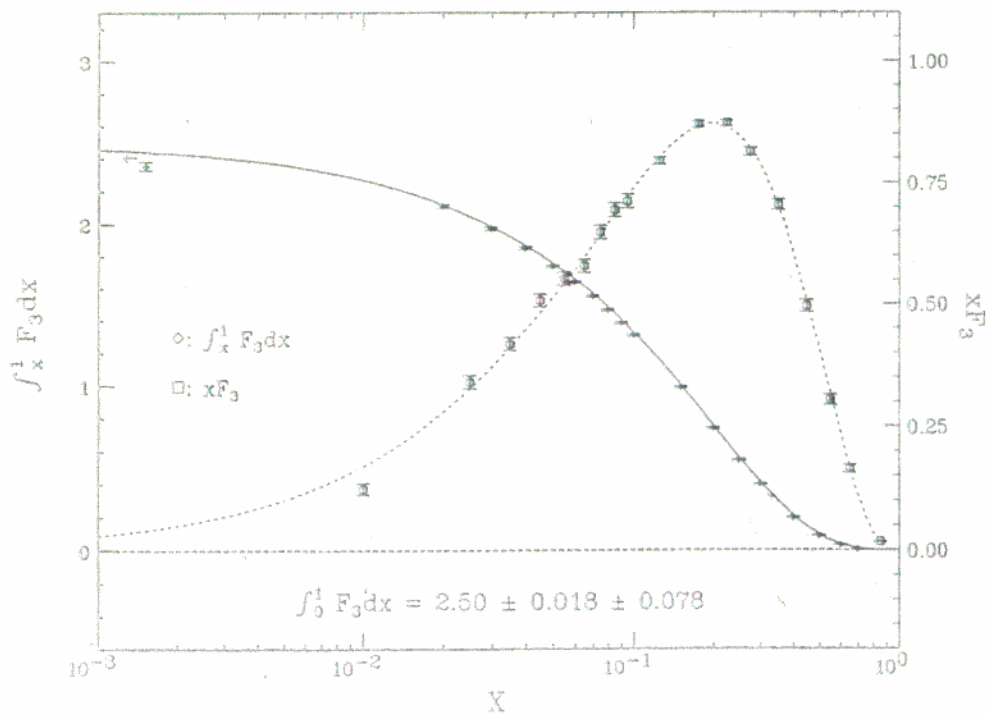


Figure 5: *CCFR test of the Gross-Llewellyn Smith sum rule. The dashed line indicates the parametrisation of the data used to evaluate the integral and the solid line illustrates the convergence of the integral at small x.*

$$\Rightarrow \alpha_s(M_Z) = 0.114 \begin{matrix} +.005 \\ -.006 \end{matrix} \begin{matrix} +.007 \\ -.009 \end{matrix} \begin{matrix} +.004 \\ -.005 \end{matrix}$$

Stat Syst Higher twist

Compare:

- Precision electroweak fits → $0.120 \pm .003$
- "all" data → $0.119 \pm .004$

1970 Many ideas still on market

- diffractive model, Harari model, generalized VMD, Veneziano-like model ... and quarks / partons:

- **CHLLS** → results true in all quark-parton models:

→ rederive formally (as sum rules for moments):

- $F_3^{\nu p} - F_3^{\nu n} = 12\,(F_1^{ep} - F_1^{en})$
- $F_2^{ep} + F_2^{en} \geq \frac{5}{18}\,(F_2^{\nu p} + F_2^{\nu n})$, equality if no strange quarks

but equivalent to $\frac{\sigma_{\gamma^*(I=0)}}{\sigma_{\gamma^*(I=1)}} = \frac{1}{9}$ - known to work for real photons: explained by VMD - nothing to do with quarks ??!

→ $\int F_2^{ep}\,dx = \sum_i Q_i^2 \langle x_i \rangle$ too big with only quarks: $= 1/3$ naive 3q, $> 2/9$ any valence + sea model, $= 0.18$ expt.

"easily reduced by adding a background of neutral constituents [which could be responsible for binding quarks]"

attacked - not in spirit of quark model

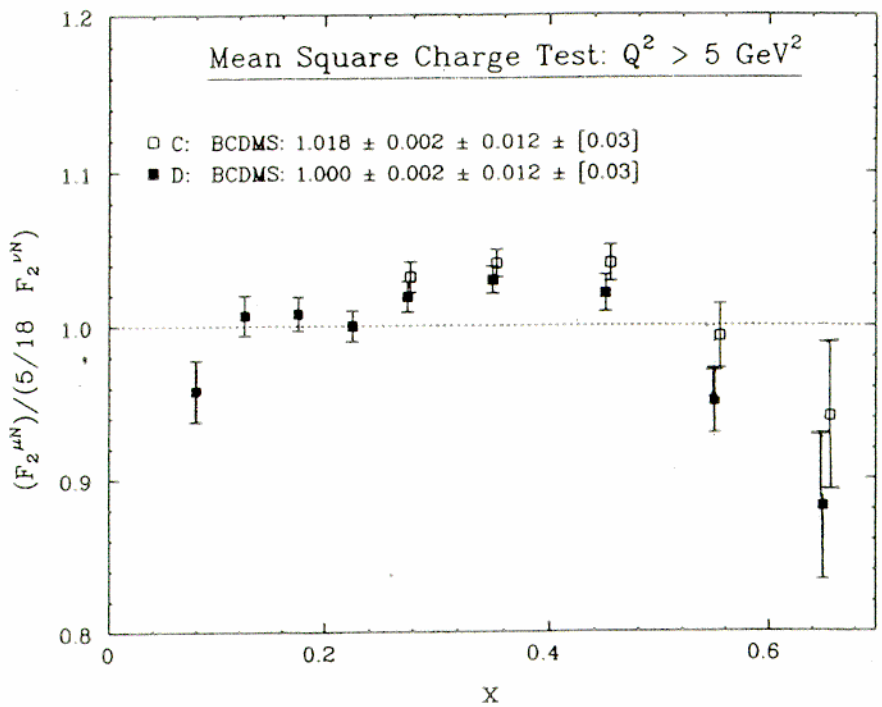


Figure 4: $F_2^{\nu}(x)$ *measured by CCFR with an iron target, compared to* $F_2^{\mu}(x)$ *measured on deuterium and carbon by BCDMS. The deuterium data are corrected for nuclear effects. In the numerical results for the average ratio, the error in square brackets accounts for the overall normalisation uncertainties of the data.*

- $F_2^{ep} \neq F_2^{en}$ Sept (?) 1970

1971 ['t Hooft proves renormalizability of non-Abelian gauge theories]

- **CHLLS**

$$\epsilon = 1 + \int \left(\frac{3}{4} \left[F_2^{\nu p} + F_2^{\nu n} \right] - \frac{9}{2} \left[F_2^{ep} + F_2^{en} \right] \right) \frac{d\omega}{\omega^2}$$

↑ momentum in glue ($\theta_{\mu\nu}$ sum rule)

$\geq 0.52 \pm 0.38$ 1971

Latest fits to all data (error < 1%):

$=$	0.39,	0.44,	0.47,	0.48
$Q^2 =$	2	20	2000	200,000

1972 [Gribov & Lipatov sum leading logs in $ep \to eX$ in Abelian gauge theory: reproduced by Christ Hasslacher & Mueller using operator prod. expansion]

- $\dfrac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} \approx \dfrac{1}{3}$ Gargamelle

1973 [Asymptotic freedom, QCD: neutral currents discovered]

- $\frac{d\sigma^{\nu}}{dy}$ from Gargamelle but high y anomaly from HPWF

1974

- First calculations of QCD scaling violations, corrections to sum rules...
- $\sigma_{\bar{e}e \to \text{Hadrons}} \xrightarrow{?}$ constant ?! (53 explanations at London conference), trimuons, high y anomaly .... J/ψ
- **Ellis-Jaffe ansatz** – below

1975 evidence for scaling violations

1977..... Perturbative QCD

13

Fig. 4 shows the ratio (R) of the total hadronic cross section to mu-pair production. The previously published points from the Frascati $\mu\pi$ and $\gamma\gamma$ groups and the Novosibirsk points[11] are included. The large spread in the values of R at low energy reflects the detection efficiency problem mentioned earlier. In the region below S = 9, R seems to lie between 2 and 3 and rises smoothly to about 6 at S = 25. These results are in violent disagreement with the predictions of the simple quark model which gives values for R which depend on the model but range from 2/3 for the simple 3 quark model to 4 for the Han-Nambu model. A large fraction of the theoretical contributions to the conference attempt to explain this peculiar variation of R.

The approximate constancy of the total cross section with energy has generated sufficient surprise to prompt many people to ask whether or not these high energy events are indeed from one photon annihilation. It is not possible to answer the question unambiguously but we can eliminate three possible explanations - background, photon-photon collisions, and heavy lepton production. The background in all these experiments was measured by separated beam runs and found to be small - approximately 3% to 5% in the CEA and SLAC-LBL experiments and 10% to 15% in the BCF experiments. These backgrounds have been subtracted from the results.

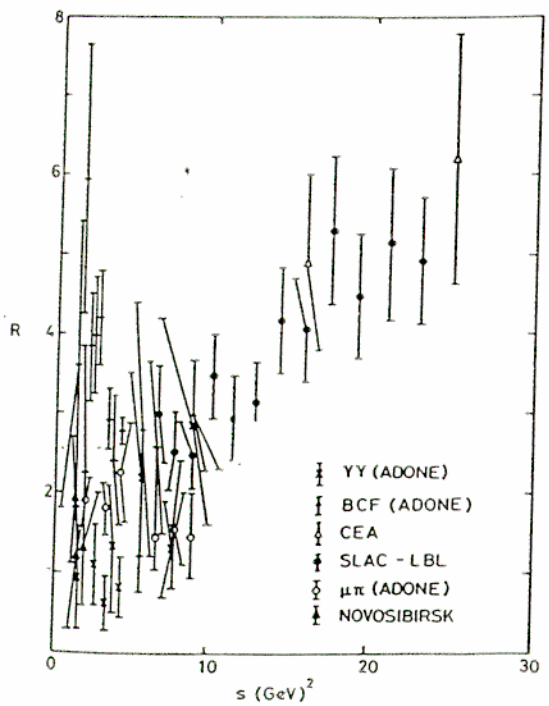


Fig. 4. All measurements of the ratio of σ_{TOT} to $\sigma_{\mu\mu}$ vs S, for $S \geq 1.5\ (\mathrm{GeV})^2$.

The photon-photon process is illustrated schematically in fig. 5. Both the incoming electron and positron radiate almost real photons which in turn react to produce the hadrons. These reactions are characterised by the presence of electrons and positrons at small angles to the beam direction in the final state as well as by the presence of hadrons. The cross section is roughly given by

$$\sigma \propto \frac{\alpha^4}{M_h^2} \left(\ln \frac{E}{m_e}\right)^2 \qquad (4)$$

where M_h is the mass of the hadron system produced. This cross section can become comparable with the one photon annihilation cross section in spite of the extra α^2, for the denominator contains M_h^2 instead of S and the numerator contains a $\log^2 \frac{E}{m_e}$. The ratio of this cross section to the mu-pair production cross section is given by

$$\frac{\sigma}{\sigma_{\mu\mu}} \propto \alpha^2 \left(\ln \frac{E}{m_e}\right)^2 \frac{S}{M_h^2} \qquad (5)$$

and is expected to be small when the sum of the hadron energy is not small. For example, Brodsky has estimated the total cross section for this process will be less than one nanobarn when the total energy of the hadron system is greater than 1 GeV[12].

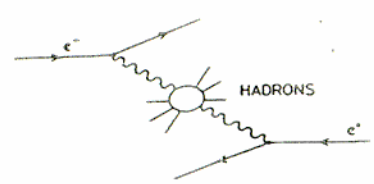


Fig. 5. Schematic of the two photon process.

Richter - London Conference 1974

X THEORY

The e^+e^- annihilation data contradict both the simple quark-parton model and the Bjorken scaling hypothesis. This has come as a shock for they were both doing so well - giving an understanding of multiplet structure, cross section relationships, decay branching ratios, deep inelastic electron, neutrino, and muon scattering, etc. Indeed, scaling was tested and found to work to 10% to 20 % over three orders of magnitude in the structure functions and for values of momentum transfer ranging up to 60 to 70 $(GeV/c)^2$ and values of inelasticity out to 100 GeV. Most of the 61 theoretical contributions to this session of the conference which range from the bizarre to the ordinary, attempt to resolve the contradiction between the success of simple models in the space-like momentum transfer region and their failure in the time-like momentum transfer region.

I am supposed to review these contributions for you. I clearly cannot review them all for time is short and I do not understand half of them. (These papers have been reviewed by Ellis and Renard in the parallel session and their reviews are included in the conference proceedings.) I will instead select a few examples of different kinds of models, try to describe their assumptions and predictions, and test them against the data where possible. I will begin with my favourite - models involving new lepton-hadron interactions.

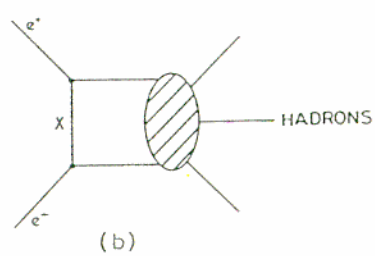
a. New Interactions

When I first saw our data in the SLAC-LBL experiment I was struck by the similarity of certain features such as the roughly constant cross section, the slope of $Ed\sigma/d^3p$ at low momentum, heavy particle yields, and mean multiplicities to similar features seen in hadronic interactions. I suggested then that perhaps electrons had strong interactions of very short range and that what we were seeing was not 1 photon annihilation but a kind of no photon annihilation. I soon discovered I had some very distinguished company for Pati and Salam had been working on such a scheme for some time. Others began working along these lines and at this conference we have contributions from Pati and Salam[23], Greenberg and Yodh[24], Nanopolous and Vlassopoulos[25], and Budini and Furlan[26]. I will describe two of these models.

Figure 21a shows the quark scheme of Pati and Salam which unifies the leptons with a standard set of quarks with colour and charm in a kind of Han-Nambu scheme. The model comes with a set of gauge bosons which couple the leptons to the quarks, and hadron production in e^+e^- interactions proceeds by the exchange of one of these bosons as shown in figure 21b. Working through the model as originally formulated, they find that their gauge bosons are too heavy to account for the cross section. On the

$$\begin{pmatrix} P_1 & P_2 & P_3 & \nu_e \\ n_1 & n_2 & n_3 & e^- \\ \lambda_1 & \lambda_2 & \lambda_3 & \mu^- \\ P_1' & P_2' & P_3' & \nu_\mu \end{pmatrix}$$

(a)



(b)

Fig. 21. Pati-Salam model: (a) The quark scheme including leptons (b) Hadron production via the exchange of a gauge boson.

1975 SLAC Conference (CH10 data) - L. Mo:

observed to simulated event rates are shown in Fig. 7.

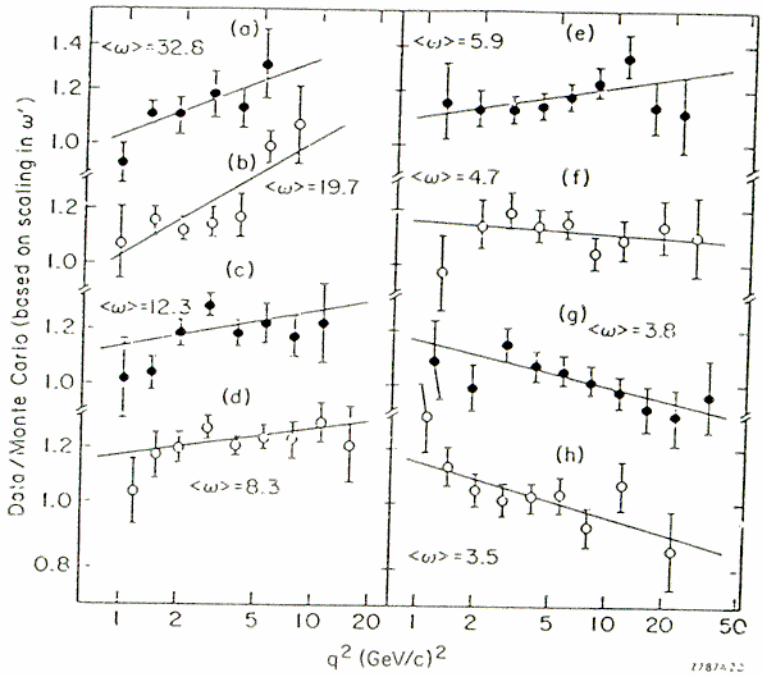


Fig. 7--Ratio of observed to "Monte Carloed" event rate vs. q^2 for eight ranges of ω.

These plots indicate the distinctive feature that the deviation from Bjorken scaling as a function of q^2 has a negative slope at values of ω less than 5, and this slope becomes positive at values of ω greater than 5. This phenomenon seems to be in accord with the prediction of Tung,[11] using conventional field theory and also asymptotic freedom theory with anomalous dimensions. This particular calculation says that at large values of ω, νW_2 should first rise and then fall as q^2 increases; at values of ω less than 5, νW_2 should continue to decrease as q^2 increases.

Vector-dominance models or generalized vector-dominance models also predict[12-14] the increase of νW_2 with q^2.

In the parton model of Drell and Chanowitz[15] scaling violation is exhibited through the introduction of a parton form factor.

We are now facing a very interesting situation. Scale invariance is totally violated in the time-like region.[16] In the space-like region, this muon experiment at Fermilab has indicated a violation of Bjorken scaling only at a level of less than ~30%. This fact alone should provide enough incentives to continue the study at higher energies.

IV. MUON SCATTERING FROM HYDROGEN AND DEUTERIUM

The experiment on muon scattering from a liquid hydrogen or deuterium target was carried out at Fermilab by a Chicago-Harvard-Illinois-Oxford collaboration.[17] The aim of the experiment was to measure the inclusive muon cross sections for the

1975 SLAC Conference (SLAC-MIT data)

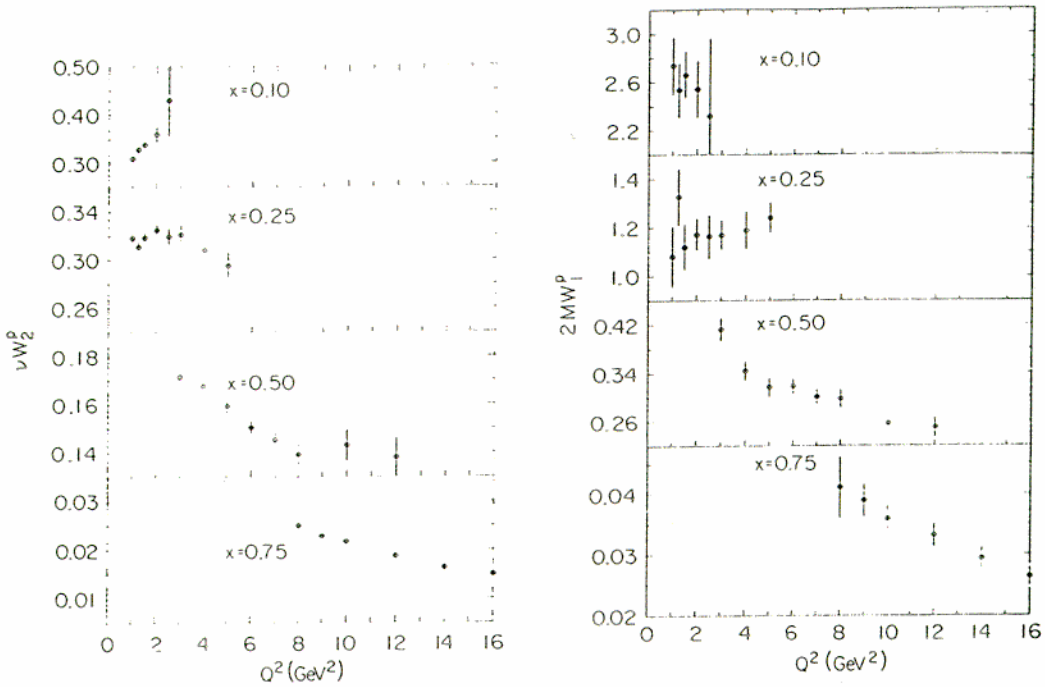


Fig. 4--νW_2^p and $2MW_1^p$ structure functions. Separated proton structure functions are illustrated for selected values of x. It is clear that neither structure function is scaling in x, i.e., that there is Q^2 dependence of the structure function at constant x.

(1) Assume scaling variables, x_a, of the form $1/x_a = \omega_a = \omega + a/Q^2$, where $\omega = 2M\nu/Q^2 = 1/x$ and a is a parameter ($x_0 = x$, $x_{M_p^2} = x'$, etc.).

(2) We allow scale breaking through a term which is linear in Q^2, say $(1 + bQ^2)$.

Then, we can write

$$\nu W_2(Q^2, W^2) = F_2(\omega_a)(1 + bQ^2) \tag{5}$$

F_2 is expressed as a power series in $(1 - x_a)$, e.g.,

$$F_2(\omega_a) = \sum_{i=3}^{i=7} \alpha_i (1 - x_a)^i \tag{6}$$

More complicated forms could be used for the variable, the breaking, or the functional form of F_2, but the present electron data do not really justify the use of more than a couple of parameters (a and b) to describe the breaking.

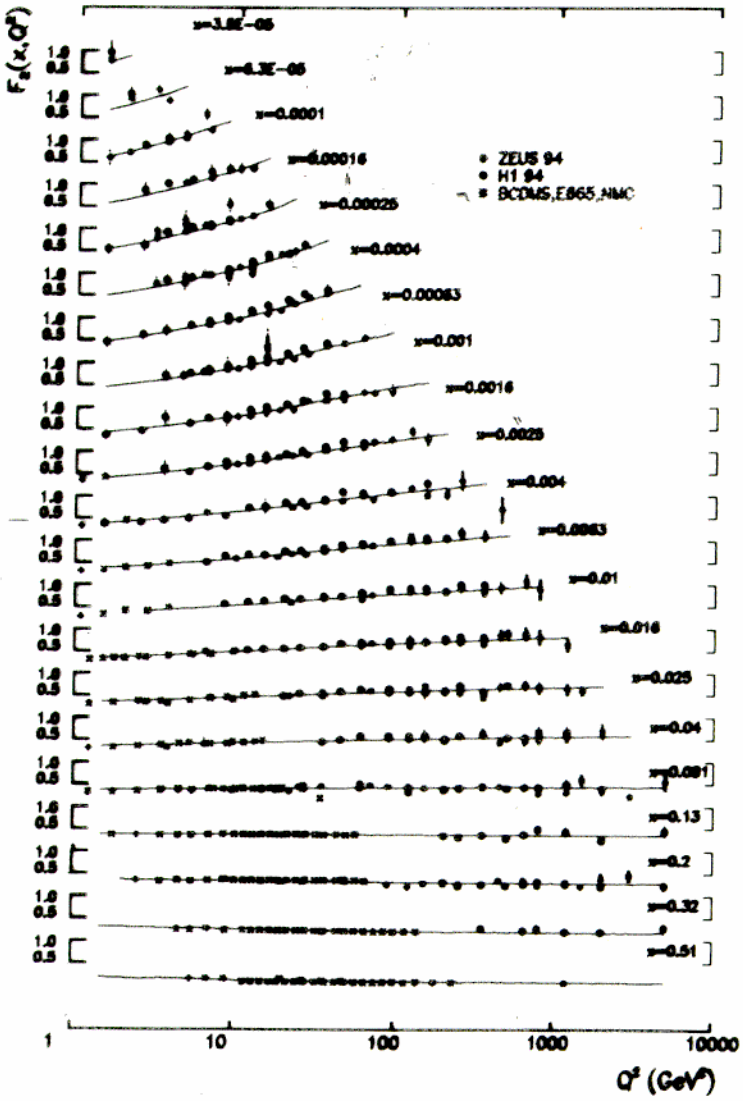
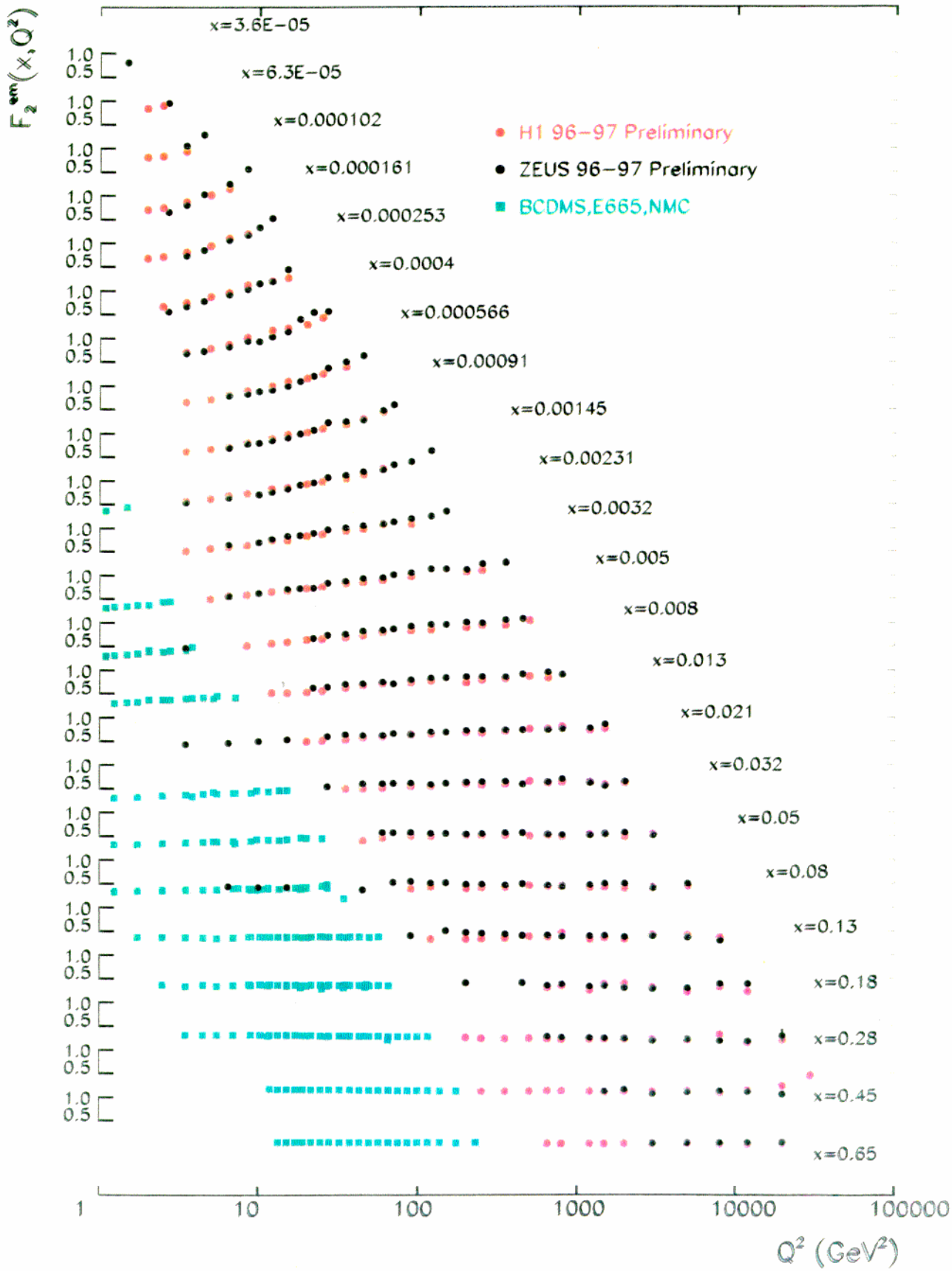
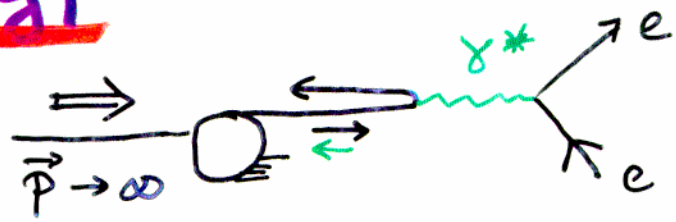


Fig. 29. F_2^p data from HERA(94) and fixed target experiments at fixed x as a function of Q^2. The curves shown are the NLO DGLAP QCD fit used to smooth the data during unfolding.


$F_2^{em}(x,Q^2)$
x=3.6E−05
x=6.3E−05
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.000566
x=0.00091
x=0.00145
x=0.00231
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.28
x=0.45
x=0.65
H1 96−97 Preliminary
ZEUS 96−97 Preliminary
BCDMS,E665,NMC
1.0
0.5
1
10
100
1000
10000
100000
Q^2 (GeV^2)

Spin Dependent Structure Function g_1

- Naive parton model:



$$\int g_1(x)\,dx = \frac{1}{2}\sum Q_i^2 \left(n_i^{\uparrow} + n_{\bar{i}}^{\downarrow} - n_i^{\downarrow} - n_{\bar{i}}^{\downarrow}\right)$$

$$Q_i^2 = \frac{2}{9} + \frac{1}{3} I_3^i + \frac{1}{6} Y^i$$

but $\langle \lambda = 1/2, p | \bar{\Psi} I_3 \gamma_\mu (1 \mp \gamma_5) \Psi | \lambda = 1/2, p \rangle$

$$= 2 p_\mu \sum I_3^i \left(n_i^{\downarrow\uparrow} - n_{\bar{i}}^{\uparrow\downarrow}\right)$$

while $\langle 1/2, p | \bar{\Psi} I_3 \gamma_\mu \gamma_5 \Psi | 1/2, p \rangle = 2 p_\mu \left(-\frac{g_A}{2 g_V}\right)$

$$\Rightarrow \int (g_1^p - g_1^n)\,dx = \frac{1}{6}\left|\frac{g_A}{g_V}\right| \quad \text{B.J}$$

To predict $\int g_1^p dx$ need: hyperon data

$\to$ Y + assumption for singlet:

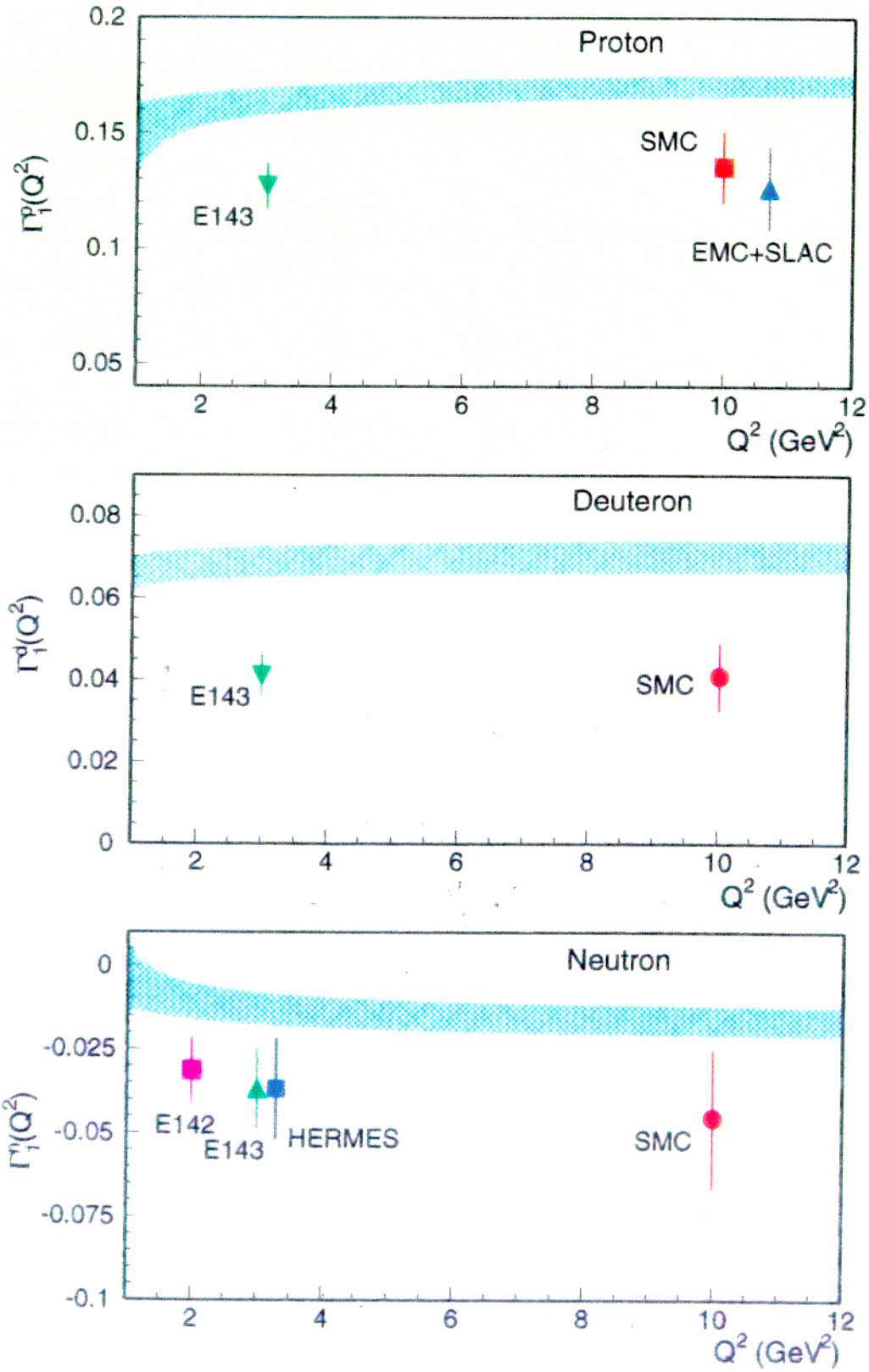
Ellis-Jaffe assumed $n_s^{\uparrow} + n_{\bar{s}}^{\uparrow} - n_s^{\downarrow} - n_{\bar{s}}^{\downarrow} = 0$

$\Rightarrow \int g_1^p dx = 0.189 \pm 0.005$ in serious contradiction with data $\to$

$\Rightarrow \Delta\Sigma = \sum_i \left(n_i^{\uparrow} + n_{\bar{i}}^{\uparrow} - n_i^{\downarrow} - n_{\bar{i}}^{\downarrow}\right) = 0.60 \pm .03$ (= 1 naively)

$= 0.27 \pm .05$ expt

The Ellis–Jaffe Sum Rules



→ What survives in QCD

(renormalization; $\int^{\infty} d^2k_T$)?

- Non-Singlet/Bj – OK

  No special renormalization for $\bar{\Psi}\gamma_\mu\gamma_5 I_3 \Psi$
  – conserved for $m_q \to 0$

- Singlet

  "$\bar{\Psi}\gamma_\mu\gamma_5\Psi$" (= twist 2 Wilson operator)
  needs special renormalization (anomaly!):
  properties depend on renorm. scheme:

→ Respecting QCD gauge invariance:

- No gluon contribution to $\int g_1\, dx$: no appropriate Wilson operator
- $\partial^\mu \left[\bar{\Psi}\gamma_\mu\gamma_5\Psi\right]_{g.i.} = \frac{\alpha_s}{2\pi} F.\tilde{F} + O(m_q)$

q^2 dep.

$\Rightarrow \int g_1^S(x, Q^2)\, dx \sim C(q^2)\, \Sigma(q^2)$

latest fits → $\Sigma(1) = 0.19^{+.04}_{-.05}$

– no help with interpretation

or → "Adler Bardeen"* scheme :

Write

$$[\bar{\Psi}\gamma_\mu\gamma_5\Psi]_{gi} = [\bar{\Psi}\gamma_\mu\gamma_5\Psi]_s + K_\mu$$

conserved as $m_q \to 0$, but gauge dependent

gauge dependent gluon operator ($O(\alpha_s)$)

Proponents → K_μ "obviously" to be identified with gluon contribution

→ $[\bar{\Psi}\gamma_\mu\gamma_5\Psi]_s$ → q^2 indep. contribution "as is necessary for quark model interpretation"

→ $$\Delta\Sigma(q^2) = \Delta\Sigma^{quark} - \frac{\alpha_s}{2\pi}\Delta g$$

above

α_s^{-1} as $q^2 \to \infty$

latest fits → $\Delta\Sigma^q = 0.38 \pm .03$

– still hard to interpret

+ physics unclear ($[\bar{\Psi}\gamma_\mu\gamma_5]_s$ and K_μ coupled to ghost Goldstone boson); this definition of $\Delta\Sigma$ and Δg looks inconsistent with definitions of higher moments of spin dependent parton distributions .....

* AB not responsible

Conclusions on spin dependent quark & gluon distributions

- In singlet case, divergences of quantum field theory really matter
  - do not know how to compare with non field theoretical low energy quark model
- Would like more large q^2 data

eg $\vec{\mu}\,\vec{\vec{p}} \to \mu c \bar{c} \ldots$ → polarised gluon distribution COMPASS

→ consistently defined* and used parton distributions

* True for A-B?

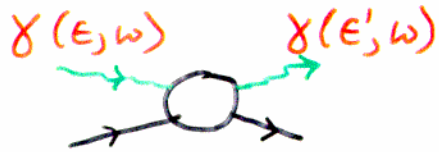
Spin Outlook

$$1/2 = 1/2\Delta\Sigma + \Delta G + L_q + L_G$$

Process/ Experiment	Leading order subprocess	Parton behaviour probed
DIS ($\ell N \to \ell X$) $g_1^{\ell p}, g_1^{\ell d}, g_1^{\ell n}$ (SLAC, EMC/SMC, HERMES)	$\gamma^* q \to q$	Two structure functions $\to$ $\sum_q e_q^2(\Delta q + \Delta\bar{q})$ $\Delta A_3 = \Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}$
$\ell p, \ell n \to \ell \pi X$ (SMC,HERMES)	$\gamma^* q \to q$ with $q = u, d, \bar{u}, \bar{d}$	$\Delta u_V, \Delta d_V, \Delta\bar{q}$
$\ell N \to c\bar{c}X$ (COMPASS, HERMES)	$\gamma g \to c\bar{c}$	ΔG $(x \approx 0.15, 0.3)$
$\ell N \to h^+ h^- X$ (COMPASS)	$\gamma g \to q\bar{q}$ $\gamma q \to qg$	ΔG $(x \approx 0.1)$
$pp \to (\gamma^*, W^\pm, Z^0)X$ (RHIC)	$q\bar{q} \to \gamma^*, W^\pm, Z^0$	$\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d}$ $(x \gtrsim 0.06)$
$pp \to$ jets X (RHIC)	$q\bar{q}, qq, qg, gg \to 2j$	ΔG (?) $(x \gtrsim 0.03)$
$pp \to \gamma X$ (RHIC)	$qg \to q\gamma$ $q\bar{q} \to g\gamma$	ΔG $(x \gtrsim 0.03)$
DIS ($e^\pm p \to \nu X$) $g_1^\pm, g_5^\pm$ (HERA)	$W^* q \to q'$	Two structure functions $\to$ $\Delta u - \Delta\bar{d} - \Delta\bar{s}$ $\Delta d + \Delta s - \Delta\bar{u}$
DIS (small x) g_1^{ep} (HERA)	$\gamma^* q \to q$	α_q, G $(\Delta\bar{q} \sim x^{\alpha_q}, \Delta G \sim x^{\alpha_G})$
$\ell N \to \ell$ jets X (HERA)	$\gamma^* g \to q\bar{q}$ $\gamma^* q \to qg$	ΔG $(x \gtrsim 0.0015)$
$pp \to \ell^+ \ell^- X$ (HERA-$\vec{N}$)	$q\bar{q} \to \gamma^*$	$\Delta\bar{q}$ $(x \gtrsim 0.15)$

Drell Hearn Gerasimov Iddings

Forward γp:



$$T(\omega) = \vec{\epsilon}'^{*}\cdot\vec{\epsilon}\, f(\omega) + i\vec{\sigma}\cdot(\vec{\epsilon}'^{*}\times\vec{\epsilon})\, g(\omega)$$

$$\mathrm{Im}\, g(\omega) \sim \sigma^{3/2}_{\gamma p} - \sigma^{1/2}_{\gamma p}$$

$\sigma_{\gamma p \to \text{all}}$ for $J_z = \pm 3/2, \pm 1/2$

$$\frac{m^2 g(\omega)}{2\alpha\omega} = -\frac{\kappa^2}{4} + O(\omega^2)$$ low energy theorem

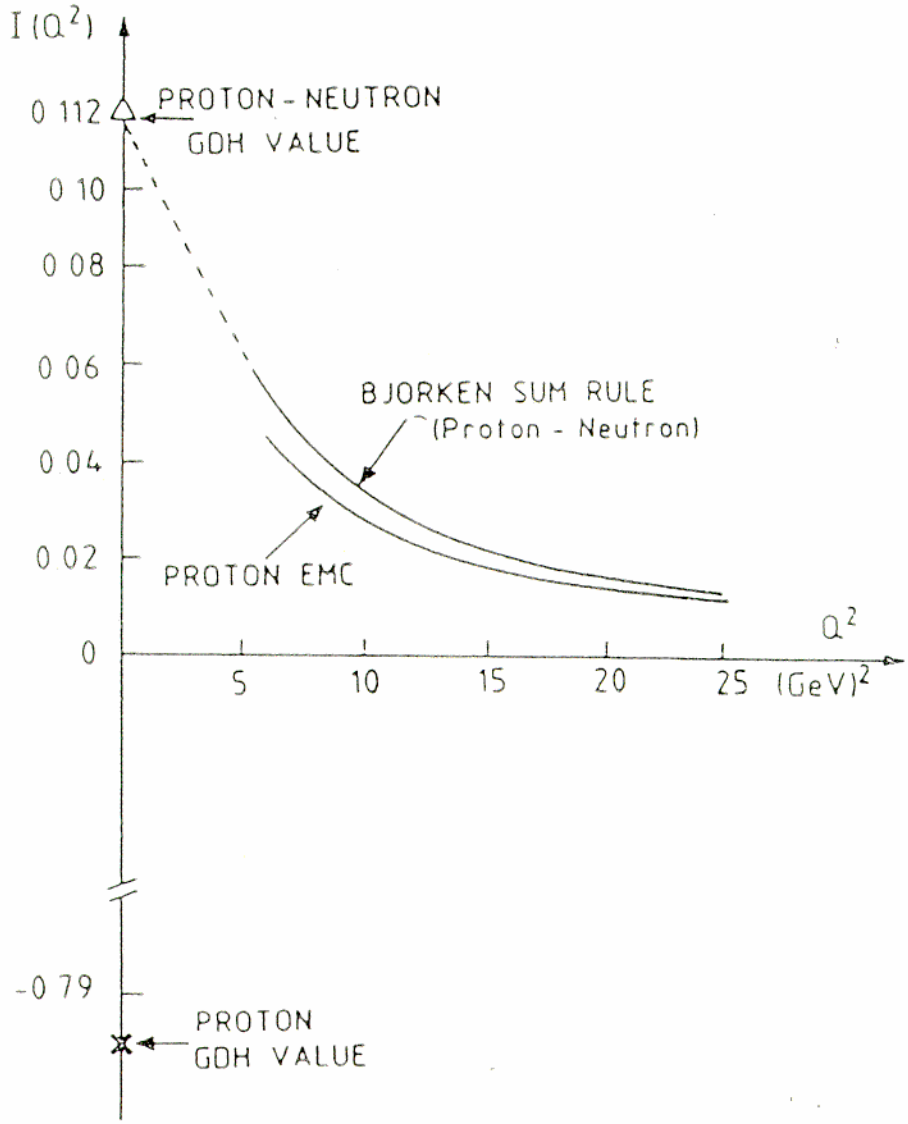
$\Leftarrow$ DGHI

$$= \underbrace{\frac{m^2}{8\pi^2\alpha}\int \frac{d\omega'}{\omega'}\left(\sigma^{3/2} - \sigma^{1/2}\right)}_{= I(0) \text{ where:}} + O(\omega^2)$$

unsubtracted dispersion relation

$$I(Q^2) = \frac{2m^2}{Q^2}\int dx\, g_1(x, Q^2)$$

- DH "Sum rule of interest because of its experimental as well as theoretical implications" – test a "formidable challenge" – not yet done.
- Is it true?
- Connection to large Q^2?
- Other implications


I(Q²)
0 112
PROTON - NEUTRON
GDH VALUE
0 10
0 08
0 06
0 04
0 02
0
BJORKEN SUM RULE
(Proton - Neutron)
PROTON EMC
Q²
5
10
15
20
25
(GeV)²
-0 79
PROTON
GDH VALUE

Is it true?

	p	n	p+n	p−n
• DHGI (μb)	−204.5	−232.8	−437.3	28.3
• "Data" (errors?)	−257	−189	−446	−68

↑ phase shift analysis of unpolarized $\gamma N \to \pi N$ + estimate of 2π contribution (Karliner: $E < 1.2$ GeV: updated, with $E < 1.7$ GeV, by Workman & Arndt)

✗ for p−n (should be most convergent)

✓ for p+n ! accident?

– known ρN, ϵN, ωN... large ⇒

• Resonances	−203	−125	−328	−78

↑ ⇐ $F_{37}(1950)$ from data + quark model (Burkert & Li)

⇒ include $\gamma N \to K\Lambda^*$, $\gamma N \to K\Sigma^*$, (Ellis-Jaffe problem → large $s\bar{s}$ contribution?) continuum

→ a lot to be learned from forthcoming DHGI tests at CEBAF → which will also →

q^2 variation :

$\sigma^{\gamma p}_{3/2} > \sigma^{\gamma p}_{1/2} \longrightarrow \sigma^{\gamma^* p}_{3/2} < \sigma^{\gamma^* p}_{1/2}$

γ prod. : DHGI deep inelastic

- γ prod. data
  - $\gamma N \to \Delta$ M1 : $\sigma^{3/2} = 3\sigma^{1/2}$
  - $\gamma N \to D_{13}(1520)$ $\sigma^{3/2}$ large
  - $\gamma N \to F_{15}(1690)$ $\sigma^{1/2} \approx 0$

Explained in harmonic oscillator quark model (Copley, Karl, Obryk: Feynman, Kisslinger Ravndal) :

- $A^{\gamma p}(D_{13}) \sim |\vec{q}_{cm}|^2 - \alpha^2/g$ ← harmonic osc. Param
- $A^{\gamma p}(F_{15}) \sim |\vec{q}_{cm}|^2 - 2\alpha^2/g$ ← quark g factor

$|\vec{q}_{cm}|^2_{F15} \simeq 2 \times |\vec{q}_{cm}|^2_{D13}$ so both amplitudes can, plausibly, be small but this will change rapidly with q^2 (Close, Gilman 1972)

Latest quark model calculations (Burkert-Li) → complex q^2 variation, different from expectations of chiral perturbation theory .....

→ **await CEBAF**

Inferences from DHGI :

$$(g-2)^2 = \frac{m^2}{8\pi\alpha}\int \frac{d\omega}{\omega}\Delta\sigma \quad , \qquad \Delta\sigma = \sigma^{3/2} - \sigma^{1/2}$$

- QED : $\Delta\sigma = O(\alpha^2)$ $\qquad \gamma e \to \gamma e$

→ $g-2 = 0 + O(\alpha)$

- Left hand side $= O(\alpha^2)$

$$\therefore \int \frac{d\omega}{\omega}\Delta\sigma_{QED} = O(\alpha^3)$$

Altarelli
Cabbibo
Maiani

generally $\int \frac{d\omega}{\omega}\Delta\sigma^{tree}_{\gamma a \to bc} = 0$ in lowest order

Brodsky
Schmidt

→ non-trivial checks on calculations
+ diagnostic for new physics
eg in $\gamma e \to W\nu$ ....

- Constraints on sub-structure :

$$\int \frac{d\omega}{\omega}\Delta\sigma_{\gamma\,lepton} = O(\alpha^3) + \int_{m^*} \frac{d\omega}{\omega}\Delta\sigma_{\gamma L \to X}$$

constituents

$O(\alpha)$

→ $g-2 = 0 + O(\alpha^2)$ ← QED

$+ O\left(\frac{m_L}{m^*}\right)$ $\qquad$ Brodsky & Drell