Fat penguins
and
PQCD

A.I. Sanda

Nagoya University

Collaborators:
Y. Keum, E. Kou, T. Kurimoto, H-n. Li, C. D. Lu,
N. Shinha, R. Shinha, K. Ukai, T. Yoshikawa
M. Yang

My friend and I

Bit of history

$$\frac{\Gamma(K^0 \to \pi^+\pi^-)}{\Gamma(K^+ \to \pi^+\pi^0)} = 447$$

Bose statistics: S wave $\pi\pi$ state I=0, 2

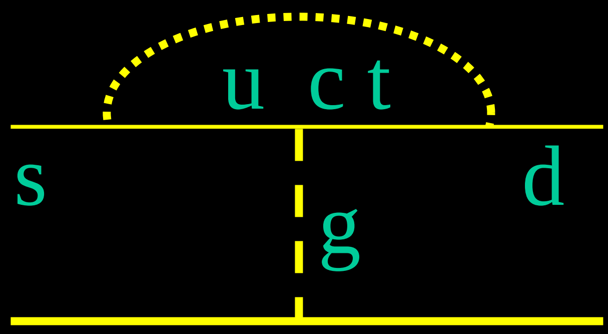
$\pi^{\square}\pi^{-}$ can be in I=0 or in I =2

But $\pi^{\square}\pi^{\square}$ can only be in I=2

→ $\Delta I = \frac{1}{2}$ rule

Gell-Man and Pais 50 years ago

Penguins came to save the day



svz

$$\left\langle \pi\pi \,|\, \bar{s}_L \gamma_\mu T^a d_L (\bar{u}\gamma^\mu T^a u + \bar{d}\gamma^\mu T^a d + \bar{s}\gamma^\mu T^a s) \,|\, K \right\rangle$$

Penguin can not cause ΔI=2 transitions

$$\langle \pi\pi \,|\, \bar{s}_L \gamma_\mu T^a d_L (\bar{u}\gamma^\mu T^a u + \bar{d}\gamma^\mu T^a d + \bar{s}\gamma^\mu T^a s) \,|\, K \rangle$$

Chiral perturbation theory

Factorization approximation

$$\langle \pi\pi \,|\, \bar{s}_L u_R (\bar{u}_R d_L + \bar{d}_R d_L + \bar{s}_R d_L) \,|\, K \rangle$$

$$= \langle \pi \,|\, \bar{s}_L \Gamma^i u_R \,|\, K \rangle \quad \langle \pi \,|\, \bar{u}_R d_L + \bar{d}_R d_L + \bar{s}_R d_L \,|\, 0 \rangle$$

$$= \frac{q^{\varpi}}{m_s + m_d} \langle \pi \,|\, \bar{s}_L \gamma_\mu{}^i u_L \,|\, K \rangle \frac{p_\pi^\mu}{m_s + m_d} \langle \pi \,|\, \bar{u}_R \gamma_\mu d_R + \bar{d}_R \gamma_\mu d_R + \bar{s}_R \gamma_\mu d_L \,|\, 0 \rangle$$

$$= \frac{(p_K - p_\pi)^\mu}{m_s + m_d} (p_K + p_\pi)_\mu f_+ \frac{p_\pi^\mu}{m_s + m_d} p_{\mu\pi} f_\pi$$

$$= \frac{M_K^2}{m_s + m_d} f_+ \frac{M_\pi^2}{m_u + m_d} f_\pi$$

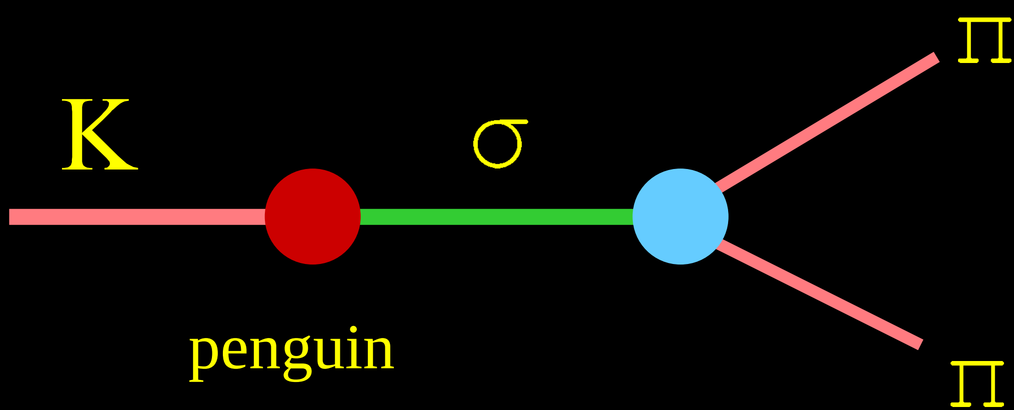
$$\langle \pi\pi \,|\, \bar{s}_L \gamma_\mu T^a d_L \bar{u}_L \gamma^\mu T^a u_L \,|\, K \rangle$$

$$= \langle \pi \,|\, \bar{s}_L \gamma_\mu T^a d_L \,|\, K \rangle \langle \pi \,|\, \bar{u}_L \gamma^\mu T^a u_L \,|\, 0 \rangle$$

$$= M_K^2 f_+ f_\pi$$

factor of 10 enhancement

My understanding of the ΔI=1/2 rule



Penguins play important role
in the $\Delta I=1/2$ rule

For B physics it also play important role

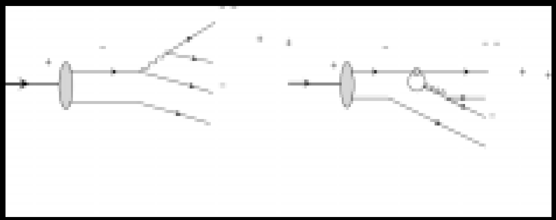
Rare decays give us chance to hunt for
physics beyond the standard model

But, they pollute CP asymmetries

What we have leaned

$$\frac{A(B \to f)}{A(\bar{B} \to \bar{f})} = \frac{V_T T + V_P P e^{i\delta}}{V_T^* T + V_P^* P e^{i\delta}}$$

$$\frac{A(B \to f)}{A(\bar{B} \to \bar{f})} = \frac{V_T}{V_T^*} \frac{1 + \frac{V_P P e^{i\delta}}{V_T T}}{1 + \frac{V_P^* P e^{i\delta}}{V_T^* T}}$$



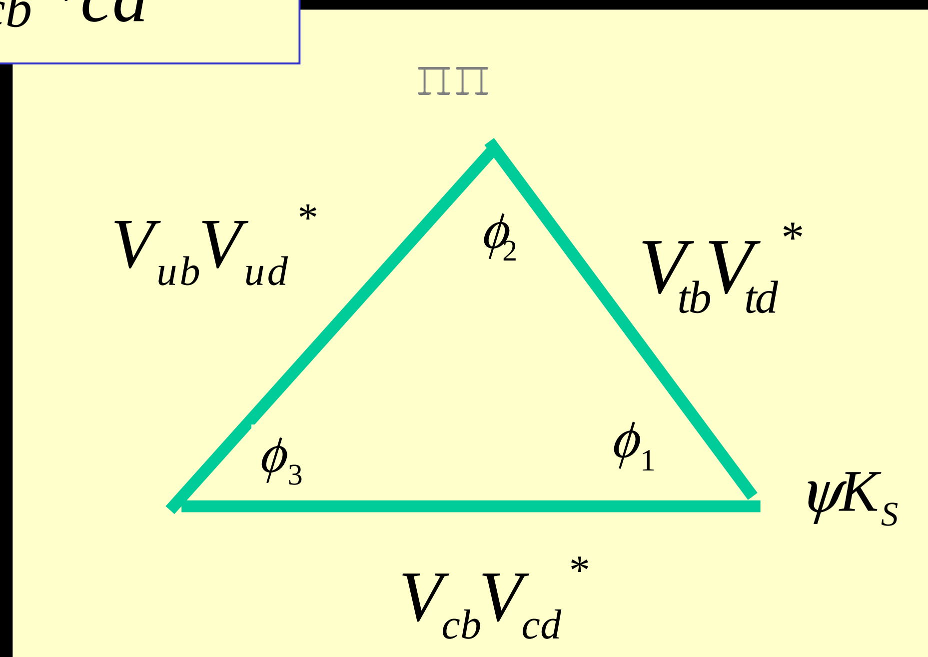
Ratio is independent of strong interaction if:

1. Penguin and Tree have same KM phase
2. Penguin is absent

Nearly 100%....

$$\sin(2\phi_1) = \mathrm{Im}\left[\frac{V_{tb}V_{td}^{*}V_{cb}^{*}V_{cd}}{V_{tb}^{*}V_{td}V_{cb}V_{cd}^{*}}\right]$$

Bj notation
Rosner&AIS
Fermilab proeedings



$$V_{cb}V_{cd}^{*} + V_{ub}V_{ud}^{*} + V_{tb}V_{td}^{*} = 0$$

Large CP Violation has been discovered!

$$B \to \psi K_S$$

$\sin 2\phi_1 = 0.82 \pm 0.12(\text{stat}) \pm 0.05(\text{syst})$ Belle

$\sin 2\phi_1 = 0.75 \pm 0.09(\text{stat}) \pm 0.04(\text{syst})$ Babar

Where do we go from here?

Penguins seems to be large in B decays

$$\frac{A(B\to K\pi)}{A(B\to\pi\pi)}=\frac{V_{ub}^{*}V_{us}T+V_{tb}^{*}V_{ts}P}{V_{ub}^{*}V_{ud}T+V_{tb}^{*}V_{td}P}=\frac{\lambda^{4}T+\lambda^{2}P}{\lambda^{3}T+\lambda^{3}P}$$

If T dominated over P,

$$\frac{Br(B\to K\pi)}{Br(B\to\pi\pi)}=\lambda^{2}\approx\frac{1}{20}$$

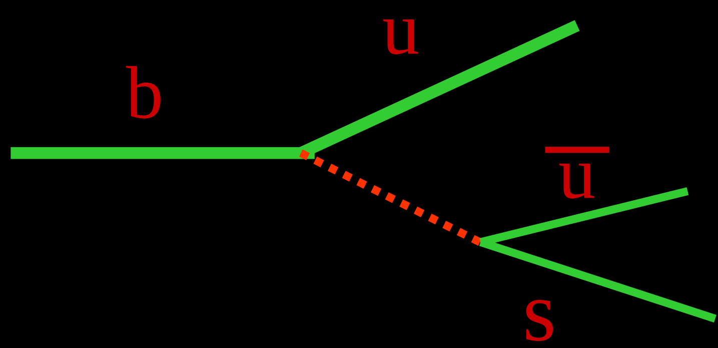
$$\frac{Br(B\to K\pi)}{Br(B\to\pi\pi)}>1 \quad\Rightarrow\quad \frac{P}{T}>\lambda$$

Fat penguins

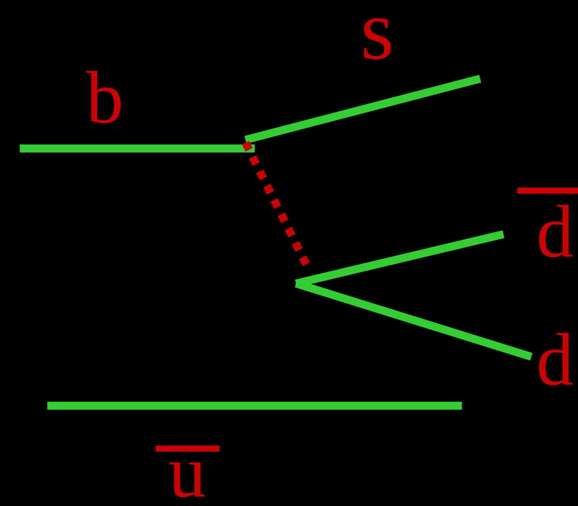
We expected

$$\frac{P}{T}=\frac{\alpha}{12\pi^{2}}\log\frac{M_{W}}{\mu}=O(\lambda^{2})$$

T



P



$P:\Delta I=0$

$B^- \rightarrow \pi^0\pi^-$ Pure T

$B^- \rightarrow \overline{K}^0\pi^-$

Pure P

$$\frac{Br(B^- \rightarrow \overline{K}^0\pi^-)}{Br(B^- \rightarrow \pi^0\pi^-)} = \left|\frac{V_{tb}V_{ts}^*P}{V_{ub}V_{ud}^*T}\right|^2 = 3$$

$$\Rightarrow \left|\frac{P}{T}\right| = .11$$

$$\left|\frac{q}{p}\frac{A(\overline{B}\to\psi K_S)}{A(B\to\psi K_S)}\right| = 0.92 \pm 0.06 \pm 0.03$$

Babar

With an assumption that |q/p|=1:

$$\left|\frac{q}{p}\right| = 0.92 \pm 0.06 \pm 0.03$$

$$G_{\pi^+\pi^-}(t) \propto 1 + C\cos\Delta Mt - \sqrt{1-C^2}\,\sin(\phi_2+\phi_\rho)\sin\Delta Mt$$

$$C = \frac{1-\left|\overline{\rho}(\pi^+\pi^-)\right|^2}{1+\left|\overline{\rho}(\pi^+\pi^-)\right|^2} \qquad \left|\overline{\rho}(\pi^+\pi^-)\right| = \frac{A(\overline{B}\to\pi^+\pi^-)}{A(B\to\pi^+\pi^-)} \neq 1$$

$$S = -2\frac{\mathrm{Im}\left[q/p\,\overline{\rho}(\pi^+\pi^-)\right]}{1+\left|\overline{\rho}(\pi^+\pi^-)\right|^2} = -2\frac{\left[\overline{\rho}(\pi^+\pi^-)\right]}{1+\left|\overline{\rho}(\pi^+\pi^-)\right|^2}\sin(\phi_2+\phi_\rho)$$

$$C^2 + S^2 = 1$$

$$C = .94^{+.25}_{-.31} \pm .09$$

$$S = -1.21^{+.38}_{-.27}\ {}^{+.16}_{-.13}$$

Belle

$$C = -0.25^{+.45}_{-..47} \pm .14$$

$$S = 0.03^{+.53}_{-.56} \pm .11$$

Babar

$$G(t) = a + be^{\Delta\Gamma t} + ce^{\Delta\Gamma t/2}\cos\Delta M t + de^{\Delta\Gamma t/2}\sin\Delta M t$$

$$a = \tfrac{1}{2}(1 + |\eta^{-1}\overline{\rho}|^2) + \mathrm{Re}(\eta^{-1}\overline{\rho}) \qquad \eta = \frac{q}{p}$$

$$\overline{a} = \tfrac{1}{2}(|\overline{\rho}|^2 + |\eta|^2) + \mathrm{Re}(\eta\overline{\rho}^*)$$

$$b = \tfrac{1}{2}(1 + |\eta^{-1}\overline{\rho}|^2) - \mathrm{Re}(\eta^{-1}\overline{\rho})$$

$$\overline{b} = \tfrac{1}{2}(|\overline{\rho}|^2 + |\eta|^2) - \mathrm{Re}(\eta\overline{\rho}^*)$$

$$c = 1 - |\eta^{-1}\overline{\rho}|^2$$

$$\overline{c} = |\overline{\rho}|^2 - |\eta|^2$$

3 unknowns

Lots of observables

$$d = -2\,\mathrm{Im}(\eta^{-1}\overline{\rho}\ )$$

$$\overline{d} = -2\,\mathrm{Im}(\eta\overline{\rho}^*)$$

We have learned that Penguins are large!

Model independent measurement
is difficult

Dynamical calculation of P and T
Should be used as guide lines
In digging for physics beyond the SM

Nonleptonic 2 body decays

$$
\begin{aligned}
B \;\rightarrow\;\; & \pi^+\pi^-,\; \pi^0\pi^0,\; \eta'\eta',\; \eta\eta',\; \pi^0\eta',\; \pi^0\eta \\
& K_S\pi^0,\; K_S\eta',\; K_S\eta,\; \rho^0 K_S,\; \phi K_S, \\
& K^0\bar{K}^0,\; K^+K^-,\; K^{*0}\bar{K}^{*0},\; K^{*+}K^{*-} \\
& \rho^0\pi^0,\; \omega\pi^0,\; \rho^0\eta,\; \rho^0\eta',\; \omega\eta,\; \omega\eta', \\
& \phi\pi^0,\; \phi\eta,\; \phi\eta',\; \rho^+\rho^-,\; \omega\omega,\; \rho^0\omega,\; \rho^0\phi',
\end{aligned}
$$

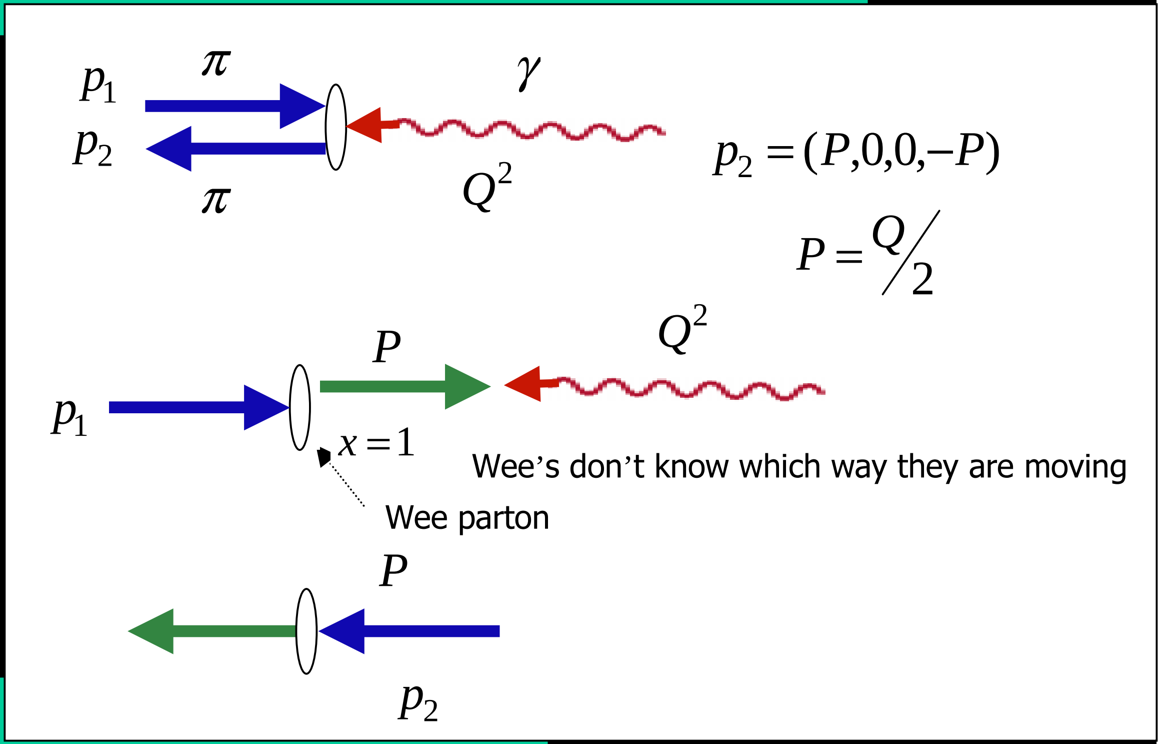
Over 70 decay modes

History of pQCD approach

- Brodsky Lepage PR D22,2157(80)
- Isgar Llewellynsmith NPB317,526(89)
- Botts Sterman NP B325, 62(89)
- Li and his collaborators
- Kroll Eur.Phys.J.C12,99(00)
- Li, Keum, AIS hep-ph/0004173 PR
  hep-ph/0004004

Feynman's Mistake?
Pion form factor

$F(Q^2)$ Probability of finding a parton near $x = 1$

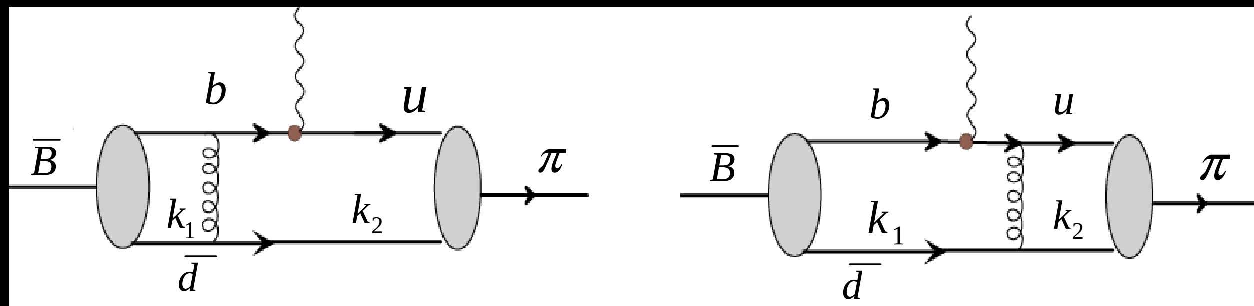


$F(Q^2)$

Depends on wee dynamics

Cannot be computed by perturbative QCD

Feynman's reasoning – Naive QCD



(a) (b)

$$\langle \pi(P_2)|J_\mu(0)|\pi(P_1)\rangle = g^2 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} d^4x\, d^4y\, e^{-ik_2 y}\langle \pi|\bar{u}_\gamma(0) d_\beta(y)|0\rangle \times e^{ik_1 x}\langle 0|\bar{d}_\alpha(x) u_\delta(0)|\pi\rangle\, T_H^{\gamma\beta;\alpha\delta},$$

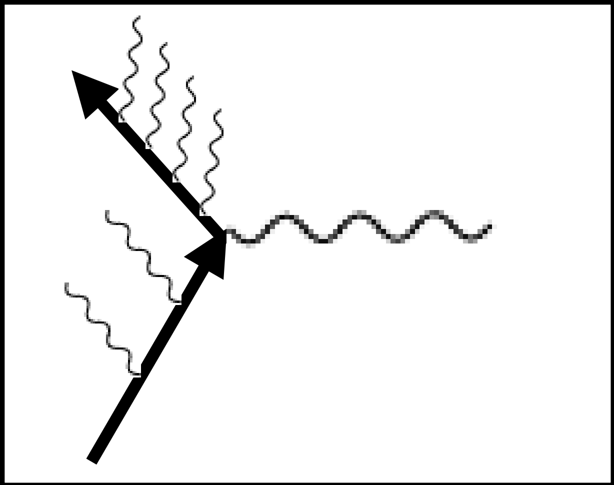
$$T_H^{\gamma\beta;\alpha\delta} = [\gamma_\sigma]^{\alpha\beta}\frac{1}{(k_2-k_1)^2}\left[\gamma_\mu \frac{\not{P}-\not{k}_2}{(P-k_2)^2}\gamma^\sigma\right]^{\gamma\delta}.$$

$$(k_2-k_1)^2 \propto x_1 x_2,$$

Infrared singularity! Infrared singularity!

◇Isgar Llewellynsmith NPB317,526(89) Isgar Llewellynsmith

Sudakov Factor in QED



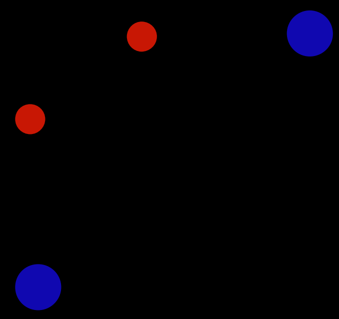
$$\left(\frac{d\sigma}{d\Omega}\right)_{measured} = \left(\frac{d\sigma}{d\Omega}\right)_0 \times \left| exp\left[-\frac{\alpha}{2\pi}\log\left(\frac{-q^2}{m^2}\right)\log\left(\frac{-q^2}{E_l^2}\right)\right]\right|^2$$

Feynman says small x and small $k_\perp$ dominates

This is not so in QCD

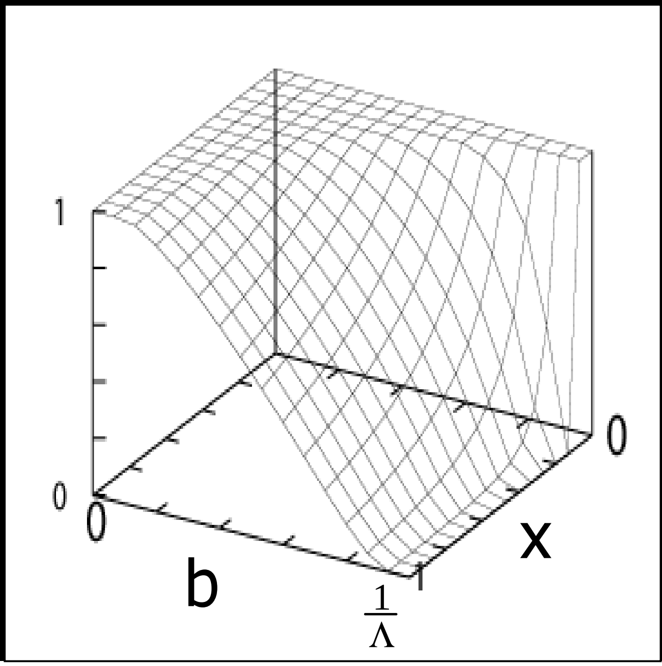
Small $x \Rightarrow$ large longtudinal separation

Small $k_\perp \Rightarrow$ large transverse separation

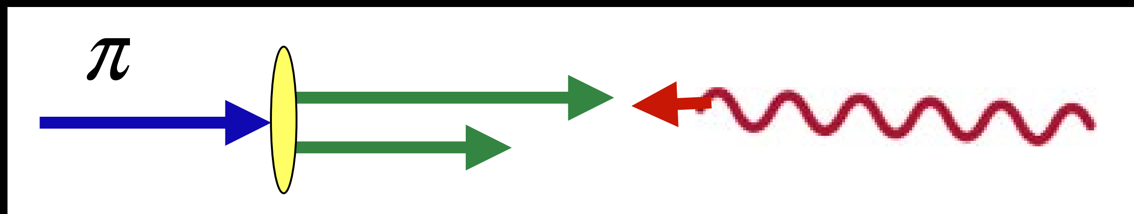


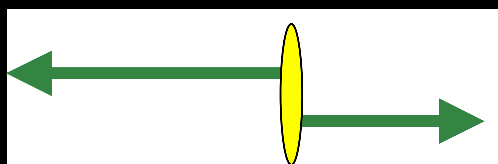
The quark and anti-quark are far apart in space

Sudakov factor suppress these regions

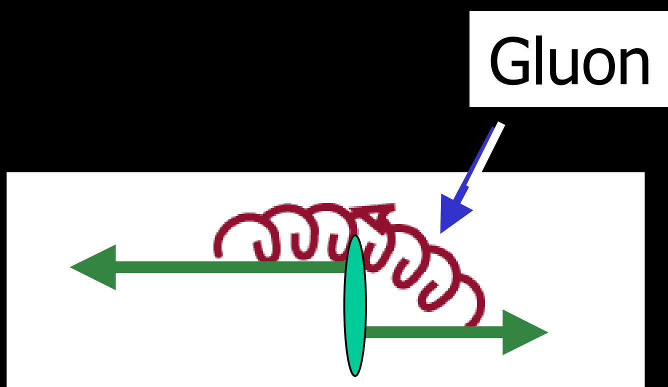


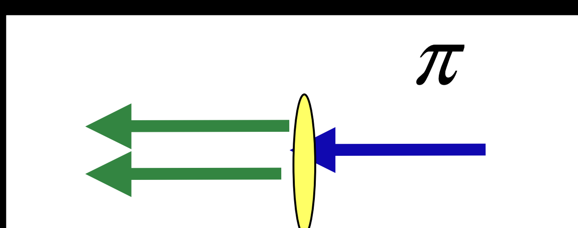
PQCD approach to pion form factor

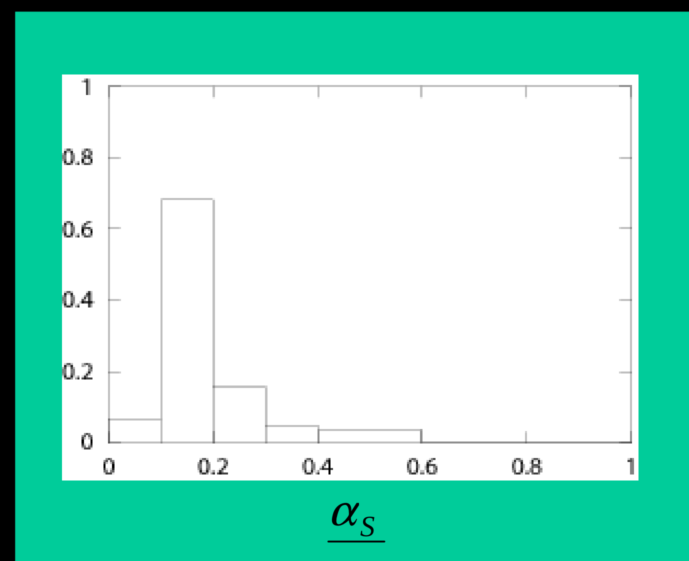




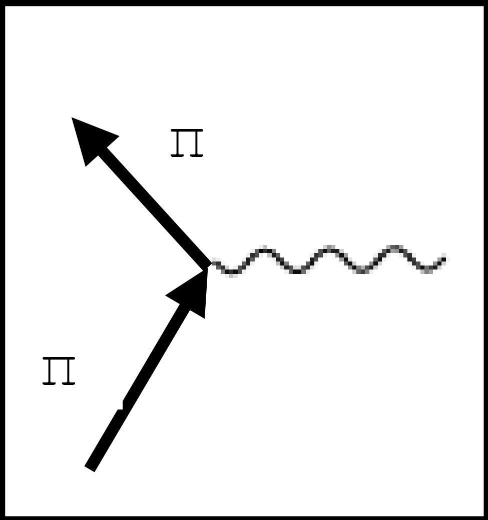
$\pi + "\gamma" \rightarrow X$

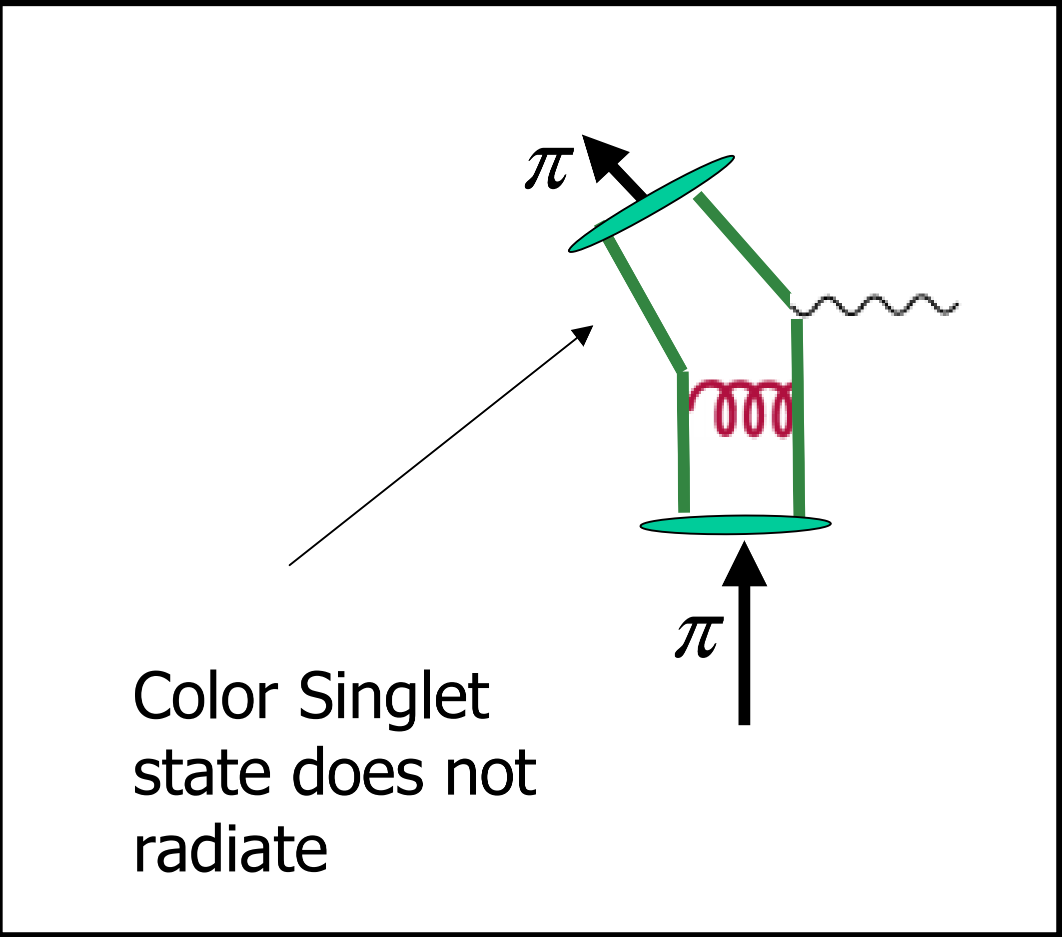






Pion form factor

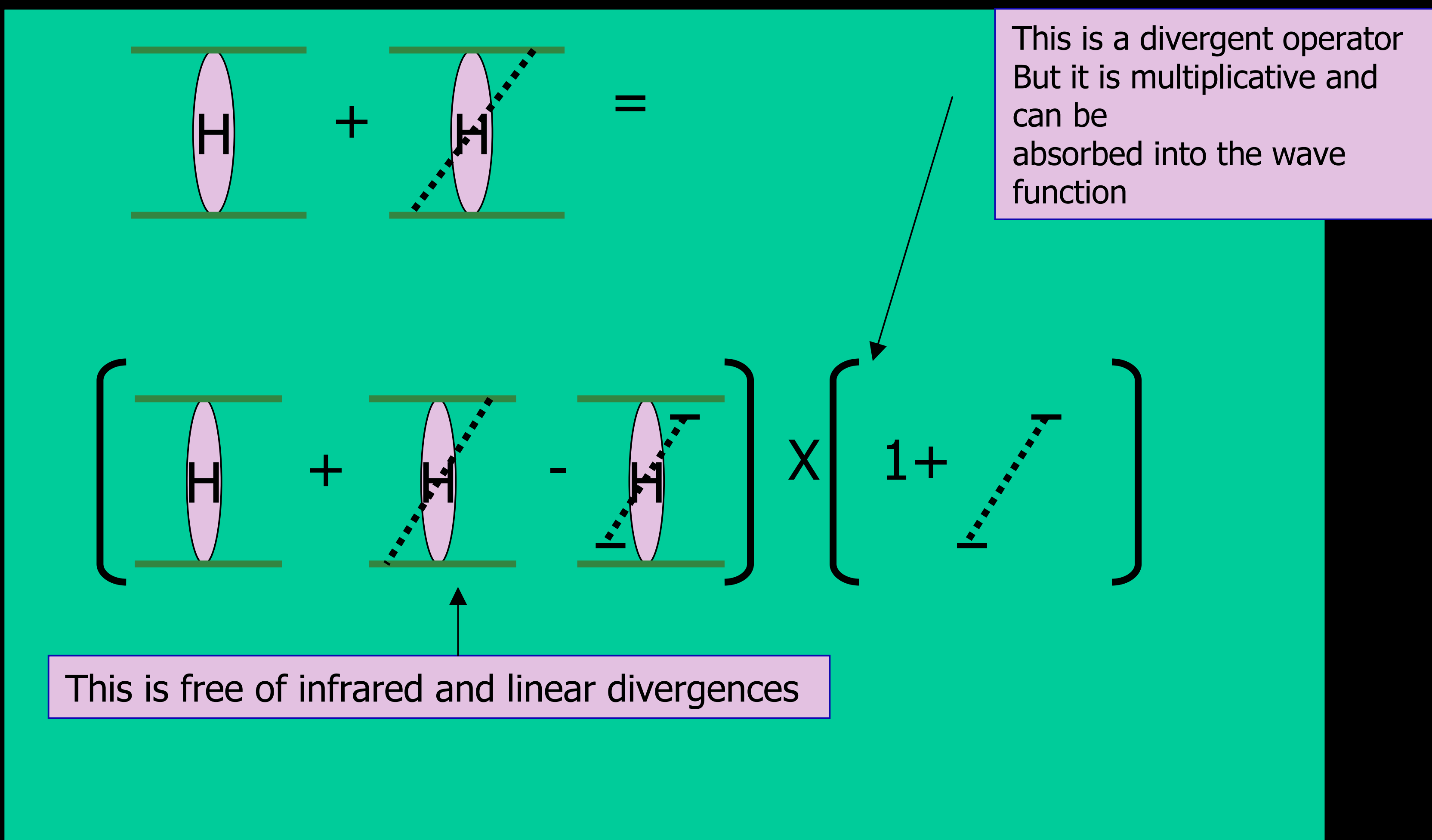




Sudakov factor

Factorization Theorem

- Brodsky Lepage PR D22,2157(80)
- Botts Sterman NP B325, 62(89)
- Li and his collaborators



Pion wave function

$$\langle \pi^-(P)|\bar{d}(z)\gamma_\mu\gamma_5 u(0)|0\rangle = -i\frac{f_\pi}{N_c}P_\mu \int_0^1 dx e^{ixP\cdot z}\phi_v(x) ,$$

$$\langle \pi^-(P)|\bar{d}(z)\gamma_5 u(0)|0\rangle = -i\frac{f_\pi}{N_c}m_0 \int_0^1 dx e^{ixP\cdot z}\phi_p(x) ,$$

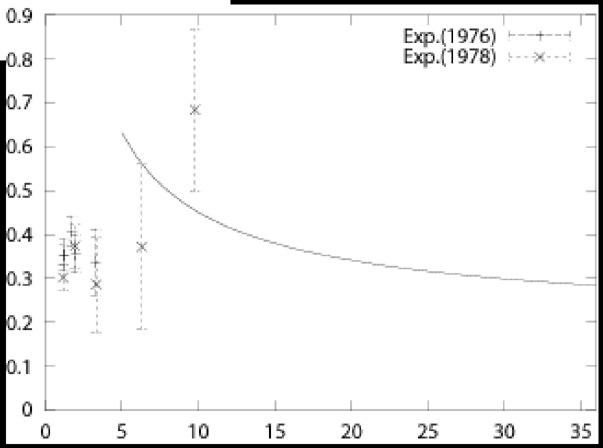
$$\langle \pi^-(P^-)|\bar{d}(z)\sigma_{\mu\nu}\gamma_5 u(0)|0\rangle = -\frac{1}{6N_c}f_\pi m_0 \epsilon_{\mu\nu} \int_0^1 dx e^{ixP\cdot z}\frac{d}{dx}\phi_\sigma(x) .$$

$$\phi_\pi(x) = \frac{3}{\sqrt{2N_c}}f_\pi x(1-x)[1+0.44C_2^{3/2}(2x-1)+0.25C_4^{3/2}(2x-1)] ,$$

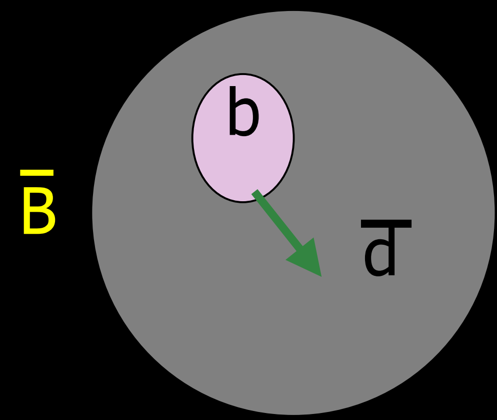
$$\phi_\pi^p(x) = \frac{f_\pi}{2\sqrt{2N_c}}[1+0.43C_2^{1/2}(2x-1)+0.09C_4^{1/2}(2x-1)] ,$$

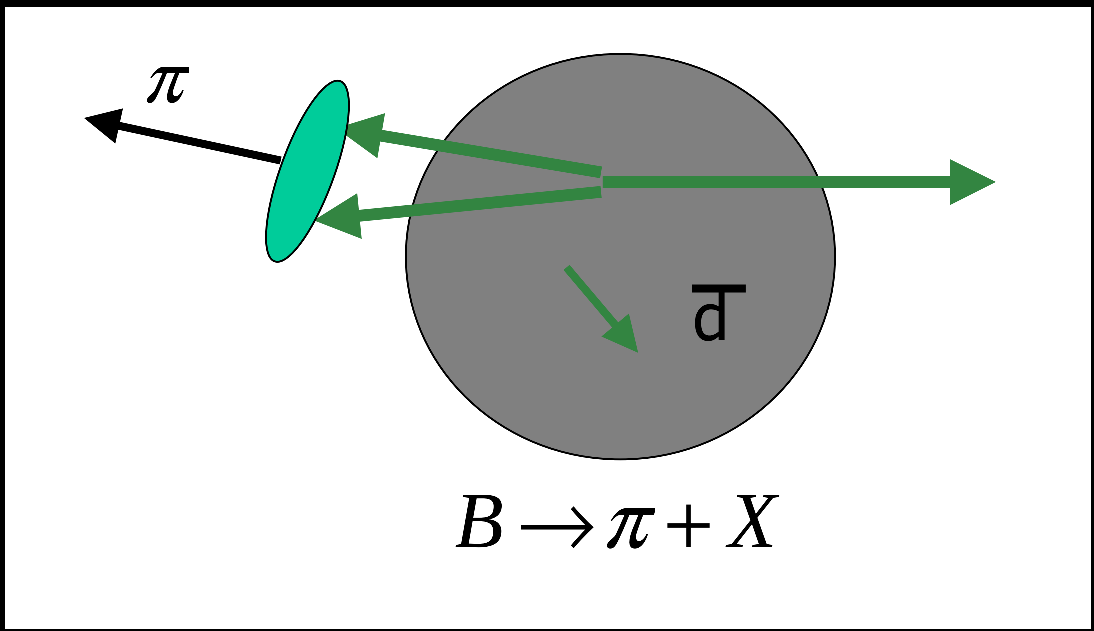
$$\phi_\pi^t(x) = \frac{f_\pi}{2\sqrt{2N_c}}(1-2x)[1+0.55(10x^2-10x+1)] ,$$

Pion formfactor

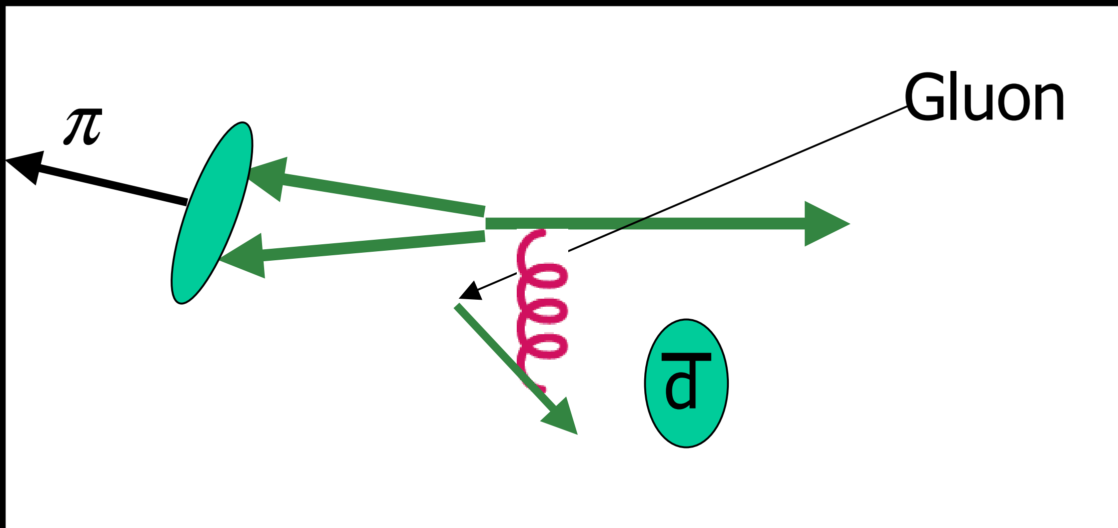


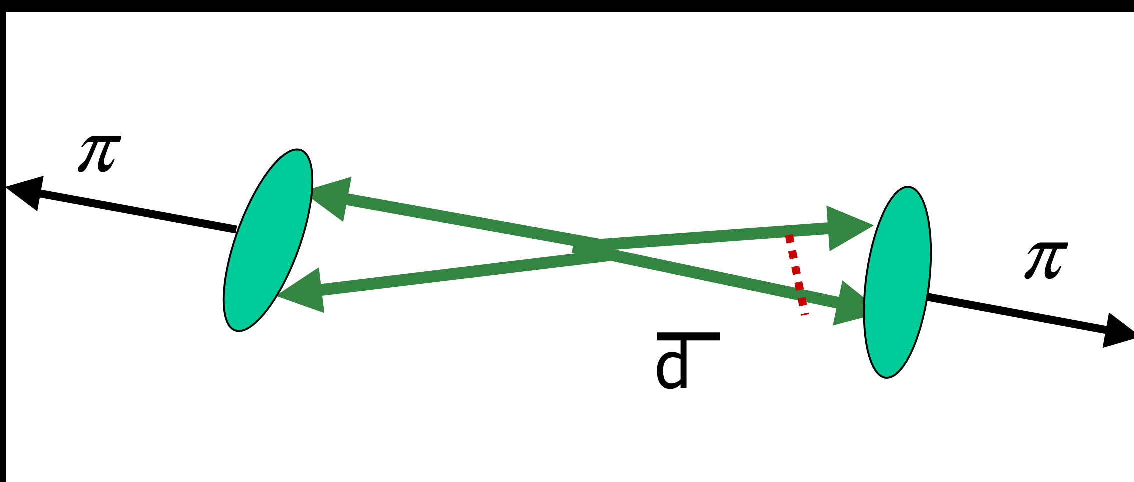
b quark decay





PQCD approach

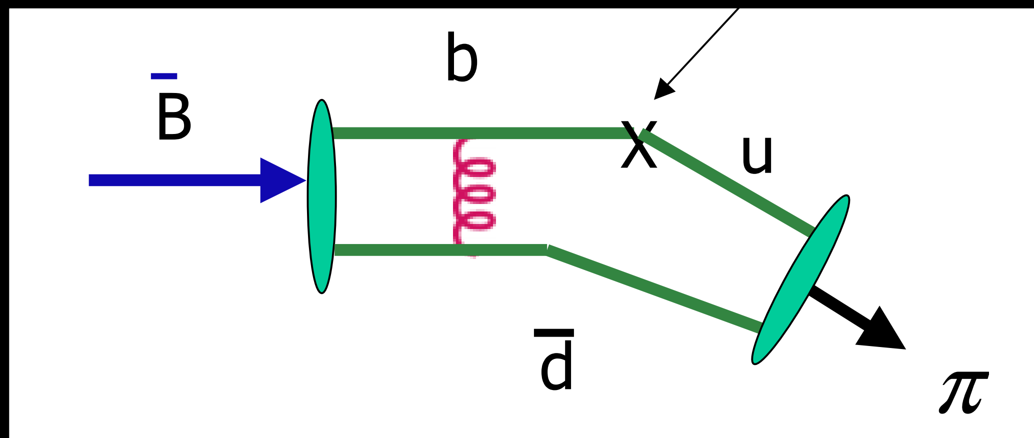




$B \to \pi + \pi$

$B \to \pi$ transition form factor

$$\frac{G_F}{\sqrt{2}} V_{ub}\, \bar{u}\gamma_\mu (1-\gamma_5) b$$



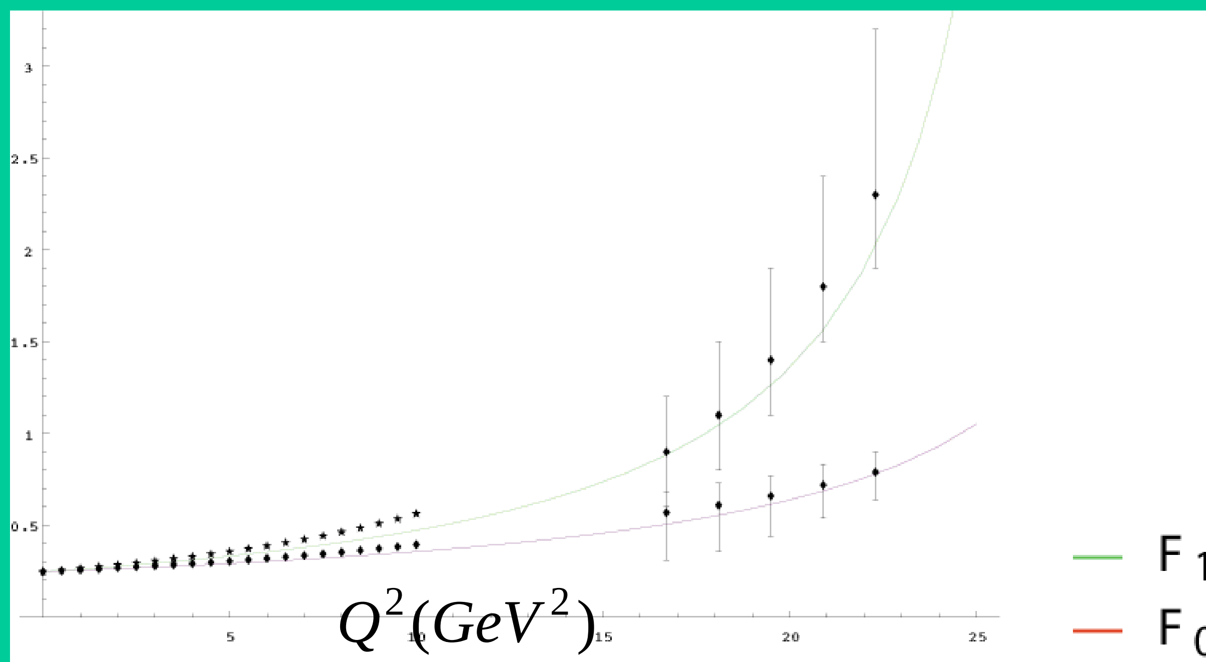
$$\langle \pi(p_2) | \bar{u}\gamma_\mu (1-\gamma_5) b | B(p_1) \rangle$$

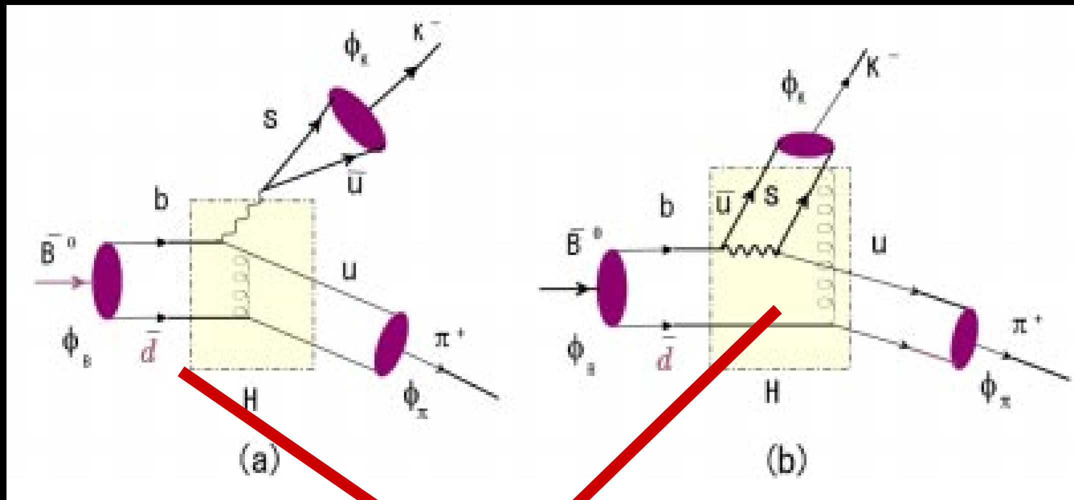
$$= \left[(p_1 + p_2)_\mu - \frac{M_B{}^2 - m_\pi{}^2}{q^2} q_\mu \right] F_1(q^2)$$

$$+ \frac{M_B{}^2 - m_\pi{}^2}{q^2} q_\mu F_2(q^2)$$

$B \to \pi$ transition form factor

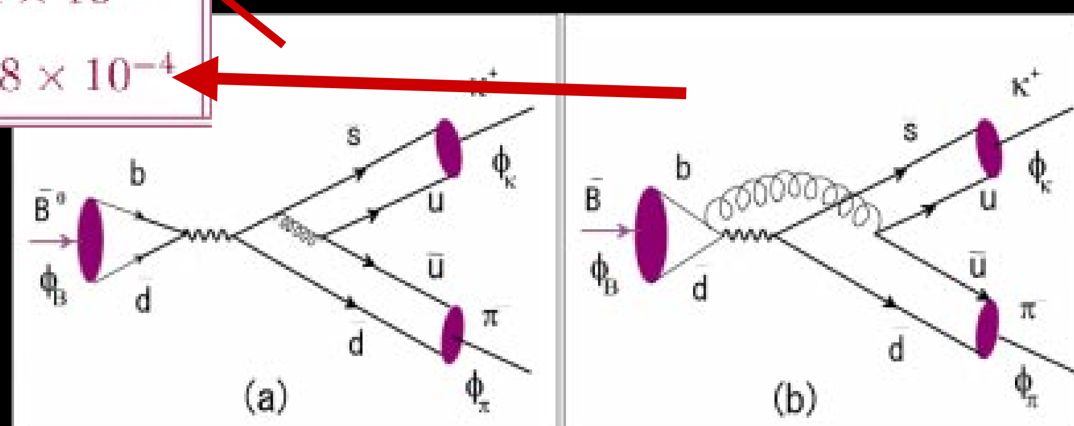
$$\int \frac{d^4 y}{(2\pi)^4} e^{ik_1 \cdot y} \langle 0 | \bar{b}_\beta(0) d_\gamma(y) | B(P_1) \rangle = -\frac{i}{\sqrt{2N_c}} \left[(\not{P}_1 + M_B)\gamma_5 \{ \phi_B(k_1) + (\not{n}_+ - \not{n}_-) \bar{\phi}_B(k_1) \} \right]_{\gamma\beta}$$





F_e	5.577×10^{-1}
F_e^P	-5.537×10^{-2}
F_a^P	$3.333 \times 10^{-3} + i\, 3.181 \times 10^{-2}$
M_e	$-0.942 \times 10^{-3} + i\, 3.385 \times 10^{-3}$
M_e^P	$2.931 \times 10^{-5} - i\, 1.304 \times 10^{-4}$
M_a^P	$-9.397 \times 10^{-5} - i\, 1.918 \times 10^{-4}$

We now know Why FA works



	Exp	PQCD
$K^{□}\pi^{\pm}$	18.4± 2.2	16.4± 3.3
$K^{\pm}\pi^{\pm}$	18.5± 1.5	15.5± 3.3
$K^{\pm}\pi^{□}$	11.5± 1.5	9.1± 1.9
$K^{□}\pi^{□}$	8.8 ± 2.2	8.6± 2.2
$\pi^{□}\pi^{-}$	4.6± 0.8	7.0± 2.0
$\pi^{□}\pi^{□}$	5.9± 1.4	3.7± 1.3
$\pi^{□}\pi^{□}$		0.3±0.1

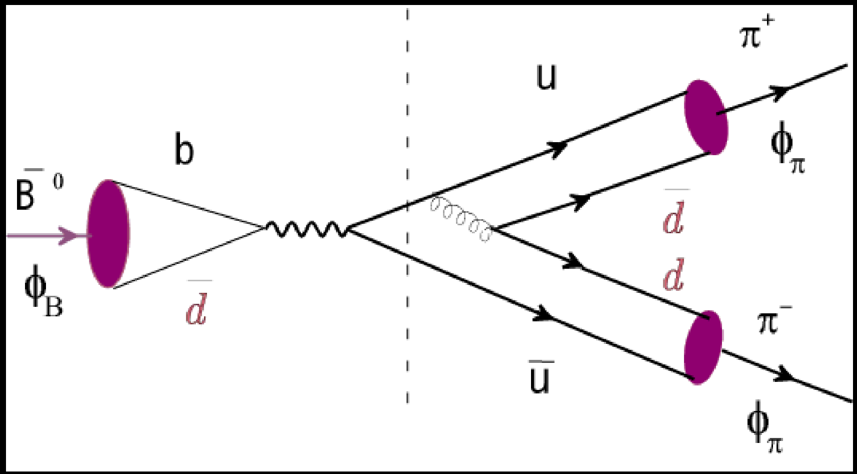
$\pi\pi$ branching ratio would agree better if penguins are larger

CP asymmetry

$$A(B \rightarrow f) = Ae^{i(\phi+\delta)} + Be^{i(\phi'+\delta')}$$

$$A(\overline{B} \rightarrow \bar{f}) = Ae^{i(\phi-\delta)} + Be^{i(\phi'-\delta')}$$

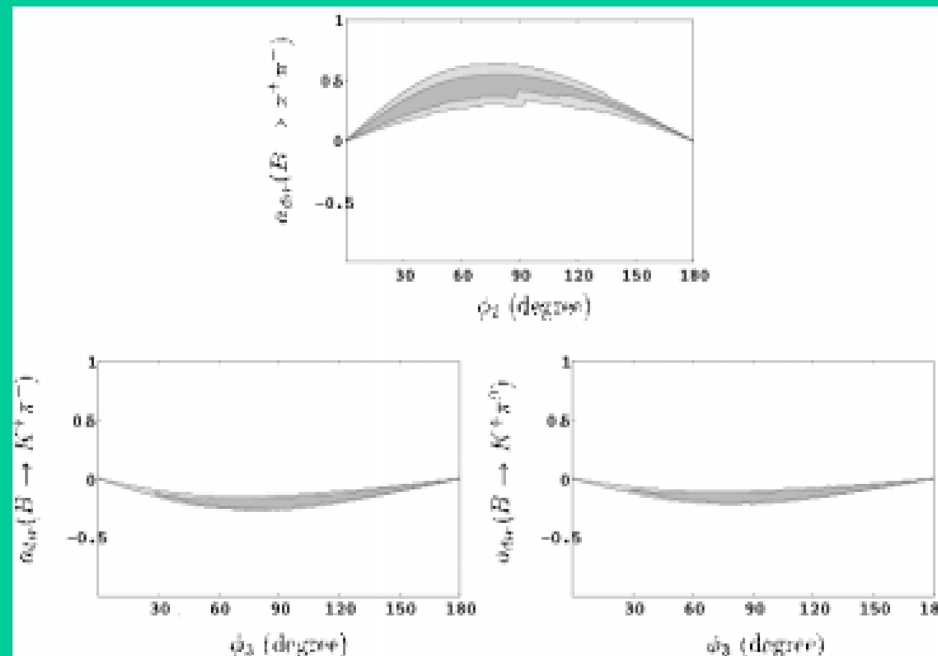
$$\frac{\Gamma(B \rightarrow f) - \Gamma(\overline{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\overline{B} \rightarrow \bar{f})} = \frac{-2AB\sin(\Delta\phi)\sin(\Delta\delta)}{|A|^2 + |B|^2 + 2AB\cos(\Delta\phi)\cos(\Delta\delta)}$$



The diagram which produces strong interaction phase -> CP violation

$$\frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})}$$

$K^{\square}\pi^{\pm}$	0.186±0.105
$K^{\pm}\pi^{\pm}$	-0.062±0.054
$K^{\pm}\pi^{\square}$	-0.087±0.115



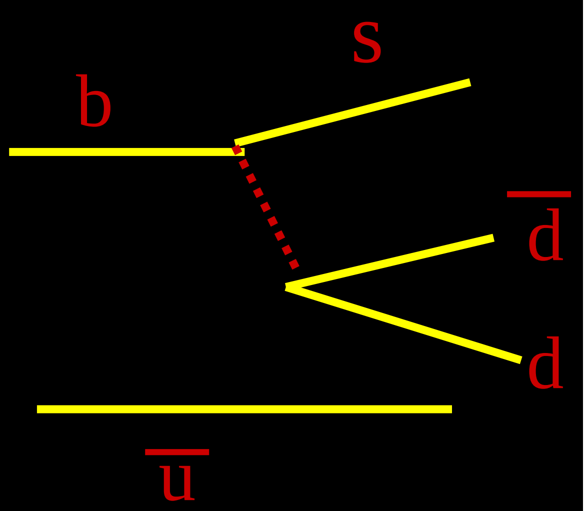
CP asymmetries will become smaller
if penguins are larger

We should not worry about the disagreement until $K^{\square}\pi^{\pm}$
asymmetry is settled

$$B^- \to \overline{K}^0 \pi^-$$

Pure P

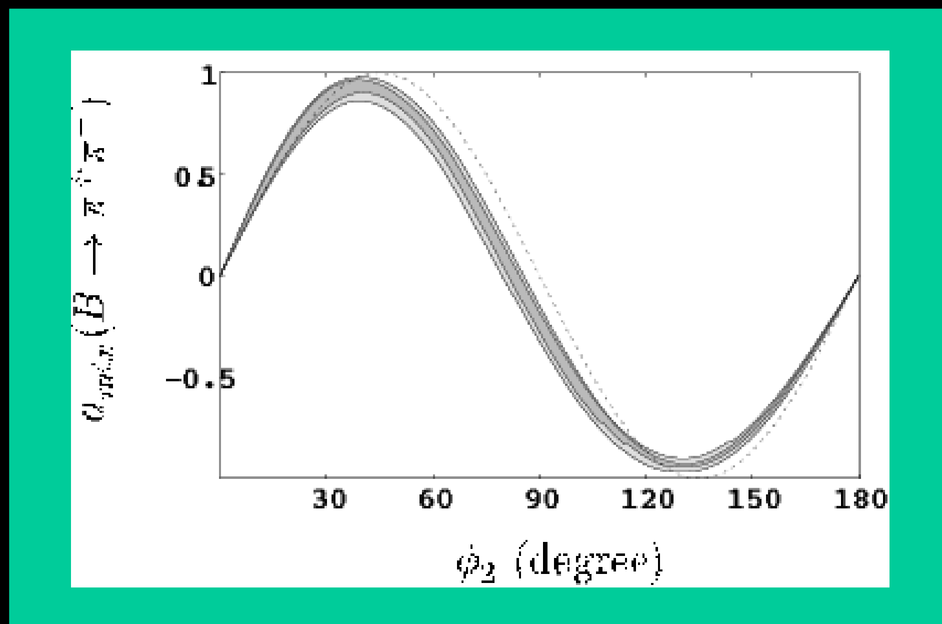
P



$$A(B \to f) = Ae^{i(\phi+\delta)} + Be^{i(\phi'+\delta')}$$

$$A(\overline{B} \to \bar{f}) = Ae^{i(\phi-\delta)} + Be^{i(\phi'-\delta')}$$

$$\frac{\Gamma(B \to f) - \Gamma(\overline{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\overline{B} \to \bar{f})} = \frac{-2AB\sin(\Delta\phi)\sin(\Delta\delta)}{|A|^2 + |B|^2 + 2AB\cos(\Delta\phi)\cos(\Delta\delta)}$$


$a_{mix}(B \to \pi^+\pi^-)$
1
0.5
0
-0.5
30
60
90
120
150
180
ϕ_2 (degree)

CP averaged branching ratios
Experimental data & PQCD

final state	exp. data(10^{-5})	min.	max.
$\pi^+\pi^-$	$0.43^{+0.16}_{-0.14} \pm 0.05$(CLEO)	0.67	1.14
	$0.56^{+0.23}_{-0.20} \pm 0.04$(BELLE)	0.67	1.14
	$0.41 \pm 0.10 \pm 0.07$(BABAR)	0.67	1.14
$\pi^0\pi^0$		0.010	0.028
$\pi^+\pi^0$		0.44	0.50
$K^+\pi^-$	$1.72^{+0.25}_{-0.24} \pm 0.12$(CLEO)	1.26	2.64
	$1.93^{+0.34}_{-0.32}{}^{+0.15}_{-0.06}$(BELLE)	1.26	2.64
	$1.67 \pm 0.16 \pm 0.13$(BABAR)	1.26	2.64
$K^+\pi^0$	$1.16^{+0.30}_{-0.27}{}^{+0.14}_{-0.13}$(CLEO)	0.88	1.70
	$1.63^{+0.35}_{-0.33}{}^{+0.16}_{-0.18}$(BELLE)	0.88	1.70
	$1.08^{+0.21}_{-0.19} \pm 0.10$(BABAR)	0.88	1.70
$K^0\pi^+$	$1.82^{+0.46}_{-0.40} \pm 0.16$(CLEO)	2.03	2.06
	$1.37^{+0.57}_{-0.48}{}^{+0.19}_{-0.18}$(BELLE)	2.03	2.06
	$1.82^{+0.33}_{-0.30} \pm 0.20$(BABAR)	2.03	2.06
$K^0\pi^0$	$1.46^{+0.59}_{-0.51}{}^{+0.24}_{-0.33}$(CLEO)	0.74	0.77
	$1.60^{+0.72}_{-0.59}{}^{+0.25}_{-0.27}$(BELLE)	0.74	0.77
	$0.82^{+0.31}_{-0.27} \pm 0.12$(BABAR)	0.74	0.77
$\pi^+\rho^-$	$2.76^{+0.84}_{-0.74} \pm 0.42$(CLEO)	2.39	3.37
	$2.89 \pm 5.4 \pm 0.43$(BABAR)	2.39	3.37
$\pi^+\rho^0$	$1.04^{+0.33}_{-0.34} \pm 0.21$(CLEO)	0.48	0.59
$\pi^0\rho^+$		0.64	1.01
$\pi^0\rho^0$		0.008	0.011
$\pi^+\omega$	$1.13^{+0.33}_{-0.29} \pm 0.14$(CLEO)	0.43	0.81
	$0.66^{+0.21}_{-0.18} \pm 0.07$(BABAR)	0.43	0.81
$\pi^0\omega$		0.010	0.028
ϕK^+	$0.55^{+0.21}_{-0.18} \pm 0.06$(CLEO)	1.01	
	$1.06^{+0.21}_{-0.19} \pm 0.22$(BELLE)	1.01	
	$0.77^{+0.16}_{-0.14} \pm 0.08$(BABAR)	1.01	
ϕK^0	$0.87^{+0.38}_{-0.30} \pm 0.15$(BELLE)	0.943	
	$0.81^{+0.31}_{-0.25} \pm 0.08$(BABAR)	0.943	

Conclusion

- PQCD is at its infant stage
- Seems very promising
- Predicts 2 body decay rates
- Input: wave function
- Predicts strong interaction phase
- Existence of CP violation at 10-20% level for some channels

Summary 2

- Are large CPV inconsistent with experiment?
- May be, but can't say until $K^+\pi^0$ CP asymmetry is in order ◊0