

A Mixing-Independent Construction of The Unitarity Triangle

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Experimental Physics Seminar

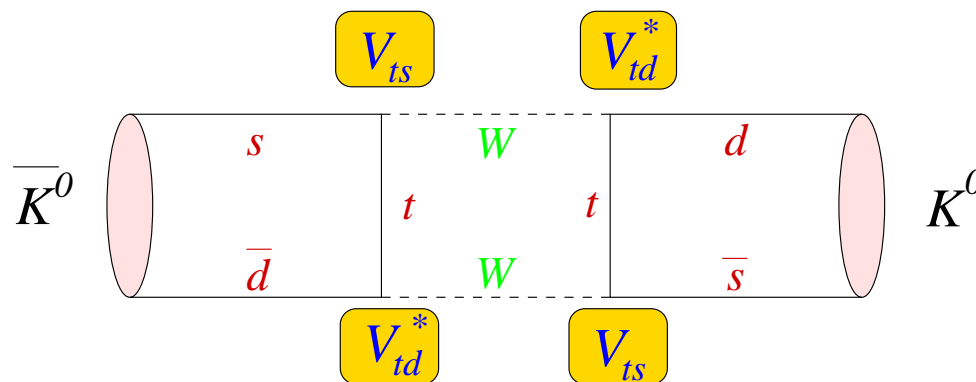
SLAC, 26 September 2002

(based on [hep-ph/0207327](https://arxiv.org/abs/hep-ph/0207327) & [hep-ph/0207002](https://arxiv.org/abs/hep-ph/0207002))

Constraints on the Unitarity Triangle

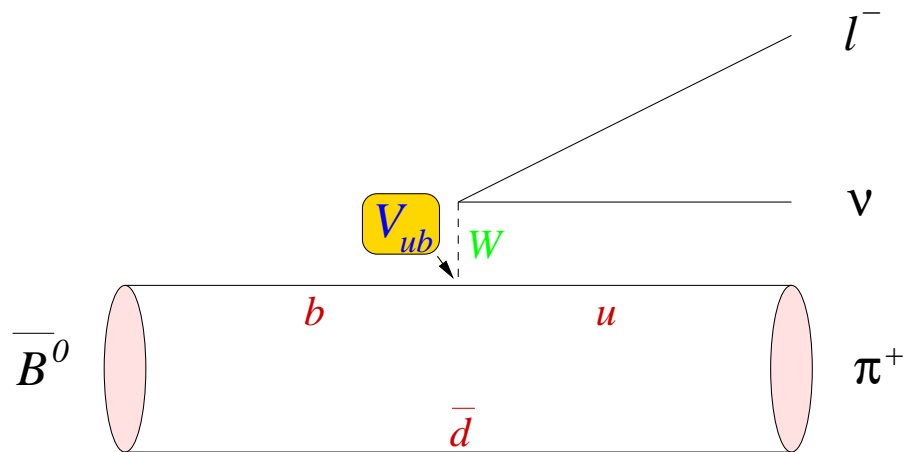
ϵ_K from CP violation in $K-\bar{K}$ mixing:

- due to CP violation, the long-lived strange meson $|K_L\rangle \approx (|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2}$ is not exactly a CP eigenstate and so can decay into two pions
- ϵ_K is sensitive to $\text{Im}[(V_{td}^* V_{ts})^2]$



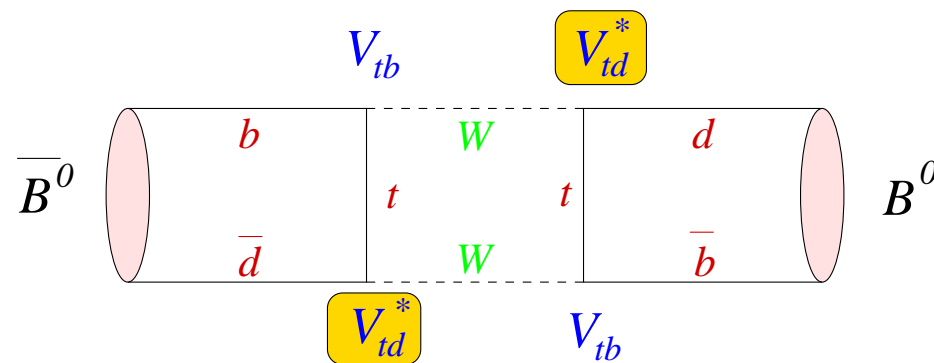
$|V_{ub}/V_{cb}|$ from semileptonic B decays:

- ratio can be measured by comparing semileptonic $b \rightarrow ul\nu$ and $b \rightarrow cl\nu$ decays



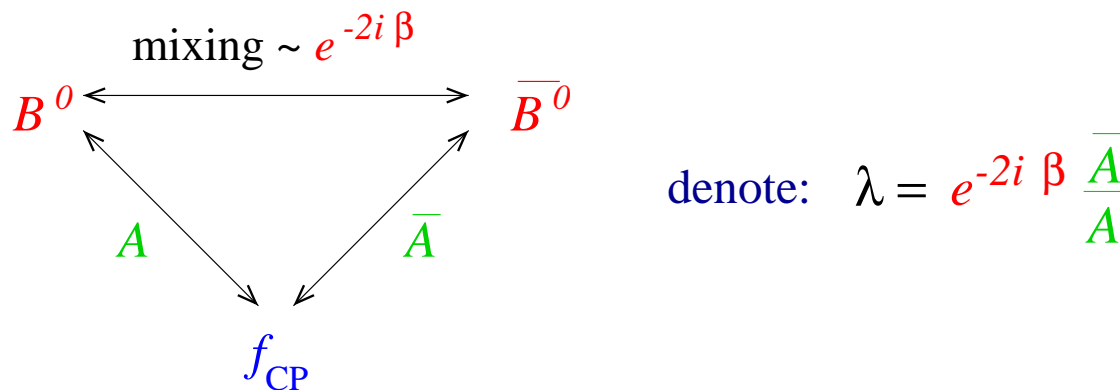
$\Delta m_{d,s}$ from $B_{d,s}-\bar{B}_{d,s}$ mixing:

- $B-\bar{B}$ mixing amplitudes are dominated by virtual production of top quarks
- $\Delta m_{d,s}$ is sensitive to $|V_{td}^* V_{tb}|^2$



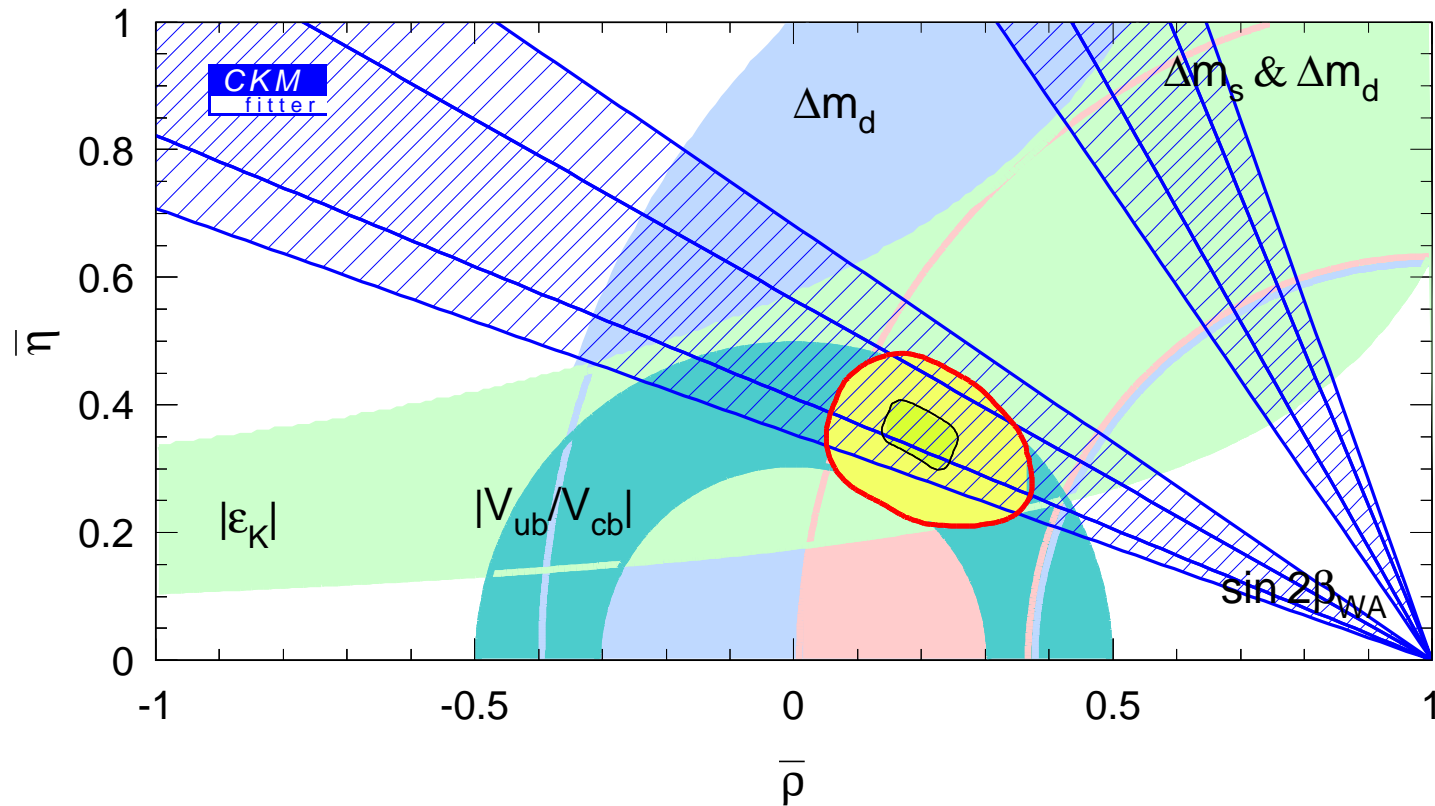
$\sin 2\beta$ from $B \rightarrow J/\psi K$ decays:

- use amplitude interference in B decays into a CP eigenstate f_{CP} :



- CP asymmetry: $A_{\text{CP}}(t) = -\sin 2\beta \sin(\Delta m_d t)$

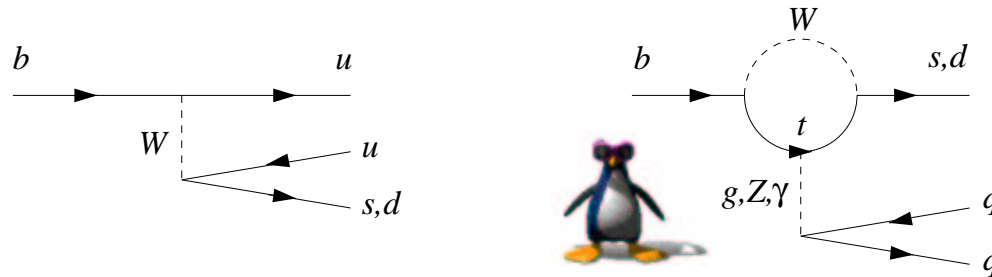
Summary of Constraints (2002)



- has established the existence of a **CP-violating phase** in the top sector ($\text{Im}(V_{td}) \neq 0$)
- with exception of $|V_{ub}|$, all other constraints are sensitive to potential New Physics in $B-\bar{B}$ or $K-\bar{K}$ mixing
- except for $\sin 2\beta$, individual constraints have large theoretical uncertainties

Rare Hadronic B Decays

- after obtaining a consistent picture of CP violation in the top sector, the next step must be to explore the complex phase $\gamma = \arg(V_{ub}^*)$ in the **bottom sector**
- γ can be probed via the **tree–penguin interference** in rare hadronic decays $B \rightarrow \pi K, \pi\pi, \dots$



- information from CP asymmetries ($\sim \sin \gamma$) and CP-averaged branching fractions ($\sim \cos \gamma$)

The Challenge

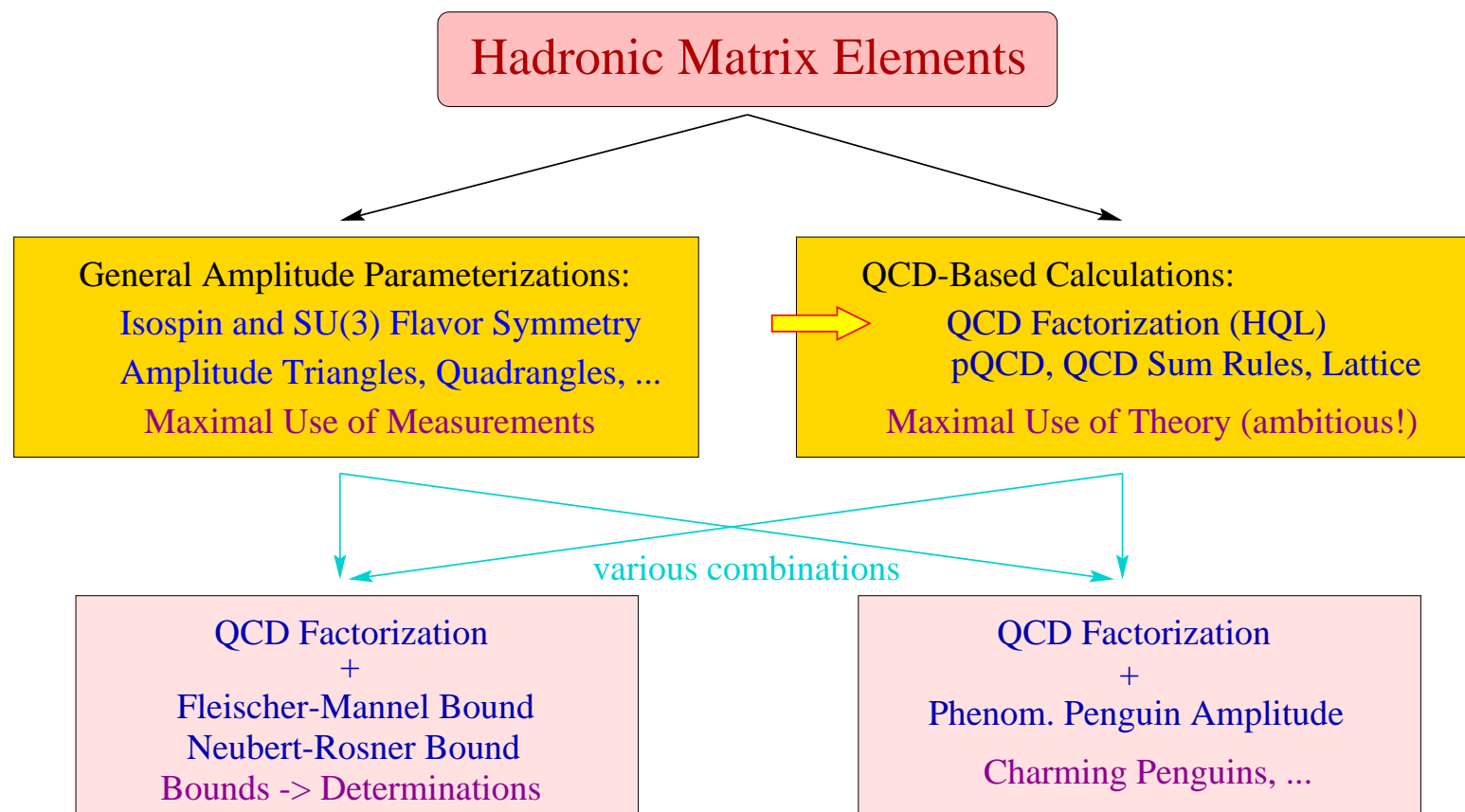
QCD, the marvellous theory of the strong interactions, has a split personality. It explains both “hard” and “soft” phenomena, the softer ones being the hardest.

(Y. Dokshitzer)

high energies \Leftrightarrow weak coupling (asymptotic freedom)

low energies \Leftrightarrow strong coupling (confinement)

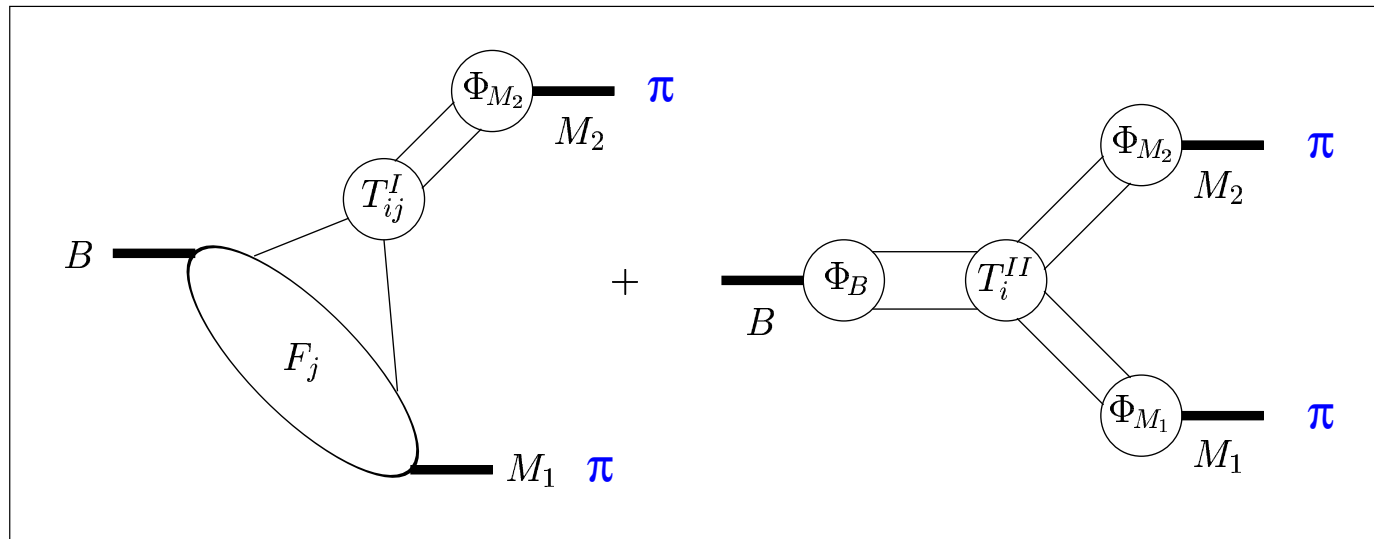
Different strategies exist for determining the relevant hadronic matrix elements:



QCD Factorization Approach

Factorization formula for hadronic B -meson decays:

[Beneke, Buchalla, MN, Sachrajda]



\Rightarrow provides a model-independent description of hadronic B -decay amplitudes (including their phases) in the heavy-quark limit

Crucial Tests

- magnitude of **tree amplitude**:
 $\text{Br}(B^\pm \rightarrow \pi^\pm \pi^0) = (5.7 \pm 0.9) \cdot 10^{-6}$ compares well with prediction $5.3^{+0.8}_{-0.4}$ (pars.) ± 0.3 (power)

- magnitude of **tree-to-penguin ratio**:

$$\epsilon_{\text{exp}} = \tan \theta_C \frac{f_K}{f_\pi} \left[\frac{2\text{Br}(B^\pm \rightarrow \pi^\pm \pi^0)}{\text{Br}(B^\pm \rightarrow \pi^\pm K^0)} \right]^{\frac{1}{2}} = 0.22 \pm 0.02$$

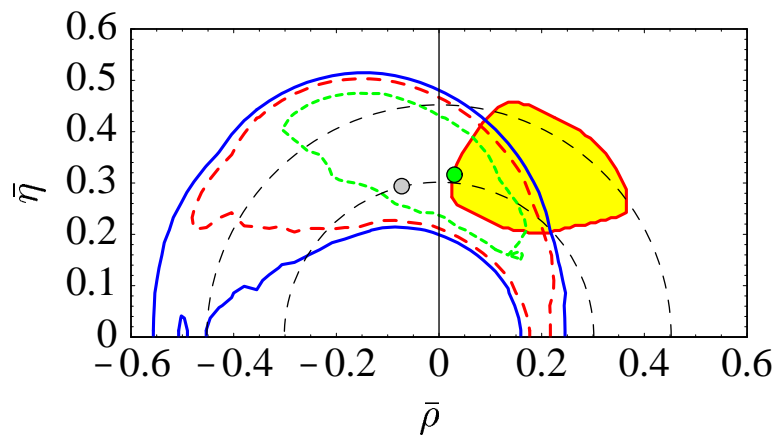
agrees with prediction

$$0.23 \pm 0.04 \text{ (pars.)} \pm 0.04 \text{ (power)} \pm 0.05 (V_{ub})$$

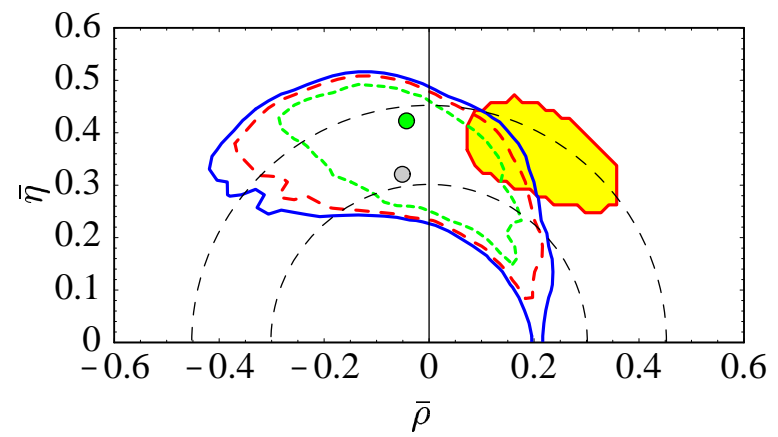
- **direct CP asymmetries** are predicted (and found) to be small

Establishing CPV in the Bottom Sector

- ratios of CP-averaged $B \rightarrow \pi K, \pi\pi$ rates exhibit strong dependence on γ and $|V_{ub}|$
- derive constraints on $\bar{\rho}$ and $\bar{\eta}$ from a global analysis of the data in the context of QCD factorization: [BBNS]



spring 2001



summer 2002

- combination of results from rare hadronic B decays with the $|V_{ub}|$ measurement in semileptonic decays excludes $\bar{\eta} = 0$ and so **establishes the existence of a CP phase in the bottom sector of the CKM matrix**
- allowed regions obtained from the fit to charmless hadronic decays are compatible with the standard fit, but tend to favor larger γ values
- same trend seen in an analysis that does not rely on QCD factorization but instead employs general amplitude parameterizations and flavor symmetries

[Fleischer, Matias]

Origins of a Possible Discrepancy?

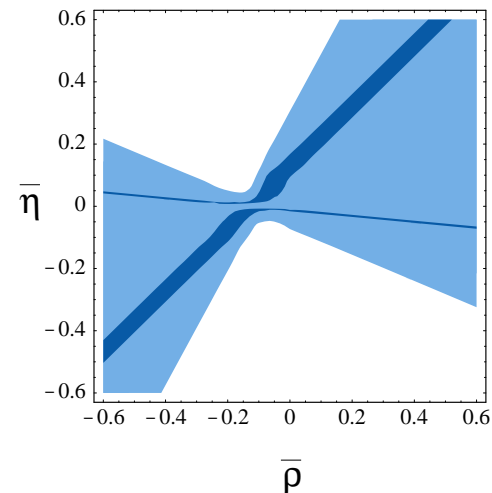
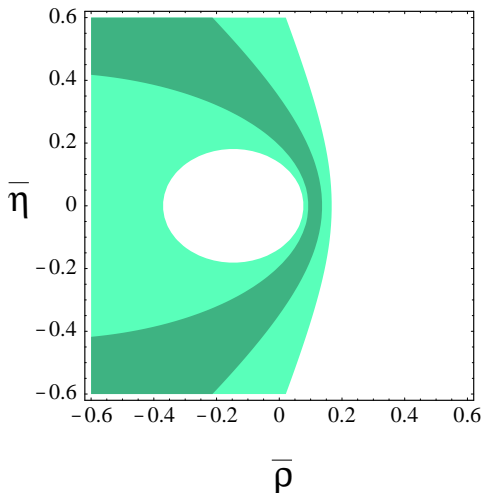
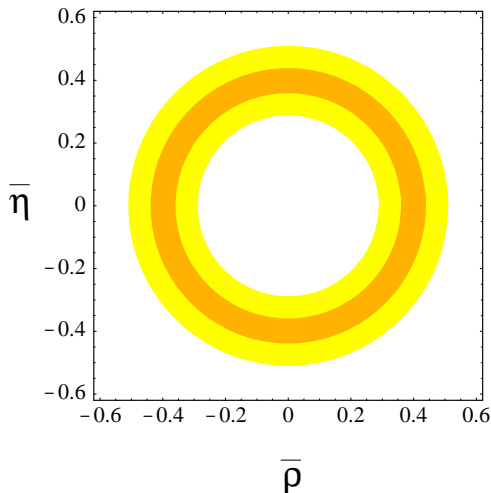
- errors in lattice calculations of matrix elements for $B_d-\bar{B}_d$ and $B_s-\bar{B}_s$ mixing may have been underestimated [Kronfeld, Ryan]
- more exciting: New Physics interpretations!
- New Physics in $B_s-\bar{B}_s$ mixing \Rightarrow check at Tevatron
- New Physics in $B_d-\bar{B}_d$ mixing
- New Physics in $b \rightarrow s$ or $b \rightarrow d$ FCNC transitions (e.g. from penguin and box graphs with exchange of new heavy particles)
 \Rightarrow clean signal would be a difference in the time-dependent CP asymmetries in $B \rightarrow \phi K_S$ and $B \rightarrow J/\psi K_S$ decays

The Future: “CP-b Triangle”

- if trend toward larger γ values persists, one will want to check compatibility with the standard analysis using measurements whose interpretation is theoretically “clean”
- propose a novel construction of the unitarity triangle which is over-determined, insensitive to potential New Physics effects in $B-\bar{B}$ or $K-\bar{K}$ mixing, and affected by smaller theoretical uncertainties than the standard analysis
- feasible with existing data

Ingredients

- $|V_{ub}/V_{cb}|$ extracted from semileptonic B decays
- ratio of the CP-averaged $B^\pm \rightarrow (\pi K)^\pm$ branching fractions (generalized Neubert–Rosner method)
- time-dependent CP asymmetry $S_{\pi\pi} = \sin 2\alpha_{\text{eff}}$ in $B \rightarrow \pi^+\pi^-$ decays (analysed using QCD factorization and $\sin 2\beta$ measurement)



I. Comments on $|V_{ub}|$

- important recent developments concerning power corrections to the universal **shape function** connecting Fermi-motion effects in $B \rightarrow X_s \gamma$ and $B \rightarrow X_u l \nu$ decays [Bauer, Luke, Mannel; Leibovich, Ligeti, Wise; MN]
- corrections can be included into weight function connecting, e.g., the photon spectrum to the lepton spectrum:

$$F_u(E_0) = \left(1 + \underbrace{\frac{2\Lambda_{\text{SL}}(E_0)}{m_b}}_{\text{residual cor.}} \right) \int_{E_0}^{M_B/2} dE_\gamma w(E_\gamma, E_0) S(E_\gamma)$$

weight function:

$$w(E_\gamma, E_0) = 2 \left(1 - \frac{E_0}{E_\gamma} \right) \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} g(E_0/E_\gamma) \right\} - \frac{8\lambda_2}{m_b^2}$$

E_0 [GeV]	NLO pert.	$1/m_b$	total	residual error
2.0	0.313 ± 0.014	-0.040 ± 0.006	0.273 ± 0.015	± 0.003
2.1	0.228 ± 0.010	-0.037 ± 0.006	0.191 ± 0.011	± 0.005
2.2	0.150 ± 0.006	-0.033 ± 0.005	0.117 ± 0.008	± 0.006
2.3	0.083 ± 0.004	-0.026 ± 0.004	0.057 ± 0.006	± 0.008

\Rightarrow method used in a recent CLEO analysis (2002),
giving $|V_{ub}| = (4.1 \pm 0.6_{\text{exp}} \pm 0.3_{\text{th}}) \times 10^{-3}$

Is shape-function sensitivity good or bad?

- often argued that one should avoid sensitivity to Fermi motion using a cut on the lepton invariant mass (“ q^2 cut”), and that the region of phase space with low hadronic mass *and* energy is theoretically favored over that with low mass but large energy
[Bauer, Ligeti, Luke]
- however, this argument ignores the problem of quark–hadron duality violations! [Bigi, Uraltsev]

- usually argue that duality holds, since an inclusive measurement includes a large number of hadronic final states with large mass and/or energy
 $M_H, E_H \gg \Lambda$ (necessity of having a hard scale!)
- any cut that eliminates the charm background restrict the invariant hadronic mass $M < m_D \sim (\Lambda m_B)^{1/2}$, but in principle still allows large energy $E_H \sim m_B$
- shape function effects result from the region where $\Lambda E_H / M_H^2 \sim 1$, corresponding to large E_H
- \Rightarrow smearing provided by Fermi motion is crucial for restoring quark–hadron duality, and so is a **good feature!**

II. Comments on generalized NR method

- without recourse to factorization, measurement of

$$R_* = \frac{\text{Br}(B^\pm \rightarrow \pi^\pm K^0)}{2\text{Br}(B^\pm \rightarrow \pi^0 K^\pm)} = 0.71 \pm 0.10$$

and of the tree-to-penguin ratio $\epsilon_{\text{exp}} = 0.22 \pm 0.02$ provide a bound on $\cos \gamma$, which can be turned into a determination of $\cos \gamma$ when information about the relevant strong phase $\phi_{\pi^0 K^-}$ is available

- QCD predicts that

$$\cos \phi_{\pi^0 K^-} = 1 - O[\alpha_s(m_b)^2, (\Lambda/m_b)^2, \alpha_s(m_b) \Lambda/m_b]$$

equals 1 in the heavy-quark limit up to **second-order** corrections

- data (!) can be used to place bounds on strong phases:

$$A_{\text{CP}}(\pi^+ K^-) = -0.05 \pm 0.05 \quad \Rightarrow \quad \phi_{\pi^+ K^-} = (8 \pm 10)^\circ$$

$\phi_{\pi^0 K^-} \simeq \phi_{\pi^+ K^-}$ to good approximation [Gronau, Rosner]
better: use precision measurement of $A_{\text{CP}}(\pi^0 K^-)$ to constrain $\phi_{\pi^0 K^-}$ directly

- \Rightarrow safe to assume that $\cos \phi_{\pi^0 K^-} > 0.8$

III. Comments on $S_{\pi\pi}$ theory

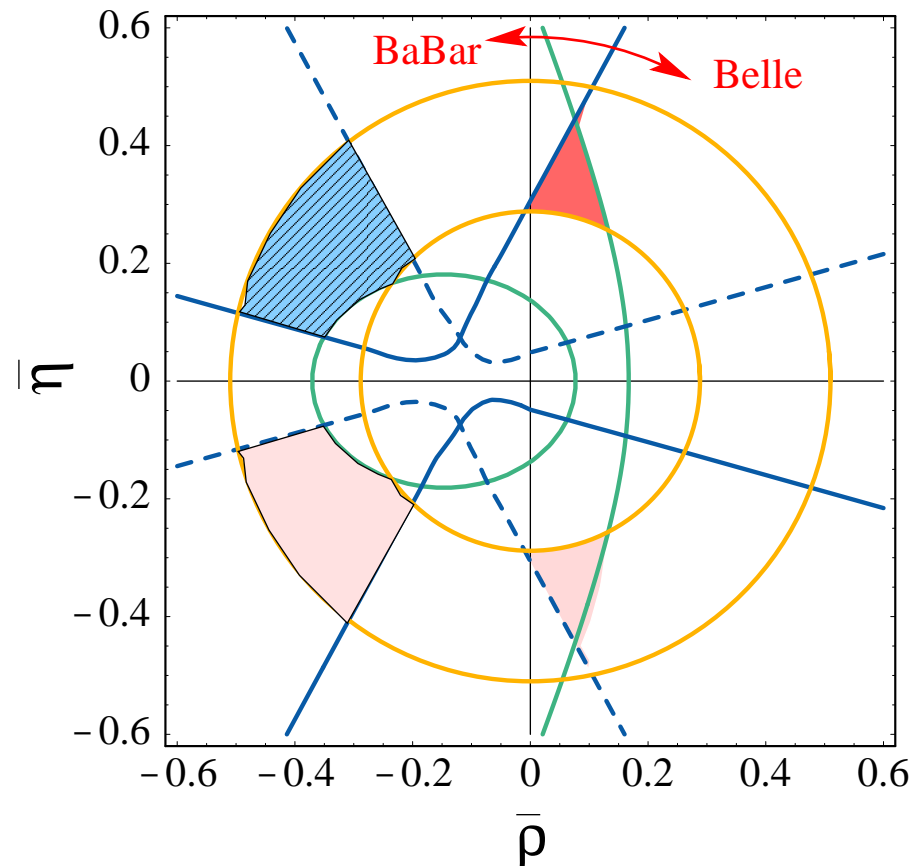
General formula ($\phi_d = 2\beta$ in SM):

$$S_{\pi\pi} = \frac{2 \operatorname{Im} \lambda_{\pi\pi}}{1 + |\lambda_{\pi\pi}|^2} \quad \text{with} \quad \lambda_{\pi\pi} = e^{-i\phi_d} \frac{e^{-i\gamma} + (P/T)_{\pi\pi}}{e^{+i\gamma} + (P/T)_{\pi\pi}}$$

- trick to get insensitive to New Physics in mixing is to use $e^{-i\phi_d} = \pm(1 - s_{\text{exp}}^2)^{1/2} - i s_{\text{exp}}$ with $s_{\text{exp}} = (\sin 2\beta)_{\text{exp}}$
- this turns circles in $(\bar{\rho}, \bar{\eta})$ plane into straight lines, which intersect $|V_{ub}|$ circles at (almost) 90° angles
- hadronic uncertainties (from QCD factorization) are large in α , but small when displayed as bands in the $(\bar{\rho}, \bar{\eta})$ plane (and that is what counts!)

Resulting CP-b Triangle

Combine three constraints and construct the resulting allowed regions for the apex of the unitarity triangle:



- if we use that ϵ_K requires positive value of $\bar{\eta}$, only two solutions in the upper half-plane remain
- one of these lies close to the standard fit (though once again somewhat larger γ values are preferred, in particular by the BaBar $S_{\pi\pi}$ result)
- a second allowed region, consistent with the constraints from ϵ_K and charmless hadronic decays, is incompatible with the constraints from $\sin 2\beta$ and $\Delta m_s/\Delta m_d$
 \Rightarrow would require a significant New Physics contribution to $B-\bar{B}$ mixing

Summary

- it is time to move beyond $\sin 2\beta$
- many alternative methods exist that provide powerful constraints on the unitarity triangle
- rare hadronic decays still favor larger γ values than the standard analysis of the unitarity triangle
- construction of the CP-b triangle reinforces this trend, but with smaller theoretical uncertainties than previous methods (large γ favored by R_* and $S_{\pi\pi}^{\text{BaBar}}$)
- if this discrepancy is real, it may imply that (after all) **New Physics** is just around the corner!