Hadronic Vacuum Polarization

Confronting Spectral Functions from e^+e^- Annihilation and τ Decays: Consequences for the Muon Magnetic Moment

LABORATOIRE DE L'ACCÉLÉRATEUR LINÉAIRE

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

Andreas Höcker (*)

LAL - Orsay

SLAC Seminar

September 3, 2002

(*) work done in collaboration with M. Davier, S. Eidelman and Z. Zhang [hep-ph/0208177]

Outline

The Muon Magnetic Anomaly

... and Hadronic Vacuum Polarization

a_μ[had]

$\Delta\alpha_{QED}$[had]

e^+e^- versus τ Data

Isospin Breaking

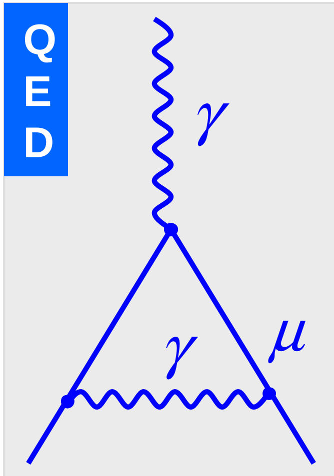
and

Radiative Corrections

Two New Standard Model Predictions

for $(g-2)_\mu$

Magnetic Anomaly



QED Prediction:

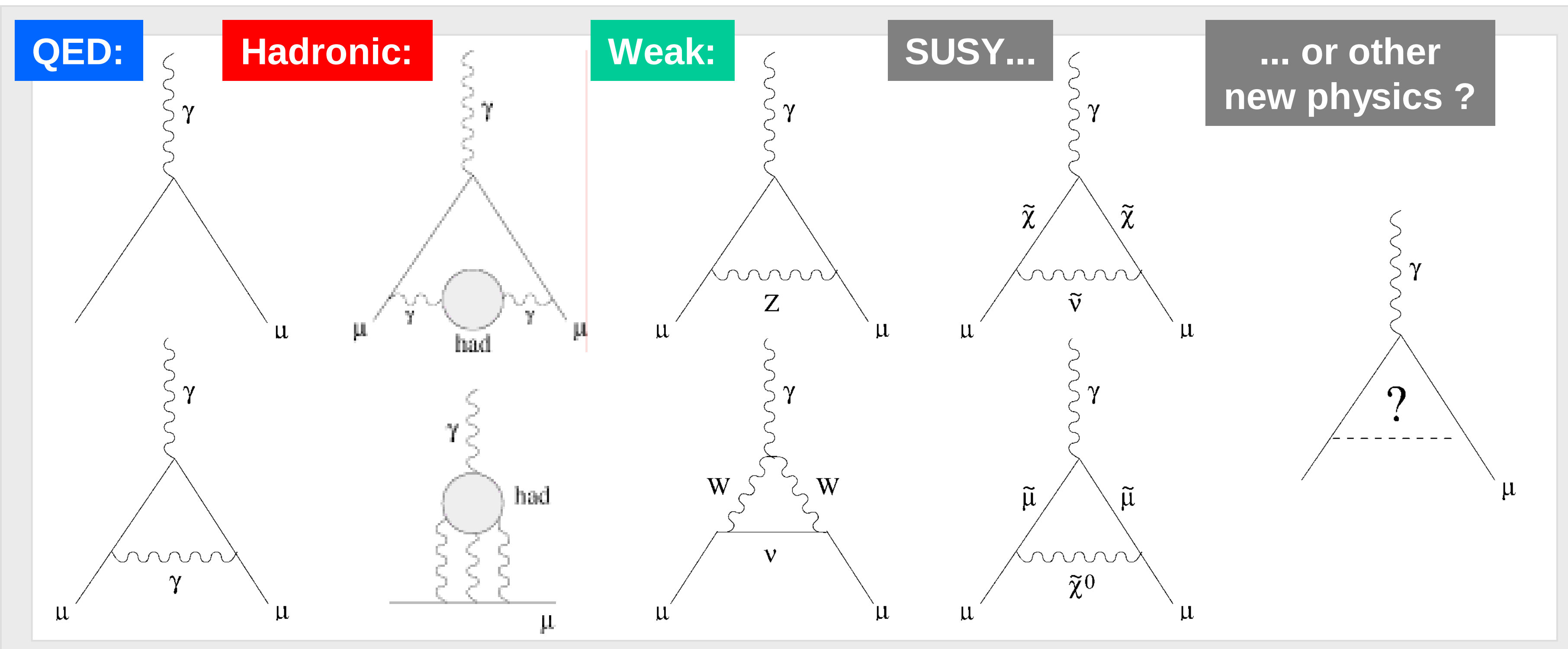
Computed up to 4th order [Kinoshita *et al.*] (5th order estimated [Mohr, Taylor])

$$\Gamma_\mu = e\gamma_\mu + a_\ell \frac{ie}{2m}\sigma_{\mu\nu}q_\nu$$

$$a_\ell = \frac{\alpha}{2\pi} = 0.001161\ldots$$

Schwinger 1947
(Nobel price 1965)

$$a_\mu^{QED} = \sum_{n=1}\left(\frac{\alpha}{\pi}\right)^n \approx \begin{pmatrix} 11614098.1 + 41321.8 \\ +3014.2 + 36.7 + 0.6 \end{pmatrix} \times 10^{-10}$$



The Muonic $(g-2)_\mu$

Contributions to the Standard Model (SM) Prediction:

$$a_\mu \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}$$

The Situation 1995

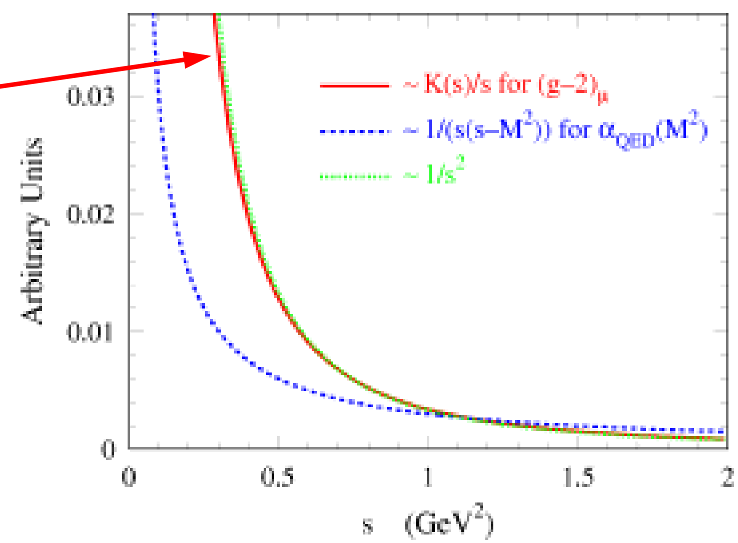
Source	$\sigma(a_\mu)$	Reference
QED	$\sim 0.3 \times 10^{-10}$	[Schwinger '48 & others]
Quarks and Hadrons	$\sim (15 \oplus 4) \times 10^{-10}$	[Eidelman-Jegerlegner '95 & others]
Z, *W* exchange	$\sim 0.4 \times 10^{-10}$	[Czarnecki *et al.* '95 & others]

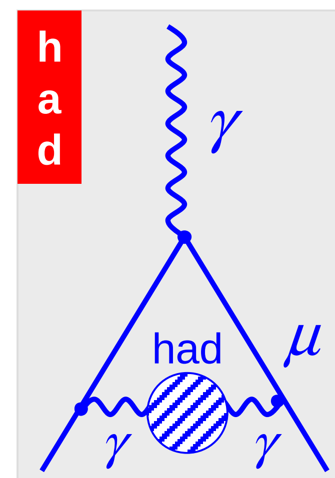
Dominant uncertainty from lowest order hadronic piece. Cannot be calculated from QCD ("first principles") – but: we can use experiment (!)

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} R(s)$$

"Dispersion relation"

...





Hadronic Vacuum Polarization

Define: photon vacuum polarization function $\Pi_\gamma(q^2)$

$$i\int d^4x\, e^{iqx} \langle 0 | T J^\mu_{\rm em}(x) \left(J^\nu_{\rm em}(x)\right)^\dagger | 0 \rangle = -\left(g^{\mu\nu} q^2 - q^\mu q^\nu\right) \Pi_\gamma(q^2)$$

***Ward identities*: only vacuum polarization modifies electron charge**

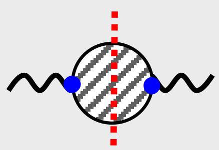
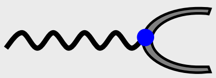
$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)} \qquad \text{with:} \quad \Delta\alpha(s) = -4\pi\alpha \,\mathrm{Re}\left[\Pi_\gamma(s) - \Pi_\gamma(0)\right]$$

Leptonic $\Delta\alpha_{\rm lep}(s)$ calculable in QED. However, quark loops are modified by long-distance hadronic physics, cannot (yet) be calculated within QCD (!)

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

Way out: Optical Theorem (*unitarity*) ...

$$12\pi\,\mathrm{Im}\,\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \text{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

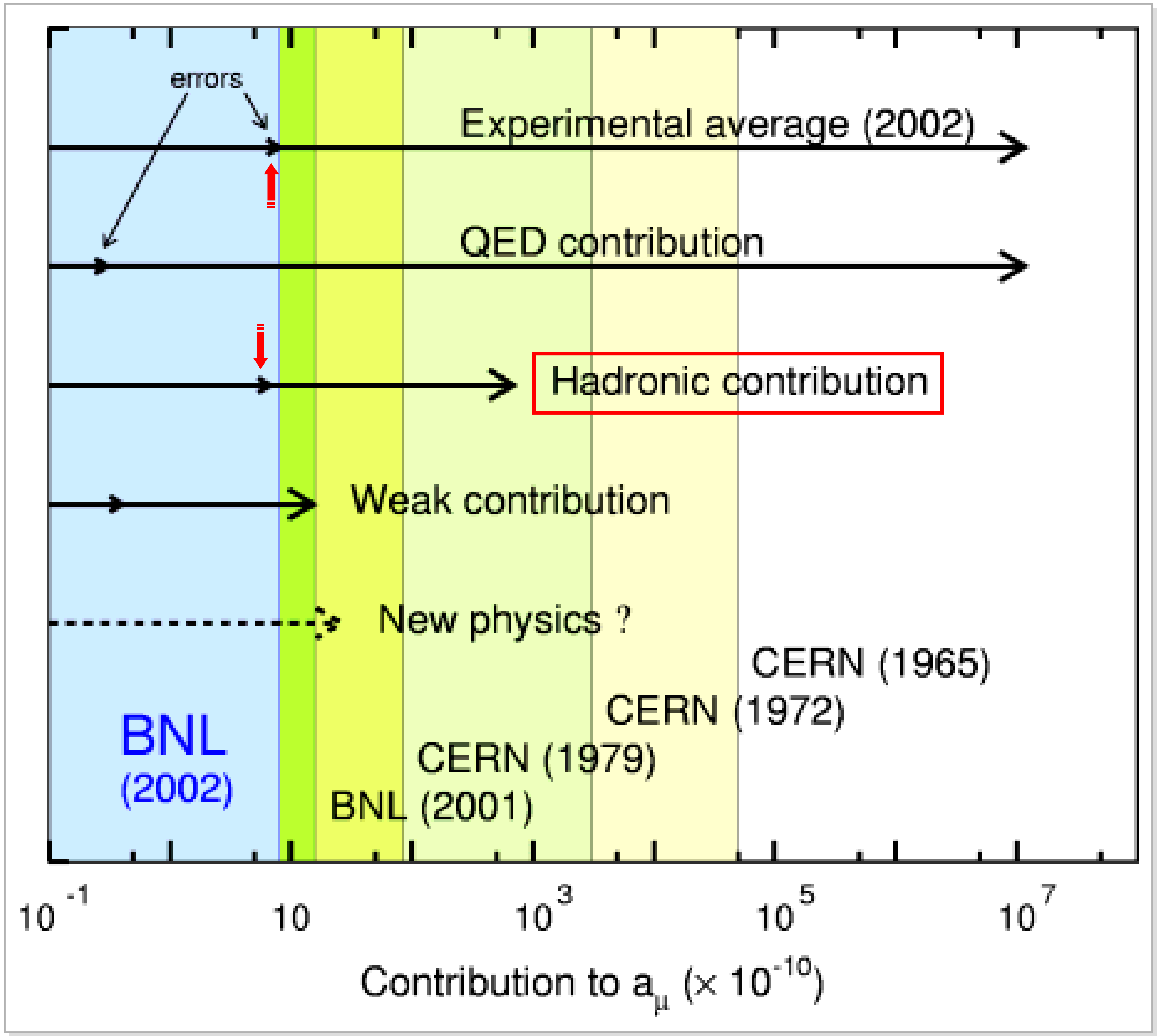
Im[] $\propto$ |  hadrons $|^2$

... and the subtracted dispersion relation of $\Pi_\gamma(q^2)$ (*analyticity*)

$$\Pi_\gamma(s) - \Pi_\gamma(0) = \frac{s}{\pi}\int_0^\infty ds' \frac{\mathrm{Im}\,\Pi_\gamma(s')}{s'(s'-s) - i\varepsilon} \quad \Rightarrow \quad \Delta\alpha_{\rm had}(s) = -\frac{\alpha s}{3\pi}\mathrm{Re}\int_0^\infty ds' \frac{R(s')}{s'(s'-s) - i\varepsilon}$$

... and equivalently for a_μ[had]

Why Do We Need to Know it so Precisely?



Experimental Progress on Precision of $(g-2)_\mu$

Aiming at:

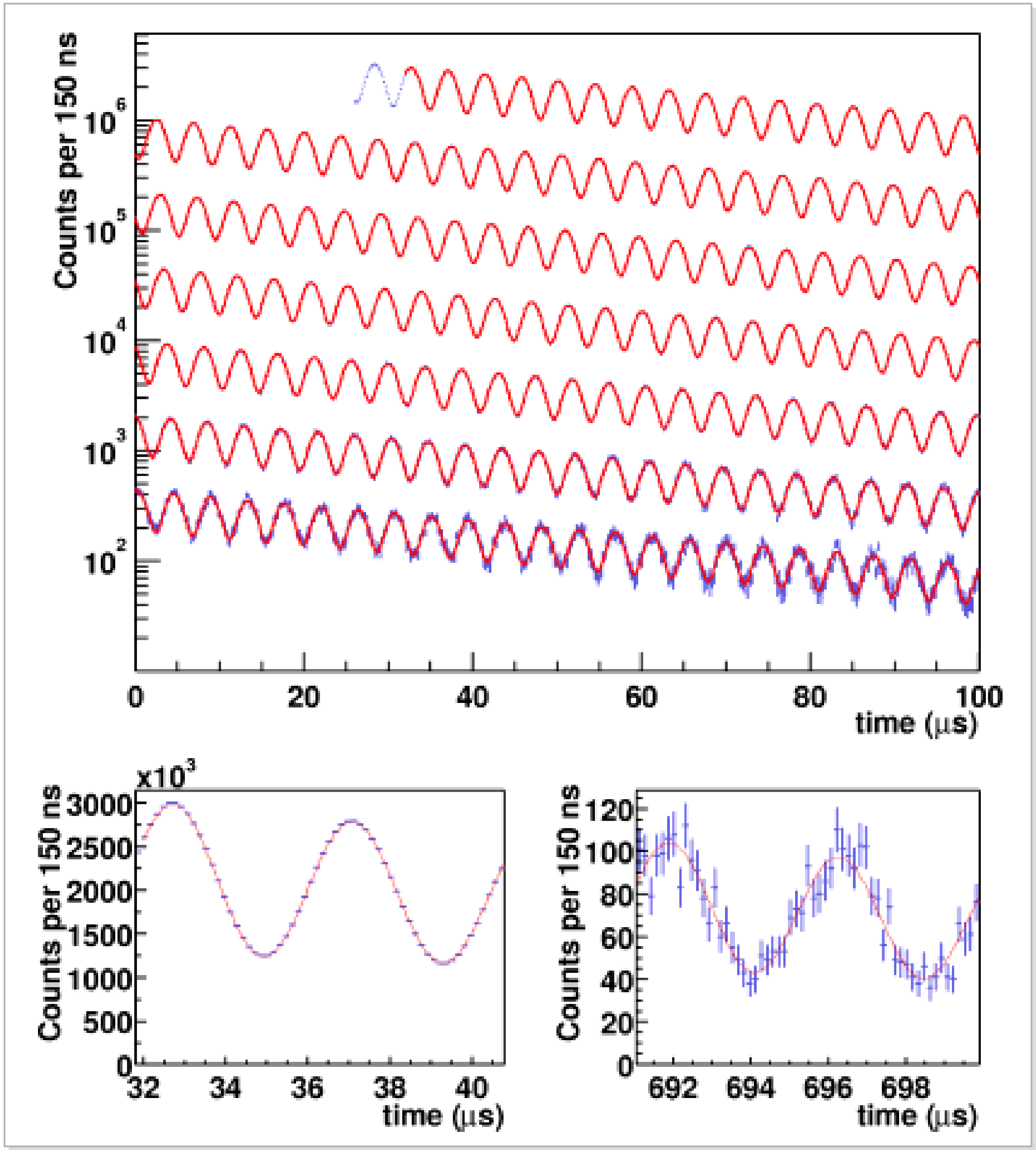
$$\sigma_{\text{exp}}(a_\mu) \sim 4\times10^{-10}$$

$$\sim \sigma(a_\mu^{\text{had}})$$

$$\sim a_\mu^{\text{weak [SM]}}$$

The fine structure constant at M_Z is an important ingredient to EW precision fits

For the Beauty of it: BNL (*g* –2) Measurement



Observed positron rate in successive 100μs periods

Spin precession frequency:

$$\vec{\omega}_a = \frac{e}{mc} a_\mu \vec{B}$$

obtained from 5-parameter fit to the function:

$$N(t)=N_0 e^{-\lambda t}[1+A\cos(\omega_a t+\phi)]$$

taken from:
E821 (*g* –2) [D.W. Hertzog]
hep-ex/0202024

2001: The First Round of BNL Results on a_μ

The E821 (g –2) experiment at BNL published early 2001 a value 3× more precise than the previous CERN and BNL exps. combined:

E821 (g –2)
hep-ex/0102017

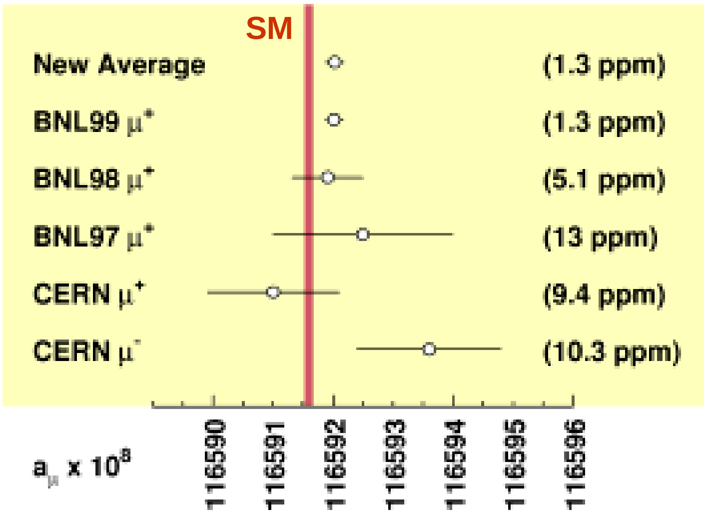
$$a_\mu(\text{exp}) = 11\,659\,202(16) \times 10^{-10}$$

BNL compares with Standard Model prediction:

$$a_\mu(\text{SM}) = 11\,659\,159.6(6.7) \times 10^{-10}$$

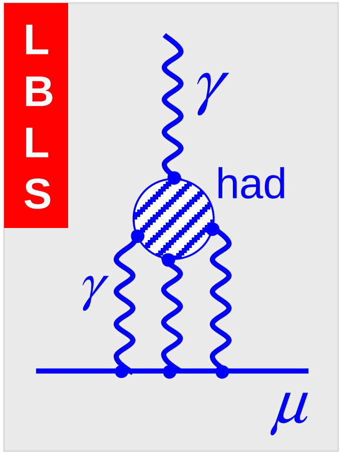
Averaging E821 with previous experiments gives:

$$a_\mu(\text{exp}) - a_\mu(\text{SM}) = 43(16) \times 10^{-10} \quad [\to 2.7\,\sigma]$$



BUT: In November 2001, Knecht & Nyffeler have corrected a sign error in the dominant (π-pole) contribution from hadronic light-by-light (LBL) scattering, reducing the above discrepancy to

$$25(16) \times 10^{-10} \quad [\to 1.6\,\sigma]$$



Knecht-Nyffeler, hep-ph/0111058; result approved by: **Hayakawa-Kinoshita**, hep-ph/0112102; **Bijnens-Pallante-Prades**, hep-ph/0112255

2002: The Second Round of BNL Results on a_μ

The new analysis, first presented at ICHEP'02, achieves 2× better precision (using 4× more statistics) than the 2001 result:

E821 (g –2)
hep-ex/0208001

$$a_\mu(\text{exp}) = 11\,659\,203(7)(5) \times 10^{-10}$$

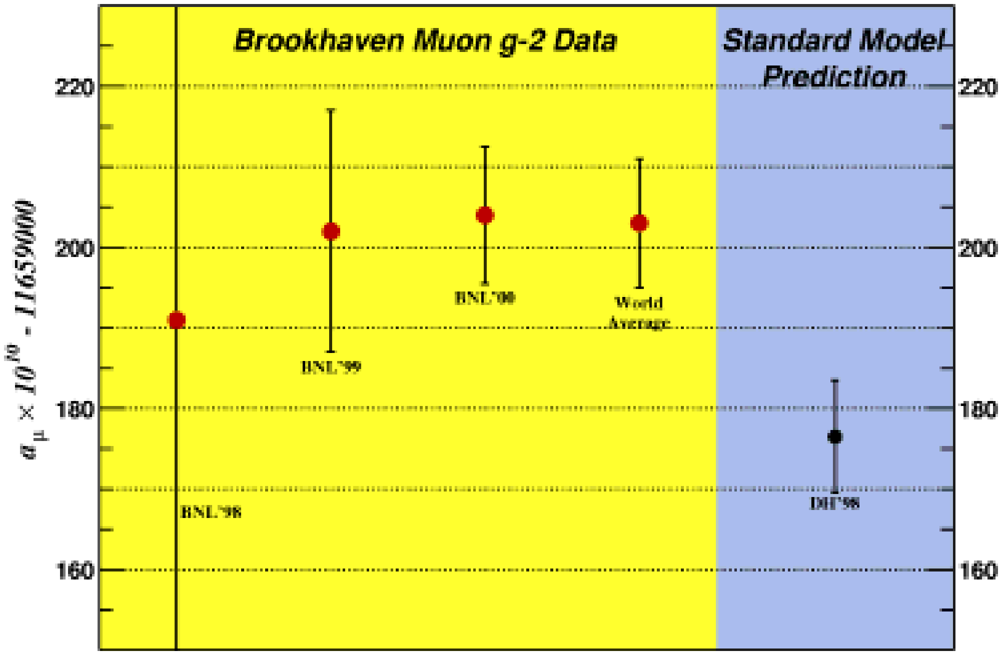
Error dominated by statistics. Systematics:

- 3.6×10^{-10} from precession frequency
- 2.8×10^{-10} from magnetic field

BNL compares WA with SM prediction (using DH'98 for hadronic vac. pol.):

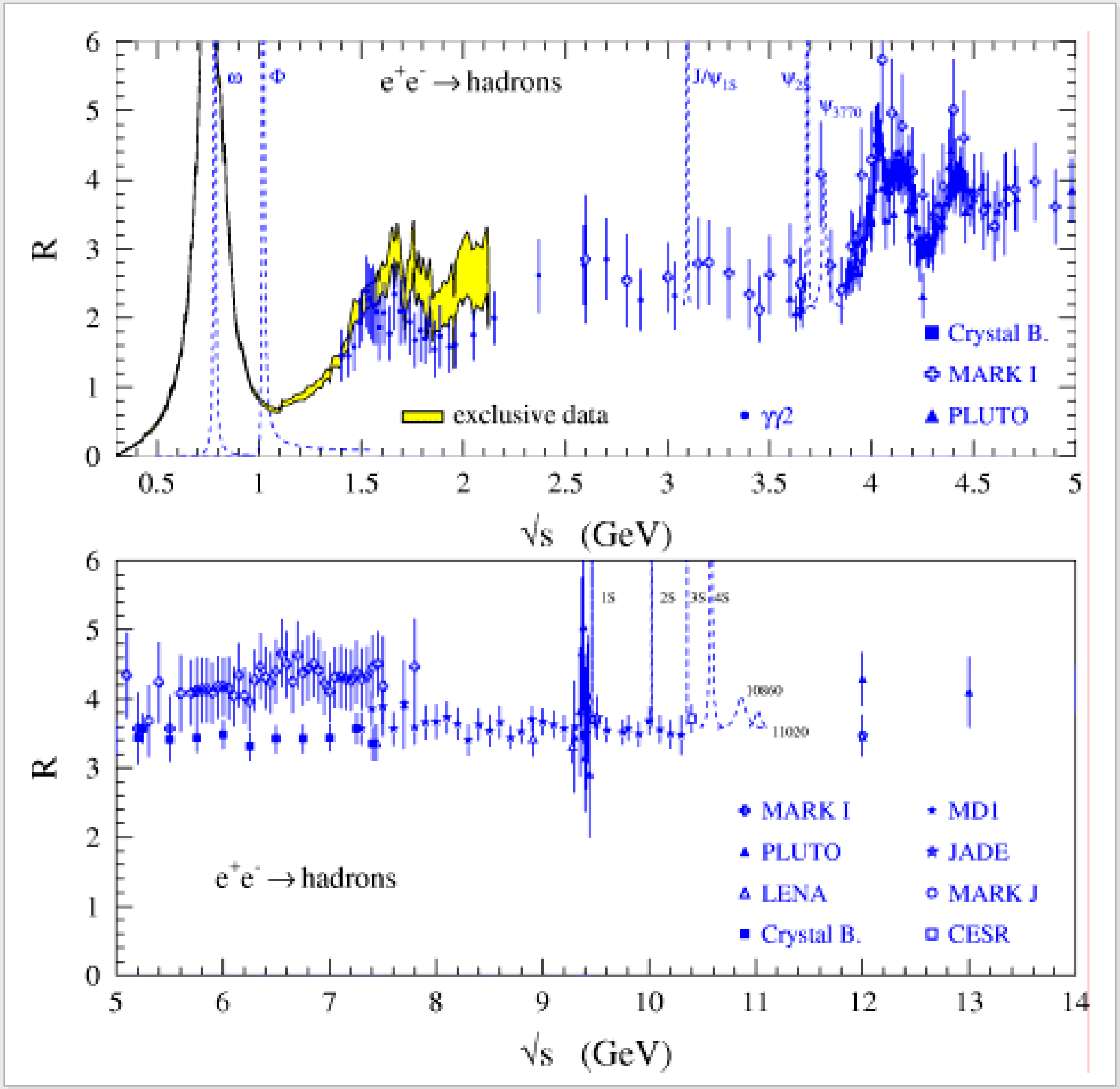
$$a_\mu(\text{exp}) - a_\mu(\text{SM}) = 25(10) \times 10^{-10}$$

Experimental and theoretical uncertainties now of similar order !



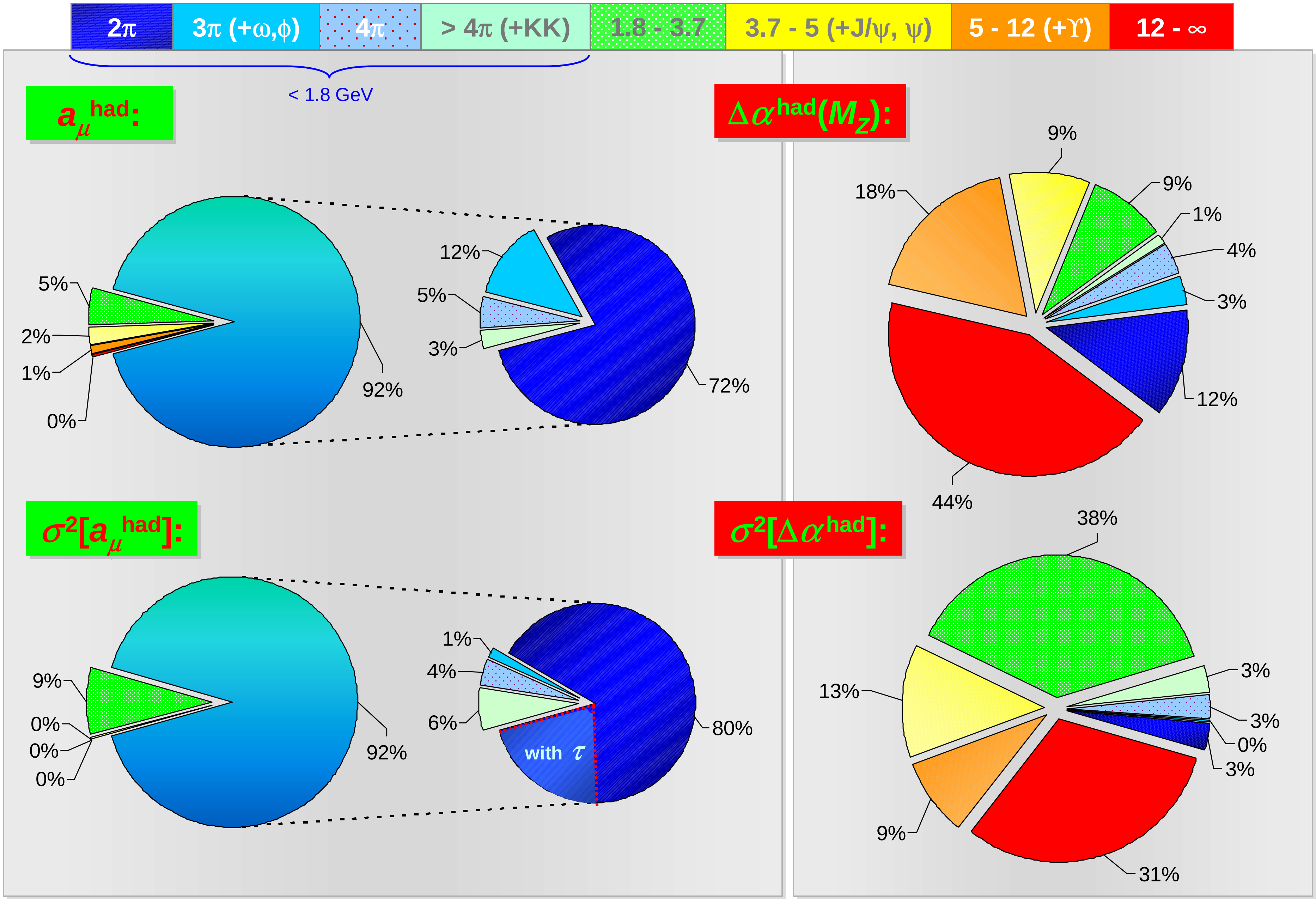
More work and, in particular, better data needed to achieve a more precise prediction of the hadronic contribution

The Data Situation (around 1995)



Data density and quality unsatisfactory in some crucial energy regions

(I) Energy-Dependent Contributions and Errors



Improved Determination of the Hadronic Contribution to $(g-2)_\mu$ and $\alpha(M_Z^2)$

Situation 1995 [Eidelman-Jegerlehner] and since ...

Eidelman-Jegerlehner
hep-ph/9502298

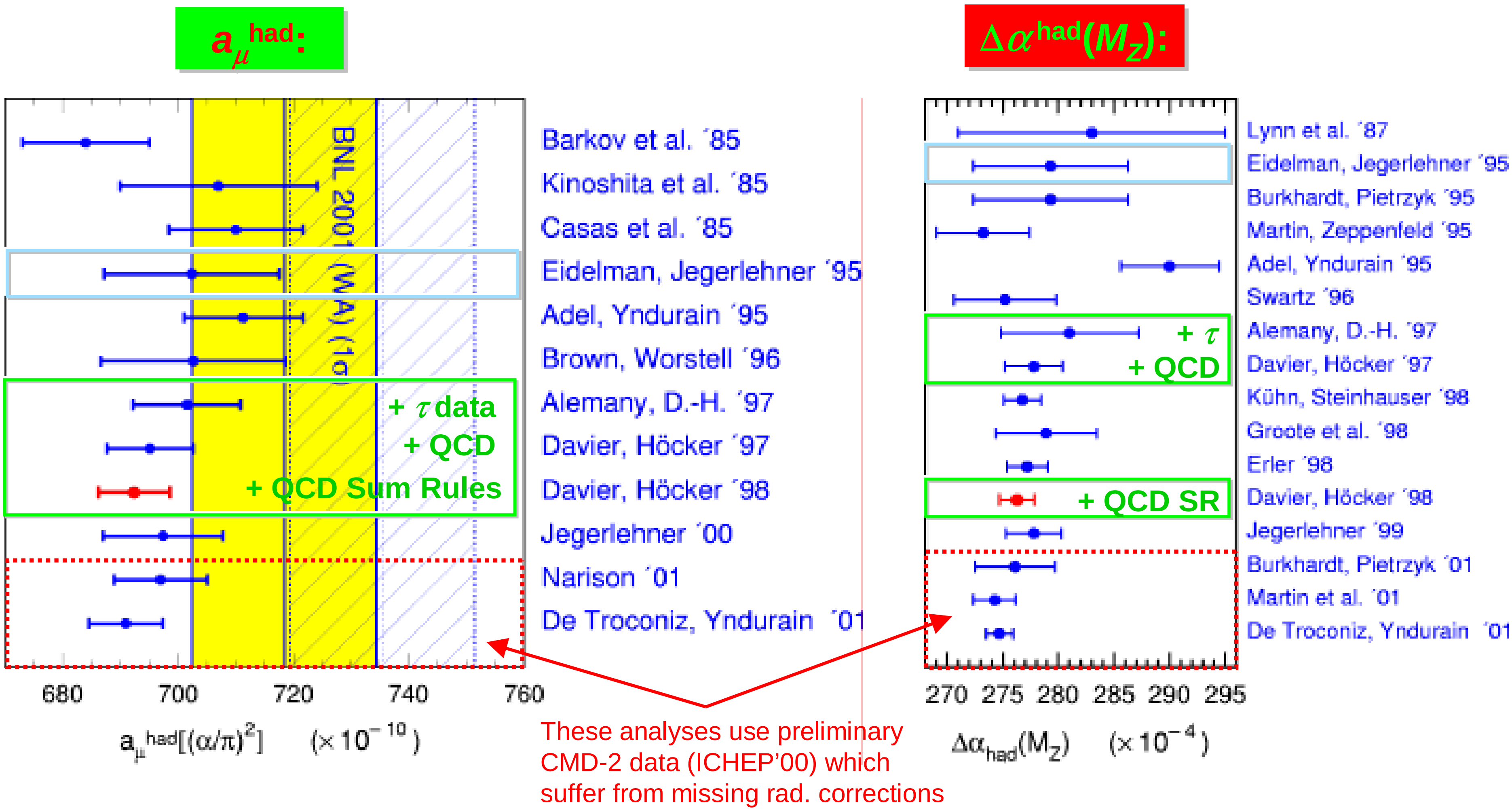
Davier-AH
hep-ph/9805470

Energy range [GeV]	Input 1995	Input after 1998
$2m_\pi$ - 1.8	**Data**	**Data (e^+e^- & τ) + QCD**
1.8 – J/ψ	**Data**	**QCD**
J/ψ - Υ	**Data**	**Data + QCD**
Υ - 40	**Data**	**QCD**
40 - ∞	**QCD**	**QCD**

Improvement in 3 Steps:

- Inclusion of precise τ data using SU(2) (*CVC*)
  Alemany-Davier-AH'97, Narison'01, Trocóniz-Ynduráin'01
- Extended use of (dominantly) perturbative QCD
  Martin-Zeppenfeld'95, Davier-AH'97, Kühn-Steinhauser'98, Erler'98, + others
- More theoretical constraints from QCD sum rules
  Groote, Körner, Schilcher, Nasrallah'98, Davier-AH'98, Martin-Outhwaite-Ryskin'00, Cvetič-Lee-Schmidt'01, + others

Situation before Summer 2002



- **Reasonable agreement among recent evaluations**
- **All recent analyses use enhanced theoretical input** (compared to EJ'95)
- **Most recent analyses employ e^+e^- and τ data**

A New Analysis of a_μ^{had}

Motivation for new work:

- **New high precision e^+e^- results (0.6% sys. error) around ρ from CMD-2 (Novosibirsk)**
- **New τ results from ALEPH using full LEP1 statistics, also: use CLEO data**
- **New *R* results from BES between 2 and 5 GeV**
- **New theoretical analysis on SU(2) breaking**

CMD-2 PL B527, 161 (2002)

ALEPH CONF 2002-19

CLEO PR D61, 112002; PR D61, 072003 (2000)

BES PRL 84 594 (2000); PRL 88, 101802 (2002)

Cirigliano-Ecker-Neufeld hep-ph/0207310

Outline of the new analysis:

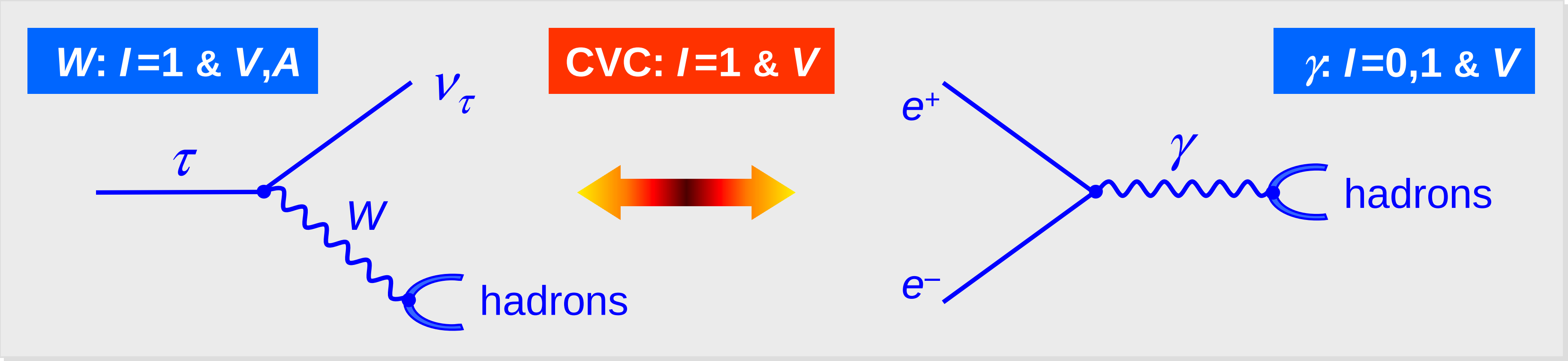
- **Include all new Novisibirsk (CMD-2, SND) and ALEPH data**
- **Apply (revisited) SU(2)-breaking corrections to τ data**
- **Identify application/non-application of radiative corrections**
- **Recompute all exclusive, inclusive and QCD contributions to dispersion integral; revisit threshold contribution and resonances**
- **Results, comparisons, disussions...**

Davier-Eidelman-AH-Zhang hep-ph/0208177

The Conserved Vector Current

The CVC property of weak decays follows from the factorization of strong physics produced through the γ and *W* propagators out of the QCD vacuum

The Conserved Vector Current – SU(2)



Hadronic physics factorizes in *Spectral Functions*:

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

Isospin symmetry (CVC) connects I=1 e^+e^- cross section to vector τ spectral functions:

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\,\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]}\;\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\;\frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching Fractions **mass spectrum** **kinematic factor (PS)**

τ Spectral Functions

Hadronic τ decays are a clean probe of hadron dynamics – experimentally in many ways complementary to $e^+e^- \to$ hadrons:

τ

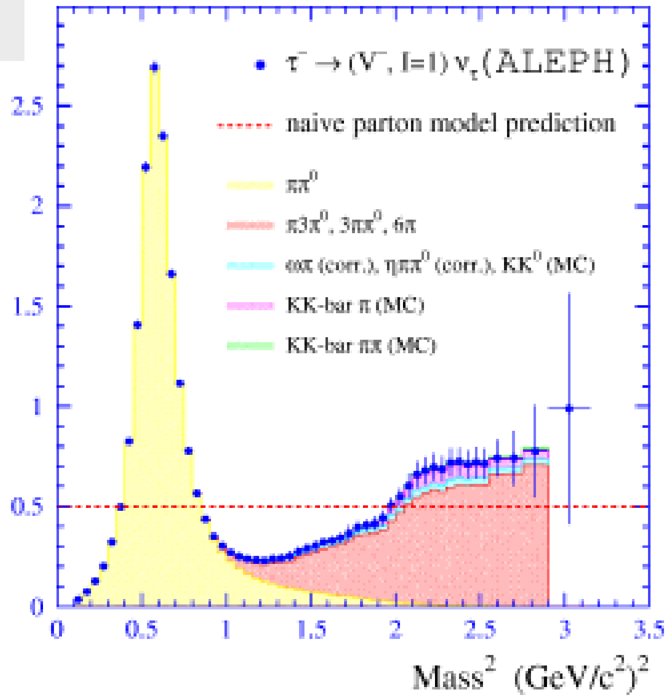
- **Excellent normalization (branching fractions) due to high statistics, large acceptance, small non-τ background**
- **Shape subject to bin-to-bin resolution corrections (*unfolding*)**

e^+e^-

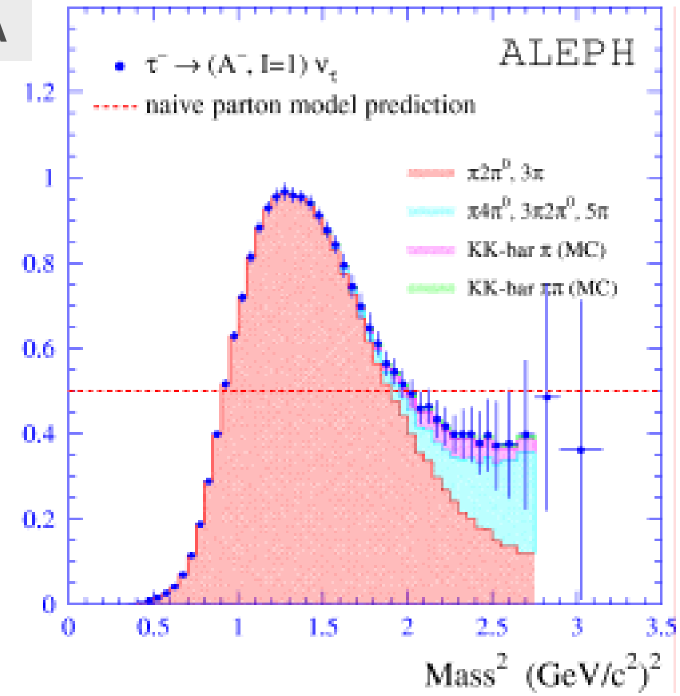
- **Excellent relative cross sections (correlated systematics)**
- **Overall normalization subject to radiative corrections, systematic uncertainties from acceptance and luminosity**

V

ALEPH
ZP C76, 15 (1997)



A



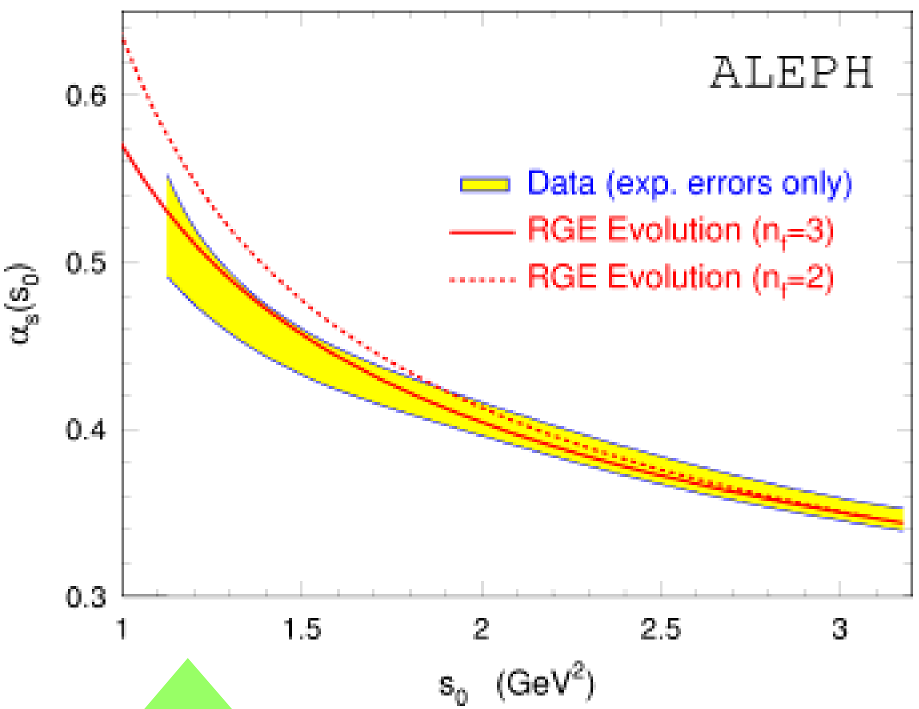
ALEPH
EPJ C4, 409 (1998)

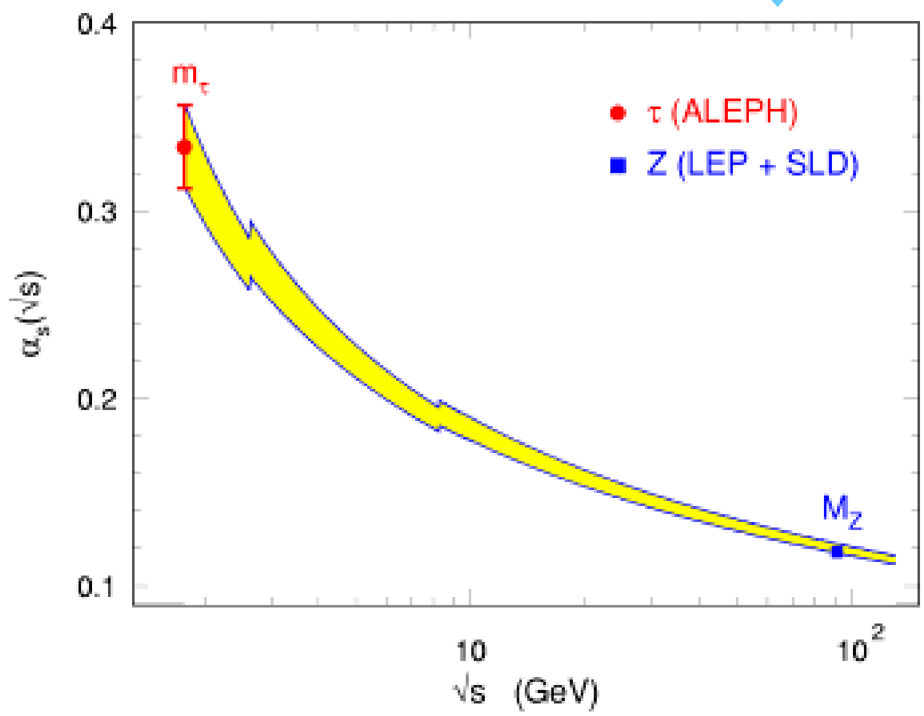
QCD Results from τ Decays

Evolution of $\alpha_s(m_\tau)$, measured using τ decays, to M_Z using RGE (4-loop QCD β-function & 3-loop quark flavor matching) shows the excellent compatibility of τ result with EW fit:

$\alpha_s(M_Z) = 0.1202 \pm 0.0027$ (ALEPH'98, theory dominated)

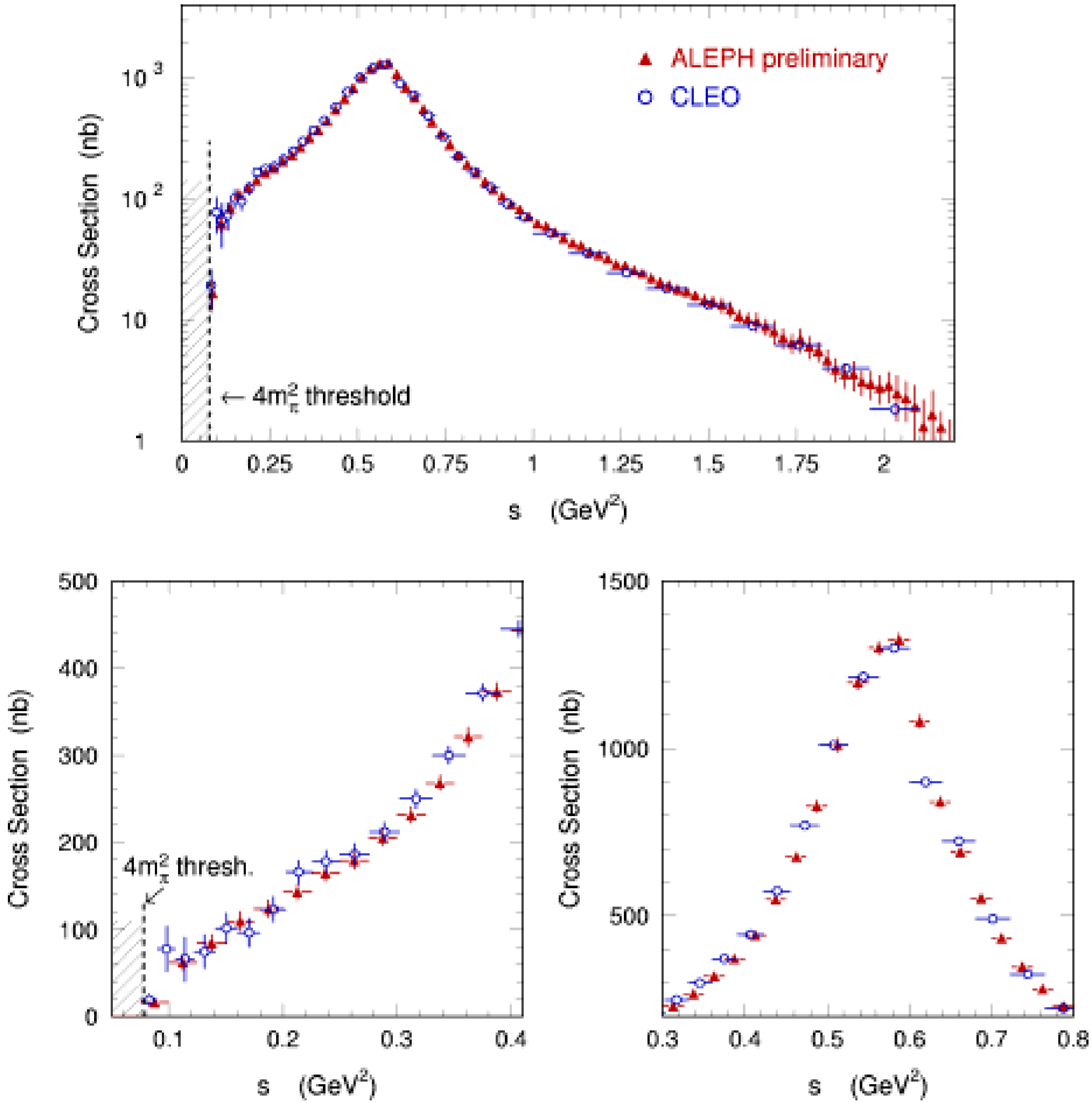
$\alpha_s(M_Z) = 0.1183 \pm 0.0027$ (LEP'00, statistics dominated)





The τ spectral function allows to directly measure the running of $\alpha_s(s_0)$ within $\sqrt{s_0} \in$ [~1...1.8 GeV]

$\tau \to \pi^- \pi^0 \nu_\tau$: Comparing ALEPH and CLEO



Spectral functions expressed as cross sections.

Shape comparison only. Both normalized to WA branching fraction.

- **Agreement observed**
- **ALEPH more precise at low *s***
- **CLEO better at high *s***

SU(2) Breaking

Electromagnetism does not respect isospin and hence we have to consider isospin breaking when dealing with an experimental precision of 0.5%

Corrections for SU(2) breaking applied to τ data for dominant $\pi^-\pi^+$ contrib.:

New Develpment

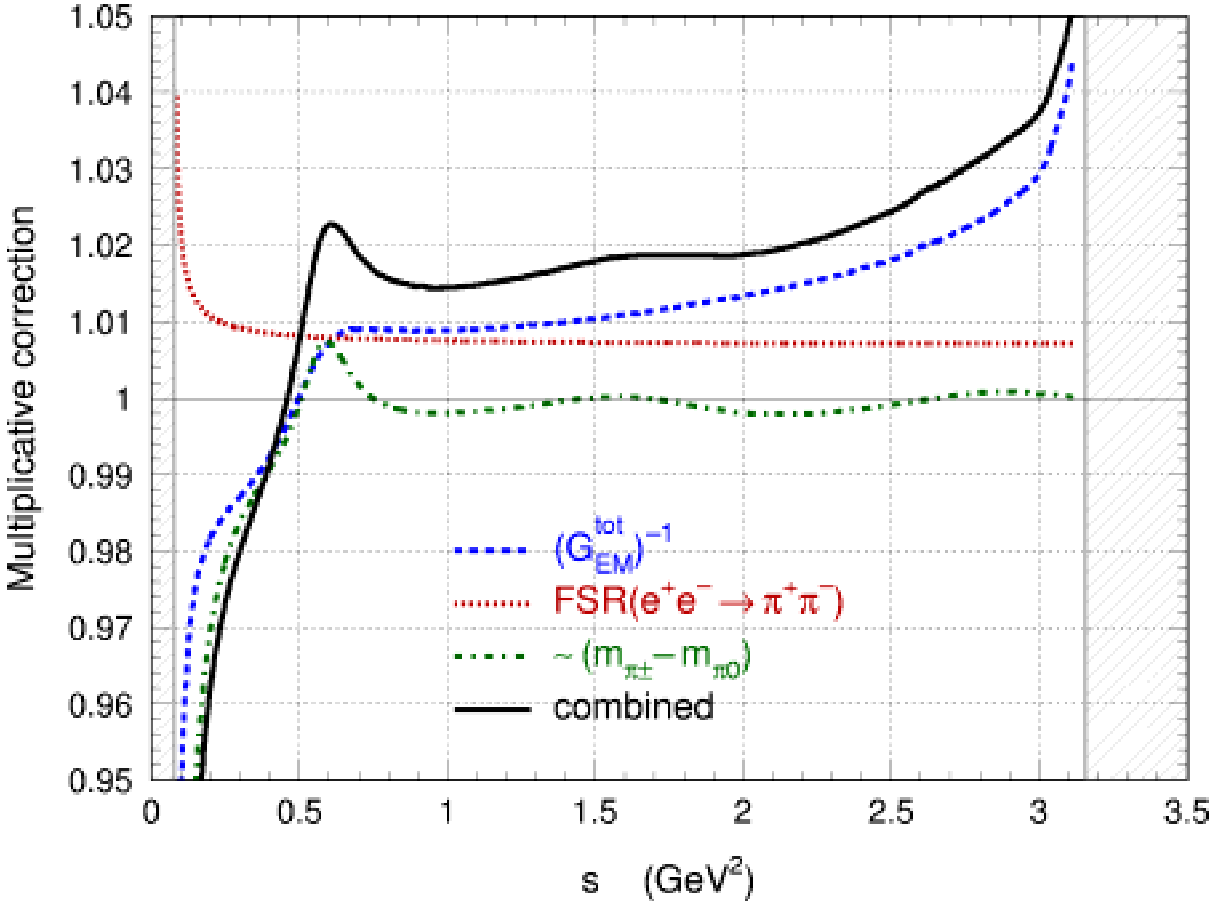
- **Electroweak radiative corrections:**
  - **dominant contribution from short distance correction S_{EW} to effective 4-fermion coupling** $\propto (1 + 3\alpha(m_\tau)/4\pi)(1+2\langle Q\rangle)\log(M_Z/m_\tau)$
  - **subleading corrections calculated and small**
  - **long distance radiative correction $G_{EM}(s)$ calculated**
    [add FSR to the bare cross section in order to obtain $\pi^-\pi^+(\gamma)$]

Cirigliano-Ecker-Neufeld, hep-ph/0207310

- **Charged/neutral mass splitting:**
  - **$m_{\pi^-} \neq m_{\pi^0}$ leads to phase space (cross sec.) and width (FF) corrections**
  - **$m_{\rho^-} \neq m_{\rho^0}$ and ρ-ω mixing** (EM $\omega \to \pi^-\pi^+$ decay) **corrected using FF model**
- **Electromagnetic decays, like:** $\rho \to \pi\pi\gamma$, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to l^+l^-$
- **Quark mass difference $m_u \neq m_d$ generating "second class currents" (negligible)**

SU(2) Breaking

Multiplicative SU(2) corrections applied to $\tau^- \to \pi^-\pi^0\nu_\tau$ spectral function:



Only β^3 and EW short distance corrections applied to 4π spectral functions

SU(2) Breaking

Corrections for isospin violation applied to τ data

Source	Δa_μ^{had} (10^{-10}) $\pi^-\pi^+$ (simple)	$\pi^-\pi^+$ (improved)	$\pi^-\pi^+2\pi^0$	$2\pi^-2\pi^+$
Short distance radiative corrections to τ decays (S_{EW} = 1.0267± 0.0027) [Marciano-Sirlin'88, Braaten-Li'90, new evaluation DEHZ'02]	**−13.8 ± 2.5**	**−13.8 ± 2.5**	**−0.49 ± 0.09**	**−0.25 ± 0.05**
Long distance corrections	-	**−1.0**	-	-
$m_{\pi^-} \neq m_{\pi^0}$ (β in cross section)	**−7.0**	**−7.0**	**+0.6 ± 0.6**	**−0.4 ± 0.4**
$m_{\pi^-} \neq m_{\pi^0}$ (β in width)	**+4.0**	**+4.2**	-	-
$m_{\rho^-} \neq m_{\rho^0}$ (± ≈1 MeV/c^2)	**+0 ± 0.2**	**+0 ± 2.0**	-	-
ρ-ω **mixing** (exp. uncertainty)	**+3.5 ± 0.6**	**+3.5 ± 0.6**	-	-
EM decay modes	**−1.4 ± 1.2**	**−1.4 ± 1.2**	-	-
Total correction	**−14.7 ± 2.9**	**−15.7 ± 2.8**	**+0.1 ± 0.6**	**−0.7 ± 0.4**

e^+e^- Radiative Corrections

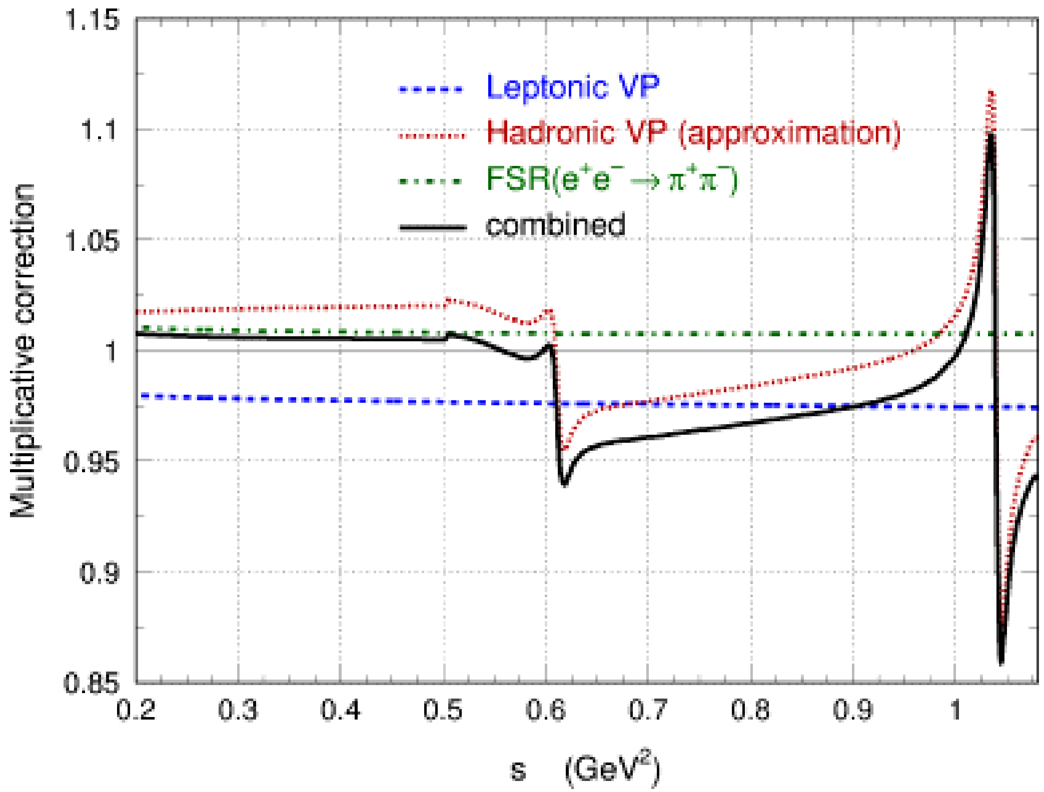
Multiple radiative corrections are applied on measured e^+e^- cross sections

Situation often unclear: whether or not - and if - which corrections were applied

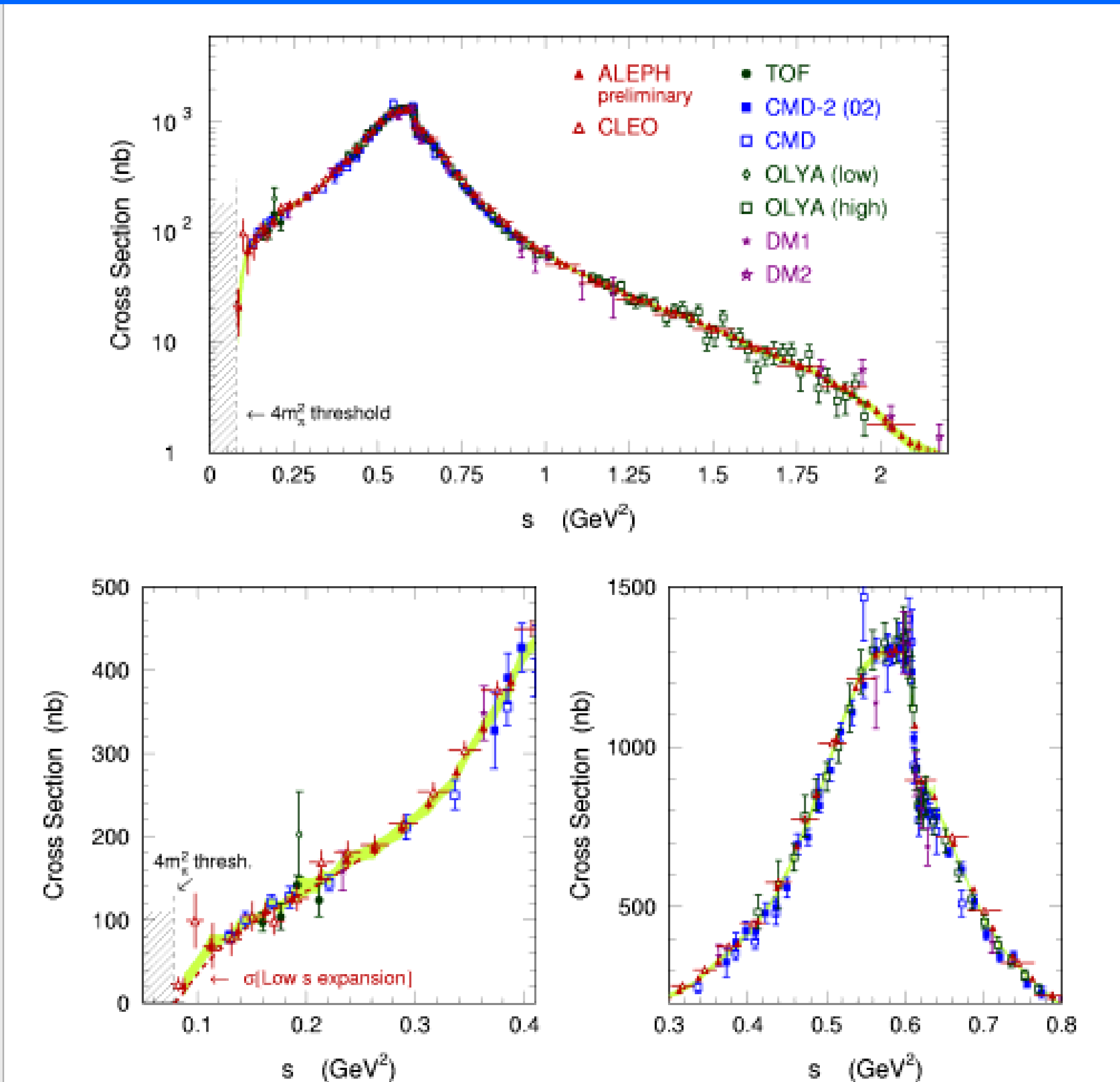
- **Vauum polarization (VP) in the photon propagator:**
  - **leptonic VP mostly corrected**
  - **hadronic VP not corrected but for CMD-2 (in principle: iterative proc.)**

- **Initial state radiation (ISR)**
  - **corrected by experiments**

- **Final state radiation (FSR) [we need $e^+e^- \to$ hadrons (γ) in dispersion integral]**
  - **mostly, experiments obtain bare cross section so that FSR has to be added "by hand"; done for CMD-2, (supposedly) not done for others**



Comparing $e^+e^- \to \pi^+\pi^-$ and $\tau \to \pi^-\pi^0\nu_\tau$

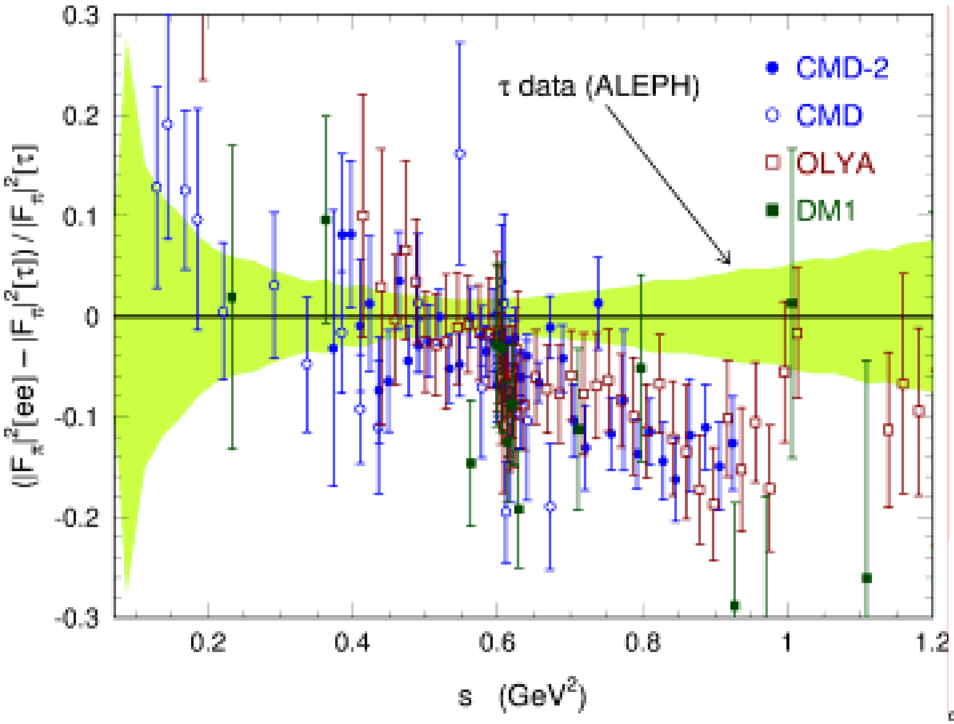


Correct τ data for missing ρ-ω mixing (taken from BW fit) and all other SU(2)-breaking sources

- **Remarkable agreement**
- **<u>But:</u> is it good enough ?**

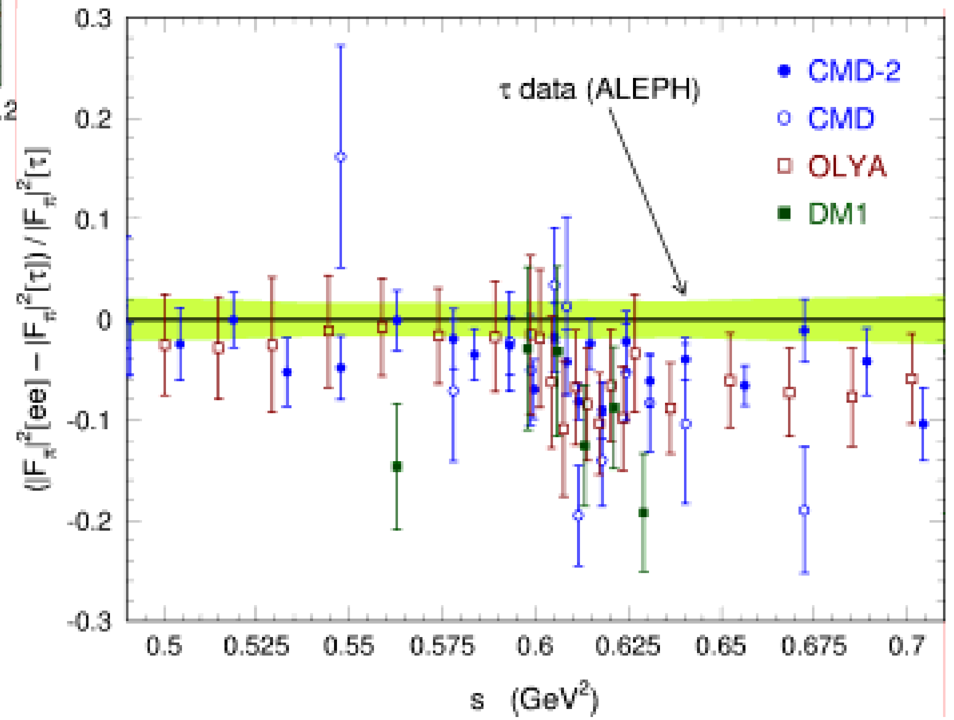
...

Comparing $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\nu_\tau$

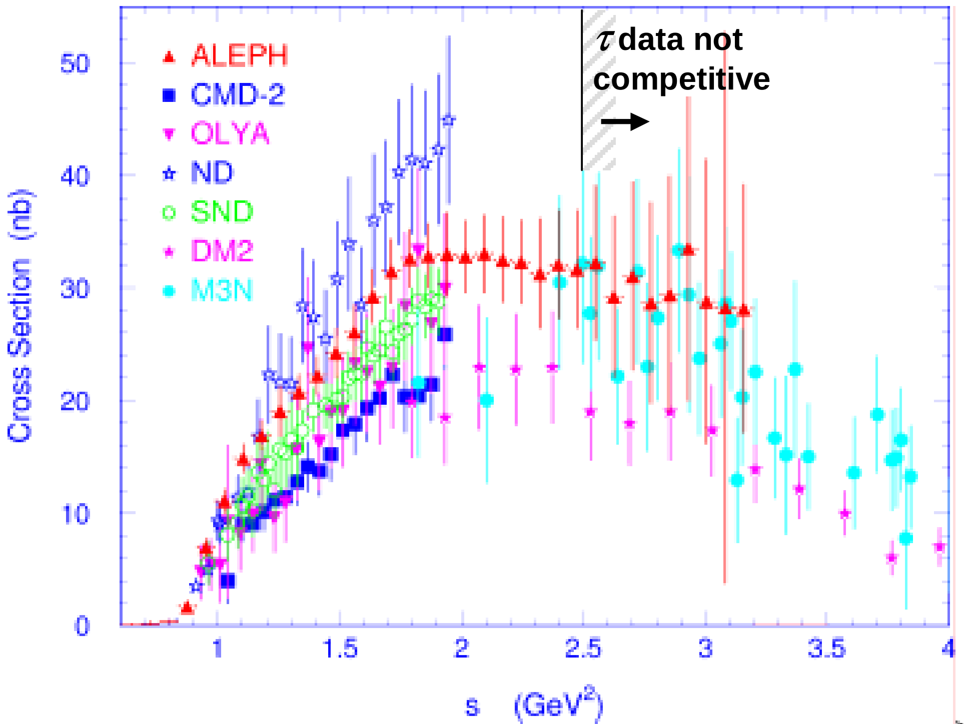


Relative difference between τ and e^+e^- data

zoom



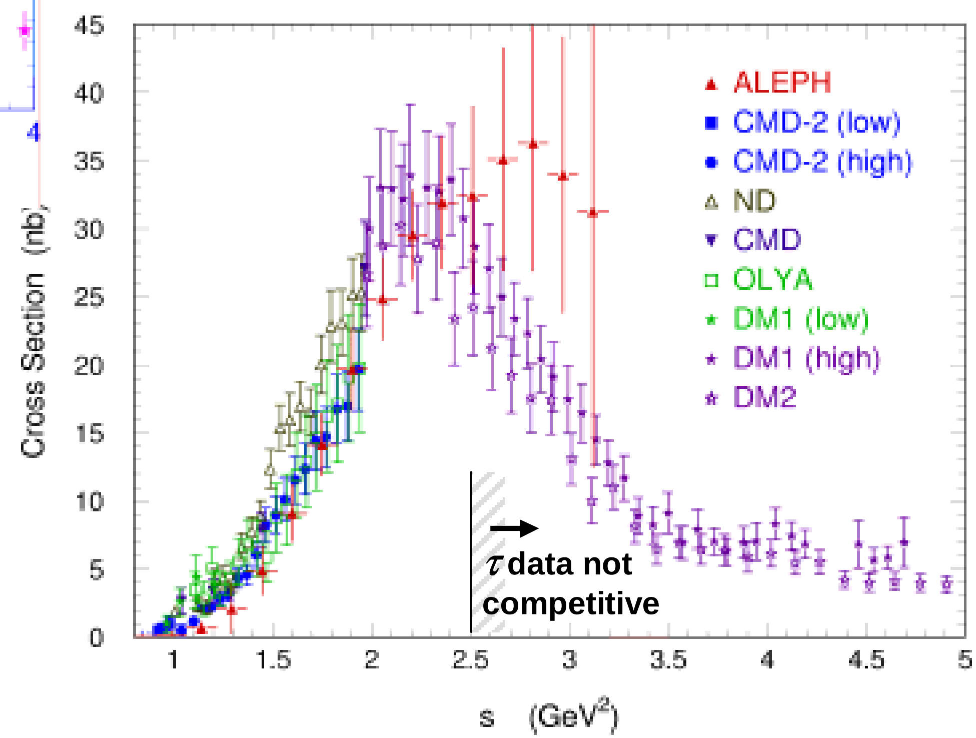
Comparing the 4π Spectral Functions



$e^+e^- \rightarrow \pi^+\pi^-2\pi^0$

Large discrepancies between experiments

$e^+e^- \rightarrow 2\pi^+2\pi^-$



CVC relations (Isospin rotation):

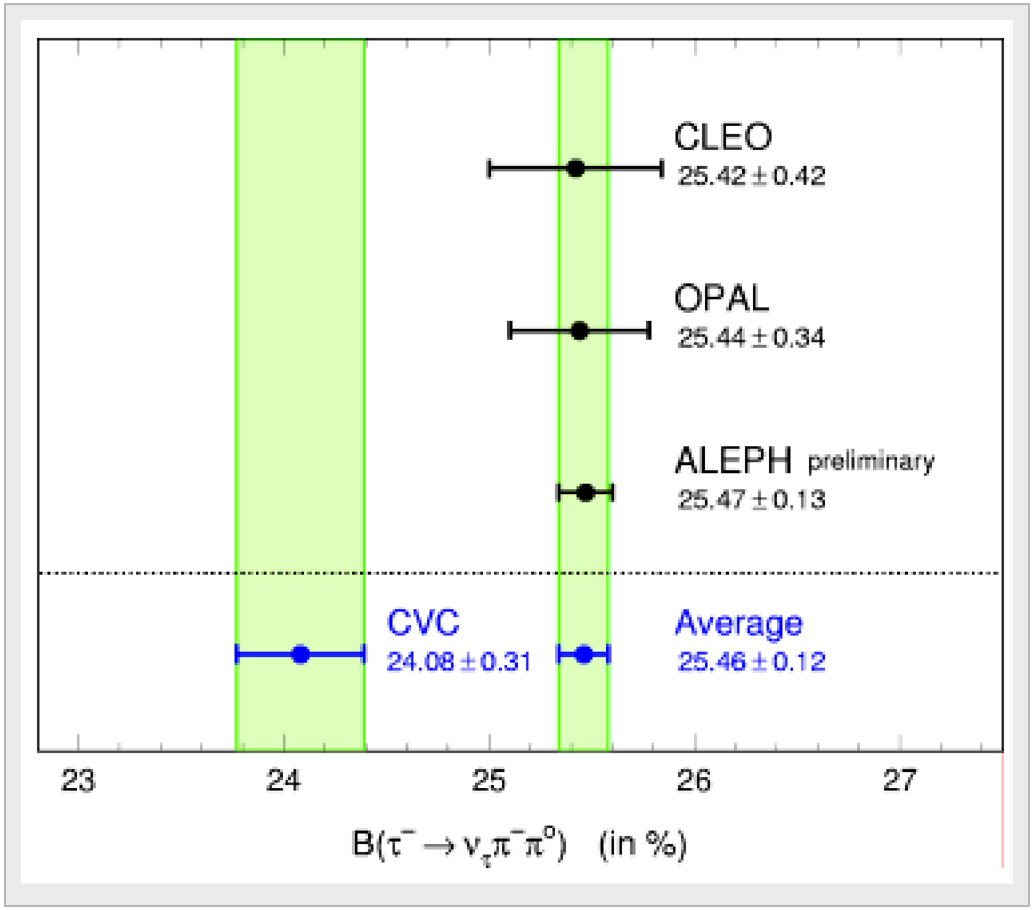
$$\sigma^{(I=1)}_{2\pi^+2\pi^-} = 2\cdot\frac{4\pi\alpha^2}{s}\upsilon_{\pi^-3\pi^0\nu_\tau}$$

$$\sigma^{(I=1)}_{\pi^+\pi^-2\pi^0} = \frac{4\pi\alpha^2}{s}\left(\upsilon_{2\pi^-\pi^+\pi^0\nu_\tau} - \upsilon_{\pi^-3\pi^0\nu_\tau}\right)$$

Testing CVC

Infer τ branching fractions from e^+e^- data:

$$\mathrm{BR}_{\mathrm{CVC}}\left(\tau^- \to \pi^-\pi^0\nu_\tau\right) = \frac{6\pi\,|V_{ud}|^2\,S_{EW}}{m_\tau^2}\int_0^{m_\tau^2} ds\ \mathrm{kin}(s)\cdot \upsilon^{\mathrm{SU(2)\text{-}corrected}}(s)$$



← $\tau^- \to \pi^-\pi^0\nu_\tau$

Difference: BR[τ] – BR[e^+e^- (CVC)]:

Mode	$\Delta(\tau - e^+e^-)$	„Sigma“
$\tau^- \to \pi^-\pi^0\nu_\tau$	+ 1.38 ± 0.33	4.0
$\tau^- \to \pi^- 3\pi^0\nu_\tau$	– 0.09 ± 0.11	0.8
$\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$	+ 0.88 ± 0.25	3.5

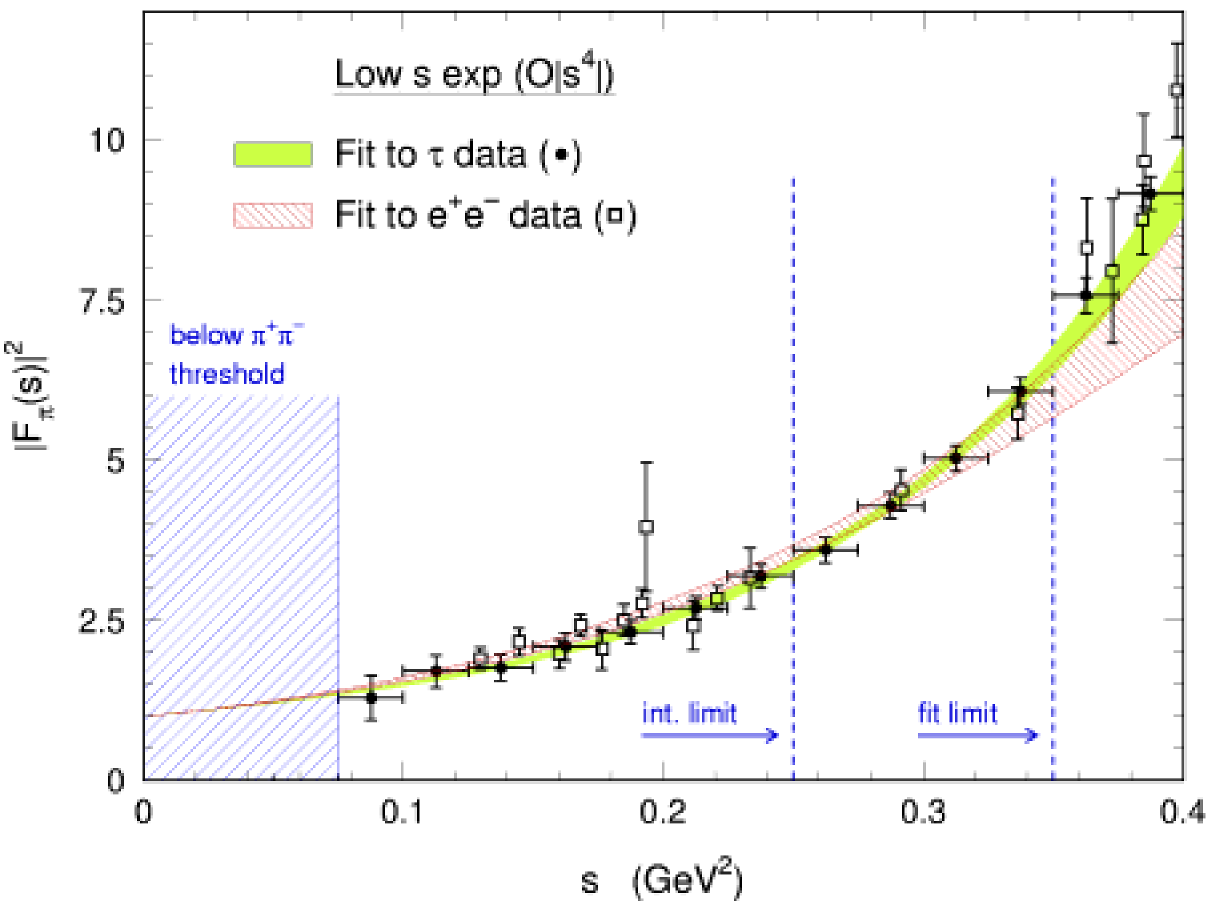
Specific Contributions

Specific Contributions: Low s Threshold

Use Taylor expansion for $\pi^+\pi^-$ threshold:

$$\sigma_{\pi\pi} = \frac{\pi\alpha^2\beta^3}{3s}\left|F_\pi\right|^2 \qquad \text{and:} \qquad F_\pi = 1 + \frac{1}{6}\left\langle r^2\right\rangle_\pi s + c_1 s^2 + c_2 s^3 + O(s^4)$$

- **exploiting precise space-like data, $\langle r^2\rangle_\pi$= (0.439 ± 0.008) fm^2, and fitting c_1 and c_2**



Fit range:

- **up to 0.35 GeV^2**

Integration range:

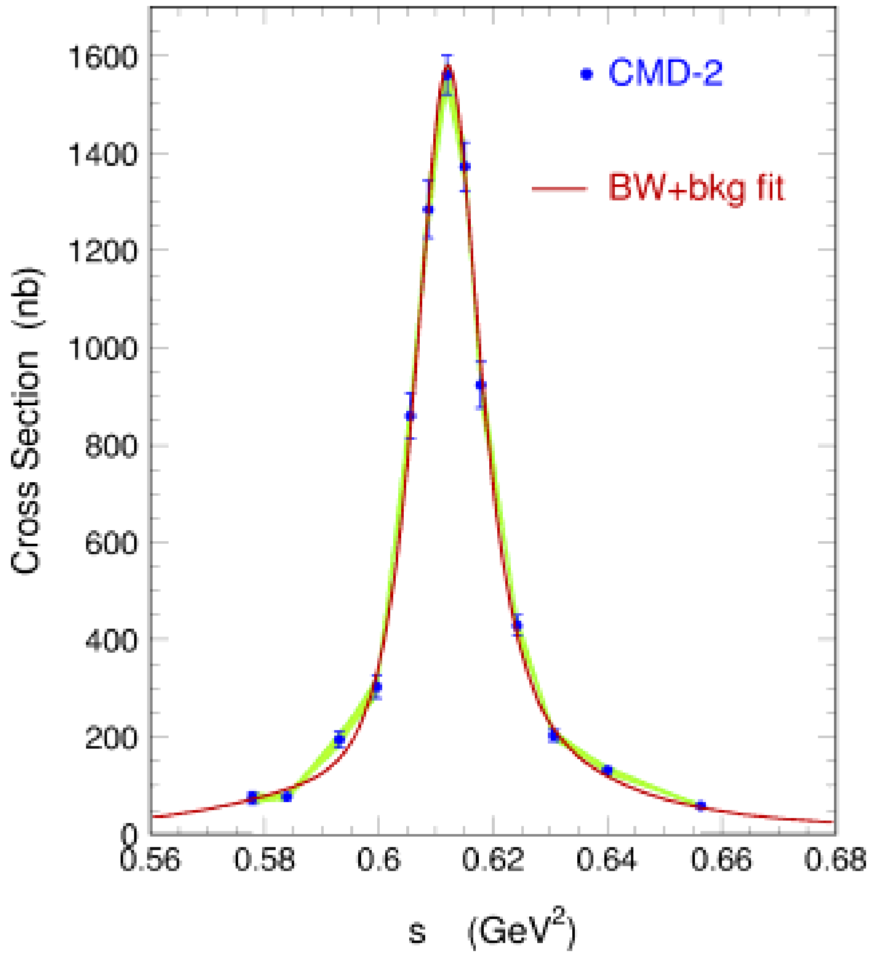
- **up to 0.25 GeV^2**

- **Excellent χ^2 for both τ and e^+e^- data**
- **strong anti-correlation between c_1 and c_2**

Specific Contributions: Narrow Resonances

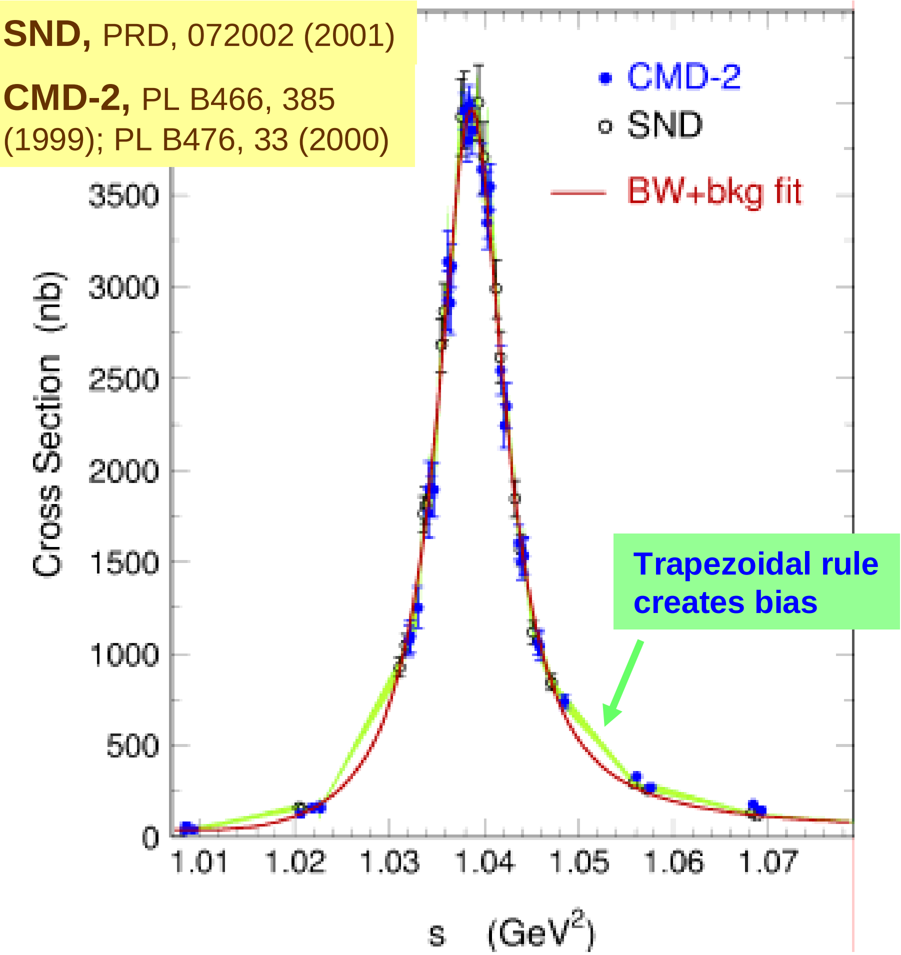
Use direct data integration for $\omega(782)$ and $\phi(1020)$ to account for non-resonant contributions. However, careful integration necessary:

- **trapezoidal rule creates systematics for functions with strong curvature**
- **use phenomenological fit**



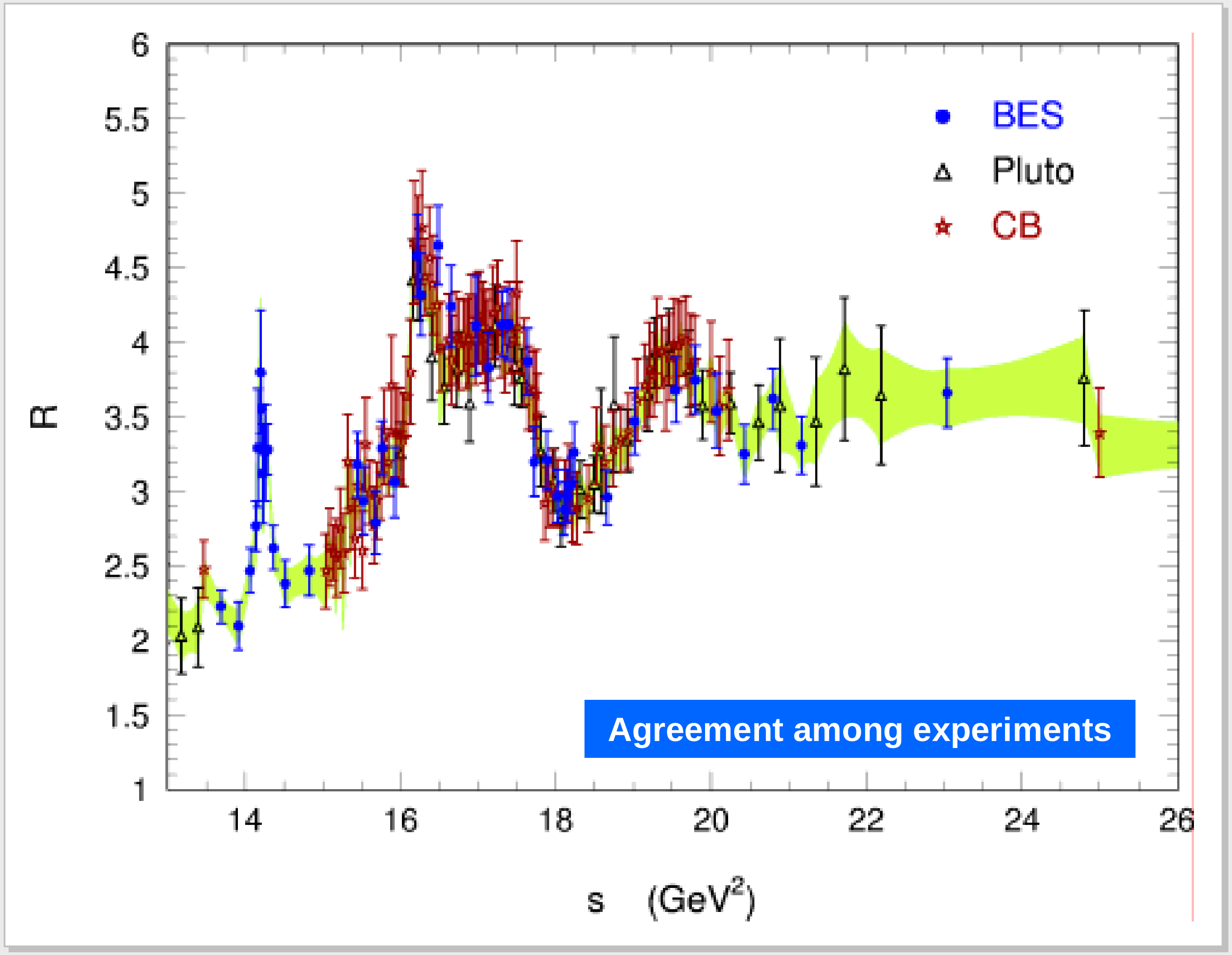
SND, PRD, 072002 (2001)

CMD-2, PL B466, 385 (1999); PL B476, 33 (2000)



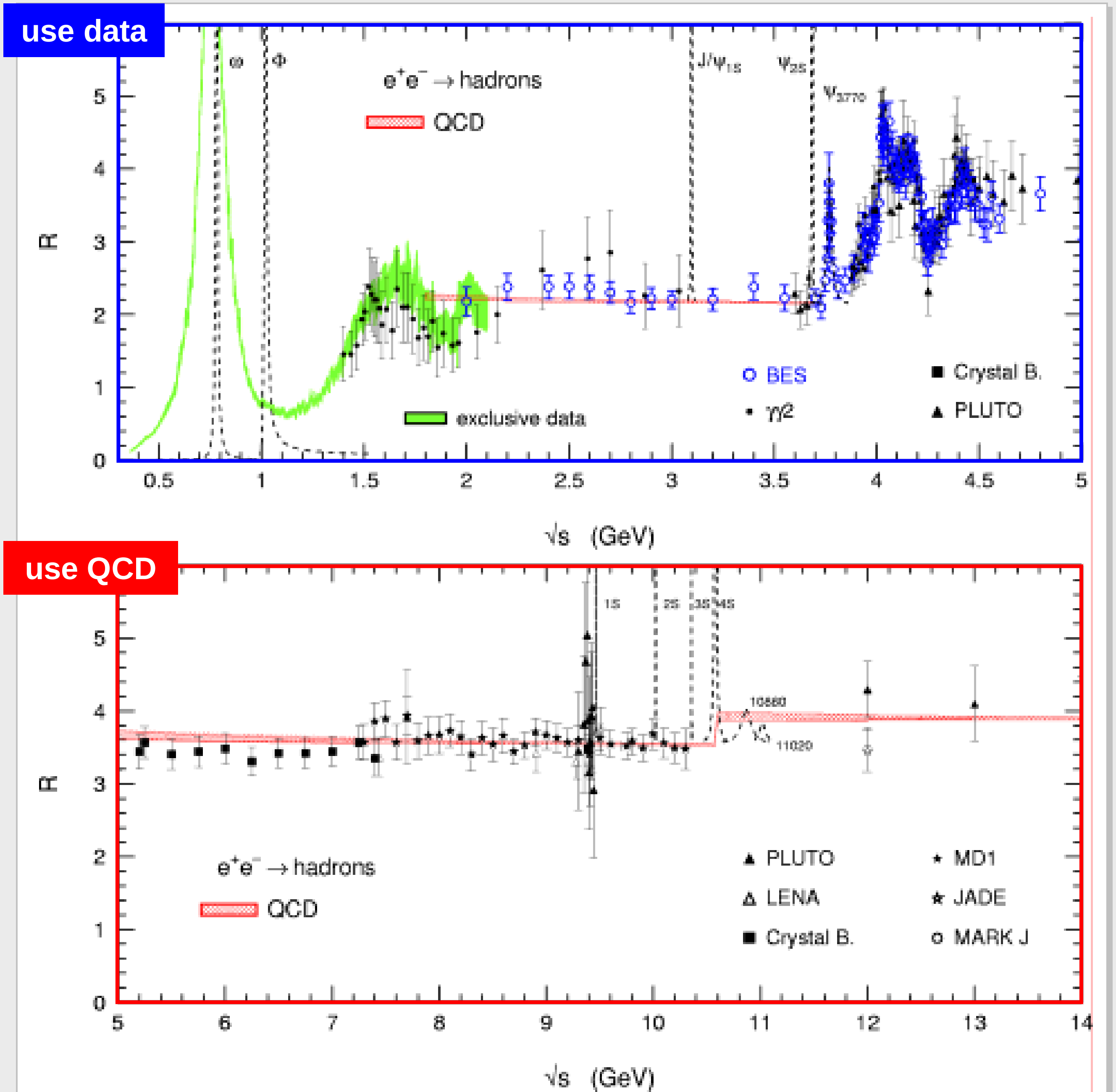
Specific Contributions: the Charm Region

New precise BES data improve $c\bar{c}$ resonance region:



Results

Results: the Data & the Theory



- **Agreement between Data (BES) and pQCD**

- **Better agreement between exclusive and inclusive ($\gamma\gamma2$) data than in previous analysis**

Results: the Compilation

Contributions to a_μ^{had} from the different energy domains:

Modes	Energy range [GeV]	a_μ^{had} (10^{-10}) e^+e^-	τ
Low s expansion	$2m_\pi$ – 0.5	$58.0 \pm 1.7 \pm 1.1_{rad}$	$54.0 \pm 1.7 \pm 0.3_{SU(2)}$
$\pi^+\pi^-$	$2m_\pi$ – 1.8	$440.8 \pm 4.7 \pm 1.5_{rad}$	$459.0 \pm 2.9 \pm 2.5_{SU(2)}$
$\pi^+\pi^- 2\pi^0$	$2m_\pi$ – 1.8	$16.7 \pm 1.3 \pm 0.2_{rad}$	$21.4 \pm 1.1 \pm 0.6_{SU(2)}$
$2\pi^+ 2\pi^-$	$2m_\pi$ – 1.8	$14.0 \pm 0.9 \pm 0.2_{rad}$	$12.3 \pm 1.0 \pm 0.4_{SU(2)}$
ω(782)	0.3 – 0.81	$36.9 \pm 0.8 \pm 0.8_{rad}$	-
ϕ(1020)	1.0 – 1.055	$34.8 \pm 0.9 \pm 0.6_{rad}$	-
Other exclusive	$2m_\pi$ – 2.0	$32.2 \pm 1.6 \pm 0.3_{rad}$	-
J/ψ, ψ(2S)	3.08 – 3.11	$7.4 \pm 0.4 \pm 0_{rad}$	-
R [data]	2.0 – 5.0	$33.9 \pm 1.7_{exp} \pm 0_{rad}$	-
R [QCD]	5.0 – ∞	$9.9 \pm 0.2_{theo}$	-
Sum	$2m_\pi$ – ∞	$684.7 \pm 6.0 \pm 3.6_{rad}$	$701.9 \pm 4.7 \pm 1.2_{rad} \pm 3.8_{SU(2)}$

Discussion

The problem of the $\pi^+\pi^-$ contribution [shifts given in units of 10^{-10}]**:**

- **Experimental conspiracy:**
  - new **CMD-2 data produce downward shift** [–1.9]**, with much better precision**
  - **new ALEPH BRs produce upward shift** [+3.5]
  - **CLEO spectral functions produce upward shift** [+2.2]
  - **previous difference was: $\Delta[\tau - e^+e^-] = (11 \pm 15)\ 10^{-4}$ → we could use average**
- **Who is wrong ?**
  - **e^+e^- is consistent with among experiments, but error dominated by CMD-2; large radiative corrections applied**
  - **τ is consistent with among experiments, but error dominated by ALEPH**
  - **SU(2) corrections: basic contributions identified and stable since long; overall correction applied to τ is (-2.2 ± 0.5) %, dominated by uncontroversial short distance piece; additional long-distance corrections found to be small**

At present, we believe that it is unappropriate to combine τ and e^+e^-

Other changes with respect to DH'98 analysis:

- **Experimental Contribution from ϕ resonance much smaller** [–4.3]
- **Constraints from isospin lead to reduced contributions from 6π modes**

Final Results

[DEHZ'02]

$a_\mu^{had}[ee] = (684.7 \pm 7.0)\ 10^{-10}$

$a_\mu^{had}[\tau] = (701.9 \pm 6.2)\ 10^{-10}$

(exp and theo errors added in quadrature)

$\Delta[\tau - e^+e^-] = (0.62 \pm 0.29)\%$

692.4 ± 6.2

[DH'98]

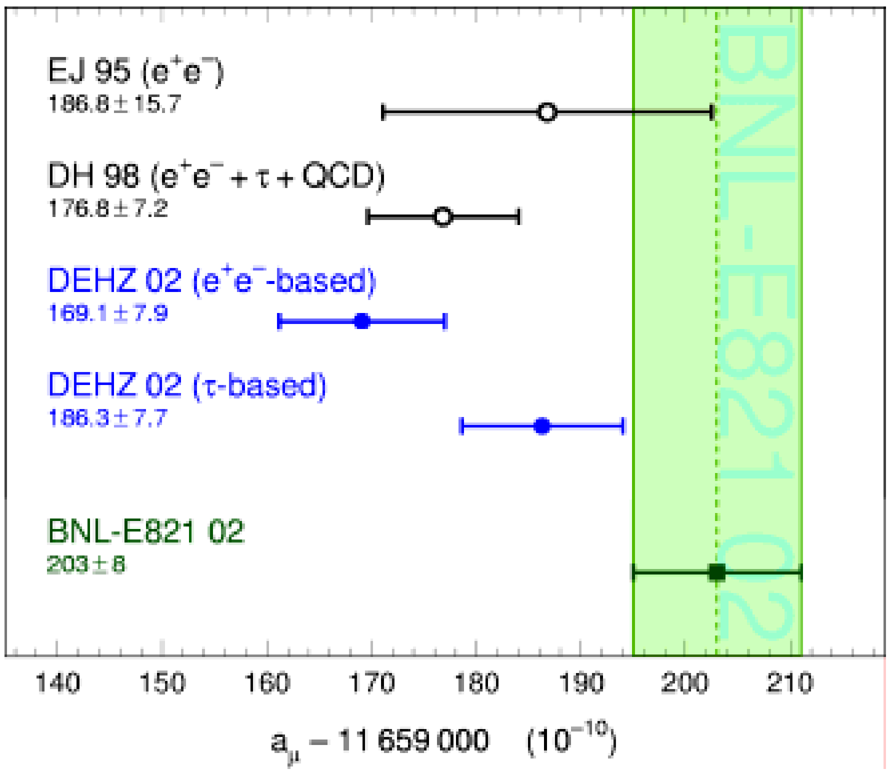
$a_\mu[ee] = (11\,659\,169.1 \pm 7.0_{had} \pm 3.5_{LBL} \pm 0.3_{QED+EW})\ 10^{-10}$

$a_\mu[\tau] = (11\,659\,186.3 \pm 6.2_{had} \pm 3.5_{LBL} \pm 0.3_{QED+EW})\ 10^{-10}$

including:

Hadronic contribution from higher order : $a_\mu^{had}[(\alpha_s/\pi)^3] = -(10.0 \pm 0.6)\ 10^{-10}$

Hadronic contribution from LBL scattering : $a_\mu^{had}[LBL] = +(8.6 \pm 3.5)\ 10^{-10}$



Observed Discrepancy:

$$a_\mu[\exp] - a_\mu[SM] \ (10^{-10}) = \begin{cases} 34 \pm 11 & [e^+e^-] \\ 17 \pm 11 & [\tau] \end{cases}$$

Effect on $\Delta\alpha_{had}(M_Z^2)$:

$\Delta[\tau - e^+e^-] = (2.37 \pm 0.62)\ 10^{-4}$

➔ $\Delta M_{Higgs} \approx -15$ GeV/c² for τ

Conclusions/Perspectives

- **Hadronic vacuum polarization creates dominant systematics for SM predictions of many precision measurements**

- **New analysis of leading hadronic contribution motivated by new, precise e^+e^-** (0.6% systematic error for e^+e^-) **and τ** (0.5% error on normalization) **data**

- **New theoretical analysis confirmed the rules to correct for SU(2) breaking**

- **Radiative (VP and FSR) corrections in e^+e^- are major source of systematics**

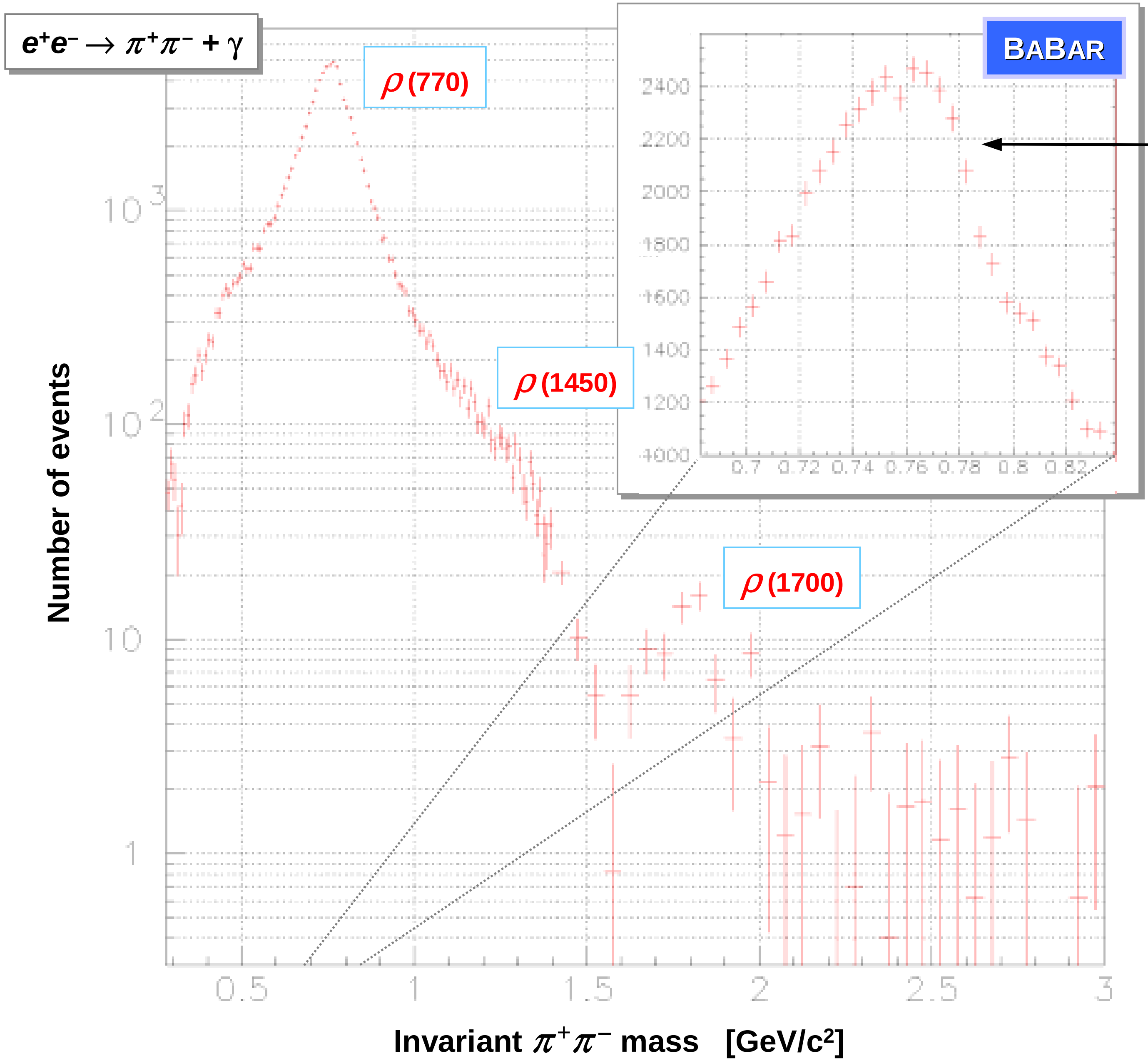
- **We have re-evaluated all exclusive and inclusive as well as resonance contributions**

- **We conclude with two incompatible numbers from e^+e^- and (mainly) τ, leading to SM predictions that differ by 3.0 σ [e^+e^-] and 1.6 σ [τ] from the experiment**

- **The key problem is the quality of the experimental data...**

- **Future experimental projects are:**
  - **PEP-N (SLAC): $e^+e^- \rightarrow$ hadrons between 1.4-3.1 GeV** ✯
  - **ISR production $e^+e^- \rightarrow$ hadrons + γ @ KLOE, BABAR, CLEO & BES as τ /charm factories ➡ (systematics?)**

Proof of Principle of ISR Method



Finite mass resolution visible in sharp ρ-ω interference. Requires unfolding.

- The data correspond to an integrated luminosity of only 22 fb^{-1}.
- Background from $e^+e^- \to \mu^+\mu^- + \gamma$ events is at the level of 1%.
- The present statistics (>80 fb^{-1}) is more than competitive with the latest results from CMD-2.
- The main concern is the control of the systematics, in particular for particle identification.

Backup Slides

how could we further improve a_μ^{had},
if there were no incompatibilities ?

Extend the Use of QCD

- The inclusion of τ data has reduces the error on a_μ^{had} by 40%
- However, no significant improvement for $\Delta\alpha^{had}(M_Z)$

...how can we further improve – in particular: $\Delta\alpha^{had}(M_Z)$?

HINT: Inclusive hadronic τ decays have shown that QCD is safely applicable at m_τ~1.8 GeV/c^2

ALEPH (1993, 1998)
CLEO (1994)
OPAL (1998)

→ **Why not also for e^+e^- ?**

...

TEST: Do equivalent QCD analysis as for τ decays using spectral moments of $\sigma[e^+e^- \to \text{hadrons}](s)$

$$R_{e^+e^-}(s_0) = \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} D(s)$$

The combined fit of spectral moment to e^+e^- data showed consistent OPE prediction and revealed small non-perturbative contributions at s_0=1.8 GeV

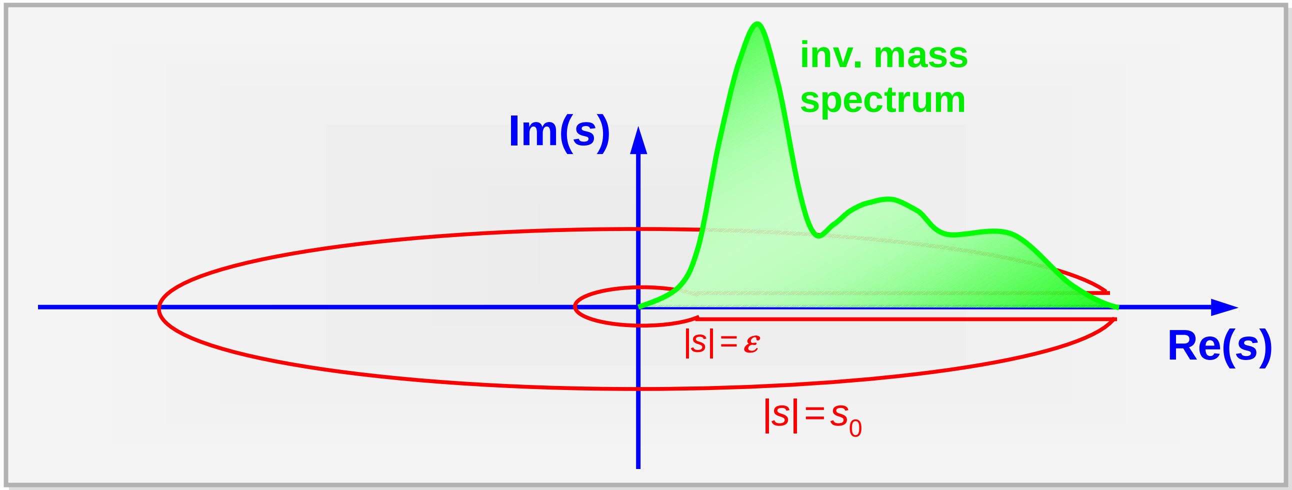
Spectral Functions and QCD

(1) Optical theorem $\upsilon(s) \propto \mathrm{Im}\,\Pi(s)$

(2) Apply Cauchy's theorem for "save" (*i.e.*, sufficiently large) s_0:

$$R(s_0) \propto \int_0^{s_0} ds\, f(s)\, \mathrm{Im}\,\Pi(s) \quad \Leftrightarrow \quad -\frac{1}{2i} \oint_{|s|=s_0} ds\, f(s)\,\Pi(s)$$

kinematic factor



(3) Use the Adler function to remove unphysical subtractions: $D(s) = s\dfrac{d\,\Pi(s)}{ds}$

(4) Use global quark-hadron duality in the framework of the Operator Product Expansion (OPE) to predict: $D(s) \sim D_{\mathrm{pert}}(s) + D_{\mathrm{q\text{-}mass}}(s) + D_{\mathrm{non\text{-}pert}}(s)$

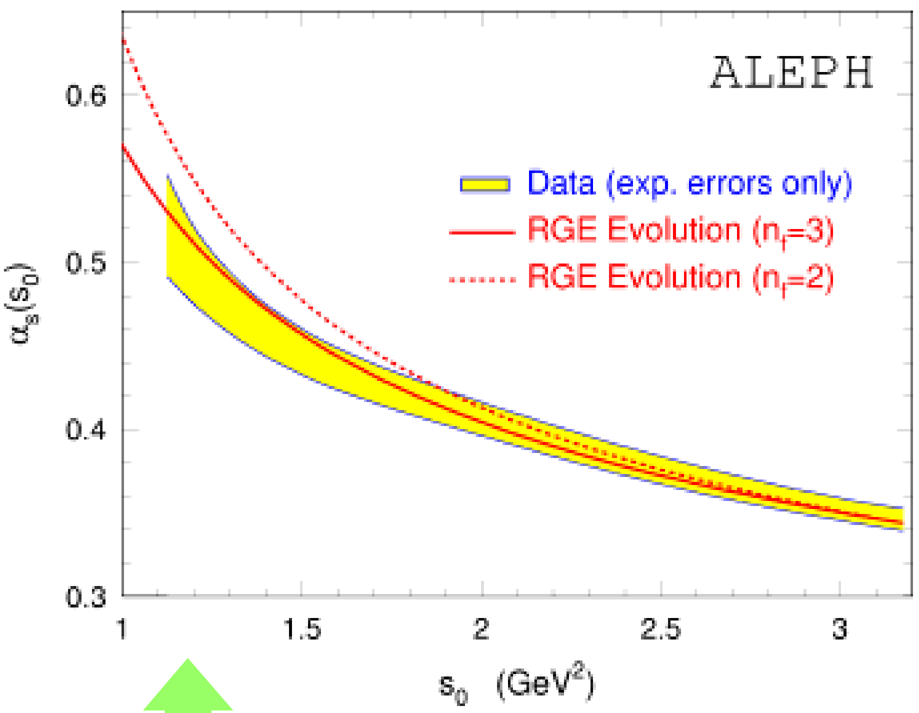
(5) Use analytical moments $f_n(s) = f(s)\cdot\mathrm{poly}_n(s)$ to fix non-perturbative parameters of the OPE ... and then fit $\alpha_s(m_\tau)$

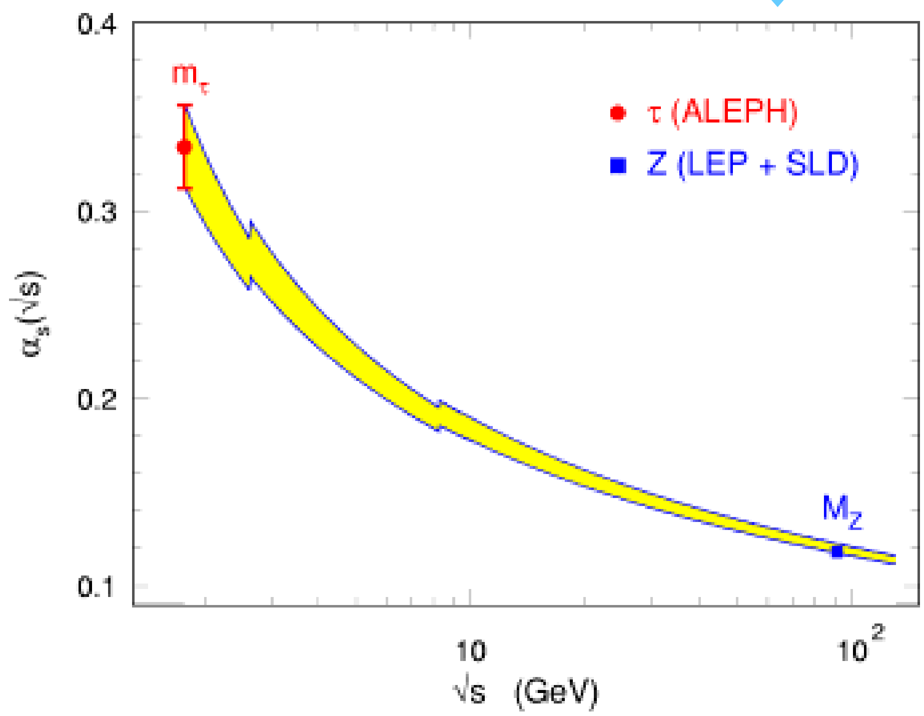
QCD Results from τ Decays

Evolution of $\alpha_s(m_\tau)$ to M_Z using RGE (4-loop QCD β-function & 3-loop quark flavor matching) shows the excellent compatibility of τ result with EW fit:

$\alpha_s(M_Z) = 0.1202 \pm 0.0027$ (ALEPH'98, theory dominated)

$\alpha_s(M_Z) = 0.1183 \pm 0.0027$ (LEP'00, statistics dominated)





The τ spectral function allows to directly measure the running of $\alpha_s(s_0)$ within $\sqrt{s_0} \in$ [~1...1.8 GeV]

Data Driven QCD Sum Rules

- The use of τ data and extended QCD reduces the error on a_μ^{had} by 51%
- and provides a 60% (!) reduction of the error on $\Delta\alpha^{\mathrm{had}}(M_Z)$

...however, there exist domains for which no theoretical constraints are used so far... we can thus still increase our information budget !

USE: Modified Dispersion Integral:

$$\int_0^{s_0} ds\, f \cdot \mathrm{Im}\Pi = \int_0^{s_0} ds\, [f - P_n]\,\mathrm{Im}\Pi + \frac{i}{2}\oint_{|s|=s_0} ds\, P_n \cdot \Pi$$

Data **Theory**

where the P_n are (analytic) polynomials that approximate the kernel $f(s)$ and thus reduce the data piece of the integral replaced by known theory. No new assumptions !

An optimization procedure minimizes the total experimental and (conservative) theoretical error