Precision measurement of the hadronic cross-section through the radiative return method

Germán Rodrigo

[H. Czyż, M. Guilleaume, A. Gerhmann-De Ridder, J.H. Kühn, M. Szopa]

- Motivation
- Radiative Return at meson factories
- PHOKHARA: a NLO generator
- New features of PHOKHARA
- Conclusions

The hadronic x-section and R in e^+e^-

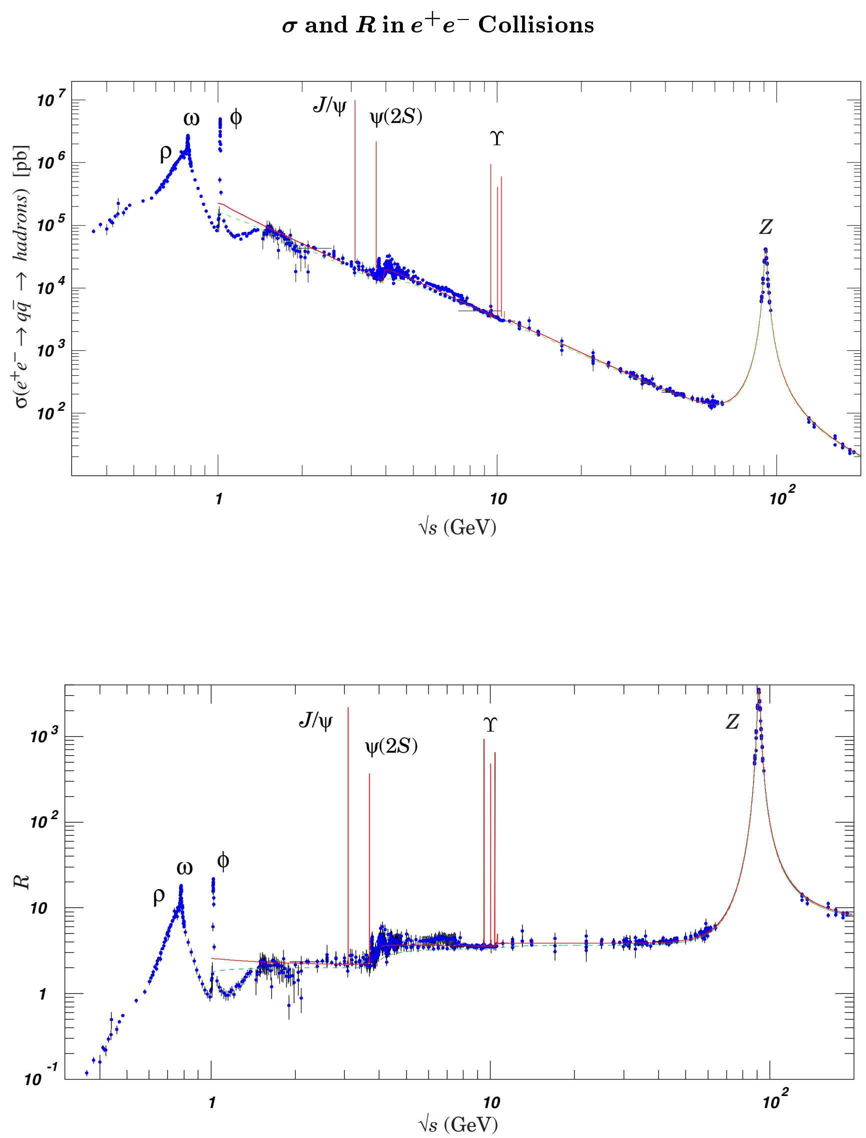


Figure 39.6, Figure 39.7: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-$, QED simple pole). The curves are an educative guide. The solid curves are the 3-loop pQCD predictions for $\sigma(e^+e^- \to hadrons)$ and the R ratio, respectively [see our Review on Quantum chromodynamics, Eq. (9.12)] or, for more details, K.G. Chetyrkin *et al.*, Nucl. Phys. **B586**, 56 (2000), Eqs. (1)–(3)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1..4$ are also shown. **Note:** The experimental shapes of these resonances are dominated by the machine energy spread and are not shown. The dashed curves are the naive quark parton model predictions for σ and R. The full list of references, as well as the details of R ratio extraction from the original data, can be found in O.V. Zenin *et al.*, `hep-ph/0110176` (to be published in J. Phys. **G**). Corresponding computer-readable data files are available at `http://wwwppds.ihep.su/≈zenin_o/contents_plots.html`. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, November 2001.)

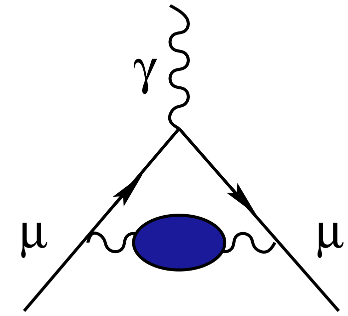
one of the fundamental observables in particle physics

- its high energy behaviour provides one of the most convincing arguments for the point-like nature of quarks
- its normalization was evidence for the existence of quarks of three different colours
- allows precise measurement of the strong coupling constant

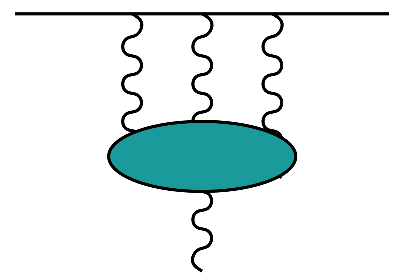
muon anomalous magnetic moment

$$a_\mu^{\mathtt{SM}} \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{\mathtt{QED}} + a_\mu^{\mathtt{had}} + a_\mu^{\mathtt{weak}}$$

		$a_\mu \times 10^{11}$	
QED	116584706 $\pm$	3	[Laporta,Remiddi], [Kinoshita *et al.*]
had (vac.pol.LO)	6924 $\pm$	62	[Davier, Höcker,98]
	6974 $\pm$	105	[Jegerlehner,01]
	6895 $\pm$	59	[Teubner *et al.*,02] 'excl.'
	6835 $\pm$	57	[Teubner *et al.*,02] 'incl.'
	6847 $\pm$	70	[Davier *et al.*,02] 'e^+e^-'
	7019 $\pm$	62	[Davier *et al.*,02] 'τ'
had (vac.pol.NLO)	101 $\pm$	6	[Krause,97]
had (light-by-light)	80 $\pm$	40	[Nyffeller,01]
weak	152 $\pm$	4	[Czarnecki *et al.*, 95]



vac. pol. LO

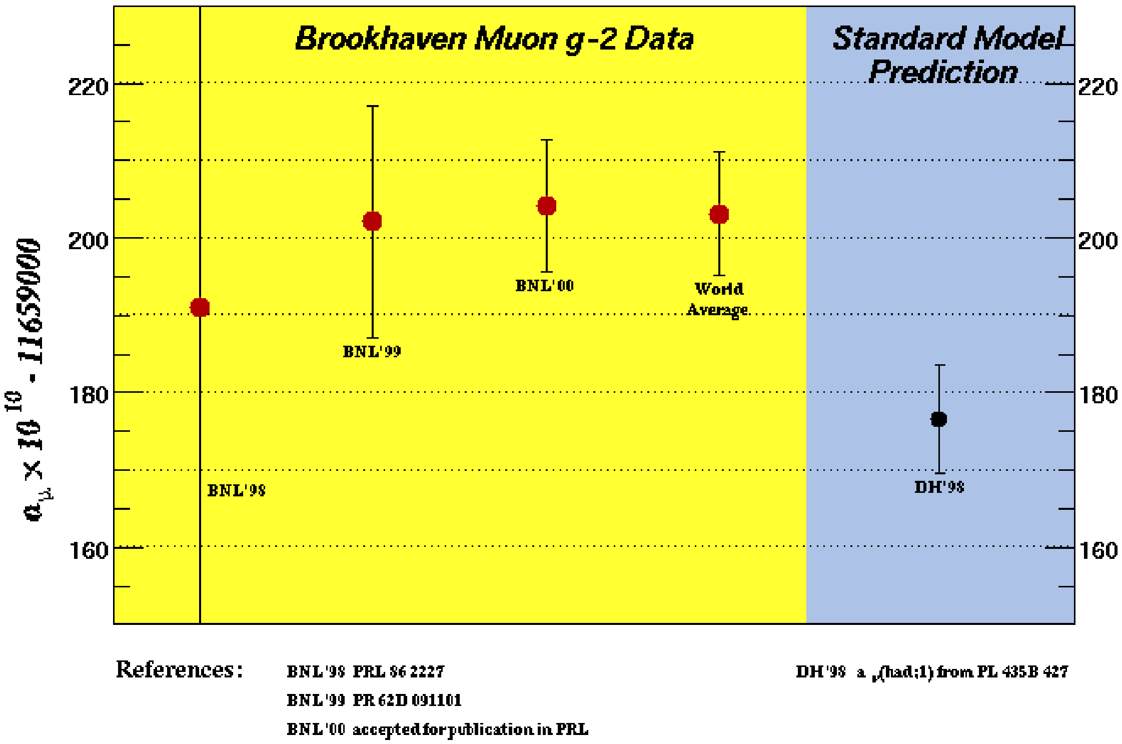

light–by–light

dispersion integral
$$a_\mu^{\rm had}(\text{vac.pol.LO}) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2}\, K(s)\ R(s)$$
67% of the error below 1 GeV

muon anomalous magnetic moment

New measurement from E821

a_{μ^+} (BNL'00)=11659204(7)(5) 10^{-10} (0.7 ppm)



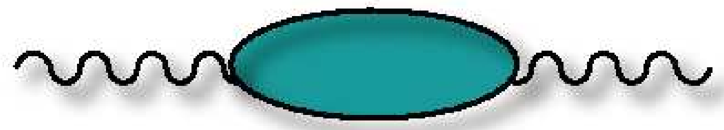
$$a_\mu(\text{exp}) - a_\mu(\text{th}) \sim 2 - 3\,\sigma$$

running QED coupling

From the Thompson limit to M_Z

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$

$$\Delta\alpha = \Delta\alpha_{\mathtt{lep}} + \Delta\alpha_{\mathtt{top}} + \Delta\alpha_{\mathtt{had}}$$



	$\Delta\alpha(\mathrm{M_Z}) \times 10^4$	
lep	314.98	[Steinhauser, 98]
top	-0.70 $\pm$ 0.05	
had	276.1 $\pm$ 3.6	[Burkhardt, Pietrzyk, 01]

dispersion integral

$$\Delta\alpha_{\mathrm{had}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} Re \int_{4m_\pi^2}^{\infty} \frac{ds}{s} \frac{R(s)}{s - M_Z^2 - i\eta}$$

17% of the error below 1 GeV

exclusive channels

[Höcker, Moriond 01]

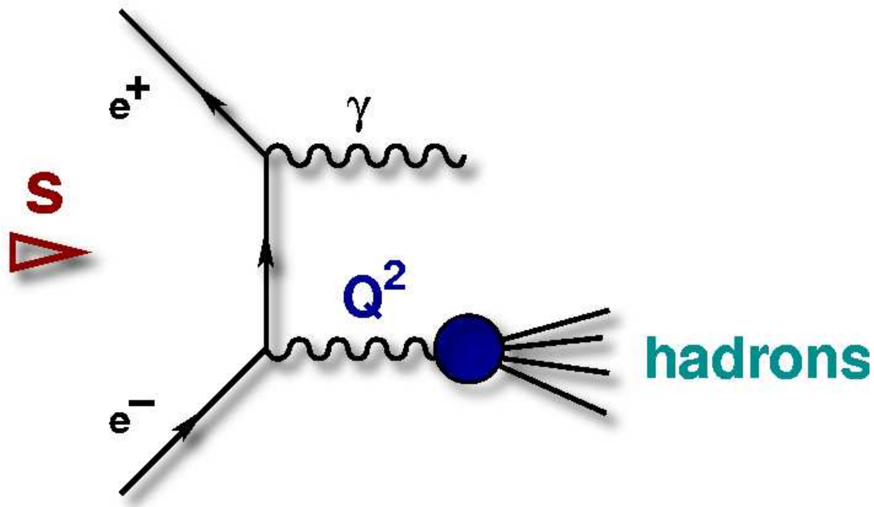
Final state	$a_\mu^{had}(\times 10^{11})$	$\Delta\alpha_{had}(M_Z^2)(\times 10^4)$	Energy(GeV)
$\pi^+\pi^-(e^+e^-\&\ \tau)$	5000.0±60.3	34.20±0.38	0.320-1.800
$\pi^+\pi^-(e^+e^-$ only)	4950.5±124.6	33.90±0.87	0.320-1.800
Resonances	844.6±15.2	19.31±0.70	0.420-11.20
$\pi^+\pi^-\pi^0$	58.0± 6.5	0.80±0.15	0.810-1.800
$\pi^+\pi^-2\pi^0(e^+e^-\&\ \tau)$	197.9± 13.2	5.00±0.48	0.897-1.800
$\pi^+\pi^-\pi^+\pi^-(e^+e^-\&\ \tau)$	145.6± 8.7	3.86±0.25	0.983-1.800
5π	32.8± 5.1	1.01±0.15	1.019-1.800
6π	61.9±24.3	2.52±0.99	1.350-1.800
$\eta\ \pi^+\pi^-$	4.4± 1.6	0.13±0.03	1.075-1.800
$K\bar{K}$	42.6± 5.8	0.83±0.10	1.055-1.800
$K\bar{K}\pi$	33.9± 6.7	0.92±0.18	1.340-1.800
$K\bar{K}\pi\pi$	27.3±10.0	0.94±0.34	1.441-1.800
TOTAL $(e^+e^-\&\ \tau)$	7011± 94	281.7±6.2	$2m_\pi$-∞
TOTAL $(e^+e^-$ only)	6950± 150	280.7±6.3	$2m_\pi$-∞

$\pi^+\pi^-$ and 4π biggest contributions to a_μ^{had} and $\Delta\alpha_{had}$

RADIATIVE RETURN

photon radiated off the initial e^+e^- (ISR) reduces the effective energy of the collision

$$d\sigma(e^+e^- \to \mathrm{hadrons} + \gamma) = H(Q^2, \theta_\gamma)\; d\sigma(e^+e^- \to \mathrm{hadrons})$$



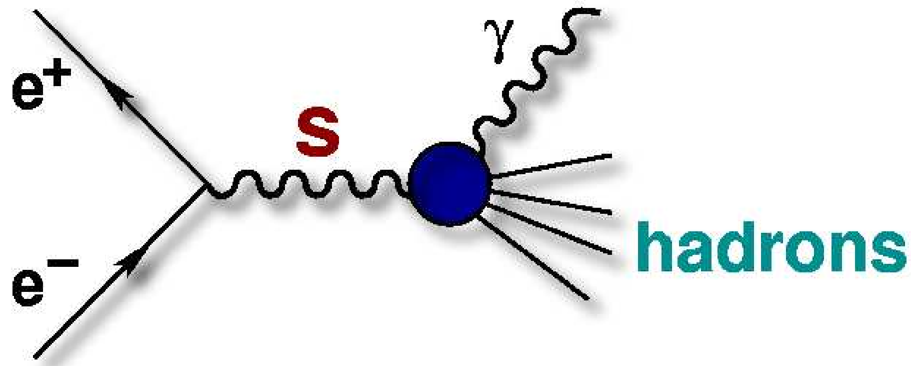
- measurement of $R(s)$ over the full range of energies, from threshold up to $\sqrt{s}$
- large luminosities of factories compensate α/π from photon radiation
- radiative corrections essential (NLO)
- advantage over energy scan (BES, CMD2, SND): systematics (e.g. normalization) only once

High precision measurement of the hadronic cross-section at DAΦNE, CLEO-C,B-factories

particularly interesting the low energy region < 2 GeV

FSR versus ISR

background for our process and Model dependent



Solutions:

❶ select configurations with dominantly ISR

❷ allow only configurations where FSR is well predicted: γ soft, collinear

❸ identify distributions which test FSR model: angular distributions, charge asymmetry

✚ ISR-FSR interference is C-odd: cancels under C-symmetric cuts

quantitative analysis
➪ Monte Carlo generator

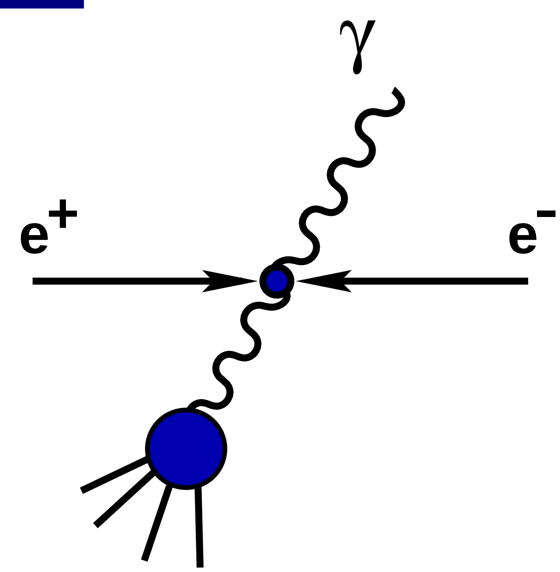
DAΦNE versus B-factories: FSR

Spectrum of photons emitted by an ultra-relativistic particle

$$d\sigma \propto d\sigma_0 \frac{dE_\gamma}{E_\gamma} \frac{d\theta_\gamma^2}{\theta_\gamma^2}$$

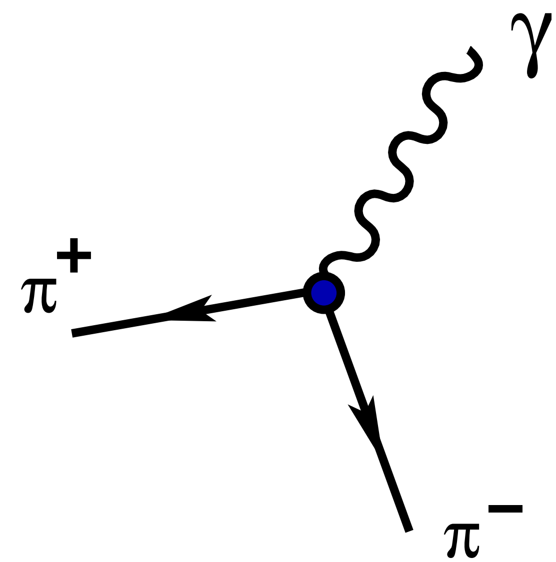
photons at small angles relative to the beam and well separated from the hadrons, dominated by **ISR**

10 GeV



very hard photon: clear kinematic separation between photon and hadrons

1 GeV

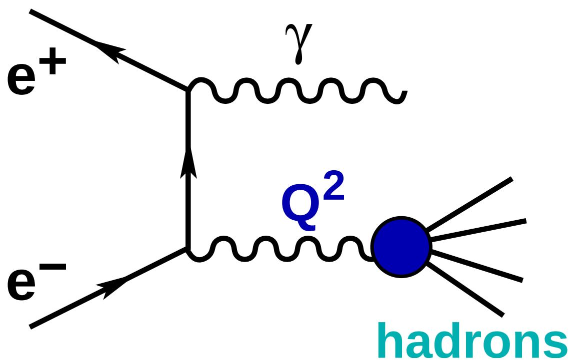


no natural kinematic separation

➪ cuts to control FSR versus ISR

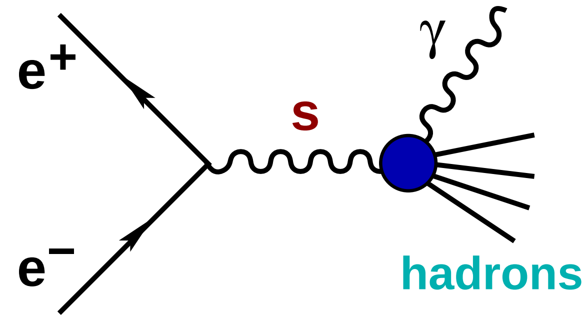
FSR at B-factories

ISR



$$\frac{1}{(Q^2)^2}\,|F(Q^2)|^2$$

FSR



$$\frac{1}{s^2}\,|F(s)|^2$$

- at DAΦNE Q^2 and s of the same order
- FSR at B-factories further suppressed by the Form Factor

MONTE CARLO generators

EVA: $e^+e^- \to \pi^+\pi^-\gamma$

- tagged photon $(\theta_\gamma > \theta_{cut})$
- ISR at LO + Structure Function
- FSR: point-like pions

[Binner et al.]

$e^+e^- \to 4\pi + \gamma$

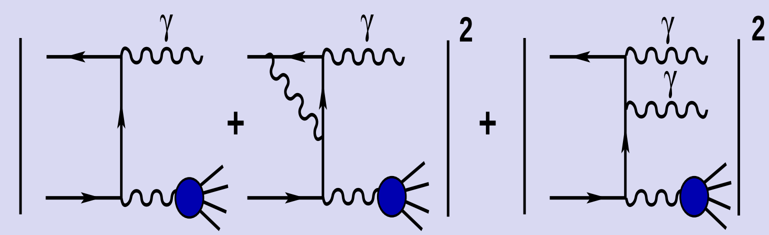
- ISR at LO + Structure Function

[Czyż, Kühn]

other exclusive channels: 3π, KK
FSR

PHOKHARA 1.0: $\pi^+\pi^-$ or $\mu^+\mu^-$

- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level



- tagged or untagged photons
- Modular structure

http://cern.ch/german.rodrigo/phokhara

LL versus NLO

LL

❶ Resums big logs $L = \log(s/m_e^2)$ to all orders

❷ Valid only in the collinear limit

❸ Extra collinear emission integrated out ➪ no momentum conservation

❹ Untagged photon: double counting

NLO

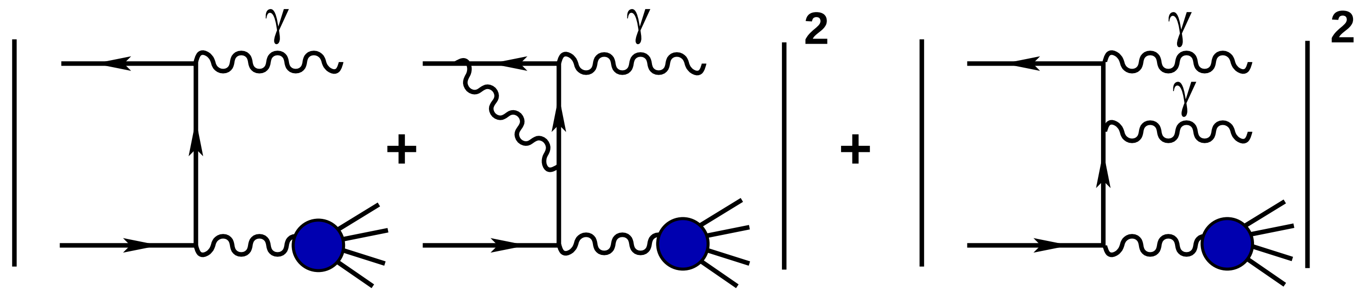
❶ LL at a fixed order + subleading terms **(1 %)**

❷ Full angular dependence

❸ Momentum conservation

❹ Tagged or untagged photon: ✔

NLO corrections to ISR



- **one-loop**
- **two photons**

UV: on-shell renormalization

IR divergences $(D = 4 - 2\epsilon)$

soft: *analytically* ➪ $E_\gamma < w\sqrt{s}$

hard: *numerically* ➪ helicity amplitude method

finite but

w - dependent

Leptonic Tensor

Differential rate cast into the product of a leptonic tensor and a hadronic tensor

$$d\sigma = \frac{dQ^2}{4\pi s} \, L_{\mu\nu} \, d\Phi_{i+1}(p_1, p_2; Q, k_1, \cdot, k_i) \, H^{\mu\nu} \, d\Phi_n(Q; q_1, \cdot, q_n)$$

facilitates the simulation of different hadronic exclusive channels

Leptonic Tensor: virtual + soft

assuming conserved currents $Q_\mu J^\mu = 0$

the leptonic tensor has the following general form

Cartesian basis

$$L^{\mu\nu} = \alpha^2 \left[a_{00}\, g^{\mu\nu} + a_{11}\, \frac{p_1^\mu p_1^\nu}{s} + a_{22}\, \frac{p_2^\mu p_2^\nu}{s} + a_{12}\, \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{s} + i\pi\, a_{-1}\, \frac{p_1^\mu p_2^\nu - p_2^\mu p_1^\nu}{s} \right]$$

Circular basis

$$L^{\mu\nu} = \alpha^2 \sum a_{ij}\, \varepsilon_i^{*\mu}\, \varepsilon_j^\mu \ , \quad i,j = L, \pm$$

ε_L and $\varepsilon_\pm$ circular polarization vectors of the virtual photon

$$a_{L-} = a_{L+} \ , \quad a_{-L} = a_{+L} = a_{L+}^* \ ,$$
$$a_{--} = a_{++} \ , \quad a_{-+} = a_{+-} \ .$$

were at the NLO: $a_{ij} = a_{ij}^{(0)} + \frac{\alpha}{\pi}\, a_{ij}^{(1)}$ $\quad a_{-1} = \frac{\alpha}{\pi}\, a_{-1}^{(1)} \propto \mathrm{Im}(a_{L+})$

trace related to the x-section after angular averaging of the hadronic tensor

$$L^{\mu\nu}(Q_\mu Q_\nu / Q^2 - g_{\mu\nu}) = a_{LL} + 2a_{++}$$

LO Leptonic Tensor

at LO

$$L_0^{\mu\nu} = \frac{(4\pi\alpha/s)^2}{q^4} \left[\left(\frac{2m^2 q^2 (1-q^2)^2}{y_1^2 y_2^2} - \frac{2q^2 + y_1^2 + y_2^2}{y_1 y_2} \right) g^{\mu\nu} + \left(\frac{8m^2}{y_2^2} - \frac{4q^2}{y_1 y_2} \right) \frac{p_1^\mu p_1^\nu}{s} + \left(\frac{8m^2}{y_1^2} - \frac{4q^2}{y_1 y_2} \right) \frac{p_2^\mu p_2^\nu}{s} - \left(\frac{8m^2}{y_1 y_2} \right) \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{s} \right]$$

with

$$y_i = \frac{2k_1 \cdot p_i}{s} = w_\gamma (1 \pm \beta \cos\theta_\gamma) \; , \qquad m^2 = \frac{m_e^2}{s} \; ,$$

$$\beta = \sqrt{1 - 4m^2} \; , \qquad q^2 = \frac{Q^2}{s}$$

NLO Leptonic Tensor

For the coefficient proportional to $g^{\mu\nu}$ at NLO , we get

$$
\begin{aligned}
a_{00}^{(1)} = a_{00}^{(0)} \Bigg[& -\log(4w^2)[1+\log(m^2)] - \frac{3}{2}\log(\frac{m^2}{q^2}) - 2 + \frac{\pi^2}{3} \Bigg] \\
& - \frac{q^2(1-q^2)}{2} - y_1 y_2 - \left[q^2 + \frac{2y_1y_2}{1-q^2} \right] \log(q^2) \\
& + \frac{y_1}{2} \left[4 - y_1 - \frac{3(1+q^2)}{1-y_2} \right] \log(\frac{y_1}{q^2}) + \frac{y_2}{2} \left[4 - y_2 - \frac{3(1+q^2)}{1-y_1} \right] \log(\frac{y_2}{q^2}) \\
& - \left[1 + (1-y_2)^2 + \frac{y_1 q^2}{y_2} \right] L(y_1) - \left[1 + (1-y_1)^2 + \frac{y_2 q^2}{y_1} \right] L(y_2) \ ,
\end{aligned}
$$

with

$$
L(y_i) = Li_2(\frac{-y_i}{q^2}) - Li_2(1-\frac{1}{q^2}) + \log(q^2+y_i)\log(\frac{y_i}{q^2}) \ ,
$$

expected **soft** and **collinear** behaviour + **subleading terms**

expression valid where the tagged photon is far from the collinear region

for arbitrary photon angles terms of the type $\boldsymbol{\frac{m_e^2}{y_i^2}}$ and $\boldsymbol{\frac{m_e^4}{y_i^3}}$ have to be included

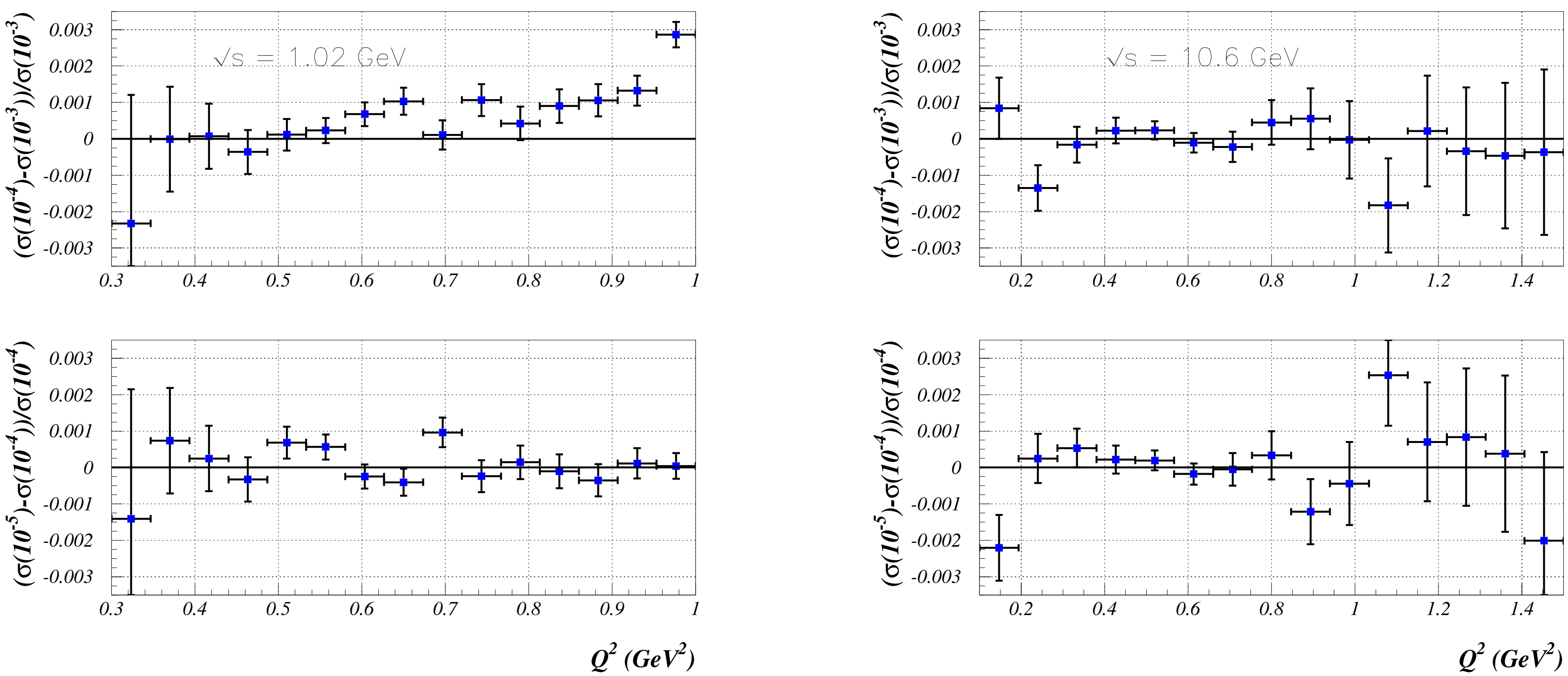
soft photon cutoff independence

w	$\sqrt{s}$ =1.02 GeV	4 GeV	10.6 GeV
10^{-3}	2.0324 (4)	0.12524 (5)	0.010564 (4)
10^{-4}	2.0332 (5)	0.12526 (5)	0.010565 (4)
10^{-5}	2.0333 (5)	0.12527 (5)	0.010565 (5)

Total cross section (nb): $e^+e^- \to \pi^+\pi^-\gamma$ at NLO

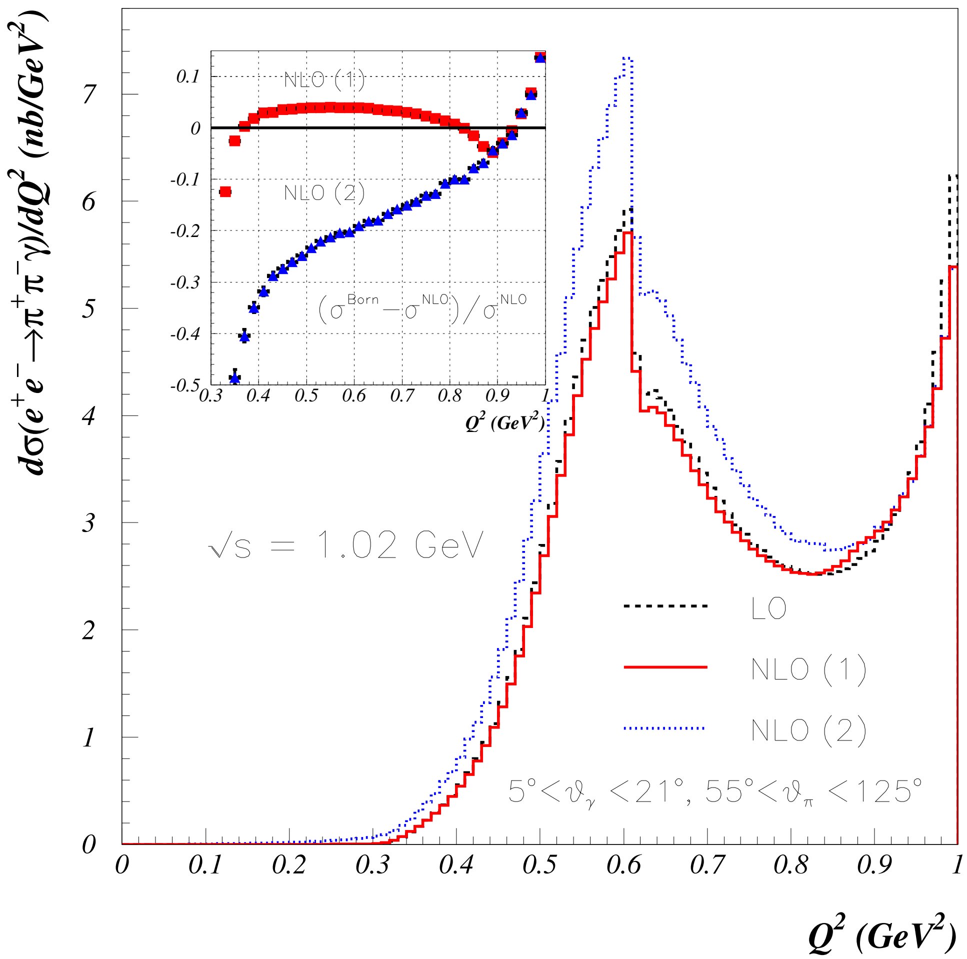
a particular value should be used for the generation:

➪ **optimal value ?**



small deviations for $w = 10^3$

NLO results: $\sqrt{s}$=1.02 GeV



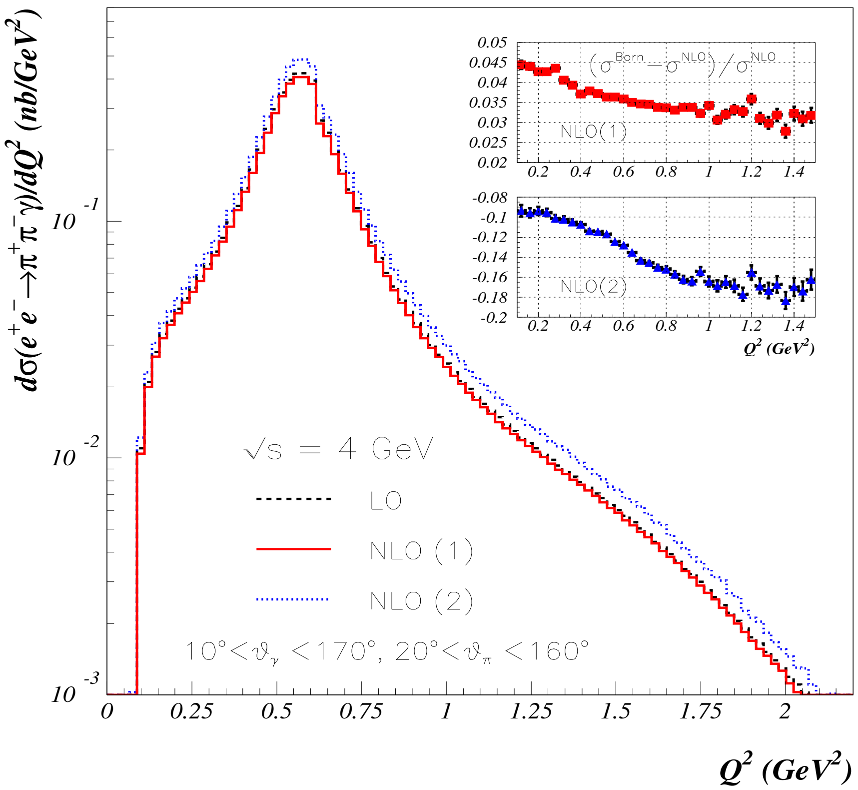
$e^+e^- \to \pi^+\pi^-\gamma(\gamma)$	σ (nb)
Born	2.1361 (4)
Collinear (SF)	2.0192 (4)
NLO (1)	2.0332 (5)
NLO (2)	2.4126 (7)

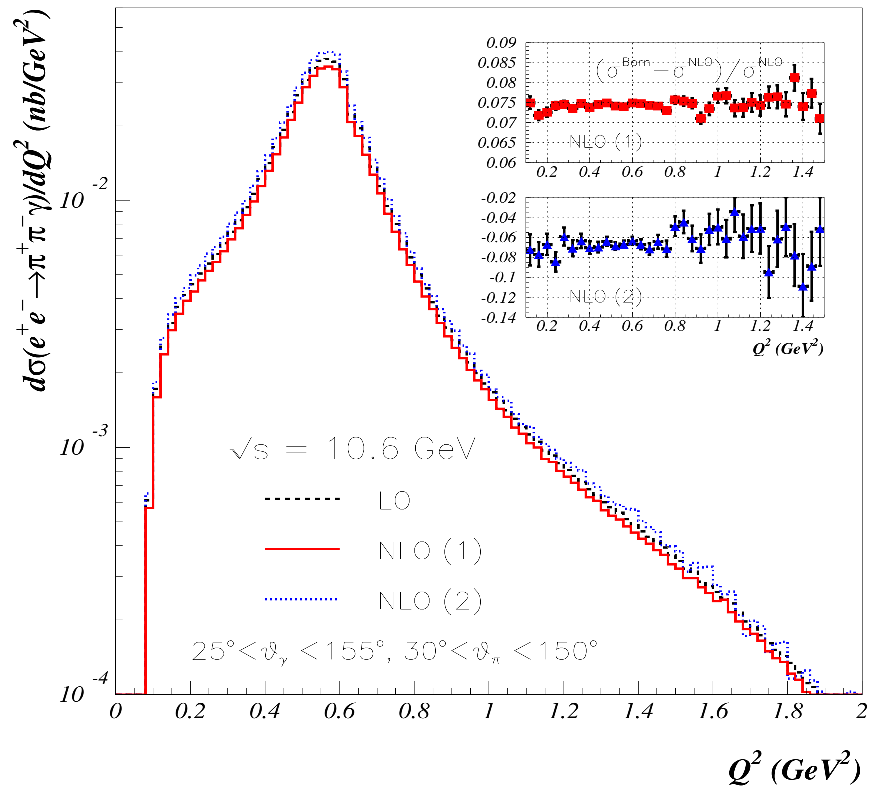
$M^2_{\pi^+\pi^-\gamma} > 0.9\ GeV^2$

NLO corrections depend on the kinematical cuts

➪ Monte Carlo generator

NLO results: $\sqrt{s}$=4 and 10.6 GeV



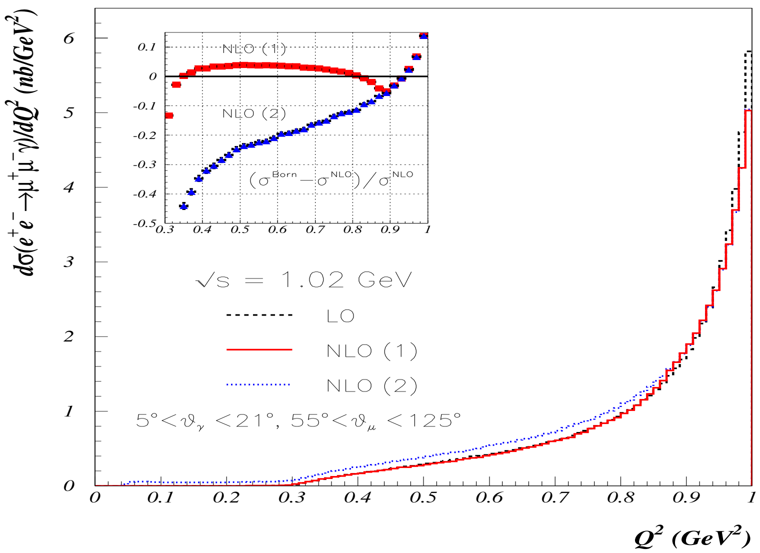


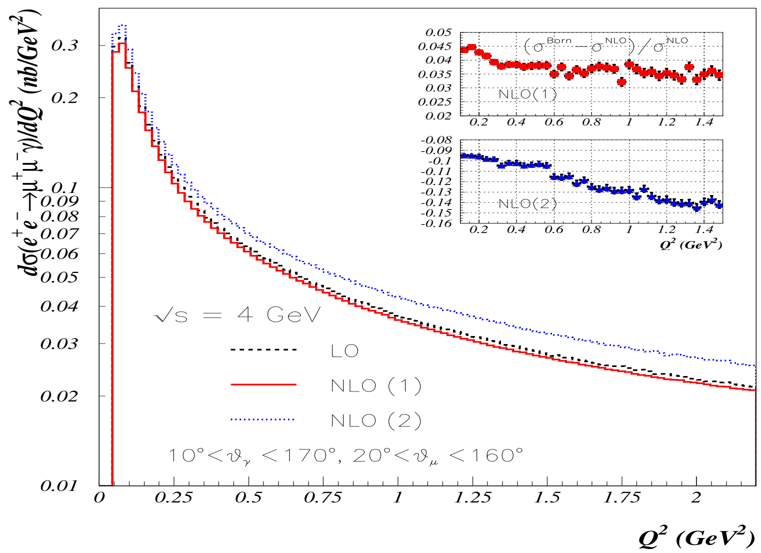
	$\sqrt{s}$ =4 GeV	10.6 GeV
Born	0.12979 (3)	0.011350 (3)
Collinear (SF)	0.12439 (5)	0.010526 (3)
NLO (1)	0.12526 (5)	0.010565 (4)
NLO (2)	0.14891 (9)	0.012158 (9)

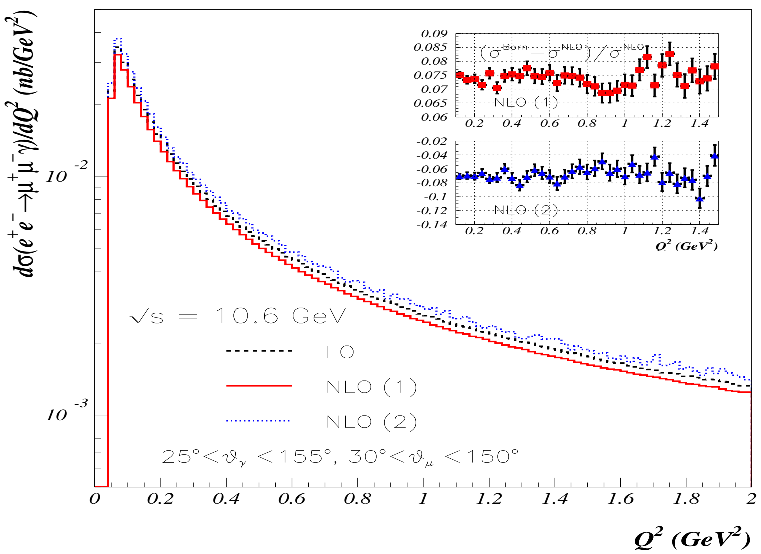
Total cross section (nb): $e^+e^- \to \pi^+\pi^-\gamma(\gamma)$

$\}$ $M^2_{\pi^+\pi^-\gamma} > 12, 90\ GeV^2$ respectively

NLO results: muons







	$\sqrt{s}$ =1.02 GeV	4 GeV	10.6 GeV
Born	0.8243(5)	0.4690(6)	0.003088(6)
NLO (1)	0.7587(5)	0.4449(6)	0.002865(6)
NLO (2)	0.8338(7)	0.4874(14)	0.00321(6)

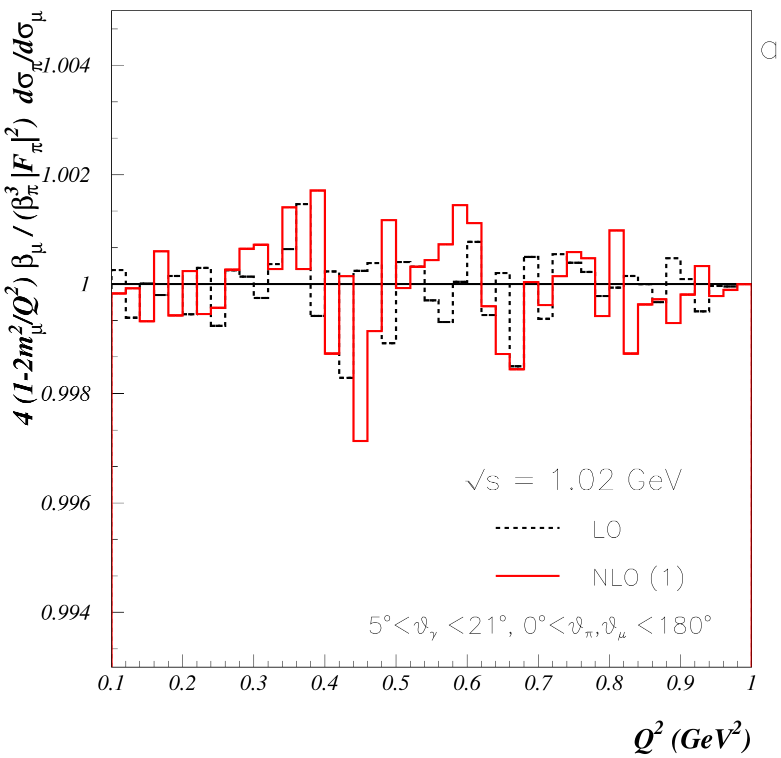
Total cross section (nb): $e^+e^- \to \mu^+\mu^-\gamma(\gamma)$

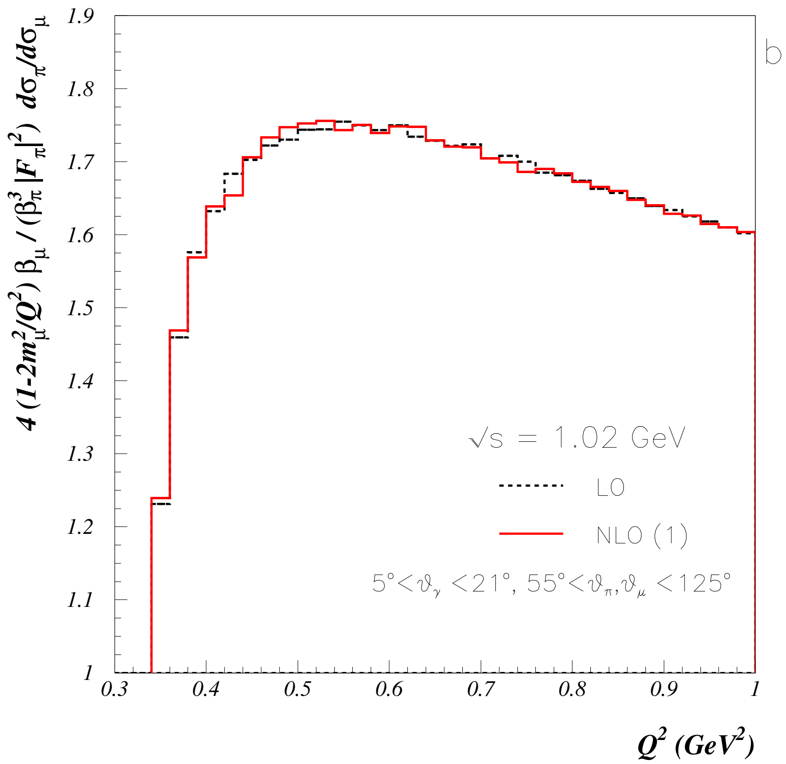
- Normalization of $R(s)$
- Narrow resonances not included (yet!)

ratio of pion to muon yields

Most of the **radiative corrections** cancel in the ratio

$$\frac{d\sigma(e^+e^- \to \pi^+\pi^-\gamma(\gamma))}{d\sigma(e^+e^- \to \mu^+\mu^-\gamma(\gamma))}$$



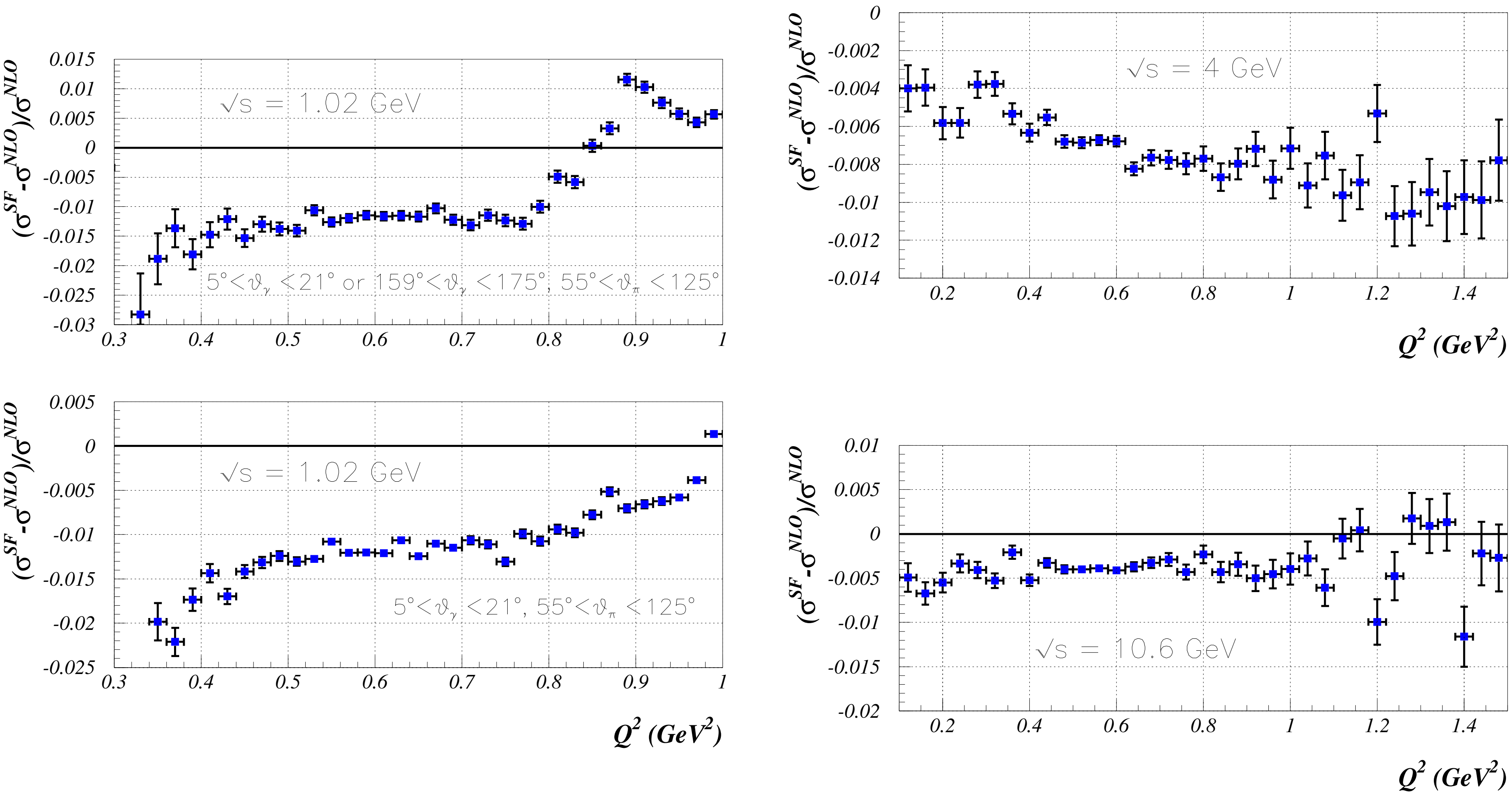


However, different angular distributions ➪ depending on the angular cuts

NLO (PHOKHARA) vs LL (EVA)

reasonable good agreement $(\theta_\gamma > \theta_{cut})$

depends on the kinematical cuts



estimation of the theoretical error

- Leading photonic NNLO: $\frac{1}{2}[\frac{3\alpha}{2\pi}\log(\frac{s}{m_e^2})]^2 \sim 0.1 - 0.2\%$
- $e^+e^- \to \gamma\gamma^* f\bar{f}$ with collinear $f\bar{f}$ in the beam pipe + fermion loop insertions. Largely absorbed by choosing a running coupling $\alpha(1GeV)$ $\frac{1}{4}(\frac{\alpha}{\pi})^2\log^2(\frac{s}{m_e^2}) < 0.1\%$
- Multiphoton emission: exponentiation $0.4\% - 0.7\%$

0.5% for ISR

Tagged versus untagged events

two complementary approaches at DAΦNE

tagged evts ($\theta_\gamma > \theta_{min}$):

✔ kinematic constraints

important for complicated

final state ($\gamma\pi^+\pi^-2\pi^0$)

untagged evts (θ_γ arbitrary):

✔ higher rates (factor 3-4)

✔ FSR less problematic

✘ $e^+e^- \to \gamma\gamma^* f\bar{f}$

Monte Carlo status

❖ tagged evts: EVA and PHOKHARA initially designed to work with tagged evts

a minimal θ_γ used for generation

❖ untagged evts: PHOKHARA ✔

inclusive analytic calculations (NLO) [Berends *et al.*] [Höfer, Gluza, Jegerlehner]

inclusive $\hat{=}$ untagged

will provide important test of the generator

untagged photons: m_e^2 corrections

the electron mass enters twofold

- through: $y_i = w_\gamma(1 \pm \beta \cos\theta_\gamma)$, $\beta = \sqrt{1-4m^2}$
- explicitely: e.g. m^2/y_i^2, $m^2/(m^2-y_i)^2$

❶

$$\int \frac{1}{y_1 y_2}\, d\cos\theta_\gamma \sim \log(m^2) + \mathcal{O}(m^2)$$

give the collinear logs

❷

$$\int \frac{m^2}{y_1^2}\, d\cos\theta_\gamma \sim \frac{1}{2} + \mathcal{O}(m^2)$$

finite corrections *relevant at very small angles*

Full result numerically unstable in the limit of small mass and angles ➪ A compact, numerically stable result, valid for an arbitrary small photon angle, is therefore required

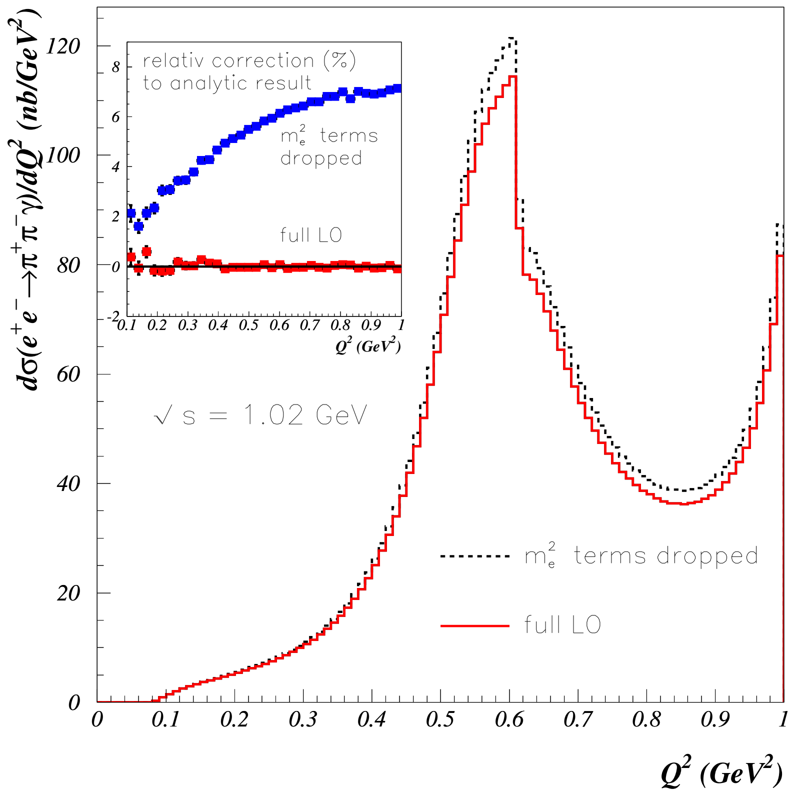
in agreement with "virtual Compton scattering" [Kuraev *et al.*]

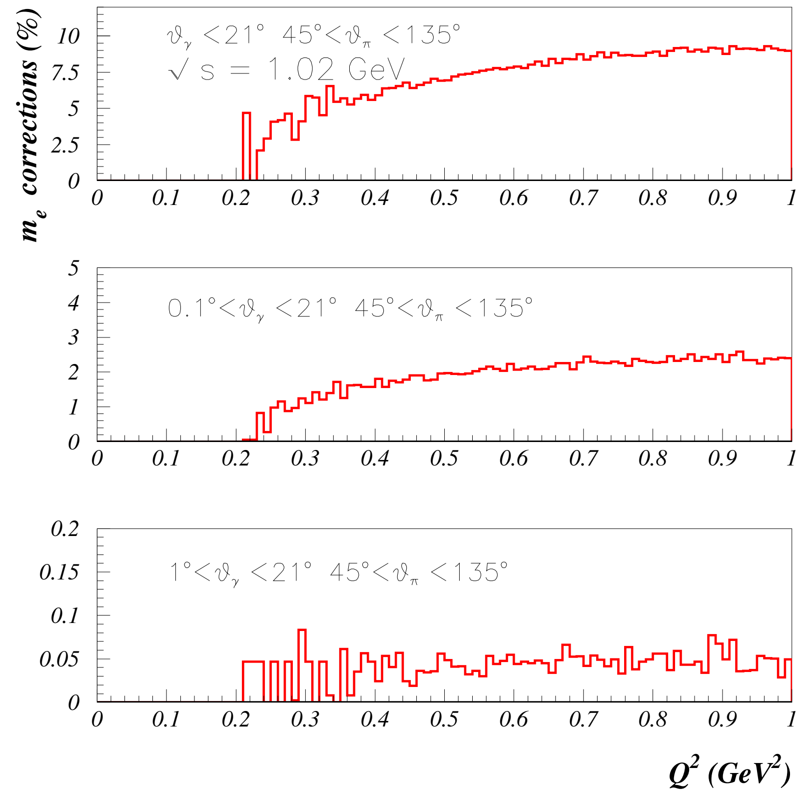
$\gamma^* e^- \to \gamma e^-$ after analytical continuation

untagged photons: m_e^2 corrections

at LO

$$Q^2 \frac{d\sigma}{dQ^2} = \underset{\downarrow}{\frac{4\alpha^3}{3s}} R(Q^2) \left\{ \frac{s^2+Q^4}{s(s-Q^2)} \left(\log \frac{s}{m_e^2} - 1 \right) \right\} , \qquad R(Q^2) \to \frac{1}{4} \left(1 - \frac{4m_\pi^2}{Q^2} \right)^{3/2} |F_\pi(Q^2)|^2$$

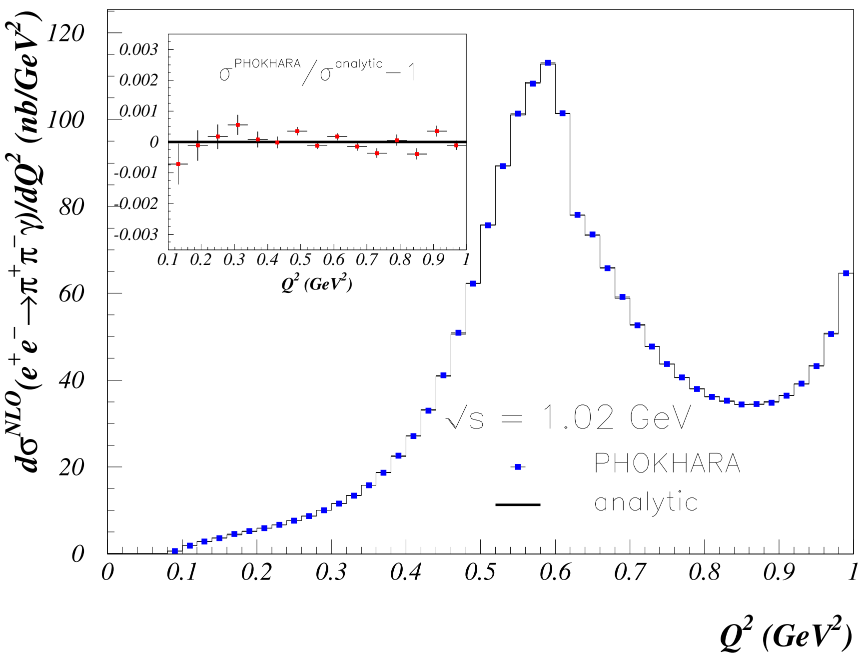


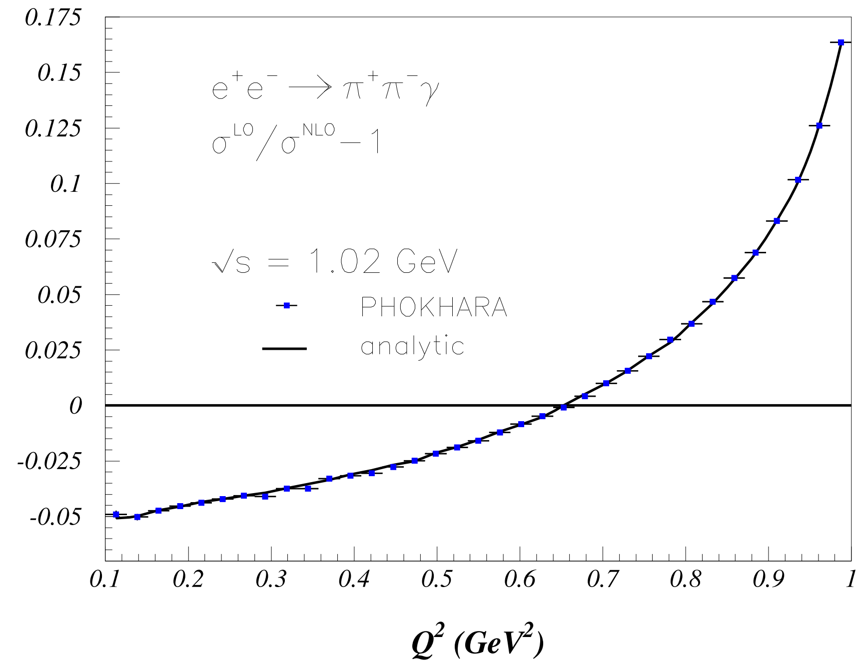


important deviation at small photon angles

untagged photons

Comparison with [Berends *et al.*] $(e^+e^- \to Z\gamma)$





✔ good performance of the MC simulation also at very small photon angles

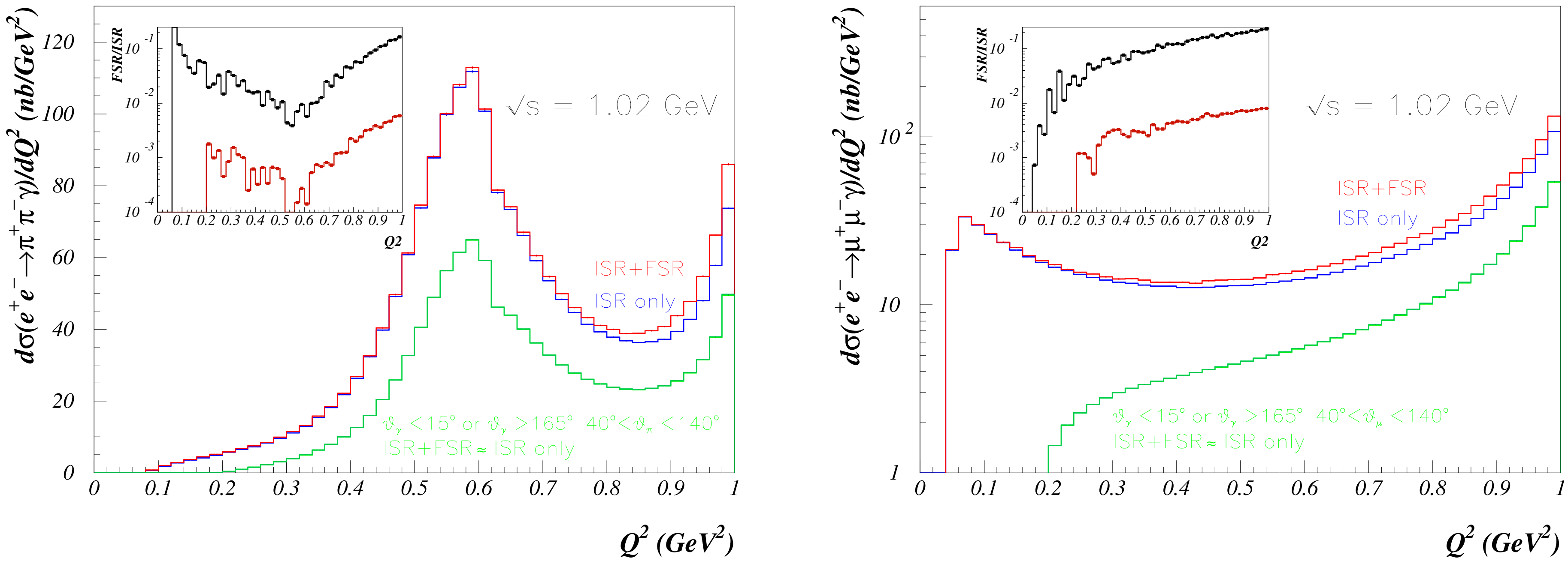
non trivial test: huge cancellations which could spoil

the numerical stability of the simulation

FSR versus ISR

PHOKHARA

simulates now FSR at LO for pions (point-like pions) and muons

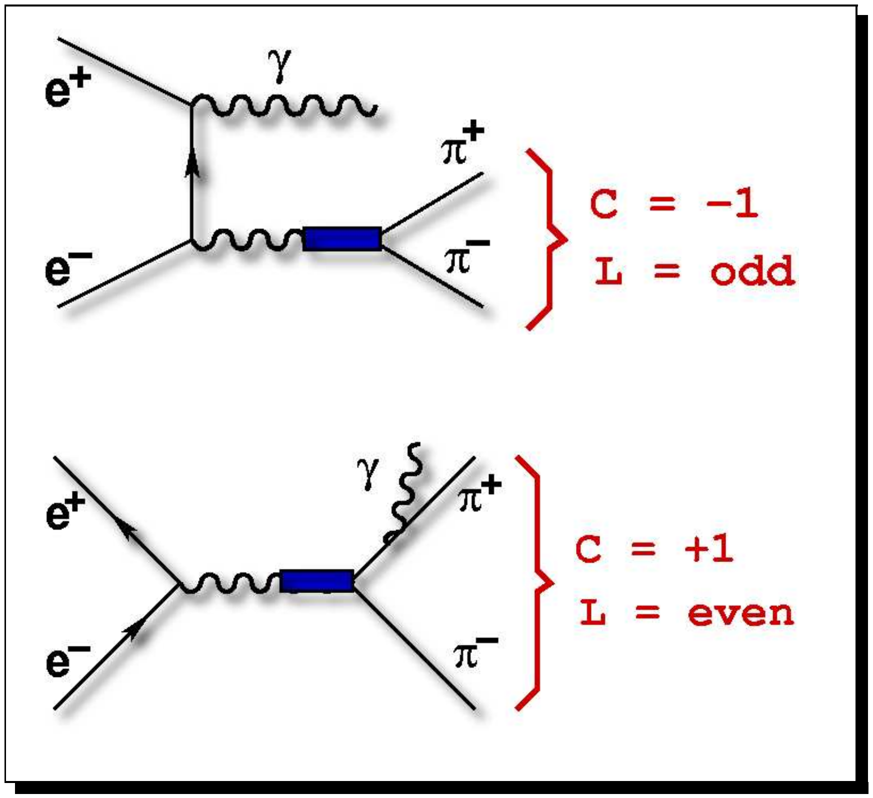


- FSR further suppressed for pions by the $\rho - \omega$ peak
- For muons suppression at threshold

Test of FSR model

- **FSR is model dependent**

point-like pions: valid for soft photons (internal structure of the pions not resolved by the photon), probably overestimated



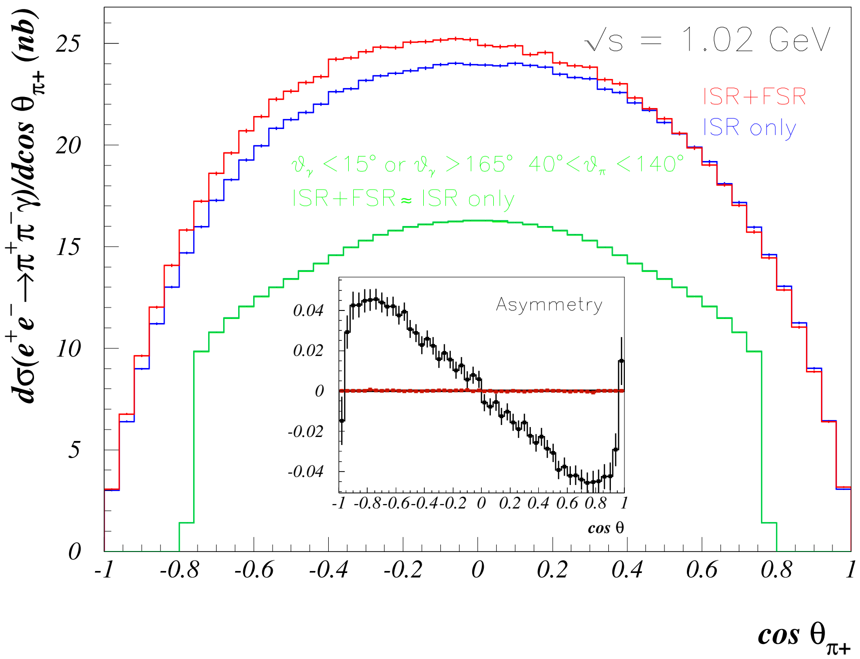
- interference odd under $\pi^+ \leftrightarrow \pi^-$
- asymmetric differential distribution:

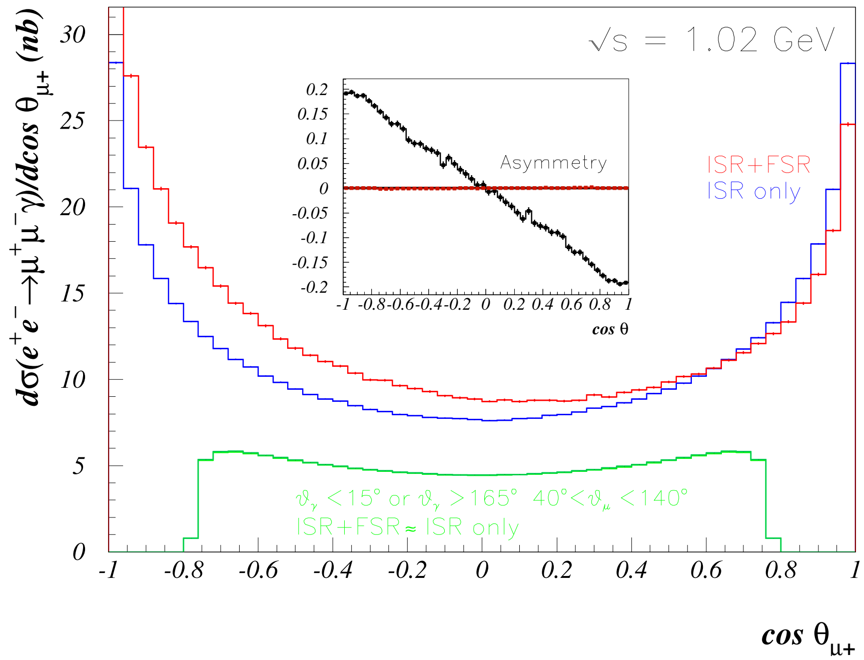
$$A(\theta) = \frac{N^{\pi^+}(\theta) - N^{\pi^-}(\theta)}{N^{\pi^+}(\theta) + N^{\pi^-}(\theta)}$$

- $\int$ interference $= 0$ under C-symmetric angular cuts

FSR/ISR interference

FSR/ISR interference modifies the angular distribution





- ❖ test of FSR model
- ❖ negligeable asymmetry if ISR dominance

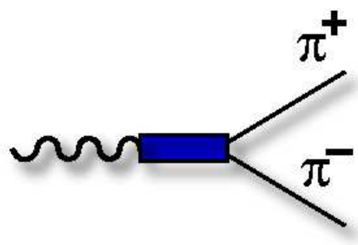
two pion Form Factor

The physics of the hadronic system enters only the hadronic tensor: $H^{\mu\nu} = J^\mu J^{\nu +}$ parametrized through form factors

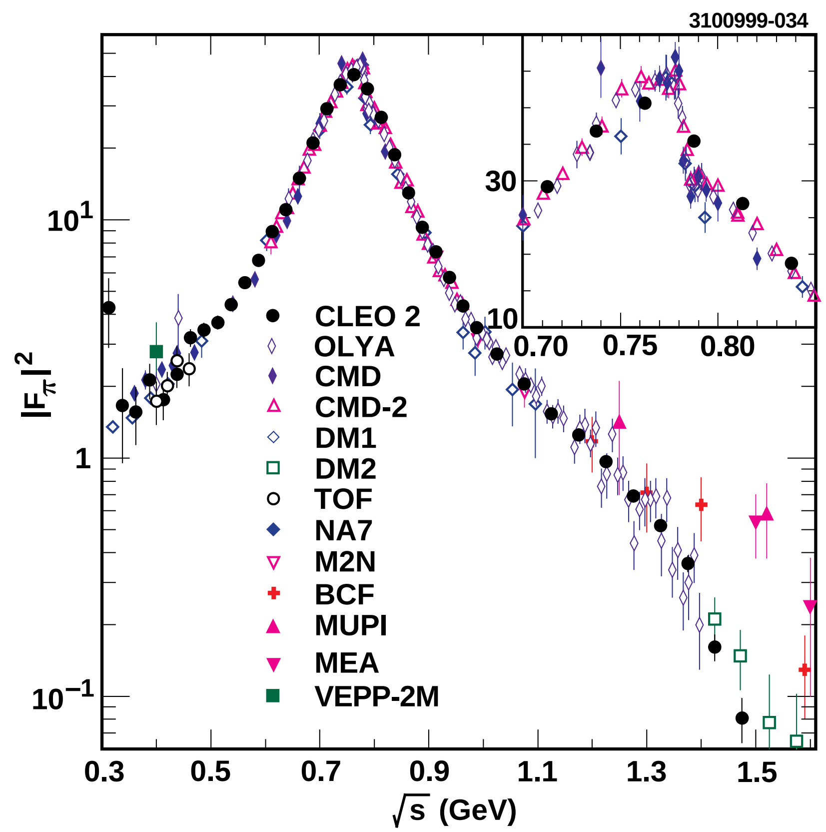
▶ two charged pions

$$J^\mu_{2\pi} = ieF_{2\pi}(Q^2)\,(q_{\pi^+} - q_{\pi^-})^\mu$$

[Kühn,Santamaria, 90]



$$F_{2\pi} = \left(\mathrm{BW}_\rho \frac{1+\alpha\,\mathrm{BW}_w}{1+\alpha} + \beta\,\mathrm{WB}_{\rho'} + \gamma\,\mathrm{BW}_{\rho''} \right) / (1+\beta+\gamma)$$

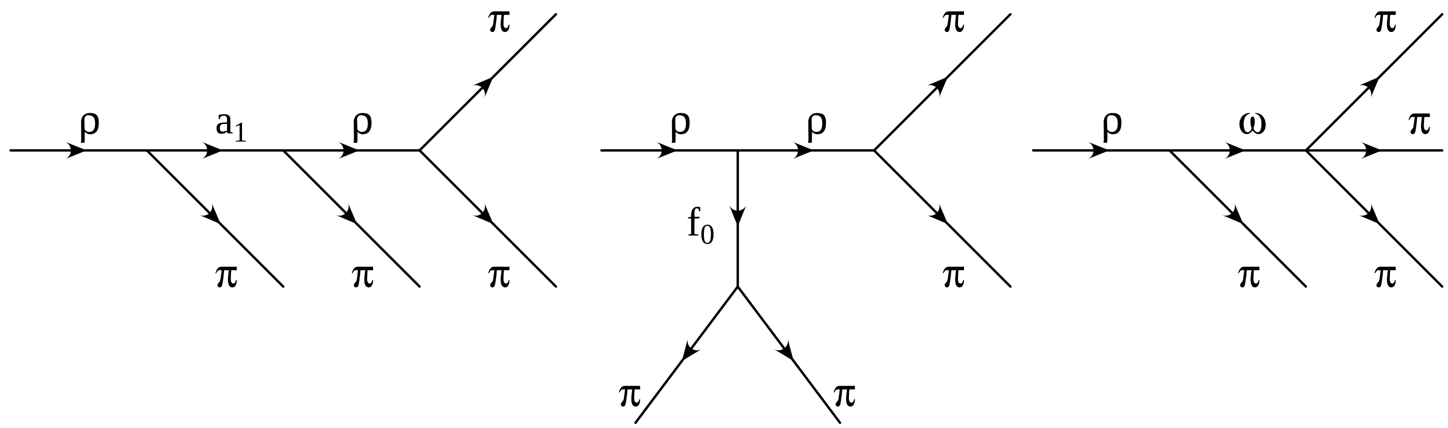


✯ other parametrizations can be easily included

four pion Form Factor

[Czyż, Kühn, 01]

modified/corrected version of model by [Decker *et al.*, 96]

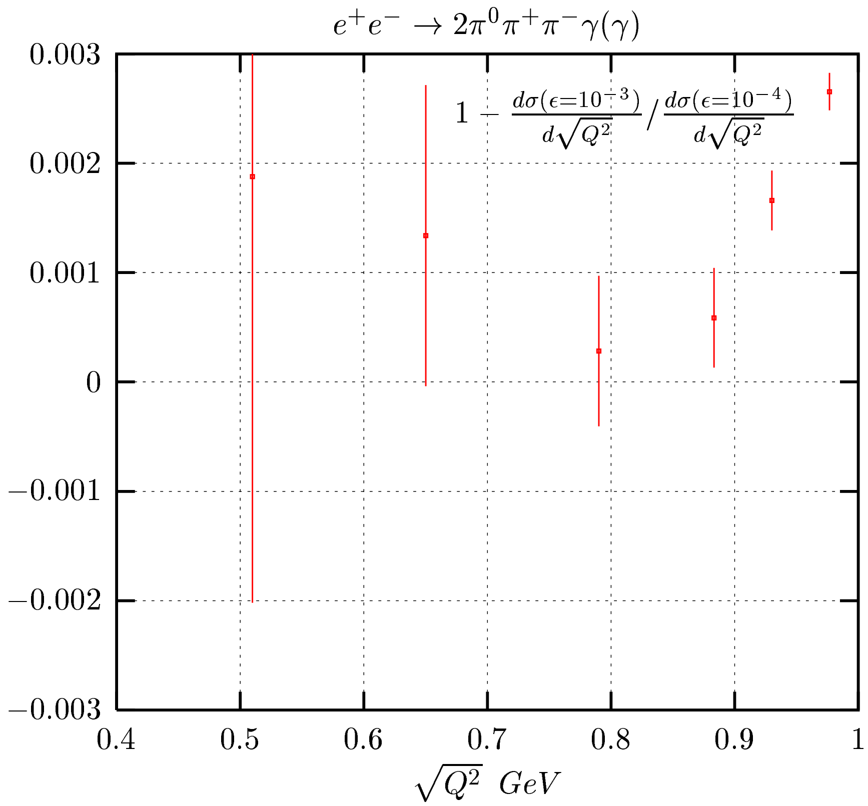


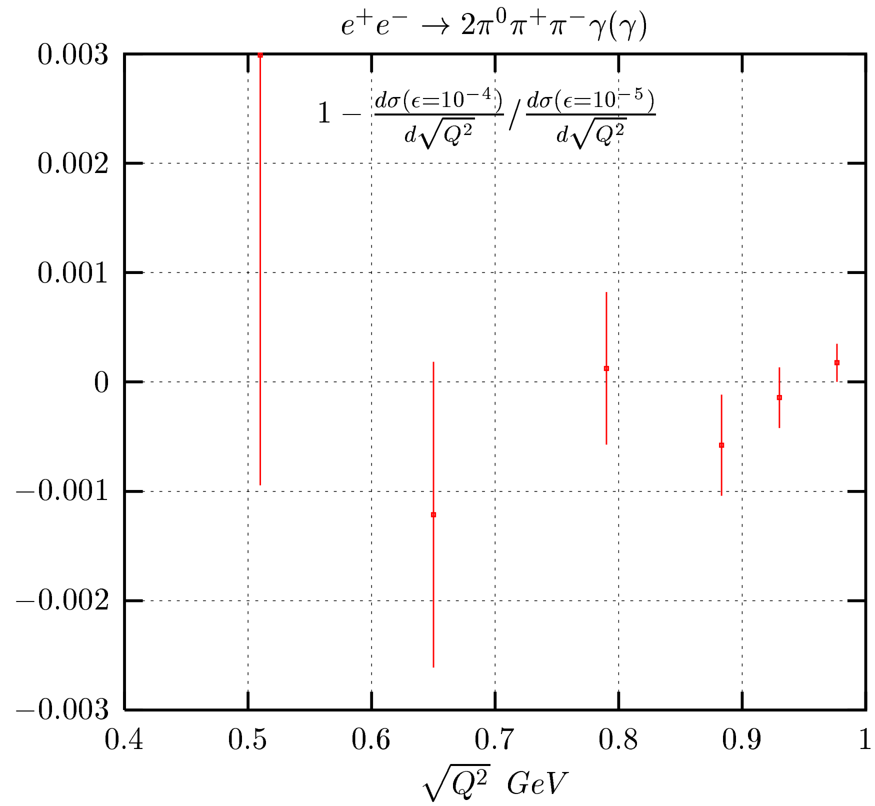
Diagrams contributing to the hadronic current.

now included in **PHOKHARA**

4π at PHOKHARA: soft photon cutoff

at 1.02 GeV



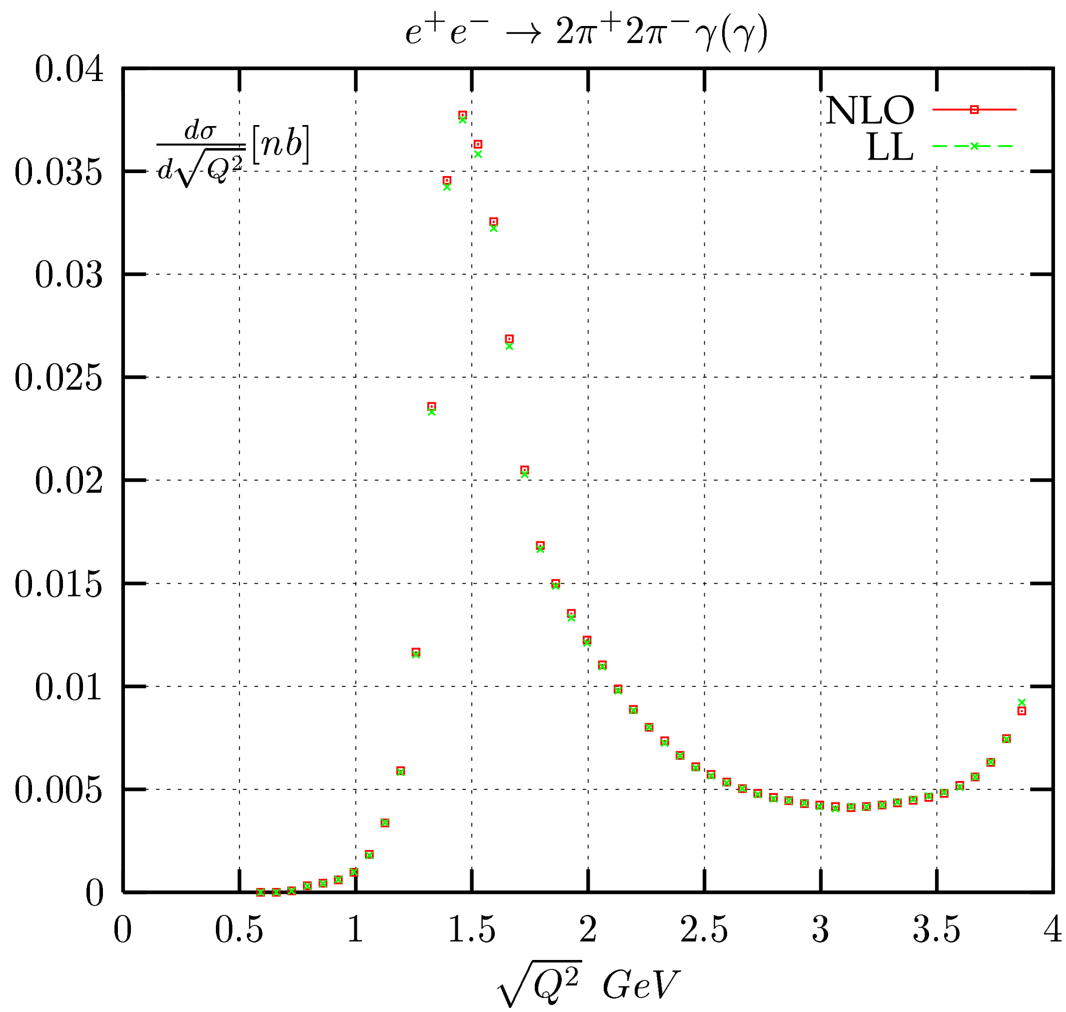


$w = 10^{-3}$	10^{-4}	10^{-5}
0.55725 (5)	0.55844 (5)	0.55845 (5)

Total x-section (nb): $e^+e^- \to \pi^+\pi^-2\pi^0\gamma(\gamma)$

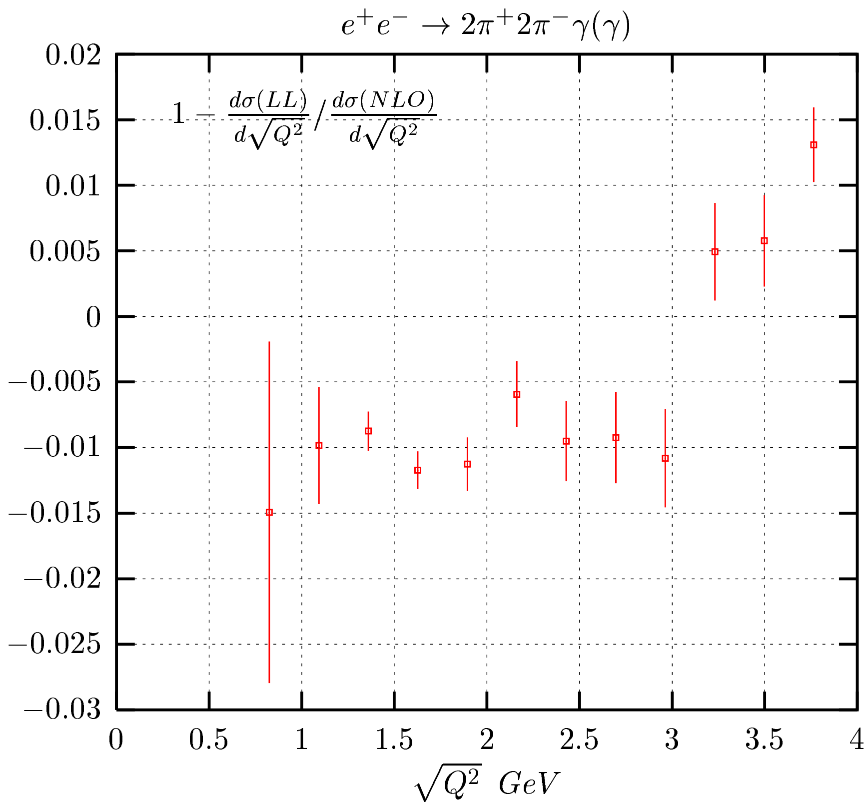
4π at PHOKHARA: x-section

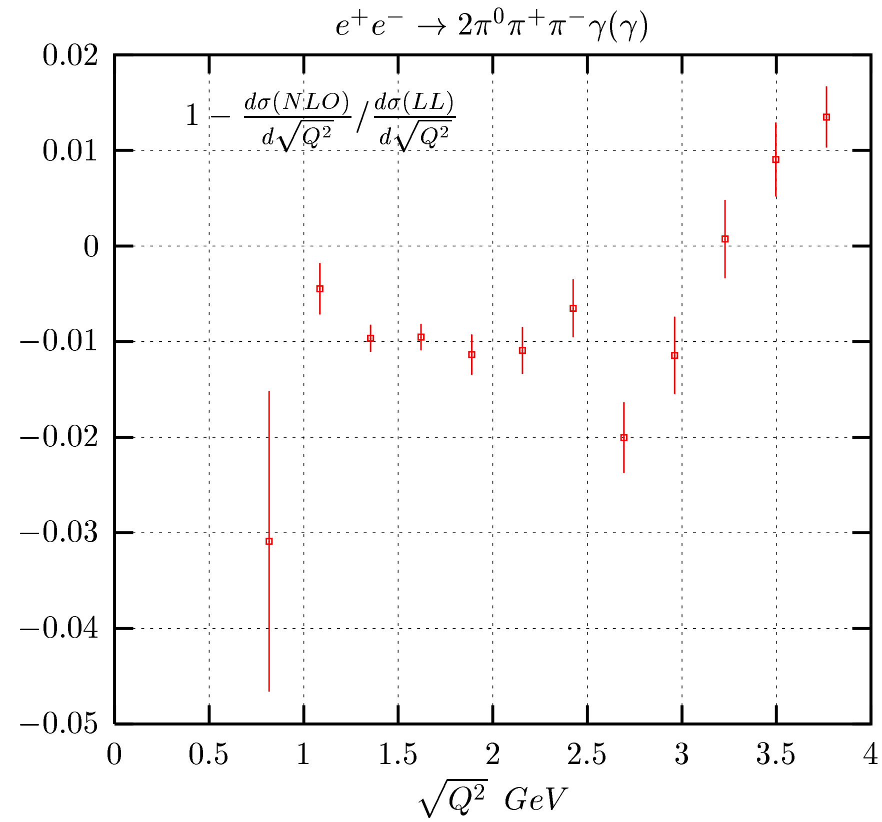
at 4 GeV



4π at PHOKHARA: LL versus NLO

at 4 GeV





Conclusions

- Radiative Return at high luminosity e^+e^- colliders (DAΦNE, CLEO-C and B-factories) will contribute to our understanding of the muon anomalous magnetic moment discrepancy.
- MC generator at NLO (PHOKHARA 1.0): $\pi^+\pi^-$ and $\mu^+\mu^-$ released in Dec 2001 virtual + soft + real corrections to ISR
- Modular structure: other hadronic channels or improved parametrizations of the hadronic current can be easily included.
- PHOKHARA: new issues
  - untagged photon generation tested
  - FSR for two pions or muons included
  - new exclusive channel: 4π
  - 3π in progress