## The New $D_{s} \pi^{0}$ State:

## The Hydrogen Atom Revisited

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## BaBar Discovery of Narrow State Decaying to $D_{s} \pi^{0}$



- $\pi^{0}$ signal $+\mathrm{D}_{s}$ signal gives peak
- $\pi^{0}$ signal $+\mathrm{D}_{s}$ sideband gives no peak
- $\pi^{0}$ sideband $+\mathrm{D}_{s}$ signal gives no peak

- $D_{s} \pi^{0}$ mass distribution for $D_{s} \rightarrow K^{+} K^{-} \pi^{+}$
- $D_{s} \pi^{0}$ mass distribution for $D_{s} \rightarrow K^{+} K^{-} \pi^{+} \pi^{0}$

- No sign of $D_{s J}^{*}(2317) \rightarrow D_{s} \gamma$
- No sign of $D_{s, J}^{*}(2317) \rightarrow D_{s}^{*} \gamma$
- Apparent structure in $D_{s} \pi^{0} \gamma$


## Heavy Quark - Light Quark Spectroscopy

## is the Hydrogen Atom

- First approximation: heavy quark is static source of potential
- Orbital angular momentum $\ell$, light-quark $\operatorname{spin} s$, separately conserved
- Add spin-orbit interaction, $\ell \cdot s$
$-j=\ell+s$ conserved, not $\ell, s$
- Add heavy quark, with spin $S$
- Interactions suppressed by $m / M$
- Spin-orbit interaction $\ell \cdot S$
- Spin-spin contact interaction $s \cdot \boldsymbol{S} \delta^{3}(\boldsymbol{r})$
- Tensor force $3 S \cdot \hat{r} s \cdot \hat{r}-S \cdot s$
$-J=j+S$ conserved, not $j=\ell+s$


## p-wave states


potential
spin-orbit
tensor force

## Canonical Approach

## DiPierro and Eichten, PRD 64, 114004 (2001)

- Dirac equation with two potentials
- Coulomb potential in fourth component of vector potential
- Linear (confining) potential in scalar potential
- Solve Dirac equation with spin-orbit included
- Add tensor force (and small spin-orbit) perturbatively
- Fix coefficients of Coulomb and linear, masses of quarks to fit data from $D, D_{s}$, $B, B_{s}$ systems


## Predictions of DiPierro and Eichten

|  | $D$ |  | $D_{s}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $J^{P}$ | $\mathrm{~m}(\mathrm{GeV})$ exp. | $\mathrm{m}(\mathrm{GeV}) \mathrm{th}$ | $\mathrm{m}(\mathrm{GeV})$ exp. | $\mathrm{m}(\mathrm{GeV}) \mathrm{th}$ |
|  |  |  |  |  |
| $S(J=0)$ | 1.865 | 1.868 | 1.969 | 1.965 |
| $S(J=1)$ | 2.007 | 2.005 | 2.112 | 2.113 |
| $P(J=0)$ | $[2.290]$ | 2.377 |  | 2.487 |
| $P(J=1)$ | 2.422 | 2.417 | 2.535 | 2.535 |
| $P(J=2)$ | 2.459 | 2.460 | 2.573 | 2.581 |
| $P(J=1)$ | $[2.400]$ | 2.490 |  | 2.605 |

- The states $D_{0}(2290)$ and $D_{1}(2400)$ are from Belle at ICHEP and were not known at the time of the predictions.
- The same potentials are used for the $D, D_{s}, B$, and $B_{s}$ systems.


## Decays and Selection Rules

- Angular momentum conserved: no $0 \rightarrow 0 \gamma$ decays
- Not weak decays: Parity conserved
$-D_{s J}^{*}(2317) \rightarrow D_{s}\left(0^{-}\right) \pi^{0}$ forces natural spin-parity for $D_{s J}^{*}(2317)\left[0^{+}, 1^{-} \ldots\right.$
$-D_{s J}^{*}(2317) \rightarrow D_{s}^{*}\left(1^{-}\right) \pi^{0}$ forbidden if $D_{s J}^{*}(2317)$ is $0^{+}$
- Isospin mostly conserved
$-D_{s J}^{*}(2317) \rightarrow D_{s}\left(0^{-}\right) \pi^{0}$ violates isospin
$-D_{s}^{*}(2112) \rightarrow D_{s}\left(0^{-}\right) \pi^{0}$ violates isospin, $5 \%$ of $D_{s}(2112) \rightarrow D_{s}\left(0^{-}\right) \gamma$
- $D_{s ?}(2460 ?) \rightarrow D_{s}\left(0^{-}\right)(\pi \pi)_{L=0}$ allowed if $1^{+}$, needs p-wave
- $D_{s 2}(2575) \rightarrow D K$ d-wave, $\rightarrow D^{*} K$ d-wave
- $D_{s 1}(2535) \rightarrow D^{*} K$ s-wave, d-wave
- Light-quark angular momentum $(j=\ell+s)$ nearly conserved
- $D_{s 1}(2535)$ mostly $j=3 / 2, D(2007)$ all $j=1 / 2$
- $D_{s 1}(2535) \rightarrow D(2007) K$ d-wave not $s$-wave $(\Gamma<2.3 \mathrm{MeV}$ )
- $D(J=1, j=1 / 2)$ should be broad
- Isospin violation
- Violated by electromagnetism
- Violated by $m_{u} \neq m_{d}$
- Scale is $\left(m_{u}-m_{d}\right) / \Lambda_{Q C D} \ll 1$


## Chiral Symmetry and Isospin Violation

$$
\Pi=\left(\begin{array}{ccc}
\frac{\pi^{0}}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^{+} & K^{+} \\
\pi^{-} & \frac{\pi^{0}}{\sqrt{2}}-\frac{\eta}{\sqrt{6}} & K^{0} \\
K^{-} & \bar{K}^{0} & -\frac{2}{\sqrt{6}} \eta
\end{array}\right) ; \quad M=\left(\begin{array}{ccc}
m_{u} & 0 & 0 \\
0 & m_{d} & 0 \\
0 & 0 & m_{s}
\end{array}\right)
$$

Mass-squared term from $\operatorname{Tr} \Pi M \Pi$ leads to $\pi^{0}-\eta$ mixing, with mixing angle

$$
\tan \theta=\frac{\sqrt{3}}{2} \frac{m_{d}-m_{u}}{m_{s}-\left(m_{u}+m_{d}\right) / 2}
$$

Use $\frac{m_{K^{+}}^{2}-m_{\pi^{+}}^{2}}{m_{K^{0}}^{2}-m_{\pi^{0}}^{2}}=\frac{m_{s}-m_{d}}{m_{s}-m_{u}} ; \quad \frac{m_{\eta}^{2}}{m_{\pi^{0}}^{2}}=\frac{\left(4 m_{s}+m_{u}+m_{d}\right) / 3}{m_{u}+m_{d}}$
to find $m_{s} / m_{d}=20, m_{u} / m_{d}=0.55, \tan \theta \approx 1 / 50$


## "Almost-Model-Independent’ Spectroscopic Predictions (J. D. Jackson \& RNC: hep-ph/0305012)

- Potential includes $V$ and $S$
- $V$ is Coulombic, as suggested by QCD
- Fourth component of vector potential
- $S$ is linear, confining
- Scalar, not fourth component of vector potential
- Determine potentials from Feynman diagram, expand in $v / c$


## Effective Interaction

$$
\begin{aligned}
& \mathcal{V}_{\text {quasi-static }}=V+S+\left(\frac{V^{\prime}-S^{\prime}}{r}\right) \boldsymbol{\ell} \cdot\left(\frac{\boldsymbol{\sigma}_{1}}{4 m_{1}^{2}}+\frac{\boldsymbol{\sigma}_{2}}{4 m_{2}^{2}}\right)+\left(\frac{V^{\prime}}{r}\right) \boldsymbol{\ell} \cdot\left(\frac{\sigma_{1}+\boldsymbol{\sigma}_{2}}{2 m_{1} m_{2}}\right) \\
&+\frac{1}{12 m_{1} m_{2}}\left(\frac{V^{\prime}}{r}-V^{\prime \prime}\right) S_{12}+\frac{1}{6 m_{1} m_{2}} \nabla^{2} V \boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2}
\end{aligned}
$$

- If $V$ is Coulombic $-V^{\prime \prime}+V^{\prime} / r=3 V^{\prime} / r$
- To order $1 / m_{2}: \quad M=\lambda \ell \cdot s_{1}+4 \tau \ell \cdot s_{2}+\tau S_{12}$

$$
\lambda=\frac{1}{2 m_{1}^{2}}\left[\frac{V^{\prime}}{r}\left(1+\frac{2 m_{1}}{m_{2}}\right)-\frac{S^{\prime}}{r}\right] \quad \tau=\frac{1}{4 m_{1} m_{2}} \frac{V^{\prime}}{r} .
$$

## Quantum Mechanics in Action

- Can diagonalize commuting variables
- One choice: $J^{2}, j^{2}, J_{z}$
- Another: $J^{2}, j^{\prime 2}, J_{z} \quad j^{\prime}=\ell+s_{2}$
- Yet another: $J^{2}, S^{2}, J_{z} \quad S=s_{1}+s_{2}$
- Must be possible to express one basis in terms of another
- Calculate $\ell \cdot s_{1}$ in first basis
- Calculate $\ell \cdot s_{s}$ in second basis
- Calculate $S_{12}$ in third


## Masses of the Four $\mathbf{P}$ states

$$
\begin{aligned}
& M_{2}=\frac{\lambda}{2}+\frac{5}{8} \tau \\
& M_{0}=-\lambda-8 \tau
\end{aligned}
$$

Masses of the two $J=1$ state from diagonalizing in the $|J j m\rangle$ basis

$$
\left(\begin{array}{cc}
\frac{\lambda}{2}-\frac{8}{3} \tau & -\frac{2 \sqrt{2}}{3} \tau  \tag{1}\\
-\frac{2 \sqrt{2}}{3} \tau & -\lambda+\frac{8}{3} \tau
\end{array}\right)
$$

The two eigenmasses for $J=1$ are then

$$
\begin{aligned}
& M_{1+}=-\frac{\lambda}{4}+\sqrt{\frac{\lambda^{2}}{16}+\frac{1}{2}(\lambda-4 \tau)^{2}} \\
& M_{1-}=-\frac{\lambda}{4}-\sqrt{\frac{\lambda^{2}}{16}+\frac{1}{2}(\lambda-4 \tau)^{2}}
\end{aligned}
$$

## Use Three Measured States to Predict Fourth

$$
\begin{aligned}
D_{2} & =M_{2}-M_{0} \\
D_{1} & =M_{1-}-M_{0}
\end{aligned}
$$

we find

$$
\tau=\frac{10}{87} D_{2}-\frac{2}{29} D_{1} \pm \sqrt{\left(\frac{10}{87} D_{2}-\frac{2}{29} D_{1}\right)^{2}+\frac{5}{232}\left(D_{1}^{2}-D_{1} D_{2}\right)}
$$

and

$$
\lambda=\frac{2}{3} D_{2}-\frac{32}{5} \tau
$$



## Masses in $D$ and $D_{s} \mathbf{P}$ States

|  | Exp. | Theory |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  |  | Sol. A | Sol. B | DiPierro-Eichten |
| $D$ mesons |  |  |  |  |
| $M\left(2^{+}\right)(\mathrm{GeV})$ | 2.459 | $[2.459]$ | $[2.459]$ | 2.460 |
| $M\left(1^{+}\right)(\mathrm{GeV})$ | 2.400 | 2.400 | 2.385 | 2.490 |
| $M\left(1^{+}\right)(\mathrm{GeV})$ | 2.422 | $[2.422]$ | $[2.422]$ | 2.417 |
| $M\left(0^{+}\right)(\mathrm{GeV})$ | 2.290 | $[2.290]$ | $[2.290]$ | 2.377 |
| $\lambda(\mathrm{MeV})$ |  | 39 | 54 | -11 |
| $\tau(\mathrm{MeV})$ |  | 11 | 9 | +11 |
| $D_{s} \mathrm{mesons}$ |  |  |  |  |
| $M\left(2^{+}\right)(\mathrm{GeV})$ | 2.572 | $[2.572]$ | $[2.572]$ | 2.581 |
| $M\left(1^{+}\right)(\mathrm{GeV})$ |  | 2.480 | 2.408 | 2.605 |
| $M\left(1^{+}\right)(\mathrm{GeV})$ | 2.536 | $[2.536]$ | $[2.536]$ | 2.535 |
| $M\left(0^{+}\right)(\mathrm{GeV})$ | 2.317 | $[2.317]$ | $[2.317]$ | 2.487 |
| $\lambda(\mathrm{MeV})$ |  | 43 | 115 | -7 |
| $\tau(\mathrm{MeV})$ |  | 20 | 9 | +11 |

## Widths in $D$ and $D_{s} \mathbf{P}$ States

|  | Exp. | DiPierro-Eichten <br> pure s-wave <br> pure d-wave |  |
| :--- | ---: | ---: | ---: |
| $D$ mesons |  |  |  |
| $D_{2}^{*}(2460) \rightarrow D(1865) \pi$ | $23 \pm 5$ |  | 16 |
| $D_{2}^{*}(2460) \rightarrow D^{*}(2007) \pi$ | $23 \pm 5$ |  | 9 |
| $D_{1}(2422) \rightarrow D^{*}(2007) \pi$ | $18.9_{-3.5}^{+4.6}$ | 94 | 10 |
| $D_{1}(2400) \rightarrow D^{*}(2007) \pi$ | $380 \pm 100 \pm 100$ | 100 |  |
| $D_{0}^{*}(2290)$ | $305 \pm 30 \pm 25$ | 100 |  |
| $D_{s}$ mesons |  |  |  |
| $D_{2}^{*}(2573) \rightarrow D(1865) K$ | $15_{-4}^{+5}$ |  | 8.9 |
| $D_{2}^{*}(2573) \rightarrow D^{*}(2007) K$ | $15_{-4}^{+5}$ |  | 1.4 |
| $D_{1}(2535) \rightarrow D^{*}(2007) K$ | $<2.3$ | 100 | 0.3 |

## Choosing Solutions

- Need nearly pure $j=3 / 2$ to suppress $D_{1}(2422), D_{s}(2535)$ decays
- Forces solutions with large $\lambda / \tau$
- Contrary to "traditional picture."


## Alternative Views: Molecules not Atoms

- Barnes, Close, and Lipkin [hep-ph/0305025]
- Bound $D K$, near threshold
- Might be isovectors, too. Look for $D_{s} \pi^{ \pm}$
- Expect to find regular $c \bar{s}$ states as well


## Alternative Views Chiral Symmetry + Heavy Quark Symmetry

- Bardeen, Eichten, and Hill [hep-ph/030549]
- If chiral symmetry good $\left(0^{-}, 1^{-}\right)$degenerate with $\left(0^{+}, 1^{+}\right)$
- Predicted roughly 340 MeV splitting, new $1^{+}$at 2460 !
- Predict
$-\Gamma\left(\left(D_{s}\left(0^{+}\right) \rightarrow D_{s}^{*}\left(1^{-}\right) \gamma\right)=1.7 \mathrm{keV}\right.$
- $\Gamma\left(\left(D_{s}\left(0^{+}\right) \rightarrow D_{s}^{*}\left(0^{-}\right) \pi^{0}\right)=22 \mathrm{keV}\right.$
- $\Gamma\left(\left(D_{s}\left(1^{+}\right) \rightarrow D_{s}\left(0^{-}\right) \gamma\right)=5 \mathrm{keV}\right.$
- $\Gamma\left(\left(D_{s}\left(1^{+}\right) \rightarrow D_{s}\left(1^{-}\right) \pi^{0}\right)=22 \mathrm{keV}\right.$
- $\Gamma\left(\left(D_{s}\left(1^{+}\right) \rightarrow D_{s}\left(0^{-}\right) \pi \pi\right)=4 \mathrm{keV}\right.$
- $\Gamma\left(\left(D_{s}\left(1^{+}\right) \rightarrow D_{s}\left(0^{+}\right) \gamma\right)=3 \mathrm{keV}\right.$


## Summary

- BaBar results on $D_{s} \pi^{0}$ contradict theoretical predictions
- Belle results on $D$ somewhat contradict theoretical predictions
- Can fit p-wave masses, but with bigger spin-orbit energy than expected
- To suppress decay rate of $1^{+}$states need to take extremely large spin-orbit energy
- BaBar results will profoundly affect heavy-quark light-quark spectroscopy

