Precision Physics with inclusive B decays

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Outline

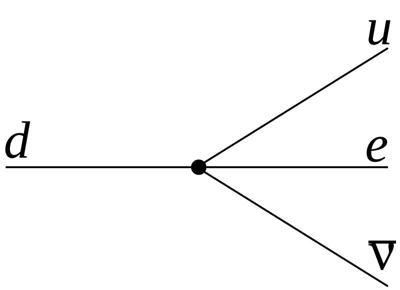
- Introduction
- Fit to inclusive shape variables
  - Discussion of available data
  - Results for m_b and $|V_{cb}|$
  - Discussion of errorrs
- BABAR hadronic moment
  - Overview of measurement
  - Relation between experiment and theory
  - Some discussions
- Conclusions

Introduction

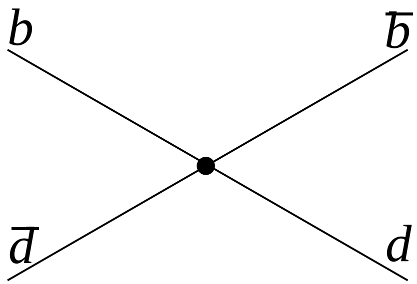
The Flavor Sector of the SM

Flavor changing processes at low energies

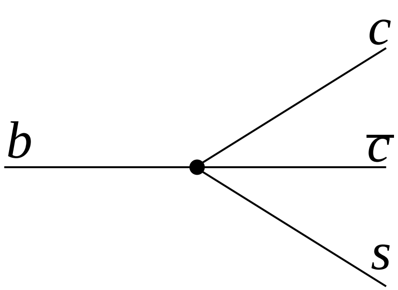
- Interactions mediated by local 4-Fermion interactions



Nuclear Beta decay



$B_d - \bar{B}_d$ *mixing*



$B \to \psi K_s$

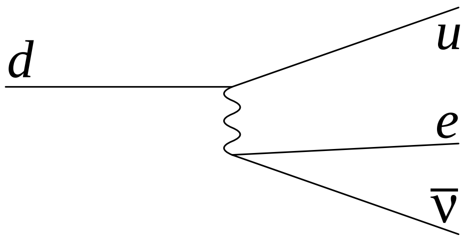
- There are hundreds of different interactions

What do we know about them?

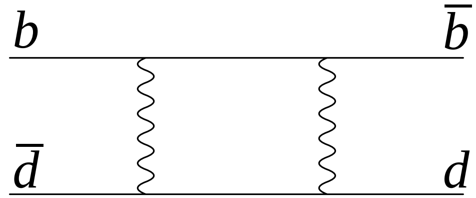
The Flavor Sector of the SM

Flavor changing processes in the Standard model

- Interactions mediated by interactions with W boson



Nuclear Beta decay



$B_d - \bar{B}_d$ *mixing*

- There are 11 parameters:
  - Fermi coupling constant G_F
  - 6 quark masses
  - 4 parameters in the CKM matrix (A, λ, ρ, η)

How to test this prediction?

Information from B decays

Most of B decays can be described by

$$m_t = 175 \pm 5, \quad m_{u,d,s} = 0, \quad |V_{us}| = 0.2169 \pm 0.0026$$

$$G_F = (1.16639 \pm 0.00001) \times 10^{-5} \mathrm{GeV}^{-2}$$

In addition to

- Two quark masses m_c and m_b
- Three additional CKM parameters

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

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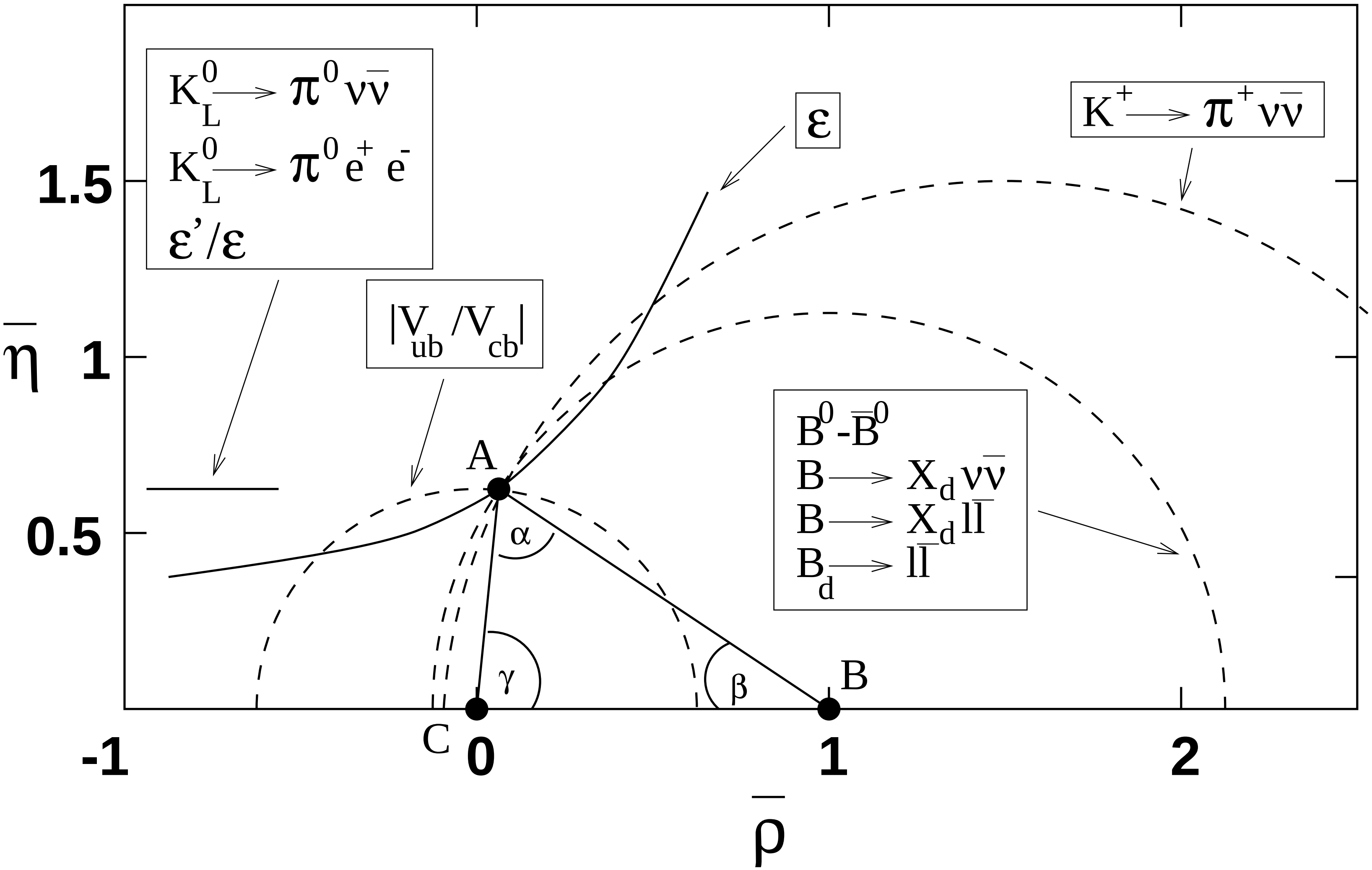
- Precise measurement of $|V_{cb}|$ crucial

The Unitarity Triangle



Checking this picture

Why is it hard to check this picture?

- Relations are between parton model amplitudes
- Measurements are done with hadrons

We have to understand the hadronization effects

Be very careful

(Don't believe everything theorists tell you)

What to believe?

Model dependent Results

- Assume model to calculate strong interaction effect
- How do we know whether to believe the model?
- How to estimate uncertainties of model?

Model independent results

- Strong interaction effect calculable in limit of QCD
- Corrections are parametrically suppressed

$$\langle O \rangle = (\text{calc}) \left[1 + \sum_k (\text{small parameter})^k \right]$$

Model independent tools

Need a parametrically small quantity $\Rightarrow$ separated scales

Model independent tools

Need a parametrically small quantity $\Rightarrow$ separated scales

chiral limit	$m_{u,d} \ll \Lambda_\chi$
heavy quark limit	$m_b \gg \Lambda_{\rm QCD}$

Model independent tools

Need a parametrically small quantity $\Rightarrow$ separated scales

chiral limit $m_{u,d} \ll \Lambda_\chi$

heavy quark limit $m_b \gg \Lambda_{\rm QCD}$

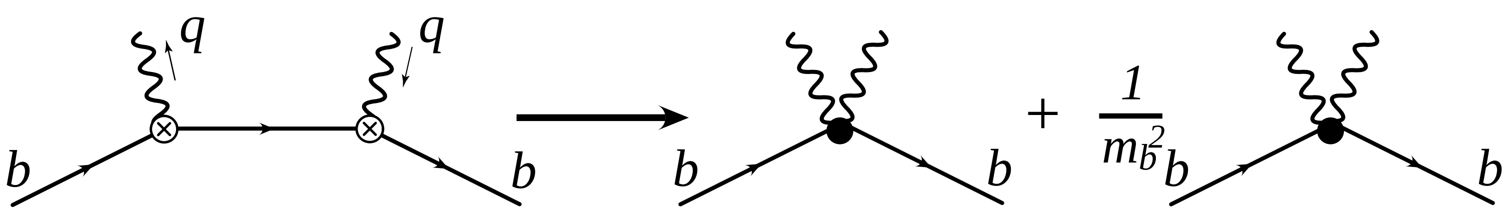
- Spin Flavor symmetry
  $\Rightarrow$ reduction of form factors
- Non-relativistic QCD
  $\Rightarrow$ Quarkonium binding effects
- Large Energy Expansion
  $\Rightarrow$ NP effects in heavy to light decays
- Operator Product Expansion
  $\Rightarrow$ NP effects in inclusive processes

Operator Product Expansion

Describe the decay $B \to X\ell\bar{\nu}$ using optical theorem

$$\Gamma \sim \sum_X |\langle B|J^\mu|X\rangle|^2 \sim \int d^4q\, e^{-iq\cdot x}\mathrm{Im}\langle B|T\{J^{\mu\dagger}(x)J^\nu(0)\}|B\rangle$$

Intermediate state off shell $\to$ propagates short distance



Expand in terms of local operators (OPE)

Similar to Deep Inelastic Scattering

Inclusive B decays

Typical OPE result for differential spectrum looks like

$$\frac{d\Gamma}{dX} = \frac{d\Gamma_{\rm part}}{dX} + \frac{\bar\Lambda}{m_M} f_\Lambda(X) + \frac{\lambda_i}{m_M^2} f_{\lambda_i}(X) + \frac{\rho_i}{m_M^3} f_{\rho_i}(X) + \dots$$

Typical OPE result for moments of spectra looks like

$$\langle X \rangle = \langle X \rangle_{\rm part} + \frac{\bar\Lambda}{m_M} F_\Lambda + \frac{\lambda_i}{m_M^2} F_{\lambda_i} + \frac{\rho_i}{m_M^3} F_{\rho_i} + \dots$$

All inclusive results given in terms of 9 parameters:

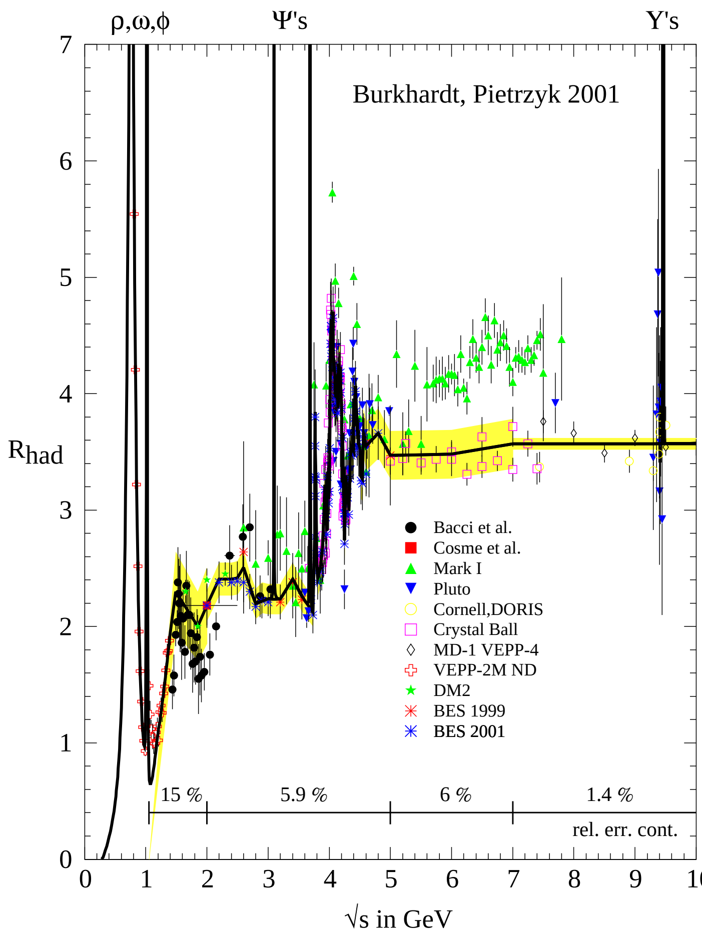
$\{\bar\Lambda, \lambda_1, \lambda_2, \rho_1, \rho_2, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4\}$

Three relations amongst parameters $\Rightarrow$ 6 parameters

Differential Spectra in the OPE

Which quantities can be calculated in OPE?

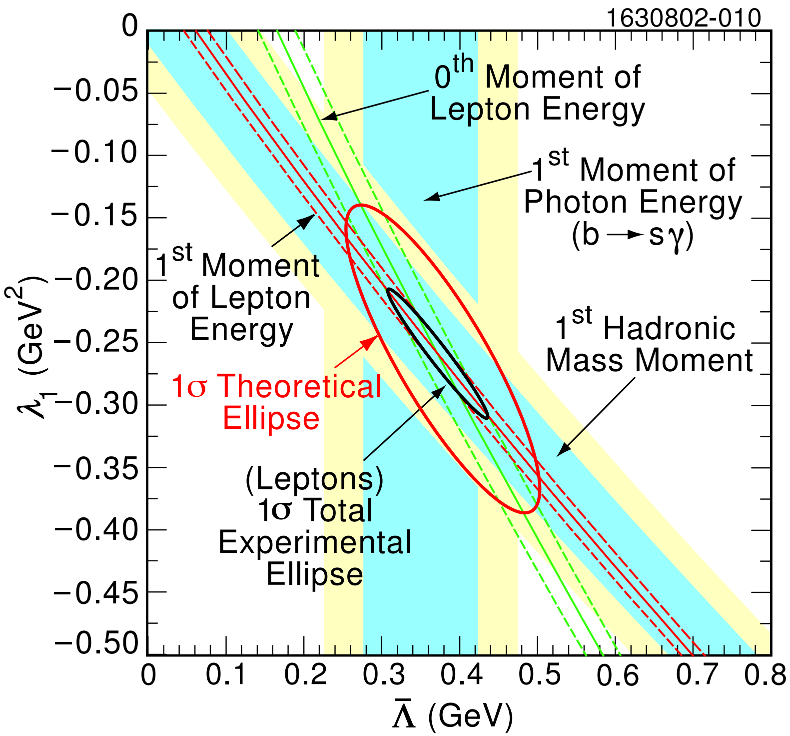
- To compare OPE results with data, have to smear result
- Smearing has to be over "many resonances"



Moments of distributions are calculable

CLEO analysis of V_{cb}

- Measure total inclusive decay rate $\Gamma_{\rm sl}$
- Measure $\bar\Lambda$ and λ_1 from moments of the decay spectra
- Determine $|V_{cb}|$



$$V_{\rm cb}^{\rm CLEO} = [40.4 \pm 0.5_\Gamma \pm 0.4_{m_b,\lambda_1} \pm 0.9_{\rm th}] \times 10^{-3}$$

Issues with the OPE

To what accuracy can we trust the OPE?

- Accuracy depends on size of higher order terms
  - Value of the six parameters ρ_i, $\mathcal{T}_i$ unknown
  - Dimensional analysis: $\rho_i,\ \mathcal{T}_i \sim \Lambda_{\rm QCD}^3$
  - How big is $\Lambda_{\rm QCD}$?
- How much do we trust the OPE itself?
  - Duality violation
  - Separate from previous question?

Can we address this question experimentally?

A Global Fit

Why do a Global Fit?

- Use more data $\Rightarrow$ reduce uncertainties
- See inconsistencies between different measurments
- Investigate the effect of theoretical uncertainties
- Include theoretical correlations between different observables

Available Data

- Lepton energy moments from CLEO

CLEO ('02)

$$R_0^{\mathrm{C}} = 0.6187 \pm 0.0021$$
$$R_1^{\mathrm{C}} = (1.7810 \pm 0.0011)\,\mathrm{GeV}$$
$$R_2^{\mathrm{C}} = (3.1968 \pm 0.0026)\,\mathrm{GeV}^2$$

- Lepton energy moments from DELPHI

DELPHI ('02)

$$R_1^{\mathrm{D}} = (1.383 \pm 0.015)\,\mathrm{GeV}$$
$$R_2^{\mathrm{D}} - (R_1^{\mathrm{D}})^2 = (0.192 \pm 0.009)\,\mathrm{GeV}^2$$

Available Data

- Hadron invariant mass moments from CLEO

CLEO ('01)

$$
\begin{aligned}
S_1(1.5\,\mathrm{GeV}) &= (0.251 \pm 0.066)\,\mathrm{GeV}^2 \\
S_2(1.5\,\mathrm{GeV}) &= (0.576 \pm 0.170)\,\mathrm{GeV}^4
\end{aligned}
$$

- Hadron invariant mass moments from DELPHI

DELPHI ('02)

$$
\begin{aligned}
S_1(0) &= (0.553 \pm 0.088)\,\mathrm{GeV}^2 \\
S_2(0) &= (1.26 \pm 0.23)\,\mathrm{GeV}^4
\end{aligned}
$$

Available Data

- Hadron invariant mass moments from BABAR

BABAR ('02)

$$S_1(1.5\,\mathrm{GeV}) = (0.354 \pm 0.080)\,\mathrm{GeV}^2$$
$$S_1(0.9\,\mathrm{GeV}) = (0.694 \pm 0.114)\,\mathrm{GeV}^2$$

- Photon energy moments from CLEO

CLEO ('01)

$$T_1(2\,\mathrm{GeV}) = (2.346 \pm 0.034)\,\mathrm{GeV}$$
$$T_2(2\,\mathrm{GeV}) = (0.0226 \pm 0.0069)\,\mathrm{GeV}^2$$

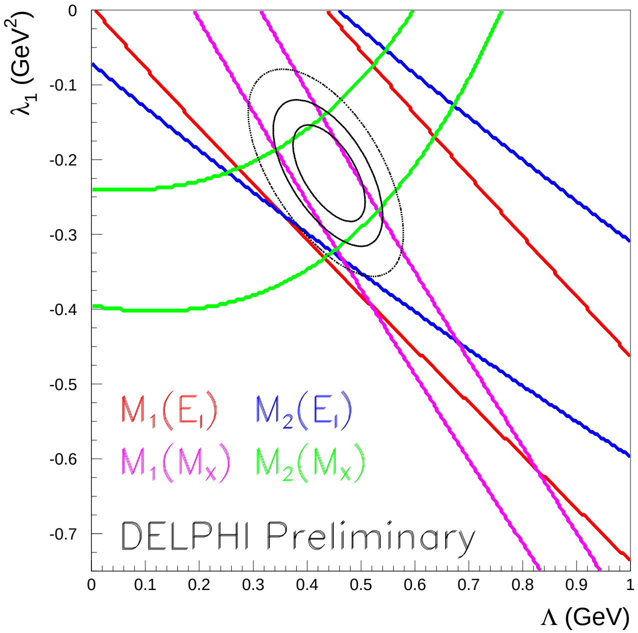
- Avarage of semileptonic decay width

PDG ('02)

$$\Gamma(B \to X\ell\bar{\nu}) = (42.7 \pm 1.4) \times 10^{-12}\,\mathrm{MeV}$$

Higher Hadron Moments

- Second hadron moment seems to give orthogonal information to most other moments



- Convergence of this moment questioned in literature

Falk, Luke ('97)

$$\left\langle s^2 - \langle s \rangle^2 \right\rangle = 0.73 \frac{\bar{\Lambda}^2}{\Lambda_{\rm QCD}^2} - 0.96 \frac{\lambda_1}{\Lambda_{\rm QCD}^2} - 0.56 \frac{\rho_1}{\Lambda_{\rm QCD}^3} + \ldots$$

Higher Hadron Moments

- From dimensional analysis

$$\frac{\langle s^2 - \langle s \rangle^2 \rangle}{m_B^4} = \mathcal{O}(1)\frac{\bar{\Lambda}^2}{\bar{m}_B^2} + \mathcal{O}(1)\frac{\lambda_1}{\bar{m}_B^2} + \mathcal{O}(1)\frac{\rho_1}{\bar{m}_B^3} + \ldots$$

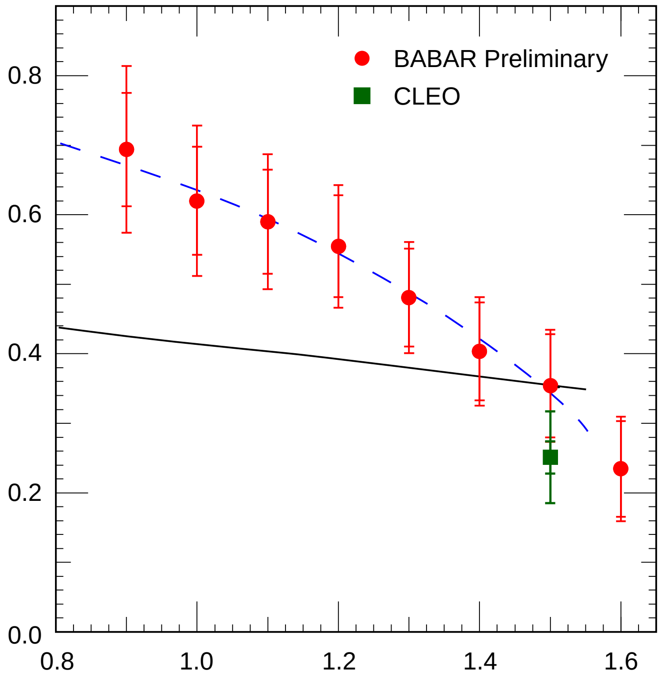
- Breakdown of OPE: some coeffs $\gg \mathcal{O}(1)$, growing
- The previous expression is

$$\frac{\langle s^2 - \langle s \rangle^2 \rangle}{m_B^4} = 0.01\frac{\bar{\Lambda}^2}{\bar{m}_B^2} - 0.14\frac{\lambda_1}{\bar{m}_B^2} - 0.86\frac{\rho_1}{\bar{m}_B^3} + \ldots$$

- Large cancellation from $\bar{\Lambda}$ and λ_1 term

Moment is well behaved, but sensitive to ρ_1

Comment on the BABAR measurement



- First measurement of a moment as a function of the cut
- Many more data points than used in this analysis
- Data points highly correlated, therefore only used two in fits

Much more in second half of talk

Correlations in the Data

- Obviously, different moments of the same spectrum are correlated
- Since most measurements have some assumptions $\Rightarrow$ correlation between diefferent experiments
- Only used publically available data
- Worthwile do redo the fits with all correlations included
- Central value and error in $|V_{cb}|$ stable

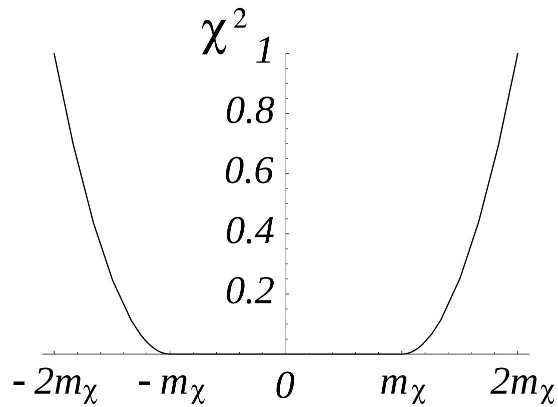
Theoretical Uncertainties

Originate from unknown higher order terms in expansion

Two different kind of terms

- Unknown $1/m_b^3$ matrix elements
  - generic size $\Lambda_{\rm QCD}^3$
  - There is no favorite value
  - Don't use Gaussian with width $(0.5\ \mathrm{GeV})^3$
  - In our fits we add

$$\Delta\chi^2(m_\chi, M_\chi) = \begin{cases} 0\,, & |\langle\mathcal{O}\rangle| \le m_\chi^3 \\ \frac{\left[|\langle\mathcal{O}\rangle| - m_\chi^3\right]^2}{(0.5\,\mathrm{GeV})^6} & |\langle\mathcal{O}\rangle| > m_\chi^3 \end{cases}$$



  - We vary $0.5\,\mathrm{GeV} < m_\chi < 1\,\mathrm{GeV}$

Theoretical Uncertainties

Originate from unknown higher order terms in expansion

Two different kind of terms

- Uncomputed higher order terms
  - For quantity of mass dimension m_B^n
    - $(\alpha_s/4\pi)^2\, m_B^n \sim 0.0003\, m_B^n$
    - $(\alpha_s/4\pi)\Lambda_{\rm QCD}^2/m_b^2\, m_B^n \sim 0.0002\, m_B^n$
    - $\Lambda_{\rm QCD}^4/(m_b^2 m_c^2)\, m_B^n \sim 0.001\, m_B^n$
  - Can underestimate perturbative uncertainties
  - Better estimate might be to relate to last term computed
  - We add $\sqrt{(0.001 m_B^n)^2 + (\text{last computed})^2/4}$

The Result

- One fit including and one fit excluding BABAR data
- This allows to investigate effect of BABAR data

m_χ [GeV]	χ^2	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]
0.5	5.0	40.8 ± 0.9	4.74 ± 0.10
1.0	3.5	41.1 ± 0.9	4.74 ± 0.11
0.5	12.9	40.8 ± 0.7	4.74 ± 0.10
1.0	8.5	40.9 ± 0.8	4.76 ± 0.11

- BABAR data makes fit considerably worse
- More on this later

Error analysis

What is included in error?

- Best estimate of perturbative uncertainties
- Best estimate of uncomputed $1/m^4$ and α_s^2/m^2 terms
- Very conservative estimate of $1/m^3$ uncertainties
- All publically available experimental uncertainties

What is not included in error?

- Unknown experimental correlations
- Uncertainties from "Duality violations"

More on Theoretical Error

- $1/m_b^3$ uncertainty

m_χ [GeV]	$\lvert V_{cb}\rvert \times 10^3$	m_b^{1S} [GeV]
0.5	40.8 ± 0.9	4.74 ± 0.10
1.0	41.1 ± 0.9	4.74 ± 0.11

- Theoretical correlations

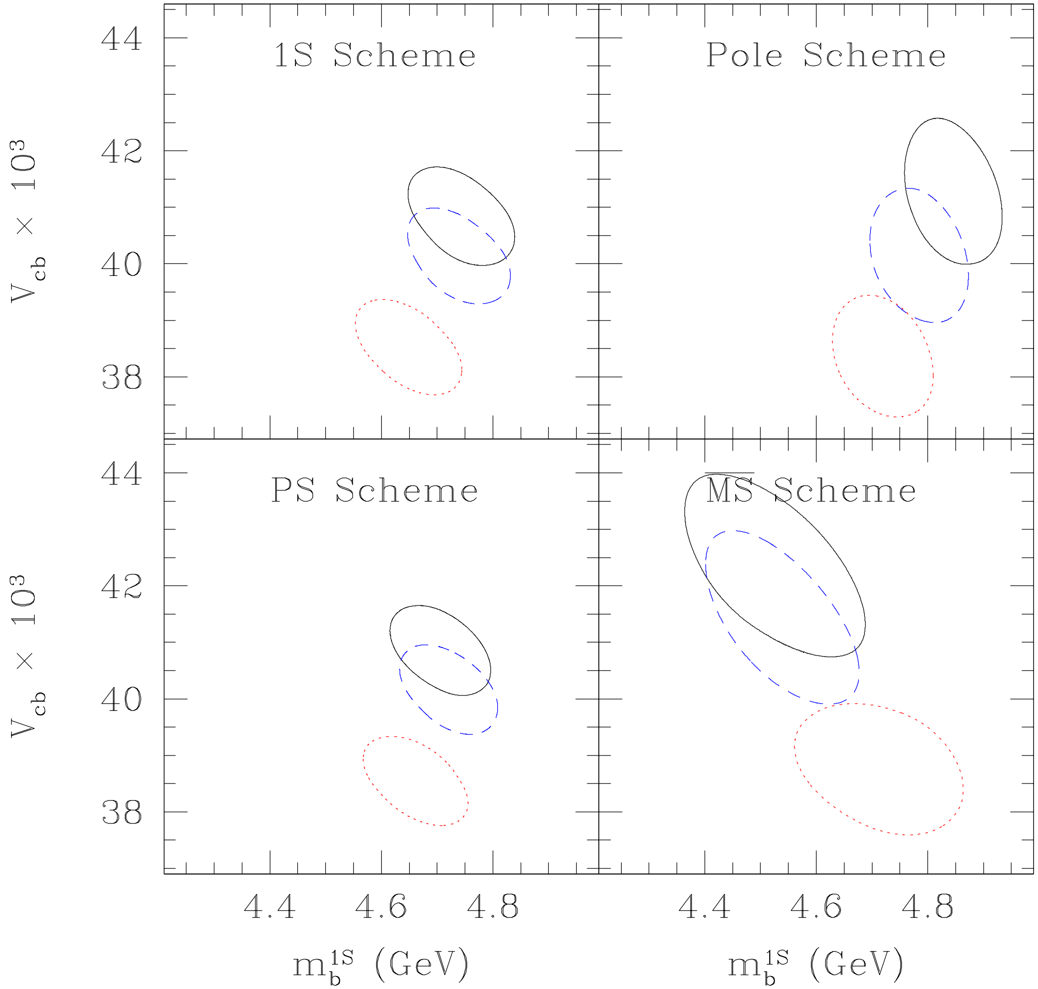
$\delta(\lambda_1)$	$\delta\left(\lambda_1 + \frac{\mathcal{T}_1+3\mathcal{T}_2}{m_b}\right)$
± 0.38	± 0.22

- Theoretical limitations

$\delta(\lvert V_{cb}\rvert) \times 10^3$	$\delta(m_b^{1S})$ [MeV]
± 0.35	± 35

Different mass schemes

tree level, order α_s, order $\alpha_s^2\beta_0$



Better convergence for 1S and PS scheme

Experimental correlations

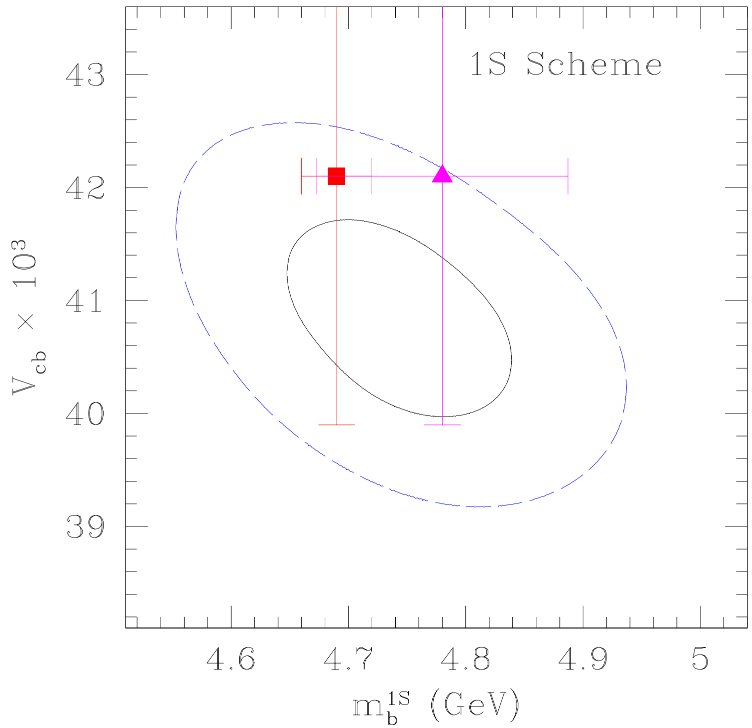
How important are experimental correlations?

- Remove DELPHI measurements from fit
- Increase all errors (except Γ_{sl}) by 2

	$\vert V_{cb}\vert \times 10^3$	m_b^{1S} [GeV]
Original Fit	40.8 ± 0.9	4.74 ± 0.10
Excluding DELPHI	40.6 ± 0.9	4.79 ± 0.09
2 $\times$ errors	40.8 ± 1.2	4.74 ± 0.24

Fit should be good for $|V_{cb}|$, but for confidence in $\delta(m_b)$ one should include all correlations

Result once again

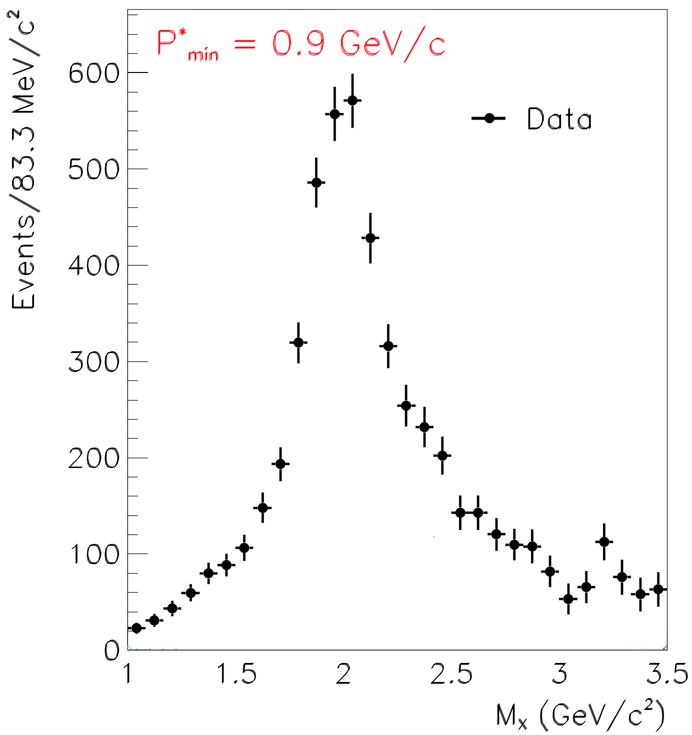


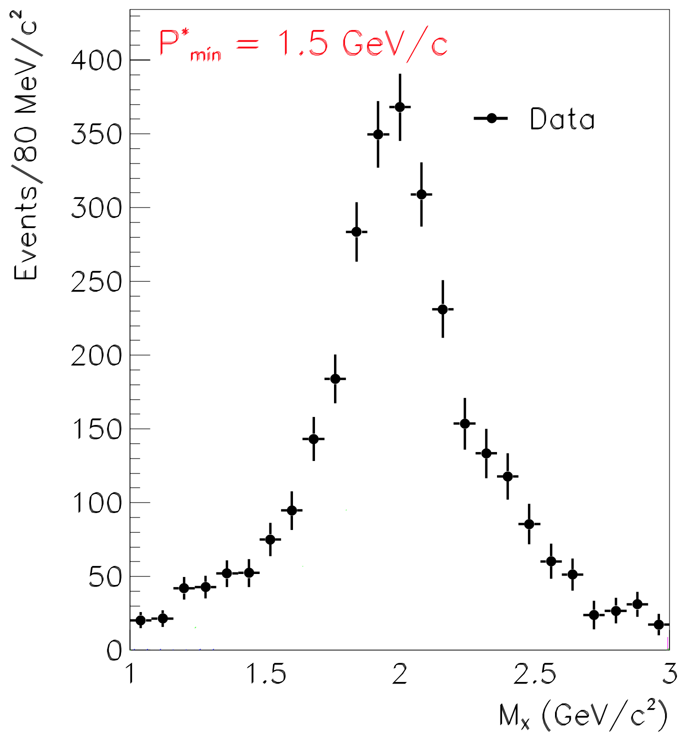
$$
\begin{aligned}
|V_{cb}| &= (40.8 \pm 0.9) \times 10^{-3} \\
m_b^{1S} &= (4.74 \pm 0.10)\,\mathrm{GeV} \\
\overline{m}_b(\overline{m}_b) &= 4.22 \pm 0.09\,\mathrm{GeV}
\end{aligned}
$$

The BABAR hadronic moments

Review of the measurement

The measured distibution

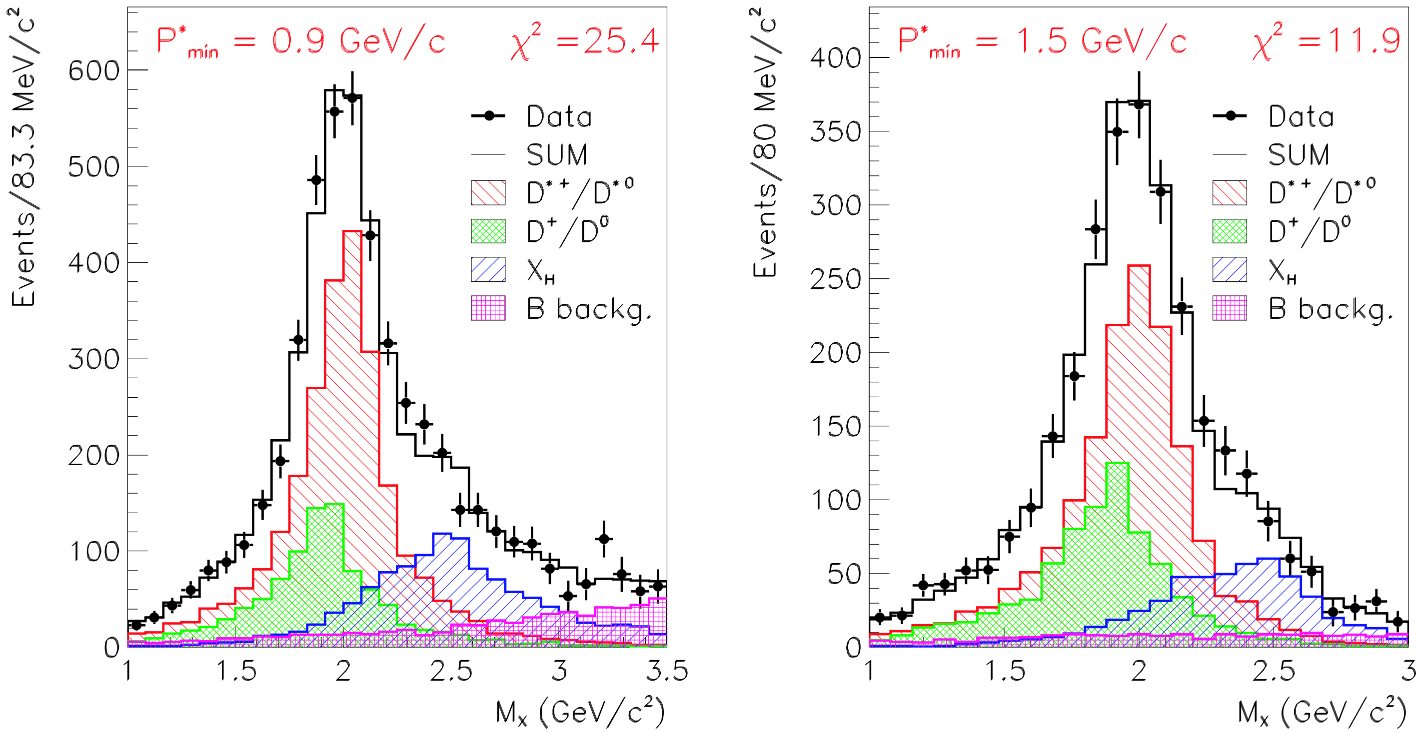




- Uses one fully reconstructed B decay
- Done as a function of the lepton energy cut

Review of the measurement

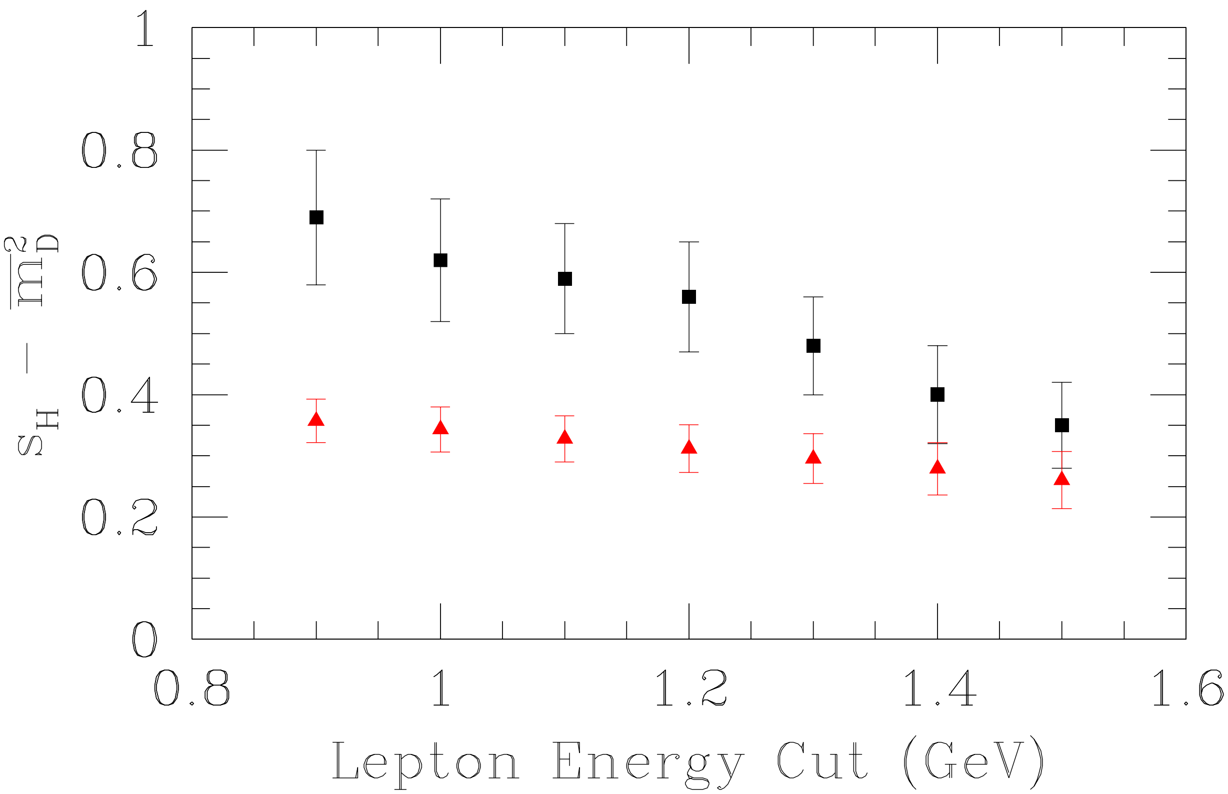
Obtaining the hadronic moments



- Fit to four distributions: D, D^*, X_H, Background
- Fit determines the fraction of D, D^*, X_H distros

Review of the measurement

Results



Significant disagreement with our fit results

Why fit to the 3 distros?

The measured differential spectrum is

$$\frac{d\Gamma}{ds_L} = \int ds_L \left\{ P_D(s, s_L)\frac{d\Gamma_D}{ds} + P_{D^*}(s, s_L)\frac{d\Gamma_{D^*}}{ds} + P_X(s, s_L)\frac{d\Gamma_X}{ds} \right\}$$

What is calculated is

$$\langle s - \bar{m}_D^2 \rangle = \int ds \, (s - \bar{m}_D^2) \left\{ \frac{d\Gamma_D}{ds} + \frac{d\Gamma_{D^*}}{ds} + \frac{d\Gamma_X}{ds} \right\}$$

Can we take the detector resolution into account theoretically?

Calculate the differential spectrum

Instead of just calculating moments, we can also calculate the differential hadronic invariant mass spectrum

$$\frac{d\Gamma_D}{ds} = \frac{\mathrm{Br(D)}}{\tau_B}\delta(s - m_D^2)\,, \qquad \frac{d\Gamma_D}{ds} = \frac{\mathrm{Br(D^*)}}{\tau_B}\delta(s - m_{D^*}^2)$$

$$\frac{d\Gamma}{ds} = \Gamma(E_{\rm cut})\left[\delta(s-\bar{m}_D^2) + A(E_{\rm cut})\delta'(s-\bar{m}_D^2) + B(E_{\rm cut})\delta''(s-\bar{m}_D^2) + \ldots\right] + \frac{\alpha_s}{\pi}P(s, E_{\rm cut})$$

$$A(E_{\rm cut}) \sim \Lambda_{\rm QCD}/m_b\,, \quad B(E_{\rm cut}) \sim \Lambda_{\rm QCD}^2/m_b^2\,, \quad \ldots$$

Convolute with detector resolution

Convolution formula was

$$\frac{d\Gamma}{ds_L} = \int ds_L \left\{ P_D(s, s_L)\frac{d\Gamma_D}{ds} + P_{D^*}(s, s_L)\frac{d\Gamma_{D^*}}{ds} + P_X(s, s_L)\frac{d\Gamma_X}{ds} \right\}$$

Can now be calculated using

$$\frac{d\Gamma_X}{ds} = \frac{d\Gamma}{ds} - \frac{d\Gamma_D}{ds} - \frac{d\Gamma_{D^*}}{ds}$$

Careful

- Theoretical distribution is singular
- Smearing functions has to have width $\sim \sqrt{\Lambda_{\rm QCD} m_b}$

Need further smearing $\Rightarrow$ Moments

Facts about convolutions

Consider the simple convolution $G(x) = \int dy\, c(x-y) g(y)$

$$
\begin{aligned}
G_N &= \int dx\, x^N G(x) = \int dy\, g(y) \int dx\, x^N c(x-y) \\
&= \int dy\, g(y) \int dz\, (z+y)^N c(z) \\
&= \int dy\, g(y) \int dz \sum_{n=0}^{N} \binom{N}{n} y^n z^{N-n} c(z) \\
&= \sum_{n=0}^{N} \binom{N}{n} c_{(N-n)} g_n
\end{aligned}
$$

Moment of convolution is product of moments

Implications for BABAR measurement

- Measured spectrum is convolution of true spectrum and detector resolution $P(s_L - s)$
- Moments of measured spectrum given in terms of true moments
- Take into account different resolution functions for D, D^* and X

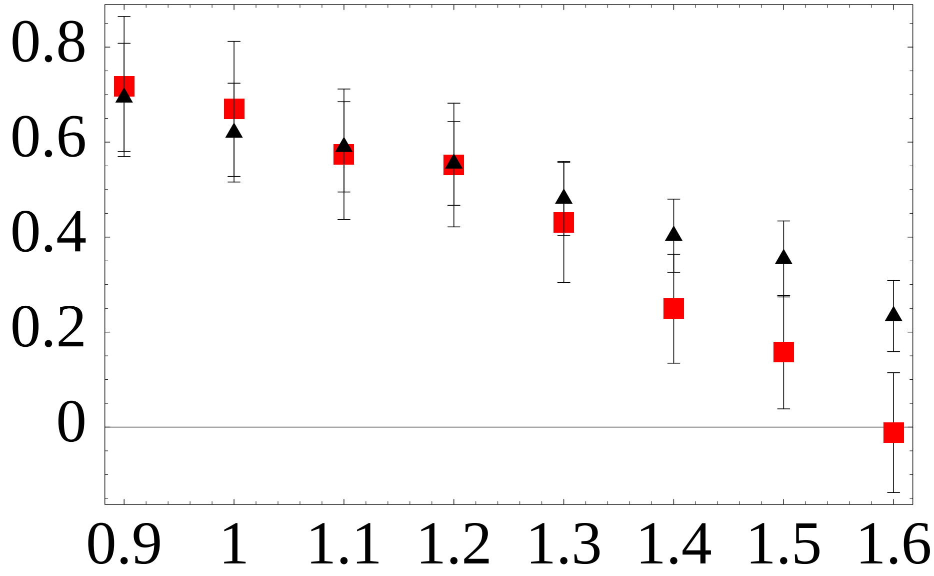
$$\langle s - \bar{m}_D^2 \rangle_{\text{meas}} - \langle s - \bar{m}_D^2 \rangle_{\text{theo}}$$
$$= P_1^X + (P_1^D - P_1^X)\mathrm{Br}(D) + (P_1^{D^*} - P_1^X)\mathrm{Br}(D^*)$$

Difference between calculated and measured moments is determined by mean of resolution functions

Some numbers

Thanks to Oliver Buchmüller and Henning Flächer

A plot of $\langle s - \bar{m}_D^2 \rangle$ ("measured" moments, orig. data)



- Corrections $P_1^{D,D^*,X}$ not yet included
- Should be positive

Eliminating Goity-Roberts does not eliminate discrepancy

Conclusions

- OPE predicts all inclusive B meson shape variables in terms of 6 parameters
- Precise knowledge of these parameters required for inclusive determination of $|V_{cb}|$
- Fit to all available data is best way to extract $V_{cb}|$
- Fit should be done in well behaved mass scheme
- Find $|V_{cb}| = (40.8 \pm 0.9) \times 10^{-3}$
- Recent measurements of BABAR show some disagreement with fit predictions
- Eliminating most of the model dependence from measurement does not solve the problem
- Should be interesting times ahead of us

Higher Moments?

- The same trick works for higher moments.
- Assume universal distribution function

$$\left\langle (s - \bar{m}_D^2)^N \right\rangle_{\rm meas} = \sum_{n=0}^{N} \binom{N}{n} P_{(N-n)} \left\langle (s - \bar{m}_D^2)^n \right\rangle_{\rm theo}$$

- Can easily take into account different resolution functions
- All measured higher moments can be related to calculated moments and moments of resolution function